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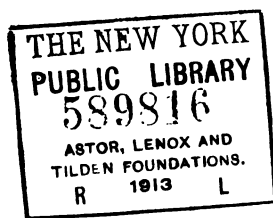
BY CHAS. H. HASWELL,

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An examination of facts is the foundation of science.

**NEW YORK:
HARPER & BROTHERS, PUBLISHERS,
FRANKLIN SQUARE.**

1893.



BY CHAS. H. HASWELL,

*Civil, Marine, and Mechanical Engineer, M. Am. Soc. C. E.
and Ins'n of C. E.*

MENSURATION.

For Tuition and Reference, containing Tables of Weights and Measures; Mensuration of Surfaces, Lines, and Solids, and Conic Sections, Centres of Gravity, &c. To which is added, Tables of the Areas of Circular Segments, Sines of a Circle, Circular and Semi-elliptical Arcs, &c. By CHAS. H. HASWELL, Civil and Marine Engineer, &c. Sixth Edition. 12mo, Sheep, 90 cents.

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TO

CAPTAIN JOHN ERICSSON, I.L.D.,

**AS A SLIGHT TRIBUTE TO HIS GENIUS AND ATTAINMENTS,
AND IN TESTIMONY OF THE SINCERE REGARD
AND ESTEEM OF HIS FRIEND.**

THE AUTHOR.

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passed



P R E F A C E

To the Forty-fifth Edition.

THE First Edition of this work, consisting of 284 pages, was submitted to the Mechanics and Engineers of the United States by one of their number in 1843, who designed it for a convenient reference to Rules, Results, and Tables connected with the discharge of their various duties.

The Twenty-first Edition was published in 1867, consisted of 664 pages, and, in addition to the original design of the work, it was essayed to embrace some general information upon Mechanical and Physical subjects.

The Tables of Areas and Circumferences of Circles have been extended, and together with those of Weights of Metals, Balls, Tubes, Pipes, etc., of this and some preceding editions were computed and verified by the author.

This edition is a revision and an entire reconstruction of all preceding, embracing amended and much new matter, as Masonry, Strength of Girders, Floor Beams, Logarithms, etc., etc.

To the young Mechanic and Engineer it is recommended to cultivate a knowledge of Physical Laws and to note results of observations and of practice, without which eminence in his profession can never be attained; and if this work shall assist him in the attainment of these objects, one great purpose of the author will be well accomplished.

NOTE 1.—*Mechanical and Physical subjects, commencing at p. 427 and ending at p. 870, are given in alphabetical order.*

2.—*Tons are given and computed at 2240 lbs.*

3.—*Degrees of temperature are given by the Scale of Fahrenheit.*

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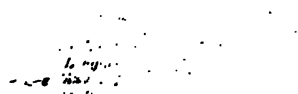
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EXPLANATIONS OF CHARACTERS AND SYMBOLS

Used in Formulas, Computations, etc., etc.

$=$ *Equal to*, signifies equality; as 12 inches $=$ 1 foot, or $8 \times 8 = 16 \times 4$.

$+$ *Plus*, or *More*, signifies addition; as $4 + 6 + 5 = 15$.

$-$ *Minus*, or *Less*, signifies subtraction; as $15 - 5 = 10$.

\times *Multiplied by*, or *Into*, signifies multiplication; as $8 \times 9 = 72$. $a \times c$ or $a \cdot c$, or ad , also signify that a is to be multiplied by d .

\div *Divided by*, signifies division; as $72 \div 9 = 8$.

$:$ *Is to*, $::$ *So is*, $:$ *To*, signifies *Proportion*, as $2 : 4 :: 8 : 16$; that is, as 2 is to 4, so is 8 to 16.

\therefore signifies *Therefore* or *Hence*, and \because *Because*.

$\overline{}$ *Vinculum*, or *Bar*, signifies that numbers, etc., over which it is placed, are to be taken together; as $8 - 2 + 6 = 12$, or $3 \times 5 + 3 = 24$.

\cdot *Decimal point*, signifies, when prefixed to a number, that that number has some power of 10 for its denominator; as $.1$ is $\frac{1}{10}$, $.15$ is $\frac{15}{100}$, etc.

∞ *Difference*, signifies, when placed between two quantities, that their difference is to be taken, it being unknown which is greater.

$\sqrt{}$ *Radical sign*, which, prefixed to any number or symbol, signifies that square root or that number, etc., is required; as $\sqrt{9}$, or $\sqrt{a+b}$. The degree of the root is indicated by number placed over the sign, which is termed *index of the root or radical*; as $\sqrt[3]{}$, $\sqrt[4]{}$, etc.

$>$, $<$, \lceil signify *Inequality*, or *greater*, or *less than*, and are put between two quantities; as $a \rceil b$ reads a greater than b , and $a \lfloor b$ reads a less than b .

$()$ $[\]$ *Parentheses* and *Brackets* signify that all figures, etc., within them are to be operated upon as if they were only one; thus, $(3 + 2) \times 5 = 25$ $[8 - 2] \times 5 = 30$.

\pm \mp signify that the formula is to be adapted to two distinct cases, a $c \mp v = a$, either diminished or increased by v . Here there are expressed two values: first, the difference between c and v ; second, the sum of c and v .

In this and like expressions, the upper symbol takes preference of the lower.

p or π is used to express ratio of circumference of a circle to its diameter $= 3.1416$; $\frac{1}{4}p = .7854$, and $\frac{1}{6}p = .5236$.

$^{\circ}$ $'$ $''$ signify *Degrees*, *Minutes*, *Seconds*, and *Thirds*.

$''$ set *superior* to a figure or figures, signify, in denoting dimensions, *Feet* and *Inches*.

a' a'' a''' signify *a prime*, *a second*, *a third*, etc.

$_1$, $_2$, added to or set *inferior* to a symbol, reads *sub 1* or *sub 2*, and is used to designate corresponding values of the same element, as h , h_1 , h_2 , etc.

2 , 3 , 4 , added or set *superior* to a number or symbol, signify that that number, etc., is to be *squared*, *cubed*, etc.; thus, 4^2 means that 4 is to be multiplied by 4; 4^3 , that it is to be *cubed*, as $4^3 = 4 \times 4 \times 4 = 64$. The power or number of times a number is to be multiplied by itself, is shown by number added, as 2 , 3 , 4 , 5 , etc.

NATIONS OF CHARACTERS AND SYMBOLS

Used in Formulas, Computations, etc., etc.

signifies equality; as 12 inches = 1 foot, or $8 \times 8 = 16 \times 4$.

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Less, signifies subtraction; as $15 - 5 = 10$.

by, or Into, signifies multiplication; as $8 \times 9 = 72$. $a \times d$ signify that a is to be multiplied by d .

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that it is to be cubed, as $4^3 = 4 \times 4 \times 4 =$

as a number is to be multiplied by itself

2, 3, 4, 5, etc.

ILLUSTRATION.—When a number, hyp. log. of which = a given figure or number, is required.

Multiply figure or number (hyp. log.) by .434 294 (modulus of com. log.) = com. log. of figure.

Thus, Required the number, hyp. log. of which = .02. $.02 \times .434\ 294 = .00\ 868\ 588$, com. log., and 1.0202 = number.

Log. $100^{.059} = .059 \times \log. \text{ of } 100 = .059 \times 2 = .118$; the number corresponding to log. .118, is 1.3122; hence, $100^{.059} = 1.3122$. That is, if 100 is raised to 59th power, and the 1000th root is extracted, the result will be 1.3122.

Differential and Integral Calculus.—In Equation, $u = 3x^2 - 2x$, u is termed a function of x . If it is desired to indicate the fact that u thus depends for its value upon value of x , without expressing exact value of u in terms of x , following notation is used:

$$u = f(x), \quad u = F(x), \quad \text{or } u = \phi(x).$$

Each of these notations is read, u is a function of x . If in such function of x value of x is assumed to commence with 0 and to increase uniformly, the notation indicating rate of increase is dx , and is read "the differential of x ."

Differentiation. d is its symbol, and it is the process of ascertaining the ratio existing between the rate of increase or decrease of a function of a variable and the rate of increase or decrease of the variable itself. If $y = 3x^2$, y or its equal $3x^2$ is the function of x , and x is the independent variable, while the exponent of the variable or the primitive exponent is 2.

By the operation of Calculus, such expressions are differentiated by diminishing the exponent of the variable by unity, multiplying by the primitive exponent, and attaching the dx .

Hence, $dy = 2 \times 3x dx = 6x dx$. This indicates the relation between the differential of y , the function of x , and the differential of x itself.

Assume that x increasing at rate of 3 per second becomes 4; that is, $x = 4$, and $dx = 3$; hence $dy = 6 \times 4 \times 3 = 72$. That is, if x is increasing at rate of 3 per second, at the time that $x = 4$, the function itself is increasing at rate of 72 per second.

To differentiate an expression of two or more terms, it is necessary to differentiate them separately and connect the results with the signs with which the terms are connected.

Thus, differentiating $u = 3x^2 - 2x$, we have $du = d(3x^2 - 2x) = 6x dx - 2 dx = (6x - 2) dx$.

Assuming $x = 4$ and $dx = 3$, we have $du = (6 \times 4 - 2) \times 3 = 66$. This indicates that when $x = 4$, and is increasing at rate of 3 per second, the function u , or $3x^2 - 2x$, is at same instant increasing at rate of 66 per second.

Integration. Its symbol \int was originally letter S, initial of *sum*, the symbol of an operation the reverse of differentiation; and when the operation of integration is to be performed twice, thrice, or more times, it is written \iint , \iiint , etc.

By the operation of Calculus, expressions are integrated by increasing the exponent of the variable by unity, dividing by the new exponent, and detaching the dx .

Hence, integrating the differential $6x dx$, we have $\int 6x dx = 3x^2$. This is the function, the differential of which is $6x dx$.

To integrate an expression of two or more terms, it is necessary to integrate the terms separately and connect the results with the signs with which the terms are connected.

Thus, $\int (6x - 2) dx$, we have $\int (6x - 2) dx = \int (6x dx - 2 dx)$ It is the function the differential of which is $(6x - 2) dx$.

the exponent 0, as x^0 or 3^0 , is equal to unity.

The operation of summation may also be illustrated in use of the symbol \int . Assuming $x=4$, the former of the preceding results becomes $\int 6x dx = 3x^2 = 48$, the latter $\int (6x-2) dx = 3x^2 - 2x = 40$.

Here x is assumed to commence at 0 and to continue to increase by infinitely small increments of dx until it becomes 4. The summation is the addition of all these values of x from 0 to 4.

Arithmetically.—The first formula may be written

$6(x' + x'' + \text{etc.}) dx$. If then x is to advance from 0 to 4 by increments of 1, we have $6(0 + 1 + 2 + 3 + 4) \times 1 = 60$, which exceeds 48. If dx is assumed to be .5, the result is 54. The correct result is obtained only when dx is taken infinitely small. By Arithmetic this is approximated, but it is reached by the operations of Calculus alone.

The second formula may be written

$(6[x' + x'' + x''' + \text{etc.}] - 2[x^0 + x^{0'} + x^{0''} \text{etc.}]) dx$. Assuming $x=4$, and $dx=1$, we have $(6[1 + 2 + 3 + 4] - 2[1 + 1 + 1 + 1]) \times 1 = 52$, which exceeds 40. If $dx=.25$, the result would be 43, and if .125 it would be 41.5, ever approaching but never reaching 40, so long as a finite value is assigned to dx .

Δ , *Delta*, when put before a quantity, signifies an absolute and finite increment of that quantity, and not simply the rate of increase.

Σ , *Sigma*, signifies the summation of finite differences or quantities. Thus, $\Sigma y^2 \Delta x = (y^2 + y'^2 + y''^2 + \text{etc.}) \Delta x$. Assume $y'=6$, $y''=8$, $y'''=4$, and Δx the common increment of $x=5$, then $\Sigma y^2 \Delta x = (36 + 64 + 16) \times 5 = 360$.

NOTATION.

| | | |
|-----------|-----------------|------------------------------|
| 1 = I. | 20 = XX. | 1 000 = M, or CIO. |
| 2 = II. | 30 = XXX. | 2 000 = MM. |
| 3 = III. | 40 = XL. | 5 000 = V, or IOO. |
| 4 = IV. | 50 = L. | 6 000 = VI. |
| 5 = V. | 60 = LX. | 10 000 = X, or CCIOO. |
| 6 = VI. | 70 = LXX. | 50 000 = L, or IOOO. |
| 7 = VII. | 80 = LXXX. | 60 000 = LX. |
| 8 = VIII. | 90 = XC. | 100 000 = C, or CCCIOOO. |
| 9 = IX. | 100 = C. | 1 000 000 = M, or CCCCIOOOO. |
| 10 = X. | 500 = D, or IO. | 2 000 000 = MM. |

As often as a character is repeated, so many times is its value repeated, as CC = 200.

A less character before a greater diminishes its value, as IV = V - I.

A less character after a greater increases its value, as XI = X + I.

For every O annexed to IO the sum as 500 is increased 10 times.

If C is placed on left side of I as many times as O is on the right, the number is doubled.

A bar, thus —, over any number, increases it 1000 times.

Illustration 1.—1880, MDCCCLXXX. 18 560, XVIIIIDLX.

2. — IO = 500. CIO = $500 \times 2 = 1000$. IOO = $500 \times 10 = 5000$. CCIOO = $5000 \times 2 = 10000$. IOOO = $500 \times 10 \times 10 = 50000$. CCCIOOO = $50000 \times 2 = 100000$.

LOGICAL ERAS.—MEASURES AND WEIGHTS.

CHRONOLOGICAL ERAS AND CYCLES FOR 1892.

—692, or the 117th year of the Independence of the United States of America, corresponds to

- the year 7400—13 of the Byzantine Era;
- “ 6606 of the Julian Period;
- “ 5652—53 of the Jewish Era;
- “ 2668 of the Olympiads, or the last year of the 667th Olympiad, commencing in July (1892), the era of the Olympiads being placed at 775.5 years before Christ, or near the beginning of July of the 3938th year of the Julian Period;
- “ 2645 since the foundation of Rome, according to Varro;
- “ 2204 of the Grecian Era, or the Era of the Seleucidæ;
- “ 1608 of the Era of Diocletian.

The year 1310 of the Mohammedan Era, or the Era of the Hegira, begins on the 26th of July, 1892.

The first day of January of the year 1892 is the 2,412,101st day since the commencement of the Julian Period.

| | | | |
|------------------------|------|-----------------------------------|----|
| Dominical Letters..... | C, B | Lunar Cycle or Golden Number..... | 12 |
| Epact..... | 1 | Solar Cycle..... | 25 |

Roman Indiction was a period of 15 years, in use by the Romans. The precise time of its adoption is not known beyond the fact that the year 313 A.D. was a first year of a Cycle of Indiction.

Julian Period is a cycle of 7980 years, product of the Lunar and Solar Cycles and the Indiction ($19 \times 28 \times 15$), and it commences at 4714 years B.C.

$6513 + (\text{given year} - 1800) = \text{year of Julian Period, extending to } 3267.$

NOTE.—If year of Julian Period is divided by 19, 28, 15, or 32, the remainders will respectively give the *Lunar* and *Solar Cycles*, the *Indiction*, and the *Year of the Dionysian*.

MEASURES OF LENGTH.

Standard of measure is a brass scale 82 inches in length, and the yard is measured between the 27th and 63d inches of it, which, at temperature of 62° , is standard yard.

Lineal.

| | | | | | | |
|------------|--------------|---------|--------|--------|-------|-------|
| 12 inches | = 1 foot. | Inches. | Feet. | Yards. | Rods. | Furl. |
| 3 feet | = 1 yard. | 36 | = 3. | | | |
| 5.5 yards | = 1 rod. | 108 | = 16.5 | = 5.5. | | |
| 40 rods | = 1 furlong. | 720 | = 660 | = 220 | = 40. | |
| 8 furlongs | = 1 mile. | 6360 | = 5280 | = 1760 | = 320 | = 8. |

Inch is sometimes divided into 3 *barleycorns*, or 12 *lines*.

A hair's breadth is .02083 (48th part) of an inch.

1 yard = .000568, and 1 inch = .0000158 of a mile.

Gunter's Chain.

7.92 inches = 1 link. | 100 links = 1 chain, 4 rods, or 22 yards.
80 chains = 1 mile.

Ropes and Cables.

1 fathom = 6 feet. | 1 cable's length = 120 fathoms.

Geographical and Nautical.

Assuming the Equatorial radius at 6967 459.893 yards (3958.784
ft. U. S. Coast Survey, = 69.094 Statute miles.
or 6080.27 feet.
miles.

Log Lines.

making a mile at 6080.27 feet, and using a 30" glass,

knot = 50 feet 8.03 inches. | 1 fathom = 5 feet .08 inch.

28" glass is used, and 8 divisions, then

1 knot = 47 feet 5 inches. | 1 fathom = 5 feet 11.25 inches.

line should be about 150 fathoms long, having 10 fathoms between chip and out for stray line.

—This estimate of a mile or knot is that of U. S. Coast Survey, assuming the radius of Earth to be 6967 459.893 yards and a Meter to be 39.370 432 of the Troughton scale at 62°.

Cloth.

1 nail = 2.25 inches. | 1 quarter = 4 nails. | 5 quarters = 1 ell.

Pendulum.

6 points = 1 line. | 12 lines = 1 inch.

Shoemakers'.

1 is 4.125 inches, and every succeeding number is .333 of an inch.

There are 28 numbers or divisions, in two series or numbers—viz., from 1 and 1 to 15.

Miscellaneous.

lines or 72 points = 1 inch. | 1 hand = 4 inches.

palm = 3 inches. | 1 span = 9 inches.

1 cubit = 18 inches.

Vernier Scale.

Vernier Scale is $\frac{1}{10}$, divided into 10 equal parts; so that it divides a scale into 100ths when two lines of the two scales meet.

Metric, by Act of Congress of July 28, 1866.

'Measurement is the METER, which by this Act is declared to be 39.37 ins.

| Designations. | Meters. | Inches. | Feet. | Yards. | Miles. |
|-------------------|---------|---------|-------------|-------------|---------|
| 1 meter..... | .001 | .0394 | — | — | — |
| 1 decimeter..... | .01 | .3937 | — | — | — |
| 1 centimeter..... | .1 | 3.937 | .328 083 | — | — |
| 1 millimeter..... | 1. | 39.37 | 3.280 83 | 1.093 61 | — |
| 1 meter..... | 10. | 393.7 | 32.808 33 | 10.936 11 | — |
| 1 decimeter..... | 100. | — | 328.083 33 | 109.361 11 | — |
| 1 centimeter..... | 1 000. | — | 3280.833 33 | 1093.611 11 | .621 37 |
| 1 meter..... | 10 000. | — | — | — | 6.213 7 |

220 system, values of the base of each measure—viz., Meter, Liter, Stere, Gramme—are decreased or increased by following prefix. Thus,

| | | |
|--------------------|---------------------------|-------------------------|
| 10th part or .001. | Deci, 10th part or .1. | Hekto, 100 times value. |
| 10th " .01. | Deka, 10 times value. | Kilo, 1000 " |
| | Myria, 10000 times value. | |

The Meter, as adopted by England, France, Belgium, Prussia, and Russia, terminated by Capt. A. R. Clarke, R.E., F.R.S., 1866, which at 62° in terms of standard at 62° F. is 39.370 432 inches or 1.093 623 11 yards, its legal value by Metric Act of 1864 being 39.3708 inches, the same as adopted in

Kater's comparison, and the one formerly adopted by the U. S. Ordnance = 39.370 797 1 inches, or 3.280 899 76 feet, and the one adopted by the Survey, as above noted, is = 39.370 432 35 inches.

| | | |
|------------|------------|--------------------|
| In. | | |
| 3.94 | .32 inch | = 8.13 millimeters |
| 39.4 | 3.2 inches | = 81.3 " |

MEASURES OF SURFACE.

12 inches = 1 square foot. | 9 square feet = 1 square yard.
Architect's Measure, 100 square feet = 1 square.

Land.

| | | | | |
|-------------------|------------------|-----------|-------------------|--------|
| 4840 square yards | = 1 square rod. | Yards. | Rods. | Roods. |
| 4 square rods | = 1 square rod. | 1210. | | |
| 4 square rods | } = 1 acre. | 4840 | = 160. | |
| 4 square chains | | 3 097 600 | = 102 400 = 2560. | |
| 1 square mile | = 1 square mile. | | | |

feet, 69,570 109 yards square, or 220 by 198 feet square = 1 Acre.

Paper.

quire. | 20 quires = 1 ream. | 21.5 quires = 1 printer's ream.
 48 sheets = 1 bundle. | 5 bundles = 1 bale.

Drawing.

| | |
|----------------------|--------------------------------|
| 13 × 16 inches. | Columbier 23 × 34 inches. |
| 15 × 20 " | Atlas 26 × 34 " |
| 17 × 22 " | Theorem 28 × 34 " |
| 19 × 23 " | Doub. Elephant, 27 × 40 " |
| l 19 × 27 " | Antiquarian ... 31 × 53 " |
| 22 × 30 " | Emperor 40 × 60 " |
| 23 × 28 " | Uncle Sam 48 × 120 " |
| Peerless | 18 × 52 inches. |

Tracing.

| | |
|-------------------------------------|---|
| 1 20 × 30 inches. | Grand Royal 18 × 24 inches. |
| own .. 30 × 40 " | Grand Aigle 27 × 40 " |
| Crown, 40 × 60 " | Vellum Writing, 18 to 28 ins. in width. |
| Mounted on cloth, 38 ins. in width. | |

Miscellaneous.

| | |
|------------------|-------------------------|
| sheet = 4 pages. | 1 duodecimo = 24 pages. |
| quarto = 8 " | 1 eighteenmo = 36 " |
| octavo = 16 " | 1 bundle = 2 reams. |

1 piece wall-paper, 20 ins. by 12 yards.
 1 " " French, 4.5 sq. yards.
 Roll of Parchment = 60 sheets.

Copying.

100 Words = 1 Folio.

by Act of Congress of July 28, 1866.

Unit of Surface is Are or Square Dekameter.

are (39.27²) = 1549.9969 sq. ins., but by this Act is declared to be
 1550 sq. ins.

| sq. Meters. | Sq. Inches. | Sq. Feet. | Sq. Yards. | Acres. |
|--------------|-------------|-------------|------------|--------|
|0001 | .155 | — | — | — |
|01 | 15.50 | .107 638 | — | — |
| 1 | 1550. | 20.763 888 | 2.196 | — |
| 100. | — | 2076.388 88 | 219.6 | — |
| 10000. | — | — | 21 960. | — |

Equivalent Values in Metric Denominations of U. S.

| Denominations. | Sq. Meters. | Denominations. | Sq. Meters. | Sq. Hectares. | Sq. Area. |
|----------------|-------------|----------------|--------------|---------------|----------------|
| Sq. Inch | .000 645 16 | Sq. Chain ... | 404.686 47 | — | 4.046 865 |
| " Foot | .092 903 23 | " Rod | 1011.716 175 | — | 10.117 162 |
| " Yard | .836 129 07 | " Acre | 4046.864 699 | .404 686 | 40.468 647 |
| " Rod | 25.292 904 | " Mile | — | 258.999 34 | 25 899.934 074 |

Approximate Equivalents of Old and Metric U. S.
Square Measures.

| | |
|--------------------------------------|---|
| 6.5 square centimeters = 1 sq. inch. | 1 acre = 1.16 per cent. over 4000 sq. meters. |
| 1 " meter = 10.75 sq. feet. | 1 square mile = 259 hectares. |

MEASURES OF VOLUME.

Standard gallon measures 231 cube ins., and contains 8.338 882 2 avoirdupois pounds, or 58 373 Troy grains of distilled water, at temperature of its maximum density (39.1°), barometer at 30 ins.

Standard bushel is the *Winchester*, which contains 2150.42 cube ins., or 77.627 413 lbs. avoirdupois of distilled water at its maximum density. Its dimensions are 18.5 ins. diameter inside, 19.5 ins. outside, and 8 ins. deep; and when heaped, the cone must not be less than 6 ins. high, equal 2747.715 cube ins. for a true cone.

A struck bushel contains 1.244 45 cube feet.

Liquid.

| | | |
|----------------------|-----------|---------------|
| 4 gills = 1 pint. | Cube Ins. | |
| 2 pints = 1 quart. | 28.875 | Gills. Pinta. |
| 4 quarts = 1 gallon. | 57.75 | 8. |
| | 231. | 32 = 8. |

Dry.

| | | |
|----------------------|-----------|-----------------------|
| 2 pints = 1 quart. | Cube Ins. | |
| 4 quarts = 1 gallon. | 67.2006 | Pinta. Quarts. Galla. |
| 2 gallons = 1 peck. | 268.8025 | 8. |
| 4 pecks = 1 bushel. | 537.605 | 16 = 8. |
| | 2150.42 | 64 = 32 = 8. |

Cube.

| | |
|----------------------------|---------|
| 1728 cube inches = 1 foot. | Inches. |
| 27 cube feet = 1 yard. | 46 656 |

NOTE.—A cube foot contains 2200 cylindrical inches, or 3300 spherical inches.

Fluid.

| | |
|---------------------|------------------------|
| 60 minims = 1 dram. | Minims. Drams. Ounces. |
| 8 drams = 1 ounce. | 480. |
| 16 ounces = 1 pint. | 7 680 = 128. |
| 8 pints = 1 gallon. | 61 240 = 1024 = 128. |

Nautical.

lacement in salt water = 35 cube feet.
tered internal capacity = 40 " "

Dimensions of a Barrel.

7 ins.; bung, 19 ins.; length, 28 ins.; volume, 7689 cube ins.

Approximate Equivalents of Old and Metric U. S. Measures of Volume.

| | |
|--|--|
| 1 Gallon.....= 4.5 <i>liters</i> . | 1 cube meter.....= 1.35 <i>cube yards</i> . |
| 1 Liter.....= .26 <i>gallon</i> . | 1 " yard.....= .75 " <i>meter</i> . |
| 1 cube foot.....= 28.3 <i>liters</i> . | 1 " kiloliter = 2240 <i>lbs. nearly of water</i> . |

MEASURES OF WEIGHT.

Standard *avoirdupois* pound is weight of 27.7015 cube inches of distilled water weighed in air, at (39.83°) barometer at 30 inches.

A cube inch of such water weighs 252.6937 grains.

Avoirdupois.

| | Drams. | Ounces. | Pounds. |
|----------------------------------|------------------------|----------|---------|
| 16 drams = 1 ounce. | 256. | | |
| 16 ounces = 1 pound. | 28 672 = | 1 792. | |
| 112 pounds = 1 cwt. | 573 440 = | 35 840 = | 2240. |
| 20 cwt. = 1 ton. | | | |
| 1 pound = 14 oz. 11 dwt. 16 grs. | Troy, or 7000 grains. | | |
| 1 ounce = 18 dwt. 5.5 grains | Troy, or 437.5 grains. | | |
| 1 dram = 1 dwt. 3.343 75 grains | Troy, or 53.5 grains. | | |
| 1 stone = 14 pounds. | | | |

Troy.

| | Grains. | Dwt. |
|------------------------------|------------------------------|------|
| 24 grains = 1 dwt. | 480. | |
| 20 dwt. = 1 ounce. | 5760 = | 240. |
| 12 ounces = 1 pound. | | |
| 7000 Troy grains = | 1 lb. <i>avoirdupois</i> . | |
| 437.5 " " = | 1 oz. " | |
| 27.343 75 Troy grains = | 1 dram " | |
| 175 Troy pounds = | 144 lbs. " | |
| 175 " ounces = | 192 oz. " | |
| 1 " ounce = | 480 grs. " | |
| 1 " pound = | .822 857 lb. | |
| 1 <i>avoirdupois</i> pound = | 1.215 278 lbs. <i>Troy</i> . | |

Apothecaries.

| | Grains. | Scruples. | Drams. |
|---|---------|-----------|--------|
| 20 grains = 1 scruple. | 60. | | |
| 3 scruples = 1 dram. | 480 = | 24. | |
| 8 drams = 1 ounce. | 5760 = | 288 = | 96. |
| 12 ounces = 1 pound. | | | |
| 45 drops = 1 teaspoonful or a fluid dram. | | | |
| 2 tablespoonfuls = 1 ounce. | | | |

The pound, ounce, and grain are the same as in Troy weight.

Diamond.

| | |
|-----------------------------|-----------------------------|
| 1 grain = 16 parts. | 4 grains = 3.2 Troy grains. |
| 16 parts = .8 Troy grain. | 1 carat = 4 grains. |
| 15.5 carats = 1 Troy ounce. | |

Lead.

Lead of lead = 8 pigs.

6.5 to 7.5 feet in width and from 30 to 35 feet in length.

Wheat.

Wheat per Bushel.

| Lbs. | Lbs. | Lbs. |
|--|------|------|
| 56 Oats..... 32 Barley.... 43 | | |

Miscellaneous.

COAL.

| | |
|--------------------------|------------------------------------|
| Intracite | 1 cube foot = 1.75 broken. |
| " | 50 to 55 lbs. per cube foot. |
| " | 41 to 45 cube feet = 1 ton broken. |
| lituminous | 70 to 78 lbs. per heaped bushel. |
| " | 40 to 50 lbs. per cube foot. |
| " Cumberland | 53 " " " |
| " Cannel | 50.3 lbs. per cube foot. |
| " Welsh | 43 cube feet = 1 ton. |
| " Lancashire | 44 " " = 1 " |
| " Newcastle | 45 " " = 1 " |
| " Scotch | 43 " " = 1 " |
| " R. N. allowance | 48 " " = 1 " |
| Charcoal, hardwood | 18.5 lbs. per cube foot. |
| " pine wood | 18 " " " " |

WOOD.

in pine . . 2700 lbs. = 1 cord. | Southern pine . 3300 lbs. = 1 cord.

EARTH.

and . . 21 cube feet = 1 ton. | Marl or Clay, 28 cube feet = 1 ton.
 gravel, 23 " " = 1 " | Mold 33 " " = 1 "

tric, by Act of Congress of July 28, 1866.

Weight is the GRAM, which is weight of one cube centimeter of pure water
 held in vacuo at temperature of 4° C., or 39.2° F., which is about its tem-
 perature of maximum density = 15.432 grains.

| Measures. | Values. | Grains. | Ounces. | Lbs. | Ton. |
|-----------|--------------------|---------|---------|---------|----------|
| 1..... | 1 cube millimeter | — | — | — | — |
| 10..... | 1 " " " | .154 32 | — | — | — |
| 1..... | 1 " centimeter | 1.543 2 | — | — | — |
| 1..... | 1 " " " | 15.432 | .035 27 | — | — |
| 10..... | 1 " " " | — | .352 7 | — | — |
| 1..... | 1 deciliter | — | 3.527 | .220 46 | — |
| 1..... | 1 liter | — | 35.27 | 2.204 6 | — |
| 10..... | 1 " " " | — | — | 22.046 | — |
| 1..... | 1 hektoliter | — | — | 220.46 | .008 419 |
| 1..... | 1 cube meter | — | — | 2204.6 | .984 196 |

≈ 2.679 17 lbs. Troy, or 2 lbs. 8 oz. 3 dwt. .3072 grain.

Metric Values in Metric Denominations of U. S.

| Measures. | Grams. | Dekagrams. | Denominations. | Grams. | Kilograms. |
|-----------|----------|------------|----------------|----------|-------------|
| | .064 8 | — | Ounce | 28.3502 | .028 35 |
| | 1.296 | — | " Troy | 31.1042 | .031 1 |
| 1..... | 1.555 2 | — | Pound | 453.6028 | .453 6 |
| | 1.771 87 | 17.7187 | " Troy | 373.2504 | .373 25 |
| 10..... | 3.888 | 38.88 | Ton | — | 1016.057 28 |

Metric Equivalents of Old and New U. S.

Measures of Weight.

and the gram are at nearly equal distances above and below the

Thus,

.. = 1 016 057.28 grams. | 1 kilogram ..

1 nearly 15.5 grains (about .5 per cent. less)

1 about 2.2 pounds avoirdupois (about .25

grams, or a metric ton, nearly 1 Engl. ton (a

Weight of Men and Women.

average weight of 20000 men and women, weighed in Boston, 1864, was 141.5 lbs.; women, 124.5 lbs. Average of men, women, and children, 105.5 lbs. A mass of people, densely packed, weighs 85 lbs. per sq. foot, occupying .8 of one sq. foot of area = 54.450 per acre.

Weight of Horses.—(U. S.)

Weight of horses ranges from 800 to 1200 lbs.

WEIGHT OF CATTLE.**To Compute Dressed Weight of Cattle.**

E.—Measure as follows in feet:

Girth close behind shoulders, that is, over crop and under plate, immediately behind elbow.

Length from point between neck and body, or vertically above point of cervical and dorsal processes of spine, along back to bone at end in a vertical line with rump.

1. multiply square of girth in feet by length, and multiply product of girths in following table, and quotient will give dressed weight of lbs.

| Position. | Heifer, Steer, or Bullock. | Bull. | Condition. | Heifer, Steer, or Bullock. | Bull. |
|-----------|-------------------------------|-------|--------------------|-------------------------------|-------|
| | 3.15 | 3.36 | Very prime fat ... | 3.64 | 3.85 |
| fat..... | 3.36 | 3.5 | Extra fat..... | 3.78 | 4.06 |
| t..... | 3.5 | 3.64 | | | |

EXAMPLE.—Girth of a prime fat bullock is 7 feet 2 ins., and length measured 4 feet 5 ins.

7.17 , and $7.17^2 = 51.4$, which $\times 4' 5''$ and by $3.5 = 794.5$ lbs. Exact weight 799 lbs.

1. Quarters of a beef exceed by a little, half weight of living animal weighs about eighteenth part, and tallow twelfth part of animal.

Comparative Weights of Live Cattle and of Beef.

| | Lbs. | Per cent. | | Lbs. | Per cent. |
|-------|------|-----------|---------------|------|-----------|
| | 2800 | 72 to 78 | Bullocks..... | 1550 | 61 to 64 |
| | 2600 | 70 to 76 | Heifers..... | 1550 | |
| | 2600 | | Bullocks..... | 1260 | 58 to 61 |
| | 2400 | 66 to 70 | Heifers..... | 1200 | |
| | 2400 | | Bullocks..... | 1050 | 57 to 58 |
| | 2100 | 64 to 68 | Heifers..... | 1050 | |
| | 2100 | | Bullocks..... | 980 | |
| | 1800 | 63 to 66 | Heifers..... | 950 | 50 to 56 |

Weight of Offal in a Beef and Sheep.

| | BEEF. | SHEEP. | |
|----------|----------|----------|--------------------------|
| | Lbs. | Lbs. | |
| Air.... | 56 to 98 | 8 to 16* | Kidneys, Heart, } |
| | 42 " 140 | 5 " 14 | Liver, etc. } |
| Tongue.. | 28 " 49 | 6 " 11† | Stomach, Entrails, etc., |
| | 21 " 35 | 2 " 3 | Blood..... |

* Weighs 2 to 6 lbs. for fleece.

† Including a te.

To Compute Equivalents of Old and New U. S. and of Metric Denominations.

By Act of Congress, July 28, 1866.

RULE.—Divide fourth term by second, multiply quotient by first term, and divide product by third term.

Or, Ascertain relative ratio of first and second terms, and multiply result by ratio of third and fourth terms.

NOTE.—When result is required in French or other Metric denominations than those of U. S., use exact denominations, as, 61.025 387 for 61.022, 39.370 432 for 39.37, etc.

EXAMPLE 1.—If one gallon (1st), per sq. foot, yard, acre, etc. (2d); how many liters, (3d), per sq. foot, yard, acre, etc. (4th)?

$$\frac{1}{1} \times 231 \div 61.022 \dots \dots = 3.7851 \text{ liters or } 3.7848 \text{ liters.}$$

$$\text{Or, } \frac{231}{144} = 1.604, \text{ and } \frac{144}{61.022} = 2.3598; \text{ hence, } 1.604 \times 2.3598 = 3.7851 \text{ liters.}$$

NOTE.—In computing ratios, first term is to be divided by second, and fourth by third.

EXAMPLE 2.—If one ton per cube foot, how many kilograms per cube decimeter?

$$\frac{61.022}{1728} \times 2240 \div 2.2046 = 35.881 \text{ liters, or } 35.882 \text{ litres.}$$

MEASURES.

By Act of Congress of U. S.

By Metric Computation.

| | | | |
|------------------------------|-------------------------------|----|------------------|
| 1 Liter per sq. foot, etc. = | .2642 Gallon per sq. foot, | or | .264 2 gallon. |
| 1 Liter per sq. meter = | .0245 Gallon per sq. foot, | or | .024 5 gallon. |
| 1 Gallon per sq. foot = | 40.746 Liters per sq. meter, | or | 40.745 4 litres. |
| 1 Sq. foot per acre ... = | .2296 Sq. meters per hectare, | or | 2.296 09 metres. |

WEIGHTS AND PRESSURES.

By Act of Congress of U. S.

By Metric Computation.

| | | |
|----------------------|-----------------------------|-------------------------|
| Per sq. inch. | Per sq. inch. | |
| 1 Centimeter = | .3937 Ins. | or .393 704 32 Ins. |
| 1 Atmosphere = | 6.6679 Kilograms, | or 6.667 8 kilogrammes. |
| 1 Inch mercury .. = | 2.54 Centimeters, | or 2.54 centimetres. |
| 1 Pound = | 453.6029 Grams, | or 453.592 6 grammes. |
| 1 Kilogram = | 317.4624 Lbs. per sq. foot, | or 317.465 lbs. |

NOTE.—30 ins. of mercury at 62° = 14.7 lbs. per sq. inch; hence, 1 lb. = 2.0408 ins., and a centimeter of mercury = 30 ÷ .3937 for U. S. computation, and 30 ÷ .393 704 32 for French or Metric.

POWER AND WORK.

$$1 \text{ Horse-power} = \text{Cheval or Cheval-vapeur} = 4500 \text{ k} \times m = 33\,000 \div (4500 \times 2.2046 \times 39.37 \div 12) = 1.013\,88 \text{ chevaux.}$$

$$1 \text{ Cheval or Cheval-vapeur } (75 \text{ k} \times m \text{ per second}) = \text{horse-power.}$$

$$(4500 \times 2.2046 \times 39.37 \div 12) \div 33\,000 = .9863 \text{ horse-power.}$$

By Act of Congress of U. S.

By Metric Computation.

| | |
|-----------------|--|
| 1 Horse-power = | 7.233 foot-lbs.; hence, |
| | = .138 26 Kilogrammeter, or .138 25 kilogramme. |
| | .0279 Cube meter per cheval, or .0279 cheval. |
| | .47 38 Kilogram per cheval, or .447 38 kilogramme. |
| | 1.44 per lb., or 35.8058 lb. |

TEMPERATURES.

oric or French unit = 3.968 *Heat-units*, and 1 heat-unit = $1 \div 3.968$ *caloric*.

S. Mechanical equivalent (772 foot-lbs.) = $772 \div 7.233 = 106.733$ *mmeters* and 106.733 *kilogrammetres*.

mch Mechanical equivalent ($423.55 k \times m$) = $3.280833 \times 2.2046 \times$
= 3063.505 *foot-lbs.*, or 3063.566 *foot-lbs. Metric*.

it-unit per pound = .5556 *Kilogram*, or .5556 *kilogramme*.

it-unit per sq. foot = .2715 *Caloric per sq. meter*, or .2713 *per sq. metre*.

VELOCITIES.

t per second..... = .3047 *Meter per second*, or .3047 *metres*.

s per hour..... = .447 " " " or .447 "

MEASURES OF TIME.

| | | |
|-----------------------|--|------------------------|
| io thirds = 1 second. | | 60 minutes = 1 degree. |
|-----------------------|--|------------------------|

| | | |
|------------------------|--|----------------------|
| io seconds = 1 minute. | | 30 degrees = 1 sign. |
|------------------------|--|----------------------|

360 degrees = 1 circle.

or *apparent time* is that deduced from observations of the Sun, time as that shown by a properly adjusted sun-dial.

Solar time is deduced from time in which the Earth revolves *axis*, as compared with the Sun; assumed to move at a mean *axis orbit*, and to make 365.242 218 revolutions in a mean Solar *year*.

al time is period which elapses between time of a fixed star *meridian* of a place and time of its return to that place.

rd unit of time is the *sidereal day*.

l day = 23 h. 56 m. 4.092 *sec.* in *solar* or *mean time*.

l year, or revolution of the earth, 365 d. 5 h. 48 m. 47.6 *sec.* in *solar*
time = 365.242 24 *solar days*.

ry, mean = 24 h. 3 m. 56.555 *sec.* in *sidereal time*.

var (Equinoctial, Calendar, Civil or Tropical) = 365.242 218 *solar*
is d. 5 h. 48 m. 47.6 *sec.*

ry commences at midnight. *Astronomical day* commences at
he civil day, having same designation, that is, 12 hours later
ivil day.

or *sea day* commences 12 hours before *civil time* or 1 day
ronomical time.

'e was introduced in England in 1752.

a Russia days are reckoned by Old Style, a
orian record.

MEASURES OF VALUE.

10 mills = 1 cent.

10 cents = 1 dime.

10 dimes = 1 dollar.

10 dollars = 1 eagle.

Standard of gold and silver is 900 parts of pure metal and 100 of alloy in 1000 parts of coin.

Fineness expresses quantity of pure metal in 1000 parts.

Remedy of the Mint is allowance for deviation from exact standard fineness and weight of coins.

Nickel cent (old) contained 88 parts of copper and 12 of nickel.

Bronze cent contains 95 parts of copper and 5 of tin and zinc.

Pure Gold 23.22 grains = \$1.00. Hence value of an ounce is \$20.67.183+.

Standard Gold, \$18.60.465+ per ounce.

WEIGHT, FINENESS, ETC., OF U. S. COINS.

Gold.

| Denomination. | Weight of Coin. | | of Pure Metal. | Denomination. | Weight of Coin. | | of Pure Metal. |
|-----------------|--------------------|------|-------------------|-----------------|--------------------|-----|-------------------|
| | Oz. | Gr. | Gr. | | Oz. | Gr. | Gr. |
| Dollar..... | .053 75 | 25.8 | 23.22 | Half Eagle..... | .268 75 | 129 | 116.1 |
| Quarter Eagle.. | .134 375 | 64.5 | 58.05 | Eagle..... | .537 5 | 258 | 232.2 |
| Three Dollar... | .161 25 | 77.4 | 69.66 | Double Eagle... | 1.075 | 516 | 464.4 |

Silver.

| | | | | | | | |
|-----------------|------------|-------|--------|------------------|----------|-------|--------|
| Dime..... | .080 375 | 38.58 | 34.722 | Half Dollar.... | .401 875 | 192.9 | 173.61 |
| 20 Cent..... | .160 75 | 77.16 | 69.444 | Trade Dollar... | .875 | 420 | 378 |
| Quarter Dollar. | .200 937 5 | 96.45 | 86.805 | Silver Dollar... | .859 375 | 412.5 | 371.25 |

Copper and Nickel.

| | Weight. | Copper. | Tin and Zinc. | | Weight. | Copper. | Tin and Zinc. |
|--------------|---------|-----------|------------------|--------------|---------|-----------|------------------|
| | Grains. | Per cent. | Per cent. | | Grains. | Per cent. | Per cent. |
| One Cent.... | 48 | 95 | 5 | Three Cents. | 30 | 75 | 25 |
| Two Cents... | 96 | 95 | 5 | Five Cents.. | 77.16 | 75 | 25 |

Tolerance.—*Gold*, Dollar to Half Eagle, .25 grains. Eagles, .5 grains.
—*Silver*, 1.5 grains for all denominations. — *Copper*, 1 to 3 cents, 2 grains;
5 cents, 3 grains.

Legal Tenders.—*Gold*, unlimited. — *Silver*. Dollars of 412.5 grains unlimited; for subdivisions of dollar, \$10. (Trade dollars [420 grains] are not legal tender.)—*Copper* or *cents*, 25 cents.

NOTE.—Weight of dollar up to 1837 was 416 grains, thence to 1873, 412.5. Weight of \$1000, @ 412.5 gr. = 859.375 oz.

BRITISH standards are: *Gold*, $\frac{23}{24}$ of a pound,* equal to 11 parts pure gold and 1 of alloy; *Silver*, $\frac{11}{12}$ of a pound, or 37 parts pure silver and 3 of alloy
~35 fine.

* ounce of standard gold is coined into £3 17s. 10d. 2s., and an
silver into 5s. 6d. 1 lb. silver is coined into 66 shillings.
Proportion of 2 shillings to pound avoirdupois.

86.65; hence $\frac{1}{10}$ of this = value of 1 penny =

be divided into 24 equal parts or carats, hence the pro-
1

To Compute Value of Coins.

- Divide product of weight in grains and fineness, by 480 (an ounce), and multiply result by value of pure metal per ounce, or multiply weight in ounces by fineness and by value of pure metal per ounce.

1.—When fine gold is \$20.67.183+ per oz., what is value of a British Sovereign, p. 40. Sovereign weighs .2567 oz., and .2567 X 480 = 123.216 has a fineness of .9165.

$$\text{Hence, } \frac{123.216 \times .9165}{480} \times 20.67.183+ = \$4.86.34.$$

2.—When fine silver is \$1.15.5 per oz., what is value of U. S. Trade dollar? p. 38, Dollar weighs .875 oz. and has a fineness of .900.

$$\text{Hence, } .875 \times .900 \times 1.15.5 = 90.95625 \text{ cents.}$$

3.—A 4-Florin (Austrian) weighs 49.92 grains and has a fineness of .900. value?

$$\frac{49.92 \times .900}{480} \times 20.67.183+ = \$1.93.49.$$

Convert U. S. to British Currency and Contrariwise.

—Divide Cents by 2.02771— (2.02770833), or, Multiply by (.49311826), and result is Pence.

Multiply Pence by 2.02771—, or divide by .49312—, and result is Cents.

—What are 100 cents in pence?

$$100 \times .49312 = 49.312 \text{ pence} = 4s. 1.312d.$$

—What is a Pound sterling in cents?

$$20 \times 12 = 240 \text{ pence, which } \times 2.02771 = \$4.86.65.$$

FOREIGN MEASURES OF VALUE.

t, Fineness, and Mint Values of Foreign Silver and Gold Coins.

of Congress, Regulations of the Mint, and Reports of its Directors.

Value of silver coins is necessarily omitted, as the value of variable element. Hence, in order to compute current value of coin, the price of fine or a given standard of silver being

as per above rule to compute value of coins.

of silver should be taken as that of the London market for standard (925 fine), it being recognized as the standard value, and rates in all countries.

—If it is required to determine value of a Mexican

57.5 oz. .903 fine. Value of Silver in London is 5s. 6d.

$$\frac{57.5 \times .903}{925} = .846867 \text{— and } 106.9616 \times .846867.$$

Weight and Mint Values of Foreign Coin.
 (The Value of the Silver Coins is based on their Value on January, 1888.)
Countries given in Italics have not a National Coinage.

| Country and Denomination. | Weight. | Fines- ness. | Pure Silver or Gold. | Current or Nominal. | V A L U E. | |
|------------------------------|---------|-----------------|-------------------------------|---------------------------|------------|-------------------|
| | | | | | U. S. | Gold. British. |
| | Oz. | Thous's. | Grains. | Cents. | \$ c. | £ s. d. |
| <i>Arabia.</i> | | | | | | |
| Piastre or Mocha Dollar..... | — | — | — | 83.14 | — | — |
| <i>Argentine Republic.</i> | | | | | | |
| Dollar = 100 Centesimos.... | — | — | — | 50.69 | — | — |
| Peso, Double..... | — | — | — | — | 96.5 | — |
| <i>Australasia.</i> | | | | | | |
| Same as British. | | | | | | |
| <i>Australia.</i> | | | | | | |
| Sovereign, 1855..... | .256.5 | 916 | — | — | 4.85.7 | 19 11 3 |
| Pound, 1852..... | .281 | 916.5 | — | — | 5.32.37 | 1 11 1 |
| <i>Austria.</i> | | | | | | |
| Kreutzer (copper)..... | — | — | — | .41 | — | .3 |
| Florin, new..... | .397 | 900 | 171.47 | — | 34.5 | — |
| Dollar, "..... | .596 | 900 | 257.47 | — | — | — |
| 4 Florins..... | .104 | 900 | — | — | 1.93.49 | 7 11 |
| Ducat..... | .112 | 986 | — | — | 2.28.3 | 9 4 6 |
| Souverein..... | .363 | 900 | — | — | 6.75.4 | 1 7 9 1 |
| <i>Belgium.</i> | | | | | | |
| Same as France. | | | | | | |
| <i>Bolivia.</i> | | | | | | |
| Centena..... | — | — | — | .75 | — | .3 |
| Dollar, new..... | .801 | 900 | 346.03 | — | 69.9 | — |
| Doubloon, 1827-36..... | .867 | 870 | 362.06 | — | 15.59.3 | 3 4 1 |
| <i>Brazil.</i> | | | | | | |
| Rei..... | — | — | — | .547 | — | .7 |
| Milreis..... | .028.8 | 916.66 | 12.67 | — | .54.59 | 56 9 |
| Double Milreis..... | .82 | 918.5 | 393.6 | — | — | — |
| 20 Milreis, 1854-56..... | .575 | 917.5 | — | — | 10.90.6 | 2 4 9 6 |
| Moldore, 4000 Reis..... | .261 | 914 | — | — | 4.92 | 1 0 2 6 |
| <i>Canada.</i> | | | | | | |
| Mil, sterling..... | — | — | — | .1 | — | .5 |
| Cent "..... | — | — | — | 1.01 | — | .5 |
| 20 Cent, currency..... | .15 | 925 | 66.6 | — | — | — |
| 25 "..... | .187.5 | 925 | 83.25 | — | — | — |
| Penny "..... | — | — | — | 1.52 | — | .75 |
| Shilling "..... | — | — | — | — | — | — |
| Dollar, sterling..... | — | — | — | — | 1 | 4 8 |
| 4 " = 20 shillings, currency | — | — | — | — | 3.97.43 | 16 4 |
| Pound "..... | — | — | — | — | 3.99.97 | 10 5 8 |
| <i>Cape of Good Hope.</i> | | | | | | |
| Same as British. | | | | | | |
| <i>Central America.</i> | | | | | | |
| 4 Reals..... | .027 | 875 | 11.34 | — | — | — |
| Dollar..... | .866 | 850 | 353.33 | — | — | — |
| 2 Escudos..... | .209 | 853.1 | — | — | 3.68.8 | 15 1.8 |
| Doubloon ante 1834..... | .869 | 833 | — | — | 14.96.39 | 3 1 5 9 |
| <i>Chile.</i> | | | | | | |
| Centaro..... | — | — | — | .9 | — | .4 |
| Dollar, new..... | .801 | 900.5 | 346.22 | — | 91.2 | — |
| 10 Pesos..... | .492 | 900 | — | — | 9.15.4 | 1 17 7.4 |
| Doubloon..... | .867 | 870 | — | — | 15.59.3 | 3 4 1 |
| <i>China.</i> | | | | | | |
| Cash, Lo..... | — | — | — | .14 | — | .5 |
| 10 Cents, Leang..... | .087 | 901 | 37.98 | — | — | — |
| Dollar..... | .866 | 901 | 374.63 | — | — | — |
| <i>Cochin China.</i> | | | | | | |
| Mas, 60 Saepks..... | — | — | — | 6.75 | — | .3 |
| 10 Mas, 1 Quan..... | — | — | — | 67.52 | — | 2 1 |

Weight and Mint Values.

| Country and Denomination. | Weight. | Fine- ness. | Pure Silver or Gold. | Current or Nominal. | V A L U E. Gold. | |
|-------------------------------------|---------|----------------|-------------------------------|---------------------------|---------------------|---------------------|
| | | | | | U. S. ¢. | British. £ s. d. |
| Cuba. | Oz. | Thous'. | Grains. | Centa. | \$ c. | £ s. d. |
| Same as Spain. | — | — | — | — | — | — |
| Colombia. | — | — | — | — | — | — |
| Centaro..... | — | — | — | 1.01 | — | .5 |
| Peso, new..... | .801 | 900 | 346.03 | — | 69.9 | — |
| 4 Escudos..... | .433 | 844 | — | — | 7.55.5 | 1 11 0.58 |
| Doublon, old..... | .867 | 870 | — | — | 15.59.3 | 3 4 1 |
| Costa Rica. | — | — | — | — | — | — |
| Same as Mexico. | — | — | — | — | — | — |
| Denmark. | — | — | — | — | — | — |
| Mark, 16 Skilling..... | — | — | — | 8.94 | — | 4.39 |
| Crown..... | .025 | 900 | — | — | 26.8 | 13.22 |
| 3 Rigsdaler..... | .927 | 877 | 390.23 | — | — | — |
| 10 Thaler..... | .427 | 895 | — | — | 7.90 | 1 12 5.6 |
| East Indies. | — | — | — | — | — | — |
| See Hindostan and Japan. | — | — | — | — | — | — |
| Ecuador. | — | — | — | — | — | — |
| Centaro..... | — | — | — | 1.01 | — | .5 |
| Peso..... | .801 | 900 | 346.03 | — | 69.9 | — |
| England. | — | — | — | — | — | — |
| Penny..... | .304 | — | — | 2.02 1/2* | — | 1 |
| Groat..... | .060.4 | 925 | 26.82 | — | — | — |
| Shilling, new..... | .182.5 | 924.5 | 80.99 | — | — | — |
| " average..... | .178 | 925 | 79.03 | — | — | — |
| Half Crown..... | .454.5 | 925 | 201.8 | — | — | — |
| Florin..... | .363.6 | 925 | 161.44 | — | — | — |
| Sovereign or Pound, new... | .256.7 | 916.5 | — | — | 4.86.65 | 1 0 0 |
| " average..... | .256.2 | 916.5 | — | — | 4.85.1 | 1 0 0 |
| Egypt. | — | — | — | — | — | — |
| Piastre, 40 Paras..... | .04 | 755 | 14.5 | — | 4.9 | — |
| Guinea, Bedidlik..... | .275 | 875 | — | — | 5. 0.52 | 1 0 6.84 |
| Pound..... | .275 | 875 | — | — | 4.97.4 | 1 0 5.3 |
| Purse, 5 Guineas..... | 1.375 | 875 | — | — | 25. 2.6 | 5 2 10.2 |
| France. | — | — | — | — | — | — |
| Centime..... | .032 | — | — | .2 | — | .1 |
| Sou, 5 Centimes..... | .161 | — | — | 1.01 | — | .5 |
| Franc, 100 Centimes..... | .161 | 900 | 69.55 | — | 19.3 | — |
| 5 Francs..... | .804 | 900 | 347.76 | — | 96.45 | — |
| 20 Francs, Napoleon, new... | .207.5 | 899 | — | — | 3.85.8 | 15 10.26 |
| 25 Francs 20 centimes = £1 Stg. | — | — | — | — | — | — |
| Germany. | — | — | — | — | — | — |
| Groschen, 10 Pfennig..... | — | — | — | 2.38 | — | 1.175 |
| Mark, 10 Groschen..... | .012.8 | 900 | — | — | 23.8 | 11.74 |
| 10 Marks..... | .128 | 900 | — | — | 2.38.24 | 9 9.5 |
| Thaler..... | .595 | 900 | 257.04 | — | — | — |
| Ducat..... | .112 | 986 | — | — | 2.28.38 | 9 4.63 |
| Greece and Ionian Islands. | — | — | — | — | — | — |
| Same as France. | — | — | — | — | — | — |
| Drachma, 100 Lepta..... | .010.4 | 900 | — | — | 19.3 | 9.5 |
| 5 Drachmas..... | .719 | 900 | 310.61 | — | — | — |
| 20 Drachmas..... | .185 | 900 | — | — | 44.2 | 14 1.75 |
| Pound..... | — | — | — | — | 5. 6.11 | 1 0 9.6 |
| Guatemala. | — | — | — | — | — | — |
| Same as Mexico. | — | — | — | — | — | — |
| Guiana, British, French, and Dutch. | — | — | — | — | — | — |
| Same as that of their Countries. | — | — | — | — | — | — |
| Hanse Towns. | — | — | — | — | — | — |
| Mark..... | .012.8 | 900 | — | — | 23.8 | 11.74 |
| Holland. | — | — | — | — | — | — |
| Canal..... | — | — | — | .4 | — | — |

* 2.027 71 cents.

D*

Weight and Mint Values.

| Country and Denomination. | Weight. | Fino- ness. | Pure Silver or Gold. | Current or Nominal. | VALU ^{ES} Gold. | |
|-------------------------------|---------|------------------------|-------------------------------|---------------------------|-----------------------------|----------|
| | | | | | U. S. | British. |
| | Oz. | Thous ^{ths} . | Grains. | Cents. | \$ c. | £ s. d. |
| Holland. | | | | | | |
| Florin or Guilder, 100 cents. | .021.6 | 900 | — | — | 41.2 | 1 8 |
| 10 Guilders | .215 | 899 | — | — | 3.99-7 | 16 5.11 |
| Hindustan. | | | | | | |
| Rupee | .374 | 916.5 | 164.53 | — | — | 1 10.5 |
| Honduras. | | | | | | |
| Same as Mexico. | | | | | | |
| Italy. | | | | | | |
| Same as France. | | | | | | |
| Lira, 100 Centimes | .16 | 835 | 65.12 | — | 10.3 | — |
| Scudo | .864 | 900 | 373.24 | — | — | — |
| Indian Empire. | | | | | | |
| Pic, nominal | — | — | — | .25 | — | .105 |
| Anna | — | — | — | 3.03 | — | 2.5 |
| Rupee, * 16 Annas | .375 | 916.5 | 165 | — | 35.2 | — |
| 10 Rupees, and 4 Annas | — | — | — | — | 4.86.65 | 1 0 0 |
| Mohur, 15 Rupees | .375 | 916.5 | — | — | 6.84.36 | 1 8 1.5 |
| Japan. | | | | | | |
| Sen | — | — | — | 1 | — | .5 |
| Itzebu, new | .279 | 890 | 119.19 | — | — | — |
| Yen, 100 Sen | .866.7 | 900 | 374.4 | — | 75.3 | — |
| " " | .053.6 | 900 | — | — | 99.72 | 4 1.18 |
| Cobang, old | .289 | 572 | — | — | 3.57.6 | 14 8.35 |
| " new | .362 | 568 | — | — | 4.44 | 18 2.0 |
| 20 Yen | 1.072 | 900 | — | — | 19.94.4 | 4 1 11.0 |
| Java. | | | | | | |
| Same as Holland. | | | | | | |
| Liberia. | | | | | | |
| U. S. Currency. | | | | | | |
| Malta. | | | | | | |
| 12 Scudi = 1 Sovereign | — | — | — | — | 4.86.65 | 1 0 0 |
| Mexico. | | | | | | |
| Peso, new | .867.5 | 903 | 377.17 | — | 75.9 | 4 2 |
| Maximilian | .861 | 902.5 | 372.98 | — | — | — |
| Doubloon, new | .867.5 | 870.5 | — | — | 15. 6.1 | 3 4 1.88 |
| 20 Pesos, Republic | 1.081 | 873 | — | — | 19.51.5 | 4 0 2.4 |
| Morocco. | | | | | | |
| Ounce, 4 Blanckeels | — | — | — | — | — | — |
| 10 Ounces, Mitkeel | — | — | — | — | — | — |
| Naples. | | | | | | |
| Scudo | .844 | 830 | 336.25 | — | — | — |
| 6 Ducati | .245 | 996 | — | — | 5. 4.4 | 1 0 8.75 |
| Netherlands. | | | | | | |
| Same as Holland. | | | | | | |
| New Brunswick. | | | | | | |
| Same as Canada. | | | | | | |
| Newfoundland. | | | | | | |
| Same as Canada. | | | | | | |
| New Granada. | | | | | | |
| Dollar 1857 | .803 | 896 | — | — | — | — |
| Doubloon, Popayan | .867 | 858 | — | — | 15.37.8 | 3 3 3.39 |
| Norway. | | | | | | |
| Like Denmark. | | | | | | |
| Mark, 24 Skillingen | — | — | — | 21.63 | — | 10.66 |
| Nova Scotia. | | | | | | |
| Same as Canada. | | | | | | |
| Persia. | | | | | | |
| Rum | — | — | — | 22.81 | — | 11.05 |

2. Wages starting.

Weight and Mint Values.

| Denomination. | Weight. | Finess. | Pure Silver or Gold. | Current or Nominal. | Value of Gold. | |
|-------------------|---------|------------------------|----------------------|---------------------|----------------|-----------|
| | | | | | U. S. | British. |
| | Oz. | Thous ^{ths} . | Grains. | Cents. | \$ c. | £ s. d. |
| | .766 | 900 | 341.01 | — | — | — |
| | .802 | 900 | 346.46 | — | 69.9 | — |
| old | .867 | 868 | — | — | 15.55-7 | 3 3 11.22 |
| 10000 Reils | .308 | 912 | — | — | 10.81.78 | 2 4 5.5 |
| | .095 | 912 | — | — | 10.8 | — |
| | .322 | 835 | 129.06 | — | — | — |
| | — | 500 | — | .77 | — | .38 |
| Rouble | .667 | 875 | 277.73 | — | 55.9 | — |
| | .21 | 916.6 | — | — | 3.97.6 | 16 4.8 |
| unds. | | | | | | |
| ancy. | | | | | | |
| | .16 | 835 | 65.12 | — | — | — |
| | — | — | — | .19 | — | .095 |
| 5. Peseta | .16 | 835 | 64.13 | — | 19.3 | — |
| sela | .8 | 900 | 345.6 | — | 92.6 | — |
| | .268 | 806 | — | — | 4.96.4 | 1 0 4.8 |
| ns=1 U.S. Dollar. | .270.8 | 806 | — | — | 5.1.5 | 1 0 7.32 |
| 100 Ore | .273 | 750 | 98.28 | — | — | — |
| | 1.092 | 750 | 393.12 | — | — | — |
| Francs | .104 | 900 | — | — | 1.93.5 | 7 11.42 |
| nce. | | | | | | |
| 5 Cents | — | — | — | 6.33 | — | 3.125 |
| Karubs | — | — | — | 11.83 | — | 5.83 |
| | .511 | 898.5 | 220.38 | — | — | — |
| | .161 | 900 | — | — | 2.99.5 | 12 3.7 |
| Paras | — | — | — | 4.39 | — | 2.16 |
| | .77 | 830 | 306.77 | — | — | — |
| Medjidie | .231 | 915 | — | — | 4.36.9 | 18 0 |
| equin | .112 | 900 | — | — | 2.31.3 | 9 6.1 |
| Mahbub | — | — | — | 74.3 | 63 | 3 0.89 |
| Centimes | — | — | — | — | — | — |
| British. | | | | | | |
| England. | | | | | | |
| | — | — | — | 1 | — | .5 |
| ranc | — | — | — | — | 19.3 | — |

Memoranda.

Ronze coins 9.5 copper, 4 tin, and 1 zinc.

sa.—Monetary system same as that of German Empire.

n.—The Centime is termed a Rappe.

Peseta piece is 10s. 9.5d. Stg.; Real vellon was 2.5d. Stg.

coins same weight and fineness as those of France.

ari and 4 Grani = 1 Shilling Sterling.

Para = .061 5d. Sterling, and 97.22 Piastres = 1 Sovereign.

12.—1 Lac Rupees = £10000 Sterling. In Ceylon, Rupee = 1

ENGLISH AND FRENCH MEASURES AND WEIGHTS.

MEASURES OF LENGTH.

LEGATION.—Imperial standard yard is referred to a natural standard, which is a pendulum 39.1393 ins. in length vibrating seconds in vacuum in London, at level of sea; measured between two marks on a brass bar, at temperature of 62°.

NOTE.—In consequence of destruction of standard by fire in 1834, and difficulty of replacing it by measurement of a pendulum, the present standard is held to be about 1 part in 17240 less than that of U. S., equal to 3.07 ins. in a mile.

Miscellaneous.

Land.—Woodland pole or perch or Fen = 13 feet.

Forest pole = 21 "

Irish mile = 2240 yards. | Scotch mile = 1984 yards.

100 fathoms, or 1000 fathoms, or 6080.27 feet, or 1.1516 Statute miles.

1 Admiralty or Nautical mile or knot = 6080 feet.

3 miles = 1 league. 60 Nautical or 60.004 Statute miles or 20 Leagues = 1 degree.

Mean length of a minute of Latitude at mean level of the sea = 1.1451 statute miles.

Nautical mile is taken as length of a minute at the Equator.

Nautical fathom is 1000th part of a nautical mile, and averages about .0125 longer than the common fathom.

FRANCE.—Standard Metre or unit of measurement is defined as the ten-millionth part of the terrestrial meridian, or the distance from the Equator to the Pole, passing through Paris. Actual standard is a platinum bar deposited in the Palais des Archives, Paris.

| | Metres. | Inches. | Feet. | Varia. | Miles. |
|--------------|---------|----------|------------|------------|-----------|
| 1 Metre | 1 | 3.937 | — | — | — |
| 1 Decimetre | .1 | .3937 | — | — | — |
| 1 Centimetre | .01 | .393704 | — | — | — |
| 1 Millimetre | .001 | .3937043 | .00328087 | .000328087 | — |
| 1 Decimetre | .1 | — | .3937043 | .3937043 | — |
| 1 Centimetre | .01 | — | .03937043 | .03937043 | — |
| 1 Millimetre | .001 | — | .003937043 | .003937043 | — |
| 1 Decimetre | .1 | — | .3937043 | .3937043 | .62136 |
| 1 Centimetre | .01 | — | .03937043 | .03937043 | .0062136 |
| 1 Millimetre | .001 | — | .003937043 | .003937043 | .00062136 |

For further details see p. 27.

Old Measures.

| | | | |
|----------|-------------------|----------------------|--------------------|
| 1 Fathom | = 640 metres. | 1 Terrestrial league | = 4.44 Kilometres. |
| 1 Mile | = 645 Kilometres. | 1 Nautical league | = 5.55 " |
| 1 Arpent | = 875 " | 1 Arpent | = 900 sq. toises. |

MEASURES OF SURFACE.

ENGLAND.—Same as that of United States of America.

Miscellaneous.

Builders. 1 square foot = 1 square inch.

1 square foot = 1 inch.

1 square foot = square foot.

1 square foot = 9 inches deep, and

FRENCH.

Metrio Surfaces in Square Inches, Feet, etc.

| Denomination. | Sq. Inches. | Sq. Feet. | Sq. Yards. | Sq. Acres. |
|-----------------------------|--------------|--------------|---------------|--------------|
| 1 are millimetre..... | .001 55 | — | — | — |
| 16 centimetre..... | .155 003 | — | — | — |
| 16 decimetre..... | 15.500 309 | .107 641 | — | — |
| 16 Metre or Centiare | 1550.030 916 | 10.764 104 | 1.196 01 | — |
| 16 dekametre or are..... | — | 1076.410 358 | 119.601 15 | .024 711 |
| 16 hektometre or heclare .. | — | — | 11 960.115 09 | 2.471 098 |
| 16 kilometre..... | — | — | — | 247.109 816 |
| 16 myrametre*..... | — | — | — | 24 710.981 6 |

* Equal 38.610 908 sq. miles.

Old System.

square inch = 1.135 87 inches.
 toise = 6.394 6 feet.
 arpent (Paris) = 900 square toises = 4089 square yards.
 arpent (woodland) = 100 square royal perches = 6108.24 square yards.

MEASURES OF VOLUME.

Imperial gallon measures 277.123 cube ins., but by Act of Parliament its volume is 277.274 cube ins., equal to 10 lbs. avoirdupois of led water, weighed in air, at temperature of 62°, barometer at 30 in. 6.2355 gallons in a cube foot.

Imperial bushel, 18.5 ins. internal diameter, 19.5 external, and 8.25 depth, contains 2218.192 cube ins., and when heaped in form of a cone, at least .75 depth of the measure, must contain 2815.4872 ins. or 1.6293 cube feet.

win.—1 quarter = 8 bushels or 10.2694 cube feet.

vcls.—1 ton displacement = 35 cube feet; 1 ton freight by measure = 40 cube feet.

in internal capacity = 100 cube feet, and 1 ton ship-builders = 94 feet.

fish standard No. 5 is .008 grain heavier than the pound, and U. S. pound is 16 lighter than English.

Wine and Spirit Measures.

| | |
|--------------------------------|--------------------------|
| 4 quarts (231 cube ins.) | = .8333 Imperial gallon. |
| 10 gallons | = 1 anchor. |
| 18 " (15 imperial) | = 1 runlet. |
| 31.5 " 26.25 " | = 1 barrel. |
| 42 " 35 " | = 1 tierce. |
| 63 " 52.5 " | = 1 hogshead. |
| 84 " 70 " | = 1 puncheon. |
| 126 " 105 " | = 1 pipe or butt. |
| 2 pipes or } | |
| 3 puncheons } | = 1 tun. |

Ale and Beer Measures.

| | | | |
|-----------------------|---------------------|-------------------------|--------------------|
| 16 (282 cube ins.) .. | Imp'l gall. = 1.017 | 2 kilderkins = 1 barrel | Imp'l gall. = 6.12 |
| 16 = 1 firkin | = 9.153 | 54 gallons = 1 hogshe | |
| 16 = 1 kilderkin ... | = 18.306 | 108 " = 1 butt .. | |

Apothecaries' or Fluid Measures.

| | |
|--------------------------------|--------------------------------------|
| 1 drop = 1 grain. | 4 drachms = 1 tablespoon. |
| 60 drops = 1 drachm. | 2 ounces (875 grains) = 1 wineglass. |

Coal Measures.

| | |
|--|--|
| 50 pounds = 1 <i>cube foot.</i> | 12 sacks = 1 <i>chaldron.</i> |
| 88 " = 1 <i>bushel.</i> | 1 chaldron = 58.6548 <i>cube ft.</i> |
| 9 bushels = 1 <i>vat.</i> | 5.25 chaldrons . . = 1 <i>room.</i> |
| 80 or 84 pounds = { 1 <i>London or</i> | 1 London chaldron = 26.5 <i>cwt.</i> |
| | 1 Newcastle " = 53 " |
| 90 or 94 " = 1 <i>Cornish ".</i> | 1 ton = 44.5 <i>cube feet.</i> |
| 93 pounds = 1 <i>Welsh bushel.</i> | 1 room = 7 <i>tons.</i> |
| 3 heaped bush. = 1 <i>sack.</i> | 21 chaldrons = 1 <i>score.</i> |
| 10 sacks = 1 <i>ton.</i> | 1 barge or keel . . = 21.2 <i>tons.</i> |

Miscellaneous.

| | | | |
|---------------------------|-------------------|---------------------------|--------------|
| 1 last corn | = 80 bushels. | 1 truss old hay | = 50 pounds. |
| 1 ton water | = 35.9 cube feet. | 1 " new " | = 60 " |
| 1 dicker hides | = 10 skins. | 1 bushel oats | = 40 " |
| 1 last hides | = 20 dickers. | 1 " barley | = 47 " |
| 1 barrel tar | = 26.5 gallons. | 1 " wheat | = 60 " |
| 6 bushels wheat | = 1 sack flour. | 1 cube yard new hay = | 84 " |
| 1 clove | = 7 pounds. | 1 " old " | = 126 " |
| 1 score | = 20 " | 1 quintal | = 100 " |
| 1 sack flour | = 28.2 " | 1 boll | = 140 " |
| 1 truss straw | = 36 " | 1 sack wool | = 362 " |

35.9 cube feet = 1 ton water.

LIQUID.

| | | | | | |
|-------------------|---------|------------------|-----------------------|---------|-----------------|
| 1 wine gallon = | 231 | <i>cube ins.</i> | 1 hogshead wine .. = | 52.5 | <i>gallons.</i> |
| 1 beer " = | 282 | " | 1 " beer ... = | 54.918 | " |
| 1 litre = | .22009 | <i>gallon.</i> | 1 puncheon wine .. = | 70 | " |
| 1 gallon . . . = | 4.544 | <i>litres.</i> | 1 pipe or butt wine = | 105 | " |
| 1 cube foot .. = | 6.32121 | <i>gallons.</i> | 1 " " beer = | 109.836 | " |
| 1 anker = | 8.333 | " | 1 tun = | 210 | " |

1 ton water $62^{\circ} = 224$ gallons.

BUILDERS.

| | | | | | |
|----------------------------|------|-------------------|-----------------------|-----------|-------------------|
| 1 solid part..... | = 12 | <i>cube ins.</i> | 1 square | = 100 | <i>sq. feet.</i> |
| 12 " parts | = 1 | <i>"inch."</i> | 1 bundle laths | = 120 | <i>laths.</i> |
| 12 "inches"..... | = 1 | <i>cube foot.</i> | 1 rod brickwork | = 360 | <i>cube feet.</i> |
| 1 load timber, rough | = 40 | <i>"feet."</i> | 1 rood masonry ... | = 648 | <i>"</i> |
| 1 " hewn | = 50 | <i>"</i> | Batten, in section .. | = 7 × 2.5 | <i>ins.</i> |
| 1 " lime | = 32 | <i>brushels.</i> | Deal, " " .. | = 9 × 3 | <i>"</i> |
| 1 " sand | = 36 | <i>"</i> | Plank, " " .. | = 11 × 3 | <i>"</i> |

Metric Volumes in Cube Inches, Feet, etc.

| Denominations. | Litres. | Gills. | Pints. | Quarts. | Gallons. | Bushels. | Quarters. |
|----------------|---------|--------|--------|---------|----------|----------|-----------|
| Centilitre... | | | | | — | — | — |
| Decilitre... | | | | | — | — | — |
| | | | | | 2.2001 | — | — |
| | | | | | 2.2009 | .275 11 | — |
| | | | | | 2.2017 | .275 13 | .3439 |
| | | | | | 2.2025 | .275 15 | .3439 |

Wood Measure.

Stere or cube metre = 35.3150 *cube feet* or 1.308 *cube yards*.
 Voie de bois (Paris) = 70.6312 *cube feet*; 1 voie de charbon (charcoal)
 .063 *cube feet*; 1 corde = 4 cube metres = 141.26 *cube feet*.

MEASURES OF WEIGHT.

158.—1 Troy grain = .003 961 cube inches of distilled water.
 1 Troy pound = 22.815 689 cube inches of water.
 1 Avoir. drachm = 27.343 75 Troy grains.

Avoirdupois.

| | | | | |
|-----------|---------|------------|----------|--------------------------|
| achms, or | } | = 1 ounce. | 8 pounds | .. = 1 stone (for meat). |
| 5 grains | | | 14 " | .. = 1 stone. |
| nces, or | } | = 1 pound. | 28 " | .. = 1 quarter. |
| grains | | | 112 " | .. = 1 cwt. |

20 hundredweights = 1 ton.

1 grain, of which there are 7000 to the pound avoirdupois, is same as grain, of which there are by the revised table 7000 to the Troy pound.
 1 Troy pound is equal with the Avoirdupois pound.
 Wales, the iron ton is 20 cwt. of 120 lbs. each.

Troy.

| | | | |
|---------------|---------------|---------------------------|--------------|
| ins | } .. = 1 dwt. | 16 ounces | = 1 pound. |
| myweights, or | | 25 pounds | = 1 quarter. |
| 7.5 grains | | 4 quarters, or 100 pounds | = 1 cwt. |

this are weighed gold, silver, jewels, and such liquors as are sold by

old Troy ounce to the Avoirdupois ounce was as 480 grains, the
 of the former, to 437.5 grains, weight of the latter; or, as 1 to .9115.

Apothecaries.*

437.5 grains = 1 ounce. | 16 ounces = 1 pound.

FRENCH.

Metric Weights in Avoirdupois.

| Nominations. | Grammes. | Grains. | Ounces. | Pounds. | Ton. |
|--------------|-----------|---------------|---------|-------------|-------|
| mmé | .001 | .015 43 | — | — | — |
| mmé | .01 | .154 32 | — | — | — |
| mmé | .1 | 1 543 23 | — | — | — |
| mmé | 1 | 15 432 35 | — | — | — |
| mmé | 10 | 154 323 49 | .3527 | — | — |
| mmé | 100 | 1 543 234 87 | 3.5274 | .220 46 | — |
| mmé | 1 000 | 15 432 348 74 | 35.2739 | 2.204 62 | — |
| mmé | 10 000 | — | — | 22.046 21 | — |
| mmé | 100 000 | — | — | 220.462 12 | — |
| r Ton. | 1 000 000 | — | — | 2204.621 25 | .9842 |

† Kilogramme = 2 lbs. 3 oz. 4 drachms, 10.4734 grains.

—For the values of the prefixes, as Milli, Centi, etc., see p. 27.

Old System.

| | |
|----------------------------|-----------------------------------|
| 1 .. = 0.8188 grains Troy. | 1 ounce = 1.0780 oz. Avoirdupois. |
| 1 .. = 58.9548 " | 1 livre = 1.0780 lbs. |

* As by revised Pharmacopœia.

Bagdad.

| | |
|------------------------|-------------|
| | 31.665 ins. |
| Barbary States. | |
| linen..... | 18.62 ins. |
| cloth..... | 26.49 " |
| li..... | 21.75 " |

Batavia.

| | |
|-------|-------------|
| | 12.357 ins. |
| | 27 " |
| | 27.75 " |

Bavaria.

| | |
|----------------|------------------|
| | 11.49 ins. |
| | 5.745 36 ft. |
| | 3.1918 yds. |
| | 8060 " |
| adrat..... | 10.1876 sq. yds. |
| r Tagwerk..... | .8416 acre. |
| ube..... | 4.097 cub. yds. |
| | 15.058 56 galls. |
| | 6.119 " |
| | 1 0196 bush. |
| | 8642 grains. |

*Also Decimal System.***Belgium.**

| | |
|-------|------------|
| | 2.132 yds. |
|-------|------------|

*Also Decimal System.***Benares.**

| | |
|------------|---------|
| lor's..... | 33 ins. |
|------------|---------|

1, Bombay, and Calcutta.

| | |
|-------------|-----------------|
| | 3 ins. |
| | 9 " |
| bar..... | 10.46 ins. |
| | 18 " |
| bay..... | 27 " |
| gal..... | 36 " |
| intum..... | 3.417 ft. |
| gal..... | 1.136 miles. |
| utta..... | 1.2273 " |
| | 9.8175 sq. yds. |
| engal..... | .3306 acre. |
| bombay..... | .8114 " |
| ory..... | .68 cub. ins. |
| mbay..... | 12.704 cub. ft. |
| ibay..... | 1.234 pints. |
| | 4.4802 galls. |
| | 112.0045 " |

*and Grain measured by weight.***Bohemia.**

| | |
|------------|------------|
| guc..... | 11.88 ins. |
| erial..... | 12.45 " |

*Also same as Austria.***ia, Chili, and Peru.**

| | |
|-------|---------------|
| | 33.333 ins. |
| | 1.5888 acres. |
| | .74 gall. |
| | 1.572 " |
| | 1.014 lbs. |
| | 25.36 " |

*in Spain; now Decimal in Chili and Peru.***Brasil.**

| | |
|-------------------|--------------|
| Palmo, Bahia..... | 8.5592 ins. |
| Vara..... | 3.566 ft. |
| Braca..... | 7.132 " |
| Geira..... | 1.448 acres. |

*Also same as Portugal, and sometimes as in England.***Buenos Ayres.**

| | |
|--------------------------|-------------------|
| Vara..... | 2.84 ft. |
| Legua..... | 3.226 miles. |
| Suertes de Estancia..... | 27 000 sq. varas. |

*Also same as Spain.***Burmah.**

| | |
|--------------|------------|
| Paulgat..... | 1 inch. |
| Dain..... | 4.277 yds. |
| Viss..... | 3.6 lbs. |
| Taim..... | 5.5 " |
| Seading..... | 22 " |

*Also same as England.***Canary Isles.**

| | |
|---------------------|-------------|
| Onza..... | .927 inch. |
| Pic, Castilian..... | 11.128 ins. |
| Almude..... | .0416 acre. |
| Fanegada..... | .5 " |
| Libra..... | 1.0148 lbs. |

*Also same as Spain.***Cape of Good Hope.**

| | |
|-------------|-----------------|
| Foot..... | 11.616 ins. |
| Morgen..... | 2.116 54 acres. |

*Also same as in England.***Ceylon.**

| | |
|-------------|-------------|
| Seer..... | 1 quart. |
| Parrah..... | 5.62 galls. |

*Also same as in England.***China.**

| | |
|-----------------------|---------------|
| Li..... | .486 inch. |
| Chih, Engineer's..... | 12.71 ins. |
| " or Covid..... | 13.125 " |
| " " legal..... | 14.1 " |
| Chang..... | 131.25 " |
| " legal..... | 141 " |
| Pu..... | 4.05 ft. |
| Chang, fathom..... | 10.9375 ft. |
| Li..... | 486 yds. |
| Pá or Kung..... | 3.32 sq. yds. |
| King, 100 Mau..... | 16.485 acres. |
| Tau..... | 1.13 galls. |
| Tael..... | 1.333 oz. |
| Catty..... | 1.333 lbs. |

Cochin China.

| | |
|---------------------|--------------|
| Thuoc or Cubit..... | 13.2 ins. |
| Sao..... | 64 sq. yds. |
| Mao..... | 1.32 acres. |
| Hao..... | 6.222 galls. |
| Shita..... | 12.444 " |
| Nen..... | .8594 lb. |

Colombia and Venezuela.

| | |
|------------|------------|
| Libra..... | 1.102 lbs. |
| Oncha..... | 1.102 lbs. |

Also

Denmark,* Greenland, Iceland, and Norway.

| | |
|-------------------|----------------|
| Tomme..... | 1.0297 ins |
| Fod..... | 1.0297 ft. |
| Favn, 3 Alen..... | 6.1783 " |
| Mil..... | 4.68055 miles. |
| " nautical..... | 4.61072 " |
| Anker..... | 8.0709 galls. |
| Skeppe..... | .478 bush. |
| Fjerdjngkar..... | .9558 " |
| Pund..... | 1.1023 lbs |
| Lispund..... | 17.367 " |
| Centner..... | 110.23 " |

* Also Decimal System.

**Ecuador.
Decimal System.****Genoa, Sardinia, and Turin.**

| | |
|----------------------------|---------------|
| Palmo..... | 9.8076 ins. |
| Piede, Manual, 8 oncie.... | 13.488 " |
| " Liprando, 12 "..... | 20.23 " |
| Trabuco or Tessa..... | 10.113 ft. |
| Miglio..... | 1.3835 miles. |
| Starello..... | .9804 acre. |
| Giomaba..... | .9394 " |

Germany.

The old measures of the different States differ very materially; generally, however,

| | |
|----------------------|--------------|
| Foot, Rhineland..... | 12.357 ins. |
| Meile..... | 4.603 miles. |

Decimal System made compulsory in 1872.

Greece.

| | |
|--------------|------------|
| Stadium..... | 6155 mile. |
|--------------|------------|

Also Decimal System.

Guinea.

| | |
|--------------|--------|
| Jachtan..... | 12 ft. |
|--------------|--------|

Hamburg.

| | |
|-------------------------|----------------|
| Fuss..... | 11.2788 ins. |
| Klafter..... | 5.6413 ft. |
| Morgen..... | 2.386 acres. |
| Cube Fuss..... | .8311 cub. ft. |
| Tehr..... | 99.73 " |
| Viertel..... | 1.5947 galls. |
| Pfund (500 grammes).... | 1.10232 lbs. |
| Ton..... | 2135.8 lbs. |

Also Decimal System.

Hanover.

| | |
|-------------|-------------|
| Fuss..... | 11.5 ins. |
| Morgen..... | .6476 acre. |

Hindostan.

| | |
|-------------|------------|
| Borrel..... | 1.311 ins. |
| Gerah..... | 2.387 " |
| Haut..... | 20.23 " |
| Kobe..... | " |
| Com..... | " |

*Tada....
Candy....*

Hungary.

| | |
|------------|-------------|
| Fuss..... | 12.445 ins. |
| Elle..... | 30.67 " |
| Meile..... | 9.139 yds |

Also as in Vienna.

Indian Empire.

| | |
|-------------|--------------------|
| Guz..... | 27.125 ins. |
| Cowrie..... | 1 sq. yd. |
| Sen..... | 61.025 39 cub. ins |
| "..... | 2.204737 lbs |

Uniform standard of multiples of the Sen adopted in 1871.

Italy.**Milan and Venice.
Decimal System.**

The Metre is termed Metra; the Aro, Ara; the Stere, Stero; the Litro, Litro; the Gramme, Gramma, and the Tonneau, Tonnelata de Mare.

Naples and Two Sicilies.

| | |
|-------------------|---------------|
| Palmo..... | 10.381 ins. |
| Canua..... | 6.921 ft. |
| Miglio..... | 1.1506 miles. |
| Migliago..... | .7467 acre. |
| Moggia..... | .86 " |
| Pezza, Roman..... | .6529 " |

Roman States.

Old Measure.

| | |
|--------------------|--------------|
| Foot..... | 11.592 ins. |
| " Architect's..... | 11.73 " |
| Braccio..... | 30.73 " |
| Palmo..... | 8.347 " |
| Miglio..... | 1628 yds. |
| Quarta..... | 1.1414 acres |

Lucca and Tuscany.

| | |
|---------------|---------------|
| Pie..... | 11.94 ins. |
| Palmo..... | 11.49 " |
| Braccio..... | 22.98 " |
| Passetto..... | 3.829 ft. |
| Passo..... | 5.74 " |
| Miglio..... | 1.0277 miles. |
| Quadrato..... | .8413 acre. |
| Saccato..... | 1.324 " |

Japan.

| | |
|--------------------------|-----------------------|
| Sun, .30303 Metre.... | 1.193* ins. |
| Shaku, 3.0303 Metres.... | 11.9305* ins. |
| Jo, 30.303 "..... | 9.9421* ft. |
| Ken, 5.5 "..... | 5.9653* " |
| Ri, 11880 "..... | 2.4403 miles |
| Kai-ri..... | 6080 feet.† |
| Hiro..... | 4.971* feet. |
| Momme..... | 3.7565217 grammes Fr. |
| Hiyaku-me..... | .82817 lbs. |
| Kwan-me..... | 8.28171 " |
| Hiyak-kin..... | 132.50732 " |
| Man's load..... | 57.972 " |
| Koku..... | 331.26831 " |
| Hiyak-koku..... | 33126.8308 " |

*are as equivalent as they are practicable.
by Jach.*

| | |
|--------------|---------------|
| Java. | |
| | 1.3 ins. |
| | 27.08 " |
| | 7.015 acres. |
| | 328 galls. |
| | 593.6 grains. |
| | 61.034 lbs. |
| | 122.068 " |
| | 1.356 " |

| | |
|----------------|-------------|
| Madras. | |
| | 10.46 ins. |
| | 18.6 " |
| | 33 " |
| | 20.92 ft. |
| | 3472 yds. |
| | 338 galls. |
| | 2.704 " |
| | 180 grains. |
| | .625 lbs. |
| | 3.086 " |
| | 24.686 " |

| | |
|-----------------|------------|
| Malabar. | |
| | 10.46 ins. |

| | |
|-----------------|-------------|
| Malacca. | |
| 1..... | 18.125 ins. |
| | 6 ft. |
| | 80 yds. |

| | |
|--------------------------|--------------|
| Malta. | |
| | 10.3125 ins. |
| | 11.167 " |
| | 82.5 " |
| | 4.44 acres. |
| <i>'so as in Sicily.</i> | |

| | |
|------------------|------------|
| Moldavia. | |
| | 8 ins. |
| | 24.86 ins. |
| | 8 ft. |

| | |
|----------------------|-------------|
| Loca Islands. | |
| | 18.333 ins. |

| | |
|-----------------|----------------|
| Morocco. | |
| | 2.81025 ins. |
| | 20.34 ins. |
| | 21 " |
| | 3.08135 galls. |
| | 3.356 " |
| | 1.12 lbs. |

than oil are sold by weight.

| | |
|----------------|-----------|
| Mysore. | |
| | 2.12 ins. |
| | 19.1 " |
| | 38.2 " |
| | 500 lbs. |

| | |
|---------------------------|----------------|
| Netherlands. | |
| | 39.370432 ins. |
| <i>System since 1817.</i> | |

| | |
|----------------|------------|
| Persia. | |
| | 2.375 ins. |
| | 25 " |
| | 37.5 " |

| | |
|----------------------------|-----------------|
| Archin, Schah | 31.55 ins. |
| " Arish..... | 38.27 " |
| Parasang..... | 6076 yds. |
| Chenica..... | 80.26 cub. ins. |
| Artaba..... | 1.809 bush. |
| Miscal..... | 71 grains. |
| Ratel..... | 2.1136 lbs. |
| Batman Mauld..... | 6.49 " |

Liquids are measured by weight.

| | |
|------------------|---------------|
| Poland. | |
| Trewico..... | 14.03 ins. |
| Precikow..... | 17 ins. |
| Pretow..... | 4.7245 yds. |
| Mile, short..... | 6075 yds. |
| Morgen..... | 1.3843 acres. |

| | |
|---------------------------------|---------------|
| Portugal and Mozambique. | |
| Foot..... | 13 ins. |
| Milha..... | 1.2788 miles. |
| Almude..... | 3.7 galls. |
| Fanga..... | 1.488 bush. |
| Alguieri..... | 3.6 " |
| Libra..... | 1.012 lbs. |

Also Decimal System.

| | |
|-------------------|----------------|
| Prussia. | |
| Fuss..... | 12.358 ins. |
| Ruthe..... | 4.1192 yds. |
| Meile..... | 24 000 feet. |
| Quadrat Fuss..... | 1.0603 sq. ft. |
| Morgen..... | .63103 acres. |
| Cube Fuss..... | 1.092 cub. ft. |
| Scheffel..... | 1.5121 bush. |
| Anker..... | 7.559 galls. |
| Pound..... | 7217 grains. |
| Zollfund..... | 1.1023 lbs. |
| Centner..... | 113.43 lbs. |

| | |
|-------------------|---------------|
| Russia. | |
| Verahok..... | 1.75 ins. |
| Foot..... | 12 ins. |
| Arschine..... | 28 " |
| Rhein Fuss..... | 1.03 ft. |
| Sajene..... | 7 ft. |
| Verst..... | 3500 " |
| Mila..... | 5.5574 miles. |
| Dessatina..... | 2.4954 acres. |
| Vedro..... | 2.7049 galls. |
| Tschel-werha..... | 1.4424 " |
| Pajak..... | 1.4426 bush. |
| Tschetwert..... | 5.7704 " |
| Pound..... | 6317 grains. |
| Funt..... | .90285 lbs. |

Decimal System adopted in 1872.

| | |
|---------------|--------------|
| Siam. | |
| K'up..... | 9.75 ins. |
| Covid..... | 18 ins. |
| Ken..... | 39 " |
| Jod..... | .09848 mile. |
| Roengeng..... | 2.462 miles. |

| | |
|-----------------|---------------|
| Silesia. | |
| Fuss..... | 12.358 ins. |
| Ruthe..... | 4.1192 yds. |
| Meile..... | 24 000 feet. |
| Morgen..... | 1.3843 acres. |

Singapore.

| | |
|---------------------|---------|
| Hasta or Cubit..... | 18 ins. |
| Dessa..... | 6 ft. |
| Orlong..... | 80 yds. |

Smyrna.

| | |
|-------------|------------|
| Pic..... | 26.48 ins. |
| Indise..... | 24.648 " |
| Berri..... | 1828 yds. |

Spain, Cuba, Malaga, Manilla, Guatemala, Honduras, and Mexico.

| | |
|------------------------|-----------------|
| Pie..... | 11.128 ins. |
| Vara..... | 33.384 " |
| Milla..... | .865 mile. |
| Legua, 8000 varas..... | 4.2151 miles. |
| Fanegada..... | 1.6374 acres. |
| Vara, cubo..... | 21.531 cub. ft. |
| Cuartilla..... | .888 gall. |
| Arroba, Castile..... | 3.554 galls. |
| Fanega..... | 1.5077 bush. |
| Libra..... | 1.0144 lbs. |
| Tonelada..... | 2026.2 lbs. |

Also Decimal System.

Stettin.

| | |
|----------------------|---------------|
| Fuss..... | 11.12 ins. |
| Foot, Rhineland..... | 12.357 " |
| Elle..... | 25.6 ins. |
| Morgen..... | 1.5729 acres. |

Sumatra.

| | |
|---------------------|--------|
| Jankal or Span..... | 9 ins. |
| Elle..... | 18 " |
| Hailoh..... | 36 " |
| Fathom..... | 6 ft. |
| Tung..... | 4 yds. |

Surat.

| | |
|--------------------|------------|
| Tussoo, cloth..... | 1.161 ins. |
| Guz, "..... | 27.864 " |
| Hath..... | 20.9 " |
| Covid..... | 18.5 " |
| Biggah..... | .51 acre. |

Sweden.

| | |
|-------------|---------------|
| Fot..... | 11.6928 ins. |
| Ref..... | 32.4703 yds. |
| Faden..... | 5.845 ft. |
| League..... | 3.3564 miles. |
| Melle..... | 6.6417 " |

Holland.

Denominations corresponding to the French are as follows:

Length.—Millimetre, Streep; centimetre, Duim; decimetre, Palm; metre, El; decametre, Roede; kilometre, Mijle.

Surface.—Square millimetre, Vierkante Streep; square centimetre, Vierkante Duim; and so on. Hectare, Vierkante Bander.

Cube Measure.—Millistere, Kubieke Streep, and so on.

Capacity.—Centilitre, Vingerhoed; decilitre, Maatje; liquid litre, Kan; dry litre, Kop; decallitre, Schepel; liquid hectolitre, Vat or Ton; dry hectolitre, Mud or Zak; 70 hectolitres = 1 Last = 10.723 quarters.

Weight.—Decigramme, Koekegramme, Wigteje; decagramme, Lood; hectogramme, Kilogramme.

11.—The
retro.

| | |
|---------------|--------------|
| Tunnland..... | 1.2198 acres |
| Anker..... | 8.641 galls. |
| Spann..... | 1.962 bush. |
| Centner..... | 112.05 lbs. |

Also Decimal System.

Switzerland.

| | |
|---------------------|--------------|
| Fuss, Berne..... | 11.52 ins. |
| "..... | 11.54 " |
| Vaud..... | 11.81 " |
| Klafter..... | 5.77 ft. |
| Melle..... | 4.8568 miles |
| Juchart, Berne..... | .85 acre. |
| Maas..... | 2.6412 pinta |
| Elmer..... | 8.918 galls. |
| Malter..... | 4.1268 bush. |
| Pfund..... | 1.1023 lbs. |

Also Decimal System.

Tripoli.

| | |
|-------------------|-----------------|
| Pik, 3 palmi..... | 26.42 ins. |
| Almud..... | 319.4 cub. ins. |
| Killow..... | 2023 " |
| Barile..... | 14.267 galls. |
| Temer..... | .7383 bush. |
| Rottol..... | 7680 grains. |
| Oke..... | 2.8286 lbs. |

Turkey.

| | |
|-----------------|--------------|
| Pic, grent..... | 27.9 ins. |
| " small..... | 27.06 " |
| Berri..... | 1.828 yds. |
| Alma..... | 1.154 galls. |

Also Decimal System.

Württemberg.

| | |
|----------------|-----------------|
| Fuss..... | 11.29 ins. |
| Elle..... | 2.015 ft. |
| Melle..... | 8146.25 yds. |
| Morgen..... | .7793 acre. |
| Cube Fuss..... | .83045 cub. ft. |
| Eimer..... | 64.721 galls. |
| Scheffel..... | 4.878 bush. |
| Pound..... | 7217 grains. |

Zurich.

| | |
|-------------------|--------------|
| Fuss..... | 11.812 ins. |
| Elle..... | 23.625 " |
| Klafter..... | 5.9062 ft. |
| Melle..... | 4.8568 miles |
| Jachart..... | .808 acre. |
| Cube Klafter..... | 144 cub. ft. |

Belgium.

substituted for kilogramme, Litron for litre,

SCRIPTURE AND ANCIENT LINEAR MEASURES.

Scripture.

| | |
|--|-------------------------------|
|912 inch. | Span, 3 palms..... 10.944 ina |
| 40 digits..... 3.648 ina | Cubit, 2 spans..... 21.888 " |
| Fathom, 4 cubits..... 7 feet 3.552 ina | |

Hebrew and Egyptian.

| | |
|-------------------------------------|---------------------------------|
| 1 cubit..... 1.475 feet | Babylonian foot..... 1.140 feet |
| "..... 1.721 6 " | Hebrew "..... 1.212 " |
| ian finger..... .661 45 " | " cubit..... 1.817 " |
| Hebrew sacred cubit..... 2.002 feet | |

Grecian.

| | | |
|--------------------------------|---|-------------|
|7554 inch. | Ancient Greek foot } (16 Egyptian fingers) } | .9841 foot. |
| foot)..... 1.0073 feet | | |
| 1.1332 " | Arabian foot..... | 1.095 feet. |
| or natural foot..... .814 foot | Stadium..... | 604.0375 " |
| or Olympic "..... 1.009 foot | Olympic stadium..... | 606.29 " |

Mile, 8 stadium..... 4835 feet.

Alexandrian or Phileterian stadium (600 Phil. feet) = 708.65 feet.

Volume.—Keramon or Metretes..... 8.488 gallons.

Jewish.

| | |
|-----------------------------|-----------------------------------|
| 1.824 feet | Mile, 4000 cubits..... 7296 feet. |
| h day's journey.... 3648. " | Day's journey..... 33.164 miles |

Roman Long Measures.

| | |
|--------------------|------------------------------|
|725 75 ina. | Cubit..... 1.4505 feet. |
| inch)..... .967 " | Passus..... 4.835 " |
| st)..... 11.604 " | Mile, milliarium..... 4842 " |

ANCIENT WEIGHTS.

Hebrew and Egyptian.

| | Troy grains. | | Troy grains. |
|------------------------|--------------|----------------------|----------------------|
| olus..... | { 8.2* | Denarius, Roman..... | { 51.9* |
| | { 9.1† | " Nero..... | { 62.5† |
| achma..... | { 51.9* | Shekel..... | { 92.62 |
| | { 54.6† | Ounce..... | { 415.1* |
| mina..... | { 69† | | { 437.2† |
| mina..... | 3.892 | Drachm..... | 431.2† |
| mina..... | 5.46 | Libra..... | 146.5 |
| 1 mina..... | 8.326* | Libra..... | 4086.1 |
| le "..... | 8.085* | Pound..... | 12 Roman ounces. |
| rian "..... | 9.992* | Talub..... | 581.71 ounces. |
| | 4.63 | | |
| Talent (60 minae)..... | | | 56 lbs. avoirdupois. |

Grecian.

| | Troy grains. | | Troy ounces. |
|--------------|--------------|--------------|--------------|
| ancient..... | 8.33 | Mina..... | 10.41 |
| | 11.57 | " great..... | 14.472 |
| | 23.15 | Talent..... | 625.19 |
| | 50.01 | " Attic..... | 868.32 |
| great..... | 69.47 | | |

Roman.

| | |
|----------------------|------------|
| 416.82 grains. | Pound..... |
|----------------------|------------|

* Christian.

† Arbuthnot.

E*

‡ P†

GEOGRAPHIC MEASURES AND DISTANCES.

To Reduce Longitude into Time.

RULE.—Multiply degrees, minutes, and seconds by 4, and product is the time.

EXAMPLE.—Required time corresponding to $50^{\circ} 31'$. $50^{\circ} 31' \times 4 = 3^h. 22^m. 48^s$.

To Reduce Time into Longitude.

RULE.—Reduce hours to minutes and seconds, divide by 4, and quotient is the longitude. Or, Multiply them by 15.

EXAMPLE.—Required longitude corresponding to $5^h. 8^m. 11.28^s$.

$5^h. 8^m. 11.28^s = 308^m. 11.28^s$, which $\div 4 = 77^{\circ} 2' 45.5''$.

Or, multiplying by 15: $5^h. 8^m. 11.28^s \times 15 = 77^{\circ} 2' 45.5''$.

Table of Departures for a Distance run of 1 Mile.

| Course. | Departure. | Course. | Departure. | Course. | Departure. |
|-------------|------------|-------------|------------|-------------|------------|
| 3.5 points. | .773 | 4.5 points. | .634 | 5.5 points. | .471 |
| 4 " | .707 | 5 " | .556 | 6 " | .383 |

Thus, if a vessel holds a course of 4 points, that is without leeway, for distance of 1 mile, she will make .707 of a mile to windward.

Or, a vessel sailing E. N. E. upon a course of 6 points for 100 miles will make 38.3 ($100 \times .383$) miles of longitude.

Degrees, Minutes, and Seconds of each Point of the Compass with Meridian.

| NORTH. | SOUTH. | Points. | O' " | Sin. A.* | Cos. A.* | Tan. A.* |
|-----------------|-----------------|---------|----------|----------|----------|----------|
| N..... | S..... | .25 | 2 48 45 | .0489 | .9988 | .0491 |
| | | .5 | 5 37 30 | .098 | .9952 | .0985 |
| | | .75 | 8 26 15 | .1467 | .9891 | .1484 |
| | | 1 | 11 15 | .195 | .9808 | .1989 |
| N. by E..... | S. by E. | 1.25 | 14 3 45 | .2429 | .97 | .2504 |
| N. by W..... | S. by W..... | 1.5 | 16 52 30 | .2903 | .9569 | .3034 |
| | | 1.75 | 19 41 15 | .3368 | .9415 | .3578 |
| | | 2 | 22 30 | .3827 | .9239 | .4142 |
| N. N. E..... | S. S. E..... | 2.25 | 25 18 45 | .4275 | .904 | .4729 |
| N. N. W..... | S. S. W..... | 2.5 | 27 7 30 | .4714 | .8819 | .5345 |
| | | 2.75 | 30 56 15 | .5141 | .8577 | .5994 |
| | | 3 | 33 45 | .5556 | .8315 | .6682 |
| N. E. by N. ... | S. E. by S. ... | 3.25 | 36 33 45 | .5957 | .8032 | .7416 |
| N. W. by N. ... | S. W. by S. ... | 3.5 | 39 22 30 | .6344 | .773 | .8207 |
| | | 3.75 | 42 11 15 | .6715 | .7409 | .9063 |
| | | 4 | 45 | .7071 | .7071 | 1 |
| N. E. | S. E. | 4.25 | 47 48 45 | .7404 | .6715 | 1.103 |
| N. W. | S. W. | 4.5 | 50 37 30 | .773 | .6344 | 1.218 |
| | | 4.75 | 53 26 15 | .8032 | .5957 | 1.348 |
| | | 5 | 56 15 | .8315 | .5556 | 1.497 |
| N. E. by E. ... | S. E. by E. ... | 5.25 | 59 3 45 | .8577 | .5141 | 1.668 |
| N. W. by W. ... | S. W. by W. ... | 5.5 | 61 52 30 | .8819 | .4714 | 1.871 |
| | | 5.75 | 64 41 15 | .904 | .4275 | 2.114 |
| | | 6 | 67 30 | .9239 | .3827 | 2.414 |
| E. N. E. | E. S. E. | 6.25 | 70 18 45 | .9415 | .3368 | 2.795 |
| W. N. W. | W. S. W. | 6.5 | 73 7 30 | .9569 | .2903 | 3.296 |
| | | 6.75 | 75 56 15 | .97 | .2429 | 3.941 |
| | | 7 | 78 45 | .9808 | .195 | 5.027 |
| E. by N. | E. by S. | 7.25 | 81 33 45 | .9891 | .1467 | 6.741 |
| W. by N. | W. by S. | 7.5 | 84 22 30 | .9952 | .098 | 10.153 |
| | | | 87 11 15 | .9988 | .0489 | 20.555 |
| | | | | 1 | .0000 | ∞ |

Or Wind...

S. by E...

... from the meridian.

GEOGRAPHIC LEVELLING.

Curvature and Refraction.

Correction for Curvature of Earth, to be subtracted from reading of levelling-staff, is determined as follows:

vide square of distance in feet from level to staff, by Earth's Equatorial diameter—viz., 41 852 124 feet.

Two thirds of square of distance in statute miles equal the curvature in feet.

Correction for Refraction is to be subtracted from reading, and as a mean be taken at about one sixth of that for curvature.

Correction for Curvature and Refraction combined, is to be added to reading on staff.

Formulas of Capt. T. J. Lee, U. S. Engineers.

$\frac{D^2}{2R}$ = correction for curvature, $\frac{D^2}{R} m$ = correction for refraction, and

$2m \frac{D^2}{2R}$ = correction for curvature and refraction. D representing distance, R radius of earth, and m a coefficient of refraction = .075, all in feet.

EXAMPLE.—A distance is 3 statute miles, what is correction for curvature and refraction?

$$(1 - 2 \times .075) \frac{5280 \times 3^2}{41\,852\,124} = .85 \times 5.996 = 5.097 \text{ feet.}$$

approximately, $\frac{2}{3} D^2$ = curvature in feet.

Levelling by Boiling Point of Water.

Compute Height Above or Below Level of Sea.

$$517 (212^\circ - T) + (212^\circ - T)^2 = \text{Height.}$$

EXAMPLE.—What is height of an elevation, when boiling point of water is 182° ?

$$517 \times \overline{212^\circ - 182^\circ} + \overline{212^\circ - 182^\circ}^2 = 517 \times 30 + 30^2 = 16\,410 \text{ feet.}$$

Corrections for Temperature to be made in Connection with Formula.

| sero- tion. | Temp. | Correc- tion. | Temp. | Correc- tion. | Temp. | Correc- tion. | Temp. | Correc- tion. | Temp. | Correc- tion. |
|----------------|-------|------------------|-------|------------------|-------|------------------|-------|------------------|-------|------------------|
| | ° | | ° | | ° | | ° | | ° | |
| 936 | 18 | .972 | 36 | 1.008 | 54 | 1.046 | 72 | 1.083 | 90 | 1.12 |
| 94 | 20 | .976 | 38 | 1.012 | 56 | 1.05 | 74 | 1.087 | 92 | 1.124 |
| 944 | 22 | .98 | 40 | 1.016 | 58 | 1.054 | 76 | 1.091 | 94 | 1.128 |
| 948 | 24 | .984 | 42 | 1.02 | 60 | 1.058 | 78 | 1.096 | 96 | 1.132 |
| 952 | 26 | .988 | 44 | 1.024 | 62 | 1.062 | 80 | 1.1 | 98 | 1.136 |
| 956 | 28 | .992 | 46 | 1.028 | 64 | 1.066 | 82 | 1.104 | 100 | 1.14 |
| 96 | 30 | .996 | 48 | 1.032 | 66 | 1.071 | 84 | 1.108 | 102 | 1.144 |
| 964 | 32 | 1 | 50 | 1.036 | 68 | 1.075 | 86 | 1.11 | | 1.148 |
| 968 | 34 | 1.004 | 52 | 1.041 | 70 | 1.079 | 88 | 1.114 | | |

EXAMPLE.—Assume temperature in preceding illustration

Then $16\,410 \times 1.1 = 18\,051 \text{ feet.}$

Apparent Level of Objects at or upon Surface of Land or Sea, and Differences between True and Apparent Levels, Curvatures, etc.
Distances in Geographic or Nautical Miles.

| HEIGHT of Curvature above Land. | | | | HEIGHT of Line of Sight. | | | | HEIGHT of Curvature above Land. | | | | HEIGHT of Curvature and Refraction. | | | | Apparent Level including Curvature and Refraction. | | | |
|---|-------|------|--------|-----------------------------------|-------|------|--------|---|-------|------|--------|--|-------|------|--------|--|-------|------|--------|
| Feet. | Land. | Sea. | Miles. | Feet. | Land. | Sea. | Miles. | Feet. | Land. | Sea. | Miles. | Feet. | Land. | Sea. | Miles. | Feet. | Land. | Sea. | Miles. |
| 1 | .667 | | | | | | | | | | | | | | | | | | |
| 2 | 1.08 | | | | | | | | | | | | | | | | | | |
| 3 | 1.69 | | | | | | | | | | | | | | | | | | |
| 4 | 2.31 | | | | | | | | | | | | | | | | | | |
| 5 | 2.98 | | | | | | | | | | | | | | | | | | |
| 6 | 3.67 | | | | | | | | | | | | | | | | | | |
| 7 | 4.37 | | | | | | | | | | | | | | | | | | |
| 8 | 5.06 | | | | | | | | | | | | | | | | | | |
| 9 | 5.76 | | | | | | | | | | | | | | | | | | |
| 10 | 6.45 | | | | | | | | | | | | | | | | | | |
| 11 | 7.15 | | | | | | | | | | | | | | | | | | |
| 12 | 7.84 | | | | | | | | | | | | | | | | | | |
| 13 | 8.54 | | | | | | | | | | | | | | | | | | |
| 14 | 9.23 | | | | | | | | | | | | | | | | | | |
| 15 | 9.93 | | | | | | | | | | | | | | | | | | |
| 16 | 10.62 | | | | | | | | | | | | | | | | | | |
| 17 | 11.32 | | | | | | | | | | | | | | | | | | |
| 18 | 12.01 | | | | | | | | | | | | | | | | | | |
| 19 | 12.71 | | | | | | | | | | | | | | | | | | |
| 20 | 13.40 | | | | | | | | | | | | | | | | | | |
| 21 | 14.10 | | | | | | | | | | | | | | | | | | |
| 22 | 14.79 | | | | | | | | | | | | | | | | | | |
| 23 | 15.49 | | | | | | | | | | | | | | | | | | |
| 24 | 16.18 | | | | | | | | | | | | | | | | | | |
| 25 | 16.88 | | | | | | | | | | | | | | | | | | |
| 26 | 17.57 | | | | | | | | | | | | | | | | | | |
| 27 | 18.27 | | | | | | | | | | | | | | | | | | |
| 28 | 18.96 | | | | | | | | | | | | | | | | | | |
| 29 | 19.66 | | | | | | | | | | | | | | | | | | |
| 30 | 20.35 | | | | | | | | | | | | | | | | | | |

NOTE 1.—Height or elevation in second column of table is also curvature of Earth at Ocean.
 2. D. Refraction is variable, and more variable at sunrise and sunset, and consequently, difference between true and apparent level of objects at sea, and differences between true and apparent levels, curvatures, etc.

STRATION.—Curvature of Earth independent of refraction is computed at 8.004 ins. for 1 geographical mile, and as refraction on land is taken as .04 or 1.248 ins., and on ocean at .099 foot or 1.188 ins., relative visible distance of an object, including curvature and refraction, for an elevation of

| | | | |
|-----------|------|-------------------------|--------------------|
| .667 foot | is | 1.09 miles on land, and | 1.08 miles at sea. |
| 1 | " | 1.33 | " " " 1.32 |
| 9 | feet | " " " 4 | " " " 3.98 |
| 1 | mile | 104.03 | " " " 103.54 |

ference between two levels in feet is as square of their distance in

STRATION.—At what elevation can an object be seen, at surface of ocean, when miles distant?

$$1^2 : 2^2 :: .568 : 2.272 \text{ feet} = 2 \text{ feet } 3.25 + \text{ ins.}$$

ference between two distances in miles is as square root of their heights

STRATION I.—At an elevation of 9 feet above level of sea, at what distance object be seen upon its surface?

$$\sqrt{.568} = .754 : 1 :: \sqrt{9} : 3.98 \text{ miles.}$$

If a man at the fore-topgallant mast-head of a vessel, 100 feet from water, sees a large vessel "hull to," how far are the vessels apart?

ge vessel's bulwarks are at least 20 feet from water.

100, by table, 100 feet..... = 13.27

20 " = 5.93

Distance..... 19.20 miles.

When an observation for distance is taken from an elevation, as from a house, a vessel's mast, etc., of an object that intervenes between observer and horizon, or contrariwise, observer being at a horizon to an object, distance of observer from intervening object can be ascertained by finding its elevation from horizon, and adding its distance from whole distance between observer and from which observation is taken, and remainder will give distance of object from observer.

STRATION.—Top of smoke-pipe of a steamer, assumed to be 50 feet above surface of water, is in range with horizon from an elevation of 100 feet; what is distance of steamer from elevation?

100 feet..... = 13.27

50 " = 9.38

Distance..... 3.89 miles.

STRATION.—Curvature less Refraction = .566 D² for land and .563 D² for sea. Distance in miles.

MAGNETIC VARIATION OF NEEDLE.

STRATION.—Needle reached a Westerly maximum in 1660, and then returned to East until 1800, when it reversed to West.

STRATION (Eng.).—From 1576 to 1815 variation ranged from 11° 15' 24° 27' West, when it receded gradually to 21° in 1865.

STRATION (W. I.).—No variation from year 1660.

STRATION.—There is a small diurnal variation, being greatest in summer (15'), and least in winter (7' 30"), added to which a variation affects a needle.

Variation in U. S.—Professor Loomis concludes that the Western variation is increasing and Easterly diminishing in every part of United States; that this change occurred between 1793 and 1819, and the present annual change is about 2' in Southern and Western States, from 3' to 4' in Middle States, and 5' to 7' in Eastern States.

Rules for computation of variation are empirical, except in each particular locality, as the annual and diurnal variations of the needle added to local attraction, render it altogether unreliable.

Decennial Variation of Needle.

Mr. Schott, U. S. Coast and Geodetic Survey.

From January 1, 1790, to January 1, 1880.

| LOCATION. | 1790. | 1800. | 1810. | 1820. | 1830. | 1840. | 1850. | 1860. | 1870. | 1880. |
|-----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | W. ° | W. ° | W. ° | W. ° | W. ° | W. ° | W. ° | W. ° | W. ° | W. ° |
| Halifax, N. S. | 15.1 | 15.9 | 16.7 | 17.4 | 18.1 | 18.7 | 19.3 | 19.8 | 20.1 | 20.5 |
| Quebec, Can. | — | — | 11.2 | 12.3 | 13.4 | 14.4 | 15.3 | 16 | 16.4 | — |
| Portland, Me. | 8.5 | 8.9 | 9.4 | 10 | 10.6 | 11.23 | 11.82 | 12.35 | 12.8 | 13.3 |
| Burlington, Vt. | 7.7 | 7.52 | 7.39 | 7.58 | 8.17 | 8.94 | 9.62 | 10.21 | 10.97 | 11.6 |
| Newburyport, M's. | 7.2 | 7.4 | 7.8 | 8.4 | 9 | 9.6 | 10.23 | 10.83 | 11.4 | 11.9 |
| Portsmouth, N. H. | 7.8 | 8 | 8.4 | 8.8 | 9.35 | 9.94 | 10.55 | 11.15 | 11.7 | 12.2 |
| Rutland, Vt. | 6.5 | 6.2 | 6.14 | 6.39 | 6.9 | 7.64 | 8.53 | 9.53 | 10.54 | 11.5 |
| Salem, Mass. | 6.2 | 6.2 | 6.5 | 7 | 7.8 | 8.7 | 9.8 | 10.9 | 11.9 | 12.1 |
| Boston, Mass. | 6.7 | 7 | 7.4 | 7.9 | 8.43 | 9.05 | 9.69 | 10.32 | 10.9 | 11.5 |
| Cambridge, Mass. . | 6.9 | 7.1 | 7.5 | 8 | 8.64 | 9.33 | 10.03 | 10.67 | 11.21 | 11.8 |
| Providence, R. I. . | 6.24 | 6.37 | 6.45 | 6.73 | 7.43 | 8.31 | 9.09 | 9.65 | 10.21 | 10.8 |
| Hartford, Conn. . | 5.2 | 5.16 | 5.24 | 5.46 | 5.8 | 6.24 | 6.77 | 7.36 | 7.99 | 8.6 |
| New Haven, Conn. . | 4.8 | 4.7 | 4.8 | 5 | 5.43 | 5.99 | 6.67 | 7.41 | 8.18 | 8.9 |
| New York, N. Y. . | 4.29 | 4.28 | 4.3 | 4.47 | 4.91 | 5.59 | 6.34 | 6.96 | 7.43 | 7.9 |
| Philadelphia, Pa. . | 2.4 | 2.1 | 2.1 | 2.28 | 2.71 | 3.33 | 4.11 | 4.99 | 5.89 | 6.8 |
| Baltimore, Md. | — | — | .6 | .8 | 1.2 | 1.7 | 2.4 | 2.9 | — | — |
| Albany, N. Y. | — | — | 5.4 | 5.79 | 6.32 | 6.97 | 7.7 | 8.47 | 9.2 | 9.9 |
| | | E. | | | | | | | | |
| Buffalo, N. Y. | .14 | .01 | .05 | .3 | .74 | 1.33 | 2.05 | 2.85 | 3.68 | 4.5 |
| | E. | E. | E. | E. | E. | | | | | |
| Erie, Pa. | .03 | .35 | .49 | .43 | .17 | .25 | .83 | 1.5 | 2.23 | 3 |
| | | | | | | E. | E. | | | |
| Cleveland, O. | 2.2 | 2 | 1.8 | 1.5 | 1.05 | .6 | .14 | .31 | .72 | 1.1 |
| | | | | | | | | E. | E. | |
| Detroit, Mich. | — | 3.18 | 3.11 | 2.9 | 2.55 | 2.09 | 1.56 | .99 | .41 | — |
| | | W. | W. | W. | W. | W. | W. | W. | W. | |
| Washington, D. C. | .1 | — | .3 | .6 | 1 | 1.49 | 1.99 | 2.47 | 2.9 | 3.4 |
| | | E. | E. | E. | E. | E. | E. | E. | E. | |
| Acapulco, Mex. | 7.2 | 7.8 | 8.3 | 8.68 | 8.88 | 8.91 | 8.79 | 8.5 | 8.06 | 7.7 |
| Charleston, S. C. . | 5.1 | 4.9 | 4.5 | 4.04 | 3.44 | 2.78 | 2.12 | 1.52 | 1 | — |
| Havana, Cuba. | — | 6.2 | 6.26 | 6.22 | 6.12 | 5.94 | 5.71 | 5.44 | 5.1 | — |
| Kingston, W. I. . | 6.3 | 6 | 5.7 | 5.4 | 5 | 4.6 | 4.2 | 3.8 | 3.4 | — |
| San Diego, Cal. | 11 | 11.1 | 11.3 | 11.6 | 11.9 | 12.2 | 12.54 | 12.88 | 13.2 | 13.5 |
| Savannah, Ga. | — | — | 4.9 | 4.8 | 4.5 | 4.14 | 3.65 | 3.08 | 2.48 | 1.9 |
| Mobile, Ala. | — | 7.1 | 7.2 | 7.3 | 7.2 | 7.1 | 7 | 6.8 | — | 6.5 |
| Key West, Fla. | — | — | — | 6.9 | 6.52 | 6.03 | 5.47 | 4.86 | 4.24 | 3.6 |
| Monterey, Cal. | 11.4 | 12 | 12.6 | 13.3 | 13.9 | 14.44 | 14.95 | 15.42 | 15.79 | 16.1 |
| Mexico, Mex. | 7.1 | 7.7 | 8.3 | 8.6 | 8.8 | 8.9 | 8.76 | 8.48 | 8.04 | 7.6 |
| New Orleans, La. . | 7 | 7.5 | 7.9 | 8.1 | 8.2 | 8.14 | 7.94 | 7.61 | 7.15 | 6.7 |
| San Blas, Mex. | 7.41 | 7.88 | 8.28 | 8.61 | 8.84 | 8.97 | 9.9 | 8.91 | — | — |
| San Francisco, Cal. . | 12.8 | 13.4 | 13.9 | 14.42 | 14.92 | 15.38 | 15.78 | 16.11 | 16.36 | 16.6 |
| Sitka, Alaska. | — | 26.12 | 27.11 | 27.89 | 28.48 | 28.88 | 29.08 | 29.08 | 28.88 | 28.6 |
| Vera Cruz, Mex. | 8.37 | 8.95 | 9.32 | 9.48 | 9.42 | 9.14 | 8.66 | 7.98 | 7.15 | — |

For variation in other locations in United States and North America see Treatises of J. B. Stone, C.E., New York, and Heller and Briggs Philadelphia, 1878.

for Reducing Observed Daily Variation of Needle
to Mean Variation of the Day.

U. S. Coast and Geodetic Survey, 1878.

| No. | Needle East of Mean Magnetic Meridian. | | | | | Needle West of Mean Magnetic Meridian. | | | | | | | |
|-------|--|------|------|------|------|--|-------|------|------|------|------|------|------|
| | A.M. | A.M. | A.M. | A.M. | A.M. | A.M. | NOON. | P.M. | P.M. | P.M. | P.M. | P.M. | P.M. |
| | h. | h. | h. | h. | h. | h. | Noon. | h. | h. | h. | h. | h. | h. |
| | 6 | 7 | 8 | 9 | 10 | 11 | 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| | 3 | 4 | 4 | 3 | 1 | 1 | 4 | 5 | 5 | 4 | 3 | 2 | 1 |
| | 4 | 5 | 5 | 4 | 1 | 2 | 4 | 5 | 5 | 4 | 3 | 2 | 1 |
| | 2 | 3 | 3 | 2 | — | 2 | 3 | 4 | 3 | 2 | 1 | — | — |
| | 1 | 1 | 2 | 2 | 1 | — | 2 | 3 | 3 | 2 | 1 | — | — |

on of Needle at Locations in United States and
Canada, 1875.

U. S. Coast and Geodetic Survey.

EAST.

| LOCATION. | Variation. | LOCATION. | Variation. |
|-------------------|------------|----------------------------|------------|
| | ° ' " | | ° ' " |
| T. | 21 30 | Montgomery, Ala. | 5 2 |
| la. | 2 28 | Natchez, Miss. | 7 26 |
| E. | 9 15 | Nebraska, Neb. | 11 20 |
| Dak. | 16 6 | New Orleans, La. | 6 50 |
| L. | 5 | Olympia, W. T. | 22 8 |
| O. | 2 55 | Omaha, Neb. | 11 |
| prings, Col. | 14 18 | Oregon City, Or. | 20 55 |
| S. O. | 1 45 | Paducah, Kan. | 6 2 |
| O. | 1 8 | Portland, Or. | 21 4 |
| Dak. | 16 20 | Port Townsend, W. T. | 23 |
| h. | 14 45 | Sacramento, Cal. | 17 4 |
| ich. | 3 | Salt Lake City, Utah. | 17 |
| in. | 10 12 | San Antonio, Tex. | 9 17 |
| Tex. | 8 13 | Santa Barbara, " | 14 58 |
| Wis. | 6 | Santa Fé, N. Mex. | 13 18 |
| ex. | 27 | Springfield, Ill. | 6 3 |
| lis, Ind. | 3 38 | St. Augustine, Fla. | 8 55 |
| iss. | 7 | St. Louis, Mo. | 6 30 |
| lle, Fla. | 3 | St. Paul, Minn. | 10 30 |
| an. | 9 20 | Tallahassee, Fla. | 4 14 |
| a. | 7 55 | Toledo, O. | 1 2 |
| k, Ark. | 8 5 | Topeka, Kan. | 10 12 |
| Ky. | 4 | Vincennes, Ind. | 5 |
| Wis. | 5 48 | Yazoo, Miss. | 7 2 |

WEST.

| | | | |
|-----------------|-------|-------------------------|------|
| Mc. | 14 34 | Newburgh, N. Y. | 8 |
| e. | 16 | Newport, R. I. | 10 4 |
| Y. | 4 40 | Norfolk, Va. | 2 35 |
| e. | 15 22 | Ogdensburgh, N. Y. | 9 25 |
| t, Conn. | 8 12 | Oswego, N. Y. | 6 8 |
| | 18 | Ottawa, Can. | 9 38 |
| N. H. | 11 42 | Pittsburgh, Pa. | 1 28 |
| L. | 4 12 | Raleigh, N. C. | 24 |
| r, Mass. | 10 30 | Richmond, Va. | 1 48 |
| , Can. | 2 55 | Rochester, N. Y. | 5 20 |
| g, Pa. | 4 18 | Saratoga, N. Y. | 9 40 |
| Y. | 8 48 | Stamford, Conn. | 8 |
| Me. | 14 | Syracuse, N. Y. | 7 |
| asa. | 11 15 | Toronto, Can. | 3 50 |
| r, Vt. | 12 5 | Trenton, N. J. | 6 8 |
| Can. | 12 20 | Troy, N. Y. | 9 25 |
| ord, Mass. | 10 30 | Utica, N. Y. | 8 |
| on, Conn. | 9 15 | Wilmington, Del. | 4 52 |
| J. | 7 18 | Wilmington, N. C. | 12 |

Dip of Horizon.

Approximate, $57.4 \sqrt{H} = \text{dip in seconds}$, varying with temperature of air. H representing height of observer's eye in feet.

$.667 n^2 = H$: $.493 s^2 = H$: $1.42 \sqrt{H} = s$: $1.23 \sqrt{H} = n$
 n representing distance in geographical miles and s in statute.

Measurement of Heights with a Sextant.

| Multi-plier. | Angle. | Multi-plier. | Angle. | Multi-plier. | Angle. | Multi-plier. | Angle. | Multi-plier. | Angle. |
|--------------|--------|--------------|--------|--------------|--------|--------------|--------|--------------|--------|
| 1 | 45 0 | 2.5 | 68 11 | 4 | 75 58 | 5.5 | 79 42 | 8 | 82 52 |
| 1.5 | 56 18 | 3 | 71 34 | 4.5 | 77 29 | 6 | 83 32 | 9 | 83 40 |
| 2 | 63 26 | 3.5 | 74 4 | 5 | 78 41 | 7 | 85 52 | 10 | 84 17 |

Operation. — Set sextant to any angle in table, and height will equal distance multiplied by number opposite to it.

ILLUSTRATION. — When sextant is set at $80^\circ 32'$, and horizontal distance from object in a vertical line is 100 feet, what is its height?

$$100 \times 6 = 600 \text{ feet.}$$

By Trigonometry: $1 : 100 :: 5.997$ (tan. angle) : 599.7 feet.

To Reduce a Sounding to Low Water.

$\frac{h}{2} (1 \mp \cos. \frac{180t}{t}) = h'$. h representing vertical rise of tide, and h' sounding or depth at low water, both in feet; t time between high and low water, and t' time from time of sounding to low water, in hours. — \cos , when $\frac{180t}{t} < 90^\circ$, and $+$ \cos . when $> 90^\circ$.

ILLUSTRATION. — Low water occurring at 3.45, and high water at 10.15 P.M., a sounding taken at 5.30 P.M. was 18.25 feet; what was depth at low water, vertical rise being 10 feet?

$$h = 10 \text{ feet; } t' = 5h. 30m. - 3h. 45m. = 1h. 45m. = 1.75 \text{ hours.}$$

$$t = 10h. 15m. - 3h. 45m. = 6h. 30m. = 6.5 \text{ hours.}$$

$$\text{Then } \frac{10}{2} (1 \mp \cos. \frac{180 \times 1.75}{6.5}) = 5 (1 - \cos. 48^\circ 27' 41'') = 5 \times (1 - .663124) = 1.68438 \text{ feet}$$

$$\text{Sounding } 18.25 \text{ feet} - \text{Reduction } 1.68407 \text{ feet} = 16.56593 \text{ feet.}$$

Lengths of a Degree of Longitude on parallels of Latitude, for each of its Degrees from Equator to Pole.

| Lat. | Miles. | Lat. | Miles. | Lat. | Miles. | Lat. | Miles. | Lat. | Miles. | Lat. | Miles. |
|------|--------|------|--------|------|--------|------|--------|------|--------|------|--------|
| 1° | 59.99 | 16° | 57.67 | 31° | 51.43 | 46° | 41.68 | 61° | 29.09 | 76° | 14.52 |
| 2 | 59.96 | 17 | 57.38 | 32 | 50.88 | 47 | 40.92 | 62 | 28.17 | 77 | 13.5 |
| 3 | 59.92 | 18 | 57.06 | 33 | 50.32 | 48 | 40.15 | 63 | 27.74 | 78 | 12.48 |
| 4 | 59.85 | 19 | 56.73 | 34 | 49.74 | 49 | 39.36 | 64 | 26.3 | 79 | 11.45 |
| 5 | 59.77 | 20 | 56.38 | 35 | 49.15 | 50 | 38.57 | 65 | 25.36 | 80 | 10.42 |
| 6 | 59.67 | 21 | 56.01 | 36 | 48.54 | 51 | 37.76 | 66 | 24.4 | 81 | 9.38 |
| 7 | 59.55 | 22 | 55.63 | 37 | 47.92 | 52 | 36.94 | 67 | 23.44 | 82 | 8.35 |
| 8 | 59.42 | 23 | 55.23 | 38 | 47.28 | 53 | 36.11 | 68 | 22.48 | 83 | 7.31 |
| 9 | 59.26 | 24 | 54.81 | 39 | 46.63 | 54 | 35.27 | 69 | 21.5 | 84 | 6.27 |
| 10 | 59.09 | 25 | 54.38 | 40 | 45.96 | 55 | 34.41 | 70 | 20.52 | 85 | 5.23 |
| 11 | 58.89 | 26 | 53.93 | 41 | 45.28 | 56 | 33.45 | 71 | 19.53 | 86 | 4.18 |
| 12 | 58.69 | 27 | 53.46 | 42 | 44.59 | 57 | 32.68 | 72 | 18.54 | 87 | 3.14 |
| 13 | 58.46 | 28 | 52.97 | 43 | 43.88 | 58 | 31.79 | 73 | 17.54 | 88 | 2 |
| 14 | 58.22 | 29 | 52.48 | 44 | 43.16 | 59 | 30.9 | 74 | 16.54 | 89 | 1.05 |
| 15 | 57.95 | 30 | 51.96 | 45 | 42.43 | 60 | 30 | 75 | 15.53 | 90 | .00 |

Note. — Degrees of longitude at latitudes.

as Cosines of their

Elements of Figure of the Earth.

Capt. A. R. Clarke, 1866.

| | Feet. | Miles. |
|---|-----------|------------|
| r semi-axis of Equator (longitude 15° 34' E.) | 20926 350 | 3963.324 |
| r " " " " (" 105° 34' E.) | 20919 972 | 3962.115 |
| r " " " " " " " " " " " " | 20853 429 | 3949.513 |
| torial semi-axis | 20926 062 | 3963.269 |
| ifference, mean | — | 24 898.562 |
| eter, " " | — | 7 916. |

BOARD AND TIMBER MEASURE.

BOARD MEASURE.

Board Measure, all boards are assumed to be 1 inch in thickness.

To Compute Measure or Surface.

When all Dimensions are in Feet.

R.—Multiply length by breadth, and product will give surface in 3 feet.

When either of Dimensions are in Inches.

PLR.—What are number of square feet in a board 15 feet in length and 16 in width?

$$15 \times 16 = 240, \text{ and } 240 \div 12 = 20 \text{ sq. feet.}$$

When all Dimensions are in Inches.

L.—Multiply as before, and divide product by 144.

TIMBER MEASURE.

To Compute Volume of Round Timber.

When all Dimensions are in Feet.

L.—Add together squares of diameters of greater and lesser ends, duct of the two diameters; multiply sum by .7854, and product third of length.

$$+ a' + a'' \times \frac{l}{3} = V, \text{ and } c^2 + c'^2 + c \times c' \times .07958 \times \frac{l}{3} = V. \text{ } a \text{ and } c \text{ representing areas of ends, } a'' \text{ area of mean proportional, } l \text{ length, and } c \text{ circumference of ends.}$$

Mean proportional is square root of product of areas of both ends.

EXAMPLE.—Diameters of a log are 2 and 1.5 feet, and length 15 feet.

$$2^2 + 1.5^2 + 2 \times 1.5 = 9.25, \text{ which } \times .7854 \text{ and } \frac{15}{3} = 36.3245 \text{ cube feet.}$$

Length in Feet, and Areas or Circumferences in Inches.

—Proceed as above, and divide by 144.

When all Dimensions are in Inches.

—Proceed as before, and divide by 1728.

Ordinary rule of Hutton, Ordnance Manual of U. S., and Molesworth, of gives a result of about .25 less than exact volume or what it would be was hewn or sawed to a square. *c* represents circumference.

To Compute Volume of Squared Timber.

When all Dimensions are in Feet.

RULE.—Multiply product of breadth by depth, by length, and product will give volume in cube feet.

When either Dimension is in Inches.

RULE.—Multiply as above, and divide product by 12.

When any two Dimensions are in Inches.

RULE.—Multiply as before, and divide by 144.

EXAMPLE.—A piece of timber is 15 inches square, and 20 feet in length; required its volume in cube feet.

$$\frac{15 \times 15 \times 20}{144} = 31.25 \text{ cube feet.}$$

Allowance is to be made for bark, by deducting from each girth from .5 inch in logs with thin bark, to 2 inches in logs with thick bark.

Measures of Timber.—(*English.*)

100 superficial feet
of planking } = 1 square.

50 cube feet of squared timber } = 1 load.

120 deals = 1 hundred.

40 feet of unhewn timber = 1 load.

600 superficial feet of inch planking = 1 load.

Deals.

Deals.—Boards exceeding 7 ins. in width, and if less than 6 feet in length, are termed deal ends.

Battens are similar to deals, but only 7 inches in width.

Balk.—Roughly squared log or trunk of a tree.

Planks are boards 12 ins. in width.

Local Standards.

| Country. | Long. | Bread. | Thick. | Volume. | Country. | Long. | Bread. | Thick. | Volume. |
|------------|-------|--------|--------|----------|------------|-------|--------|--------|----------|
| | Ft. | Ins. | Ins. | Cub. ft. | | Ft. | Ins. | Ins. | Cub. ft. |
| Russia and | | | | | Norway .. | 12 | 9 | 3 | 2.25 |
| Prussia .. | 12 | 11 | 1.5 | 1.375 | Christiana | 11 | 9 | 1.25 | .859 |
| Sweden ... | 14 | 9 | 3 | 2.625 | Quebec ... | 12 | 11 | 2.5 | 2.29 |

100 Petersburg standard deals equal 60 Quebec deals.

SPARS AND POLES.

Pine and Spruce Spars, from 10 to 4.5 inches in diameter inclusive, are to be measured by taking their diameter, clear of bark, at one third of their length from abut or large end.

Spars are usually purchased by the inch diameter; all under 4 inches are termed *Poles*.

Spars of 7 inches and less should have 5 feet in length for every inch of diameter, and those above 7 inches should have 4 feet in length for every inch of diameter.

Loss or Waste in Hewing or Sawing of Timber.

(C. Mackrow.)

| | | | | | |
|-------------------|-----|-----------|------------------------------|-----|-----------|
| Oak, English..... | 200 | per cent. | Yellow Pine from planks..... | 10 | per cent. |
| " African..... | 100 | " " | Teak..... | 15 | " " |
| " Bantz..... | 50 | " " | Elm, English..... | 200 | " " |
| " Amer..... | " | " " | " American..... | 15 | " " |

CISTERNS.

Capacity of Cisterns in Cube Feet and Gallons.
For each 10 Inches in Depth.

| L. | Cub. ft. | Gallons. | Diam. | Cub. ft. | Gallons. | Diam. | Cub. ft. | Gallons. |
|----|----------|----------|-------|----------|----------|-------|----------|----------|
| | | | | | | | | |
| | | | Feet. | | | Feet. | | |
| | 2.618 | 19.58 | 9.5 | 59.068 | 441.8 | 17 | 189.15 | 1414.94 |
| | 4.091 | 30.6 | 10 | 65.449 | 489.6 | 17.5 | 200.432 | 1499.33 |
| | 5.89 | 44.07 | 10.5 | 72.158 | 539.78 | 18 | 212.056 | 1586.28 |
| | 8.018 | 59.97 | 11 | 79.194 | 592.4 | 19 | 236.274 | 1767.45 |
| | 10.472 | 78.33 | 11.5 | 86.558 | 647.5 | 20 | 261.797 | 1958.3 |
| | 13.254 | 99.14 | 12 | 94.248 | 705 | 21 | 288.632 | 2159.11 |
| | 16.362 | 122.4 | 12.5 | 102.265 | 764.99 | 22 | 316.776 | 2369.64 |
| | 19.798 | 148.1 | 13 | 110.61 | 827.4 | 23 | 346.23 | 2589.97 |
| | 23.562 | 176.24 | 13.5 | 119.282 | 892.29 | 24 | 376.992 | 2820.09 |
| | 27.652 | 206.84 | 14 | 128.281 | 959.6 | 25 | 409.062 | 3059.8 |
| | 32.07 | 239.88 | 14.5 | 137.608 | 1029.38 | 26 | 442.44 | 3309.67 |
| | 36.816 | 275.4 | 15 | 147.262 | 1101.6 | 27 | 477.13 | 3569.17 |
| | 41.888 | 313.33 | 15.5 | 157.243 | 1176.26 | 28 | 513.126 | 3838.44 |
| | 47.288 | 353.72 | 16 | 167.552 | 1253.37 | 29 | 550.432 | 4117.51 |
| | 53.014 | 396.55 | 16.5 | 178.187 | 1332.93 | 30 | 589.048 | 4406.08 |

Excavation and Lining of Wells or Cisterns.
For each 10 Inches in Depth.

| Excavation. | Bricks. | | Masonry. | | Diameter. | Excavation. | Bricks. | | Masonry. | |
|-------------|----------|-----------|-----------------|---------------|-----------|-------------|----------|-----------|-----------------|---------------|
| | Num-ber. | Laid dry. | 8 inches thick. | 1 foot thick. | | | Num-ber. | Laid dry. | 8 inches thick. | 1 foot thick. |
| Cub. ft. | | Cub. ft. | Cub. ft. | Cub. ft. | Feet. | Cub. ft. | Cub. ft. | Cub. ft. | Cub. ft. | Cub. ft. |
| 12.29 | 126 | 5.24 | 6.4 | 10.47 | 8.5 | 356 | 14.83 | 16 | 24.87 | |
| 15.29 | 147 | 6.11 | 7.27 | 11.78 | 9 | 377 | 15.71 | 16.87 | 26.18 | |
| 18.62 | 168 | 6.98 | 8.14 | 13.09 | 9.5 | 398 | 16.58 | 17.75 | 27.49 | |
| 22.27 | 188 | 7.85 | 9.02 | 14.4 | 10 | 419 | 17.45 | 18.62 | 28.8 | |
| 26.25 | 209 | 8.73 | 9.89 | 15.71 | 10.5 | 440 | 18.33 | 19.49 | 30.11 | |
| 30.50 | 230 | 9.6 | 10.76 | 17.02 | 11 | 461 | 19.2 | 20.36 | 31.42 | |
| 35.2 | 251 | 10.47 | 11.64 | 18.33 | 12 | 482 | 20.04 | 22.11 | 34.03 | |
| 40.16 | 272 | 11.34 | 12.51 | 19.63 | 13 | 503 | 20.92 | 23.85 | 36.65 | |
| 45.45 | 293 | 12.22 | 13.38 | 20.94 | 14 | 524 | 21.8 | 25.6 | 39.27 | |
| 51.07 | 314 | 13.09 | 14.25 | 22.25 | 15 | 545 | 22.69 | 27.34 | 41.89 | |
| 57.02 | 335 | 13.96 | 15.13 | 23.56 | 16 | 566 | 23.58 | 29.09 | 44.51 | |

ber of bricks and width of curb are taken at dimensions of ordinary viz., 8 by 4 by 2.25 ins. = 72 cube ins. Computing number of bricks required, an addition of 5 per cent. should be made for waste. It is to be considered, also, that diameter of excavation slightly exceeds that of masonry.

SHINGLES.

lly of white Cedar and Cypress; 27 inches in length and 6 to 7 in width, dressed to light .25 inch at point and .3125 inch at

in three thicknesses and courses of about 8 inches.
of a shingle is exposed to air, or about 2.25 sh
er square foot of roof.

les, alike to Slates, are laid upon boards or batter

less
re-

SLATES AND SLATING.

A *Square* of Slate or Slating is 100 superficial feet.

Gauge is distance between the courses of the slates.

Lap is distance which each slate overlaps the slate lengthwise next but one below it, and it varies from 2 to 4 inches. Standard is assumed to be 3 inches.

Margin is width of course exposed or distance between tails of the slates.

Pitch of a slate roof should not be less than 1 in height to 4 of length.

To Compute Surface of a Slate when laid, and Number of Squares of Slating.

RULE.—Subtract lap from length* of slate, and half remainder will give length of surface exposed, which, when multiplied by width of slate, will give surface required.

Divide 14 400 (area of a square in inches) by surface thus obtained, and quotient will give number of slates required for a square.

EXAMPLE.—A slate is 24 × 12 inches, and lap is 3 inches; what will be number required for a square?

24 — 3 = 21, and 21 ÷ 2 = 10.5, which × 12 = 126 inches; and 14 400 ÷ 126 = 114.29 slates.

Dimensions of Slates.

[AMERICAN.]

| Inch. | Inch. | Inch. | Inch. | Inch. | Inch. | Inch. |
|--------|---------|---------|---------|---------|---------|---------|
| 14 × 7 | 14 × 10 | 16 × 10 | 18 × 11 | 20 × 11 | 22 × 12 | 24 × 13 |
| 14 × 8 | 16 × 8 | 18 × 9 | 18 × 12 | 20 × 12 | 22 × 13 | 24 × 14 |
| 14 × 9 | 16 × 9 | 18 × 10 | 20 × 10 | 22 × 11 | 24 × 12 | 24 × 16 |

ENGLISH.

| | Inch. | | Inch. | | Inch. |
|------------------|---------|----------------|---------|----------------|---------|
| Doubles | 13 × 10 | Ladies | 12 × 8 | Marchioness .. | 22 × 22 |
| " | 13 × 7 | | 14 × 8 | Duchess | 24 × 12 |
| Small doubles .. | 11 × 6 | | 14 × 12 | Imperial | 30 × 24 |
| " | 10 × 5 | | 15 × 8 | Rags | 36 × 24 |
| Plantations .. { | 12 × 10 | Countess | 16 × 8 | Queens | 36 × 24 |
| " | 13 × 10 | | 16 × 10 | Empress | 26 × 15 |
| Viscountess ... | 18 × 10 | | 20 × 10 | Princess | 24 × 14 |

Thickness of slates ranges from .125 to .3125 of an inch, and their weight varies from 2 to 4.53 lbs. per sq. foot.

Weight of One Square Foot of Slating.

| | | | |
|-------------------------------|-----------|-------------------------------|-----------|
| .125 in. thick on laths..... | 4.75 lbs. | .25 in. thick on laths..... | 9.25 lbs. |
| " " " " 1 in. boards.. | 6.75 " | " " " " 1 in. boards.. | 11.25 " |
| .1875 in. thick on laths..... | 7 " | .3125 in. thick on laths..... | 11.15 " |
| " " " " 1 in. boards.. | 9 " | " " " " 1 in. boards.. | 14.10 " |

Slate weighs from 167 to 181 lbs. per cube foot, and in consequence of laps, it requires an average of nearly 2.5 square feet of slate to make one of slating.

Weights per 1000 and Number Required to Cover a Square.

| | No. | | Lbs. | No. |
|---------------|-----|--------------|---------|------|
| Doubles | 12 | Countess ... | 20 × 10 | 6720 |
| Ladies | 1 | Duchess ... | 24 × 12 | 4480 |
| | | | | 171 |
| | | | | 125 |

a nail-hole to tail.

PILING OF SHOT AND SHELLS.

To Compute Number of Shot.

Triangular Pile. RULE.—Multiply continually together, number of shot in one side of bottom course, and that number increased by 1, and again by 2, and one sixth of product will give number.

EXAMPLE.—What is number of shot in a triangular pile, each side of base containing 30 shot?

$$\frac{30 \times 30 + 1 \times 30 + 2}{6} = \frac{2970}{6} = 4950 \text{ shot.}$$

Square Pile. RULE.—Multiply continually together, number in one side of bottom course, and that number increased by 1, double same number increased by 1, and one sixth of product will give number.

EXAMPLE.—How many shells are there in a square pile of 30 courses?

$$\frac{30 \times 30 + 1 \times 30 \times 2 + 1}{6} = \frac{5670}{6} = 945 \text{ shells.}$$

Oblong Pile. RULE.—From 3 times number in length of base course subtract one less than number in breadth of it; multiply remainder by number in breadth, and again by breadth, increased by 1, and one sixth of product will give number.

EXAMPLE.—Required number of shells in an oblong pile, numbers in base course being 16 and 7?

$$\frac{16 \times 3 - 7 - 1 \times 7 \times 7 + 1}{6} = \frac{2352}{6} = 392 \text{ shells.}$$

Incomplete Pile. RULE.—From number in pile, considered as complete, subtract number conceived to be in that portion of pile which is wanting, and remainder will give number.

FRAUDULENT BALANCES.

To Detect Them.—After an equilibrium has been established between weight and article weighed, transpose them, and weight will preponderate if article weighed is lighter than weight, and contrariwise if it is heavier.

To Ascertain True Weight. RULE.—Ascertain weight which will produce equilibrium after article to be weighed and weight have been transposed; reduce these weights to same denomination, multiply them together, and square root of their product will give true weight.

EXAMPLE.—If first weight is 32 lbs., and second, or weight of equilibrium after transposition, is 24 lbs. 8 oz., what is true weight?

$$24 \text{ lbs. } 8 \text{ oz.} = 24.5 \text{ lbs.}$$

Then $32 \times 24.5 = 784$, and $\sqrt{784} = 28$ lbs.

Or, when a represents longest arm, b shortest arm, A greatest weight, and B least weight.

Then $Wa = Ab$, and $Wb = Ba$; multiplying these two equations, $W^2 ab = ABab$, or $W^2 = AB$, and $W = \sqrt{AB}$.

ILLUSTRATION.— $A = 32$; $B = 24.5$; $W = 28$. Assume length of longest arm = 10.

$$\text{Then } 32 : 28 :: 10 : 8.75.$$

Hence, $a = 10$, $b = 8.75$, or $28^2 = 32 \times 24.5$, and $\sqrt{32 \times 24.5} = 28$.

Weighing without Scales.

To Ascertain

Weight of a Bar, Beam, etc., by Aid of a known Weight.

OPERATION
it and end of
and move bar
in equilibrio;
longest arm
divided from

ance bar, etc., over a fulcrum, and note distance between
gest arm. Suspend a known weight from longest arm,
pon fulcrum, so that bar with attached weight will be
ct distance between the two positions of fulcrum from
tained; multiply this remainder by weight suspended,
stance between fulcrums, and quotient will give weight.

EXAMPLE.—A 1
when 13 feet from
pended from
crum is 12 fe

of tapered timber 24 feet in length is balanced over a fulcrum
end; but when the body of a man weighing 210 lbs. is sus-
of longest arm, the piece and weight are balanced when ful-
is end. What is weight of the timber?

13—12=

and 13—1 = 12 feet. Then $12 \times 210 \div 1 = 2520$ lbs.

1 pound of paint
6 yards for each

yards for a first coat and about

Proportions

Colors

Paints.—By Weight.

| COLORS. | White Lead. | Lamp-black. | Red Lead. | Red Ochre. |
|-------------|-------------|-------------|-----------|------------|
| White | 100 | — | — | — |
| Black | — | 100 | — | — |
| Green | 25 | — | — | 75 |

| COLORS. | White Lead. | Lamp-black. | Red Lead. | Red Ochre. | Vermilion. | Spanish Brown. |
|---------|-------------|-------------|-----------|------------|------------|----------------|
| | 98 | 2 | — | — | — | — |
| | — | — | 50 | 50 | — | — |
| late.. | — | 4 | — | — | — | 96 |

These are the colors alone, to which boiled seed oil, litharge, Japan varnish, and spirits turpentine are to be added according to the application of the paint.

Lamp-black and litharge are ground separately with oil, then stirred into the lead and oil.

Thus for black paint: Lamp-black 25 parts, litharge 1, Japan varnish 1, boiled linseed oil 72, and spirits turpentine 1.

Tar Paint.—Coal tar 9 gallons, slaked lime 13 lbs., turpentine or naphtha 2 or 3 quarts.

| A GALLON OF PAINT WILL COVER | Superficial feet. | A GALLON OF PAINT WILL COVER | Superficial feet. |
|--------------------------------|-------------------|----------------------------------|-------------------|
| On stone or brick, about | 190 to 225 | On well-painted surface or iron | 600 |
| On composite, etc., from | 300 " 375 | One gallon tar, first coat | 90 |
| On wood, from | 375 " 525 | " " " second coat | 160 |

Boiled Oil.—Raw linseed oil 91 parts, copperas 3, and litharge 6.

Put litharge and copperas in a cloth bag and suspend in middle of a kettle. Boil oil four hours and a half over a slow fire, then let it stand and deposit the sediment.

White Paint.

| | Inside work. | Outside work. | | Inside work. | Outside work. |
|-----------------------|--------------|---------------|---------------------|--------------|---------------|
| White lead, in oil .. | 80 | 80 | Raw oil | — | 9 |
| Boiled oil | 14.5 | 9 | Spirits turpentine. | 8 | 4 |

New wood-work requires 1 lb. to square yard for three coats.

Coats for 100 Square Yards New White Pine.

| INSIDE. | White lead. | Raw oil. | Turpen- tine. | Drier. | OUTSIDE. | White lead. | Raw oil. | Boiled oil. | Turpen- tine. |
|---------------|-------------|----------|------------------|--------|-----------------|-------------|----------|-------------|------------------|
| | Lbs. | Pts. | Pts. | Lbs. | | Lbs. | Pts. | Pts. | Lbs. |
| Priming | 16 | — | — | 20 | Priming | 18.5 | 2 | 2 | — |
| 2d coat | 15 | 3.2 | — | — | 2d and 3d coats | 15 | 2 | 2 | .5 |
| 3d " | 13 | 2 | — | — | | | | | |

1 lb. of drier with p

HYDROMETERS.

U. S. Hydrometer (Tralle's) ranges from 0 (water) to 100 (pure spirit); it has not any subdivision or standard termed "Proof," but 50, upon stem of instrument, at a temperature of 60°, is basis upon which computations of duties are made.

In connection with this instrument, a Table of Corrections, for differences in temperature of spirits, becomes necessary; and one is furnished by the Treasury Department, from which all computations of value of a spirit are made.

ILLUSTRATION.—A cask contains 100 gallons of whiskey at 70°, and hydrometer sinks in the spirit to 25 upon its stem.

Then, by table, under 70°, and opposite to 25, is 22.99, showing that there are 22.99 gallons of pure spirit in the 100.

Commercial Hydrometer (Gendar's) has a "Proof" at 60°, which is equal to 50 upon U. S. Instrument and its gradations, run up to 100 with it, and down to 10 below proof, at 0 upon U. S. Instrument; or 0 of the Commercial Instrument is at 50 upon U. S. Instrument, from which it progresses numerically each way, each of its divisions being equal to two of latter.

In testing spirits, Commercial standard of value is fixed at proof; hence any difference, whether higher or lower, is added or subtracted, as case may be, to or from value assigned to proof.

A scale of Corrections for temperature being necessary, one is furnished with a Thermometer.

Application of Thermometer.—Elevation of the mercury indicates correction to be added or subtracted, to or from indication upon stem of hydrometer.

When elevation is above 60°, subtract correction; and when below, add it.

ILLUSTRATION.—A hydrometer in a spirit indicates upon its stem 50 below proof, and thermometer indicates 4 above 60° in appropriate column.

Then $50 - 4 = 46 = \text{strength below proof.}$

To Compute Strength of a Spirit, or Volume of its Pure Spirit, by Commercial Hydrometer, and Convert it to Indication of a U. S. Hydrometer.

When Spirit is above Proof. **RULE.**—Add 100 to indication, and divide sum by 2.

When Spirit is below Proof. **RULE.**—Subtract indication from 100, and divide remainder by 2.

EXAMPLE.—A spirit is 11 above proof by a Commercial Hydrometer; what proportion of pure spirit does it contain?

$$11 + 100 \div 2 = 55.5 \text{ per cent.}$$

To Compute Strength, etc., by a U. S. Hydrometer.

When Spirit is above Proof. **RULE.**—Multiply indication by 2, and subtract 100.

When Spirit is below Proof. **RULE.**—Multiply indication by 2, and subtract it from 100.

EXAMPLE.—A spirit is 55.5; what is its per centage above proof?

$$55.5 \times 2 - 100 = 11 \text{ per cent.}$$

Commercial practice of reducing indications of a hydrometer is as follows:

Multiply number of gallons of spirit by per centage or number of degrees above or below proof, divide by 100, and quotient will give number of gallons to be added or subtracted, as case may be.

ILLUSTRATION.—50 gallons of whiskey are 11 per cent. above proof.

Then $50 \times 11 \div 100 = 5.5$, which added to 50 = 55.5 gallons.

HYGROMETER.

Dew-point.—When air is gradually lowered in its temperature at a constant pressure, its density increases, and ratio of increase is sensibly same for the vapor as for the air with which it is combined, until a point is reached at which the density of the vapor becomes equal to the maximum density corresponding to the temperature.

This temperature is termed *dew-point* of given mass, and any further reduction of it will induce the condensation of a portion of the vapor in form of dew, rain, snow, or frost, according as temperature of surface is above or below freezing point.

Mason's or like Hygrometer.

To Ascertain Dew-point.

RULE.—Subtract absolute dryness from temperature of air, and remainder is dew-point.

EXAMPLE.—Temperature of air 57° , and absolute dryness 7° .

Hence $57^{\circ} - 7^{\circ} = 50^{\circ}$ dew-point.

To Ascertain Absolute Existing Dryness.

RULE.—Subtract temperature of wet bulb from temperature of air, as indicated by a dry bulb, add excess of dryness from following table, multiply sum by 2, and product will give absolute dryness in degrees.

EXAMPLE.—Temperature of air 57° , wet bulb 54° .

Then $57^{\circ} - 54^{\circ} = 3^{\circ}$, and $3^{\circ} + .5^{\circ}$ (from table) $\times 2 = 7^{\circ}$ absolute dryness.

| Observed Dryness. | Excess of Dryness. | Observed Dryness. | Excess of Dryness. | Observed Dryness. | Excess of Dryness. | Observed Dryness. | Excess of Dryness. | Observed Dryness. | Excess of Dryness. |
|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|-------------------|--------------------|
| .5 | .083 | 5 | .833 | 9.5 | 1.583 | 14 | 8.333 | 18.5 | 3.083 |
| 1 | .166 | 5.5 | .9165 | 10 | 1.666 | 14.5 | 2.4165 | 19 | 3.166 |
| 1.5 | .2495 | 6 | 1 | 10.5 | 1.7495 | 15 | 2.5 | 19.5 | 3.2495 |
| 2 | .333 | 6.5 | 1.083 | 11 | 1.833 | 15.5 | 2.583 | 20 | 3.333 |
| 2.5 | .4165 | 7 | 1.166 | 11.5 | 1.9165 | 16 | 2.666 | 20.5 | 3.4165 |
| 3 | .5 | 7.5 | 1.2495 | 12 | 2 | 16.5 | 2.7495 | 21 | 3.5 |
| 3.5 | .583 | 8 | 1.333 | 12.5 | 2.083 | 17 | 2.833 | 21.5 | 3.583 |
| 4 | .666 | 8.5 | 1.4165 | 13 | 2.166 | 17.5 | 2.9165 | 22 | 3.666 |
| 4.5 | .7495 | 9 | 1.5 | 13.5 | 2.2495 | 18 | 3 | 22.5 | 3.7495 |

To Compute Volume of Vapor in Atmosphere.

By a Hygrometer.

When temperature of atmosphere in shade, and of dew-point are given.—If temperature of air and dew-point correspond, which is the case when both thermometers are alike, and air consequently saturated with moisture, then in table* opposite to temperature will be found corresponding weight of a cube foot of vapor in grains.

ILLUSTRATION.—Assume temperature of air and dew-point 70° . Then opposite temperature weight of a cube foot of vapor = 8.392 grains.

But if temperature of air is different from dew-point, a correction is necessary to obtain exact weight.

ILLUSTRATION.—Assume dew-point 70° as before, but temperature of air in shade 80° , then the vapor has suffered an expansion due to an excess of 10° , which requires a correction.

In table of corrections for 10° is 1.0203. Then divide 8.392 grains at dew-point—viz., 70° by correction corresponding to degrees of absolute dryness—viz., 10° .

$\frac{8.392}{1.0203} = 8.221$ grains of existing vapor, which, subtracted from weight of vapor corresponding to temperature of 80° , will give number of grains required for saturation at that temperature.

11.111 grains at temperature of 80° — 8.221 contained in the air = 3.112 required for saturation.

* For table see Leslie and

ascertain relations of these conditions on natural scale of humidity (complete saturation being 1000), divide weight of vapor at dew-point by weight at temperature of air, and quotient will give degrees of saturation.

ILLUSTRATION.—Dew-point = 70° , weight = 8.392.

then $8.392 \div 11.333$ (at 80°) = .7405 degrees of humidity; saturation = 1000.

To Compute Weight of Vapor in a Cube Foot of Air.

See Pressures, Temperatures, Volumes, and Density of Steam, p. 708.

Thus, Required weight of vapor in a cube foot of saturated air at 212° .

a temperature of 212° density or weight of 1 cube foot of air = .038 lb.

density is required for any temperatures not in table, see rule, p. 706.

Humidity.—Condition of air in respect to its moisture involves amount of vapor present in air and ratio of it to amount which would saturate it at its temperature, and it is this element which is denoted by term *humidity*, and expressed as a per centage; thus, if weight of vapor present is .7 of that required for saturation, the humidity is 70.

Wet Air is air, humidity of which is below zero, but it is customary to say it is dry when its humidity is below the average proportion.

Dry Air—Air in a highly heated space contains as much vapor (when weight of it is equal) as a like volume of external air, but it is drier as its capacity for vapor is less.

SUN-DIAL.

To Set a Sun-dial.

column on which dial is to be placed perpendicular to horizon. Ascertain by level that upper surface is perfectly horizontal; screw on plate loosely by means of the screw, and bring gnomon as nearly as practicable to its proper direction.

On a bright day set dial at 9 A.M. and 3 P.M. exactly, with a correctly regulated watch, observe difference between them, and correct dial to half difference. Proceed in the same manner till watch and dial are found to agree perfectly. Then fix firmly in that situation, and dial will be correctly set.

It is obvious; for, if there were any defects, the Sun's shadow would not agree with time indicated by watch, both before and after he passed meridian. Take care, however, to allow for equation of time, or you may set dial wrong. Best day to set a dial is 15th of June, as there is no equation to allow for, and no variation arises from change of declination. A dial may be set without a watch, by drawing a circle around centre, and marking spot where top of shadow of an upright piece of wire, placed in centre, just touches circle in A.M., and again in P.M. Lines should be drawn from one spot to the other, and bisected exactly; then a line drawn from centre of dial through that bisection will be a true meridian line, and the XII hours' mark should be set.

CHAINING OVER AN ELEVATION.

$L = L$, and $C = \cos. \text{angle}$.

representing length of line chained, C cos. angle of elevation with horizon, length of line reduced to horizontal.

ILLUSTRATION.—Length of an elevation at an angle of $30^{\circ} 17'$ is 100 feet; what is the horizontal distance?

Use of Cosines, $30^{\circ} 17' = .86354$. Hence, $100 \times .86354 = 86.354 \text{ feet}$.

to find out a Right Angle with a Chain, Tape-line, etc.

40 links on chain or feet of line for base, 30 links or feet for perpendicular, or hypotenuse, or in this ratio for any length or distance.

USEFUL NUMBERS IN SURVEYING.

| Converting | Multiplier. | Converse. | For Converting |
|------------|-------------|-----------|---------------------|
| to Links.. | 1.515 | .66 | Square feet into ac |
| " " .. | 4.545 | .22 | Square yards " |

Converse.
43.56
4848

CHRONOLOGY.

Solar day is measured by rotation of the Earth upon its axis with respect to the Sun.

Motion of the Earth, on account of ellipticity of its orbit, and of perturbations produced by the planets, is subject to an acceleration and retardation. To correct this fluctuation, timepieces are adjusted to an average or mean solar day (*mean time*), which is divided into hours, minutes, and seconds.

In *Civil* computations day commences at midnight, or A.M., and is divided into two portions of 12 hours each.

In *Astronomical* computations and in *Nautical* time day commences at M., or 12 hours later than the civil day, and it is counted throughout the 24 hours.

Solar Year, termed also *Equinoctial*, *Tropical*, *Civil*, or *Calendar Year*, is the time in which the Sun returns from one Vernal Equinox to another; and its average time, termed a *Mean Solar Year*, is 365.242218 solar days, or 365 days, 5 hours, 48 minutes, and 47.6 seconds.

Year is divided into 12 *Calendar* months, varying from 28 to 31 days.

Mean Lunar Month, or lunation of the Moon, is 29 days, 12 hours, 44 minutes, 2 seconds, and 5.24 thirds.*

Bissexile or *Leap Year* consists of 366 days; correction of one year in four is termed *Julian*; hence a mean *Julian* year is 365.25 days.

In year 1582 error of Julian computation of a year had amounted to a period of 10 days, which, by order of Pope Gregory VIII., was suppressed in the Calendar, and 5th of October reckoned as 15th.

Error of Julian computation, 1.00776 days, is about 1 day in 128.79 years, and adoption of this period as a basis of intercalation is termed *Gregorian Calendar*, or *New Style*,† *Julian Calendar* being termed *Old Style*.

Error of Gregorian year (365.2425 days) amounts to 1 day in 3571.4286 years.

New Style was adopted in England in 1752 by reckoning 3d of September as 14th.

By an English law, the years 1900, 2100, 2200, etc., and any other 100th year, excepting only every 400th year, commencing at 2000, are not to be reckoned bissexile years.

Dominical or *Sunday Letter* is one of the first seven letters of alphabet, and is used for purpose of determining day of week corresponding to any given date. In *Ecclesiastical Calendar* letter A is placed opposite to 1st day of year, January 1st; B to second; and so on through the seven letters; then the letter which falls opposite to first Sunday in year will also fall opposite to every following Sunday in that year. See table, p. 73.

NOTE.—In bissexile years two Dominical letters are used, one before and the other after the intercalary day.

In *Ecclesiastical Year* the intercalary day is reckoned upon 24th of February; hence 24th and 25th days are denoted by same letter, the dominical letter being set back one place.

In *Civil Year* the intercalary day is added at end of February, the change of letter taking place at 1st of March.

Dominical Cycle is a period of 400 years, when the same order of dominical letters and days of the week will return.

Cycle of the Sun, or *Sunday Cycle*, is the 28 years before same order of Dominical letters return to same days of month, and it is considered as having commenced 9 years before the era of Julian Calendar.

To Compute Cycle of the Sun.

RULE.—Add 9 to given year; divide sum by 28; quotient is number of cycles that have elapsed, and remainder is number or years of cycle.

Use of this computation is determination of dominical letter for any given *Julian Calendar* for each of the 28 years of a cycle.

† Now adopted in every Christian country except Russia and Greece.

7 adoption of *Gregorian Calendar*, order of the letters is necessarily interrupted
uppression of the century bissextile years in 1000, 2100, etc., and a table of Do-
lical letters must necessarily be reconstructed for following century.

Lunar Cycle, or *Golden Number*, is a period of 19 years, after which the new
moons fall on same days of the month of Julian year, within 1.5 hours.

Year of birth of Jesus Christ is reckoned first of the Lunar Cycle.

To Compute Lunar Cycle, or Golden Number.

RULE.—Add 1 to given year; divide sum by 19, and remainder is Golden Number.

NOTE.—If 0 remain, it is 19.

EXAMPLE.—What is Golden Number for 1879?

$1879 + 1 \div 19 = 98$, and remainder = 18 = *Golden Number*.

Fact for any year is a number designed to represent age of the moon on 1st day
January of that year. See table, p. 73.

To Compute the Roman Indiction.

RULE.—Add 3 to given year; divide sum by 15, and remainder is Indiction.

NOTE.—If 0 remain, Indiction is 15.

Number of Direction is the number of days that Easter-day occurs after 21st of
March.

Easter-day is first Sunday after first full moon which occurs upon or next after
1st of March; and if full moon occurs upon a Sunday, then Easter-day is Sunday
and it is ascertained by adding number of direction to 21st of March. It is
before March $N + 21$, or April $N - 10$.

ILLUSTRATION.—If Number of Direction is 19, then for March, $19 + 21 = 40$, and
for April, $19 - 10 = 9$ = 9th of April.

NOTE.—Moon upon which Easter immediately depends is termed *Paschal Moon*.

Paschal Moon is 14th day of moon, that is, 13 days after preceding day of new moon.

Days of the Roman Calendar.

Ides were the first 6 days of a month, *Nones* following 9 days, and *Ides* remain-
ing.

In March, May, July, and October, *Ides* fell upon 15th and *Nones* began upon 7th.
In other months *Ides* commenced upon 13th and *Nones* upon 5th.

Roman Indiction and Julian Period see p. 26.

Chronology.

Creation of World (according to Julius Africanus, Sept. 1, 5508; Samaritan
Pentateuch, 4700; Septuagint, 5872; Josephus, 4658; Talmudists, 5344; Sca-
liger, 3950; Petavius, 3984; Hales, 5411).

Deluge (according to Hales, 3154).

Bricks made and Cement first used.

Tower of Babel finished.

Chinese Monarchy.

First Egyptian Pyramid and Canal.
Gold and Silver Money first intro-
duced.

Letters first used in Egypt.

Ammon invents the Egyptian Al-
phabet.

Rockery introduced.

Wheel, Wedge, Wimble, Lever, Masts
and Sails invented by Daedalus
of Athens.

City destroyed.

Mariner's Compass discovered in
China.

Foundation of Rome.

Hales asserts Earth to be spherical.
Geometry, Maps, etc., first intro-
duced.

576. Money coined at Rome.

562. First Comedy performed at Athens.

480. First recorded Map by Aristagoras.

420. First Theatre built at Athens.

336. Calippus calculates the revolution of
the Eclipses.

320. Aristotle writes first work on Me-
chanics.

310. Aqueducts and Baths introduced in
Rome.

306. First Light-house in Alexandria.

289. First Sun-dial.

267. Ptolemy constructs a Canal from the
Nile to the Red Sea.

224. Archimedes demonstrates
properties of Mechanical Powers
the Art of measuring Sur-
faces, and Sections.

219. Hannibal crossed the Alps.

219. Surveying first introduced.

202. Printing introduced in Chi.

B.C.

198. Books with leaves of vellum first introduced by Attalus.
 170. Paper invented in China.
 168. An eclipse of the Moon which was predicted by Q. S. Gallus.
 162. Hipparchus locates the first degree of Longitude and the Latitude at Ferro.

A.D.

69. Destruction of Jerusalem.
 79. Destruction of Herculaneum and Pompeii.
 214. Grist-mills introduced.
 622. Year of Hegira, commencing 16th July; Glazed windows first introduced into England in this cent'y.
 667. Glass discovered.
 670. Stone buildings introduced into England.
 842. Lands first enclosed in England.
 933. Printing said to have been invented by the Chinese.
 991. Arabic Numerals introduced.
 1066. Battle of Hastings.
 1111. Mariner's Compass discovered.
 1180. Mariner's Compass introduced in Europe.
 1368. Chimneys first introduced into Rome from Padua.
 1383. Cannon introduced.
 1390. Woollens first made.
 1434. Printing invented at Mayence.
 1460. Wood-engraving invented and First Almanac.
 1471. Printing in England by Caxton.
 1477. Watches first introduced at Nuremberg.
 1492. America discovered.
 1497. Vasco de Gama discovers passage to India.
 1500. Variation of Mariner's Compass observed.
 1522. F. de Magellan circumnavigates the Globe.
 1530. Incas conquered by Pizarro.
 1545. Needles first introduced.
 1586. Potato introduced into Ireland from America.
 1590. Telescopes invented by Jansen and used in London in 1608.
 1616. Tobacco first introduced into Virginia.
 1620. Thermometer invented by Drebel.
 1627. Barometer invented.
 1629. First Printing press in America.
 1639. First Printing-office in America at Cambridge.
 Otto Van Guericke constructed first electric machine.
 Roads with wooden rails introduced near Newcastle.
 Newspaper Advertisement.
 Paper in America.
 First made at Bristol, Eng.

B.C.

159. Clepsydra, or Water-clock, invented.
 146. Carthage destroyed.
 70. First Water-mill described.
 51. Caesar invaded Britain.
 45. First Julian Year by Caesar.
 8. Augustus corrects the Calendar.

A.D.

1752. Benjamin Franklin demonstrated identity of the electric spark and lightning, by aid of a kite.
 1752. New Style, introduced into Britain; Sept. 3 reckoned Sept. 14.
 1753. First Steam-engine in America.
 1769. James Watt—First design and patent of a Steam-engine with separate vessel of condensation.
 1772. Oliver Evans—Designed the Non-condensing Engine. 1792. Applied for a patent for it. 1801. Constructed and operated it.
 1774. Spinning-jenny invented by Robert Arkwright.
 1776. Iron Railway at Sheffield, England.
 1783. First Balloon ascension, and Vessel's bottoms coppered.
 1790. Water-lines first introduced in models of Vessels in the U. S.
 1797. John Fitch—Propelled a yawl-boat by application of Steam to side-wheels, and also to a screw-propeller, upon Collect Pond, New York.
 1807. Robert Fulton—First Passenger Steamboat.
 1824. Compound marine steam-engines first introduced by James P. Allan, New York.
 1825. Introduction of steam towing by Mowatt, Bros. & Co., of New York, by steam-boat "Henry Eckford," New York to Albany.*
 1826. Voltaic Battery discovered by Alex. Volta, and First Horse-railroad.
 1827. First Railroad in U. S., from Quincy to Neponset.
 1829. First Lucifer Match and first Locomotive in America.
 1830. Liverpool and Manchester Railroad opened. First Steel Pen and first Iron Steamer.
 1832. S. F. B. Morse invents the Magnetic Telegraph.
 1836. Robert L. Stevens first burned Anthracite Coal in furnace of boiler of steamboat "Passaic."
 1840. First steam-boiler constructed for burning Anthracite Coal in steamboat "North America," N. Y.
 1844. Telegraph line from Washington to Baltimore, Md.
 1846. First complete Sewing-machine. Elias Howe, inventor.
 1866. Submarine Telegraph laid from Valencia to Newfoundland, N.S.

* Witnessed by author.

of Day of Week, corresponding to Day determined by following Table.

| ry, | February,* | May. | January, | January,* | September, | |
|------|------------|------|----------|-----------------|------------|-------|
| ber. | August. | | October. | April, July. | December. | June. |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3 | 9 | 10 | 11 | 12 | 13 | 14 |
| 5 | 16 | 17 | 18 | 19 | 20 | 21 |
| 7 | 23 | 24 | 25 | 26 | 27 | 28 |
| 9 | 30 | 31 | | | | |

s, if Monday is the day determined by the year given, the following dates are ndays in that year.

ots, Dominical Letters, and an Almanac, from 1800 to 1901.

OF TABLE.—To ascertain day of the week on which any given day of the falls in any year from 1800 to 1901.

STRATION.—The great fire occurred in New York on 16th of December, 1835; was day of the week?

osite 1835 is Sunday; and by preceding table, under December, it is ascertained 16th was Sunday; consequently, 16th was Wednesday.

| Days. | Dom. Let- ters. | Ex- act. | Years. | Days. | Dom. Let- ters. | Ex- act. | Years. | Days. | Dom. Let- ters. | Ex- act. |
|-------------|-----------------------|-------------|--------|-------------|-----------------------|-------------|--------|-------------|-----------------------|-------------|
| Saturday. | E | 4 | 1834 | Saturday. | E | 20 | 1868 | Sunday.* | ED | 6 |
| Sunday. | D | 15 | 1835 | Sunday. | D | 1 | 1869 | Monday. | C | 17 |
| Monday. | C | 26 | 1836 | Tuesday.* | CB | 12 | 1870 | Tuesday. | B | 28 |
| Tuesday. | B | 7 | 1837 | Wednesday. | A | 23 | 1871 | Wednesday. | A | 9 |
| Thursday.* | AG | 18 | 1838 | Thursday. | G | 4 | 1872 | Friday.* | GF | 20 |
| Friday. | F | 29 | 1839 | Friday. | F | 15 | 1873 | Saturday. | E | 1 |
| Saturday. | E | 11 | 1840 | Sunday.* | ED | 26 | 1874 | Sunday. | D | 12 |
| Sunday. | D | 22 | 1841 | Monday. | C | 7 | 1875 | Monday. | C | 23 |
| Tuesday.* | CB | 3 | 1842 | Tuesday. | B | 18 | 1876 | Wednesday.* | BA | 4 |
| Wednesday. | A | 14 | 1843 | Wednesday. | A | 29 | 1877 | Thursday. | G | 15 |
| Thursday. | G | 25 | 1844 | Friday.* | GF | 11 | 1878 | Friday. | F | 26 |
| Friday. | F | 6 | 1845 | Saturday. | E | 22 | 1879 | Saturday. | E | 7 |
| Sunday.* | ED | 17 | 1846 | Sunday. | D | 3 | 1880 | Monday.* | DC | 18 |
| Monday. | C | 28 | 1847 | Monday. | C | 14 | 1881 | Tuesday. | B | 29 |
| Tuesday. | B | 9 | 1848 | Wednesday.* | BA | 25 | 1882 | Wednesday. | A | 11 |
| Wednesday. | A | 20 | 1849 | Thursday. | G | 6 | 1883 | Thursday. | G | 22 |
| Friday.* | GF | 1 | 1850 | Friday. | F | 17 | 1884 | Saturday.* | FE | 3 |
| Saturday. | E | 12 | 1851 | Saturday. | E | 28 | 1885 | Sunday. | D | 14 |
| Sunday. | D | 23 | 1852 | Monday.* | DC | 9 | 1886 | Monday. | C | 25 |
| Monday. | C | 4 | 1853 | Tuesday. | B | 20 | 1887 | Tuesday. | B | 6 |
| Wednesday.* | BA | 15 | 1854 | Wednesday. | A | 1 | 1888 | Thursday.* | AG | 17 |
| Thursday. | G | 26 | 1855 | Thursday. | G | 12 | 1889 | Friday. | F | 28 |
| Friday. | F | 7 | 1856 | Saturday.* | FE | 23 | 1890 | Saturday. | E | 9 |
| Saturday. | E | 18 | 1857 | Sunday. | D | 4 | 1891 | Sunday. | D | 20 |
| Sunday.* | DC | 29 | 1858 | Monday. | C | 15 | 1892 | Tuesday.* | CB | 1 |
| Tuesday. | B | 11 | 1859 | Tuesday. | B | 26 | 1893 | Wednesday. | A | 12 |
| Wednesday. | A | 22 | 1860 | Thursday.* | AG | 7 | 1894 | Thursday. | G | 23 |
| Thursday. | G | 3 | 1861 | Friday. | F | 18 | 1895 | Friday. | F | 4 |
| Friday.* | FE | 14 | 1862 | Saturday. | E | 29 | 1896 | Sunday. | | 15 |
| Saturday. | D | 25 | 1863 | Sunday. | D | 11 | 1897 | Monday | | |
| Sunday. | C | 6 | 1864 | Tuesday.* | CB | 22 | 1898 | Tuesday | | |
| Monday. | B | 17 | 1865 | Wednesday. | A | 3 | 1899 | Wednesday | | |
| Tuesday.* | AG | 28 | 1866 | Thursday. | G | 14 | 1900 | Thursday | | |
| Wednesday. | F | 9 | 1867 | Friday. | F | 25 | 1901 | Friday | | |

* In leap-year, January and February must be taken in columns marked

To Ascertain Year or Years of Coincidences of a given Day of the Week with a given Day of a Month.

Look in preceding table and ascertain day of week opposite to year of occurrence, and every year in which same day is given will be year of coincidences required.

ILLUSTRATION.—If a child was born on Saturday, 19th Sept. 1829, when could and can his birthdays be celebrated, that occurred or are to occur on same day of week and date of month?

Opposite to 1829 is Sunday, and in preceding table the Sundays for September of that year were 6th, 13th, 20th; hence, if 20th was Sunday, the 19th was Saturday.

Hence, every year in table opposite to which is Sunday are the years of the coincidence required, as 1835, 1840, 1846, 1857, 1863, 1868, 1874, 1885, etc.

MOON'S AGE.

To Compute Moon's Age.

RULE.—To day of month add *Epact* and *Number of month*; from sum subtract 29 days, 12 hours, 44 min. and 2 sec., as often as sum exceeds this period, and result will give Moon's age approximately at 6 o'clock A.M. in United States, east of Mississippi River.

Numbers of the Months.

| | d. h. | | d. h. | | d. h. | | d. h. |
|---------------|-------|------------|-------|--------------|-------|---------------|-------|
| January..... | 5 | April..... | 1 21 | July..... | 4 7 | October..... | 7 16 |
| February..... | 1 22 | May..... | 2 8 | August..... | 5 18 | November..... | 9 4 |
| March..... | 9 | June..... | 3 19 | September... | 7 5 | December.... | 9 15 |

EXAMPLE.—Required age of Moon on 25th February, 1877?

Given day 25 + epact 15 + number of month 1.22 = 41 d. 22 h. — 29 d. 12 h. 44 m. 2 sec. = 12 d. 9 h. 15 min. 58 sec.

In Leap-years add 1 day to result after 28th February.

To Compute Age of Moon at Mean Noon at any other Location than that Given.

RULE.—Ascertain age, and add or subtract difference of longitude or time, according as place may be West or East of it, to or from time given.

Or, when time of new Moon is ascertained for a location, and it is required to ascertain it for any other, *add* difference of longitude or time of the place, if East, and *subtract* it if it is West of it.

Moon's Southing, as usually given in United States Almanacs, both Civil and Nautical, is computed for Washington.

To Compute Time of High-water by Aid of American Nautical Almanac.

RULE.—Ascertain time of transit of Moon for Greenwich, preceding time of the high-water required.

For any other location (west of Greenwich), multiply the time in column "diff. for one hour" by longitude of location west of Greenwich, expressed in hours, and add product to time of transit.

NOTE.—It is frequently necessary to take the transit for preceding astronom. day, as the latter does not end until noon of day under computation.

EXAMPLE.—Required time of high-water at New York on 25th of August.

Time of New York from Greenwich = 4 h. 56 m. 1.65 sec., which, multiplied by the difference for 1 hour = 10.71 min. for correction to be added to obtain time of transit at New York.

e of transit, 18 h. 38.8 m.; then 18 h. 38.8 m. + 10.71 m. = 18 hours 49.51 min.
 e of transit at New York, 24 d. 18 h. 50 m.
 Establishment of the Port, $\frac{8}{13}$

25 d. 3 h. 3 m. = time of high-water.

E. — Time of 25th at 3 h. 3 m. Astronomical computation = 25th at 3 h. 3 m. civil Time.

Compute Time of High-water at Full and Change of Moon.

Time of High-water and Age of Moon on any Day being given.

LE.—Note age of Moon, and opposite to it, in last column of following table, which subtract from time of high-water at this age of Moon, added to 12 h. 26 m., or 24 h. 52 m., as case may require (when sum to be subtracted is greatest), and remainder is time required.

MPLE.—What is time of high-water at full and change of Moon at New York? e of high-water at Governor's Island on 25th of Jan. 1864, was 9 h. 20 m. A.M. time. Age of Moon at 12 m. on that day was 16 d. 8 h. 59 m.

osite to 16 days, in following table, is 13 h. 28 m., and difference between 16 d. 5 d. 12 h. = (16.5 — 16, or 13.53 — 13.28) is 25 m.; hence, if 12 h. = 25 m., 16 d. m. — 16 d. = 8 h. 59 m. = 18.71 or 19 m., which, added to 13 h. 28 m. = 13 h.

29 h. 20 m. + 12 h. 26 m. (as sum to be subtracted is greater than time) — 13 h. = 21 h. 46 m. — 13 h. 47 m. = 7 h. 59 m.

is a difference of but 13 minutes from *Establishment of Port.*

ne after apparent Noon before Moon next passes Meridian, Age at Noon being given.

(S. H. Wright, A.M., Ph.D.)

| Moon at Meridian. | Age of Moon. | Moon at Meridian. | Age of Moon. | Moon at Meridian. | Age of Moon. | Moon at Meridian. | Age of Moon. | Moon at Meridian. |
|-------------------|--------------|-------------------|--------------|-------------------|--------------|-------------------|--------------|-------------------|
| H. M. | Days. | H. M. | Days. | H. M. | Days. | H. M. | Days. | H. M. |
| P. M. | | P. M. | | P. M. | | A. M. | | A. M. |
| 0 | 6 | 5 3 | 12 | 10 6 | 18 | 15 8 | 24 | 20 11 |
| 25 | 6.5 | 5 28 | 12.5 | 10 31 | 18.5 | 15 34 | 24.5 | 20 37 |
| 50 | 7 | 5 53 | 13 | 10 56 | 19 | 15 59 | 25 | 21 2 |
| 1 16 | 7.5 | 6 19 | 13.5 | 11 21 | 19.5 | 16 24 | 25.5 | 21 27 |
| 1 41 | 8 | 6 44 | 14 | 11 47 | 20 | 16 49 | 26 | 21 52 |
| | | | | A. M. | | | | |
| 2 6 | 8.5 | 7 9 | 14.5 | 12 12 | 20.5 | 17 15 | 26.5 | 22 17 |
| 2 31 | 9 | 7 34 | 15 | 12 37 | 21 | 17 40 | 27 | 22 43 |
| 2 57 | 9.5 | 7 59 | 15.5 | 13 2 | 21.5 | 18 5 | 27.5 | 23 8 |
| 3 22 | 10 | 8 25 | 16 | 13 28 | 22 | 18 30 | 28 | 23 33 |
| 3 47 | 10.5 | 8 50 | 16.5 | 13 53 | 22.5 | 18 56 | 28.5 | 23 58 |
| 4 12 | 11 | 9 15 | 17 | 14 28 | 23 | 19 21 | 29 | 24 24 |
| 4 38 | 11.5 | 9 40 | 17.5 | 14 43 | 23.5 | 19 46 | 29.5 | 24 48 |

Tidal Phenomena.

levation of a tidal wave towards the Moon slightly exceeds that of the opposite, and the intensity of it diminishes from Equator to the Poles.

sun by its action twice elevates and depresses the sea every day, following on of the Moon, but with less effect.

g Tides arise from the combined action of the Sun and Moon when they are on opposite sides of the Earth.

Tides are the consequence of the divided action of the Sun and Moon, when on opposite sides of the Earth, and the greatest elevations and depressions occur until the 2d or 3d day after a full or a new Moon.

Sun and Moon are in conjunction, and the time is near to the Equinoxes are fullest. The mean effect of the Moon on the tidal wave

he Sun. If, therefore, the Moon caused a tide of 6 feet, the Sun 33 feet; hence a spring tide will be 7.33 feet, and a neap tide 1.67 feet. In locations as to contour of shores, straits, capes, and river channels, shoals, etc., disturb these general rules.

LATITUDE AND LONGITUDE.

Latitude and Longitude of Principal Locations and Observatories.

Compiled from Records of U. S. Coast and Geodetic Survey and Topographical Engineer Corps, Imperial Gazetteer, and Bowditch's Navigator.

Longitude computed from Meridian of Greenwich.

A., *represents Academy*; Az., *Azimuth*; A.S., *Astronomical Station*; C., *College*; Cap., *Capitol*; Ch., *Church*; C.H., *City Hall*; C.S., *Coast Survey*; Ct., *Court-house*; Cy., *Chimney*; F.S., *Flagstaff*; G.S., *Geodetic Station*; Hos., *Hospital*; L., *Light-house*; Obs., *Observatory*; S.H., *State-house*; Sp., *Spire*; Sq., *Square*; S.S., *Signal Station*; T., *Telegraph*; T.H., *Town Hall*; U., *University*; Un., *Union*; B., *Baptist*; Con., *Congregational*; E., *Episcopal*; P., *Presby.*; and M.Ch., *Meth. Churches.*

| LOCATION. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|------------------------------|-----------|------------|-----------------------------|-----------|------------|
| NORTH AND SOUTH AMERICA. | N. | W. | NORTH AND SOUTH AMERICA. | N. | W. |
| Acapulco Mex. | 0 15 50 | 99 49 | Canandaigua . . . N. Y. | 42 54 | 9 77 17 |
| Albany, P. Ch. N. Y. | 42 39 3 | 73 45 24 | Cape Ann, S. L. . . Mass. | 42 38 11 | 70 34 10 |
| Ann Arbor. Mich. | 42 16 48 | 83 43 3 | Cape Breton Va. | 45 57 | 59 48 5 |
| Annapolis. Md. | 38 58 42 | 76 29 6 | Cape Canaveral . . . Fla. | 28 27 30 | 80 33 3 |
| Apalachicola, F. S. Fla. | 29 43 30 | 84 59 6 | Cape Cod, L. P. L. . . Ma. | 42 2 | 70 9 40 |
| Astoria, F. S. Or. | 40 11 49 | 123 49 42 | Cape Fear N. C. | 33 48 | 77 57 |
| Atlanta, C. H. Ga. | 33 44 57 | 84 23 22 | Cape Flattery, L. W. T. | 48 23 15 | 124 43 54 |
| Auburn. N. Y. | 42 55 | 76 28 | Cape Florida, L. . . Fla. | 25 39 54 | 80 9 2 |
| Augusta. Ga. | 33 28 | 81 54 | Cape Hancock, Colo. R. | 46 16 35 | 124 1 45 |
| Augusta, B. Ch. . . . Me. | 44 18 52 | 69 46 37 | Cape Hatteras, L., N. C. | 35 15 2 | 75 30 54 |
| Austin. Tex. | 30 16 21 | 97 44 12 | Cape Henlopen, L., Del. | 38 46 6 | 75 4 7 |
| Balize. La. | 29 8 5 | 89 1 4 | Cape Henry, L. . . . Va. | 36 55 30 | 76 0 2 |
| Baltimore, Mon't. Md. | 39 17 48 | 76 36 59 | Cape Horn, S. Pt., Her- | | |
| Bangor, Tho's Hill. Me. | 44 48 23 | 68 46 59 | mit's Island. | 55 59 | 67 16 |
| Barbadoes, S. Pt., W. I. | 13 3 | 59 37 | | N. | |
| Barnegat, L. N. J. | 39 46 | 74 6 | Cape May, L. N. J. | 38 55 48 | 74 57 18 |
| Bath, W. S. Ch. Me. | 43 54 55 | 69 49 | Cape Race. N. S. | 46 39 24 | 53 4 3 |
| Baton Rouge. La. | 30 26 | 91 18 | Cape Sable. S. C. | 43 24 | 65 36 |
| Beaufort, Ct. N. C. | 34 43 5 | 76 39 48 | Cape Sable, C. S. . . Fla. | 25 6 53 | 81 15 |
| Beaufort, E. Ch. . . . S. C. | 32 26 2 | 80 40 27 | | S. | |
| Belfast, M. Ch. Me. | 44 25 29 | 69 19 | Cape St. Roque, Brazil | 5 28 | 35 17 |
| Benicia, Ch. Cal. | 38 3 5 | 122 9 23 | | N. | |
| Benington. Vt. | 42 40 | 73 18 | Carthagena. N. G. | 10 26 | 75 38 |
| Bismarck, S. S. . . . Neb. | 40 48 | 100 38 | Castine. Me. | 44 22 30 | 68 45 |
| Boston, L. Mass. | 42 19 6 | 70 53 6 | Cedar Keys, Depot Isl. | 29 7 30 | 83 2 45 |
| Boston, S. H. | 42 21 30 | 71 3 30 | Chagres. N. G. | 9 20 | 80 1 21 |
| Brazos Santiago, Tex. | 26 6 | 97 12 | Charleston, C. Ch., S. C. | 32 46 44 | 79 55 39 |
| Bridgeport. Conn. | 41 10 30 | 73 11 4 | Charlestown, Mon., Ms. | 42 22 36 | 71 3 18 |
| Bristol. R. I. | 41 40 11 | 71 16 5 | Cheboygan, L. . . . Mich. | 45 40 9 | 84 24 37 |
| Brooklyn, C. H. . . . N. Y. | 40 41 31 | 73 59 27 | Chicago, C. Ch. . . . Ill. | 41 53 48 | 87 37 47 |
| Brownsville, S. S. . . Tex. | 26 | 97 30 | Chickasaw Miss. | 35 53 30 | 88 6 25 |
| Brunswick, A. Ga. | 31 8 51 | 81 29 26 | Cincinnati, Obs. . . . O. | 39 6 26 | 84 29 45 |
| Brunswick, C. Sp. . . Me. | 43 54 29 | 69 57 24 | Cleveland, Hos. . . . " | 41 30 25 | 81 40 30 |
| Buffalo, L. N. Y. | 42 50 | 78 59 | Colorado Springs. Col. | 38 50 | 104 49 48 |
| Burlington. N. J. | 40 4 52 | 74 52 37 | Columbia, S. H. . . . S. C. | 33 59 58 | 81 2 3 |
| Burlington, C. Vt. | 44 28 52 | 78 10 | Columbus, Cap. . . . O. | 39 57 40 | 82 59 40 |
| Burlington, Pub. Sq., Ia. | 40 48 22 | 91 6 25 | Concord, S. H. . . . N. H. | 43 12 29 | 71 29 47 |
| Bushnell. Ill. | 41 13 54 | 103 52 57 | Corpus Christi. . . Tex. | 27 47 18 | 97 27 2 |
| Cairo. Neb. | 36 59 48 | 89 11 14 | Council Bluffs, Neb. T. | 41 30 | 95 48 |
| Calais, C. S. Obs. . . Me. | 45 11 5 | 67 16 5 | Crescent City, L. . . Cal. | 41 44 34 | 124 11 22 |
| | S. | | Cumberland. Md. | 39 39 14 | 78 45 25 |
| | N. | | Darien, W. H. Ga. | 31 21 54 | 81 25 39 |
| F. S. Peru | 12 4 | 77 13 | Davenport, S. S. . . Ia. | 41 32 | 90 38 |
| | 22 52 | 71 7 43 | Dayton. O. | 39 44 | 84 11 |
| | 34 17 | 80 33 | Deadwood, S. S. . . Dak. | 44 22 | 103 34 |
| | 40 40 | 90 33 | Decatur, S. S. . . Tex. | 33 10 | 97 39 |

Latitude and Longitude—Continued.

| LOCATION. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|------------------------|-----------|------------|--|-----------|------------|
| ETH AND SOUTH AMERICA. | N. | W. | NORTH AND SOUTH AMERICA. | N. | W. |
| ver, S. H. Sp. Col. | 39 45 | 104 59 33 | Lockport, N. Y. | 43 11 | 78 46 |
| Moines, C. H. Ia. | 41 35 | 93 37 16 | Los Angeles, Cal. | 34 3 5 | 118 14 32 |
| oit, St. P. Ch., Mich. | 42 19 46 | 83 2 23 | Louisville, Ky. | 38 3 | 85 30 |
| r, Del. | 39 10 | 75 30 | Lowell, St. Ann's Ch., Mass. | 42 38 46 | 71 19 2 |
| r, N. H. | 43 13 | 70 54 | Machias, Th., Me. | 44 43 1 | 67 27 21 |
| ique, Ia. | 42 29 55 | 90 39 57 | Macon, Arsenal, Ga. | 32 50 25 | 83 37 36 |
| th, S. S., Min. | 46 48 | 92 8 | Madison, Dome., Wis. | 43 4 33 | 89 24 3 |
| ort, Con. Ch., Me. | 44 54 15 | 66 59 14 | Marblehead, L., Mass. | 42 30 14 | 70 50 39 |
| ton, C. H., N. C. | 36 3 24 | 76 36 31 | Martintiquo, S. P. t. W. I. | 14 27 | 60 55 |
| beth City, Ct. | 36 17 58 | 76 13 23 | Matagorda, G. S., Tex. | 28 41 29 | 95 57 56 |
| L., Penn. | 42 8 43 | 80 4 12 | Matamoras, S. S., Tenn. | 35 7 | 97 27 50 |
| ca, M. Ch., Cal. | 40 48 11 | 124 9 41 | Matanzas, Cuba | 23 3 | 81 40 |
| St. Anth'y, Minn. | 44 58 40 | 93 10 30 | Memphis, S. S., Tenn. | 35 7 | 90 7 |
| ndina, A. S., Fla. | 30 40 18 | 81 27 47 | Mexico, Mex. | 19 25 45 | 99 5 6 |
| nce, Ala. | 34 47 13 | 87 41 40 | Milwaukee, Mich. | 43 2 24 | 87 54 4 |
| ibson, Ind. T. | 35 47 35 | 95 15 10 | Minneapolis, U. C., Min. | 44 58 38 | 93 14 8 |
| Henry, Tenn. | 36 30 22 | 88 3 40 | Mississippi City, G. S., Miss. | 30 22 54 | 89 1 57 |
| Laramie, Wyo. T. | 42 12 10 | 104 47 43 | Mobile, E. Ch., Ala. | 30 41 26 | 88 2 18 |
| avenworth, Ks. | 39 21 14 | 94 44 | Monterey, Az. S., Cal. | 36 35 21 | 121 52 59 |
| fort, Ky. | 38 14 | 84 40 | | | |
| rick, Md. | 39 24 | 77 18 | Montevideo, Rat Is'd | 34 53 | 56 13 |
| ricksburg, E. Ch., Va. | 38 18 6 | 77 27 38 | Montgomery, S. H., Ala. | 32 22 45 | 86 18 |
| iction, N. B. | 46 3 | 66 38 15 | Montreal, C. E. | 45 31 | 73 32 56 |
| ston, Cath'l, Tex. | 29 18 17 | 94 47 26 | Mound City, Ill. | 37 4 47 | 89 12 |
| town, Ber. | 32 22 2 | 64 37 6 | Nantucket, L., Mass. | 41 23 24 | 70 2 24 |
| town, E. Ch., S. C. | 33 22 8 | 79 16 49 | Nantucket, S. Tower, Mass. | 41 16 57 | 70 5 57 |
| ster, U. Ch., Ma. | 42 36 46 | 70 39 59 | Nashville, U., Tenn. | 36 9 33 | 86 49 3 |
| Haven, S. S., Mich. | 43 5 | 86 18 | Nassau, L., N. P. | 25 5 2 | 77 21 2 |
| t, Obs., N. S. | 44 39 4 | 63 35 | Natchez, Miss. | 31 34 | 91 24 42 |
| burg, Penn. | 40 16 | 76 50 | Nebraska, Junction of Forks of Platte Riv. | 41 5 5 | 101 21 24 |
| rd, S. H., Conn. | 41 45 59 | 72 40 45 | New Bedford, B. Ch., Mass. | 41 38 10 | 70 55 36 |
| l, Moro., Cuba | 23 9 | 82 21 23 | New Haven, Col., Conn. | 41 18 28 | 72 55 45 |
| the Wall, L., Bahamas | 25 51 5 | 77 10 6 | New London, P. Ch. | 41 21 16 | 72 5 29 |
| 's Hole, Ch., Ms. | 41 27 13 | 70 35 59 | New Orleans, Mint. Ia. | 29 57 46 | 90 3 28 |
| l., N. Y. | 42 14 | 73 46 | New York, C. H., N. Y. | 40 42 44 | 74 24 |
| illo, Ala. | 34 36 | 86 57 | Newborn, E. Sp., N. C. | 35 6 21 | 77 5 |
| opolis, Ind. | 39 55 | 86 5 | Newburg, A. Sp., N. Y. | 41 30 6 | 74 33 |
| la, G. S., Tex. | 28 32 28 | 96 31 1 | Newburyport, L., Mass. | 42 48 30 | 70 52 28 |
| l., Miss. | 32 23 | 90 8 | New Castle, E. Ch., Del. | 39 39 36 | 75 33 48 |
| ville, M. Ch., Fla. | 30 19 43 | 81 39 14 | Newport, Sp., R. I. | 41 29 12 | 71 18 49 |
| Mex. | 19 30 8 | 96 54 30 | Norfolk, C. H., Va. | 36 50 47 | 76 7 22 |
| n City, Mo. | 38 36 | 92 8 | Norwalk, Conn. | 41 2 50 | 73 25 35 |
| ity, Gas Ch'y. | 40 43 28 | 74 2 24 | Norwich, Conn. | 41 33 | 72 7 |
| M. Ch., W. T. | 46 26 | 122 50 39 | Ocracoke, L., N. C. | 35 6 28 | 75 50 51 |
| , S. S., Ia. | 40 23 | 91 25 | Ogdensburg, L., N. Y. | 44 45 | 75 30 |
| st, T. Obs., Fla. | 24 33 31 | 81 48 31 | Old Point Comfort, Va. | 37 21 | 76 18 6 |
| l., Jamaica | 17 58 | 76 46 | Olympia, Wash. T. | 47 3 | 122 55 |
| , C. H., C. W. | 44 8 | 76 28 37 | Omaha, P. Ch., Neb. | 41 15 43 | 95 56 14 |
| le, Tenn. | 35 59 | 83 54 | Oswego, S. S., N. Y. | 43 28 32 | 76 35 5 |
| e, Ct. S., Wis. | 43 58 50 | 91 14 48 | Ottawa, Can. | 45 23 | 75 42 |
| r, Penn. | 40 2 36 | 76 20 33 | Panama, Cath'l., N. G. | 8 57 9 | 79 27 17 |
| A. S., Tex. | 28 37 36 | 96 37 21 | Parkersburg, W. Va. | 39 26 | 81 34 12 |
| orth, S. S., Ks. | 39 29 | 94 58 | Pascagoula, Miss. | | |
| n, Ky. | 38 6 | 84 18 | Pensacola, Sq're., Fla. | | |
| Peru | 12 3 | 77 6 | Petersburg, C. H., Va. | | |
| Ark. | 34 40 | 92 12 | | | |

Latitude and Longitude—Continued.

| LOCATION. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|--|-----------|------------|---|-----------|------------|
| NORTH AND SOUTH AMERICA. | N. | W. | NORTH AND SOUTH AMERICA. | N. | W. |
| Philadelphia, S. H., Pa. | 39 56 53 | 75 9 3 | St. Augustine, P. Ch., Fla. | 29 53 20 | 81 18 41 |
| Pike's Peak, S. S., Col. | 38 48 | 104 59 | St. Bartholomew, S. Point, W. I. | 17 53 30 | 62 56 54 |
| Pittsburg, Penn. | 40 32 | 80 2 | St. Christopher, N. Pl., W. I. | 17 24 | 62 50 |
| Plattsburg, Sp. N. Y. | 44 41 57 | 73 26 54 | St. Domingo, " | 17 44 30 | 64 40 42 |
| Plymouth, Pier, Ms. | 41 58 44 | 70 39 12 | St. Eustatia, Town, " " | 18 29 | 63 |
| Point Hudson, W. T. | 48 7 3 | 122 44 33 | St. Jago de Cuba, En- trance, W. I. | 19 58 | 75 52 |
| Port au Prince, W. I. | 18 33 | 72 16 3 | St. John, N. B. | 45 14 6 | 66 3 30 |
| Port Townsend, A. S., Wash. T. | 48 6 56 | 122 44 58 | St. Joseph, L. Cal. | 23 3 13 | 109 40 44 |
| Portland, C. H., Me. | 43 39 28 | 70 15 1 | St. Louis, W. U., " | 38 8 3 | 90 12 4 |
| Portland, S. S., O. | 45 30 | 122 27 30 | St. Mark's, Fort, Fla. | 30 9 1 | 84 12 30 |
| Porto Bello, N. G. | 9 34 | 79 40 | St. Martin's, Fort, W. I. | 18 5 | 63 3 |
| Porto Cabello, Mara- caibo | 10 28 | 68 7 | St. Mary's, M. H., Ga. | 30 43 12 | 81 32 53 |
| Portsmouth, L., N. H. | 43 4 16 | 70 42 34 | St. Paul, Minn. | 44 52 46 | 95 4 54 |
| Prairie du Chien, Wis. | 43 2 | 91 8 35 | St. Thomas, Fort Ch'n, W. I. | 18 21 | 64 55 18 |
| Princeton, S. Cap., N. J. | 40 20 40 | 74 39 55 | St. Vincent's, S. Point, W. I. | 13 9 | 61 14 |
| Providence, U. Ch., R. I. | 41 49 26 | 71 24 19 | Staunton, Va. | 38 8 51 | 79 4 15 |
| Provincetown, Sp., Mass. | 42 3 | 70 11 18 | Stockton, S. S., Tex. | 30 50 | 102 50 |
| Puebla de los Angeles, Mex. | 19 15 | 98 2 21 | Stonington, L., Conn. | 41 19 36 | 71 54 |
| Quebec, Citadel, Can'a | 46 49 12 | 71 12 15 | Sweetwater River, Mouth of, Wyo. T. | 42 27 18 | 107 45 27 |
| Queenstown, " | 43 9 | 79 8 | Sydney, S. S., N. S. | 46 12 | 60 12 |
| Raleigh, Square, N. C. | 35 46 50 | 78 38 5 | Syracuse, N. Y. | 43 3 | 76 9 16 |
| Richmond, Cap., Va. | 37 32 16 | 77 26 4 | Tallahassee, Fla. | 30 28 | 84 36 |
| Rio de Janeiro, S. Loaf, N. | 22 56 | 43 9 | Tampa Bay, E. Key, " " | 27 36 | 82 45 15 |
| Rochester, R. H., N. Y. | 43 8 17 | 77 51 | Tampico, Bar., Mex. | 22 15 30 | 97 51 51 |
| Rockland, E. Ch., Me. | 44 6 6 | 69 6 52 | Taunton, T. C. Ch., Mass. | 41 54 11 | 71 5 53 |
| Sackett's Harbor, N. Y. | 43 55 | 75 57 | Tobago, N. E. Pr., W. I. | 11 20 | 60 27 |
| Sacramento, Cal. | 38 34 41 | 121 27 44 | Toronto, Can. | 43 39 35 | 79 23 21 |
| Salem, So., Mass. | 42 31 10 | 70 53 58 | Trenton, P. Ch., N. J. | 40 13 10 | 74 45 50 |
| Salt Lake City, Obs., Utah | 40 46 4 | 111 53 47 | Trinidad, Fort, W. I. | 10 39 | 61 32 |
| Saltillo, Mex. | 25 26 22 | 101 4 45 | Troy, D. Ch., N. Y. | 42 43 | 73 2 16 |
| San Antonio, Tex. | 29 25 22 | 98 29 15 | Tuscaloosa, Ala. | 33 12 | 87 42 |
| San Buenaventura, G. S., Cal. | 34 15 46 | 119 15 56 | Utica, Dut. Ch., N. Y. | 43 6 49 | 75 13 |
| San Diego, A. S., C. S. | 32 43 68 | 117 9 40 | Valparaiso, Fort., Chill N. | 33 2 | 71 41 |
| San Francisco, C. S. Station, Cal. | 37 48 | 122 23 19 | Vandalia, Ill. | 39 50 | 89 2 |
| San José, Sp., " | 37 19 58 | 121 53 39 | Vera Cruz, Mex. | 19 11 52 | 96 8 36 |
| San Luis Obispo, " " | 35 10 38 | 120 43 31 | Vicksburg, S. S., Miss. | 32 23 | 90 54 |
| San Pedro, " | 33 43 20 | 118 16 3 | Victoria, Tex. | 28 46 57 | 97 1 |
| Sandusky, L., O. | 41 32 30 | 82 42 15 | Vincennes, Ind. | 38 43 | 87 25 |
| Sandy Hook, L., N. J. | 40 27 40 | 74 9 | Virginia City, S. S., M. T. | 45 20 | 112 3 |
| Santa Barbara, M. Ch., Cal. | 34 26 10 | 119 42 42 | WASHINGTON, Capitol | 38 53 20 | 77 36 |
| Santa Clara, C. Ch., " " | 37 20 40 | 121 26 56 | Watertown, Ars'l., Ms. | 42 21 41 | 71 9 45 |
| Santa Cruz, F. S., " " | 36 57 31 | 122 1 29 | West Point, N. Y. | 41 23 26 | 73 57 1 |
| Santa Fe, N. Mex. | 35 41 6 | 106 1 22 | Wheeling, Va. | 40 7 | 80 42 |
| Savannah, Sp., Ga. | 32 4 52 | 81 5 26 | Wilmington, E. Ch., N. C. | 34 14 2 | 77 56 38 |
| Schenectady, N. Y. | 42 48 | 73 55 | Wilmington, T. H., Del. | 39 44 27 | 75 33 3 |
| Scranton, R. D., Wy. | 41 7 50 | 105 23 33 | Worcester, Ant. H., Ms. | 42 16 17 | 71 48 13 |
| Seattle, W. I. | 32 30 | 93 45 | Yankton, S. S., Dak. | 42 45 | 97 45 |
| Shanghai, N. C. | 33 54 58 | 78 1 8 | Yazoo, Miss. | 33 5 | 90 20 |
| Shanghai, 1885, Ill. | 39 47 57 | 89 39 20 | York, Penn. | 39 58 | 76 40 |
| Shanghai, 1885, Ill. | 39 47 57 | 89 39 20 | Yorktown, Va. | 37 13 | 76 34 |

Latitude and Longitude—Continued

| N. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|-----------|-----------|------------|-------------------------|-----------|------------|
| | N. | E. | | N. | E. |
| AFRICA, | | | EUROPE, ASIA, AFRICA, | | |
| ASIA, | | | AND THE OCEANS. | | |
| | 0 11 | 37 10 | Genoa..... | 44 24 | 8 53 |
| | 31 12 | 29 53 | W. | | |
| | 36 47 | 3 4 | Gibraltar..... | 36 7 | 5 22 |
| | 52 22 | 4 53 | Glasgow..... | 55 52 | 4 16 |
| | 51 13 | 4 24 | GREENWICH..... | 51 28 38 | — |
| | 64 32 | 40 33 | | | E. |
| | 37 58 | 23 44 | Hamburg..... | 53 33 | 9 58 |
| | 41 23 | 2 11 | Havre..... | 49 29 | 6 |
| | 6 8 | 106 50 | S. | | W. |
| rt, Su'a. | 3 48 | 102 19 | Hawaii or Owyhee.... | 20 23 | 155 54 |
| | N. | | N. | | E. |
| | 52 30 16 | 13 23 45 | Hongkong..... | 22 16 30 | 114 14 45 |
| | 18 56 | 72 54 | Honolulu..... | 21 18 12 | 157 30 36 |
| | S. | | | | W. |
| l Roads. | 34 2 | 151 13 | Hood Isl'd, Gallapagos. | 1 23 | 89 46 |
| | N. | | Hood's Island, Mar- | S. | E. |
| | 53 5 | 8 49 | quesas..... | 9 26 | 138 57 |
| | W. | | N. | | |
| | 51 27 | 2 35 | Jeddo or Tokio..... | 35 40 | 139 40 |
| | E. | | Jerusalem..... | 31 48 | 37 20 |
| | 50 51 11 | 4 22 | Leghorn, L..... | 43 32 | 10 18 |
| | 30 30 | 48 | Leipsic..... | 51 20 20 | 12 22 |
| | W. | | Leyden..... | 52 9 28 | 4 29 15 |
| | 36 32 | 6 18 | | | W. |
| | E. | | Lisbon..... | 38 42 | 9 9 |
| | 30 3 | 31 18 | Liverpool, Obs..... | 53 24 48 | 3 |
| | 50 58 | 1 51 | | | E. |
| | 22 34 | 88 20 | Madras..... | 14 4 9 | 80 15 45 |
| | 35 31 | 25 8 | W. | | |
| | 23 7 | 113 14 | Madrid..... | 40 25 | 3 42 |
| | W. | | | | E. |
| | 51 26 | 9 29 | Majoren, Castle..... | 39 34 | 2 23 |
| | S. | | W. | | |
| ne, Obs.. | 33 56 | 3 18 28 45 | Malaga..... | 36 43 | 4 26 |
| Mad'r.. | 25 39 | 45 7 | | | E. |
| | N. | | Malta, Valetta..... | 35 54 | 14 30 |
| edro... | 9 49 | 80 23 | Manila..... | 14 36 | 121 2 |
| | 59 55 | 10 43 | Marseilles..... | 43 18 | 5 22 |
| | S. | | Messina, L..... | 38 12 | 15 35 |
| | 6 8 | 12 9 | Mocha..... | 13 20 | 43 12 |
| | N. | | Moscow..... | 55 40 | 35 33 |
| St. S.. | 41 1 | 28 59 | Muscat..... | 23 37 | 58 35 |
| | 55 41 | 12 34 | Naples, L..... | 40 50 | 14 16 |
| | 37 54 | 22 52 | | | W. |
| | 59 59 | 29 47 | New Castle..... | 54 58 | 1 37 |
| | 51 8 | 1 19 | New Hebrides, Table | S. | E. |
| | W. | | Island..... | 15 28 | 167 7 |
| | 53 23 12 | 6 20 30 | Nippon, Cape Idron, | N. | |
| | 55 57 | 3 12 | Japan..... | 34 36 3 | 138 50 35 |
| ids, St. | S. | | Odessa..... | 46 28 | 30 44 |
| | 15 55 | 5 45 | Palermo, L..... | 38 8 | 13 22 |
| | N. | | Paris, Obs..... | 48 50 13 | 2 20 |
| nt..... | 38 30 | 28 42 | Pekin..... | 39 54 | 116 28 |
| Ovolau, | S. | | | | W. |
| | 17 41 | 178 53 | Plymouth..... | 50 21 | 4 9 |
| | N. | | | | |
| | 43 46 | 11 16 | Port Jackson...N.S.W. | | |
| ira..... | 32 38 | 16 55 | Porto Praya, Cape Ver | | |
| | E. | | Islands..... | | |
| | 46 11 59 | 6 9 15 | Prince of Wales Island. | | |

Latitude and Longitude—Continued.

| LOCATION. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|--|--------------|------------|--|-----------------|------------|
| EUROPE, ASIA, AFRICA, AND THE OCEANS. | N. | W. | EUROPE, ASIA, AFRICA, AND THE OCEANS. | N. | E. |
| Queenstown..... | 51 47 " | 8 19 " | St. Petersburg..... | 59 56 " | 30 19 " |
| Rome, St. Peter's..... | 41 54 | 12 27 | Suez..... | 29 59 | 32 34 |
| Rotterdam..... | 5 54 | 4 29 | Surat, Castle..... | 21 11 | 72 47 |
| Santa Cruz..... | Ten'fe 28 28 | 16 16 | Sydney..... | N.S.W. 33 33 42 | 151 23 |
| Scilly, St. Agnes, L..... | 49 54 | 6 21 | Tahiti or Otaheite.... | 17 45 | 149 30 |
| Senegal, Fort..... | 16 1 | 16 32 | Tangier..... | N. | E. |
| Sevastopol..... | 44 37 | 33 30 | Toulon..... | 35 47 | 5 54 |
| Seville..... | 36 59 | 5 58 | Tripoli..... | 43 7 | 5 22 |
| Siam..... | 14 55 | 100 | Tunis, City..... | 34 54 | 13 11 |
| Sierra Leone..... | 8 30 | 13 18 | Venice..... | 36 47 | 10 6 |
| Singapore..... | N. 1 17 | 103 50 | Vienna..... | 40 50 | 14 26 |
| Smyrna..... | 38 26 | 27 7 | Warsaw, Obs..... | 48 13 | 16 23 |
| Southampton..... | 51 | 1 30 | Wellington...New Z'd | 52 13 5 | 21 2 9 |
| St. Helena..... | S. 15 55 | 5 45 | Yokohama..... | N. 41 14 | 174 44 |
| | | | Zanzibar Island, Sp... | N. 35 26 | 139 39 |
| | | | | S. 6 28 | 39 33 |

Observatories.—Not included in previous Table.

Longitude given in Time.

| LOCATION. | Latitude. | Longitude. | LOCATION. | Latitude. | Longitude. |
|------------------------------------|-------------|------------|---------------------------------|-------------|------------|
| | N. | W. | | N. | E. |
| Albany, Dudley .. | 42 39 49.55 | 4 54 59.52 | Madras..... | 13 4 8.1 | 5 20 57.3 |
| Alleghany, Penn. | 40 27 36 | 5 20 2.9 | Marseilles..... | 43 17 50 | 21 29 |
| Birr Castle, Earl of Rosse..... | 53 5 47 | 31 40.9 | Mitchell's, Cin., O. | 39 6 26 | 5 37 59 |
| Cambridge, U.S. . | 42 22 52 | 4 44 30.9 | Moscow..... | 55 45 19.8 | 2 30 16.06 |
| Cambridge, Eng.. | 52 12 51.6 | 22.75 | Munich, Bogenh'n | 48 8 45 | 46 26.5 |
| Cape of G. Hope.. | 33 56 3 | 1 13 55 | Palermo..... | 38 6 44 | 53 24.17 |
| Copenhagen, Un'y. | 55 40 53 | 50 19.8 | Portsmouth..... | 50 48 3 | 4 23.9 |
| Crescent City, A. | 41 44 43 | 8 16 49.1 | Quebec..... | 46 48 30 | 4 44 49.02 |
| S. Cal..... | 53 23 13 | 25 22 | Rome, College.... | 41 53 52.2 | 49 54.7 |
| Dublin..... | 55 57 23.2 | 12 43.6 | Salt Lake City, Utah | 40 46 4 | 7 27 35.1 |
| Edinburgh..... | 43 46 41.4 | 45 3.6 | San Francisco, Sq., Cal. | 37 47 55 | 8 9 38.1 |
| Florence..... | 46 11 59.4 | 24 37.7 | Santiago de Chili. | 33 26 24.8 | 4 42 18.9 |
| Geneva..... | 38 53 39 | 5 8 12.5 | St. Croix, W. I.... | N. 17 44 30 | 4 18 42.8 |
| Georgetown, U.S. | 32 47 7 | 5 19 44.7 | St. Petersburg, A.. | 59 56 29.7 | 2 1 13.5 |
| Gibbes's, Charles- ton, U. S. | 51 28 38 | — | Stockholm..... | 59 20 31 | 1 12 24.8 |
| Greenwich..... | 53 33 5 | 39 54.1 | Sydney..... | 33 51 41.1 | 10 4 59.86 |
| | 21 20.2 | 49 28.5 | Tiff's, Key West. | N. 24 33 31 | 5 27 14.1 |
| | 9 28.2 | 17 57.5 | Unkrechtsberg, Ol- mutz..... | 49 35 40 | 1 0 0.1 |
| | 47.8 | 12 0.11 | Washington..... | 38 53 39 | 5 8 12.03 |
| | | | West Point, N.Y.. | 41 23 26 | 4 55 48 |

DIFFERENCE IN TIME.

rence in Time at following Locations.

ngitude computed both from New York and Greenwich.

ifference of Time between New York and Greenwich is 4 h. 56 m.
: in following table 2 seconds are given when the decimal in any
xceeds .5 seconds.

F representing Fast, and S Slow.

| L. | New York. | Greenwich. | LOCATION. | New York. | Greenwich. |
|----------|------------|------------|----------------------|------------|------------|
| | A. M. S. | A. M. S. | | A. M. S. | A. M. S. |
| | 1 43 15 S. | 6 39 17 S. | Cedar Keys..... | 36 9 S. | 5 32 11 S. |
| | 1 F. | 4 55 2 | Chagres..... | 24 3 | 5 20 5 |
| Egypt | 6 55 34 | 1 59 32 F. | Charleston..... | 23 41 | 5 19 43 |
| | 5 8 18 | 12 16 | Charlestown..... | 11 48 F. | 4 44 13 |
| | 5 16 5 | 19 32 | Cheboygan..... | 41 37 S. | 5 37 38 |
| | 5 13 38 | 17 36 | Chicago..... | 54 30 | 5 50 31 |
| | 43 54 S. | 5 39 56 S. | Chickasaw..... | 56 24 | 5 52 26 |
| | 3 19 17 | 8 15 19 | Cincinnati..... | 41 57 | 5 37 59 |
| | 41 32 | 5 37 33 | Cleveland..... | 30 40 | 5 26 42 |
| | 9 50 | 5 5 52 | Colorado Springs.. | 2 3 15 | 6 59 17 |
| ...Ga. | 31 34 | 5 27 36 | Columbia..... | 28 7 | 5 24 8 |
| ...Me. | 16 55 F. | 4 39 6 | Columbus..... | 35 57 | 5 31 59 |
| | 1 34 55 S. | 6 30 27 | Concord..... | 10 6 F. | 4 45 56 |
| | 10 26 | 5 6 28 | Constantinople.... | 6 51 58 | 1 55 56 F. |
| | 20 54 F. | 4 35 8 | Copenhagen..... | 5 46 18 | 50 16 |
| ...Pt. | 57 34 | 3 58 28 | Corpus Christi..... | 1 33 47 S. | 6 29 48 S. |
| | 22 S. | 4 56 24 | Council Bluffs.... | 1 27 10 | 6 23 12 |
| | 16 46 F. | 4 39 16 | Crescent City..... | 3 20 44 | 8 16 45 |
| | 1 9 10 S. | 6 5 12 | Darien..... | 29 41 | 5 25 43 |
| ...N.C. | 10 38 | 5 6 39 | Davenport..... | 1 12 30 | 6 2 32 |
| ...S.C. | 26 39 | 5 22 40 | Dayton..... | 40 42 | 5 36 44 |
| | 20 1 F. | 4 36 1 | Deadwood..... | 1 58 30 | 6 54 32 |
| | 3 12 36 S. | 8 8 38 | Denver..... | 2 3 57 | 6 59 58 |
| | 5 49 37 F. | 53 35 F. | Detroit..... | 36 76 | 5 32 10 |
| | 1 46 30 S. | 6 42 32 S. | Dover.....Del. | 35 58 | 5 2 |
| | 9 47 38 F. | 4 51 36 F. | Dover.....N.H. | 12 26 F. | 4 43 36 |
| | 11 47.6 | 4 44 14 S. | Dublin..... | 4 30 40 | 25 22 |
| | 5 31 18 | 35 16 F. | Dubuque..... | 1 6 38 S. | 6 2 40 |
| | 3 17 | 4 52 44 S. | Duluth..... | 1 12 10 | 6 8 32 |
| Yard. | 4 | 4 55 58 | Eastport..... | 28 6 F. | 4 27 56 |
| ...Me. | 16 12 | 4 39 50 | Edenton..... | 10 24 S. | 5 6 26 |
| ...Ga. | 29 56 S. | 5 25 58 | Edinburgh..... | 4 43 14 F. | 12 48 |
| | 5 13 30 F. | 17 28 F. | Elizabeth City, N.C. | 8 52 S. | 5 4 54 |
| | 1 2 34 | 3 53 28 S. | Erie..... | 24 15 | 5 20 17 |
| | 19 54 S. | 5 15 56 | Eureka..... | 3 20 37 | 8 16 39 |
| ...Ia. | 1 8 24 | 6 4 26 | Falls St. Anthony.. | 1 16 40 | 6 12 42 |
| ...N.J. | 3 29 | 4 59 30 | Fernandina..... | 29 50 | 5 25 51 |
| ...Vt. | 16 38 | 5 12 40 | Fire Island, L..... | 3 10 F. | 4 52 51 |
| ...Neb. | 1 59 30 | 6 55 32 | Florence.....Ala. | 54 45 S. | 5 50 47 |
| | 4 30 50 F. | 25 12 | Fort Gibson..... | 1 24 59 | 6 21 1 |
| | 7 1 14 | 2 5 12 F. | Fort Henry, Tenn.. | 56 13 | 5 52 15 |
| ...Ill. | 1 43 S. | 5 56 45 S. | Fort Laramie..... | 2 3 9 | 6 59 11 |
| ...Me. | 26 57 F. | 4 29 4 | Fort Leavenworth.. | 1 22 54 | 6 18 56 |
| | 10 49 22 | 5 53 20 F. | Frederick..... | 13 10 | 5 9 12 |
| | 12 50 S. | 5 8 52 S. | Fredericksb'g., Va. | 13 49 | 5 9 51 |
| ...Mass. | 12 30 F. | 4 44 31 | Fredericton, N.B.. | 29 29 F. | 4 20 33 |
| | 12 28 58 | 7 32 56 F. | Funchal..... | 3 48 22 | 1 7 40 |
| ...Mi. | 1 2 10 S. | 5 58 12 S. | Galveston..... | 1 23 8 S. | 6 19 10 |
| Hope. | 6 9 57 F. | 1 13 55 F. | Geneva..... | 5 20 20 F. | 22 27 F. |
| | 26 58 | 4 20 4 S. | Geneva.....N.Y. | 1 | |
| | 2 56 S. | 4 58 58 | Genoa..... | 5 | |
| | 1 23 46 F. | 3 32 16 | Georgetown...Ber. | | |
| | 6 30 S. | 5 2 32 | Georgetown...S.C. | | |
| | 21 2 F. | 4 35 | Gibraltar..... | 1 | |

Difference in Time—Continued.

| LOCATION. | New York. | Greenwich. | LOCATION. | New York. | Greenwich. |
|--------------------|------------|-------------|--------------------|-------------|------------|
| | A. M. S. | A. M. S. | | A. M. S. | A. M. S. |
| Glasgow..... | 4 38 58 F. | 17 4 S. | Milwaukee..... | 55 35 S. | 5 51 36 S. |
| Gloucester..... | 13 22 | 4 42 40 | Minneapolis..... | 1 16 55 | 6 12 57 |
| Grafton..... | 24 5 S. | 5 20 7 | Mississippi City.. | 1 6 | 5 56 8 |
| Grand Haven..... | 49 10 | 5 45 12 | Mobile..... | 56 7 | 5 52 9 |
| GREENWICH..... | 4 56 1.6 | — | Montauk Point... | 18 22 F. | 4 37 40 |
| Halifax..... | 41 42 F. | 4 14 40 | Monterey..... | 3 11 30 S. | 8 7 32 |
| Hamburg..... | 5 35 54 | 39 52 F. | Montevideo..... | 1 11 10 F. | 3 44 52 |
| Harrisburg..... | 11 18 S. | 5 7 20 S. | Montgomery..... | 49 10 S. | 5 45 12 |
| Hartford..... | 5 19 F. | 4 50 43 | Montreal..... | 1 50 F. | 4 54 12 |
| Havana, Morro... | 33 24 S. | 5 29 26 | Montserrat..... | 47 14 | 4 8 48 |
| Havre..... | 4 56 26 F. | 24 F. | Moscow..... | 7 18 14 | 2 22 12 F. |
| Hawaii or Owyhee | 5 27 34 S. | 10 23 36 S. | Mound City..... | 1 26 S. | 5 56 28 S. |
| Hongkong..... | 12 27 1 F. | 7 36 59 F. | Nantucket..... | 15 38 F. | 4 40 24 |
| Honolulu..... | 15 27 30 | 10 31 28 | Naples..... | 5 53 6 | 57 4 F. |
| Hudson..... | 1 12 | 4 54 40 S. | Nashville..... | 51 15 S. | 5 47 16 S. |
| Huntsville..... | 51 46 S. | 5 47 48 | Nassau..... | 13 23 | 5 9 24 |
| Indianapolis..... | 48 18 | 5 44 20 | Natchez..... | 1 9 37 | 6 5 39 |
| Indianola..... | 1 30 2 | 6 26 4 | Nebraska..... | 1 49 24 | 6 45 30 |
| Jackson..... | 1 4 30 | 6 0 32 | New Bedford..... | 12 19 F. | 4 43 48 |
| Jacksonville..... | 30 35 | 5 26 37 | New Haven..... | 4 19 | 4 51 43 |
| Jalapa..... | 1 31 36 | 6 27 38 | New London..... | 7 40 | 4 48 22 |
| Jeddo or Tokio... | 14 16 2 F. | 9 20 F. | New Orleans..... | 1 4 12 S. | 6 14 |
| Jefferson City.... | 1 12 30 S. | 6 8 32 S. | NEW YORK..... | — | 4 56 1.6 |
| Jersey City..... | 8 | 4 56 10 | Newbern..... | 12 18 | 5 8 20 |
| Jerusalem..... | 7 25 22 F. | 2 29 20 F. | Newburg..... | 1 | 4 56 8 |
| Kalama..... | 3 15 21 S. | 8 11 23 S. | Newburyport..... | 12 32 F. | 4 43 30 |
| Keokuk..... | 1 9 38 | 6 5 40 | New Castle..... | 4 49 34 | 6 28 |
| Key West..... | 31 13 | 5 27 14 | New Castle... Del. | 6 13 S. | 5 2 15 |
| Kingston... Can. | 9 53 | 5 5 54 | Newport..... | 10 46 F. | 4 45 45 |
| Kingston... Jam. | 11 2 | 5 7 4 | Norfolk..... | 9 8 S. | 5 5 9 |
| Knoxville..... | 39 34 | 5 35 36 | Norwalk..... | 2 19 F. | 4 53 42 |
| La Crosse..... | 1 8 58 | 6 4 59 | Norwich..... | 7 34 | 4 48 28 |
| La Guayra..... | 27 54 F. | 4 28 8 | Ocracoke..... | 7 54 S. | 5 35 |
| Lancaster..... | 19 21 S. | 5 15 22 | Odessa..... | 6 58 58 F. | 2 2 56 F. |
| Lavaca..... | 1 30 27 | 6 26 29 | Ogdensburg..... | 5 58 S. | 5 2 8 |
| Leavenworth..... | 1 23 40 | 6 19 52 | Old Point Comfort | 9 11 | 5 5 12 |
| Lehigh..... | 5 37 14 F. | 4 12 12 F. | Olympia..... | 3 15 38 | 8 11 40 |
| Lexington..... | 41 10 S. | 5 37 12 S. | Omaha..... | 1 27 43 | 6 23 45 |
| Lima..... | 12 22 | 5 8 24 | Oswego..... | 10 19 | 5 6 20 |
| Lisbon..... | 4 19 26 F. | 36 36 F. | Ottawa..... | 6 46 | 5 2 48 |
| Little Rock..... | 1 12 46 S. | 6 8 48 S. | Paducah..... | 58 22 | 5 54 24 |
| Liverpool..... | 4 44 2 F. | 12 | Palermo..... | 5 49 30 F. | 53 28 F. |
| Lockport..... | 19 2 S. | 5 15 4 | Panama..... | 1 21 48 S. | 5 17 49 S. |
| Los Angeles..... | 2 56 16 | 7 52 18 | Paris..... | 5 5 22 F. | 9 20 F. |
| Louisville..... | 45 58 | 5 42 | Parkersburg..... | 30 15 S. | 5 26 37 S. |
| Lowell..... | 10 45 F. | 4 45 16 | Pekin..... | 12 41 54 F. | 7 45 53 F. |
| Machias Bay..... | 26 12 | 4 29 49 | Pensacola..... | 52 50 S. | 5 48 52 S. |
| Macon..... | 38 28 S. | 5 34 30 | Petersburg..... | 13 35 | 5 9 37 |
| Madison..... | 1 1 35 | 5 57 36 | Philadelphia..... | 4 34 6 | 5 30 9 |
| Madrid..... | 4 41 14 F. | 14 48 | Pike's Peak..... | 2 3 50 | 6 59 52 |
| Malaga..... | 4 38 18 | 17 44 | Pittsburg..... | 24 6 | 5 20 8 |
| Malta..... | 5 54 2 | 58 F. | Plattsburg..... | 2 14 F. | 4 53 48 |
| Manila..... | 13 10 | 8 4 8 | Plymouth..... | 4 39 26 | 16 36 |
| Maraculbo..... | 9 2 | 4 47 S. | Plymouth... Mass. | 13 25 | 4 42 37 |
| Marblehead, L.... | 12 41 | 4 43 21 | Port Au Prince, | | |
| Marbella..... | 5 17 20 | 21 28 F. | St. Domingo..... | 16 34 | 4 39 28 |
| Martha's Vineyard | 52 22 | 4 3 40 S. | Port Townsend..... | 3 14 58 S. | 8 11 |
| | 7 50 S. | 6 23 52 | Portland..... | 15 2 F. | 4 41 |
| | 3 30 | 6 20 51 | Portland..... Or. | 3 13 48 S. | 8 9 30 |
| | 3 38 | 5 20 40 | Porto Praya..... | 3 23 50 F. | 1 32 18 |
| | 46 | 6 0 28 | Porto Rico..... | 33 26 | 4 22 26 |
| | | 6 36 20 | Portsmouth..... | 13 11 | 4 42 51 |

Difference in Time—Continued.

| LOCATION. | New York. | Greenwich. | LOCATION. | New York. | Greenwich. |
|-----------------------|-----------------|-----------------|---------------------|-----------------|-----------------|
| | <i>h. m. s.</i> | <i>h. m. s.</i> | | <i>h. m. s.</i> | <i>h. m. s.</i> |
| Prairie du Chien..... | 1 8 33 S. | 6 4 34 S. | St. Louis..... | 4 47 S. | 6 48 S. |
| Princeton..... | 2 38 | 4 58 40 | St. Mark's..... | 40 48 | 5 36 50 |
| Providence..... | 10 24 F. | 4 45 37 | St. Mary's..... | 30 10 | 5 26 12 |
| Provincetown..... | 15 16 | 4 40 45 | St. Paul..... | 11 24, 18 | 6 20 20 |
| Quebec..... | 11 13 | 4 44 49 | St. Petersburg.... | 6 57 18 F. | 2 1 16 F. |
| Queenstown, L..... | 4 42 46 | 33 16 | St. Thomas, Fort.. | 36 20 | 4 19 41 S. |
| Raleigh..... | 18 31 S. | 5 14 32 | Staunton..... | 20 15 S. | 5 16 17 |
| Richmond..... | 13 43 | 5 9 44 | Stockholm..... | 6 8 26 F. | 1 12 25 F. |
| Rio de Janeiro..... | 2 3 26 F. | 2 52 36 | Stonington..... | 8 26 | 4 37 36 S. |
| Rochester..... | 15 22 S. | 5 11 24 | Suez..... | 7 6 18 | 2 10 16 F. |
| Rockland..... | 19 34 F. | 4 36 27 | Sweetwater River, | | |
| Rome..... | 5 45 50 | 49 48 F. | Mouth of..... | 2 15 S. | 7 11 2 S. |
| Rotterdam..... | 5 13 58 | 17 56 | Sydney..... N.S. | 8 2 F. | 4 48 |
| Sackett's Harbor..... | 7 46 S. | 5 3 48 S. | Sydney..... N.S.W. | 15 1 34 | 10 5 32 F. |
| Sacramento..... | 3 9 49 | 8 5 51 | Syracuse..... | 8 35 S. | 5 4 37 S. |
| Salem..... | 12 26 F. | 4 43 36 | Tahiti or Otaheite. | 14 44 2 F. | 9 48 F. |
| Salt Lake City..... | 2 31 34 S. | 7 27 35 | Tallahassee..... | 42 22 S. | 5 38 24 S. |
| Saltillo..... | 1 48 17 | 6 44 19 | Tampa Bay..... | 34 59 | 5 31 1 |
| San Antonio..... | 1 37 55 | 6 33 57 | Tampico Bar..... | 1 35 25 | 6 31 27 |
| San Buenaventura..... | 3 1 2 | 7 57 4 | Taunton..... | 11 38 F. | 4 44 24 |
| San Diego..... | 2 52 37 | 7 47 39 | Toronto..... | 21 32 S. | 5 17 33 |
| San Francisco, | | | Toulon..... | 5 17 30 F. | 21 28 F. |
| C. S. S. | 3 13 32 | 8 9 33 | Trenton..... | 3 2 S. | 4 59 3 S. |
| San Francisco, P. | 3 13 50 | 8 9 51 | Tripoli..... | 5 48 36 F. | 5 24 44 F. |
| San José..... | 3 11 33 | 8 7 35 | Troy..... | 3 54 | 4 52 9 S. |
| Sandusky..... | 34 47 | 5 30 49 | Tunis..... | 5 36 26 | 40 24 F. |
| Sandy Hook..... | 1 F. | 4 56 1 | Turk's Island..... | 11 22 | 4 44 40 S. |
| Santa Barbara..... | 3 2 49 S. | 7 58 50 | Tuscaloosa..... | 54 46 S. | 5 50 48 |
| Santa Clara..... | 3 9 46 | 8 5 48 | Utica..... | 4 50 | 5 52 |
| Santa Cruz..... | 3 12 4 | 8 8 6 | Valparaiso..... | 9 18 | 4 46 44 |
| Santa Cruz, Ten'fe | 3 50 58 F. | 1 5 4 | Vandalia..... | 1 6 | 5 56 8 |
| Santa Fé..... | 2 8 4 S. | 7 4 5 | Venice..... | 5 53 46 F. | 5 57 44 F. |
| Savannah..... | 28 20 | 5 24 22 | Vera Cruz..... | 1 28 33 S. | 6 24 34 S. |
| Schenectady..... | 22 F. | 4 55 40 | Vicksburg..... | 1 7 34 | 6 3 36 |
| Seville..... | 4 32 10 | 23 52 | Victoria..... Tex. | 1 32 2 | 6 28 4 |
| Sherman..... | 2 5 33 S. | 7 1 34 | Vienna..... | 6 1 34 F. | 1 5 32 F. |
| Shreveport..... | 1 18 58 | 6 15 | Vincennes..... | 53 38 S. | 5 49 40 S. |
| Siam..... | 11 36 2 F. | 6 40 F. | Virginia City..... | 2 32 10 | 7 28 12 |
| Sierra Leone..... | 4 2 50 S. | 53 12 S. | Warsaw..... | 6 20 11 F. | 1 24 9 F. |
| Singapore..... | 11 51 22 F. | 6 55 20 F. | WASHINGTON, Obs.. | 12 10 S. | 5 8 12 S. |
| Smithville..... | 16 3 S. | 5 12 5 S. | West Point..... | 14 F. | 4 55 48 |
| Smyrna..... | 6 44 30 F. | 1 48 28 F. | Wheeling..... | 26 46 S. | 5 22 48 |
| Southampton..... | 4 50 2 | 6 S. | Wilmington, Del. | 6 11 | 5 2 12 |
| Springfield..... Ill. | 1 2 36 S. | 5 58 37 | Wilmington, N.C. | 15 45 | 5 11 47 |
| Springfield, Mass. | 5 39 F. | 4 50 24 | Worcester..... | 8 49 F. | 4 47 13 |
| St. Augustine..... | 29 13 S. | 5 25 15 | Yankton..... | 1 33 58 S. | 6 30 |
| St. Croix, Obs..... | 37 19 F. | 4 18 43 | Yazoo..... | 1 5 18 | 6 1 20 |
| St. Helena..... | 4 33 2 | 23 | Yeddo..... | 14 14 42 F. | 9 18 40 F. |
| St. Jago de Cuba..... | 7 26 S. | 5 3 28 | Yokohama..... | 14 14 43 | 9 18 41 |
| St. John..... | 31 48 F. | 4 24 14 | York..... | 10 38 S. | 5 6 40 S. |
| St. Joseph, L. Cal. | 2 22 41 S. | 7 18 43 | Yorktown..... | 10 14 | 5 6 16 |

To Compute Difference of Time between New York and Greenwich and any Location not given in Table.

RULE. — Reduce longitude of location to time, and if it is W. of assumed meridian it is *Slow*; if E., it is *Fast*.

If difference for New York is required, and it exceeds 4 h. 56 m. 2 sec., subtract *this time*, and remainder will give difference of time, S.; and if it (4 h. 56 m. 2 sec.) does not exceed it, subtract difference from *W* ad remainder will give difference of time, F.

TIDES.

TIDES.

Tide-Table for Coast of United States.

Showing Time of High-water at Full and New Moon, termed Establishment of the Port, being Mean Interval between Time of Moon's Transit and Time of High-water. (U. S. Coast and Geodetic Survey.)

| LOCATIONS AND TIME. | | Spring. | Neap. | LOCATIONS AND TIME. | | Spring. | Neap. |
|---|-------|---------|-------|--|-------|---------|-------|
| COAST FROM EASTPORT TO NEW YORK. | | | | CHEESAPEAKE BAY AND RIVERS. | | | |
| Eastport.....Me. | A. M. | 15 | Feet. | Old Pt. Comforts...Va. | A. M. | Feet. | Feet. |
| Campo Bello*....." | " | 25 | " | Cape Henry*....." | 7 51 | 3 | 2 |
| Portland....." | " | 25 9.9 | 7.6 | Point Lookout...Md. | 12 58 | 1.9 | .7 |
| Cape Ann*....." | " | 11 30 | 11 | Annapolis....." | 17 4 | 1 | .8 |
| Portsmouth.....N.H. | " | 11 23 | 9.9 | Bodkin Light....." | 18 8 | 1.3 | .8 |
| Newburyport. Mass. | " | 11 22 | 9.1 | Baltimore....." | 18 59 | 1.5 | .9 |
| Salem....." | " | 11 13 | 10.6 | James R. (City Pt.), Va | 14 37 | 3 | 2.5 |
| Cape Cod*....." | " | 11 30 | 6 | Richmond....." | 16 58 | 3.4 | 2.3 |
| Boston Light....." | " | 11 12 | 10.9 | COASTS OF N. AND S. CAROLINA, GEORGIA, AND FLORIDA. | | | |
| Boston....." | " | 11 27 | 10.3 | Hatteras Inlet...N.C | 7 4 | 2.2 | 1.8 |
| Nantucket....." | " | 12 24 | 3.6 | Cape Hatteras....." | 9 1 | 5 | |
| Edgartown....." | " | 12 16 | 2.5 | Beaufort....." | 7 26 | 3.3 | 2.2 |
| Holmes's Hole....." | " | 11 43 | 1.8 | Smithville (C. Fear) " | 7 19 | 5.5 | 3.8 |
| Tarpanin Cove....." | " | 8 4 | 2.8 | Charleston (C. H.) " | 7 26 | 6 | 4.1 |
| Wood's Hole, n. side. | " | 7 50 | 4.7 | Wharf.....S.C. | | | |
| N. Bedford (Dump- ing Rock) | " | 7 57 | 4.6 | Fort Pulaski....Ga. | 7 20 | 8 | 5.9 |
| New York.....N.Y. | " | 8 13 | 5.4 | Savannah....." | 8 13 | 7.6 | 5.5 |
| Albany*....." | " | 3 30 | 1 | St. Augustine....Fla. | 8 21 | 4.9 | 3.6 |
| LONG ISLAND SOUND. | | | | Cape Florida....." | 8 84 | 1.8 | 1.2 |
| Newport.....R.I. | " | 7 45 | 4.6 | Key West....." | 9 22 | 1.6 | 1 |
| Point Judith....." | " | 7 32 | 3.7 | Tampa Bay....." | 11 21 | 1.8 | 1 |
| Montauk Point. N.Y. | " | 8 20 | 2.4 | Cedar Keys....." | 13 15 | 3.2 | 1.6 |
| Watch Hill.....R.I. | " | 9 20 | 3.1 | WESTERN COAST. | | | |
| Providence*....." | " | 8 25 | 5 | San Diego.....Cal. | 9 38 | 5 | 2.3 |
| Stonington.....Ct. | " | 9 7 | 3.2 | San Pedro....." | 9 39 | 4.7 | 2.2 |
| Little Gull Isl'd. N.Y. | " | 9 38 | 2.9 | Cuyler's Harbor. " | 9 25 | 5.1 | 2.8 |
| New London.....Ct. | " | 9 28 | 3.1 | San Luis Obispo. " | 10 8 | 4.8 | 2.4 |
| New Haven....." | " | 11 16 | 6.2 | Monterey....." | 10 22 | 4.3 | 2.5 |
| Bridgeport....." | " | 11 11 | 8 | South Farnallone " | 10 37 | 4.4 | 2.8 |
| Oyster Bay.....N.Y. | " | 11 7 | 9.2 | San Francisco...." | 12 6 | 4.3 | 2.8 |
| Sand's Point....." | " | 11 13 | 8.9 | Mare Island....." | 13 40 | 5.2 | 4.1 |
| New Rochelle....." | " | 11 22 | 8.6 | Benicia....." | 14 10 | 5.1 | 3.7 |
| Throg's Neck....." | " | 11 20 | 9.2 | Ravenswood....." | 12 36 | 7.3 | 4.9 |
| Hell Gate*....." | " | 9 35 | 6 | Bodega....." | 11 17 | 4.7 | 2.7 |
| COAST OF NEW JERSEY. | | | | Humboldt Bay...." | 12 2 | 5.5 | 3.5 |
| Cold Spring Inlet..N.J. | " | 7 32 | 5.4 | Astoria.....Or. | 12 42 | 7.4 | 4.6 |
| Sandy Hook.....N.J. | " | 7 29 | 5.6 | Nee-ah Harbor, Wash. | 12 33 | 7.4 | 4.8 |
| Amboy....." | " | 8 15 | 5 | Port Townsend...." | 3 49 | 5.5 | 4 |
| Cape May Landing " | " | 8 19 | 6 | MISCELLANEOUS. | | | |
| Egg Harbor*....." | " | 9 34 | 5 | Bay of Fundy*..N.S. | 11 | 60 | |
| WARE BAY AND NEW. | | | | Blue Hill Bay*.." | 12 | 12 | |
| Water....." | " | 8 | 4.5 | St. John's*....." | 12 | 30 | |
| Way....." | " | 8 33 | 6.2 | Kingston*.....Jam. | 2 30 | 2 | |
| N.J....." | " | 9 4 | 7 | Halifax*.....N. S. | | 7.5 | .5 |
| Cal....." | " | 9 3 | 6.9 | Pensacola*.....Fla. | | 1.5 | .4 |
| | " | | 6.8 | Galveston*.....Tex. | | 1.6 | .4 |

¹ of this name.

4251 500 D. 84.

- mean lunar day) for some ports in Del-
-ber the month; hence 12 A. 26 min. is
- that, to give the interval required.

Bench Marks referred to in preceding Table.

† BOSTON. — Top of wall or quay, at entrance to dry-dock in Charlestown navy-yard, 14.76 feet above mean low-water.

‡ NEW YORK. — Lower edge of a straight line, cut in a stone wall, at head of wooden wharf on Governor's Island, 14.56 feet above mean low-water.

§ OLD POINT COMFORT, Va. — A line cut in wall of light-house, one foot from ground, on southwest side, 11 feet above mean low-water.

|| CHARLESTON, S. C. — Outer and lower edge of embrasure of gun No. 3, at Castle Pinckney, 10.13 feet above mean low-water.

Establishment of the Port for several Locations in Europe, etc.

| PORT. | TIME. | PORT. | TIME. | PORT. | TIME. |
|---------------------|-------|--------------------|-------|---------------------|-------|
| | A. M. | | A. M. | | A. M. |
| Amsterdam..... | 3 | Chatham..... | 1 2 | Liverpool..... | 11 16 |
| Antwerp..... | 4 25 | Cherbourg..... | 7 49 | London Bridge.... | 2 7 |
| Beachy Head..Eng. | 11 | Clear Cape..... | 4 | Newcastle..... | 1 22 |
| Belfast..... | 10 | Cowes..... | 10 46 | Portsmouth D.-yard, | |
| Bordeaux..... | 6 50 | Dover Pier..... | 11 12 | Eng..... | 11 41 |
| Bremen..... | 6 | Dublin Bar..... | 11 12 | Quebec..... | 8 |
| Brest Harbor..... | 3 47 | Funchal..... | 11 30 | Ramsgate Pier..... | 10 27 |
| Bristol..... | 7 21 | Gravesend.....Eng. | 1 14 | Rye Bay.....Eng. | 11 20 |
| Bristol Quay..... | 6 27 | Greenock..... | 8 | Sheerness..... | 57 |
| Cadiz..... | 1 40 | Holyhead..... | 10 11 | Sierra Leone..... | 8 15 |
| Calais..... | 11 49 | Hull.....Eng. | 6 29 | Southampton..Eng. | 11 40 |
| Calf of Man..... | 11 5 | Land's End..... | 3 57 | Thames R. mo'th " | 12 |
| Cape St. Vincent... | 2 30 | Lisbon..... | 2 30 | Woolwich..... | 2 15 |

Rise and Fall of Tides in Gulf of Mexico.

| LOCATIONS. | Mean. | Spring. | Neap. | LOCATIONS. | Mean. | Spring. | Neap. |
|--------------------------------------|-------|---------|-------|---------------------------------------|-------|---------|-------|
| | Feet. | Feet. | Feet. | | Feet. | Feet. | Feet. |
| St. George's Island... Fla. | 1.1 | 1.8 | .6 | Isle Dernière..... La. | 1.4 | 1.2 | .7 |
| Port Morgan (Mobile Bay)..... Ala. } | 1 | 1.5 | .4 | Entrance to Lake Calcasieu..... La. } | 1.5 | 1.1 | .6 |
| Cat Island..... Miss. | 1.3 | 1.9 | .6 | Aransas Pass..... " | 1.1 | 1.8 | .6 |
| Southwest Pass..... La. | 1.1 | 1.4 | .5 | Brazos Santiago..... " | .9 | 1.2 | .5 |

Tides of Gulf of Mexico.

On Coast of Florida, from Cape Florida to St. George's Island, near Cape San Blas, the tides are of the ordinary kind, but with a large daily inequality. From St. George's Island, Apalachicola entrance, to Dernière Isle, the tides are usually of the single-day class, ebbing and flowing but once in 24 (lunar) hours. At Calcasieu entrance, double tides reappear, and except for some days about the period of Moon's greatest declination, tides are double at Galveston, Texas. At Aransas and Brazos Santiago the single-day tides are as perfectly well marked as at St. George's, Pensacola, Fort Morgan, Cat Island, and the mouths of the Mississippi. For some 3 to 5 days, however, about the time when the Moon's declination is nothing, there are generally two tides at all these places in 24 hours, the rise and fall being quite small.

Highest high and lowest low waters occur when greatest declination of Moon happens at full or change. Least tides when Moon's declination is nothing at first or last quarter.

Tides of Pacific Coast.

On Pacific coast there is, as a general rule, one large and one small tide during each day, heights of two successive high-waters occurring, one A.M., and other P.M. of same 24 hours, and intervals from next preceding transit of Moon are very different. These inequalities depend upon Moon's declination. When Moon's declination is nothing, they disappear, and when it is greatest, either North or South, they are greatest. The inequalities for low water are not same as for high, though they disappear, and have greatest value at nearly same time.

When Moon's declination is North, highest of two high tides of the 24 hours occurs at San Francisco, about 11.5 hours after Moon's southing (transit); and when declination is South, lowest of the two high tides occurs about this interval.

Lowest of two low-waters of the day is the one which follows next highest high water.

STEAMING DISTANCES.

Distances between various Ports of United States and Canada.

By Lake, River, and Canal.

| LOCATIONS. | Lake and River. | Canal. | Total. | LOCATIONS. | Lake and River. | Canal. | Total. |
|--------------------------------------|-----------------|--------|--------|---|-----------------|--------|--------|
| | Miles. | Miles. | Miles. | | Miles. | Miles. | Miles. |
| Duluth to Buffalo... | 1024 | 1 | 1025 | Chicago to New York, via Oswego..... | 1195 | 232 | 1427 |
| Chicago to Buffalo... | 925 | — | 925 | Chicago to Montreal. | 1190 | 71 | 1261 |
| Duluth to Oswego... | 1133 | 27 | 1160 | Buffalo to Colborne, via Welland Canal. | — | 26.77 | 26.77 |
| Chicago to Oswego... | 1034 | 26 | 1060 | Buffalo to New York. | 142 | 352 | 494 |
| Duluth to New York, via Buffalo..... | 1166 | 353 | 1519 | Welland Canal to Montreal..... | 304.5 | 70.5 | 375 |
| via Oswego..... | 1294 | 233 | 1527 | Montreal to Kingston. | 126.25 | 120 | 246.25 |
| Duluth to Montreal. | 1289 | 72 | 1361 | Ottawa to Kingston. | — | 126.25 | 126.25 |
| Chicago to New York, via Buffalo. | 1067 | 352 | 1419 | | | | |

Distances between various Ports and New York and London.

Not included in preceding Table.

| PORTS. | Miles. | Miles. | PORTS. | Miles. | Miles. | PORTS. | Miles. | Miles. |
|---------------|--------|---------|----------------|--------|---------|---------------|--------|---------|
| | N. Y. | London. | | N. Y. | London. | | N. Y. | London. |
| Alexandria... | 4893 | 3102 | Cape Race.... | 1004 | 2249 | New Orleans. | 1790 | 4730 |
| Amsterdam... | 3291 | 262 | Cowes..... | 3093 | 200 | Norfolk..... | 308 | 3447 |
| Barbadoes... | 1855 | 3812 | Funchal..... | 2760 | 1303 | Pensacola... | 1623 | 4654 |
| Batavia..... | 8972 | 11492 | Galway..... | 2720 | 721 | Philadelphia. | 262 | 3404 |
| Bermudas... | 682 | 3142 | Gibraltar... | 3260 | 1325 | Quebec..... | 1360 | 3080 |
| Bombay..... | 8522 | 10703 | Glasgow..... | 2913 | 765 | Queenstown. | 2780 | 551 |
| Boston..... | 356 | 3030 | Halifax..... | 590 | 2706 | Rio Janeiro. | 4070 | 5200 |
| Bremen..... | 3428 | 408 | Havana..... | 1161 | 4197 | St. Johns.... | 1064 | 2214 |
| Bristol..... | 2979 | 501 | Hobart Town. | 9187 | 11368 | Southampton | 3103 | 211 |
| Buenos Ayres | 6010 | 6280 | Kingston, Jam. | 1456 | 4305 | Swan River.. | 8480 | 10661 |
| Cadiz..... | 3125 | 1115 | Lima..... | 10053 | 10149 | Tortugas..... | 1151 | 4182 |
| Calcutta..... | 9350 | 11531 | Madras..... | 8707 | 10888 | Washington.. | 461 | 3612 |

Distances between various Ports of England, Canada, United States, etc.

Not included in preceding Table.

| PORTS. | Miles. | PORTS. | Miles. | PORTS. | Miles. |
|--------------------|--------|----------------------|--------|----------------------|--------|
| Halifax to | | Liverpool to | | Panama to | |
| Liverpool..... | 2563 | Havana..... | 4100 | San Diego..... | 2897 |
| St. Thomas..... | 1503 | Portland..... | 2770 | Monterey..... | 3198 |
| St. Johns, N. F.. | 520 | Baltimore..... | 3400 | San Francisco.. | 3240 |
| Quebec to Glasgow. | 2563 | N. Orleans to Havana | 570 | San Francisco to | |
| Liverpool to | | Cape Race to | | San Juan del Sud. | 2685 |
| Boston..... | 2955 | Fastnet..... | 1711 | Acapulco..... | 1841 |
| Quebec..... | 2855 | Halifax..... | 457 | Manzanilla..... | 1543 |
| Philadelphia..... | 3147 | Boston..... | 835 | San Diego..... | 474 |
| Callao..... | 11379 | St. Johns, N. F., to | | Monterey..... | 105 |
| Fastnet..... | 283 | Quebec..... | 891 | Humboldt..... | 200 |
| Cape Race..... | 1992 | Boston..... | 890 | Columbia R. Bar.. | 530 |
| Aspinwall..... | 4650 | Greenock..... | 1848 | Vancouver..... | 638 |
| Dartmouth..... | 3290 | Bermudas to Nassau. | 804 | Portland..... | 650 |
| Ne..... | 13290 | Panama to | | Port Townsend.. | 732 |
| Pro..... | 5125 | San Juan del Sud. | 570 | Victoria..... | 715 |
| Quebec..... | 13800 | Gulf of Fonseca... | 739 | Yokohama..... | 4750 |
| San Francisco..... | 7378 | Acapulco..... | 1416 | Honolulu..... | 2080 |
| Opoc..... | 6400 | Manzanilla..... | 1724 | Honolulu to Callao.. | 5145 |

| NEW YORK TO | | NAUTICAL MILES. | | LONDON TO | | NAUTICAL MILES. | | BATAVIA | |
|----------------------------|--------|----------------------------|--------|----------------------------|--------|----------------------------|--------|----------------------------|-------|
| Canton via Pacific R. R. | 10 500 | Canton via Pacific R. R. | 10 500 | Canton via Pacific R. R. | 10 500 | Shanghai to Yokohama | 1210 | Sydney to Cape Horn | 5470 |
| " Panama | 11 500 | " Panama | 11 500 | Canton via Suez Canal | 9 650 | Manila via Suez Canal | 9 650 | Aspinwall to Panama | 51 |
| " Suez Canal | 11 500 | " Suez Canal | 11 500 | Hongkong via Yokohama | 9 780 | Queensdown | 605 | Portsmouth to | 4050 |
| Callao via Panama | 3 860 | San Blas via Panama | 3 860 | Bermuda | 3 055 | Rio de Janeiro | 5 500 | Halifax to Gt. Hope | 2165 |
| Yokohama via San Francisco | 8 100 | Callcutta via Panama | 13 400 | Port Said | 3 365 | Liverpool to | 11 355 | Constantinople | 8400 |
| Calcutta via Panama | 12 720 | Melbourne via Cape Horn | 11 355 | Melbourne via Cape Horn | 11 355 | Southernmost to Gibraltar | 1 100 | Rio de Janeiro | 5 500 |
| Melbourne via Cape Horn | 12 720 | Rio de Janeiro | 5 500 | Hongkong to Yokohama | 1 600 | Shanghai | 1 800 | Honolulu | 4 356 |
| Manila | 1 965 | Valparaiso | 12 900 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 |
| Puerto Cabello (Honduras) | 2 114 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 | Brest | 3 000 |
| Cape Race | 2 004 | Aspinwall | 2 000 | Fernandina | 830 | Calcutta | 17 500 | Port Said | 3 365 |
| Aspinwall | 2 000 | Valparaiso | 12 900 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 |
| Fernandina | 830 | Calcutta | 17 500 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 |
| Calcutta | 17 500 | Valparaiso | 12 900 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 |
| Valparaiso | 12 900 | Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 | Brest | 3 000 |
| Galway | 2 371 | Callao | 17 800 | St. Thomas | 1 593 | Brest | 3 000 | Hamburg | 3 590 |
| Callao | 17 800 | St. Thomas | 1 593 | Brest | 3 000 | Hamburg | 3 590 | Port Said | 3 365 |
| St. Thomas | 1 593 | Brest | 3 000 | Hamburg | 3 590 | Port Said | 3 365 | New Orleans to Tehuantepec | 787 |
| Brest | 3 000 | Hamburg | 3 590 | Port Said | 3 365 | New Orleans to Tehuantepec | 787 | Aspinwall | 1 400 |
| Hamburg | 3 590 | Port Said | 3 365 | New Orleans to Tehuantepec | 787 | Aspinwall | 1 400 | New Orleans | 5165 |
| Port Said | 3 365 | New Orleans to Tehuantepec | 787 | Aspinwall | 1 400 | New Orleans | 5165 | New York | 1790 |
| New Orleans to Tehuantepec | 787 | Aspinwall | 1 400 | New Orleans | 5165 | New York | 1790 | Port Jackson | 12900 |
| Aspinwall | 1 400 | New Orleans | 5165 | New York | 1790 | Port Jackson | 12900 | St. Petersburg | 13650 |
| New Orleans | 5165 | New York | 1790 | Port Jackson | 12900 | St. Petersburg | 13650 | San Francisco | 11350 |
| New York | 1790 | Port Jackson | 12900 | St. Petersburg | 13650 | San Francisco | 11350 | Aspinwall | 14000 |
| Port Jackson | 12900 | St. Petersburg | 13650 | San Francisco | 11350 | Aspinwall | 14000 | Constantinople | 8400 |
| St. Petersburg | 13650 | San Francisco | 11350 | Aspinwall | 14000 | Constantinople | 8400 | Cape Horn | 3610 |
| San Francisco | 11350 | Aspinwall | 14000 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 |
| Aspinwall | 14000 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 |
| Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 |
| Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 |
| Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 |
| Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 |
| Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 |
| Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 |
| Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 |
| Aspinwall to Panama | 51 | Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 |
| Portsmouth to | 4050 | Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 |
| Halifax to Gt. Hope | 2165 | Constantinople | 8400 | Cape Horn | 3610 | Aspinwall to Panama | 51 | Portsmouth to | 405 |

FRACTIONS.

FRACTION, or broken number, is one or more parts of a UNIT.

ILLUSTRATION.—12 inches are 1 foot. Here, 1 foot is unit, and 12 inches its parts; these therefore, are one fourth of a foot, for 3 is fourth or quarter of 12.

Vulgar Fraction is a fraction expressed by two numbers placed one over the other, with a line between them; as, 50 cents is the $\frac{1}{2}$ of a dollar.

The upper number is termed *Numerator*, the lower *Denominator*. Terms of a fraction express numerator and denominator; as, 6 and 9 are terms of $\frac{6}{9}$.

Proper fraction has numerator equal to, or less than denominator; as, $\frac{1}{2}$, etc.

Improper fraction is reverse of a proper one; as, $\frac{3}{2}$, etc.

Mixed fraction is a compound of a whole number and a fraction; as, $5\frac{1}{2}$, etc.

Compound fraction is fraction of a fraction; as, $\frac{1}{2}$ of $\frac{3}{4}$, etc.

Complex fraction is one that has a fraction for its numerator or denominator, such as, $\frac{\frac{1}{2}}{\frac{3}{4}}$, or $\frac{\frac{5}{6}}{\frac{2}{3}}$, or $\frac{2}{\frac{3}{4}}$, or $\frac{3\frac{1}{2}}{6}$, etc.

DEF.—A Fraction denotes division, and its value is equal to quotient, obtained by dividing numerator by denominator; thus, $\frac{12}{4}$ is equal to 3, and $\frac{2}{\frac{1}{2}}$ is equal to 4.

Reduction of Fractions.

Compute Common Measure or greatest Number that will divide Two or more Numbers without a remainder.

EX.—Divide greater number by less; then divide divisor by remainder; and so dividing always last divisor by last remainder, until there is no remainder, and divisor is greatest common measure required.

EXAMPLE 1.—What is greatest common measure of 1908 and 936?

$$\begin{array}{r} 1908 \ 2 \\ 936 \ 2 \\ \hline 1872 \\ 36 \ 936 \ 26 \\ \hline 72 \\ 216. \end{array} \text{ Hence } 36.$$

How many squares can there be obtained in an area of 90 by 160 feet? 180 is greatest common measure.

$$\text{Hence, } \frac{160}{10} = 16, \text{ and } \frac{90}{5} = 18; 16 \times 18 = 288, \text{ and } \frac{90 \times 160}{288} = 50.$$

Compute least Common Multiple of Two or more Numbers.

EX.—Divide given numbers by any number that will divide the greatest number without a remainder, and set quotients with undivided numbers in a second line.

Repeat second line in same manner, and so on, until there are no two numbers that can be divided; then the continued product of divisors and last quotients will be common multiple required.

EXAMPLE.—What is least common multiple of 40, 50, 60?

$$\begin{array}{r} 40 \ 50 \ 60 \\ 5 \ 8 \ 10 \ 5 \\ \hline 2 \ 8 \ 2 \ 1 \\ \hline 4 \ 1 \ 1 \end{array}$$

Then $5 \times 5 \times 2 \times 4 \times 1 \times 1 = 200$.

To Reduce a Fraction to its Lowest Term.

—Divide terms by any number or series of numbers that will divide them a remainder, or by their greatest common measure.

EXAMPLE.—Reduce $\frac{12}{10}$ of a foot to its lowest terms.

$$\frac{12}{10} \div 2 = \frac{6}{5} \div 3 = \frac{2}{5}, \text{ or } 9 \text{ ins.}$$

To Reduce a Mixed Fraction to its Equivalent, an Improper Fraction.

RULE.—Multiply whole number by denominator of fraction and to product add numerator; then set that sum above denominator.

EXAMPLE 1.—Reduce $23\frac{2}{3}$ to a fraction. $\frac{23 \times 6 + 2}{6} = \frac{140}{6} = \frac{70}{3}$.

2.—Reduce $12\frac{3}{4}$ inches to its value in feet. $12 \div 6 = 20\frac{3}{4} = 1 \text{ foot } 8\frac{1}{2} \text{ ins.}$

To Reduce a Complex Fraction to a Simple one.

RULE.—Reduce the two parts both to a simple fraction, multiply numerator of reduced fraction by denominator of reduced denominator, and denominator of numerator fraction by denominator of denominator fraction.

EXAMPLE.—Simplify complex fraction $\frac{2\frac{3}{4}}{4\frac{1}{2}}$. $\frac{2\frac{3}{4}}{4\frac{1}{2}} = \frac{\frac{11}{4}}{\frac{9}{2}} = \frac{11 \times 2}{4 \times 9} = \frac{11 \times 5}{3 \times 24} = \frac{5}{9}$

To Reduce a Whole Number to an Equivalent Fraction having a given Denominator.

RULE.—Multiply whole number by given denominator, and set product over said denominator.

EXAMPLE.—Reduce 8 to a fraction, denominator of which shall be 9.
 $8 \times 9 = 72$; then $72/9$ result required.

To Reduce a Compound Fraction to an Equivalent Simple one.

RULE.—Multiply all numerators together for a numerator, and all denominators together for a denominator.

NOTE.—When there are terms that are common, they may be cancelled.

EXAMPLE.—Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction.

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$. Or, $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$, by cancelling 2's and 3's.

To Reduce Fractions of different Denominations to Equivalents having a Common Denominator.

RULE.—Multiply each numerator by all denominators except its own for new numerators; and multiply all denominators together for a common denominator.

NOTE.—In this, as in all other operations, whole numbers, mixed or compound fractions, must first be reduced to form of simple fractions.

2. When many of denominators are same, or are multiples of each other, ascertain their least common multiple, and then multiply the terms of each fraction by quotient of least common multiple divided by its denominator.

EXAMPLE.—Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

$$\begin{array}{l} 1 \times 3 \times 4 = 12 \\ 2 \times 2 \times 4 = 16 \\ 3 \times 2 \times 3 = 18 \\ 2 \times 3 \times 4 = 24 \end{array} \left\} = \frac{12}{24} = \frac{16}{24} = \frac{18}{24} \right.$$
or $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$

Addition.

RULE.—If fractions have a common denominator, add all numerators together, and place sum over denominator.

NOTE.—If fractions have not a common denominator, they must be reduced to also, compound and complex must be reduced to simple fractions.

EXAMPLE 1.—Add $\frac{1}{4}$ and $\frac{3}{4}$ together. $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$.

of $\frac{2}{3}$ of $\frac{1}{10}$ to $2\frac{1}{2}$ of $\frac{2}{3}$.

$\times \frac{2}{3} \times \frac{1}{10} = \frac{2}{30}$. $2\frac{1}{2}$ of $\frac{2}{3} = 1\frac{1}{3} \times \frac{2}{3} = \frac{2}{3}$.

$\frac{2}{3} = \frac{400}{600} + \frac{576}{600} = 1\frac{31}{30}$, reduced to equivalent fractions having denominator and thence to its lowest terms.

Subtraction.

RE.—Prepare fractions same as for other operations, when necessary; then subtract one numerator from the other, and set remainder over common denominator.

EXAMPLE.—What is difference

between $\frac{3}{8}$ and $\frac{5}{8}$?

$$\left. \begin{array}{r} 6 \times 9 = 54 \\ 3 \times 8 = 24 \\ 8 \times 9 = 72 \end{array} \right\} = \frac{54}{72} - \frac{24}{72} = \frac{30}{72} = \frac{5}{12}.$$

Multiplication.

RE.—Prepare fractions as previously required; multiply all numerators together for a new numerator, and all denominators together for a new denominator.

EXAMPLE 1.—What is product of $\frac{3}{4}$ and $\frac{2}{3}$? $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$.

—What is product of 6 and $\frac{2}{3}$ of 5? $6 \times \frac{2}{3}$ of 5 = $\frac{6}{1} \times \frac{10}{3} = \frac{60}{3} = 20$.

Division.

RE.—Prepare fractions as before; then divide numerator by the numerator, denominator by the denominator, if they will exactly divide; but if not, invert terms of divisor, and multiply dividend by it, as in multiplication.

EXAMPLE 1.—Divide $\frac{3}{5}$ by $\frac{2}{3}$. $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$.

Divide $\frac{5}{6}$ by $\frac{2}{15}$. $\frac{5}{6} \div \frac{2}{15} = \frac{5}{6} \times \frac{15}{2} = \frac{15}{2} \times \frac{5}{6} = \frac{75}{6} = \frac{25}{2} = 12\frac{1}{2}$.

Application of Reduction of Fractions.

Compute Value of a Fraction in Parts of a Whole Number.

RE.—Multiply whole number by numerator, and divide by denominator; then, if anything remains, multiply it by the parts in next inferior denomination, and by denominator, as before, and so on as far as necessary; so shall the quotient in order be value of fraction required.

EXAMPLE 1.—What is value of $\frac{1}{3}$ of $\frac{2}{3}$ of 9?

$$\frac{1}{3} \text{ of } \frac{2}{3} = \frac{2}{9}, \text{ and } \frac{2}{9} \times 9 = \frac{18}{9} = 2.$$

Reduce $\frac{3}{4}$ of a pound to an avoirdupois ounce.

$$\begin{array}{l} 4) 3 \text{ (or lbs.} \\ \underline{16} \text{ ounces in a lb.} \\ 4) 48 \text{ (12 ounces.} \end{array}$$

Reduce a Fraction from one Denomination to another.

RE.—Multiply number of required denomination contained in given denominator numerator if reduction is to be to a less name, but by denominator if to a

EXAMPLE 1.—Reduce $\frac{1}{4}$ of a dollar to fraction of a cent.

$$\frac{1}{4} \times 100 = \frac{100}{4} = 25.$$

Reduce $\frac{1}{6}$ of an avoirdupois pound to fraction of an ounce.

$$\frac{1}{6} \times 16 = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}.$$

Reduce $\frac{3}{8}$ of $\frac{3}{4}$ of a mile to the fraction of a foot.

$$\frac{3}{8} \text{ of } \frac{3}{4} = \frac{9}{32} \times 5280 = \frac{47520}{32} = 1485.$$

of Three in Vulgar Fractions, see Decimals, page 94.

DECIMALS.

A DECIMAL is a fraction, having for its denominator a UNIT with as many ciphers annexed as the numerator has places; it is usually expressed by writing the numerator only, with a point at the left of it. Thus, $\frac{4}{10}$ is .4; $\frac{85}{100}$ is .85; $\frac{0075}{10000}$ is .0075; and $\frac{125}{100000}$ is .00125. When there is a deficiency of figures in the numerator, prefix ciphers to make up as many places as there are ciphers in denominator.

Mixed numbers consist of a whole number and a fraction; as, 3.25, which is the same as $3\frac{25}{100}$, or $3\frac{1}{4}$.

Ciphers on right hand make no alteration in their value; for .4, .40, .400 are decimals of same value, each being $\frac{4}{10}$, or $\frac{2}{5}$.

Addition.

RULE.—Set numbers under each other according to value of their places, as in whole numbers, in which position the decimal points will stand directly under each other; then begin at right hand, add up all the columns of numbers as in integers, and place the point directly below all the other points.

EXAMPLE.—Add together 25.125 and 293.7325.

$$\begin{array}{r} 25.125 \\ 293.7325 \\ \hline 318.8575 \text{ sum.} \end{array}$$

Subtraction.

RULE.—Set numbers under each other as in addition; then subtract as in whole numbers, and point off decimals as in last rule.

EXAMPLE.—Subtract 15.15 from 89.1759.

$$\begin{array}{r} 89.1759 \\ 15.15 \\ \hline 74.0259 \text{ remainder.} \end{array}$$

Multiplication.

RULE.—Set the factors, and multiply them together same as if they were whole numbers; then point off in product just as many places of decimals as there are decimals in both factors. But if there are not so many figures in product, supply deficiency by prefixing ciphers.

EXAMPLE.—Multiply 1.56 by .75.

$$\begin{array}{r} 1.56 \\ .75 \\ \hline 780 \\ 1092 \\ \hline 1.1700 \text{ product.} \end{array}$$

By Contraction.

To Contract the Operation so as to retain only as many Decimal places in Product as may be required.

RULE.—Set unit's place of multiplier under figure of multiplicand, the place of which is same as is to be retained for the last in product, and dispose of the rest of figures in contrary order to which they are usually placed.

In multiplying, reject all figures that are more to right hand than each multiplier, and set down the products, so that their right-hand figures may fall in a line directly below each other, and increase first figure of line with what would have arisen from figures thus, add 1 for every result from 5 to 14, 2 from 15 to 29, 3 from 30 to 39, 4 from 40 to 49, etc., and the sum will be the product as required.

As 13.574 93 by 46.2051, and retain only 5 in the product.

$$\begin{array}{r} 13.574 \ 93 \\ 1 \ 502.64 \\ \hline 54 \ 299 \ 72 \\ 8 \ 144 \ 96 + 2 \text{ for } 18 \\ 271 \ 50 + 2 \text{ " } 18 \\ 6 \ 79 + 4 \text{ " } 35 \\ 14 + 1 \text{ " } 5 \\ \hline 627.23 \ 11 \end{array}$$

Is required, increase last figure with what would have arisen from all the

Division.

RULE.—Divide as in whole numbers, and point off in quotient as many places for decimals as decimal places in dividend exceed those in divisor; but if there are not so many places, supply deficiency by prefixing ciphers.

EXAMPLE. Divide 53 by 6.75. $6.75 \overline{) 53.0000} (= 7.851+)$.

Here 5 ciphers are annexed to dividend to extend division.

By Contraction.

RULE.—Take only as many figures of divisor as will be equal to number of figures, both integers and decimals, to be in quotient, and ascertain how many times they may be contained in first figures of dividend, as usual.

Let each remainder be a new dividend; and for every such dividend leave out one figure more on right-hand side of divisor, carrying for figures cut off as in Contraction of Multiplication.

NOTE.—When there are not so many figures in divisor as there are required to be in quotient, continue first operation until number of figures in divisor are equal to those remaining to be found in quotient, after which begin the contraction.

EXAMPLE.—Divide 2508.92806 by 92.41035 , so as to have only four places of decimals in quotient.

| | | | |
|-----------------------------------|-----------|----------|-----------|
| $92.4103 \overline{) 2508.92806}$ | 27.1498 | 13.849 | 912 |
| $1848207 + 1$ | | 9241 | $832 + 4$ |
| 660721 | | 4608 | 80 |
| $646872 + 2$ | | 3696 | $74 + 2$ |
| 13849 | | 912 | 6 |

Reduction of Decimals.

To Reduce a Vulgar Fraction to its Equivalent Decimal.

RULE.—Divide numerator by denominator, annexing ciphers to numerator to extent that may be necessary.

EXAMPLE.—Reduce $\frac{4}{5}$ to a decimal. $5 \overline{) 4.0}$
 $\phantom{5 \overline{) 4.0}} \underline{8}$

To Compute Value of a Decimal in Terms of an Inferior Denomination.

RULE.—Multiply decimal by number of parts in next lower denomination, and cut off as many places for a remainder, to right hand, as there are places in given decimal.

Multiply that remainder by the parts in next lower denomination, again cutting off for a remainder, and so on through all the parts of integer.

EXAMPLE 1.—What is value of .875 dollars?

$.875$
 $ \overline{100}$
Cents, 87.500
 $ \overline{10}$
Mills, 5.000 = 87 cents 5 mills.

2.—What is volume of .140 cube feet in inches?

$.140$
 $ \overline{1728}$ cube inches in a cube foot.
241.920 cube ins.

3.—What is value of .00129 of a foot?

$.01548$ ins.

To Reduce a Decimal to an Equivalent Decimal of a Higher Denomination.

RULE.—Divide by number of parts in next higher denomination, continuing operation as far as required.

EXAMPLE 1.—Reduce 1 inch to decimal of a foot.

$12 \overline{) 1.00000}$
 $\phantom{12 \overline{) 1.00000}} \underline{.08333} + \text{foot.}$

2.—Reduce $14''$ $12'''$ to decimal of a minute.

$60 \overline{) 14.12}$
 $\phantom{60 \overline{) 14.12}} \underline{85.2}$
 $\phantom{60 \overline{) 14.12}} \underline{14.2}$
 $\phantom{60 \overline{) 14.12}} \underline{.23666} + \text{minute.}$

When there are several numbers, to be reduced all to decimal of highest denomination. — Reduce them all to lowest denomination, and proceed as for one denomination.

EXAMPLE. — Reduce 5 feet 10 inches and 3 barleycorns to decimal of a yard.

| Feet. | Inch. | Bc. |
|---------|-------|--------|
| 5 | 10 | 3 |
| 12 | | |
| 70 | | |
| 3 | | |
| 3 219. | | |
| 12 | | |
| 71. | | |
| 3 | | |
| 5.9166 | | |
| 1.9722+ | | yards. |

Rule of Three.

RULE. — Prepare the terms by reducing vulgar fractions to decimals, compound numbers to decimals of the highest denomination, first and third terms to same denomination; then proceed as in whole numbers.

EXAMPLE. — If .5 of a ton of iron cost .75 of a dollar, what will .625 of a ton cost?

$$\begin{array}{r} .5 : .75 :: .625 \\ .625 \\ 5) .46875 \\ .9375, \text{ dollar.} \end{array}$$

DUODECIMALS.

In Duodecimals, or Cross Multiplication, the dimensions are taken in feet, inches, and twelfths of an inch.

RULE. — Set dimensions to be multiplied together one under the other, feet under feet, inches under inches, etc.

Multiply each term of multiplicand, beginning at lowest, by feet in multiplier, and set result of each immediately under its corresponding term, carrying 1 for every 12 from one term to the other. In like manner, multiply all multiplicand by inches of multiplier, and then by twelfth parts, setting result of each term one place farther to right hand for every multiplier. And sum of products will give result.

EXAMPLE. — How many square inches are there in a board 35 feet 4.5 inches long and 12 feet $3\frac{1}{2}$ inches wide?

$$\begin{array}{r} 35 \text{ feet } 4.5 \text{ inches} \\ \times 12 \text{ feet } 3\frac{1}{2} \text{ inches} \\ \hline 420 \text{ sq. ft. } 60 \text{ sq. ins.} \\ 120 \text{ sq. ft. } 54 \text{ sq. ins.} \\ \hline 432 \text{ sq. ft. } 114 \text{ sq. ins.} \end{array}$$

| Feet. | Inch. | Twelfths. |
|-------|-------|-----------|
| 35 | 4 | 6 |
| 12 | 3 | 4 |
| 424 | 6 | 0 |
| 8 | 10 | 1 |
| 11 | 9 | 6 |
| 434 | 3 | 11 0 0 |

Value of Duodecimals in Square Feet and Inches.

| | Sq. Ft. | Sq. Ins. | | Sq. Ft. | Sq. Ins. |
|----------------|-------------------|----------|---------------------------------------|---------------------|--------------------|
| 1 Foot..... | = 1 | or 144 | $\frac{1}{12}$ of 1 twelfth = | $\frac{1}{1728}$ | or .00058333, etc. |
| 1 Inch..... | = $\frac{1}{12}$ | " 12. | $\frac{1}{12}$ of $\frac{1}{12}$ of " | = $\frac{1}{20736}$ | " .00000486, etc. |
| 1 Twelfth..... | = $\frac{1}{144}$ | " 1. | | | |

ILLUSTRATION. — What number of square inches are there in a floor .100 feet 6 inches long and 25 feet 6 inches and 6 twelfths broad?

$$2566 \text{ feet } 11 \text{ ins. } 3 \text{ twelfths} = 2566 \text{ feet } 1135 \text{ ins.}$$

MEAN PROPORTION.

MEAN PROPORTION is proportion to two given numbers or terms.

— Multiply two numbers or terms together, and extract square root of their

— What is mean proportionate velocity to 16 and 81?

$$16 \times 81 = 1296, \text{ and } \sqrt{1296} = 36 \text{ mean velocity.}$$

RULE OF THREE.

RULE OF THREE.—It is so termed because three terms or numbers are given to ascertain a fourth.

It is either **DIRECT** or **INVERSE**.

It is **Direct** when more requires more, or less requires less; thus, if 3 barrels of flour cost \$18, what will 10 barrels cost?

In this case Proportion is *Direct*, and stating must be,

As 3 : 10 :: 18 : 60.

It is **Inverse** when more requires less, or less requires more; thus, if 6 men build a certain quantity of wall in 10 days, in how many days will 8 men build like quantity.

Or, if 3 men dig 100 feet of trench in 7 days, in how many days will 2 men dig same work?

In the Proportion is *Inverse*, and stating must be,

As 8 : 6 :: 10 : 7.5, and 2 : 3 :: 7 : 10.5.

The fourth term is always ascertained by multiplying 2d and 3d terms together, and dividing their product by 1st term.

The three given numbers necessary for the stating, two of them contain the situation, and the third a demand.

EX.—State question by setting down in a straight line the three necessary terms in following manner:

1st term be that of *supposition*, of same denomination as the result, or 4th term is to be, making *demanding* number 2d term, and the other number 1st term question is in *Direct Proportion*, but contrariwise if in *Inverse Proportion*; 3d let *demanding* number be 1st term.

Multiply 2d and 3d terms together, and divide by 1st, and product will give required 4th term sought, of same denomination as 2d term.

—If first and third terms are of different denominations, reduce them to same. If, after division is any remainder, reduce it to next lower denomination, divide by divisor as before, and it will be of this last denomination.

Sometimes two or more statings are necessary, which may always be known by the question.

EXAMPLE 1.—If 20 tons of iron cost \$225, what will 30 tons cost?

| | | |
|-------|--------|---------------|
| Tons. | Tons. | Dolla. |
| 20 : | 30 :: | 225 |
| | | <u>500</u> |
| 20 | 11 250 | 0 |
| | | 5625 dollars. |

A wall that is to be built to height of 36 feet, was raised 9 feet by 16 men in 4 days; how many men could finish it in 6 days at same rate of working?

| | | | |
|-------|-------|------|------|
| Days. | Days. | Men. | Men. |
| 4 : | 6 :: | 16 : | 24 |

, if 9 feet requires 24 men, what will 36 feet require?

| | | | |
|-----|-------|------|-----|
| 9 : | 36 :: | 24 : | 144 |
|-----|-------|------|-----|

COMPOUND PROPORTION.

COMPOUND PROPORTION is a rule by means of which such questions as require two or more statings in simple proportion (Rule of Three) are resolved in one.

It is, however, but little used, and not easily acquired, it is deemed preferable to omit it here, and to show the operation by two or more statings in Simple Proportion.

PROBLEM 1.—How many men can dig a trench 135 feet long in 8 days, if 54 feet in 6 days?

| | | | | | |
|---------------|-------|--------|------|------|---|
| First.....As | Feet. | Feet. | Men. | Men. | |
| | 54 : | 135 :: | 16 : | 40 | 1 |
| | | | | | 5 |
| Second.....As | Days. | Days. | Men. | Men. | |
| | 8 : | 6 :: | 40 : | 30 | |

96 COMPOUND PROPORTION.—INVOLUTION.—EVOLUTION.

2.—If a man travel 130 miles in 3 days of 12 hours each, how many days of 10 hours each would he require to travel 360 miles?

| | Miles. | Miles. | Days. | Days. |
|---------------|--------|----------|--------|----------|
| First.....As | 130 | : 360 :: | 3 | : 8.307+ |
| | | | Hours. | Hours. |
| Second.....As | 10 | : 12 :: | 8.307 | : 9.9684 |

3.—If 12 men in 15 days of 12 hours build a wall 30 feet long, 6 wide, and 3 deep, in how many days of 8 hours will 60 men build a wall 300 feet long, 8 wide, and 6 deep?

By Cancellation.

RULE.—On right of a vertical line put the number of same denomination as that of required answer.

Examine each simple proportion separately, and if its terms demand a *greater result than 3d term*, put *larger number on right and lesser on left of line*; but if its terms demand a *less result than 3d term*, put *smaller number on right and larger on left of line*.

Then *Cancel* the numbers divisible by a common divisor, and evolve the 4th term or result required.

Take Illustration 1, page 95: 3d term, or term of supposition of same denomination as required result, 16 men.

| Statement. | | | Result by Cancellation. |
|------------|---|---|-------------------------|
| 16 | 135 feet require more men than 54 feet, | | 16 2 |
| 54 | and 8 days less men than 6 days. | | 2 54 135 5 |
| 8 | | $2 \times 5 \times 3 = 30 \text{ men.}$ | 8 8 3 |

ILLUSTRATION 3.—3d term, 15 days.

| Statement. | | | Result by Cancellation. |
|------------|---|--|-------------------------|
| 15 | 60 men require less days than 12 men, | | 15 |
| 60 | 8 hours more days than 12 hours, 300 feet | | 4 60 12 3 |
| 12 | more days than 30 feet, 8 feet more days | | 8 8 12 4 |
| 8 | than 6 feet, and 6 feet more days than | | 30 300 10 |
| 30 | 3 feet. | | 8 8 |
| 6 | | $3 \times 4 \times 10 = 120 \text{ days.}$ | 3 8 |
| 3 | | | |

INVOLUTION.

INVOLUTION is multiplying any number into itself a certain number of times. Products obtained are termed *Powers*. The number is termed the *Root*, or first power.

When a number is multiplied by itself once, product is *square* of that number; twice, *cube*; three times, *biquadrate*; etc. Thus, of the number 5.

| | |
|--------------------------------------|---|
| 5 | is the Root, or 1st power. |
| $5 \times 5 = 25$ | " Square, or 2d power, and is expressed 5 ² . |
| $5 \times 5 \times 5 = 125$ | " Cube, or 3d power, and is expressed 5 ³ . |
| $5 \times 5 \times 5 \times 5 = 625$ | " Biquadrate, or 4th power, and is expressed 5 ⁴ . |

The lesser figure set superior to number denotes the power, and is termed the *Index* or *Exponent*.

ILLUSTRATION 1.—What is cube of 9?

2.—What is cube of $\frac{3}{4}$?

3.—What is 4th power of 1.5?

729.
 $\frac{27}{64}$.
5.0625.

EVOLUTION.

EVOLUTION is ascertaining *Root* of any number.

✓ placed before any number indicates that *square root* of that number is re-

s shown.

Characte-

any other root by placing the index above it.

$\sqrt{25} = 5$; $4 + 2 = \sqrt{36}$.

$\sqrt[3]{27} = 3$, and $\sqrt[3]{64} = 4$.

are termed *Surd Roots*.

To Extract Square Root.

—Point off given number from units' place, into periods of two figures each. Take greatest square in left-hand period, and place its root in quotient; subtract square number from this period, and to remainder bring down next period dividend.

Take this root for a divisor; ascertain how many times it is contained in dividend exclusive of right-hand figure, which, when multiplied by number to be put to the right of this divisor, product will be equal to, or next less than dividend; multiply divisor by last quotient figure, and subtract product from dividend; bring down next period, and proceed as before.

—Mixed decimals must be pointed off both ways from units.

—Mixed decimals must be pointed off both ways from units.

PLATE I.—What is square root of 2?

$$\begin{array}{r} 1 \overline{) 2.000000} \quad (1.414213562 \\ 1 \overline{) 1} \\ \hline 1 \overline{) 100} \\ 4 \overline{) 96} \\ \hline 281 \overline{) 400} \\ 1 \overline{) 281} \\ \hline 282 \overline{) 11900} \end{array}$$

2. What is square root of 144?

$$\begin{array}{r} 1 \overline{) 144} \quad (12 \\ 1 \overline{) 1} \\ \hline 22 \overline{) 044} \\ 44 \end{array}$$

Square Roots of Fractions.

—Reduce fractions to their lowest terms, and that fraction to a decimal, proceed as in whole numbers and decimals.

—When terms of fractions are squares, take root of each and set one above the other; as the root of $\frac{4}{9}$.

PLATE.—What is square root of $\frac{2}{3}$?

.8660254.

1. Compute 4th or 8th Root of a Number, etc.

—For the 4th root extract square root twice, and for 8th root thrice, etc.

To Extract Cube Root.

—From table of roots (page 272) take nearest cube to given number, and the assumed cube.

As given number added to twice assumed cube, is to assumed cube added to given number, so is root of assumed cube to required root, *nearly*; and by like manner the root thus found as an assumed cube, and proceeding in like manner, another root will be found still nearer; and in like manner as far as seemed necessary.

TABLE.—What is cube root of 10517.9?

1 cube, page 272; 10648, root 22.

$$\begin{array}{r} 10648. \quad 10517.9 \\ \underline{2} \quad \underline{2} \\ 21296 \quad 21035.8 \\ \underline{10517.9} \quad \underline{10648.} \\ 31813.9 : 31683.8 :: 22 : 21.9+ \end{array}$$

Determine or to Compute the Square or Cube Roots of Integers, Whole Numbers, and of Integers and Decimals, Table of Squares and Cubes, and Rules, pp. 272, 300.

To Extract any Root whatever.

Assumed number.

" Index of the power.

Let A represent assumed power, r its root.
R " required root of P.

Sum of $n+1 \times A$ and $n-1 \times P$ is to sum of $n+1 \times P$ and $n-1 \times A$, as assumed root r to required root R.

EXAMPLE.—What is cube root of 1500?

1 cube, page 272, is 1331, root 11.

$$\begin{array}{l} P = 1500, n = 3, A = 1331, r = 11; \\ \text{then, } n+1 \times A = 5324, n+1 \times P = 6000 \\ n-1 \times P = 3000, n-1 \times A = 2662 \\ \hline 8324 \quad 8662 \\ \text{I} \end{array}$$

To Compute the Root of an Even Power greater than any given in Table of Square and Cube Roots.

RULE.—Extract square or cube root of it, which will reduce it to half the given power; then square or cube root of that power; and so on until required root is attained.

EXAMPLE 1.—Suppose a 12th power is given; the square root of that reduces it a 6th power, and the square root of 6th power to a cube.

2.—What is biquadrate, or 4th root, of 2 560 000?

$$\sqrt[4]{2\,560\,000} = 1600, \text{ and } \sqrt{1600} = 40.$$

NOTE.—For other rules for extraction of roots see pp. 301-4.

PROPERTIES OF NUMBERS.

1. A *Prime Number* is that which can only be measured (divided without remainder) by 1 or unity.

2. A *Composite Number* is that which can be measured by some number greater than unity.

3. A *Perfect Number* is that which is equal to the sum of all its divisors or quotients; as $6 = \frac{6}{1}, \frac{6}{2}, \frac{6}{3}$.

4. If sum of the digits constituting any number be divisible by 3 or 9, the number is divisible by them.

5. A square number cannot terminate with an odd number of ciphers.

6. No square number can terminate with two equal digits, except two ciphers or two fours.

7. No number, the last digit of which is 2, 3, 7, or 8, is a square number.

Powers of the first Nine Numbers.

| 1st. | 2d. | 3d. | 4th. | 5th. | 6th. | 7th. | 8th. | 9th. |
|------|-----|-----|------|-------|--------|---------|----------|-----------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 | 65536 | 262144 |
| 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 | 390625 | 1953125 |
| 6 | 36 | 216 | 1296 | 7776 | 46656 | 279936 | 1679616 | 10077696 |
| 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 | 5764801 | 40353607 |
| 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 |
| 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 |

POSITION.

POSITION is of two kinds, SINGLE and DOUBLE, and it is determined by the number of SUPPOSITIONS.

Single Position.

RULE.—Take any number, and proceed with it as if it were the correct one; the result is to given sum, so is supposed number to number required.

EXAMPLE 1.—A commander of a vessel, after sending away in boats $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of his crew, had left 300; what number had he in command?

Suppose he had 600.

$\frac{1}{3}$ of 600 is 200

$\frac{1}{4}$ of 600 is 150

$\frac{1}{5}$ of 600 is 120

450

150 : 300 :: 600 : 1200 men.

2.—A person asked his age, replied, if $\frac{2}{3}$ of my age be multiplied by 2, and that product added to half the years I have lived, the sum will be 75. How old was he?
37.5 years.

Double Position.

RULE.—Assume any two numbers, and proceed with each according to conditions of question; multiply results or *errors* by contrary supposition; that is, first position by last error, and last position by first error.

If errors are too great, mark them +; and if too little, —.

Then, if errors are *alike*, divide *difference* of products by *difference* of errors; but if they are *unlike*, divide *sum* of the products by *sum* of errors.

EXAMPLE 1.—A asked B how much his boat cost; he replied, that if it cost him 6 times as much as it did, and \$30 more, it would have cost him \$300. What was price of the boat?

| | |
|-----------------------------------|------------------|
| Suppose it cost him..... 60 | 30 |
| 6 times. | 6 times. |
| <u>360</u> | <u>180</u> |
| and 30 more | and 30 more |
| <u>390</u> | <u>210</u> |
| <u>300</u> | <u>300</u> |
| 90+ | 90— |
| 30 2d position. | 60 1st position. |
| <u>90</u> 2700 | <u>5400</u> |
| <u>90</u> 5400 | |
| 180 8100 (45 dollars.) | |

2.—What is length of a fish when the head is 5 inches long, tail as long as its head and half its body, and body as long as both head and tail? 6 feet.

FELLOWSHIP.

FELLOWSHIP is a method of ascertaining gains or losses of individuals engaged in joint operations.

Single Fellowship.

RULE.—As the whole stock is to the whole gain or loss, so is each share to the gain or loss on that share.

EXAMPLE.—Two men drew a prize in a lottery of \$9500. A paid \$3, and B \$2 for the ticket; how much is each share?

$$\begin{aligned} 5 : 9500 :: 3 : 5700, \text{ A's share.} \\ 5 : 9500 :: 2 : 3800, \text{ B's share.} \end{aligned}$$

Double Fellowship,

Or Fellowship with Time.

RULE.—Multiply each share by time of its interest; then, as sum of products is to product of each interest, so is whole gain or loss to each share of gain or loss.

EXAMPLE.—A cutter's company take a prize of \$10000, which is to be divided according to their rate of pay and time of service on board. The officers have been on board 6 months, and the crew 3 months; pay of lieutenants is \$100, ensigns \$50 and crew \$10 per month; and there are 2 lieutenants, 4 ensigns, and 50 men; what is each one's share?

$$\begin{aligned} 2 \text{ lieutenants} & \dots\dots\dots \$100 = 200 \times 6 = 1200 \\ 4 \text{ ensigns} & \dots\dots\dots 50 = 200 \times 6 = 1200 \\ 50 \text{ men} & \dots\dots\dots 10 = 500 \times 3 = 1500 \\ & \dots\dots\dots 3900 \end{aligned}$$

$$\begin{aligned} \text{Lieutenants} & \dots\dots\dots 3900 : 1200 :: 10000 : 3076.92 \div 2 = 1538.46 \text{ doll.} \\ \text{Ensigns} & \dots\dots\dots 3900 : 1200 :: 10000 : 3076.92 \div 4 = 769.23 \text{ " } \\ \text{Men} & \dots\dots\dots 3900 : 1500 :: 10000 : 3846.16 \div 50 = 76.92 \text{ " } \end{aligned}$$

589817

PERMUTATION.

PERMUTATION is a rule for ascertaining how many different ways any given number of numbers of things may be varied in their position.

Permutation of the three letters *abc*, taken *all together*, are 6; taken *two* and *two*, are 6; and taken *singly*, are 3.

RULE.—Multiply all the terms continually together, and last product will give result.

EXAMPLE 1.—How many variations will the nine digits admit of?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880.$$

2.—How many years would there be required to elapse before 10 persons could be seated in a varied position collectively, each day at dinner, including one day in every 4 years for a leap year?

9935 years, 42 days.

When only part of the Numbers or Elements are taken at once. RULE.—Take a series of numbers, beginning with number of things given, decreasing by 1, until number of terms equals number of things or quantities to be taken at a time, and product of all the terms will give sum required.

EXAMPLE 1.—How many changes can be made with 2 events in 5?

$5 - 1 = 4$, and $4 \times 5 = 2$ terms. Hence, $5 \times 4 = 20$ changes.

2.—How many changes of 2 will 3 playing cards admit of?

$3 - 1 = 2$, and $2 \times 3 = 2$ terms. Hence, $2 \times 3 = 6$ changes.

3.—How many changes can be rung with 4 bells (taken 4 and 4 together) out of 6?

$4 - 1 = 3$, and $3 \times 4 \times 5 \times 6 = 4$ terms or changes.

Hence, $3 \times 4 \times 5 \times 6 = 360$ changes.

When several of the Elements are alike. RULE.—Ascertain the permutations of all the numbers or things, and of all that can be made of each separate kind or division; divide number of permutations of whole by product of the several partial permutations, and quotient will give number of permutations.

EXAMPLE.—How many permutations can be made out of the letters of the word persevere (9 letters, having 4 e's and 2 r's)?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880;$$

$$1 \times 2 \times 3 \times 4 = 24 \text{ for the e's; } 1 \times 2 = 2 \text{ for the r's, and } 24 \times 2 = 48.$$

$$\text{Hence, } 362880 \div 48 = 7560.$$

Or, Add logarithms of all the terms together, and number for the sum will give result.

EXAMPLE 1.—How many permutations can be made with three letters or figures?

$$\text{Log. } 1 = .00$$

$$2 = .30103$$

$$3 = .4771213$$

$$.7781513 = \text{log. of number 6.}$$

2.—How many variations will 15 numbers in 16 places admit of?

Add logarithms of numbers 1 to 16 and take logarithm of their sum—

$$\text{viz., } 13.32066197 = 20922789888000.$$

Number of positions of the blocks in the "15 puzzle" is as above for their 16 permutations.

Permutations,

Whereby any questions of Permutation may be solved by Inspection, number of terms not exceeding 20.

| | | | | | |
|----|-----------|----|----------------|----|---------------------|
| 9 | 362880 | 13 | 6227020800 | 17 | 355687428096000 |
| 10 | 3628800 | 14 | 87178291200 | 18 | 6402373705728000 |
| 11 | 39916800 | 15 | 1307674368000 | 19 | 121645100408832000 |
| 12 | 479001600 | 16 | 20922789888000 | 20 | 2432902008176640000 |

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is a series of numbers increasing or decreasing by a constant number or difference; as, 1, 3, 5, 7, 12, 9, 6, 3. The which form the series are designated *Terms*; the first and last *Extremes*, and the others *Means*.

If three of following elements are given, the remaining two can be ascertained. First term, Last term, Number of terms, Common Difference, and Sum of terms.

To Compute First Term.

First term, Number of terms, and Sum of series are given. RULE.—From twice sum of series, divided by number of terms, subtract last term.

$$; \frac{S}{n} - \frac{d(n-1)}{2}; \text{ and } \sqrt{\left(\frac{l + .5d}{2}\right)^2 - 2dS} \pm .5d = a \quad a \text{ represents first term, } n \text{ number of, and } S \text{ sum of all terms, and } d \text{ common difference.}$$

PROB.—A man travelled 390 miles in 12 days, travelling 60 miles last day. How far did he travel first day?

$$\frac{390 \times 2}{12} = 65, \text{ and } 65 - 60 = 5 \text{ first term.}$$

To Compute Last Term.

First term, Common Difference, and Number of terms are given. RULE.—From first term, multiply by common difference, and to product add first term.

PROB.—A man travelled for 12 days, at the rate of 5 miles first day, 10 second, how far did he travel the last day?

$$12 - 1 \times 5 = 55, \text{ and } 55 + 5 = 60 \text{ miles.}$$

First term, Number of terms, and Sum of series are given. RULE.—Divide sum of series by number of terms, and from quotient subtract first term.

$$-a; \quad \sqrt{2dS + (a - .5d)^2} \pm .5d; \quad \text{and } \frac{S}{n} + \frac{d(n-1)}{2} = l.$$

PROB.—A man travelled 360 miles in 12 days, commencing with 5 miles first day, how far did he travel last day?

$$\frac{360 \times 2}{12} = 65, \text{ and } 65 - 5 = 60 \text{ miles.}$$

To Compute Number of Terms.

First term, Common Difference, and Extremes, or First and Last term, are given. RULE.—Divide difference of extremes by common difference, and add 1 to quotient.

PROB.—A man travelled 3 miles first day, 5 second, 7 third, and so on, till he travelled 27 miles in one day; how many days had he travelled at close of last day?

$$27 - 3 \div 2 = 12, \text{ and } 12 + 1 = 13 \text{ days.}$$

First term, Common Difference, and Sum of series are given. RULE.—Divide twice sum of series by number of terms, and from quotient subtract first term.

$$\text{Or, } \frac{l-a}{d} + 1; \quad \sqrt{\frac{2S}{d} + \left(\frac{a-d}{2}\right)^2} \pm \frac{d-a}{2};$$

$$\text{and } \sqrt{\left(\frac{2l+d}{2}\right)^2 - \frac{2S}{d}} \pm \frac{2l+d}{2} = n$$

PROB.—A man travelled 840 miles, walking 3 miles first day, how many days was he travelling?

$$\frac{840 \times 2}{3 + 57} = \frac{1680}{60} = 28 \text{ days.}$$

- last.

To Compute Common Difference.

When Number of terms and Extremes are given. RULE.—Divide difference between extremes by 1 less than number of terms.

$$\text{Or, } \frac{2S - 2an}{n(n-1)}; \quad \frac{l+a \times l-a}{2S-l-a}; \quad \text{and } \frac{2nl-2S}{n(n-1)} = d.$$

ILLUSTRATION.—Extremes are 3 and 15, and number of terms 7; what is the difference?

$$15 - 3 \div (7 - 1) = \frac{12}{6}, \text{ and } \frac{12}{6} = 2 \text{ com. dif.}$$

To Compute Sum of the Series or of all Terms.

When Extremes and Number of terms are given. RULE.—Multiply number of terms by half sum of extremes.

$$\text{Or, } 2a + d(n-1) \times .5n; \quad \frac{l+a \times (l-a)}{2d} + \frac{l+a}{2};$$

$$\text{and } 2l - (d \times n - 1) \times .5n = S.$$

ILLUSTRATION.—How many times does hammer of a clock strike in 12 hours?

$$12 \times 12 + 1 = 156, \text{ and } 156 \div 2 = 78 \text{ times.}$$

To Compute any Number of Arithmetical Mean Terms between two Extremes.

RULE.—Subtract less extreme from greater, and divide difference by than number of means or terms required to be ascertained, and then proceed in rule.

To Compute Two Arithmetical Means or Terms between two given Extremes.

RULE.—Subtract less extreme from greater, and divide difference by 3, which will be common difference, which being added to less extreme, or taken from greater, will give means.

EXAMPLE 1.—Compute two arithmetical means between 4 and 16.

$$16 - 4 \div 3 = 4 \text{ com. dif.}$$

$$4 + 4 = 8 \text{ one mean.}$$

$$16 - 4 = 12 \text{ second mean.}$$

2.—Compute four arithmetical means between 5 and 30.

$$30 - 5 = 25, \text{ and } 25 \div 4 + 1 = 5 = \text{com. dif.}$$

$$5 + 5 = 10 = 1^{\text{st}} \text{ mean.} \quad 15 + 5 = 20 = 3^{\text{d}} \text{ mean.}$$

$$10 + 5 = 15 = 2^{\text{d}} \text{ "} \quad 20 + 5 = 25 = 4^{\text{th}} \text{ "}$$

Miscellaneous Illustrations.

1. A steamer having been purchased upon following terms—viz.: \$500 transfer of bill of sale and balance in monthly instalments, commencing at for first month, and decreasing \$500 in each month, until whole sum is paid.

1st. How many months must elapse before final payment?

2d. What was amount of purchase-money, or sum of series?

Here are first and last terms—viz., 500 and 5000, and common difference Hence, To compute number of terms and amount of purchase,

$$5000 - 500 \div 500 = 9, \text{ and } 9 + 1 = 10 = \text{number of terms or months, at}$$

$$\frac{5000 + 500}{2} = 10 \times 2750 = \$27500, \text{ amount of purchase.}$$

2. If 100 stones are placed in a right line, one yard apart; how many yards will a person walk, to take them up one at a time and put them into a basket, on the first stone?

1st term 1, last term 200, and number of terms 100.

$$\text{Hence, } 100 \times \frac{200 + 1}{2} = 10100 \text{ yards.}$$

3. If in the sinking of curb of a well, \$3 is to be given for first foot in depth, \$5 for second, \$7 for third, and increasing in like manner to a depth of 20 feet, what would it cost?

First term 3, common difference 2; and number of terms 20.

Hence, $20 - 1 \times 2 + 3 = 41$, last term.

Then, $3 + 41 \times \frac{20}{2} = 440$, sum of all terms, or cost of curb.

4. If a contractor engaged to sink a curb to depth of 20 feet for \$400, and the contract was annulled when he had reached a depth of 8 feet; how much had he earned?

$400 \div 20 =$ number of terms. But inasmuch as 400 may be divided into 20 terms in arithmetical proportion in many different ways, according to value of 1st term, it becomes necessary to assume the value of the first foot as value of 1st term.

Assuming it at \$5, the required proportion will be, 1st term 5, number of terms 20, sum of series 400.

Hence, $\frac{400 - 5 \times 20 \times 2}{20 \times (20 - 1)} = \frac{600}{380} = 1\frac{1}{19}$, common difference.

Then, $5 + 1\frac{1}{19} \times 7 = 16\frac{1}{19}$ = 1st term + product of common difference and 8th term less 1, which added to $5 = 21\frac{1}{19}$, and $\times 4 =$ half number of terms for which cost is sought = $84\frac{4}{19}$ dollars, sum earned.

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is any series of numbers continually increasing by a constant multiplier, or decreasing by a constant divisor, as 1, 2, 4, 8, 16, etc., and 15, 7.5, 3.75, etc.

The constant multiplier or divisor is the *Ratio*.

When any three of following elements are given, remaining two can be computed, viz: First term, Last term, Number of Terms, Ratio, and Sum of all Terms.

To Compute First Term.

When Ratio, Last Term, and Number of Terms are given. RULE.—Divide last term by ratio raised to a power denoted by number of terms less 1.

Or, $\frac{S(r-1)}{r^n - 1}$ and $rl - S(r-1) = a$ a representing 1st term, l last, n number of, S sum of all terms, and r ratio.

ILLUSTRATION.—Last term is 4374, number of terms 8, and ratio 3; what is first term?

$$\frac{4374}{3^8 - 1} = \frac{4374}{2187} = 2, \text{ first term.}$$

To Compute Last Term.

When First Term and Ratio are Equal. RULE.—Write a few of leading terms of series and place their indices over them, beginning with a unit. Add together the most convenient and least number of indices to make the index to term required.

Multiply terms of the series of these indices together, and product will give term required.

Or, Multiply first term by ratio raised to a power, denoted by number of terms less 1.

EXAMPLE 1.—First term is 2, ratio 2, and number of terms 13; what is last term?

Indices, 1 2 3 4 5
Terms, 2, 4, 8, 16, 32.

Then, $5 + 5 + 3 = 13 =$ sum of indices, and $32 \times 32 \times 8 = 8192 =$ last term.

Or, $2 \times 2^{13-1} = 8192$. Also by inspection of table, page 105, 13th term = 8192

2.—The price of 12 horses being 4 cents for first, 16 for second, and 64 for third, and so on; what is price of last horse?

Indices, 1 2 3 4
Terms, 4, 16, 64, 256.

Then, $4 + 4 + 4 = 12 = \text{sum of indices}$, and $256 \times 256 \times 256 = 256^3 = \$167,772.16$

When First Term and Ratio are Different. RULE.—Write a few of leading terms of series, and place their indices over them, beginning with a cipher. Add together the most convenient indices to make an index less by 1 than term sought.

Multiply terms of these series belonging to these indices together, and take the product for a dividend.

Or, Raise first term to a power, index of which is 1 less than number of terms multiplied; take result for a divisor; proceed with their division, and quotient will give term required.

EXAMPLE I.—First term is 1, ratio 2, and number of terms 23; what is the last term?

Indices, 0 1 2 3 4 5
Terms, 1, 2, 4, 8, 16, 32.

Then, $5 + 5 + 5 + 5 + 2 = 22 = \text{sum of indices}$, and $32 \times 32 \times 32 \times 32 \times 4 = 4,194,304$, and $4,194,304 \div \text{the 5th power } (6-1) \text{ of } 1 = 1 = 4,194,304$.

Or, $1 \times 2^{23-1} = 4,194,304$. By inspection of table, page 105, 23d term = 4,194,304.

2.—If 1 cent had been put out at interest in 1630, what would it have amounted to in 1834, if it had doubled its value every 12 years?

$1834 - 1630 = 204$, which $\div 12 = 17$, and $17 + 1 = 18 = \text{number of terms}$.

Indices, 0 1 2 3 4 7
Terms, 1, 2, 4, 8, 16, 128.

Then, $7 + 4 + 3 + 2 + 1 = 17$, and $128 \times 16 \times 8 \times 4 \times 2 \times 1 = 131,072$, and $131,072 \div 1$, the 4th power $(5-1)$ of 1 = \$1310.72.

When First Term, Ratio, and Sum of the series are given. RULE.—From sum of series subtract quotient of first term subtracted from sum of series, divided by ratio.

Or, $a \times r^n - r = L$

EXAMPLE.—First term is 2, ratio 3, and sum of series 2186; what is last term?

$$2186 - \frac{2186 - 2}{3} = 2186 - 728 = 1458, \text{ last term.}$$

To Compute Number of Terms.

When Ratio, First, and Last Terms are given. RULE.—Divide logarithm of quotient of product of ratio and last term, divided by first term, by logarithm of ratio.

$$\text{Or, } \frac{\log. (a + S r - 1) - \log. a}{\log. r}; \quad \frac{\log. l - \log. a}{\log. (S - a) - \log. (S - l)} + 1;$$

$$\text{and } \frac{\log. l - \log. (r l - r - 1 S)}{\log. r} + 1 = n.$$

EXAMPLE.—Ratio is 2, and first and last terms are 1 and 131,072; what is number of terms?

$$\log. \frac{2 \times 131,072}{1} = \log. 262,144 = 5.41854, \text{ and } 5.41854 \div \log. 2 = \frac{5.41854}{.30103} = 18.$$

To Compute Sum of Series.

When First Term, Ratio, and Number of Terms are given. RULE.—Raise ratio to r index of which is equal to number of terms, from which subtract 1; then remainder by ratio less 1, and multiply quotient by first term.

$$\frac{l^{n-1} - a^{n-1}}{r - 1} \text{ and } \frac{l^{n-1} - 1}{r - 1} = S.$$

PROBLEM 1.—First term is 2, ratio 2, and number of terms 13; what is sum?

$1 - 1 = 8192 - 1 = 8191$, and $8191 \div (2 - 1) = 8191$, and $8191 \times 2 = 16382$.

If a man were to buy 12 horses, giving 2 cents for first horse, 6 cents for second, and so on, what would they cost him? \$5314.40.

To Compute Ratio.

First Term, Last Term, and Numbers of Terms are given. RULE.—Divide last term by first, and quotient will be equal to ratio raised to power denoted by number of terms; then extract root of this quotient.

$$\text{Or, } \frac{S - a}{S - l} = r.$$

PROBLEM.—First term is 2, last term 4374, and number of terms 8; what is

$$\frac{4374}{2} = 2187, \text{ and } \sqrt[8]{2187} = 3, \text{ ratio.}$$

Miscellaneous Illustrations.

What is 9th term in geometrical progression 3, 9, 27, 81, etc.? and what is sum of terms?

1st term = 3, number of terms 9, and ratio 3.

by rule to compute last term, 1st term and ratio being equal—

Indices, 1 2 3 4
Terms, 3, 9, 27, 81.

$1 + 3 + 9 = 13$ = sum of indices, and $9 \times 27 \times 81 = 19683$ = last term.

to compute sum of terms—

$$\frac{3^9 - 1}{3 - 1} \times 3 = \frac{19682}{2} = 9841 \times 3 = 29523, \text{ sum of terms.}$$

1st term is 1, ratio 2, and last term 131072; what is sum of series?

$$131072 \times 2 - 1 = 262143, \text{ and } 262143 \div 2 - 1 = 262143.$$

What are the proportional terms between 2 and 2048?

$$4 + 2 = 6, \text{ and } 6 - 1 = 5, \text{ and } \sqrt[5]{\frac{2048}{2}} = 4.$$

Hence, 2 : 8 : 32 : 128 : 512 : 2048.

What is sum of series is 6560, ratio 3, and number of terms 8; what is first term?

$$6560 \times \frac{3 - 1}{3^8 - 1} = 6560 \times \frac{2}{6560} = 2, \text{ first term.}$$

Geometrical Progressions,

any questions of Geometrical Progression and of Double Ratio may be solved by Inspection, number of terms not exceeding 56.

| | | | | | |
|----|-----------|----|---------------|----|-------------------|
| 15 | 16384 | 29 | 268435456 | 43 | 4398046511104 |
| 16 | 32768 | 30 | 536870912 | 44 | 8796093022208 |
| 17 | 65536 | 31 | 1073741824 | 45 | 17592186044416 |
| 18 | 131072 | 32 | 2147483648 | 46 | 35184372088832 |
| 19 | 262144 | 33 | 4294967296 | 47 | 70368744177664 |
| 20 | 524288 | 34 | 8589934592 | 48 | 140737488355328 |
| 21 | 1048576 | 35 | 17179869184 | 49 | 281474976710656 |
| 22 | 2097152 | 36 | 34359738368 | 50 | 562949953421312 |
| 23 | 4194304 | 37 | 68719476736 | 51 | 1125899906842624 |
| 24 | 8388608 | 38 | 137438953472 | 52 | 2251799813685248 |
| 25 | 16777216 | 39 | 274877906944 | 53 | 4503599627370496 |
| 26 | 33554432 | 40 | 549755813888 | 54 | 9007199254740992 |
| 27 | 67108864 | 41 | 1099511627776 | 55 | 18014398509481984 |
| 28 | 134217728 | 42 | 2199023255552 | 56 | 36028797018963968 |

NOTE.—12th power of 2 = 4096, and 7th root of 128 = 2.

ALLIGATION.

ALLIGATION is a method of finding mean rate or quality of different materials when mixed together.

To Compute Mean Price of a Mixture.

When Prices and Quantities are known. RULE.—Multiply each quantity by its rate, divide sum of products by sum of quantities, and quotient will give rate of the composition.

EXAMPLE.—If 10 lbs. of copper at 20 cents per lb., 1 lb. of tin at 5 cents, and 1 lb. of lead at 4 cents, be mixed together, what is value of composition?

$$\begin{array}{r} 10 \times 20 = 200 \\ 1 \times 5 = 5 \\ 1 \times 4 = 4 \\ \hline 12 \quad) \quad 209 \quad (17.416 \text{ cents.} \end{array}$$

To Compute Quantity of each Article.

When Prices and Mean Price are given. RULE.—Write prices of ingredients one under the other in order of their values, beginning with least, and set mean price at left. Connect with a line each price that is less than mean rate with one or more that is greater.

Write difference between mixture rate and that of each simples opposite price with which it is connected; then sum of differences against any price will express quantity to be taken of that price.

EXAMPLE.—How much gunpowder, at 72, 54, and 48 cents per pound, will compose a mixture worth 60 cents a pound?

$$\begin{array}{r} 48 \\ 54 \\ 72 \end{array} \quad \begin{array}{l} 12, \text{ at } 48 \text{ cents.} \\ 12, \text{ at } 54 \text{ cents.} \\ 12 + 6 = 18, \text{ at } 72 \text{ cents.} \end{array}$$

Here, $72 - 60 = 12$ at 48, $72 - 60 = 12$ at 54, $60 - 48 = 12$, and $60 - 54 = 6 = 12 + 6 = 18$ at 72.

Then $12 \times 48 + 12 \times 54 + 18 \times 72 = 2520$, and $2520 \div 12 + 12 + 12 + 6 = 60$ cents.

NOTE.—Should it be required to mix a definite quantity of any one article, the quantities of each, determined by above rule, must be increased or decreased in proportion they bear to defined quantity.

Thus, had it been required to mix 18 pounds at 48 cents, result would be 18 at 48, 18 at 54, and 27 at 72 cents per pound.

When the whole Composition is limited. RULE.—As sum of relative quantities, as ascertained by above rule, is to whole quantity required, so is each quantity so ascertained to required quantity of each.

EXAMPLE.—Required 100 pounds of above mixture

$$\begin{array}{l} \text{Then, } 12 + 12 + 18 = 42. \text{ Then, } 42 : 100 :: 12 : 28.571 \text{ pounds.} \\ 42 : 100 :: 12 : 28.571 \text{ pounds.} \\ 42 : 100 :: 18 : 42.857 \text{ pounds.} \end{array}$$

When Price of Several Articles and Quantity of one of them is given. RULE.—Ascertain proportionate quantities of ingredients by previous rule.

Then, as number opposite ingredients, quantity of which is given, is to given quantity; so is number opposite to each ingredient to quantity required of that ingredient.

EXAMPLE.—Having 35 lbs. of tobacco, worth 60 cents per pound, how much of other qualities, worth 65, 70, and 75 cents per pound, must be mixed with it, so as to sell mixture at 68 cents per pound?

By previous rule, it is ascertained there must be 7 lbs. at 60, 2 at 65, 3 at 70, and 8 at 75 cents; but as there are 35 lbs. at 60 cents to be taken, other quantities and kinds must be increased in like manner.

$$\begin{array}{l} \text{Hence, } 7 : 35 :: 2 : 10 = 10 \text{ at } 65 \text{ cents.} \\ 7 : 35 :: 3 : 15 = 15 \text{ at } 70 \text{ cents.} \\ 7 : 35 :: 8 : 40 = 40 \text{ at } 75 \text{ cents.} \end{array}$$

SIMPLE INTEREST.

pute Interest on any Given Sum for a Period of One or more Years.

Multiply given sum or *principal* by rate per cent. and number of years; figures to right of product, and result will give interest in dollars and cents.

—What is interest upon \$1050 for 5 years at 7 per cent.?

$$1050 \times 7 \times 5 = 36750, \text{ and } 367.50 = \$367.50.$$

is less than One Year. **RULE.**—Proceed as before, multiplying by months or days, and dividing by following units—viz., 12 for months, 360, as the case may be, for days.

—What is interest upon \$1050 for 5 months and 30 days at 7 per cent.?

$$\text{and 30 days} = 183 \text{ days. } \frac{1050 \times 7 \times 183}{360} = 3685, \text{ and } 36.85 = \$36.85.$$

tion of computing interest may be performed thus :

Interest upon any sum at 6 per cent. = 1 per cent. for 2 months.

5 per cent. is $\frac{1}{4}$ th less than at 6 per cent.

7 per cent. is $\frac{1}{4}$ th greater than at 6 per cent.

preceding example—2 months = 1 per cent. = 10.50

$$2 \text{ " } = 1 \text{ " } 10.50$$

$$1 \text{ " } = \frac{1}{2} \text{ " } 5.25$$

$$30 \text{ days } = 1 \text{ month } = 5.25$$

$$31.50$$

$$\text{Add } \frac{1}{4} \text{ for 7 per cent.} = 5.25$$

$$\$36.75$$

difference between this amount and preceding arises from 183 days being taken in one case, or 182.5 days, in the other.

In computation of interest there are four elements—viz., Principal, Time, Interest or Amount, any three of which being given, remaining one can be found.

To Compute Principal.

Rate per Cent., and Interest are given. **RULE.**—Divide given interest by rate per cent., for given rate and time.

—What sum of money at 6 per cent. will in 14 months produce \$14?

$$14 \div .07 = 200 \text{ dollars.}$$

To Compute Rate per Cent.

Principal, Interest, and Time are given. **RULE.**—Divide given interest by given sum, for time, at 1 per cent.

—If \$32.66 was discounted from a note of \$400 for 14 months, what rate per cent.?

$$\$400 \text{ for 14 months at 1 per cent.} = 4.66.$$

$$\text{Then } 32.66 \div 4.66 = 7 \text{ per cent.}$$

To Compute Time.

Principal, Rate per Cent., and Interest are given. **RULE.**—Divide given interest of sum, at rate per cent. for one year.

—In what time will \$208 produce \$11.34, at 7 per cent.?

$$\$208 \text{ for one year is } 7.56.$$

$$11.34 \div 7.56 = 1.5 \text{ years.}$$

PROB. 1.—If an amount of \$2175 is returned for a principal having been 7 per cent., what was principal invested in 18 months will produce \$2000, what is rate?

COMPOUND INTEREST.

If any Principal be multiplied by number (in following table) opposite years, and under rate per cent., sum will be amount of that principal at compound interest for time and rate taken.

EXAMPLE.—What is amount of \$500 for 10 years at 6 per cent. ?

Tabular number.... 1.790 84, and $1.790\ 84 \times 500 = 895.42$ dollars.

| Years. | 3 | 4 | 5 | 6 | Years. | 3 | 4 | 5 | 6 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
| 1 | 1.03 | 1.04 | 1.05 | 1.06 | 13 | 1.468 53 | 1.665 07 | 1.885 64 | 2.132 98 |
| 2 | 1.0609 | 1.081 6 | 1.102 5 | 1.123 6 | 14 | 1.515 29 | 1.731 67 | 1.979 93 | 2.260 9 |
| 3 | 1.092 73 | 1.124 86 | 1.157 62 | 1.191 01 | 15 | 1.557 97 | 1.800 95 | 2.078 92 | 2.396 55 |
| 4 | 1.125 51 | 1.169 86 | 1.215 5 | 1.262 47 | 16 | 1.604 71 | 1.872 98 | 2.182 87 | 2.540 35 |
| 5 | 1.159 27 | 1.216 68 | 1.276 98 | 1.338 22 | 17 | 1.652 85 | 1.947 99 | 2.292 01 | 2.692 77 |
| 6 | 1.194 05 | 1.265 32 | 1.34 | 1.418 51 | 18 | 1.702 44 | 2.025 81 | 2.406 61 | 2.854 33 |
| 7 | 1.229 87 | 1.315 93 | 1.407 1 | 1.503 63 | 19 | 1.753 5 | 2.106 84 | 2.526 95 | 3.025 59 |
| 8 | 1.266 77 | 1.368 57 | 1.477 45 | 1.593 84 | 20 | 1.806 11 | 2.191 13 | 2.653 29 | 3.207 13 |
| 9 | 1.304 77 | 1.423 31 | 1.551 32 | 1.689 47 | 21 | 1.860 29 | 2.278 76 | 2.785 96 | 3.399 56 |
| 10 | 1.343 92 | 1.480 24 | 1.628 89 | 1.790 84 | 22 | 1.916 1 | 2.369 92 | 2.925 26 | 3.603 53 |
| 11 | 1.384 24 | 1.539 45 | 1.710 33 | 1.898 29 | 23 | 1.973 6 | 2.464 21 | 3.071 52 | 3.819 74 |
| 12 | 1.425 76 | 1.601 03 | 1.795 85 | 2.012 19 | 24 | 2.032 79 | 2.563 3 | 3.225 09 | 4.048 73 |

For any other Rate or Period.—Multiply logarithm of rate + 1 by period, and number for logarithm will give tabular amount as above.

ILLUSTRATION.—What is tabular number for 4 per cent. for 10 years?

Log. of 1.04 = .017 033 3, which $\times 10 = .170\ 333$, and number for log. = 1.480 24.

Time in Years in which a Sum of Money will be doubled at Several Rates of Interest.

| Rate. | Time. | Rate. | Time. | Rate. | Time. | Rate. | Time. |
|-----------|-------|-----------|-------|-----------|-------|-----------|-------|
| Per cent. | | Per cent. | | Per cent. | | Per cent. | |
| 1 | 69.68 | 4 | 17.67 | 7 | 10.34 | 10 | 7.27 |
| 2 | 35 | 5 | 14.21 | 8 | 9.01 | 20 | 3.8 |
| 3 | 23.44 | 6 | 11.88 | 9 | 8.04 | 30 | 2.64 |

Value of \$1, etc., Computed Semi-annually for a Period of 12 Years.

| Years. | 3 | 4 | 5 | 6 | Years. | 3 | 4 | 5 | 6 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
| .5 | 1.015 | 1.02 | 1.025 | 1.03 | 6.5 | 1.2134 | 1.2936 | 1.3785 | 1.4684 |
| 1 | 1.0302 | 1.0404 | 1.0506 | 1.0609 | 7 | 1.2317 | 1.3195 | 1.413 | 1.5102 |
| 1.5 | 1.0457 | 1.0612 | 1.0769 | 1.0927 | 7.5 | 1.2502 | 1.3459 | 1.4483 | 1.558 |
| 2 | 1.0614 | 1.0824 | 1.1038 | 1.1255 | 8 | 1.269 | 1.3728 | 1.4845 | 1.6047 |
| 2.5 | 1.0773 | 1.1041 | 1.1314 | 1.1593 | 8.5 | 1.288 | 1.4002 | 1.5216 | 1.6588 |
| 3 | 1.0934 | 1.1262 | 1.1597 | 1.1941 | 9 | 1.3073 | 1.4282 | 1.5597 | 1.7024 |
| 3.5 | 1.1098 | 1.1487 | 1.1887 | 1.2299 | 9.5 | 1.3269 | 1.4568 | 1.5987 | 1.7535 |
| 4 | 1.1265 | 1.1717 | 1.2184 | 1.2668 | 10 | 1.3469 | 1.486 | 1.6386 | 1.8061 |
| 4.5 | 1.1434 | 1.1951 | 1.2489 | 1.3048 | 10.5 | 1.3671 | 1.5157 | 1.6796 | 1.8603 |
| 5 | 1.1604 | 1.219 | 1.2801 | 1.3439 | 11 | 1.3876 | 1.546 | 1.7216 | 1.9161 |
| 5.5 | 1.178 | 1.2434 | 1.3121 | 1.3842 | 11.5 | 1.4084 | 1.5769 | 1.7606 | 1.9736 |
| 6 | 1.1956 | 1.2689 | 1.3449 | 1.4258 | 12 | 1.4295 | 1.6084 | 1.8087 | 2.0356 |

ILLUSTRATION.—What is amount of \$500 at semi-annual interest of 5 per cent. compounded for 10 years?

Tabular number 1.6386. Then, $500 \times 1.6386 = \$819.44.5$.

To Compute Interest on any Given Sum.

n Period of Years. $P(1+r)^n = A$; $\frac{A}{(1+r)^n} = P$; $\sqrt[n]{\frac{A}{P}} - 1 = r$;
 $\log \frac{P}{A} = n$. P representing principal, r rate per cent. per annum, n unit of principal and interest.

ANNUITIES.

To Compute Amount of Annuity.

When Time and Ratio of Interest are Given. RULE.—Raise the ratio to a power denoted by time, from which subtract 1; divide remainder by ratio less 1, and quotient, multiplied by annuity, will give amount.

NOTE.—§ 1 added to given rate per cent. is ratio, and preceding table in Compound Interest is a table of ratios.

EXAMPLE.—What is amount of an annual pension of \$100, interest 5 per cent., which has remained unpaid for four years?

1.05 ratio; then $1.05^4 - 1 = 1.21550625 - 1 = .21550625$, and $.21550625 \div (.05 - 1) = 4.310125$, which $\times 100 = \$431.01.25$.

To Compute Present Worth of an Annuity.

When Time and Rate of Interest are Given. RULE.—Ascertain amount of it for whole time; divide by ratio, involved to time, and result will give worth.

EXAMPLE.—What is present worth of a pension or salary of \$500, to continue 10 years at 6 per cent. compound interest?

\$500, by last rule, is worth \$6590.3975, which, divided by 1.06¹⁰ (by table, page 108, is 1.79084) = \$3680.05.

Or, Multiply tabular amount in following table by given annuity, and product will give present worth.

ILLUSTRATION 1.—As above; 10 years at 6 per cent. = 7.36008, and $7.36008 \times 500 = 3680.04$ dollars.

2. What is present worth of \$150 due in one year at 6 per cent. interest per annum?
 $.94339 \times 150 = \$141.50.85$.

Present Worth of an Annuity of \$1, at 4, 5, and 6 Per Cent. Compound Interest for Periods under 25 Years.

| Years. | 4 Per Cent. | 5 Per Cent. | 6 Per Cent. | Years. | 4 Per Cent. | 5 Per Cent. | 6 Per Cent. |
|--------|-------------|-------------|-------------|--------|-------------|-------------|-------------|
| 1 | .96154 | .95238 | .94339 | 13 | 9.98562 | 9.39357 | 8.85268 |
| 2 | 1.88609 | 1.85941 | 1.83339 | 14 | 10.56307 | 9.89864 | 9.29498 |
| 3 | 2.7751 | 2.72325 | 2.67301 | 15 | 11.11843 | 10.37966 | 9.71225 |
| 4 | 3.6299 | 3.54595 | 3.4651 | 16 | 11.65128 | 10.83778 | 10.10589 |
| 5 | 4.45203 | 4.32948 | 4.21236 | 17 | 12.16626 | 11.27407 | 10.47726 |
| 6 | 5.24215 | 5.07569 | 4.91732 | 18 | 12.65926 | 11.68958 | 10.8276 |
| 7 | 6.00203 | 5.78637 | 5.58238 | 19 | 13.13388 | 12.08532 | 11.15811 |
| 8 | 6.73176 | 6.46321 | 6.20979 | 20 | 13.59029 | 12.46221 | 11.46992 |
| 9 | 7.4364 | 7.10782 | 6.80169 | 21 | 14.02912 | 12.82115 | 11.76407 |
| 10 | 8.11085 | 7.72173 | 7.36008 | 22 | 14.45112 | 13.163 | 12.04158 |
| 11 | 8.76044 | 8.30641 | 7.88687 | 23 | 14.85682 | 13.48807 | 12.30338 |
| 12 | 9.38505 | 8.86325 | 8.38384 | 24 | 15.24695 | 13.79804 | 12.55035 |

For a Rate of Interest and Term of Years not given in either Table.

$$\frac{P}{r} \left[1 - \frac{1}{(1+r)^n} \right] = A. \text{ Notation as preceding.}$$

ILLUSTRATION.—Take \$1 at 4 per cent. for 24 years.

Log. 1.04 = .017033, which $\times 24 = .408799$. log. .408799 = 2.5633 = ratio raised to power of 24.

$$\text{Then, } \frac{1}{.04} \times \left(1 - \frac{1}{2.5633} \right) = 25 \times 1 - .390122 = \$15.24.695.$$

To Compute Yearly Amount that will Liquidate a Debt in Given Number of Years at Compound Interest.

ILLUSTRATION.—What is amount of an annual payment that in 6 years at 5 per cent. compound interest?

$$.55)^6 \text{ per table, page 108, } \frac{100 \times .05 (1 + .05)^6}{(1 + .05)^6 - 1} = \frac{5 \times 1.34}{1.34 - 1} = \frac{6.7}{.34} = \$19.76.$$

Annuities do not commence till a certain period of time, they are said to be reversion.

Compute Present Worth of an Annuity in Reversion.

—Take two amounts under rate in above table—viz., that opposite sum of 30 times and that of time of reversion; multiply their difference by annuity product will give present worth.

EX.—What is present worth of the reversion of a lease of \$40 per annum, due for 6 years, but not to commence until end of 2 years, at rate of 6 per

$$\begin{aligned} 6 + 2 = 8 \text{ years} & \dots\dots\dots 6.20979 \\ 2 \text{ " } & \dots\dots\dots 1.83339 \\ \hline & 4.37640 \times 40 = \$175.05.6. \end{aligned}$$

Amount of Annuity of \$1, etc., Compound Interest, from 1 to 20 Years.

| 4 | 5 | 6 | 7 | Years. | 4 | 5 | 6 | 7 |
|-------|-----------|-----------|-----------|--------|-----------|-----------|-----------|-----------|
| Cent. | Per Cent. | Per Cent. | Per Cent. | | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
| | 1. | 1. | 1. | 11 | 13.48635 | 14.20679 | 14.97164 | 15.7836 |
| 24 | 2.05 | 2.06 | 2.07 | 12 | 15.0258 | 15.91713 | 16.86094 | 17.88845 |
| 121 | 3.1525 | 3.1836 | 3.2149 | 13 | 16.62684 | 17.71298 | 18.88214 | 20.14064 |
| 246 | 4.31012 | 4.37462 | 4.43994 | 14 | 18.29191 | 19.59863 | 21.01507 | 22.55049 |
| 416 | 5.52563 | 5.63709 | 5.75074 | 15 | 20.02359 | 21.57856 | 23.27597 | 25.12902 |
| 532 | 6.80191 | 6.97532 | 7.15329 | 16 | 21.82453 | 23.65749 | 25.67253 | 27.88805 |
| 598 | 8.14201 | 8.39384 | 8.65402 | 17 | 23.69751 | 25.84037 | 28.21288 | 30.84022 |
| 212 | 9.54911 | 9.89747 | 10.2598 | 18 | 25.64541 | 28.13238 | 30.90565 | 33.99903 |
| 582 | 11.02656 | 11.49132 | 11.97799 | 19 | 27.67123 | 30.539 | 33.75999 | 37.37896 |
| 206 | 12.57789 | 13.18079 | 13.81645 | 20 | 29.77808 | 33.06595 | 36.78559 | 40.99549 |

EXTRATION.—What is amount of \$1000 for 20 years at 5 per cent.?

per cent. for 20 years = 33.06595; hence, $1000 \times 33.06595 = \$33,065.95$.

Compute Amount of an Annuity for any Period and Rate.

—From table for Compound Interest, page 108, take value for rate per cent. **EX.**, and raise it to a power determined by time in years, from which subtract 1 remainder by rate, and quotient multiplied by annuity will give amount.

EX.—What will an annuity of \$50, payable yearly, amount to in 4 years, at 5 per cent.?

ble, page 108, $1.05^4 = 1.2155$.

$$1.2155 - 1 \div (1.05 - 1) = 4.31, \text{ and } 4.31 \times 50 = \$215.50.$$

For Half-yearly and Quarterly Payments.

ly annuity for given time by amount in following table:

| mt. | Half-yearly. | Quarterly. | Rate percent. | Half-yearly. | Quarterly. |
|-----|--------------|------------|---------------|--------------|------------|
| | 1.007445 | 1.011181 | 5.5 | 1.013567 | 1.020395 |
| | 1.008675 | 1.013031 | 6 | 1.014781 | 1.022227 |
| | 1.009902 | 1.014877 | 6.5 | 1.015993 | 1.024055 |
| | 1.011126 | 1.016729 | 7 | 1.017204 | 1.02588 |
| | 1.012348 | 1.018559 | 7.5 | 1.018414 | 1.027704 |

EXTRATION 1.—Annuity as determined in previous case = \$215.50.

215.50×1.012348 from above table = \$218.16 for half yearly payments.

EXTRATION 2.—A person 30 years of age has an annuity for 10 years, present worth \$3550. What is annuity worth, if he may live for 10 years. What is annuity worth, if he dies out of every 3550, between the ages of 30 and 40, die annuity

$$3550 - 600 (50 \times 10) = 2950 \text{ would therefore be living.}$$

$$\text{And, } 3550 : 2950 :: 1000 = \$830.98.$$

PERPETUITIES.

PERPETUITIES are such Annuities as continue forever.

To Compute Value of a Perpetual Annuity.

RULE.—Divide annuity by rate per cent., and multiply quotient by unit in preceding table.

EXAMPLE.—What is present worth of an annuity for \$ 100, payable semi-annually, at 5 per cent.?

$$100 \div .05 = 2, \text{ and } 2 \times 1.012348, \text{ from preceding table} = 2.02470.$$

To Compute Value of a Perpetuity in Reversion.

RULE.—Subtract present worth of annuity for time of reversion from worth of annuity to commence immediately.

EXAMPLE.—What is present worth of an estate of \$50 per annum, at 5 per cent, to commence in 4 years?

$$\begin{array}{r} 50 \div .05 \dots\dots\dots = 1000 \\ \$50, \text{ for 4 years, at 5 per cent.} = 3.54595 \text{ (from table, page 110)} \times 50 = 177.2975 \\ \hline 822.7025 \end{array}$$

which in 4 years, at 5 per cent. compound interest, would produce \$1000.

COMBINATION.

COMBINATION is a rule for ascertaining how often a less number of numbers or things can be chosen varied from a greater, or how many different collections may be formed without regard to order of each collection.

Combinations of any number of things signify the different collections which may be formed of their quantities, without regard to the *order* of their arrangement.

Thus, 3 letters, *a, b, c*, taken *all together*, form but one combination, *abc*. Taken *two and two*, they form 3 combinations, as *ab, ac, bc*.

NOTE.—Class of the combination is determined by number of elements or things to be taken; if two are taken, the combination is of 2d class, and so on.

RULE.—Multiply together natural series 1, 2, 3, etc., up to the number to be taken at a time. Take a series of as many terms, decreasing by 1, from number out of which combination is to be made, ascertain their continued product, and divide this last product by former.

EXAMPLE 1.—How many single combinations, as ab , ac , may be made of 2 letters out of 3? $1 \times 2 = 2 \quad 2 \times 3 = 6$

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6} = \frac{6}{2} = 3.$$

2.—How many combinations may be made of 7 letters out of 12?

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6} = \frac{5040}{3991680}, \text{ and } \frac{3991680}{5040} = 792.$$

3.—How many different hands of cards may be held, as at whist, combinations
 13 out of 52? 635 013 559 600.

When two Numbers or Things are Combined.

RULE.—Multiply together natural series 1, 2, 3, etc., to one less term than number of combinations; ascertain their continued product, and proceed as before.

EXAMPLE.—There are 3 cards in a box, out of which two are to be drawn in a required order. How many combinations are there?

Here there are 2 terms; hence, $2 - 1 = 1$, and $\frac{1}{3 \times 2} = \frac{1}{6} = 6 \div 1 = 6$.

To Compute Number of Ways in which any Number of 'not Objects can be Divided among any Number.

multiply together numbers equal to number given, as often as objects
 are divided among them.

Different ways can 10 different cards be divided among

Combinations with Repetitions.

is case the repetition of a term is considered a new combination. Thus, nits of but one combination, if not repeated; if repeated, however, it admits combinations, as 1, 1; 1, 2; 2, 2.

—To number of terms of series add number of class of combination, less 1; y sum by successive decreasing terms of series, down to last term of series; divide this product by number of permutations of the terms, denoted by class combination.

PLK—How many different combinations of numbers of 6 figures can be out of 11?

$(6 - 1) = 16 = \text{sum of number of terms, and number of class, less 1.}$

$15 \times 14 \times 13 \times 12 \times 11 = 5765760 = \text{product of sum, and successive terms to n.}$

$\times 3 \times 4 \times 5 \times 6 = 720 \text{ permutations of class of combination.}$

Then, $\frac{5765760}{720} = 8008.$

Variations with Repetitions.

r different arrangement of individual number or things, including repetition termed a Variation.

of Variation is denoted by number of individual things taken at a time.

—Raise number denoting the individual things to a power, the exponent h is number expressing class of variation.

PLK 1.—How many variations with 4 repetitions can be made out of 5 figures?

$$5^4 = 625.$$

How many different combinations of 4 places of figures can be made out of 9 digits?

$$9 + (4 - 1) = 12, \text{ and } \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4} = \frac{11880}{24} = 495.$$

Combination without Repetitions.

1.—From number of terms of series subtract number of class of combination, multiply this remainder by successive increasing terms of series, up to last term of series; then divide this product by number of permutations of the terms, divided by class of combination.

PLK 1.—How many combinations can be made of 4 letters out of 10, excluding repetition of them in any second combination?

$(4 - 1) = 7 = \text{number of terms} - \text{number of class, less 1.}$

$7 \times 9 \times 10 = 5040 = \text{prod. of remainder 7, and successive terms up to last term.}$

$\times 3 \times 4 = 24 = \text{permutations of class of combination.}$

Then, $\frac{5040}{24} = 210.$

How many combinations of the 5th class, without repetitions, can be made of different articles?

$$12 - (5 - 1) = 8, \text{ and } \frac{8 \times 9 \times 10 \times 11 \times 12}{1 \times 2 \times 3 \times 4 \times 5} = \frac{85040}{120} = 792.$$

CIRCULAR MEASURE.

of Circular Measure is an angle which is subtended at the centre equal to radius of that circle, being equal to

$$\frac{180^\circ}{3.1416} = 57.296^\circ.$$

the measure of an angle is equal to a fraction which has for numerator the angle subtended by that angle at centre of any circle, and for denominator the circumference of that circle.

Wrought Iron, Steel, Copper, and Brass Wire

American Gauge. f. full, l. light.

| No. of Gauge. | Diameter. | PER LINEAL FOOT. | | | |
|---------------|--------------------------------|------------------|-------------|-------------|--------|
| | | Iron. | Steel. | Copper. | Br. |
| | Inch. | Lbs. | Lbs. | Lbs. | Lbs. |
| 0000 | .46 or $\frac{7}{16}$ f. | .560 74 | .566 03 | .640 513 | .605 1 |
| 000 | .409 64 | .444 683 | .448 879 | .507 946 | .479 9 |
| 00 | .364 8 or $\frac{3}{8}$ l. | .352 659 | .355 986 | .402 83 | .380 6 |
| 0 | .324 86 or $\frac{5}{16}$ f. | .279 665 | .282 303 | .319 451 | .301 8 |
| 1 | .289 3 | .221 789 | .223 891 | .253 342 | .239 7 |
| 2 | .257 63 or $\frac{1}{4}$ f. | .175 888 | .177 548 | .200 911 | .189 8 |
| 3 | .229 42 | .139 48 | .140 796 | .159 323 | .150 1 |
| 4 | .204 31 or $\frac{3}{8}$ l. | .110 616 | .111 66 | .126 353 | .119 2 |
| 5 | .181 94 or $\frac{3}{16}$ l. | .087 72 | .088 548 | .100 2 | .094 1 |
| 6 | .162 02 | .069 565 | .070 221 | .079 462 | .075 0 |
| 7 | .144 28 | .055 165 | .055 685 | .063 013 | .059 7 |
| 8 | .128 49 or $\frac{1}{8}$ f. | .043 751 | .044 164 | .049 976 | .047 1 |
| 9 | .114 43 | .034 699 | .035 026 | .039 636 | .037 1 |
| 10 | .101 89 or $\frac{1}{10}$ f. | .027 512 | .027 772 | .031 426 | .029 1 |
| 11 | .090 742 | .021 82 | .022 026 | .024 924 | .023 1 |
| 12 | .080 808 | .017 304 | .017 468 | .019 766 | .018 1 |
| 13 | .071 961 | .013 722 | .013 851 | .015 674 | .014 1 |
| 14 | .064 084 | .010 886 | .010 989 | .012 435 | .011 1 |
| 15 | .057 068 | .008 631 | .008 712 | .009 859 | .009 1 |
| 16 | .050 82 or $\frac{1}{20}$ f. | .006 845 | .006 909 | .007 819 | .007 1 |
| 17 | .045 257 | .005 427 | .005 478 | .006 199 | .005 1 |
| 18 | .040 303 | .004 304 | .004 344 | .004 916 | .004 1 |
| 19 | .035 89 | .003 413 | .003 445 | .003 899 | .003 1 |
| 20 | .031 961 | .002 708 | .002 734 | .003 094 | .002 1 |
| 21 | .028 462 | .002 147 | .002 167 | .002 452 | .002 1 |
| 22 | .025 347 | .001 703 | .001 719 | .001 945 | .001 1 |
| 23 | .022 571 | .001 35 | .001 363 | .001 542 | .001 1 |
| 24 | .020 1 or $\frac{1}{20}$ f. | .001 071 | .001 081 | .001 223 | .001 1 |
| 25 | .017 9 | .000 849 1 | .000 857 1 | .000 969 9 | .000 1 |
| 26 | .015 94 | .000 673 4 | .000 679 7 | .000 769 2 | .000 1 |
| 27 | .014 195 | .000 534 | .000 539 1 | .000 609 9 | .000 1 |
| 28 | .012 641 | .000 423 5 | .000 427 5 | .000 483 7 | .000 1 |
| 29 | .011 257 | .000 335 8 | .000 338 9 | .000 383 5 | .000 1 |
| 30 | .010 025 or $\frac{1}{100}$ f. | .000 266 3 | .000 268 8 | .000 304 2 | .000 1 |
| 31 | .008 928 | .000 211 3 | .000 213 2 | .000 241 3 | .000 1 |
| 32 | .007 95 | .000 167 5 | .000 169 1 | .000 191 3 | .000 1 |
| 33 | .007 08 | .000 132 8 | .000 134 1 | .000 151 7 | .000 1 |
| 34 | .006 304 | .000 105 3 | .000 106 3 | .000 120 4 | .000 1 |
| 35 | .005 614 | .000 083 66 | .000 084 45 | .000 095 6 | .000 1 |
| 36 | .005 or $\frac{1}{200}$ | .000 066 25 | .000 066 87 | .000 075 7 | .000 1 |
| | .453 | .000 052 55 | .000 053 04 | .000 060 03 | .000 1 |
| | z | .000 041 66 | .000 042 05 | .000 047 58 | .000 1 |
| | | .000 033 05 | .000 033 36 | .000 037 75 | .000 1 |
| | | .000 026 2 | .000 026 44 | .000 029 92 | .000 1 |
| | | 7.774 | 7.847 | 8.88 | 8. |
| | | .87 | 490.45 | 554.988 | 524. |
| | | 312 | .2838 | .3212 | . |

† computations of these weights were by
Harpe, Providence, R. I.

number of cases which favor drawing of a white ball from both bags is $5 \times 7 = 35$. Every one of the 5 white balls in one bag may be drawn in combination with every one of the 7 in the other. For a like cause, number of cases which favor drawing of a black ball from 1st bag and a black one from 2d is $5 \times 3 = 15$; a black ball from 1st and a white ball from 2d is $7 \times 2 = 14$; and a black ball from both is $3 \times 2 = 6$. Probability, therefore, of drawing is as

$$\frac{35}{70} = \frac{35}{70} = \frac{1}{2} = 1 \text{ to } 1, \text{ a white ball from both bags. } \frac{5 \times 3}{70} = \frac{15}{70} = \frac{3}{14} = 3 \text{ to } 11,$$

$$\text{a white ball from 1st, and a black from 2d. } \frac{7 \times 2}{70} = \frac{14}{70} = \frac{1}{5} = 1 \text{ to } 4, \text{ a black}$$

$$\text{from 1st, and a white from 2d. } \frac{3 \times 2}{70} = \frac{6}{70} = \frac{3}{35} = 3 \text{ to } 32, \text{ a black ball from}$$

$$\frac{5 \times 3 + 2 \times 7}{70} = \frac{29}{70} = 29 \text{ to } 41, \text{ a white ball from one, and a black from other,}$$

$$\text{both 2d and 3d cases favor this result; hence, } \frac{1}{5} + \frac{3}{14} = \frac{29}{70} \quad \frac{5 \times 7 + 5 \times 3 + 2 \times 7}{70}$$

$$= \frac{32}{35} = 32 \text{ to } 3, \text{ at least one white ball, for the 1st, 2d, and 3d cases favor this}$$

$$\text{; hence, } \frac{1}{2} + \frac{3}{14} + \frac{1}{5} = \frac{32}{35}.$$

Now, if number of white and black balls in each bag are same, say 5 white and 5 black, $5 + 2 \times 5 + 2 = 49$, then probability of drawing is as

$$\frac{5}{49} = \frac{25}{49} = 25 \text{ to } 24, \text{ a white ball from both. } \frac{5 \times 2}{49} = \frac{10}{49} = 10 \text{ to } 39, \text{ a white ball}$$

$$\text{1st, and a black from 2d. } \frac{2 \times 5}{49} = \frac{10}{49} = 10 \text{ to } 39, \text{ a black ball from 1st, and a}$$

$$\text{black from 2d. } \frac{2 \times 2}{49} = \frac{4}{49} = 4 \text{ to } 45, \text{ a black ball from both.}$$

When two dice are thrown, probability that sum of numbers on upper sides is a given number, say 7, is as follows:

Every one of the six numbers on one die may come up alike to, or in combination with the other, number of throws is $6 \times 6 = 36$.

Number 7 may be a combination of $\begin{pmatrix} 1 \text{ and } 6 \\ 2 \text{ " } 5 \\ 3 \text{ " } 4 \end{pmatrix}$; and as these numbers may be

thrown on either die, there are $3 \times 2 = 6$ throws in favor of the combination of 7; hence

$$\text{probability of throwing 7 is } \frac{6}{36} = \frac{1}{6}, \text{ or as 1 to 5.}$$

Probability of a player's partner at Whist holding a given card is as follows:

Number of cards held by the other 3 players is $3 \times 13 = 39$; probability, therefore, that it is held by partner is $\frac{1}{39}$, but it may be one of the 13 cards which he

$$\text{hence probability is } \frac{1}{39} \times 13 = \frac{13}{39} = \frac{1}{3}, \text{ or as 1 to 2.}$$

Probability of a player's partner at Whist holding two given cards is as follows:

Number of combinations of 39 things, taken 2 and 2 together, is $\frac{39 \times 38}{1 \times 2} = 7$

$$\text{hence, probability that these 2 cards are in partner's hand is } \frac{1}{\frac{39 \times 38}{1 \times 2}} = \frac{1}{39 \times 19}$$

or 1 to 740; but they may be any 2 cards in partner's hand; therefore, since

$$\text{number of combinations of 13 cards, taken 2 and 2 together, is } \frac{13 \times 12}{1 \times 2} = 78$$

$$\text{probability required is } \frac{78}{740} = \frac{2}{19}, \text{ or as 2 to 19.}$$

$$\text{hence, probability that he holds any 3 given cards is as } \frac{22}{703}, \text{ or}$$

124 WEIGHT AND STRENGTH OF WIRE, IRON, ETC.

WEIGHT AND STRENGTH OF WIRE, IRON, ETC.

Weight and Strength of Warrington Iron Wire.

Manufactured by Rylands Brothers. (England.)

Weight per 100 Lineal Feet.

| No. | Diameter. | Weight | Breaking Weight. | No. | Diameter. | Weight. | Breaking Weight. |
|--------|-----------------|--------|------------------|--------|-----------|---------|------------------|
| | Inch. | Lbs. | Lbs. | | Inch. | Lbs. | Lbs. |
| Gauge. | | | | Gauge. | | | |
| 7/0 | $\frac{1}{16}$ | 64.46 | 3490 | 9 | .146 | 5.5 | 298 |
| 6/0 | $\frac{15}{64}$ | 56.66 | 3066 | 10 | .133 | 4.43 | 247 |
| 5/0 | $\frac{7}{32}$ | 49.36 | 2673 | 10.5 | .125 | 4.03 | 218 |
| 4/0 | $\frac{11}{64}$ | 42.53 | 2303 | 11 | .117 | 3.53 | 191 |
| 3/0 | $\frac{9}{32}$ | 36.26 | 1963 | 12 | .1 | 2.66 | 145 |
| 2/0 | $\frac{11}{32}$ | 30.46 | 1653 | 13 | .09 | 2.1 | 113 |
| 0 | .326 | 27.36 | 1486 | 14 | .079 | 1.6 | 87 |
| 1 | .3 | 23.3 | 1257 | 15 | .069 | 1.23 | 66 |
| 2 | .274 | 19.36 | 1046 | 16 | .0625 | .96 | 53 |
| 3 | .25 | 16.13 | 873 | 17 | .053 | .73 | 39 |
| 4 | .229 | 13.53 | 732 | 18 | .047 | .56 | 31 |
| 5 | .209 | 11.26 | 610 | 19 | .041 | .43 | 23 |
| 6 | .191 | 9.4 | 509 | 20 | .036 | .33 | 18 |
| 7 | .174 | 7.8 | 422 | 21 | .03125 | .26 | 14 |
| 8 | .159 | 6.53 | 353 | 22 | .028 | .2 | 11 |

To Compute Length of 100 Pounds of Wire of a Given Diameter.

RULE.—Divide following numbers by square of diameter, in parts of an inch, and quotient is length in feet.

37.68 for wrought iron.
37.45 for steel.

33.42 for copper.
34.41 for brass.
13.64 for platinum.

28 for silver.
15.3 for gold.

WINDOW GLASS.

Thickness and Weight per Square Foot.


| No. | Thickness. | Weight. | No. | Thickness. | Weight. | No. | Thickness. | Weight. |
|-----|------------|---------|-----|------------|---------|-----|------------|---------|
| | Inch. | Oz. | | Inch. | Oz. | | Inch. | Oz. |
| 12 | .059 | 12 | 17 | .083 | 17 | 26 | .125 | 26 |
| 13 | .063 | 13 | 19 | .091 | 19 | 32 | .154 | 32 |
| 15 | .071 | 15 | 21 | .1 | 21 | 36 | .167 | 36 |
| 16 | .077 | 16 | 24 | .111 | 24 | 42 | .2 | 42 |

Terne Plates.

Terne Plates—Are of iron covered with an amalgam of lead.

Thickness and Weight of Galvanized Sheet Iron.

Sheet 2 Feet in Width by from 6 to 9 Feet in Length (M. Lefferts).

|  | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. |
|---|----------------------------|----------------|----------------------------|----------------|----------------------------|----------------|----------------------------|----------------|----------------------------|----------------|----------------------------|----------------|
| | No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. |
| | 26 | 15 | 23 | 20 | 20 | 27 | 17 | 36 | 14 | 53 | | |
| | 25 | 16 | 22 | 22 | 19 | 30 | 16 | 42 | 13 | 67 | | |
| | 24 | 18 | 21 | 24 | 18 | 35 | 15 | 46 | 12 | 70 | | |

WROUGHT IRON.

Weight of Square Rolled Iron,

From .125 Inch to 10 Inches. ONE FOOT IN LENGTH.

| In. | Weight. | Side. | Weight. | Side. | Weight. | Side. | Weight. |
|-----|---------|-------|---------|-------|---------|-------|---------|
| | Lbs. | | Lbs. | | Lbs. | | Lbs. |
| .25 | .053 | 2.125 | 15.263 | 4.125 | 57.517 | 6.25 | 132.04 |
| 5 | .211 | .25 | 17.112 | .25 | 61.055 | .5 | 142.816 |
| 75 | .475 | .375 | 19.066 | .375 | 64.7 | .75 | 154.012 |
| | .845 | .5 | 21.12 | .5 | 68.448 | 7 | 165.632 |
| 25 | 1.32 | .625 | 23.292 | .625 | 72.305 | .25 | 177.672 |
| 5 | 1.901 | .75 | 25.56 | .75 | 76.264 | .5 | 190.136 |
| 75 | 2.588 | .875 | 27.939 | .875 | 80.333 | .75 | 203.024 |
| | 3.38 | 3 | 30.416 | 5 | 84.48 | 8 | 216.336 |
| 15 | 4.278 | .125 | 33.01 | .125 | 88.784 | .25 | 230.068 |
| 5 | 5.28 | .75 | 35.704 | .25 | 93.168 | .5 | 244.22 |
| 5 | 6.39 | .375 | 38.503 | .375 | 97.657 | .75 | 258.8 |
| | 7.604 | .5 | 41.408 | .5 | 102.24 | 9 | 273.792 |
| 5 | 8.926 | .625 | 44.418 | .625 | 106.953 | .25 | 289.22 |
| | 10.352 | .75 | 47.534 | .75 | 111.756 | .5 | 305.056 |
| 5 | 11.883 | .875 | 50.756 | .875 | 116.671 | .75 | 321.33 |
| | 13.52 | 4 | 54.084 | 6 | 121.664 | 10 | 327.92 |

EXAMPLE.—What is weight of a bar 1.5 inches, by 12 inches in length? column 1st, find 1.5; opposite to it is 7.604 lbs., which is 7 lbs. and .604 of a lb. If denominator of ounces is required, result is obtained as follows:

Multiply remainder by 16, point off the decimals, and the figures remaining left of the point will give number of ounces.

Thus, .604 of a lb. = $.604 \times 16 = .9664 = 7 \text{ lbs. } 9.664 \text{ ounces.}$

Compute Weight for less than a Foot in Length.

EXAMPLE.—What is weight of a bar 6.25 inches square and 10.5 inches long?

Column 7th, opposite to 6.25 is 132.04, which is weight for a foot in length.

$$6.25 \times 12 \text{ inches} = 132.04$$

$$6 \text{ ins.} = .5 = 66.02$$

$$3 \text{ " } = .25 = 33.01$$

$$1.5 \text{ " } = .125 = 16.505$$

$$115.535 \text{ lbs.}$$

Weight of Angle Iron,

From 1.25 to 4.5 Inches. ONE FOOT IN LENGTH.

Thickness measured in Middle of each Side.

| EQUAL SIDES. | | | L UNEQUAL SIDES. | | | L UNEQUAL SIDES. | | |
|--------------|-------------|--------|------------------|-------------|---------|------------------|-------------|---------|
| In. | Thick-ness. | Weight | Sides. | Thick-ness. | Weight. | Sides. | Thick-ness. | Weight. |
| 1.25 | .1875 | 1.5 | 3 X 2.5 | .375 | 6.25 | 6 X 3.5 | .625 | 18 |
| 1.5 | .1875 | 2 | 3.5 X 3 | .4375 | 7.75 | 6 X 4.5 | .625 | 20 |
| 1.75 | .25 | 3 | 3.5 X 3 | .4375 | 9.6 | T | | |
| 2 | .25 | 3.5 | 4 X 3 | .5 | 11 | | | |
| 2.25 | .3125 | 4.5 | 4 X 3.5 | .5 | 11.5 | 2 X 2.375* | .375 | 5.5 |
| 2.5 | .3125 | 5 | 4 X 3.5 | .5 | 11.75 | 2.5 X 2.875 | .375 | 6.5 |
| 3 | .375 | 7 | 4.5 X 3 | .5 | 11.75 | 3.5 X 3.5 | .4375 | 10.5 |
| 3.5 | .4375 | 9 | 5 X 3 | .5 | 12.05 | 4 X 3.5 | .4375 | 12 |
| 4 | .5 | 12.5 | 5 X 3 | .5625 | 13.7 | | .75 | |
| 4.5 | .5 | 14 | 5.5 X 3.5 | .5 | 14.5 | | | |
| 4.5 | .5625 | 16 | 5.5 X 3.5 | .5625 | 15.6 | | | |

* This column gives depth of web added to the thickness of base or flange.

L*

WEIGHTS OF IRON, STEEL, COPPER, ETC.

Wrought Iron, Steel, Copper, and Brass

SOFT ROLLED. (*American Gauge.*)

| No. of Gauge. | Thickness. | PER SQUARE FOOT. | | |
|---------------|--------------------------------|------------------|---------|----------|
| | | Iron. | Steel. | Copper. |
| | Inch. | Lbs. | Lbs. | Lbs. |
| 0000 | .46 or $\frac{7}{16}$ full | 18.4575 | 18.7036 | 20.838 |
| 000 | .409 64 | 16.4368 | 16.6559 | 18.556 7 |
| 00 | .364 8 or $\frac{3}{8}$ light | 14.6376 | 14.8328 | 16.525 4 |
| 0 | .324 86 or $\frac{1}{2}$ " | 13.0351 | 13.2088 | 14.716 2 |
| 1 | .289 3 | 11.6082 | 11.7629 | 13.105 3 |
| 2 | .257 63 or $\frac{1}{4}$ full | 10.3374 | 10.4752 | 11.670 6 |
| 3 | .229 42 | 9.2055 | 9.3283 | 10.392 7 |
| 4 | .204 31 or $\frac{1}{4}$ full | 8.1979 | 8.3073 | 9.255 2 |
| 5 | .181 94 or $\frac{3}{8}$ light | 7.3004 | 7.3977 | 8.241 9 |
| 6 | .162 02 | 6.5011 | 6.5878 | 7.339 5 |
| 7 | .144 28 | 5.7892 | 5.8664 | 6.535 9 |
| 8 | .128 49 or $\frac{1}{8}$ full | 5.1557 | 5.2244 | 5.820 6 |
| 9 | .114 43 | 4.5915 | 4.6527 | 5.183 7 |
| 10 | .101 89 or $\frac{1}{10}$ full | 4.0884 | 4.1428 | 4.615 6 |
| 11 | .090 742 | 3.641 | 3.6896 | 4.110 6 |
| 12 | .080 808 | 3.2424 | 3.2856 | 3.660 6 |
| 13 | .071 961 | 2.8874 | 2.9259 | 3.259 8 |
| 14 | .064 084 | 2.5714 | 2.6057 | 2.903 |
| 15 | .057 068 | 2.2899 | 2.3204 | 2.585 2 |
| 16 | .050 82 or $\frac{1}{20}$ full | 2.0392 | 2.0664 | 2.302 1 |
| 17 | .045 257 | 1.8159 | 1.8402 | 2.050 1 |
| 18 | .040 303 | 1.6172 | 1.6387 | 1.825 7 |
| 19 | .035 89 | 1.44 | 1.4593 | 1.625 8 |
| 20 | .031 961 | 1.2824 | 1.2995 | 1.447 8 |
| 21 | .028 462 | 1.142 | 1.1573 | 1.289 3 |
| 22 | .025 347 | 1.017 | 1.0306 | 1.148 2 |
| 23 | .022 571 | .9057 | .9177 | 1.022 5 |
| 24 | .021 1 | .8065 | .8173 | .910 53 |
| 25 | .017 9 | .7182 | .7278 | .810 87 |
| 26 | .015 94 | .6396 | .6481 | .722 08 |
| 27 | .014 195 | .5696 | .5772 | .643 03 |
| 28 | .012 641 | .5072 | .514 | .572 64 |
| 29 | .011 257 | .4517 | .4577 | .509 94 |
| 30 | .010 025 | .4023 | .4076 | .454 13 |
| 31 | .008 928 | .3582 | .363 | .404 44 |
| 32 | .007 95 | .319 | .3232 | .360 14 |
| 33 | .007 08 | .2841 | .2879 | .320 72 |
| 34 | .006 304 | .2529 | .2563 | .285 57 |
| 35 | .005 614 | .2253 | .2283 | .254 31 |
| 36 | .005 | .2006 | .2033 | .226 5 |
| 37 | .004 453 | .1787 | .181 | .201 72 |
| 38 | .003 965 | .1591 | .1612 | .179 61 |
| 39 | .003 531 | .1417 | .1436 | .159 95 |
| 40 | .003 144 | .1261 | .1278 | .142 42 |

| | | | | |
|---------------------------|--------|--------|--------|---|
| Specific Gravities | 7.704 | 7.806 | 8.698 | |
| Weights of a Cube Foot .. | 481.75 | 487.75 | 543.6 | 5 |
| " " Inch .. | .278 7 | .2823 | .314 6 | |

Wrought Iron, Steel, Copper, and Brass Plates.

(*Birmingham Gauge.*)

| No. of Gauge. | Thickness. | PER SQUARE FOOT. | | | |
|---------------|------------------------------|------------------|---------|---------|---------|
| | | Iron. | Steel. | Copper. | Brass. |
| | Inch. | Lbs. | Lbs. | Lbs. | Lbs. |
| 0000 | .454 or $\frac{7}{16}$ full | 18.2167 | 18.4596 | 20.5662 | 19.4312 |
| 000 | .425 | 17.0531 | 17.2805 | 19.2525 | 18.19 |
| 00 | .38 or $\frac{3}{8}$ full | 15.2475 | 15.4508 | 17.214 | 16.264 |
| 0 | .34 or $\frac{1}{2}$ " | 13.6425 | 13.8244 | 15.402 | 14.552 |
| 1 | .3 | 12.0375 | 12.198 | 13.59 | 12.84 |
| 2 | .284 | 11.3955 | 11.5474 | 12.8652 | 12.1552 |
| 3 | .259 or $\frac{1}{4}$ full | 10.3924 | 10.5309 | 11.7327 | 11.0852 |
| 4 | .238 | 9.5497 | 9.6771 | 10.7814 | 10.1864 |
| 5 | .22 | 8.8275 | 8.9452 | 9.966 | 9.416 |
| 6 | .203 or $\frac{1}{5}$ full | 8.1454 | 8.254 | 9.1959 | 8.6884 |
| 7 | .18 or $\frac{3}{16}$ light | 7.2225 | 7.3188 | 8.154 | 7.704 |
| 8 | .165 or $\frac{1}{8}$ " | 6.6206 | 6.7089 | 7.4745 | 7.062 |
| 9 | .148 or $\frac{1}{4}$ full | 5.9385 | 6.0177 | 6.7044 | 6.3344 |
| 10 | .134 | 5.3767 | 5.4484 | 6.0702 | 5.7352 |
| 11 | .12 or $\frac{1}{8}$ light | 4.819 | 4.8792 | 5.436 | 5.136 |
| 12 | .109 | 4.3736 | 4.4319 | 4.9377 | 4.6652 |
| 13 | .095 or $\frac{1}{10}$ light | 3.8119 | 3.8627 | 4.3035 | 4.066 |
| 14 | .083 | 3.3304 | 3.3748 | 3.7599 | 3.5524 |
| 15 | .072 | 2.889 | 2.9275 | 3.2616 | 3.0816 |
| 16 | .065 | 2.6081 | 2.6429 | 2.9445 | 2.782 |
| 17 | .058 | 2.3272 | 2.3583 | 2.6274 | 2.4824 |
| 18 | .049 or $\frac{1}{20}$ light | 1.9661 | 1.9923 | 2.2197 | 2.0972 |
| 19 | .042 | 1.6852 | 1.7077 | 1.9026 | 1.7976 |
| 20 | .035 | 1.4044 | 1.4231 | 1.5855 | 1.498 |
| 21 | .032 | 1.284 | 1.3011 | 1.4496 | 1.3696 |
| 22 | .028 | 1.1235 | 1.1385 | 1.2684 | 1.1984 |
| 23 | .025 or $\frac{1}{40}$ | 1.0031 | 1.0165 | 1.1325 | 1.07 |
| 24 | .022 | .8827 | .8945 | .9966 | .9416 |
| 25 | .02 or $\frac{1}{50}$ | .8025 | .8132 | .906 | .856 |
| 26 | .018 | .7222 | .7319 | .8154 | .7704 |
| 27 | .016 | .642 | .6506 | .7248 | .6848 |
| 28 | .014 | .5617 | .5692 | .6342 | .5992 |
| 29 | .013 | .5216 | .5286 | .5889 | .5564 |
| 30 | .012 | .4815 | .4879 | .5436 | .5136 |
| 31 | .01 or $\frac{1}{100}$ | .4012 | .4066 | .453 | .428 |
| 32 | .009 | .3611 | .3659 | .4077 | .3852 |
| 33 | .008 | .321 | .3253 | .3624 | .3424 |
| 34 | .007 | .2809 | .2846 | .3171 | .2996 |
| 35 | .005 or $\frac{1}{200}$ | .2006 | .2033 | .2265 | .214 |
| 36 | .004 or $\frac{1}{250}$ | .1605 | .1626 | .1812 | .1712 |

Thickness of Sheet Silver, Gold, etc.

By Birmingham Gauge for these Metals.

| No. | Inch. | No. | Inch. | No. | Inch. | No. | Inch. | No. | Inch. | No. | Inch. |
|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|
| 1 | .004 | 7 | .015 | 13 | .036 | 19 | .064 | 25 | .095 | 31 | .133 |
| 2 | .005 | 8 | .016 | 14 | .041 | 20 | .067 | 26 | .103 | 32 | .143 |
| 3 | .008 | 9 | .019 | 15 | .047 | 21 | .072 | 27 | .113 | 33 | .145 |
| 4 | .01 | 10 | .024 | 16 | .051 | 22 | .074 | 28 | .12 | 34 | .148 |
| 5 | .013 | 11 | .029 | 17 | .057 | 23 | .077 | 29 | .124 | 35 | .158 |
| 6 | .013 | 12 | .034 | 18 | .061 | 24 | .082 | 30 | .126 | 36 | |

| Thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. |
|------------|-------------|------------|-------------|------------|-------------|------------|-------------|
| Ina. | Lbs. | Ina. | Lbs. | Ina. | Lbs. | Ina. | Lbs. |
| | 3.25 | | 3.75 | | 4.5 | | 5 |
| 1.375 | 15.102 | 1.875 | 23.762 | .75 | 11.406 | 3.25 | 54.916 |
| 1.5 | 16.475 | 2 | 25.346 | 1 | 15.208 | 3.5 | 59.14 |
| 1.625 | 17.848 | 2.25 | 28.514 | 1.25 | 19.01 | 3.75 | 63.365 |
| 1.75 | 19.221 | 2.5 | 31.682 | 1.5 | 22.812 | 4 | 67.589 |
| 1.875 | 20.594 | 2.75 | 34.851 | 1.75 | 26.614 | 4.25 | 71.813 |
| 2 | 21.967 | 3 | 38.019 | 2 | 30.415 | 4.5 | 76.038 |
| 2.25 | 24.712 | 3.25 | 41.187 | 2.25 | 34.217 | 4.75 | 80.262 |
| 2.5 | 27.458 | 3.5 | 44.355 | 2.5 | 38.019 | | 5.25 |
| 2.75 | 30.204 | | 4 | 2.75 | 41.82 | | 4.436 |
| 3 | 32.95 | | 1.69 | 3 | 45.623 | .25 | 8.871 |
| | 3.6 | .125 | 3.38 | 3.25 | 49.425 | .5 | 13.307 |
| .125 | 1.479 | .25 | 6.759 | 3.5 | 53.226 | .75 | 17.742 |
| .25 | 2.957 | .75 | 10.138 | 3.75 | 57.028 | 1 | 22.178 |
| .375 | 4.436 | 1 | 13.518 | 4 | 60.83 | 1.25 | 26.613 |
| .5 | 5.914 | 1.25 | 16.897 | 4.25 | 64.632 | 1.5 | 31.049 |
| .625 | 7.393 | 1.5 | 20.277 | | 4.75 | 1.75 | 35.484 |
| .75 | 8.871 | 1.75 | 23.656 | .25 | 4.013 | 2 | 39.92 |
| .875 | 10.35 | 2 | 27.036 | .5 | 8.026 | 2.25 | 44.355 |
| 1 | 11.828 | 2.25 | 30.415 | .75 | 12.036 | 2.5 | 48.791 |
| 1.125 | 13.307 | 2.5 | 33.795 | 1 | 16.052 | 2.75 | 53.226 |
| 1.25 | 14.785 | 2.75 | 37.174 | 1.25 | 20.066 | 3 | 57.662 |
| 1.375 | 16.264 | 3 | 40.554 | 1.5 | 24.079 | 3.25 | 62.097 |
| 1.5 | 17.742 | 3.25 | 43.933 | 1.75 | 28.092 | 3.5 | 66.533 |
| 1.625 | 19.221 | 3.5 | 47.313 | 2 | 32.105 | 3.75 | 70.968 |
| 1.75 | 20.699 | 3.75 | 50.692 | 2.25 | 36.118 | 4 | 75.404 |
| 1.875 | 22.178 | | 4.25 | 2.5 | 40.131 | 4.25 | 79.839 |
| 2 | 23.656 | .125 | 1.795 | 2.75 | 44.144 | 4.5 | 84.275 |
| 2.25 | 26.613 | .25 | 3.591 | 3 | 48.157 | 4.75 | 88.71 |
| 2.5 | 29.57 | .75 | 7.181 | 3.25 | 52.17 | 5 | |
| 2.75 | 32.527 | .5 | 10.772 | 3.5 | 56.184 | | 5.5 |
| 3 | 35.485 | .75 | 14.364 | 3.75 | 60.197 | .25 | 4.647 |
| 3.25 | 38.441 | 1 | 17.953 | 4 | 64.21 | .5 | 9.294 |
| | 3.75 | 1.25 | 21.544 | 4.25 | 68.223 | .75 | 13.94 |
| .125 | 1.584 | 1.5 | 25.135 | 4.5 | 72.235 | 1 | 18.587 |
| .25 | 3.168 | 1.75 | 28.725 | | 5 | 1.25 | 23.234 |
| .375 | 4.752 | 2 | 32.316 | .25 | 4.224 | 1.5 | 27.881 |
| .5 | 6.336 | 2.25 | 35.907 | .5 | 8.449 | 1.75 | 32.527 |
| .625 | 7.921 | 2.5 | 39.497 | .75 | 12.673 | 2 | 37.174 |
| .75 | 9.505 | 2.75 | 43.088 | 1 | 16.897 | 2.25 | 41.821 |
| .875 | 11.089 | 3 | 46.679 | 1.25 | 21.122 | 2.5 | 46.468 |
| 1 | 12.673 | 3.25 | 50.269 | 1.5 | 25.346 | 2.75 | 51.114 |
| 1.125 | 14.257 | 3.5 | 53.86 | 1.75 | 29.57 | 3 | 55.761 |
| 1.25 | 15.841 | 4 | 57.45 | 2 | 33.795 | 3.25 | 60.408 |
| 1.375 | 17.425 | | 4.5 | 2.25 | 38.019 | 3.5 | 65.055 |
| 1.5 | 19.009 | .25 | 3.802 | 2.5 | 42.243 | 3.75 | 69.701 |
| 1.625 | 20.594 | .5 | 7.604 | 2.75 | 46.468 | 4 | 74.348 |
| 1.75 | 22.178 | | | 3 | 50.692 | 4.25 | 78.995 |
| | | | | | | 4.5 | 83.642 |

ILLUSTRATION.—What is weight of a bar of iron 5.25 ins. in breadth by .75 in. in thickness

In column 7, as above, find 5.25; and below it, in column, .75; and opposite to that is 13.307, which is 13 lbs. and .307 of a pound.

For parts of a pound and of a foot, operate according to rule laid down for table.

Weight of Sheet Iron. (English. D. K. Clark.)

PER SQUARE FOOT (at 480 lbs. per Cube Foot).

As by Wire-gauge used in South Staffordshire, England.

| nom. | Weight. | Square Feet in 1 ton. | | Thickness. | Weight. | Square Feet in 1 ton. | | Thickness. | Weight. | Square Feet in 1 ton. | |
|------|---------|-----------------------|-----|------------|---------|-----------------------|-----|------------|---------|-----------------------|-----|
| | | Lbs. | No. | | | Lbs. | No. | | | Lbs. | No. |
| ach. | Lbs. | | | No. Inch. | Lbs. | | | No. Inch. | Lbs. | | |
| 1125 | .5 | 4480 | 2 | .0344 | 1.38 | 1623 | 10 | .1406 | 5.63 | 398 | |
| 1141 | .562 | 3986 | 20 | .0375 | 1.5 | 1493 | 9 | .1563 | 6.25 | 358 | |
| 1156 | .625 | 3584 | 19 | .0438 | 1.75 | 1280 | 8 | .1719 | 6.88 | 326 | |
| 1172 | .688 | 3256 | 18 | .05 | 2 | 1120 | 7 | .1875 | 7.5 | 299 | |
| 1188 | .75 | 2987 | 17 | .0563 | 2.25 | 996 | 6 | .2031 | 8.13 | 276 | |
| 1203 | .813 | 2755 | 16 | .0625 | 2.5 | 896 | 5 | .2188 | 8.75 | 256 | |
| 1219 | .875 | 2560 | 15 | .075 | 3 | 747 | 4 | .2344 | 9.38 | 239 | |
| 1234 | .938 | 2388 | 14 | .0875 | 3.5 | 640 | 3 | .25 | 10 | 224 | |
| 125 | 1 | 2240 | 13 | .1 | 4 | 560 | 2 | .2813 | 11.25 | 199 | |
| 1281 | 1.13 | 1982 | 12 | .1125 | 4.5 | 498 | 1 | .3125 | 12.5 | 179 | |
| 313 | 1.25 | 1792 | 11 | .125 | 5 | 448 | | | | | |

Weight of Hoop Iron. (English.)

PER LINEAL FOOT.

| W. G. | | Weight. | Width. | | W. G. | Weight. | Width. | | W. G. | Weight. |
|-------|-------|---------|--------|--|-------|---------|--------|--|-------|---------|
| No. | Lbs. | | Inch. | | No. | Lbs. | Inch. | | No. | Lbs. |
| 21 | .067 | | 1.125 | | 17 | .21 | 1.75 | | 14 | .484 |
| 20 | .0875 | | 1.25 | | 16 | .27 | 2 | | 13 | .634 |
| 19 | .1216 | | 1.375 | | 15 | .33 | 2.25 | | 13 | .714 |
| 18 | .1636 | | 1.5 | | 15 | .36 | 2.5 | | 12 | .91 |

Weight of Black and Galvanized Sheet Iron.

n's Table, founded upon Sir Joseph Whitworth & Co.'s Standard Birmingham Wire-Gauge.) (D. K. Clark.)

—Numbers on Holtzapfel's wire-gauge are applied to thicknesses on Whit-gauge.

| and Weight of Black Sheets. | | Approximate number of Sq. Ft. in 1 ton. | | Gauge and Weight of Black Sheets. | | | Approximate number of Sq. Ft. in 1 ton | |
|--------------------------------|------|--|-------------|--------------------------------------|-------|------|---|-------------|
| | | Black. | Galvanized. | No. | Inch. | Lbs. | Black. | Galvanized. |
| Inch. | Lbs. | Sq. Ft. | Sq. Ft. | No. | Inch. | Lbs. | Sq. Ft. | Sq. Ft. |
| 3 | 12 | 187 | 185 | 17 | .06 | 2.4 | 933 | 876 |
| 28 | 11.2 | 200 | 197 | 18 | .05 | 2 | 1120 | 1038 |
| 26 | 10.4 | 215 | 212 | 19 | .04 | 1.6 | 1400 | 1274 |
| 24 | 9.6 | 233 | 229 | 20 | .036 | 1.4 | 1556 | 1403 |
| 22 | 8.8 | 254 | 250 | 21 | .032 | 1.28 | 1750 | 1558 |
| 2 | 8 | 280 | 275 | 22 | .028 | 1.12 | 2000 | 1753 |
| 18 | 7.2 | 311 | 304 | 23 | .024 | .96 | 2333 | 2004 |
| 165 | 6.6 | 339 | 331 | 24 | .022 | .88 | 2545 | 2159 |
| 15 | 6 | 373 | 363 | 25 | .02 | .8 | 2800 | 2339 |
| 135 | 5.4 | 415 | 403 | 26 | .018 | .72 | 3111 | 2552 |
| 12 | 4.8 | 467 | 452 | 27 | .016 | .64 | | |
| 11 | 4.4 | 509 | 491 | 28 | .014 | .56 | | |
| 105 | 3.8 | 589 | 566 | 29 | .013 | .52 | | |
| 85 | 3.4 | 659 | 630 | 30 | .012 | .48 | | |
| 7 | 2.8 | 800 | 757 | 31 | .01 | .4 | | |
| 65 | 2.6 | 862 | 813 | 32 | .009 | .36 | | |

Weight of English Angle and T Iron. (D. K. Co.)
ONE FOOT IN LENGTH.

NOTE.—When base or web tapers in section, mean thickness is to be measured.

| Thick- ness. | SUM OF WIDTH AND DEPTH IN INCHES. | | | | | | | | | |
|-----------------|-----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1.5 | 1.625 | 1.75 | 1.875 | 2 | 2.125 | 2.25 | 2.375 | 2.5 | 2.625 |
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .125 | .57 | .62 | .68 | .73 | .78 | .83 | .88 | .94 | .99 | 1.04 |
| .1875 | .81 | .89 | .97 | 1.05 | 1.13 | 1.21 | 1.29 | 1.37 | 1.45 | 1.52 |
| .25 | 1.04 | 1.15 | 1.25 | 1.36 | 1.46 | 1.56 | 1.67 | 1.77 | 1.88 | 1.98 |
| .3125 | 1.24 | 1.37 | 1.5 | 1.63 | 1.76 | 1.89 | 2.02 | 2.15 | 2.28 | 2.41 |
| | 2.875 | 3 | 3.125 | 3.25 | 3.375 | 3.5 | 3.625 | 3.75 | 3.875 | 4 |
| .125 | 1.14 | 1.2 | 1.25 | 1.3 | 1.45 | 1.41 | 1.46 | 1.51 | 1.56 | 1.62 |
| .1875 | 1.68 | 1.76 | 1.84 | 1.91 | 1.99 | 2.07 | 2.15 | 2.23 | 2.3 | 2.38 |
| .25 | 2.19 | 2.29 | 2.4 | 2.5 | 2.6 | 2.71 | 2.81 | 2.92 | 3.02 | 3.13 |
| .3125 | 2.67 | 2.8 | 2.93 | 3.06 | 3.19 | 3.32 | 3.45 | 3.58 | 3.71 | 3.84 |
| .375 | 3.13 | 3.28 | 3.44 | 3.59 | 3.75 | 3.91 | 4.06 | 4.22 | 4.38 | 4.53 |
| .4375 | 3.57 | 3.75 | 3.93 | 4.11 | 4.29 | 4.48 | 4.66 | 4.84 | 5.02 | 5.2 |
| | 4.5 | 4.75 | 5 | 5.25 | 5.5 | 5.75 | 6 | 6.25 | 6.5 | 6.75 |
| .1875 | 2.7 | 2.85 | 3.01 | 3.16 | 3.32 | 3.48 | 3.63 | 3.79 | 3.95 | 4.1 |
| .25 | 3.54 | 3.75 | 3.96 | 4.17 | 4.38 | 4.59 | 4.79 | 5 | 5.21 | 5.42 |
| .3125 | 4.36 | 4.62 | 4.88 | 5.14 | 5.4 | 5.66 | 5.92 | 6.18 | 6.45 | 6.71 |
| .375 | 5.16 | 5.47 | 5.78 | 6.09 | 6.41 | 6.72 | 7.03 | 7.34 | 7.66 | 7.97 |
| .4375 | 5.92 | 6.29 | 6.65 | 7.02 | 7.38 | 7.75 | 8.11 | 8.48 | 8.84 | 9.21 |
| .5 | 6.67 | 7.08 | 7.5 | 7.92 | 8.33 | 8.75 | 9.17 | 9.58 | 10 | 10.42 |
| .5625 | 7.38 | 7.85 | 8.32 | 8.79 | 9.26 | 9.73 | 10.2 | 10.66 | 11.13 | 12.6 |
| | 7.25 | 7.5 | 7.75 | 8 | 8.25 | 8.5 | 8.75 | 9 | 9.25 | 9.5 |
| .25 | 5.83 | 6.04 | 6.25 | 6.46 | 6.67 | 6.88 | 7.08 | 7.29 | 7.5 | 7.71 |
| .3125 | 7.23 | 7.49 | 7.75 | 8.01 | 8.27 | 8.53 | 8.79 | 9.05 | 9.31 | 9.57 |
| .375 | 8.59 | 8.91 | 9.22 | 9.53 | 9.84 | 10.16 | 10.47 | 10.78 | 11.09 | 11.41 |
| .4375 | 9.93 | 10.3 | 10.66 | 11.03 | 11.39 | 11.76 | 12.12 | 12.49 | 12.85 | 13.22 |
| .5 | 11.25 | 11.67 | 12.08 | 12.5 | 12.92 | 13.33 | 13.75 | 14.17 | 14.58 | 15 |
| .5625 | 12.54 | 13.01 | 13.48 | 13.94 | 14.41 | 14.88 | 15.35 | 15.82 | 16.29 | 16.76 |
| .625 | 13.8 | 14.32 | 14.84 | 15.36 | 15.89 | 16.41 | 16.93 | 17.45 | 17.97 | 18.49 |
| | 10 | 10.5 | 11 | 11.5 | 12 | 12.5 | 13 | 13.5 | 14 | 14.5 |
| .375 | 12.03 | 12.66 | 13.28 | 13.91 | 14.53 | | | | | |
| .4375 | 13.95 | 14.67 | 15.4 | 16.13 | 16.86 | 17.59 | 18.31 | 19.04 | 19.77 | 20.5 |
| .5 | 15.83 | 16.67 | 17.5 | 18.33 | 19.17 | 20 | 20.84 | 21.67 | 22.5 | 23.34 |
| .5625 | 17.7 | 18.63 | 19.57 | 20.51 | 21.44 | 22.38 | 23.31 | 24.25 | 25.19 | 26.12 |
| .625 | 19.53 | 20.57 | 21.61 | 22.66 | 23.7 | 24.74 | 25.78 | 26.83 | 27.87 | 28.91 |
| .75 | 23.13 | 24.38 | 25.63 | 26.88 | 28.13 | 29.37 | 30.63 | 31.88 | 33.13 | 34.38 |
| | 12 | 12.5 | 13 | 13.5 | 14 | 15 | 16 | 17 | 18 | 19 |
| .625 | 23.7 | 24.74 | 25.78 | 26.83 | 27.87 | 29.95 | 32.03 | 34.12 | 36.2 | 38.28 |
| -- | 28.13 | 29.37 | 30.63 | 31.88 | 33.13 | 35.63 | 38.13 | 40.63 | 41.13 | 43.63 |
| | 45 | 33.91 | 35.36 | 36.82 | 38.28 | 41.19 | 44.12 | 47.02 | 49.95 | 52.87 |
| | 67 | 38.33 | 40 | 41.67 | 43.33 | 46.67 | 50 | 53.33 | 56.67 | 60 |

American rolled is slightly heavier.

Weight of Hoop Iron. (D. K. Clark)

ONE FOOT IN LENGTH.

As by Wire-gauge used in South Staffordshire (England).

| Inch. | Width in Inches. | | | | | | | | | | |
|-------|------------------|-------|------|------|-------|------|-------|------|-------|------|------|
| | .625 | .75 | .875 | 1 | 1.125 | 1.25 | 1.375 | 1.5 | 1.625 | 1.75 | 2 |
| Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. | Lb. |
| 344 | .0716 | .0861 | .1 | .115 | .129 | .144 | .158 | .172 | .197 | .201 | .229 |
| 375 | .0781 | .0938 | .109 | .125 | .141 | .156 | .172 | .188 | .203 | .219 | .25 |
| 438 | .0911 | .109 | .128 | .146 | .164 | .182 | .2 | .219 | .238 | .257 | .292 |
| 5 | .104 | .125 | .146 | .167 | .188 | .208 | .229 | .25 | .271 | .292 | .333 |
| 563 | .117 | .141 | .164 | .188 | .211 | .234 | .258 | .281 | .305 | .328 | .375 |
| 625 | .13 | .156 | .182 | .208 | .234 | .26 | .286 | .313 | .339 | .365 | .417 |
| 75 | .156 | .188 | .219 | .25 | .281 | .313 | .344 | .375 | .307 | .438 | .5 |
| 875 | .183 | .219 | .256 | .293 | .329 | .366 | .402 | .438 | .475 | .512 | .585 |
| | .208 | .25 | .292 | .333 | .375 | .416 | .458 | .5 | .543 | .584 | .667 |
| 125 | .234 | .281 | .328 | .375 | .422 | .469 | .516 | .563 | .609 | .656 | .75 |
| 25 | .26 | .313 | .365 | .417 | .469 | .521 | .573 | .625 | .677 | .729 | .833 |
| 406 | .293 | .352 | .41 | .469 | .527 | .586 | .645 | .703 | .762 | .82 | .938 |
| 563 | .326 | .391 | .456 | .522 | .587 | .652 | .717 | .783 | .848 | .913 | 1.04 |
| 719 | .358 | .43 | .501 | .573 | .644 | .716 | .788 | .859 | .931 | 1 | 1.15 |
| 875 | .391 | .469 | .547 | .625 | .703 | .781 | .859 | .938 | 1.02 | 1.09 | 1.25 |
| 931 | .423 | .508 | .593 | .677 | .762 | .836 | .931 | 1.02 | 1.1 | 1.19 | 1.35 |
| 188 | .456 | .547 | .638 | .729 | .82 | .912 | 1 | 1.09 | 1.19 | 1.28 | 1.46 |
| 344 | .488 | .586 | .683 | .781 | .879 | .977 | 1.07 | 1.17 | 1.27 | 1.37 | 1.56 |

CAST IRON.**Compute Weight of a Cast Iron Bar or Rod.**

ascertain weight of a wrought iron bar or rod of same dimensions in the tables, or by computation, and from weight deduct $\frac{2}{3}$ th part.

As .1000 : .9257 :: weight of a wrought bar or rod : to weight required.

Thus, what is weight of a piece of cast iron $4 \times 3.75 \times 12$ inches? See table, page 128, weight of a piece of wrought iron of these dimensions 32 lbs. Then, $1000 : .9257 :: 50.692 : 46.93$ lbs.

Braziers' and Sheathing Copper.

BRASS SHEETS, 2×4 feet from 5 to 25 lbs., 2.5×5 feet from 9 to 150 lbs., and 4×6 feet, from 16 to 300 lbs. per sheet.

BRASS COPPER, 14×48 inches, and from 14 to 34 oz. per square foot.

IRON METAL, 14×48 inches, and from 16 to 34 oz. per square foot.

Weight of Corrugated Iron Roof Plates.

PER SQUARE FOOT. (Birmingham Gauge.)

| Black. | Galvanized. | No. | Black. | Galvanized. | No. | Black. | Galvanized. |
|--------|-------------|-----|--------|-------------|-----|--------|-------------|
| Oz. | Oz. | | Oz. | Oz. | | Oz. | Oz. |
| 26 | 29 | 23 | 20 | 22 | 25 | 16 | 18 |
| 22 | 24 | 24 | 18 | 20 | 26 | 14 | 16 |

METALS.**Compute Weight of Metals of any Dimensions or Form.**

As in Mensuration of Solids (page 360), ascertain weight of a cube inch, and multiply it by weight of a cube inch, and product will be weight of the solid.

WEIGHT AND STRENGTH OF WIRE, IRON, ETC.

Weight and Strength of Warrington Iron Wire.

*Manufactured by Rylands Brothers. (England.)**Weight per 100 Lineal Feet.*

| No. | Diameter. | Weight | Breaking Weight. | | No. | Diameter. | Weight. | Breaking Weight. | |
|--------|-----------------|--------|------------------|---------|--------|-----------|---------|------------------|---------|
| | | | Annealed. | Bright. | | | | Annealed. | Bright. |
| Gauge. | Inch. | Lbs. | Lbs. | Lbs. | Gauge. | Inch. | Lbs. | Lbs. | Lbs. |
| 7/0 | $\frac{1}{32}$ | 64.46 | 3490 | 5233 | 9 | .146 | 5.5 | 298 | 447 |
| 6/0 | $\frac{15}{64}$ | 56.66 | 3066 | 4603 | 10 | .133 | 4.43 | 247 | 370 |
| 5/0 | $\frac{7}{32}$ | 49.36 | 2673 | 4000 | 10.5 | .125 | 4.03 | 218 | 327 |
| 4/0 | $\frac{3}{16}$ | 42.53 | 2303 | 3457 | 11 | .117 | 3.53 | 191 | 288 |
| 3/0 | $\frac{5}{32}$ | 36.26 | 1963 | 2945 | 12 | .1 | 2.66 | 145 | 217 |
| 2/0 | $\frac{1}{8}$ | 30.46 | 1653 | 2473 | 13 | .09 | 2.1 | 113 | 169 |
| 0 | .326 | 27.36 | 1486 | 2226 | 14 | .079 | 1.6 | 87 | 130 |
| 1 | .3 | 23.3 | 1257 | 1885 | 15 | .069 | 1.23 | 66 | 99 |
| 2 | .274 | 19.36 | 1046 | 1572 | 16 | .0625 | .96 | 53 | 77 |
| 3 | .25 | 16.13 | 873 | 1309 | 17 | .053 | .73 | 39 | 59 |
| 4 | .229 | 13.53 | 732 | 1098 | 18 | .047 | .56 | 31 | 46 |
| 5 | .209 | 11.26 | 610 | 913 | 19 | .041 | .43 | 23 | 35 |
| 6 | .191 | 9.4 | 509 | 763 | 20 | .036 | .33 | 18 | 27 |
| 7 | .174 | 7.8 | 422 | 633 | 21 | .03125 | .26 | 14 | 21 |
| 8 | .159 | 6.53 | 353 | 519 | 22 | .028 | .2 | 11 | 16 |

To Compute Length of 100 Pounds of Wire of a Given Diameter.

RULE.—Divide following numbers by square of diameter, in parts of an inch, and quotient is length in feet.

37.68 for wrought iron.

37.45 for steel.

33.42 for copper.

34.41 for brass.

28 for silver.

15.3 for gold.

13.64 for platinum.

WINDOW GLASS.

Thickness and Weight per Square Foot.

| No. | Thickness. | Weight. | No. | Thickness. | Weight. | No. | Thickness. | Weight. |
|-----|------------|---------|-----|------------|---------|-----|------------|---------|
| | Inch. | Oz. | | Inch. | Oz. | | Inch. | Oz. |
| 12 | .059 | 12 | 17 | .083 | 17 | 26 | .125 | 26 |
| 13 | .063 | 13 | 19 | .091 | 19 | 32 | .154 | 32 |
| 15 | .071 | 15 | 21 | .1 | 21 | 36 | .167 | 36 |
| 16 | .077 | 16 | 24 | .111 | 24 | 42 | .2 | 42 |

Terne Plates.

Terne Plates—Are of iron covered with an amalgam of lead.

Thickness and Weight of Galvanized Sheet Iron.

Sheet 2 Feet in Width by from 6 to 9 Feet in Length (M. Lefferts).

| Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. | Wire Gauge. | Weight per Sq. Foot. |
|-------------|----------------------|-------------|----------------------|-------------|----------------------|-------------|----------------------|-------------|----------------------|-------------|----------------------|
| No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. | No. | Oz. |
| 20 | 12 | 26 | 15 | 23 | 20 | 20 | 27 | 17 | 36 | 14 | 53 |
| 19 | 13 | 25 | 16 | 22 | 22 | 19 | 30 | 16 | 42 | 13 | 61 |
| 18 | 14 | 24 | 18 | 21 | 24 | 18 | 35 | 15 | 46 | 12 | 70 |

| Size. | Thickn. | Weight. | Diameter. | Thickn. | Weight. | Diameter. | Thickn. | Weight. |
|-------|---------|---------|-----------|---------|---------|-----------|---------|---------|
| | Inch. | Lbs. | In. | Inch. | Lbs. | In. | Inch. | Lbs. |
| 5 | .875 | 157.59 | 29 | .75 | 218.7 | 40 | .875 | 350.56 |
| | I | 181.33 | | .875 | 256.23 | | I | 401.86 |
| | .625 | 114.1 | | I | 294.05 | | I.125 | 453.46 |
| | .75 | 137.84 | 30 | .75 | 226.05 | | I.25 | 505.41 |
| | .875 | 161.88 | | .875 | 264.8 | 42 | .875 | 367.69 |
| | I | 186.23 | | I | 303.86 | | I | 421.45 |
| | .625 | 120.23 | | I.125 | 343.22 | | I.125 | 472.52 |
| | .75 | 145.19 | 31 | .75 | 233.41 | | I.25 | 529.87 |
| | .875 | 170.46 | | .875 | 273.38 | 44 | .875 | 384.88 |
| | I | 196.03 | | I | 313.66 | | I | 441.1 |
| | .625 | 126.35 | | I.125 | 354.24 | | I.125 | 497.58 |
| | .75 | 152.54 | 32 | .75 | 240.75 | | I.25 | 554.42 |
| | .875 | 179.03 | | .875 | 281.95 | 46 | .875 | 402.01 |
| | I | 205.84 | | I | 323.46 | | I | 460.07 |
| | .625 | 132.48 | | I.125 | 365.27 | | I.125 | 519.64 |
| | .75 | 159.89 | 33 | .75 | 248.11 | | I.25 | 578.88 |
| | .875 | 187.61 | | .875 | 290.53 | 48 | .875 | 419.17 |
| | I | 215.64 | | I | 333.26 | | I | 480.29 |
| | .625 | 138.61 | | I.125 | 376.29 | | I.125 | 541.69 |
| | .75 | 167.24 | 34 | .75 | 255.46 | | I.25 | 603.44 |
| | .875 | 196.19 | | .875 | 299.11 | 50 | .875 | 436.43 |
| | I | 225.44 | | I | 343.06 | | I | 499.89 |
| | .625 | 144.73 | | I.125 | 387.33 | | I.125 | 563.75 |
| | .75 | 174.59 | 35 | .75 | 262.81 | | I.25 | 627.93 |
| | .875 | 204.76 | | .875 | 307.68 | 52 | .875 | 453.49 |
| | I | 235.24 | | I | 352.87 | | I | 519.5 |
| | .625 | 150.86 | | I.125 | 398.35 | | I.125 | 585.81 |
| | .75 | 181.95 | 36 | .75 | 270.16 | | I.25 | 654.42 |
| | .875 | 213.34 | | .875 | 316.26 | 55 | .875 | 479.23 |
| | I | 245.04 | | I | 362.67 | | I | 548.9 |
| | .625 | 156.98 | | I.125 | 409.28 | | I.125 | 618.91 |
| | .75 | 189.3 | | I.25 | 456.37 | | I.25 | 689.21 |
| | .875 | 221.92 | 37 | .75 | 277.51 | | I | 578.29 |
| | I | 254.85 | | .875 | 324.84 | 58 | I.125 | 651.96 |
| | .625 | 163.11 | | I | 372.47 | | I.25 | 725.93 |
| | .75 | 196.65 | | I.125 | 420.4 | | I.375 | 800.22 |
| | .875 | 230.5 | | I.25 | 468.65 | 60 | I | 597.92 |
| | I | 264.65 | 38 | .75 | 284.86 | | I.125 | 674.01 |
| | .625 | 169.23 | | .875 | 333.41 | | I.25 | 750.45 |
| | .75 | 204 | | I | 382.27 | | I.375 | 827.17 |
| | .875 | 239.07 | | I.125 | 431.41 | 63 | I | 646.93 |
| | I | 274.45 | | I.25 | 480.89 | | I.125 | 729.18 |
| | .625 | 175.36 | 39 | .75 | 292.21 | | I.25 | 811.73 |
| | .75 | 211.35 | | .875 | 341.97 | | I.375 | 894.6 |
| | .875 | 247.65 | | I | 392.08 | 70 | I | 695.92 |
| | I | 284.25 | | I.125 | 442.44 | | I.25 | 872.98 |
| | .625 | 181.49 | | I.25 | 493.14 | | I.5 | 1051.25 |

Equivalent Length of Pipe for a Socket.

$\frac{d}{15} = l$. d representing diameter of pipe and l length in inches.
 The weight of two flanges for any diameter is computed equal to the weight of the pipe.

—These weights do not include any allowance for spigot and so on or rule to compute thicknesses of pipes, flanges, etc., see page 5.

134 WEIGHT OF FLAT ROLLED BAR AND SQUARE STEEL

Weight of Flat Rolled Bar Steel. (D. K. Clark.)

From .5 Inch to 8 Inches in Width. ONE FOOT IN LENGTH.

WIDTH IN INCHES.

| Thick- ness. | .5 | .625 | .75 | .875 | 1 | 1.25 | 1.5 | 1.75 | 2 | 2.25 | 2.5 | 2.75 |
|-----------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| $\frac{1}{4}$ | .425 | .533 | .64 | .743 | .85 | 1.06 | 1.28 | 1.49 | 1.7 | 1.91 | 2.13 | 2.34 |
| $\frac{5}{16}$ | .531 | .665 | .8 | .929 | 1.06 | 1.33 | 1.59 | 1.86 | 2.13 | 2.39 | 2.66 | 2.92 |
| $\frac{3}{8}$ | .638 | .798 | .96 | 1.11 | 1.28 | 1.59 | 1.91 | 2.23 | 2.55 | 2.87 | 3.19 | 3.51 |
| $\frac{7}{16}$ | .744 | .931 | 1.12 | 1.3 | 1.49 | 1.86 | 2.23 | 2.6 | 2.98 | 3.35 | 3.72 | 4.09 |
| $\frac{1}{2}$ | .85 | 1.06 | 1.28 | 1.49 | 1.7 | 2.13 | 2.55 | 2.98 | 3.4 | 3.83 | 4.25 | 4.68 |
| $\frac{9}{16}$ | — | 1.2 | 1.44 | 1.67 | 1.91 | 2.39 | 2.87 | 3.35 | 3.83 | 4.3 | 4.78 | 5.26 |
| $\frac{5}{8}$ | — | 1.33 | 1.6 | 1.86 | 2.12 | 2.66 | 3.19 | 3.72 | 4.25 | 4.78 | 5.31 | 5.84 |
| $\frac{11}{16}$ | — | — | 1.76 | 2.04 | 2.34 | 2.92 | 3.51 | 4.09 | 4.68 | 5.26 | 5.84 | 6.43 |
| $\frac{3}{4}$ | — | — | 1.92 | 2.23 | 2.55 | 3.19 | 3.83 | 4.46 | 5 | 5.74 | 6.38 | 7.01 |
| $\frac{7}{8}$ | — | — | — | 2.41 | 2.76 | 3.45 | 4.14 | 4.83 | 5.53 | 6.22 | 6.91 | 7.6 |
| $\frac{15}{16}$ | — | — | — | 2.6 | 2.98 | 3.72 | 4.46 | 5.21 | 5.95 | 6.69 | 7.44 | 8.18 |
| 1 | — | — | — | — | 3.19 | 3.98 | 4.78 | 5.58 | 6.38 | 7.17 | 7.97 | 8.77 |
| | — | — | — | — | 3.4 | 4.25 | 5.1 | 5.95 | 6.8 | 7.65 | 8.5 | 9.35 |

WIDTH IN INCHES.

| Thick- ness. | 3 | 3.25 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 | 8 |
|-----------------|------|------|------|------|------|------|------|------|------|------|------|------|
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| $\frac{1}{4}$ | 2.55 | 2.76 | 2.98 | 3.4 | 3.82 | 4.26 | 4.68 | 5.1 | 5.52 | 5.96 | 6.38 | 6.8 |
| $\frac{5}{16}$ | 3.19 | 3.45 | 3.72 | 4.25 | 4.78 | 5.32 | 5.84 | 6.38 | 6.9 | 7.44 | 7.97 | 8.5 |
| $\frac{3}{8}$ | 3.83 | 4.14 | 4.46 | 5.1 | 5.74 | 6.38 | 7.02 | 7.66 | 8.28 | 8.92 | 9.56 | 10.2 |
| $\frac{7}{16}$ | 4.46 | 4.83 | 5.21 | 5.95 | 6.7 | 7.44 | 8.18 | 8.92 | 9.66 | 10.4 | 11.2 | 11.9 |
| $\frac{1}{2}$ | 5.1 | 5.53 | 5.95 | 6.8 | 7.66 | 8.5 | 9.36 | 10.2 | 1.1 | 11.9 | 12.8 | 13.6 |
| $\frac{9}{16}$ | 5.74 | 6.22 | 6.69 | 7.65 | 8.6 | 9.56 | 10.5 | 11.5 | 12.4 | 13.4 | 14.3 | 15.3 |
| $\frac{5}{8}$ | 6.38 | 6.91 | 7.44 | 8.5 | 9.56 | 10.6 | 11.7 | 12.8 | 13.8 | 14.9 | 15.9 | 17 |
| $\frac{11}{16}$ | 7.01 | 7.6 | 8.18 | 9.35 | 10.5 | 11.7 | 12.9 | 14 | 15.2 | 16.4 | 17.5 | 18.7 |
| $\frac{3}{4}$ | 7.65 | 8.29 | 8.93 | 10.2 | 11.5 | 12.8 | 14 | 15.3 | 16.6 | 17.9 | 19.1 | 20.4 |
| $\frac{7}{8}$ | 8.29 | 8.98 | 9.67 | 11.1 | 12.4 | 13.8 | 15.2 | 16.6 | 18 | 19.3 | 20.7 | 22.2 |
| $\frac{15}{16}$ | 8.93 | 9.67 | 10.4 | 11.9 | 13.4 | 14.9 | 16.4 | 17.9 | 19.4 | 20.8 | 22.3 | 23.8 |
| 1 | 9.56 | 10.4 | 11.2 | 12.8 | 14.3 | 15.9 | 17.5 | 19.1 | 20.8 | 22.4 | 23.9 | 25.6 |
| | 10.2 | 11.1 | 12 | 13.6 | 15.3 | 17 | 18.7 | 20.4 | 22.1 | 23.8 | 25.5 | 27.2 |

Weight of Rolled Square Steel.

From .125 Inch to 6 Inches Square. ONE FOOT IN LENGTH.

| Side. | Weight. | Side. | Weight. | Side. | Weight. | Side. | Weight. | Side. | Weight. |
|-------|---------|--------|---------|--------|---------|-------|---------|-------|---------|
| Inch. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| .125 | .053 | .75 | 1.92 | 1.375 | 6.43 | 2.125 | 15.4 | 3.75 | 47.8 |
| .1875 | .119 | .8125 | 2.24 | 1.4375 | 7.03 | 2.25 | 17.2 | 4 | 54.4 |
| .25 | .212 | .875 | 2.6 | 1.5 | 7.65 | 2.375 | 19.2 | 4.25 | 61.4 |
| .3125 | .333 | .9375 | 3.06 | 1.5625 | 8.3 | 2.5 | 21.2 | 4.5 | 68.9 |
| .375 | .478 | 1 | 3.4 | 1.625 | 8.98 | 2.625 | 23.5 | 4.75 | 76.7 |
| .4375 | .651 | 1.0625 | 3.83 | 1.6875 | 9.79 | 2.75 | 25.7 | 5 | 85 |
| .5 | .85 | 1.125 | 4.3 | 1.75 | 10.4 | 2.875 | 28.2 | 5.25 | 93.7 |
| .562 | — | 1.1875 | 4.79 | 1.8125 | 11.2 | 3 | 30.6 | 5.5 | 102.8 |
| | — | 1.25 | 5.31 | 1.875 | 11.9 | 3.25 | 35.9 | 5.75 | 112.4 |
| | — | 1.3125 | 5.86 | 1.9375 | 12.6 | 3.5 | 41.6 | 6 | 122.4 |

Weight of Round Rolled Steel.

From .125 Inch to 12 Inches Diameter. ONE FOOT IN LENGTH.

| am. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diam. | Weight. | Diam. | Weight. |
|-----|---------|-----------|---------|-----------|---------|-------|---------|-------|---------|
| h. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| 5 | .0417 | .875 | 2.04 | 1.625 | 7.05 | 2.875 | 22 | 5.75 | 88.3 |
| 75 | .0939 | .9375 | 2.35 | 1.6875 | 7.61 | 3 | 24.1 | 6 | 96.1 |
| | .167 | 1 | 2.67 | 1.75 | 8.18 | 3.25 | 28.3 | 6.5 | 113.2 |
| 25 | .26 | 1.0625 | 3 | 1.8125 | 8.77 | 3.5 | 32.7 | 7 | 130.8 |
| 5 | .375 | 1.125 | 3.38 | 1.875 | 9.38 | 3.75 | 34.2 | 7.5 | 136.8 |
| 75 | .511 | 1.1875 | 3.76 | 2 | 10.7 | 4 | 42.7 | 8 | 170.8 |
| | .667 | 1.25 | 4.17 | 2.125 | 12 | 4.25 | 48.3 | 8.5 | 193.2 |
| 25 | .845 | 1.3125 | 4.6 | 2.25 | 13.6 | 4.5 | 54.6 | 9 | 218.4 |
| 5 | 1.04 | 1.375 | 5.05 | 2.375 | 15.1 | 4.75 | 60.3 | 9.5 | 241.2 |
| 75 | 1.27 | 1.4375 | 5.18 | 2.5 | 16.7 | 5 | 66.8 | 10 | 267.2 |
| | 1.5 | 1.5 | 6.01 | 2.625 | 18.4 | 5.25 | 73.6 | 11 | 323 |
| 25 | 1.76 | 1.5625 | 6.52 | 2.75 | 20.2 | 5.5 | 80.8 | 12 | 334.3 |

Weight of Hexagonal, Octagonal, and Oval Steel.

ONE FOOT IN LENGTH.

| HEXAGONAL. | | | OCTAGONAL. | | | OVAL. | | |
|------------|-------------------|---------|-------------------|---------|-------------------|-----------------------------------|---------|---------|
| Weight. | Diam. over Sides. | Weight. | Diam. over Sides. | Weight. | Diam. over Sides. | Diam. over Sides. | Area. | Weight. |
| Lbs. | Ins. | Lbs. | Inch. | Lbs. | Ins. | Ins. | Sq. In. | Lbs. |
| .414 | 1 | 2.94 | $\frac{3}{8}$ | .396 | 1 | $\frac{3}{4} \times \frac{3}{8}$ | .251 | .853 |
| .730 | $1\frac{1}{8}$ | 3.73 | $\frac{1}{2}$ | .704 | $1\frac{1}{8}$ | $\frac{7}{8} \times \frac{1}{2}$ | .344 | 1.17 |
| 1.15 | $1\frac{1}{4}$ | 4.6 | $\frac{5}{8}$ | 1.1 | $1\frac{1}{4}$ | $1 \times \frac{1}{2}$ | .446 | 1.52 |
| 1.66 | $1\frac{3}{8}$ | 5.57 | $\frac{3}{4}$ | 1.58 | $1\frac{3}{8}$ | $1\frac{1}{4} \times \frac{5}{8}$ | .697 | 2.37 |
| 2.25 | $1\frac{1}{2}$ | 6.63 | $\frac{7}{8}$ | 2.16 | $1\frac{1}{2}$ | $1\frac{1}{2} \times \frac{3}{4}$ | .884 | 3 |

Weight of a Square Foot of Sheet Copper.

Wire Gauge of Wm. Foster & Co. (England.)

| thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. |
|------------|---------|------------|---------|------------|---------|
| Inch. | Lbs. | W. G. | Inch. | W. G. | Inch. |
| .306 | 14 | 11 | .123 | 5.65 | 21 |
| .284 | 13 | 12 | .109 | 5 | 22 |
| .262 | 12 | 13 | .098 | 4.5 | 23 |
| .24 | 11 | 14 | .088 | 4 | 24 |
| .222 | 10.15 | 15 | .076 | 3.5 | 25 |
| .203 | 9.3 | 16 | .065 | 3 | 26 |
| .186 | 8.5 | 17 | .057 | 2.6 | 27 |
| .168 | 7.7 | 18 | .049 | 2.25 | 28 |
| .153 | 7 | 19 | .044 | 2 | 29 |
| .138 | 6.3 | 20 | .038 | 1.75 | 30 |

Weight of Composition Sheathing Nails.

| length. | Number in a Pound. | No. | Length. | Number in a Pound. | No. | Length. | No. | Number in a Pound. |
|---------|--------------------|-----|---------|--------------------|-----|---------|-----|--------------------|
| Inch. | | | Ins. | | | Ins. | | |
| .75 | 200 | 4 | 1.125 | 201 | 7 | 1.125 | | |
| .875 | 260 | 5 | 1.25 | 199 | 8 | 1.25 | | |
| | 212 | 6 | 1 | 190 | 9 | 1.5 | | |

134 WEIGHT OF FLAT ROLLED

Weight of Flat Rolled

From .5 Inch to 8 Inches in Width

| Thickness. | .5 | .625 | .75 | .875 |
|------------|------|------|------|------|
| Inch. | Lb. | Lb. | Lb. | Lb. |
| 1/4 | .425 | .533 | .64 | .743 |
| 3/16 | .531 | .665 | .8 | .929 |
| 1/2 | .638 | .798 | .96 | 1.11 |
| 5/16 | .744 | .931 | 1.12 | 1.3 |
| 3/8 | .85 | 1.06 | 1.28 | 1.5 |
| 7/16 | — | 1.2 | 1.44 | 1.7 |
| 1/2 | — | 1.33 | 1.6 | 1.9 |
| 5/8 | — | — | 1.76 | 2.1 |
| 3/4 | — | — | 1.9 | 2.3 |
| 7/8 | — | — | — | 2.5 |
| 1 | — | — | — | 2.7 |
| 1 1/16 | — | — | — | 2.9 |
| 1 1/8 | — | — | — | 3.1 |
| 1 1/4 | — | — | — | 3.3 |
| 1 1/2 | — | — | — | 3.5 |
| 1 3/4 | — | — | — | 3.7 |
| 2 | — | — | — | 3.9 |
| 2 1/4 | — | — | — | 4.1 |
| 2 1/2 | — | — | — | 4.3 |
| 2 3/4 | — | — | — | 4.5 |
| 3 | — | — | — | 4.7 |
| 3 1/4 | — | — | — | 4.9 |
| 3 1/2 | — | — | — | 5.1 |
| 3 3/4 | — | — | — | 5.3 |
| 4 | — | — | — | 5.5 |
| 4 1/4 | — | — | — | 5.7 |
| 4 1/2 | — | — | — | 5.9 |
| 4 3/4 | — | — | — | 6.1 |
| 5 | — | — | — | 6.3 |
| 5 1/4 | — | — | — | 6.5 |
| 5 1/2 | — | — | — | 6.7 |
| 5 3/4 | — | — | — | 6.9 |
| 6 | — | — | — | 7.1 |
| 6 1/4 | — | — | — | 7.3 |
| 6 1/2 | — | — | — | 7.5 |
| 6 3/4 | — | — | — | 7.7 |
| 7 | — | — | — | 7.9 |
| 7 1/4 | — | — | — | 8.1 |
| 7 1/2 | — | — | — | 8.3 |
| 7 3/4 | — | — | — | 8.5 |
| 8 | — | — | — | 8.7 |
| 8 1/4 | — | — | — | 8.9 |
| 8 1/2 | — | — | — | 9.1 |
| 8 3/4 | — | — | — | 9.3 |
| 9 | — | — | — | 9.5 |
| 9 1/4 | — | — | — | 9.7 |
| 9 1/2 | — | — | — | 9.9 |
| 9 3/4 | — | — | — | 10.1 |
| 10 | — | — | — | 10.3 |
| 10 1/4 | — | — | — | 10.5 |
| 10 1/2 | — | — | — | 10.7 |
| 10 3/4 | — | — | — | 10.9 |
| 11 | — | — | — | 11.1 |
| 11 1/4 | — | — | — | 11.3 |
| 11 1/2 | — | — | — | 11.5 |
| 11 3/4 | — | — | — | 11.7 |
| 12 | — | — | — | 11.9 |
| 12 1/4 | — | — | — | 12.1 |
| 12 1/2 | — | — | — | 12.3 |
| 12 3/4 | — | — | — | 12.5 |
| 13 | — | — | — | 12.7 |
| 13 1/4 | — | — | — | 12.9 |
| 13 1/2 | — | — | — | 13.1 |
| 13 3/4 | — | — | — | 13.3 |
| 14 | — | — | — | 13.5 |
| 14 1/4 | — | — | — | 13.7 |
| 14 1/2 | — | — | — | 13.9 |
| 14 3/4 | — | — | — | 14.1 |
| 15 | — | — | — | 14.3 |
| 15 1/4 | — | — | — | 14.5 |
| 15 1/2 | — | — | — | 14.7 |
| 15 3/4 | — | — | — | 14.9 |
| 16 | — | — | — | 15.1 |
| 16 1/4 | — | — | — | 15.3 |
| 16 1/2 | — | — | — | 15.5 |
| 16 3/4 | — | — | — | 15.7 |
| 17 | — | — | — | 15.9 |
| 17 1/4 | — | — | — | 16.1 |
| 17 1/2 | — | — | — | 16.3 |
| 17 3/4 | — | — | — | 16.5 |
| 18 | — | — | — | 16.7 |
| 18 1/4 | — | — | — | 16.9 |
| 18 1/2 | — | — | — | 17.1 |
| 18 3/4 | — | — | — | 17.3 |
| 19 | — | — | — | 17.5 |
| 19 1/4 | — | — | — | 17.7 |
| 19 1/2 | — | — | — | 17.9 |
| 19 3/4 | — | — | — | 18.1 |
| 20 | — | — | — | 18.3 |
| 20 1/4 | — | — | — | 18.5 |
| 20 1/2 | — | — | — | 18.7 |
| 20 3/4 | — | — | — | 18.9 |
| 21 | — | — | — | 19.1 |
| 21 1/4 | — | — | — | 19.3 |
| 21 1/2 | — | — | — | 19.5 |
| 21 3/4 | — | — | — | 19.7 |
| 22 | — | — | — | 19.9 |
| 22 1/4 | — | — | — | 20.1 |
| 22 1/2 | — | — | — | 20.3 |
| 22 3/4 | — | — | — | 20.5 |
| 23 | — | — | — | 20.7 |
| 23 1/4 | — | — | — | 20.9 |
| 23 1/2 | — | — | — | 21.1 |
| 23 3/4 | — | — | — | 21.3 |
| 24 | — | — | — | 21.5 |
| 24 1/4 | — | — | — | 21.7 |
| 24 1/2 | — | — | — | 21.9 |
| 24 3/4 | — | — | — | 22.1 |
| 25 | — | — | — | 22.3 |
| 25 1/4 | — | — | — | 22.5 |
| 25 1/2 | — | — | — | 22.7 |
| 25 3/4 | — | — | — | 22.9 |
| 26 | — | — | — | 23.1 |
| 26 1/4 | — | — | — | 23.3 |
| 26 1/2 | — | — | — | 23.5 |
| 26 3/4 | — | — | — | 23.7 |
| 27 | — | — | — | 23.9 |
| 27 1/4 | — | — | — | 24.1 |
| 27 1/2 | — | — | — | 24.3 |
| 27 3/4 | — | — | — | 24.5 |
| 28 | — | — | — | 24.7 |
| 28 1/4 | — | — | — | 24.9 |
| 28 1/2 | — | — | — | 25.1 |
| 28 3/4 | — | — | — | 25.3 |
| 29 | — | — | — | 25.5 |
| 29 1/4 | — | — | — | 25.7 |
| 29 1/2 | — | — | — | 25.9 |
| 29 3/4 | — | — | — | 26.1 |
| 30 | — | — | — | 26.3 |
| 30 1/4 | — | — | — | 26.5 |
| 30 1/2 | — | — | — | 26.7 |
| 30 3/4 | — | — | — | 26.9 |
| 31 | — | — | — | 27.1 |
| 31 1/4 | — | — | — | 27.3 |
| 31 1/2 | — | — | — | 27.5 |
| 31 3/4 | — | — | — | 27.7 |
| 32 | — | — | — | 27.9 |
| 32 1/4 | — | — | — | 28.1 |
| 32 1/2 | — | — | — | 28.3 |
| 32 3/4 | — | — | — | 28.5 |
| 33 | — | — | — | 28.7 |
| 33 1/4 | — | — | — | 28.9 |
| 33 1/2 | — | — | — | 29.1 |
| 33 3/4 | — | — | — | 29.3 |
| 34 | — | — | — | 29.5 |
| 34 1/4 | — | — | — | 29.7 |
| 34 1/2 | — | — | — | 29.9 |
| 34 3/4 | — | — | — | 30.1 |
| 35 | — | — | — | 30.3 |
| 35 1/4 | — | — | — | 30.5 |
| 35 1/2 | — | — | — | 30.7 |
| 35 3/4 | — | — | — | 30.9 |
| 36 | — | — | — | 31.1 |
| 36 1/4 | — | — | — | 31.3 |
| 36 1/2 | — | — | — | 31.5 |
| 36 3/4 | — | — | — | 31.7 |
| 37 | — | — | — | 31.9 |
| 37 1/4 | — | — | — | 32.1 |
| 37 1/2 | — | — | — | 32.3 |
| 37 3/4 | — | — | — | 32.5 |
| 38 | — | — | — | 32.7 |
| 38 1/4 | — | — | — | 32.9 |
| 38 1/2 | — | — | — | 33.1 |
| 38 3/4 | — | — | — | 33.3 |
| 39 | — | — | — | 33.5 |
| 39 1/4 | — | — | — | 33.7 |
| 39 1/2 | — | — | — | 33.9 |
| 39 3/4 | — | — | — | 34.1 |
| 40 | — | — | — | 34.3 |
| 40 1/4 | — | — | — | 34.5 |
| 40 1/2 | — | — | — | 34.7 |
| 40 3/4 | — | — | — | 34.9 |
| 41 | — | — | — | 35.1 |
| 41 1/4 | — | — | — | 35.3 |
| 41 1/2 | — | — | — | 35.5 |
| 41 3/4 | — | — | — | 35.7 |
| 42 | — | — | — | 35.9 |
| 42 1/4 | — | — | — | 36.1 |
| 42 1/2 | — | — | — | 36.3 |
| 42 3/4 | — | — | — | 36.5 |
| 43 | — | — | — | 36.7 |
| 43 1/4 | — | — | — | 36.9 |
| 43 1/2 | — | — | — | 37.1 |
| 43 3/4 | — | — | — | 37.3 |
| 44 | — | — | — | 37.5 |
| 44 1/4 | — | — | — | 37.7 |
| 44 1/2 | — | — | — | 37.9 |
| 44 3/4 | — | — | — | 38.1 |
| 45 | — | — | — | 38.3 |
| 45 1/4 | — | — | — | 38.5 |
| 45 1/2 | — | — | — | 38.7 |
| 45 3/4 | — | — | — | 38.9 |
| 46 | — | — | — | 39.1 |
| 46 1/4 | — | — | — | 39.3 |
| 46 1/2 | — | — | — | 39.5 |
| 46 3/4 | — | — | — | 39.7 |
| 47 | — | — | — | 39.9 |
| 47 1/4 | — | — | — | 40.1 |
| 47 1/2 | — | — | — | 40.3 |
| 47 3/4 | — | — | — | 40.5 |
| 48 | — | — | — | 40.7 |
| 48 1/4 | — | — | — | 40.9 |
| 48 1/2 | — | — | — | 41.1 |
| 48 3/4 | — | — | — | 41.3 |
| 49 | — | — | — | 41.5 |
| 49 1/4 | — | — | — | 41.7 |
| 49 1/2 | — | — | — | 41.9 |
| 49 3/4 | — | — | — | 42.1 |
| 50 | — | — | — | 42.3 |
| 50 1/4 | — | — | — | 42.5 |
| 50 1/2 | — | — | — | 42.7 |
| 50 3/4 | — | — | — | 42.9 |
| 51 | — | — | — | 43.1 |
| 51 1/4 | — | — | — | 43.3 |
| 51 1/2 | — | — | — | 43.5 |
| 51 3/4 | — | — | — | 43.7 |
| 52 | — | — | — | 43.9 |
| 52 1/4 | — | — | — | 44.1 |
| 52 1/2 | — | — | — | 44.3 |
| 52 3/4 | — | — | — | 44.5 |
| 53 | — | — | — | 44.7 |
| 53 1/4 | — | — | — | 44.9 |
| 53 1/2 | — | — | — | 45.1 |
| 53 3/4 | — | — | — | 45.3 |
| 54 | — | — | — | 45.5 |
| 54 1/4 | — | — | — | 45.7 |
| 54 1/2 | — | — | — | 45.9 |
| 54 3/4 | — | — | — | 46.1 |
| 55 | — | — | — | 46.3 |
| 55 1/4 | — | — | — | 46.5 |
| 55 1/2 | — | — | — | 46.7 |
| 55 3/4 | — | — | — | 46.9 |
| 56 | — | — | — | 47.1 |
| 56 1/4 | — | — | — | 47.3 |
| 56 1/2 | — | — | — | 47.5 |
| 56 3/4 | — | — | — | 47.7 |
| 57 | — | — | — | 47.9 |
| 57 1/4 | — | — | — | 48.1 |
| 57 1/2 | — | — | — | 48.3 |
| 57 3/4 | — | — | — | 48.5 |
| 58 | — | — | — | 48.7 |
| 58 1/4 | — | — | — | 48.9 |
| 58 1/2 | — | — | — | 49.1 |
| 58 3/4 | — | — | — | 49.3 |
| 59 | — | — | — | 49.5 |
| 59 1/4 | — | — | — | 49.7 |
| 59 1/2 | — | — | — | 49.9 |
| 59 3/4 | — | — | — | 50.1 |
| 60 | — | — | — | 50.3 |
| 60 1/4 | — | — | — | 50.5 |
| 60 1/2 | — | — | — | 50.7 |
| 60 3/4 | — | — | — | 50.9 |
| 61 | — | — | — | 51.1 |
| 61 1/4 | — | — | — | 51.3 |
| 61 1/2 | — | — | — | 51.5 |
| 61 3/4 | — | — | — | 51.7 |
| 62 | — | — | — | 51.9 |
| 62 1/4 | — | — | — | 52.1 |
| 62 1/2 | — | — | — | 52.3 |
| 62 3/4 | — | — | — | 52.5 |
| 63 | — | — | — | 52.7 |
| 63 1/4 | — | — | — | 52.9 |
| 63 1/2 | — | — | — | 53.1 |
| 63 3/4 | — | — | — | 53.3 |
| 64 | — | — | — | 53.5 |
| 64 1/4 | — | — | — | 53.7 |
| 64 1/2 | — | — | — | 53.9 |
| 64 3/4 | — | — | — | 54.1 |
| 65 | — | — | — | 54.3 |
| 65 1/4 | — | — | — | 54.5 |
| 65 1/2 | — | — | — | 54.7 |
| 65 3/4 | — | — | — | 54.9 |
| 66 | — | — | — | 55.1 |
| 66 1/4 | — | — | — | 55.3 |
| 66 1/2 | — | — | — | 55.5 |
| 66 3/4 | — | — | — | 55.7 |
| 67 | — | — | — | 55.9 |
| 67 1/4 | — | — | — | 56.1 |
| 67 1/2 | — | — | — | 56.3 |
| 67 3/4 | — | — | — | 56.5 |
| 68 | — | — | — | 56.7 |
| 68 1/4 | — | — | — | 56.9 |
| 68 1/2 | — | — | — | 57.1 |
| 68 3/ | | | | |

Weight of English Angle and T Iron. (D. K. C.)
ONE FOOT IN LENGTH.

NOTE.—When base or web tapers in section, mean thickness is to be measured.

| Thick- ness. | SUM OF WIDTH AND DEPTH IN INCHES. | | | | | | | | | | |
|-----------------|-----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1.5 | 1.625 | 1.75 | 1.875 | 2 | 2.125 | 2.25 | 2.375 | 2.5 | 2.625 | 2.75 |
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .125 | .57 | .62 | .68 | .73 | .78 | .83 | .88 | .94 | .99 | 1.04 | 1.09 |
| .1875 | .81 | .89 | .97 | 1.05 | 1.13 | 1.21 | 1.29 | 1.37 | 1.45 | 1.52 | 1.6 |
| .25 | 1.04 | 1.15 | 1.25 | 1.36 | 1.46 | 1.56 | 1.67 | 1.77 | 1.88 | 1.98 | 2.08 |
| .3125 | 1.24 | 1.37 | 1.5 | 1.63 | 1.76 | 1.89 | 2.02 | 2.15 | 2.28 | 2.41 | 2.54 |
| | 2.875 | 3 | 3.125 | 3.25 | 3.375 | 3.5 | 3.625 | 3.75 | 3.875 | 4 | 4.25 |
| .125 | 1.14 | 1.2 | 1.25 | 1.3 | 1.45 | 1.41 | 1.46 | 1.51 | 1.56 | 1.62 | 1.72 |
| .1875 | 1.68 | 1.76 | 1.84 | 1.91 | 1.99 | 2.07 | 2.15 | 2.23 | 2.3 | 2.38 | 2.54 |
| .25 | 2.19 | 2.29 | 2.4 | 2.5 | 2.6 | 2.71 | 2.81 | 2.92 | 3.02 | 3.13 | 3.33 |
| .3125 | 2.67 | 2.8 | 2.93 | 3.06 | 3.19 | 3.32 | 3.45 | 3.58 | 3.71 | 3.84 | 4.1 |
| .375 | 3.13 | 3.28 | 3.44 | 3.59 | 3.75 | 3.91 | 4.06 | 4.22 | 4.38 | 4.53 | 4.84 |
| .4375 | 3.57 | 3.75 | 3.93 | 4.11 | 4.29 | 4.48 | 4.66 | 4.84 | 5.02 | 5.2 | 5.56 |
| | 4.5 | 4.75 | 5 | 5.25 | 5.5 | 5.75 | 6 | 6.25 | 6.5 | 6.75 | 7 |
| .1875 | 2.7 | 2.85 | 3.01 | 3.16 | 3.32 | 3.48 | 3.63 | 3.79 | 3.95 | 4.1 | 4.26 |
| .25 | 3.54 | 3.75 | 3.96 | 4.17 | 4.38 | 4.59 | 4.79 | 5 | 5.21 | 5.42 | 5.63 |
| .3125 | 4.36 | 4.62 | 4.88 | 5.14 | 5.4 | 5.66 | 5.92 | 6.18 | 6.45 | 6.71 | 6.97 |
| .375 | 5.16 | 5.47 | 5.78 | 6.09 | 6.41 | 6.72 | 7.03 | 7.34 | 7.66 | 7.97 | 8.28 |
| .4375 | 5.92 | 6.29 | 6.65 | 7.02 | 7.38 | 7.75 | 8.11 | 8.48 | 8.84 | 9.21 | 9.57 |
| .5 | 6.67 | 7.08 | 7.5 | 7.92 | 8.33 | 8.75 | 9.17 | 9.58 | 10 | 10.42 | 10.83 |
| .5625 | 7.38 | 7.85 | 8.32 | 8.79 | 9.26 | 9.73 | 10.2 | 10.66 | 11.13 | 12.6 | 12.07 |
| | 7.25 | 7.5 | 7.75 | 8 | 8.25 | 8.5 | 8.75 | 9 | 9.25 | 9.5 | 9.75 |
| .25 | 5.83 | 6.04 | 6.25 | 6.46 | 6.67 | 6.88 | 7.08 | 7.29 | 7.5 | 7.71 | 7.92 |
| .3125 | 7.23 | 7.49 | 7.75 | 8.01 | 8.27 | 8.53 | 8.79 | 9.05 | 9.31 | 9.57 | 9.83 |
| .375 | 8.59 | 8.91 | 9.22 | 9.53 | 9.84 | 10.16 | 10.47 | 10.78 | 11.09 | 11.41 | 11.72 |
| .4375 | 9.93 | 10.3 | 10.66 | 11.03 | 11.39 | 11.76 | 12.12 | 12.49 | 12.85 | 13.22 | 13.58 |
| .5 | 11.25 | 11.67 | 12.08 | 12.5 | 12.92 | 13.33 | 13.75 | 14.17 | 14.58 | 15 | 15.42 |
| .5625 | 12.54 | 13.01 | 13.48 | 13.94 | 14.41 | 14.88 | 15.35 | 15.82 | 16.29 | 16.76 | 17.23 |
| .625 | 13.8 | 14.32 | 14.84 | 15.36 | 15.89 | 16.41 | 16.93 | 17.45 | 17.97 | 18.49 | 19.01 |
| | 10 | 10.5 | 11 | 11.5 | 12 | 12.5 | 13 | 13.5 | 14 | 14.5 | 15 |
| .375 | 12.03 | 12.66 | 13.28 | 13.91 | 14.53 | | | | | | |
| .4375 | 13.95 | 14.67 | 15.4 | 16.13 | 16.86 | 17.59 | 18.31 | 19.04 | 19.77 | 20.5 | 21.22 |
| .5 | 15.83 | 16.67 | 17.5 | 18.33 | 19.17 | 20 | 20.84 | 21.67 | 22.5 | 23.34 | 24.17 |
| .5625 | 17.7 | 18.63 | 19.57 | 20.51 | 21.44 | 22.38 | 23.31 | 24.25 | 25.19 | 26.12 | 27.06 |
| .625 | 19.53 | 20.57 | 21.61 | 22.66 | 23.7 | 24.74 | 25.78 | 26.83 | 27.87 | 28.91 | 29.95 |
| .75 | 23.13 | 24.38 | 25.63 | 26.88 | 28.13 | 29.37 | 30.63 | 31.88 | 33.13 | 34.38 | 35.63 |
| | 12 | 12.5 | 13 | 13.5 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| .625 | 23.7 | 24.74 | 25.78 | 26.83 | 27.87 | 29.95 | 32.03 | 34.12 | 36.2 | 38.28 | 40.36 |
| .75 | 28.13 | 29.37 | 30.63 | 31.88 | 33.13 | 35.63 | 38.13 | 40.63 | 41.13 | 43.63 | 46.13 |
| .875 | 32.45 | 33.91 | 35.36 | 36.82 | 38.28 | 41.19 | 44.12 | 47.02 | 49.95 | 52.87 | 55.78 |
| 1 | 36.67 | 38.33 | 40 | 41.67 | 43.33 | 46.67 | 50 | 53.33 | 56.67 | 60 | 63.33 |

NOTE.—American rolled is slightly heavier.

136 WEIGHT OF IRON, STEEL, COPPER, ETC.

Weight of Cast and Wrought Iron, Steel, Copper, and Brass, of a given Sectional Area.

PER LINEAL FOOT.

| Sectional Area. | Wrought Iron. | Cast Iron. | Steel. | Copper. | Lead. | Brass. | Gun-metal |
|-----------------|---------------|------------|--------|---------|-------|--------|-----------|
| Sq. Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .1 | .336 | .313 | .339 | .385 | .492 | .357 | .38 |
| .2 | .671 | .626 | .677 | .771 | .984 | .713 | .75 |
| .3 | 1.007 | .939 | 1.016 | 1.156 | 1.476 | 1.07 | 1.13 |
| .4 | 1.343 | 1.251 | 1.355 | 1.542 | 1.967 | 1.427 | 1.51 |
| .5 | 1.678 | 1.564 | 1.694 | 1.927 | 2.461 | 1.783 | 1.89 |
| .6 | 2.014 | 1.877 | 2.032 | 2.312 | 2.953 | 2.14 | 2.27 |
| .7 | 2.35 | 2.19 | 2.371 | 2.698 | 3.445 | 2.497 | 2.65 |
| .8 | 2.685 | 2.503 | 2.71 | 3.083 | 3.937 | 2.853 | 3.03 |
| .9 | 3.021 | 2.816 | 3.049 | 3.469 | 4.429 | 3.21 | 3.41 |
| 1 | 3.357 | 3.129 | 3.387 | 3.854 | 4.922 | 3.567 | 3.78 |
| 1.1 | 3.692 | 3.442 | 3.726 | 4.24 | 5.414 | 3.923 | 4.17 |
| 1.2 | 4.028 | 3.754 | 4.065 | 4.625 | 5.906 | 4.28 | 4.55 |
| 1.3 | 4.364 | 4.067 | 4.404 | 5.01 | 6.398 | 4.636 | 4.93 |
| 1.4 | 4.699 | 4.38 | 4.742 | 5.396 | 6.89 | 4.993 | 5.31 |
| 1.5 | 5.035 | 4.693 | 5.081 | 5.781 | 7.383 | 5.35 | 5.69 |
| 1.6 | 5.371 | 5.006 | 5.42 | 6.167 | 7.875 | 5.707 | 6.07 |
| 1.7 | 5.706 | 5.319 | 5.759 | 6.552 | 8.367 | 6.063 | 6.45 |
| 1.8 | 6.042 | 5.632 | 6.097 | 6.937 | 8.859 | 6.42 | 6.83 |
| 1.9 | 6.378 | 5.945 | 6.436 | 7.323 | 9.351 | 6.777 | 7.21 |
| 2 | 6.714 | 6.258 | 6.775 | 7.708 | 9.843 | 7.133 | 7.59 |
| 2.1 | 7.049 | 6.57 | 7.114 | 8.094 | 10.33 | 7.49 | 7.97 |
| 2.2 | 7.385 | 6.883 | 7.452 | 8.474 | 10.83 | 7.847 | 8.35 |
| 2.3 | 7.721 | 7.196 | 7.791 | 8.864 | 11.32 | 8.203 | 8.73 |
| 2.4 | 8.056 | 7.509 | 8.13 | 9.25 | 11.81 | 8.56 | 9.11 |
| 2.5 | 8.392 | 7.822 | 8.469 | 9.635 | 12.3 | 8.917 | 9.49 |
| 2.6 | 8.728 | 8.135 | 8.807 | 10.02 | 12.8 | 9.273 | 9.87 |
| 2.7 | 9.063 | 8.448 | 9.146 | 10.41 | 13.29 | 9.63 | 10.25 |
| 2.8 | 9.399 | 8.76 | 9.485 | 10.79 | 13.78 | 9.98 | 10.63 |
| 2.9 | 9.734 | 9.073 | 9.824 | 11.18 | 14.27 | 10.34 | 11.01 |
| 3 | 10.07 | 9.386 | 10.16 | 11.56 | 14.76 | 10.7 | 11.39 |
| 3.1 | 10.41 | 9.699 | 10.5 | 11.95 | 15.26 | 11.06 | 11.77 |
| 3.2 | 10.74 | 10.01 | 10.84 | 12.33 | 15.75 | 11.41 | 12.15 |
| 3.3 | 11.08 | 10.32 | 11.18 | 12.72 | 16.24 | 11.77 | 12.53 |
| 3.4 | 11.41 | 10.64 | 11.52 | 13.1 | 16.73 | 12.13 | 12.91 |
| 3.5 | 11.75 | 10.95 | 11.86 | 13.49 | 17.22 | 12.48 | 13.29 |
| 3.6 | 12.08 | 11.26 | 12.19 | 13.87 | 17.72 | 12.84 | 13.67 |
| 3.7 | 12.42 | 11.58 | 12.53 | 14.26 | 18.21 | 13.2 | 14.05 |
| 3.8 | 12.76 | 11.89 | 12.87 | 14.64 | 18.7 | 13.55 | 14.43 |
| 3.9 | 13.09 | 12.2 | 13.21 | 15.03 | 19.19 | 13.91 | 14.81 |
| 4 | 13.43 | 12.51 | 13.55 | 15.42 | 19.69 | 14.27 | 15.19 |
| 4.1 | 13.76 | 12.83 | 13.89 | 15.8 | 20.18 | 14.62 | 15.57 |
| 4.2 | | 13.14 | 14.23 | 16.19 | 20.67 | 14.98 | 15.95 |
| 4.3 | | 13.45 | 14.57 | 16.57 | 21.16 | 15.34 | 16.33 |
| 4.4 | | 13.77 | 14.91 | 16.96 | 21.65 | 15.69 | 16.71 |
| 4.5 | | 14.08 | 15.24 | 17.34 | 22.15 | 16.05 | 17.09 |
| 4.6 | | 14.39 | 15.58 | 17.73 | 22.64 | 16.41 | 17.47 |
| 4.7 | | 14.7 | 15.92 | 18.11 | 23.13 | 16.76 | 17.85 |
| 4.8 | | 15.02 | 16.26 | 18.5 | 23.62 | 17.12 | 18.23 |
| 4.9 | | 15.33 | 16.6 | 18.88 | 24.12 | 17.48 | 18.61 |
| 5 | | 15.64 | 16.94 | 19.27 | 24.61 | 17.83 | 18.99 |

Weight of Lead and Tin Lined Pipe per Foot.*From .375 Inch to 5 Inches in Diameter. (Tatham & Bros.)*

| In. | WASTE-PIPE. | | | | BLOCK-TIN PIPE. | | | | |
|-----|-------------|-------|---------|-------|-----------------|-------|---------|-------|---------|
| | Weight. | Diam. | Weight. | Diam. | Weight. | Diam. | Weight. | Diam. | Weight. |
| s. | Lbs. | In. | Lbs. | Inch. | Lb. | Inch. | Lbs. | In. | Lbs. |
| 5 | 2 | 4 | 8 | .375 | .3594 | .625 | .5 | 1.25 | 1.25 |
| | 3 | 4.5 | 6 | .375 | .375 | .625 | .625 | 1.25 | 1.5 |
| | 3.5 | 4.5 | 8 | .375 | .5 | .75 | .625 | 1.5 | 2 |
| | 5 | 5 | 8 | .5 | .375 | .75 | .75 | 1.5 | 2.5 |
| | 5 | 5 | 10 | .5 | .5 | 1 | .9375 | 2 | 2.5 |
| | 6 | 5 | 12 | .5 | .625 | 1 | 1.125 | 2 | 3 |

WATER-PIPE.*From .375 Inch to 5 Inches in Diameter.*

| Thick-ness. | Weight. | Diam. | Thick-ness. | Weight. | Diam. | Thick-ness. | Weight. | Diam. | Thick-ness. | Weight. |
|-------------|---------|-------|-------------|---------|-------|-------------|---------|-------|-------------|---------|
| Inch. | Lbs. | In. | Inch. | Lbs. | In. | Inch. | Lbs. | In. | Inch. | Lbs. |
| .08 | .625 | .625 | .25 | 3.5 | 1.25 | .19 | 4.75 | 2.5 | .3125 | 14 |
| .12 | 1 | .75 | .1 | 1.25 | 1.25 | .25 | 6 | 2.5 | .375 | 17 |
| .16 | 1.25 | .75 | .12 | 1.75 | 1.5 | .12 | 3 | 3 | .1875 | 9 |
| .19 | 1.5 | .75 | .16 | 2.25 | 1.5 | .14 | 3.5 | 3 | .25 | 12 |
| .34 | 2.5 | .75 | .2 | 3 | 1.5 | .17 | 4.25 | 3 | .3125 | 16 |
| .07 | .0545 | .75 | .23 | 3.5 | 1.5 | .19 | 5 | 3 | .375 | 20 |
| .09 | .75 | .75 | .3 | 4.75 | 1.5 | .23 | 6.5 | 3.5 | .1875 | 9.5 |
| .11 | 1 | 1 | .1 | 1.5 | 1.5 | .27 | 8 | 3.5 | .25 | 15 |
| .13 | 1.25 | 1 | .11 | 2 | 1.75 | .13 | 4 | 3.5 | .3125 | 18.5 |
| .16 | 1.75 | 1 | .14 | 2.5 | 1.75 | .17 | 5 | 3.5 | .375 | 22 |
| .19 | 2 | 1 | .17 | 3.25 | 1.75 | .21 | 6.5 | 4 | .1875 | 12.5 |
| .25 | 3 | 1 | .21 | 4 | 1.75 | .27 | 8.5 | 4 | .25 | 16 |
| .08 | .0727 | 1 | .24 | 4.75 | 2 | .15 | 4.75 | 4 | .3125 | 21 |
| .09 | 1 | 1 | .3 | 6 | 2 | .18 | 6 | 4 | .375 | 25 |
| .13 | 1.5 | 1.25 | .1 | 2 | 2 | .22 | 7 | 4.5 | .1875 | 14 |
| .16 | 2 | 1.25 | .12 | 2.5 | 2 | .27 | 9 | 4.5 | .25 | 18 |
| .2 | 2.5 | 1.25 | .14 | 3 | 2.5 | .1875 | 8 | 5 | .25 | 20 |
| .22 | 2.75 | 1.25 | .16 | 3.75 | 2.5 | .25 | 11 | 5 | .375 | 31 |

Marks and Weight of Tin-plates. (English.)

| MARK BRAND. | Plates per Box. | Dimensions. | Weight per Box. | MARK OR BRAND. | Plates per Box. | Dimensions. | Weight per Box. |
|-------------|-----------------|--------------|-----------------|----------------|-----------------|--------------|-----------------|
| | No. | In. | No. | | No. | In. | No. |
| 1 Com. | 225 | 13.75 X 10 | 112 | DXXXX..... | 100 | 16.75 X 12.5 | 189 |
| | 225 | 13.25 X 9.75 | 105 | SDC..... | 200 | 15 X 11 | 168 |
| | 225 | 12.75 X 9.5 | 98 | SDX..... | 200 | 15 X 11 | 188 |
| | 225 | 13.75 X 10 | 119 | SDXX..... | 200 | 15 X 11 | 209 |
| | 225 | 13.75 X 10 | 157 | SDXXX..... | 200 | 15 X 11 | 230 |
| | 225 | 13.75 X 10 | 140 | SDXXXX..... | 200 | 15 X 11 | 251 |
| | 225 | 13.25 X 9.75 | 133 | SDXXXXXX..... | 200 | 15 X 11 | 272 |
| | 225 | 12.75 X 9.5 | 126 | SDXXXXXXX..... | 200 | 15 X 11 | 293 |
| | 225 | 13.75 X 10 | 161 | Leaded IC..... | 112 | 20 X 14 | 112 |
| | 225 | 13.75 X 10 | 182 | " IX..... | 112 | 20 X 14 | 140 |
| X..... | 225 | 13.75 X 10 | 203 | ICW..... | 225 | 13.75 X 10 | 112 |
| XX..... | 225 | 13.75 X 10 | 224 | IXW..... | 225 | 13.75 X 10 | 140 |
| XXX..... | 225 | 13.75 X 10 | 245 | CSDW..... | 200 | 15 X 11 | 13 |
| | 100 | 16.75 X 12.5 | 98 | CIW..... | 100 | 16.75 X 12.5 | 100 |
| | 100 | 16.75 X 12.5 | 126 | XIW..... | 100 | 16.75 X 12.5 | 100 |
| | 100 | 16.75 X 12.5 | 147 | TT..... | 450 | 13.7 | 13.7 |
| | 100 | 16.75 X 12.5 | 168 | XTT..... | 450 | 13.7 | 13.7 |

When the plates are 14 by 20 inches, there are 112 in a box.

Weight of Cast and Wrought Iron, Steel, Copper
Brass, of a given Sectional Area.

PER LINEAL FOOT.

| Sectional Area. | Wrought Iron. | Cast Iron. | Steel. | Copper. | Lead. | Brass. |
|--------------------|------------------|------------|--------|---------|-------|--------|
| Sq. Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .1 | .336 | .313 | .339 | .385 | .492 | .357 |
| .2 | .671 | .626 | .677 | .771 | .984 | .713 |
| .3 | 1.007 | .939 | 1.016 | 1.156 | 1.476 | 1.07 |
| .4 | 1.343 | 1.251 | 1.355 | 1.542 | 1.967 | 1.427 |
| .5 | 1.678 | 1.564 | 1.694 | 1.927 | 2.461 | 1.783 |
| .6 | 2.014 | 1.877 | 2.032 | 2.312 | 2.953 | 2.14 |
| .7 | 2.35 | 2.19 | 2.371 | 2.698 | 3.445 | 2.497 |
| .8 | 2.685 | 2.503 | 2.71 | 3.083 | 3.937 | 2.853 |
| .9 | 3.021 | 2.816 | 3.049 | 3.469 | 4.429 | 3.21 |
| 1 | 3.357 | 3.129 | 3.387 | 3.854 | 4.922 | 3.567 |
| 1.1 | 3.692 | 3.442 | 3.726 | 4.24 | 5.414 | 3.923 |
| 1.2 | 4.028 | 3.754 | 4.065 | 4.625 | 5.906 | 4.28 |
| 1.3 | 4.364 | 4.067 | 4.404 | 5.01 | 6.398 | 4.636 |
| 1.4 | 4.699 | 4.38 | 4.742 | 5.396 | 6.89 | 4.993 |
| 1.5 | 5.035 | 4.693 | 5.081 | 5.781 | 7.383 | 5.35 |
| 1.6 | 5.371 | 5.006 | 5.42 | 6.167 | 7.875 | 5.707 |
| 1.7 | 5.706 | 5.319 | 5.759 | 6.552 | 8.367 | 6.063 |
| 1.8 | 6.042 | 5.632 | 6.097 | 6.937 | 8.859 | 6.42 |
| 1.9 | 6.378 | 5.945 | 6.436 | 7.323 | 9.351 | 6.777 |
| 2 | 6.714 | 6.258 | 6.775 | 7.708 | 9.843 | 7.133 |
| 2.1 | 7.049 | 6.57 | 7.114 | 8.094 | 10.33 | 7.49 |
| 2.2 | 7.385 | 6.883 | 7.452 | 8.474 | 10.83 | 7.847 |
| 2.3 | 7.721 | 7.196 | 7.791 | 8.864 | 11.32 | 8.203 |
| 2.4 | 8.056 | 7.509 | 8.13 | 9.25 | 11.81 | 8.56 |
| 2.5 | 8.392 | 7.822 | 8.469 | 9.635 | 12.3 | 8.917 |
| 2.6 | 8.728 | 8.135 | 8.807 | 10.02 | 12.8 | 9.273 |
| 2.7 | 9.063 | 8.448 | 9.146 | 10.41 | 13.29 | 9.63 |
| 2.8 | 9.399 | 8.76 | 9.485 | 10.79 | 13.78 | 9.98 |
| 2.9 | 9.734 | 9.073 | 9.824 | 11.18 | 14.27 | 10.34 |
| 3 | 10.07 | 9.386 | 10.16 | 11.56 | 14.76 | 10.7 |
| 3.1 | 10.41 | 9.699 | 10.5 | 11.95 | 15.26 | 11.06 |
| 3.2 | 10.74 | 10.01 | 10.84 | 12.33 | 15.75 | 11.41 |
| 3.3 | 11.08 | 10.32 | 11.18 | 12.72 | 16.24 | 11.77 |
| 3.4 | 11.41 | 10.64 | 11.52 | 13.1 | 16.73 | 12.13 |
| 3.5 | 11.75 | 10.95 | 11.86 | 13.49 | 17.22 | 12.48 |
| 3.6 | 12.08 | 11.26 | 12.19 | 13.87 | 17.72 | 12.84 |
| 3.7 | 12.42 | 11.58 | 12.53 | 14.26 | 18.21 | 13.2 |
| 3.8 | 12.76 | 11.89 | 12.87 | 14.64 | 18.7 | 13.55 |
| 3.9 | 13.09 | 12.2 | 13.21 | 15.03 | 19.19 | 13.91 |
| 4 | 13.43 | 12.51 | 13.55 | 15.42 | 19.69 | 14.27 |
| 4.1 | 13.76 | 12.83 | 13.89 | 15.8 | 20.18 | 14.62 |
| 4.2 | 14.1 | 13.14 | 14.23 | 16.19 | 20.67 | 14.98 |
| 4.3 | 14.43 | 13.45 | 14.57 | 16.57 | 21.16 | 15.34 |
| 4.4 | 14.77 | 13.77 | 14.91 | 16.96 | 21.65 | 15.69 |
| 4.5 | 15.11 | 14.08 | 15.24 | 17.34 | 22.15 | 16.05 |
| 4.6 | 15.44 | 14.39 | 15.58 | 17.73 | 22.64 | 16.41 |
| 4.7 | 15.78 | 14.7 | 15.92 | 18.11 | 23.13 | 16.76 |
| 4.8 | 16.12 | 15.02 | 16.26 | 18.5 | 23.62 | 17.12 |
| 4.9 | 16.46 | 15.33 | 16.6 | 18.88 | 24.12 | 17.48 |
| 5 | 16.8 | 15.64 | 16.94 | 19.27 | 24.61 | 17.83 |

Weight of Lead and Tin Lined Pipe per Foot.

From .375 Inch to 5 Inches in Diameter. (Tatham & Bros.)

| m. | WASTE-PIPE. | | | | BLOCK-TIN PIPE. | | | | |
|----|-------------|-------|---------|-------|-----------------|-------|---------|-------|---------|
| | Weight. | Diam. | Weight. | Diam. | Weight. | Diam. | Weight. | Diam. | Weight. |
| 1. | Lbs. | Ins. | Lbs. | Inch. | Lb. | Inch. | Lbs. | Ins. | Lbs. |
| 5 | 2 | 4 | 8 | .375 | .3594 | .625 | .5 | 1.25 | 1.25 |
| | 3 | 4.5 | 6 | .375 | .375 | .625 | .625 | 1.25 | 1.5 |
| | 3.5 | 4.5 | 8 | .375 | .5 | .75 | .625 | 1.5 | 2 |
| | 5 | 5 | 8 | .5 | .375 | .75 | .75 | 1.5 | 2.5 |
| | 5 | 5 | 10 | .5 | .5 | 1 | .9375 | 2 | 2.5 |
| | 6 | 5 | 12 | .5 | .625 | 1 | 1.125 | 2 | 3 |

WATER-PIPE.

From .375 Inch to 5 Inches in Diameter.

| Thick- ness. | Weight. | Diam. | Thick- ness. | Weight. | Diam. | Thick- ness. | Weight. | Diam. | Thick- ness. | Weight. |
|-----------------|---------|-------|-----------------|---------|-------|-----------------|---------|-------|-----------------|---------|
| Inch. | Lbs. | Ins. | Inch. | Lbs. | Ins. | Inch. | Lbs. | Ins. | Inch. | Lbs. |
| .08 | .625 | .625 | .25 | 3.5 | 1.25 | .19 | 4.75 | 2.5 | .3125 | 14 |
| .12 | 1 | .75 | .1 | 1.25 | 1.25 | .25 | 6 | 2.5 | .375 | 17 |
| .16 | 1.25 | .75 | .12 | 1.75 | 1.5 | .12 | 3 | 3 | .1875 | 9 |
| .19 | 1.5 | .75 | .16 | 2.25 | 1.5 | .14 | 3.5 | 3 | .25 | 12 |
| .34 | 2.5 | .75 | .2 | 3 | 1.5 | .17 | 4.25 | 3 | .3125 | 16 |
| .07 | .0545 | .75 | .23 | 3.5 | 1.5 | .19 | 5 | 3 | .375 | 20 |
| .09 | .75 | .75 | .3 | 4.75 | 1.5 | .23 | 6.5 | 3.5 | .1875 | 9.5 |
| .11 | 1 | 1 | .1 | 1.5 | 1.5 | .27 | 8 | 3.5 | .25 | 15 |
| .13 | 1.25 | 1 | .11 | 2 | 1.75 | .13 | 4 | 3.5 | .3125 | 18.5 |
| .16 | 1.75 | 1 | .14 | 2.5 | 1.75 | .17 | 5 | 3.5 | .375 | 22 |
| .19 | 2 | 1 | .17 | 3.25 | 1.75 | .21 | 6.5 | 4 | .1875 | 12.5 |
| .25 | 3 | 1 | .21 | 4 | 1.75 | .27 | 8.5 | 4 | .25 | 16 |
| .08 | .0727 | 1 | .24 | 4.75 | 2 | .15 | 4.75 | 4 | .3125 | 21 |
| .09 | 1 | 1 | .3 | 6 | 2 | .18 | 6 | 4 | .375 | 25 |
| .13 | 1.5 | 1.25 | .1 | 2 | 2 | .22 | 7 | 4.5 | .1875 | 14 |
| .16 | 2 | 1.25 | .12 | 2.5 | 2 | .27 | 9 | 4.5 | .25 | 18 |
| .2 | 2.5 | 1.25 | .14 | 3 | 2.5 | .1875 | 8 | 5 | .25 | 20 |
| .22 | 2.75 | 1.25 | .16 | 3.75 | 2.5 | .25 | 11 | 5 | .375 | 31 |

Marks and Weight of Tin-plates. (English.)

| MARK BRAND. | Plates per Box. | Dimensions. | Weight per Box. | MARK OR BRAND. | Plates per Box. | Dimensions. | Weight per Box. |
|----------------|--------------------|-------------|--------------------|-------------------|--------------------|-------------|--------------------|
| | No. | Ins. | No. | | No. | Ins. | No. |
| 1 Com. | 225 | 13.75×10 | 112 | DXXXX..... | 100 | 16.75×12.5 | 189 |
| | 225 | 13.25×9.75 | 105 | SDC..... | 200 | 15×11 | 168 |
| | 225 | 12.75×9.5 | 98 | SDX..... | 200 | 15×11 | 188 |
| | 225 | 13.75×10 | 119 | SDXX..... | 200 | 15×11 | 209 |
| | 225 | 13.75×10 | 157 | SDXXX..... | 200 | 15×11 | 230 |
| | 225 | 13.75×10 | 140 | SDXXXX..... | 200 | 15×11 | 251 |
| | 225 | 13.25×9.75 | 133 | SDXXXXXX..... | 200 | 15×11 | 272 |
| | 225 | 12.75×9.5 | 126 | SDXXXXXXX..... | 200 | 15×11 | 293 |
| | 225 | 13.75×10 | 161 | Leaded IC..... | 112 | 20×14 | 112 |
| | 225 | 13.75×10 | 182 | " IX..... | 112 | 20×14 | 149 |
| X..... | 225 | 13.75×10 | 203 | ICW..... | 225 | 13.75×10 | 112 |
| XX..... | 225 | 13.75×10 | 224 | IXW..... | 225 | 13.75×10 | 140 |
| XXX..... | 225 | 13.75×10 | 245 | CSDW..... | 200 | 15 | ? |
| | 100 | 16.75×12.5 | 98 | CIW..... | 100 | 16.7 | |
| | 100 | 16.75×12.5 | 126 | XIW..... | 160 | 16.7 | |
| | 100 | 16.75×12.5 | 147 | TT..... | 450 | 13 | |
| | 100 | 16.75×12.5 | 168 | XTT..... | 450 | 13 | |

When the plates are 14 by 20 inches, there are 112 in a

Iron Welded Steam, Gas, and Water Pipe.

STANDARD DIMENSIONS.

National Tube Works Company.

| Diameter. | | | Thickness. | Circumference. | | Transverse Areas. | | | Length of Pipe per Sq. Foot of Surface. | | Length containing One Cubic Foot. | Weight. |
|------------|------------|-----------------|------------|----------------|------------|-------------------|------------|----------|---|------------|-----------------------------------|---------|
| Ex-ternal. | Ex-ternal. | Actual In'tnal. | | Ex-ternal. | In-ternal. | Ex-ternal. | In-ternal. | Metal. | Ex-ternal. | In-ternal. | | |
| Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Sq. Ins. | Sq. Ins. | Sq. Ins. | Feet. | Feet. | Feet. | Lbs. |
| .125 | .4 | .27 | .07 | 1.27 | .85 | .13 | .06 | .07 | 9.44 | 14.15 | | |
| .25 | .54 | .36 | .09 | 1.7 | 1.14 | .23 | .1 | .12 | 7.07 | 10.49 | | |
| .375 | .67 | .49 | .09 | 2.12 | 1.55 | .36 | .19 | .17 | 5.66 | 7.73 | 751.2 | |
| .5 | .84 | .62 | .11 | 2.64 | 1.96 | .55 | .3 | .25 | 4.55 | 6.13 | 472.4 | |
| .75 | 1.05 | .82 | .11 | 3.3 | 2.59 | .87 | .53 | .33 | 3.64 | 4.63 | 270 | 1.1 |
| 1 | 1.31 | 1.05 | .13 | 4.13 | 3.29 | 1.36 | .86 | .49 | 2.9 | 3.64 | 166.9 | 1.4 |
| 1.25 | 1.66 | 1.38 | .14 | 5.21 | 4.33 | 2.16 | 1.5 | .67 | 2.3 | 2.77 | 96.2 | 2.2 |
| 1.5 | 1.9 | 1.61 | .14 | 5.97 | 5.06 | 2.83 | 2.04 | .8 | 2.01 | 2.37 | 70.7 | 2.6 |
| 2 | 2.37 | 2.07 | .15 | 7.46 | 6.49 | 4.43 | 3.36 | 1.07 | 1.61 | 1.85 | 42.9 | 3.6 |
| 2.5 | 2.87 | 2.47 | .20 | 9.03 | 7.75 | 6.49 | 4.78 | 1.71 | 1.33 | 1.55 | 30.1 | 5.1 |
| 3 | 3.5 | 3.07 | .22 | 11 | 9.64 | 9.62 | 7.39 | 2.24 | 1.09 | 1.24 | 19.5 | 7.1 |
| 3.5 | 4 | 3.55 | .23 | 12.57 | 11.15 | 12.57 | 9.89 | 2.68 | .95 | 1.08 | 14.6 | 9 |
| 4 | 4.5 | 4.03 | .24 | 14.14 | 12.65 | 15.9 | 12.73 | 3.17 | .85 | .95 | 11.3 | 10.6 |
| 4.5 | 5 | 4.51 | .25 | 15.71 | 14.16 | 19.63 | 15.96 | 3.67 | .76 | .85 | 9 | 12.6 |
| 5 | 5.56 | 5.04 | .26 | 17.48 | 15.85 | 24.31 | 19.99 | 4.32 | .69 | .76 | 7.2 | 14.1 |
| 6 | 6.62 | 6.06 | .28 | 20.81 | 19.05 | 34.47 | 28.89 | 5.58 | .58 | .63 | 5 | 18.1 |
| 7 | 7.62 | 7.02 | .30 | 23.95 | 22.06 | 45.66 | 38.74 | 6.93 | .5 | .54 | 3.7 | 23.4 |
| 8 | 8.62 | 7.98 | .32 | 27.1 | 25.08 | 58.43 | 50.04 | 8.39 | .44 | .48 | 2.9 | 28.1 |
| 9 | 9.62 | 8.94 | .34 | 30.24 | 28.08 | 72.76 | 62.73 | 10.03 | .4 | .43 | 2.3 | 33.1 |
| 10 | 10.75 | 10.02 | .37 | 33.77 | 31.48 | 90.76 | 78.84 | 11.92 | .35 | .38 | 1.8 | 40.4 |
| 11 | 12 | 11.25 | .37 | 37.7 | 35.34 | 113.1 | 99.4 | 13.7 | .32 | .34 | 1.5 | 45.9 |
| 12 | 12.75 | 12 | .37 | 40.06 | 37.7 | 127.68 | 113.1 | 14.58 | .3 | .32 | 1.3 | 48.9 |
| 13 | 14 | 13.25 | .37 | 43.98 | 41.63 | 153.94 | 137.89 | 16.05 | .27 | .29 | 1 | 53.9 |
| 14 | 15 | 14.25 | .37 | 47.12 | 44.77 | 176.72 | 159.49 | 17.23 | .26 | .27 | .9 | 57.8 |
| 15 | 16 | 15.25 | .37 | 50.27 | 47.91 | 201.06 | 182.66 | 18.41 | .24 | .25 | .8 | 61.7 |
| — | 18 | 17.25 | .37 | 56.55 | 54.19 | 254.47 | 233.71 | 20.76 | .21 | .22 | .6 | 69.6 |
| — | 20 | 19.25 | .37 | 62.83 | 60.48 | 314.16 | 291.04 | 23.12 | .19 | .20 | .5 | 77.5 |
| — | 22 | 21.25 | .37 | 69.12 | 66.76 | 380.13 | 354.66 | 25.48 | .17 | .18 | .4 | 85.4 |
| — | 24 | 23.25 | .37 | 75.4 | 73.04 | 452.39 | 424.56 | 27.83 | .16 | .16 | .3 | 93.3 |

Lap-welded Steel, Semi-steel, Special Locomotive and Franklinite Boiler-Tubes.

STANDARD DIMENSIONS. National Tube Works Company.

| Diameter. | | Thickness. | Wire Gauge. | Circumference. | | Transverse Areas. | | | Length per Sq. Foot of Surface. | | Weight. |
|------------|------------|------------|-------------|----------------|------------|-------------------|------------|----------|---------------------------------|------------|---------|
| Ex-ternal. | In-ternal. | | | Ex-ternal. | In-ternal. | Ex-ternal. | In-ternal. | Metal. | Ex-ternal. | In-ternal. | |
| Ins. | Ins. | Ins. | No. | Ins. | Ins. | Sq. Ins. | Sq. Ins. | Sq. Ins. | Feet. | Feet. | Lbs. |
| 1 | .834 | .083 | 14 | 3.142 | 2.62 | .785 | .546 | .239 | 3.82 | 4.58 | .8 |
| 1.25 | 1.084 | .083 | 14 | 3.927 | 3.405 | 1.227 | .923 | .304 | 3.056 | 3.524 | 1.0 |
| | .11 | .093 | 13 | 4.712 | 4.115 | 1.767 | 1.348 | .419 | 2.546 | 2.916 | 1.4 |
| | | .109 | 12 | 5.498 | 4.813 | 2.405 | 1.843 | .562 | 2.183 | 2.493 | 1.9 |
| | | .09 | 12 | 6.283 | 5.598 | 3.142 | 2.494 | .648 | 1.91 | 2.144 | 2.3 |
| | | | 12 | 7.069 | 6.384 | 3.976 | 3.243 | .733 | 1.698 | 1.88 | 2.4 |
| | | | | 7.854 | 7.1 | 4.909 | 4.011 | .898 | 1.528 | 1.69 | 3.0 |
| | | | | 8.639 | 7.885 | 5.94 | 4.948 | .992 | 1.389 | 1.522 | 3.5 |
| | | | | 9.424 | 8.67 | 7.069 | 5.983 | 1.086 | 1.273 | 1.384 | 3 |

Lap Welded Charcoal Iron Boiler Tubes.

STANDARD DIMENSIONS.

National Tube Works Company.

| Diameter. | In- ternal. | Thickness. | Wire Gauge. | Circumference. | | Transverse Area. | | | Length per Sq. Foot of Surface. | | Weight per Foot. |
|-----------|----------------|------------|----------------|----------------|----------------|------------------|----------------|----------|---------------------------------------|----------------|------------------------|
| | | | | Ex- ternal. | In- ternal. | Ex- ternal. | In- ternal. | Metal. | Ex- ternal. | In- ternal. | |
| | Ins. | Ins. | No. | Ins. | Ins. | Sq. Ins. | Sq. Ins. | Sq. Ins. | Feet. | Feet. | Lbs. |
| | .86 | .072 | 15 | 3.14 | 2.69 | .78 | .57 | .21 | 3.82 | 4.46 | .71 |
| 25 | .98 | .072 | 15 | 3.53 | 3.08 | .99 | .76 | .24 | 3.39 | 3.89 | .8 |
| 3 | 1.11 | .072 | 15 | 3.93 | 3.47 | 1.23 | .96 | .27 | 3.06 | 3.45 | .89 |
| 3 | 1.15 | .083 | 14 | 4.12 | 3.6 | 1.35 | 1.03 | .32 | 2.91 | 3.33 | 1.08 |
| 75 | 1.21 | .083 | 14 | 4.32 | 3.8 | 1.48 | 1.15 | .34 | 2.78 | 3.16 | 1.13 |
| | 1.33 | .083 | 14 | 4.71 | 4.19 | 1.77 | 1.4 | .37 | 2.55 | 2.86 | 1.24 |
| 15 | 1.43 | .095 | 13 | 5.1 | 4.51 | 2.07 | 1.62 | .46 | 2.35 | 2.66 | 1.53 |
| 1 | 1.56 | .095 | 13 | 5.5 | 4.9 | 2.4 | 1.91 | .49 | 2.18 | 2.45 | 1.66 |
| 15 | 1.68 | .095 | 13 | 5.89 | 5.29 | 2.76 | 2.23 | .53 | 2.04 | 2.27 | 1.78 |
| | 1.81 | .095 | 13 | 6.28 | 5.69 | 3.14 | 2.57 | .57 | 1.91 | 2.11 | 1.91 |
| 15 | 1.93 | .095 | 13 | 6.68 | 6.08 | 3.55 | 2.94 | .61 | 1.8 | 1.97 | 2.04 |
| | 2.06 | .095 | 13 | 7.07 | 6.47 | 3.98 | 3.33 | .64 | 1.7 | 1.85 | 2.16 |
| 5 | 2.16 | .109 | 12 | 7.46 | 6.78 | 4.43 | 3.65 | .78 | 1.61 | 1.77 | 2.61 |
| | 2.28 | .109 | 12 | 7.85 | 7.17 | 4.91 | 4.09 | .82 | 1.53 | 1.67 | 2.75 |
| | 2.53 | .109 | 12 | 8.64 | 7.95 | 5.94 | 5.03 | .9 | 1.39 | 1.51 | 3.04 |
| 5 | 2.66 | .109 | 12 | 9.03 | 8.35 | 6.49 | 5.54 | .95 | 1.33 | 1.44 | 3.18 |
| | 2.78 | .109 | 12 | 9.42 | 8.74 | 7.07 | 6.08 | .99 | 1.27 | 1.37 | 3.33 |
| | 3.01 | .12 | 11 | 10.21 | 9.46 | 8.3 | 7.12 | 1.18 | 1.17 | 1.26 | 3.96 |
| | 3.26 | .12 | 11 | 11 | 10.24 | 9.62 | 8.35 | 1.27 | 1.09 | 1.17 | 4.28 |
| | 3.51 | .12 | 11 | 11.78 | 11.03 | 11.04 | 9.68 | 1.37 | 1.02 | 1.09 | 4.6 |
| | 3.73 | .134 | 10 | 12.57 | 11.72 | 12.57 | 10.94 | 1.63 | .95 | .92 | 5.47 |
| | 3.98 | .134 | 10 | 13.35 | 12.51 | 14.19 | 12.45 | 1.73 | .9 | .96 | 5.82 |
| | 4.23 | .134 | 10 | 14.14 | 13.29 | 15.9 | 14.07 | 1.84 | .85 | .9 | 6.17 |
| | 4.48 | .134 | 10 | 14.92 | 14.08 | 17.72 | 15.78 | 1.94 | .8 | .85 | 6.53 |
| | 4.7 | .148 | 9 | 15.71 | 14.78 | 19.63 | 17.38 | 2.26 | .76 | .81 | 7.58 |
| | 4.95 | .148 | 9 | 16.49 | 15.56 | 21.65 | 19.27 | 2.37 | .73 | .77 | 7.97 |
| | 5.2 | .148 | 9 | 17.28 | 16.35 | 23.76 | 21.27 | 2.49 | .7 | .73 | 8.36 |
| | 5.67 | .165 | 8 | 18.85 | 17.81 | 28.27 | 25.25 | 3.02 | .64 | .67 | 10.16 |
| | 6.67 | .165 | 8 | 21.99 | 20.95 | 38.48 | 34.94 | 3.54 | .55 | .57 | 11.9 |
| | 7.67 | .165 | 8 | 25.13 | 24.1 | 50.27 | 46.2 | 4.06 | .48 | .50 | 13.65 |
| | 8.64 | .18 | 7 | 28.27 | 27.14 | 63.62 | 58.63 | 4.99 | .42 | .44 | 16.76 |
| | 9.59 | .203 | 6 | 31.42 | 30.14 | 78.54 | 72.29 | 6.25 | .38 | .4 | 20.99 |
| | 10.56 | .22 | 5 | 34.56 | 33.17 | 95.03 | 87.58 | 7.45 | .35 | .36 | 25.03 |
| | 11.54 | .229 | 4.5 | 37.7 | 36.26 | 113.1 | 104.63 | 8.47 | .32 | .33 | 28.46 |
| | 12.52 | .238 | 4 | 40.84 | 39.34 | 132.73 | 123.19 | 9.54 | .29 | .3 | 32.06 |
| | 13.5 | .248 | 3.5 | 43.98 | 42.42 | 153.94 | 143.22 | 10.71 | .27 | .28 | 36 |
| | 14.48 | .259 | 3 | 47.12 | 45.5 | 176.71 | 164.72 | 11.99 | .25 | .26 | 40.3 |
| | 15.43 | .284 | 2 | 50.26 | 48.48 | 201.06 | 187.04 | 14.02 | .24 | .25 | 47.11 |

EX.—For diameters from 16 up to and including 24 ins. **NOTE.** Details conformance with the circumstances, as there is not a **as** varying.

In estimating effective heating or evaporating surface: liquids by steam, superheating steam, or transferring from one gas to another, mean surface of Tubes is to be

M *

Weight of Seamless Drawn Copper Tubes

American Tube Works. (Boston.)

BY EXTERNAL DIAMETER. ONE FOOT IN LENGTH.

Stub's W. G. From .25 Inch to 12 Ins.—*f* full, *l* light.

| No. | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | |
|-----------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|--|
| Ins. | $\frac{7}{32} f$ | $\frac{3}{64} l$ | $\frac{3}{64} f$ | $\frac{1}{16} l$ | $\frac{7}{16} f$ | $\frac{5}{64} l$ | $\frac{5}{64} f$ | $\frac{3}{32} f$ | $\frac{7}{64}$ | |
| Diamet'r. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | |
| .25 | .09 | .1 | .12 | .13 | .14 | .15 | .17 | .18 | .19 | |
| .375 | .14 | .16 | .19 | .23 | .24 | .26 | .29 | .32 | .35 | |
| .5 | .2 | .23 | .27 | .31 | .34 | .37 | .42 | .47 | .52 | |
| .625 | .25 | .29 | .34 | .4 | .44 | .48 | .55 | .61 | .69 | |
| .75 | .3 | .36 | .42 | .49 | .54 | .59 | .67 | .76 | .85 | |
| .875 | .36 | .42 | .49 | .58 | .64 | .7 | .8 | .9 | 1.02 | |
| 1 | .41 | .48 | .57 | .67 | .74 | .81 | .93 | 1.05 | 1.18 | |
| 1.125 | .46 | .55 | .64 | .76 | .83 | .92 | 1.05 | 1.19 | 1.35 | |
| 1.25 | .52 | .61 | .71 | .84 | .93 | 1.03 | 1.18 | 1.34 | 1.52 | |
| 1.375 | .57 | .68 | .79 | .93 | 1.03 | 1.14 | 1.31 | 1.48 | 1.68 | |
| 1.5 | .62 | .74 | .86 | 1.02 | 1.13 | 1.25 | 1.43 | 1.63 | 1.85 | |
| 1.625 | .68 | .8 | .94 | 1.11 | 1.23 | 1.36 | 1.56 | 1.77 | 2.02 | |
| 1.75 | .73 | .87 | 1.01 | 1.2 | 1.33 | 1.47 | 1.69 | 1.92 | 2.18 | |
| 1.875 | .78 | .93 | 1.09 | 1.29 | 1.43 | 1.58 | 1.81 | 2.06 | 2.35 | |
| 2 | .84 | 1 | 1.16 | 1.37 | 1.53 | 1.69 | 1.94 | 2.21 | 2.51 | |
| 2.125 | .89 | 1.06 | 1.24 | 1.46 | 1.63 | 1.8 | 2.07 | 2.35 | 2.68 | |
| 2.25 | .94 | 1.13 | 1.31 | 1.55 | 1.73 | 1.91 | 2.19 | 2.5 | 2.85 | |
| 2.375 | 1 | 1.19 | 1.39 | 1.64 | 1.82 | 2.02 | 2.32 | 2.64 | 3.01 | |
| 2.5 | 1.05 | 1.25 | 1.46 | 1.73 | 1.92 | 2.13 | 2.45 | 2.79 | 3.18 | |
| 2.625 | 1.1 | 1.32 | 1.54 | 1.82 | 2.02 | 2.23 | 2.57 | 2.93 | 3.35 | |
| 2.75 | 1.16 | 1.38 | 1.61 | 1.9 | 2.12 | 2.34 | 2.7 | 3.08 | 3.51 | |
| 2.875 | 1.21 | 1.45 | 1.68 | 1.99 | 2.22 | 2.45 | 2.83 | 3.22 | 3.68 | |
| 3 | 1.26 | 1.51 | 1.76 | 2.08 | 2.32 | 2.56 | 2.95 | 3.37 | 3.84 | |
| 3.25 | 1.37 | 1.64 | 1.91 | 2.26 | 2.52 | 2.78 | 3.21 | 3.66 | 4.18 | |
| 3.5 | 1.48 | 1.77 | 2.06 | 2.43 | 2.72 | 3 | 3.46 | 3.95 | 4.51 | |
| 3.75 | 1.58 | 1.9 | 2.21 | 2.61 | 2.92 | 3.22 | 3.71 | 4.24 | 4.84 | |
| 4 | 1.69 | 2.02 | 2.36 | 2.79 | 3.11 | 3.44 | 3.97 | 4.53 | 5.17 | |
| 4.25 | 1.8 | 2.15 | 2.51 | 3.14 | 3.31 | 3.66 | 4.22 | 4.82 | 5.51 | |
| 4.5 | 1.9 | 2.28 | 2.65 | 3.32 | 3.51 | 3.88 | 4.47 | 5.11 | 5.84 | |
| 4.75 | 2.01 | 2.41 | 2.8 | 3.49 | 3.71 | 4.1 | 4.73 | 5.4 | 6.17 | |
| 5 | 2.12 | 2.54 | 2.95 | 3.67 | 3.91 | 4.32 | 4.98 | 5.69 | 6.5 | |
| 5.25 | 2.23 | 2.66 | 3.1 | 3.85 | 4.11 | 4.54 | 5.23 | 5.98 | 6.84 | |
| 5.5 | 2.34 | 2.79 | 3.25 | 3.85 | 4.3 | 4.76 | 5.49 | 6.27 | 7.17 | |
| 5.75 | 2.44 | 2.92 | 3.4 | 4.02 | 4.5 | 4.98 | 5.74 | 6.56 | 7.5 | |
| 6 | 2.55 | 3.05 | 3.55 | 4.2 | 4.7 | 5.2 | 5.99 | 6.85 | 7.83 | |
| 6.25 | 2.66 | 3.18 | 3.7 | 4.38 | 4.9 | 5.41 | 6.25 | 7.14 | 8.17 | |
| 6.5 | 2.76 | 3.31 | 3.85 | 4.55 | 5.1 | 5.63 | 6.5 | 7.43 | 8.5 | |
| 6.75 | 2.87 | 3.44 | 4 | 4.73 | 5.3 | 5.85 | 6.75 | 7.72 | 8.83 | |
| 7 | 2.98 | 3.56 | 4.15 | 4.91 | 5.49 | 6.07 | 7.01 | 8.01 | 9.16 | |
| 7.25 | 3.09 | 3.69 | 4.3 | 5.09 | 5.69 | 6.29 | 7.26 | 8.30 | 9.5 | |
| 7.5 | 3.19 | 3.82 | 4.45 | 5.26 | 5.89 | 6.51 | 7.51 | 8.59 | 9.83 | |
| 8 | 3.41 | 4.08 | 4.74 | 5.62 | 6.29 | 6.95 | 8.02 | 9.17 | 10.49 | |
| 8.25 | 3.52 | 4.33 | 5.04 | 5.97 | 6.68 | 7.39 | 8.52 | 9.75 | 11.16 | |
| 8.5 | 3.63 | 4.59 | 5.34 | 6.33 | 7.08 | 7.83 | 9.03 | 10.33 | 11.82 | |
| 8.75 | 3.74 | 4.74 | 5.64 | 6.68 | 7.48 | 8.26 | 9.54 | 10.91 | 12.49 | |
| 9 | 3.85 | 4.91 | 5.84 | 7.03 | 7.87 | 8.7 | 10.05 | 11.49 | 13.15 | |
| 9.25 | 3.96 | 5.05 | 6.04 | 7.26 | 8.17 | 9.07 | 10.41 | 11.91 | 13.68 | |
| 9.5 | 4.07 | 5.17 | 6.22 | 7.48 | 8.47 | 9.41 | 10.75 | 12.31 | 14.21 | |
| 9.75 | 4.18 | 5.29 | 6.41 | 7.71 | 8.71 | 9.67 | 11.06 | 12.65 | 14.78 | |
| 10 | 4.29 | 5.41 | 6.56 | 7.94 | 8.97 | 10.02 | 11.56 | 13.23 | 15.15 | |
| 10.25 | 4.4 | 5.54 | 6.74 | 8.21 | 9.29 | 10.45 | 12.07 | 13.81 | 15.81 | |

| o. | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|--------|--------|--------|---------|--------|-------|--------|---------|-------|--------|---------|
| | 9/64 l | 9/64 f | 11/64 l | 3/16 l | 13/64 | 7/32 f | 15/64 f | 1/4 f | 9/32 f | 19/64 f |
| net'r. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 175 | .4 | .41 | .42 | .44 | — | — | — | — | — | — |
| | .61 | .64 | .67 | .71 | .73 | .75 | .76 | — | — | — |
| 25 | .81 | .86 | .92 | .99 | 1.04 | 1.09 | 1.12 | 1.13 | 1.18 | — |
| 5 | 1.01 | 1.09 | 1.17 | 1.26 | 1.35 | 1.42 | 1.49 | 1.53 | 1.61 | 1.63 |
| 75 | 1.22 | 1.31 | 1.42 | 1.53 | 1.66 | 1.76 | 1.85 | 1.92 | 2.04 | 2.09 |
| | 1.42 | 1.54 | 1.67 | 1.81 | 1.97 | 2.09 | 2.21 | 2.32 | 2.48 | 2.55 |
| 25 | 1.63 | 1.78 | 1.93 | 2.08 | 2.28 | 2.43 | 2.58 | 2.71 | 2.91 | 3 |
| 5 | 1.83 | 2 | 2.18 | 2.36 | 2.59 | 2.76 | 2.94 | 3.11 | 3.34 | 3.46 |
| 75 | 2.03 | 2.22 | 2.43 | 2.63 | 2.9 | 3.1 | 3.3 | 3.5 | 3.77 | 3.92 |
| | 2.24 | 2.44 | 2.68 | 2.91 | 3.21 | 3.43 | 3.67 | 3.9 | 4.21 | 4.38 |
| 15 | 2.44 | 2.67 | 2.93 | 3.18 | 3.52 | 3.77 | 4.03 | 4.29 | 4.64 | 4.83 |
| | 2.65 | 2.89 | 3.18 | 3.45 | 3.83 | 4.11 | 4.39 | 4.69 | 5.07 | 5.29 |
| 15 | 2.85 | 3.12 | 3.44 | 3.73 | 4.14 | 4.44 | 4.76 | 5.08 | 5.51 | 5.75 |
| | 3.06 | 3.34 | 3.69 | 4 | 4.45 | 4.78 | 5.12 | 5.48 | 5.94 | 6.21 |
| 15 | 3.26 | 3.57 | 3.94 | 4.28 | 4.75 | 5.11 | 5.48 | 5.87 | 6.37 | 6.66 |
| | 3.46 | 3.8 | 4.19 | 4.55 | 5.06 | 5.45 | 5.84 | 6.27 | 6.81 | 7.12 |
| 5 | 3.67 | 4.02 | 4.44 | 4.82 | 5.37 | 5.78 | 6.21 | 6.66 | 7.24 | 7.57 |
| | 3.87 | 4.25 | 4.69 | 5.1 | 5.68 | 6.12 | 6.57 | 7.06 | 7.67 | 8.04 |
| 5 | 4.08 | 4.47 | 4.95 | 5.37 | 6 | 6.45 | 6.93 | 7.45 | 8.1 | 8.49 |
| | 4.28 | 4.7 | 5.2 | 5.65 | 6.3 | 6.79 | 7.29 | 7.85 | 8.54 | 8.95 |
| 5 | 4.48 | 4.92 | 5.45 | 5.92 | 6.61 | 7.12 | 7.66 | 8.24 | 8.97 | 9.41 |
| | 4.69 | 5.15 | 5.7 | 6.2 | 6.92 | 7.46 | 8.02 | 8.64 | 9.4 | 9.87 |
| | 5.1 | 5.6 | 6.2 | 6.74 | 7.54 | 8.13 | 8.75 | 9.43 | 10.27 | 10.78 |
| | 5.51 | 6.05 | 6.71 | 7.29 | 8.16 | 8.8 | 9.47 | 10.22 | 11.14 | 11.7 |
| | 5.91 | 6.5 | 7.21 | 7.84 | 8.78 | 9.47 | 10.2 | 11.01 | 12 | 12.61 |
| | 6.32 | 6.95 | 7.71 | 8.39 | 9.4 | 10.14 | 10.92 | 11.8 | 12.87 | 13.53 |
| | 6.73 | 7.4 | 8.22 | 8.94 | 10.02 | 10.81 | 11.65 | 12.59 | 13.73 | 14.44 |
| | 7.14 | 7.85 | 8.72 | 9.49 | 10.64 | 11.48 | 12.37 | 13.38 | 14.6 | 15.36 |
| | 7.55 | 8.3 | 9.22 | 10.04 | 11.26 | 12.16 | 13.1 | 14.17 | 15.46 | 16.27 |
| | 7.96 | 8.75 | 9.73 | 10.58 | 11.88 | 12.83 | 13.83 | 14.96 | 16.33 | 17.19 |
| | 8.36 | 9.21 | 10.23 | 11.13 | 12.49 | 13.5 | 14.55 | 15.75 | 17.2 | 18.1 |
| | 8.77 | 9.66 | 10.73 | 11.68 | 13.11 | 14.17 | 15.28 | 16.54 | 18.06 | 19.02 |
| | 9.18 | 10.11 | 11.24 | 12.23 | 13.73 | 14.84 | 16 | 17.33 | 18.93 | 19.93 |
| | 9.59 | 10.56 | 11.74 | 12.78 | 14.35 | 15.51 | 16.73 | 18.12 | 19.79 | 20.85 |
| 10 | 11.01 | 12.24 | 13.33 | 14.97 | 16.18 | 17.46 | 18.91 | 20.66 | 21.76 | 22.68 |
| 10.41 | 11.46 | 12.75 | 13.88 | 15.59 | 16.85 | 18.18 | 19.7 | 21.53 | 22.68 | 23.59 |
| 10.82 | 11.91 | 13.25 | 14.42 | 16.21 | 17.52 | 18.91 | 20.49 | 22.39 | 23.59 | 24.51 |
| 11.22 | 12.36 | 13.75 | 14.97 | 16.83 | 18.19 | 19.63 | 21.28 | 23.26 | 24.51 | 25.42 |
| 11.63 | 12.81 | 14.26 | 15.52 | 17.45 | 18.86 | 20.36 | 22.07 | 24.13 | 25.42 | 26.34 |
| 12.04 | 13.26 | 14.76 | 16.07 | 18.07 | 19.54 | 21.08 | 22.86 | 25 | 26.34 | 27.25 |
| 12.45 | 13.71 | 15.26 | 16.62 | 18.68 | 20.21 | 21.81 | 23.65 | 25.86 | 27.25 | 28.17 |
| 12.86 | 14.17 | 15.77 | 17.17 | 19.3 | 20.88 | 22.54 | 24.44 | 26.72 | 28.17 | 29.08 |
| 13.27 | 14.62 | 16.27 | 17.71 | 19.92 | 21.55 | 23.26 | 25.23 | 27.59 | 29.08 | 30 |
| 13.67 | 15.07 | 16.77 | 18.26 | 20.54 | 22.22 | 23.99 | 26.02 | 28.45 | 30 | 30.91 |
| 14.08 | 15.52 | 17.28 | 18.81 | 21.16 | 22.89 | 24.71 | 26.81 | 29.32 | 30.91 | 31.83 |
| 14.49 | 15.97 | 17.78 | 19.36 | 21.78 | 23.56 | 25.44 | 27.6 | 30.18 | 31.83 | 32.74 |
| 14.9 | 16.42 | 18.28 | 19.91 | 22.4 | 24.23 | 26.17 | 28.39 | 31.05 | 32.74 | 33.66 |
| 15.31 | 16.87 | 18.79 | 20.46 | 23.02 | 24.9 | 26.89 | 29.18 | 31.92 | 33.66 | 34.57 |
| 15.72 | 17.32 | 19.29 | 21.01 | 23.64 | 25.57 | 27.62 | 29.97 | 32.78 | 34.57 | 35 |
| 16.12 | 17.77 | 19.79 | 21.55 | 24.26 | 26.24 | 28.34 | 30.71 | 33.61 | 35 | 35 |
| 16.94 | 18.68 | 20.8 | 22.65 | 25.5 | 27.59 | 29.79 | 32.1 | 34.15 | 35 | 35 |
| 17.76 | 19.58 | 21.81 | 23.75 | 26.73 | 28.93 | 31.25 | 33 | 35 | 35 | 35 |
| 18.57 | 20.48 | 22.81 | 24.84 | 27.97 | 30.27 | 32.7 | 35 | 35 | 35 | 35 |
| 19.39 | 21.38 | 23.82 | 25.94 | 29.21 | 31.61 | 34.15 | 35 | 35 | 35 | 35 |

By Internal Diameter.

Add following Units to Weights for External Diameter in preceding table

| | | | | | | | | | | |
|-----|------|------|------|------|------|------|-----|-----|-----|-----|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 2.21 | 1.97 | 1.66 | 1.38 | 1.18 | 1.01 | .78 | .67 | .53 | .43 |
| No. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | .35 | .20 | .22 | .17 | .13 | .11 | .08 | .06 | .05 | .03 |

ILLUSTRATION.—What is weight of a copper tube 6 ins. in internal diameter No. 3 gauge, and one foot in length?

By preceding table 6 ins. external, No. 3 gauge = 18.12, and $18.12 + 1.66 = 19.78$ lbs.

WEIGHT OF BRASS TUBES.

To Compute Weight of Brass Tubes.

American Tube Works, (Boston.)

RULE.—Deduct 5 per cent. from weight of Copper tubes.

EXAMPLE. — What is weight of a brass tube 6 ins. in external diameter, No. 1 gauge, and one foot in length?

By preceding table 6 lbs. = 18.12, from which deduct 5 per cent. = 17.21 lbs.

By Internal Diameter.

RULE.—Proceed as above for internal diameter, and deduct 5 per cent.

EXAMPLE.—Weight of a copper tube 6 ins. internal diameter, No. 3 gauge, and 1 foot in length = 10.78 lbs.

Hence, $19.78 - 5 \text{ per cent.} = 18.79 \text{ lbs.}$

NOTE.—Diameter of Tubes, as for Boilers, is given externally, and that for Pipes internally.

Weights of English are essentially alike to the preceding. (D. K. Clark)

Seamless Brass Pipes.

Made to Correspond with Iron Pipes and to Fit Iron
Pipe Fittings.

American Tube Works. (Boston.)

WEIGHT PER LINEAL FOOT.

| Diameter of Iron Pipe. | | | Diameter of Iron Pipe. | | | Diameter of Iron Pipe. | | |
|------------------------|-----------|---------|------------------------|-----------|---------|------------------------|-----------|---------|
| Internal. | External. | Weight. | Internal. | External. | Weight. | Internal. | External. | Weight. |
| Inch. | Ins. | Lbs. | Ins. | Ins. | Lbs. | Ins. | Ins. | Lbs. |
| .125 | .375 | .25 | 1 | 1.3135 | 1.7 | 3 | 3.5 | 8.3 |
| .25 | .5625 | .43 | 1.25 | 1.625 | 2.5 | 3.5 | 4 | 10.4 |
| .375 | .6875 | .63 | 1.5 | 1.875 | 3 | 4 | 4.5 | 12.7 |
| .5 | .8125 | .9 | 2 | 2.375 | 4.125 | 5 | 5.5625 | 15.7 |
| .75 | 1.0625 | 1.25 | 2.5 | 2.875 | 5.75 | 6 | 6.625 | 20.6 |

Seamless Copper pipes of like diameter are one nineteenth heavier.

Weight of Sheet Brass.

ONE SQUARE FOOT. (*Holtzapffel's Gauge.*)

| Thickness. | | | Weight. | | | Thickness. | | | Weight. | | |
|------------|-------|------|---------|-------|------|------------|-------|------|---------|-------|------|
| No. | Inch. | Lbs. | No. | Inch. | Lbs. | No. | Inch. | Lbs. | No. | Inch. | Lbs. |
| 3 | .259 | 10.9 | 9 | .148 | 6.23 | 15 | .072 | 3.03 | 21 | .032 | 1.35 |
| | .238 | 10 | 10 | .134 | 5.64 | 16 | .065 | 2.74 | 22 | .028 | 1.12 |
| | | 9.26 | 11 | .12 | 5.05 | 17 | .058 | 2.44 | 23 | .025 | 1.00 |
| | | 8.55 | 12 | .109 | 4.59 | 18 | .049 | 2.06 | 24 | .022 | .87 |
| | | 7.58 | 13 | .095 | 4 | 19 | .042 | 1.77 | 25 | .02 | .80 |
| | | 6.95 | 14 | .083 | 3.49 | 20 | .035 | 1.47 | | | |

Weight of Wrought Iron Tubes. (English.)

EXTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzapfel's Wire-Gauge. *f*, full, *l*, light.

| to. | — | — | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---------------|--------------|------------------------|-------------|---------------|----------------------|------------------------|-----------------------|
| in. | .3125 5/16 | .281 9/32 | .238 15/64 <i>f</i> | .22 7/32 | .203 13/64 | .18 3/16 <i>l</i> | .165 11/64 <i>l</i> | .148 9/64 <i>f</i> |
| am. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 7 | 21.9 | 19.8 | 16.9 | 15.6 | 14.5 | 12.9 | 11.8 | 10.6 |
| 7.5 | 23.5 | 21.3 | 18.1 | 16.8 | 15.5 | 13.8 | 12.7 | 11.4 |
| 8 | 25.2 | 22.7 | 19.3 | 17.9 | 16.6 | 14.7 | 13.5 | 12.2 |
| 8.5 | 26.8 | 24.2 | 20.6 | 19.1 | 17.6 | 15.7 | 14.4 | 12.9 |
| 9 | 28.4 | 25.7 | 21.8 | 20.2 | 18.7 | 16.6 | 15.3 | 13.7 |
| 9.5 | 30.1 | 27.1 | 23.1 | 21.4 | 19.8 | 17.6 | 16.1 | 14.5 |
| 10 | 31.7 | 28.6 | 24.3 | 22.5 | 20.8 | 18.5 | 17 | 15.3 |

| to. | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|------|----------------------|------------------------|-----------------------|-----------------------|---------------------|--------------|-----------------------|-----------------------|-----------------------|
| in. | .18 3/16 <i>l</i> | .165 11/64 <i>l</i> | .148 9/64 <i>f</i> | .134 9/64 <i>l</i> | .12 1/8 <i>l</i> | .109 7/64 | .095 3/32 <i>f</i> | .083 5/64 <i>f</i> | .072 5/64 <i>l</i> |
| am. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 25 | 1.55 | 1.44 | 1.32 | 1.22 | 1.11 | 1.02 | .9 | .797 | .7 |
| 5 | 1.78 | 1.66 | 1.51 | 1.39 | 1.26 | 1.16 | 1.3 | .906 | .794 |
| 7.5 | 2.02 | 1.88 | 1.71 | 1.57 | 1.42 | 1.3 | 1.15 | 1.01 | .888 |
| 75 | 2.25 | 2.09 | 1.9 | 1.74 | 1.58 | 1.45 | 1.27 | 1.12 | .983 |
| | 2.49 | 2.31 | 2.1 | 1.92 | 1.73 | 1.59 | 1.4 | 1.23 | 1.08 |
| 35 | 2.72 | 2.52 | 2.29 | 2.09 | 1.89 | 1.73 | 1.52 | 1.34 | 1.17 |
| 5 | 2.96 | 2.74 | 2.48 | 2.27 | 2.05 | 1.87 | 1.65 | 1.45 | 1.27 |
| 15 | 3.19 | 2.96 | 2.68 | 2.45 | 2.21 | 2.02 | 1.77 | 1.56 | 1.36 |
| | 3.43 | 3.17 | 2.87 | 2.62 | 2.36 | 2.16 | 1.9 | 1.67 | 1.45 |
| 15 | 3.67 | 3.39 | 3.06 | 2.8 | 2.52 | 2.3 | 2.02 | 1.78 | 1.55 |
| 1 | 3.9 | 3.6 | 3.26 | 2.97 | 2.68 | 2.44 | 2.14 | 1.88 | 1.64 |
| 5 | 4.14 | 3.82 | 3.45 | 3.15 | 2.83 | 2.59 | 2.27 | 1.99 | 1.74 |
| | 4.37 | 4.04 | 3.65 | 3.32 | 2.99 | 2.73 | 2.39 | 2.1 | 1.83 |
| 5 | 4.61 | 4.25 | 3.84 | 3.5 | 3.15 | 2.87 | 2.52 | 2.21 | 1.93 |
| | 4.84 | 4.47 | 4.03 | 3.67 | 3.31 | 3.02 | 2.64 | 2.32 | 2.02 |
| 5 | 5.08 | 4.68 | 4.23 | 3.85 | 3.46 | 3.16 | 2.77 | 2.43 | 2.11 |
| | 5.32 | 4.9 | 4.42 | 4.02 | 3.62 | 3.3 | 2.89 | 2.54 | 2.21 |
| | 5.79 | 5.33 | 4.81 | 4.37 | 3.94 | 3.59 | 3.14 | 2.75 | 2.4 |
| | 6.26 | 5.76 | 5.2 | 4.72 | 4.25 | 3.87 | 3.39 | 2.97 | 2.59 |
| | 6.73 | 6.19 | 5.58 | 5.07 | 4.57 | 4.16 | 3.64 | 3.19 | 2.77 |
| | 7.2 | 6.63 | 5.97 | 5.43 | 4.88 | 4.44 | 3.89 | 3.4 | 2.96 |
| | 7.67 | 7.06 | 6.36 | 5.78 | 5.2 | 4.73 | 4.13 | 3.62 | 3.15 |
| | 8.14 | 7.49 | 6.45 | 6.13 | 5.51 | 5.01 | 4.38 | 3.84 | 3.34 |
| | 8.61 | 7.91 | 7.13 | 6.48 | 5.82 | 5.3 | 4.63 | 4.06 | 3.53 |
| | 9.08 | 8.35 | 7.52 | 6.83 | 6.13 | 5.58 | 4.88 | 4.27 | 3.72 |
| 9.56 | 8.79 | 7.91 | 7.18 | 6.44 | 5.87 | 5.13 | 4.49 | 3.9 | |
| 10 | 9.22 | 8.3 | 7.53 | 6.76 | 6.15 | 5.38 | 4.71 | 4.09 | |
| 10.5 | 9.65 | 8.68 | 7.88 | 7.07 | 6.44 | 5.63 | 4.93 | 4.28 | |
| 11 | 10.1 | 9.07 | 8.23 | 7.39 | 6.73 | 5.87 | 5.14 | 4.47 | |
| 11.4 | 10.5 | 9.46 | 8.58 | 7.7 | 7.01 | 6.12 | 5.36 | 4.66 | |
| 11.9 | 10.9 | 9.85 | 8.93 | 8.02 | 7.3 | 6.37 | 5.58 | 4.85 | |
| 12.4 | 11.4 | 10.2 | 9.28 | 8.33 | 7.58 | 6.62 | 5.79 | 5.03 | |
| 12.9 | 11.8 | 10.6 | 9.63 | 8.64 | 7.87 | 6.87 | | 5.22 | |
| 13.3 | 12.2 | 11 | 9.99 | 8.96 | 8.15 | 7.12 | | | |
| 13.8 | 12.7 | 11.4 | 10.3 | 9.27 | 8.44 | 7.3 | | | |
| 14.3 | 13.1 | 11.8 | 10.7 | 9.59 | 8.72 | 7.6 | | | |
| 14.7 | 13.5 | 12.2 | 11 | 9.9 | 9.01 | 7.8 | | | |

Weight of Seamless Drawn Copper Tubes. (English)
For Diameters and Thicknesses not given in preceding Tables. (D. K. Clark.)

INTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzappel's Wire-Gauge. *f* full, *l* light.

Specific Weight = 1.16. Wrought Iron = 1.

| No. | 0000 | 000 | 00 | 0 | No. | 0000 | 000 | 00 | 0 |
|-------|---------------|------------------------|---------------------|--------------|-------|---------------|------------------------|---------------------|--------------|
| Ins. | .454 29/64 | .425 27/64 <i>f</i> | .38 3/8 <i>f</i> | .34 11/32 | Ins. | .454 29/64 | .425 27/64 <i>f</i> | .38 3/8 <i>f</i> | .34 11/32 |
| Diam. | Lbs. | Lbs. | Lbs. | Lbs. | Diam. | Lbs. | Lbs. | Lbs. | Lbs. |
| .75 | — | — | — | 4.5 | 5.75 | 34.2 | 31.9 | 28.3 | 25.2 |
| .875 | — | — | 5.79 | 5.02 | 6 | 35.6 | 33.2 | 29.5 | 26.2 |
| 1 | 8.02 | 7.36 | 6.37 | 5.53 | 6.5 | 38.4 | 35.8 | 31.8 | 28.3 |
| 1.125 | 8.71 | 8 | 6.95 | 6.05 | 7 | 41.1 | 38.3 | 34.1 | 30.3 |
| 1.25 | 9.4 | 8.65 | 7.52 | 6.57 | 7.5 | 43.9 | 40.9 | 36.4 | 32.4 |
| 1.375 | 10.1 | 9.3 | 8.1 | 7.08 | 8 | 46.6 | 43.5 | 38.7 | 34.5 |
| 1.5 | 10.8 | 9.94 | 8.68 | 7.6 | 9 | 52.1 | 48.7 | 43.3 | 38.6 |
| 1.625 | 11.5 | 10.6 | 9.26 | 8.12 | 10 | 57.7 | 53.8 | 47.9 | 42.7 |
| 1.75 | 12.1 | 11.2 | 9.83 | 8.63 | 11 | 63.2 | 59 | 52.5 | 46.8 |
| 1.875 | 12.8 | 11.9 | 10.4 | 9.15 | 12 | 68.7 | 64.2 | 57.2 | 51 |
| 2 | 13.5 | 12.5 | 11 | 9.66 | 13 | 74.2 | 69.3 | 61.8 | 55.1 |
| 2.125 | 14.2 | 13.3 | 11.6 | 10.2 | 14 | 79.7 | 74.5 | 66.4 | 59.2 |
| 2.25 | 14.9 | 13.8 | 12.1 | 10.7 | 15 | 85.2 | 79.6 | 71 | 63.4 |
| 2.375 | 15.6 | 14.5 | 12.7 | 11.2 | 16 | 90.7 | 84.8 | 75.6 | 67.7 |
| 2.5 | 16.3 | 15.1 | 13.3 | 11.7 | 17 | 96.3 | 90 | 80.2 | 71.8 |
| 2.625 | 17 | 15.8 | 13.9 | 12.2 | 18 | 101.8 | 95.1 | 84.9 | 76 |
| 2.75 | 17.7 | 16.4 | 14.5 | 12.8 | 19 | 107.3 | 100.3 | 89.5 | 80.1 |
| 3 | 19.1 | 17.7 | 15.6 | 13.8 | 20 | 112.8 | 105.5 | 94.1 | 84.2 |
| 3.25 | 20.4 | 19 | 16.8 | 14.8 | 21 | 118.3 | 110.7 | 98.7 | 88.3 |
| 3.5 | 21.8 | 20.3 | 17.9 | 15.9 | 22 | 123.8 | 115.8 | 103.3 | 92.5 |
| 3.75 | 23.2 | 21.6 | 19.1 | 16.9 | 23 | 129.3 | 120.9 | 107.9 | 96.6 |
| 4 | 24.6 | 22.9 | 20.2 | 17.9 | 24 | 134.8 | 126.1 | 112.6 | 100.6 |
| 4.25 | 25.9 | 24.2 | 21.4 | 19 | 26 | 146 | 136.4 | 121.8 | 108.8 |
| 4.5 | 27.3 | 25.4 | 22.5 | 20 | 28 | 157.2 | 146.7 | 131 | 117.7 |
| 4.75 | 28.7 | 26.7 | 23.7 | 21 | 30 | 168.4 | 157.1 | 140.2 | 125.4 |
| 5 | 30.1 | 28 | 24.8 | 22.1 | 32 | 179.6 | 167.4 | 149.5 | 133.6 |
| 5.25 | 31.5 | 29.3 | 26 | 23.1 | 34 | 190.7 | 177.7 | 158.7 | 141.9 |
| 5.5 | 32.8 | 30.6 | 27.1 | 24.1 | 36 | 201.9 | 188 | 167.9 | 150.1 |

For Diameters from 13 to 24 Inches.

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------|----------------------|-----------------------|----------------------|------------------------|----------------------|---------------|----------------------|------------------------|-----------------------|-----------------------|
| Ins. | .3 19/64 <i>f</i> | .284 9/32 <i>f</i> | .259 1/4 <i>f</i> | .238 15/64 <i>f</i> | .22 7/32 <i>f</i> | .203 13/64 | .18 3/16 <i>l</i> | .165 11/64 <i>l</i> | .148 9/64 <i>f</i> | .134 9/64 <i>l</i> |
| Diam. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 13 | 48.5 | 45.8 | 41.7 | 38.3 | 35.3 | 32.6 | 28.8 | 26.4 | 23.6 | 21.4 |
| 14 | 52.1 | 49.3 | 44.9 | 41.2 | 38 | 35.1 | 31 | 28.4 | 25.4 | 23 |
| 15 | 55.8 | 52.7 | 48 | 44.1 | 40.7 | 37.6 | 33.2 | 30.4 | 27.2 | 24.6 |
| 16 | 59.4 | 56.2 | 51.2 | 46.9 | 43.4 | 40 | 35.4 | 32.4 | 29 | 26.3 |
| 17 | 63 | 59.6 | 54.3 | 49.8 | 46 | 42.5 | 37.5 | 34.4 | 30.8 | 27.8 |
| 18 | 66.7 | 63.1 | 57.4 | 52.7 | 48.7 | 45 | 39.7 | 36.4 | 32.6 | 29.5 |
| 19 | 70.3 | 66.5 | 60.6 | 55.6 | 51.4 | 47.4 | 41.9 | 38.4 | 34.4 | 31.2 |
| 20 | 74 | 70 | 63.7 | 58.5 | 54 | 49.9 | 44.1 | 40.4 | 36.2 | 32.8 |
| 21 | 77.6 | 73.4 | 66.9 | 61.4 | 56.7 | 52.4 | 46.3 | 42.4 | 38 | 34.4 |
| | 80.3 | 76.9 | 70 | 64.3 | 59.4 | 54.9 | 48.5 | 44.4 | 39.8 | 36 |
| | 83.8 | 80.3 | 73.2 | 67.2 | 62.1 | 57.3 | 50.7 | 46.4 | 41.6 | 37.7 |
| | | 83.8 | 76.3 | 70.1 | 64.7 | 59.8 | 52.9 | 48.5 | 43.4 | 39 |

For Diameters from 13 to 24 Inches.

| | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|------|--------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | .12 7/8 l | .109 7/64 | .095 3/32 f | .083 5/64 f | .072 5/64 l | .065 1/16 f | .058 1/16 l | .049 3/64 f | .042 3/64 l | .035 1/32 f |
| Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 19.1 | 17.4 | 15.1 | 13.2 | 11.4 | 10.3 | 9.2 | 7.77 | 6.65 | 5.55 | |
| 20.6 | 18.7 | 16.3 | 14.2 | 12.3 | 11.1 | 9.9 | 8.37 | 7.16 | 5.98 | |
| 22.1 | 20 | 17.4 | 15.2 | 13.2 | 11.9 | 10.6 | 8.96 | 7.67 | 6.4 | |
| 23.5 | 21.3 | 18.6 | 16.2 | 14.1 | 12.7 | 11.3 | 9.56 | 8.18 | 6.82 | |
| 25 | 22.7 | 19.7 | 17.2 | 14.9 | 13.5 | 12.1 | 10.2 | 8.69 | 7.27 | |
| 26.4 | 24 | 20.9 | 18.2 | 15.8 | 14.3 | 12.7 | 10.7 | 9.2 | 7.69 | |
| 27.9 | 25.3 | 22 | 19.2 | 16.7 | 15.1 | 13.4 | 11.3 | 9.71 | 8.12 | |
| 29.3 | 26.6 | 23.2 | 20.2 | 17.6 | 15.9 | 14.1 | 11.9 | 10.2 | 8.54 | |
| 30.8 | 27.9 | 24.3 | 21.3 | 18.4 | 16.6 | 14.8 | 12.5 | 10.7 | 8.96 | |
| 32.3 | 29.3 | 25.5 | 22.3 | 19.3 | 17.4 | 15.5 | 13.1 | 11.2 | 9.39 | |
| 33.7 | 30.6 | 26.7 | 23.3 | 20.2 | 18.2 | 16.2 | 13.7 | 11.8 | 9.81 | |
| 35.2 | 31.9 | 27.8 | 24.3 | 21.1 | 19 | 16.9 | 14.3 | 12.3 | 10.2 | |

Weight of Wrought Iron Tubes. (English.)

Diameters and Thicknesses not given in preceding Tables. (D. K. Clark.)

INTERNAL DIAMETER. ONE FOOT IN LENGTH.

Holtzapfel's Wire-Gauge. *f* full, *l* light.

| THICKNESS IN INCHES. | | | | | | | 4 | 5 | 6 | 7 |
|----------------------|-------|-------|-------|-------|-------|------|-----------------|---------------|---------------|---------------|
| 5/8 | 9/16 | 1/2 | 7/16 | 3/8 | 5/16 | 1/4 | .238 15/64 f | .22 7/32 f | .203 13/64 | .18 3/16 l |
| Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 128.5 | 115.2 | 102.1 | 89.1 | 76.1 | 63.2 | 50.4 | 48 | 44.2 | 40.8 | 36.2 |
| 135 | 121.1 | 107.3 | 93.6 | 80 | 66.5 | 53 | 50.4 | 46.5 | 42.9 | 38 |
| 141.5 | 127 | 112.6 | 98.2 | 83.9 | 69.7 | 55.6 | 52.9 | 48.8 | 45.1 | 39.9 |
| 148.1 | 132.9 | 117.8 | 102.8 | 87.9 | 73 | 58.3 | 55.4 | 51.1 | 47.2 | 41.8 |
| 154.6 | 138.8 | 123.1 | 107.4 | 91.8 | 76.3 | 60.9 | 57.9 | 53.4 | 49.3 | 43.7 |
| 161.2 | 144.7 | 128.3 | 112 | 95.7 | 79.6 | 63.5 | 60.4 | 55.7 | 51.5 | 45.6 |
| 174.3 | 156.5 | 138.8 | 121.1 | 103.6 | 86.1 | 68.7 | 65.4 | 60.3 | 55.7 | 49.3 |
| 187.4 | 168.3 | 149.2 | 130.3 | 111.4 | 92.7 | 74 | 70.4 | 64.9 | 60 | 53.1 |
| 200.4 | 180 | 159.7 | 139.5 | 119.3 | 99.2 | 79.2 | 75.4 | 69.5 | 64.2 | 56.8 |
| 213.5 | 191.8 | 170.2 | 148.6 | 127.1 | 105.7 | 84.4 | 80.4 | 74.1 | 68.5 | 60.6 |
| 226.7 | 203.6 | 180.6 | 157.8 | 135 | 112.3 | 89.7 | 85.4 | 78.7 | 72.8 | 64.4 |
| 239.7 | 215.4 | 191.1 | 167 | 142.9 | 118.8 | 94.9 | 90.4 | 83.4 | 77 | 68.1 |

| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|------------------|----------------|----------------|--------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| .165 1 1/64 l | .148 9/64 f | .134 9/64 l | .12 1/8 l | .109 7/64 | .095 3/32 f | .083 5/64 f | .072 5/64 l | .065 1/16 f | .058 1/16 l | .049 3/64 f |
| Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 33.1 | 29.7 | 26.9 | 24 | 21.8 | 19 | 16.6 | 14.4 | 13 | 11.6 | 9.78 |
| 34.8 | 31.2 | 28.3 | 25.3 | 22.9 | 20 | 17.5 | 15.1 | 13.7 | 12.2 | 10.3 |
| 36.6 | 32.8 | 29.7 | 26.5 | 24.1 | 21 | 18.3 | 15.9 | 14.3 | 12.8 | 10.8 |
| 38.3 | 34.3 | 31.1 | 27.8 | 25.2 | 22 | 19.2 | 16.6 | 15 | 13.4 | 11.3 |
| 40 | 35.9 | 32.5 | 29.1 | 26.4 | 23 | 20.1 | 17.4 | 15.7 | 14 | 11.8 |
| 41.8 | 37.4 | 33.9 | 30.3 | 27.5 | 24 | 20.9 | 18.1 | 16.4 | 14.6 | 12.6 |
| 45.2 | 40.5 | 36.7 | 32.8 | 29.8 | 26 | 22.6 | 19.7 | | 3 | 13.4 |
| 48.7 | 43.6 | 39.5 | 35.3 | 32.1 | 28 | 24.4 | 21.2 | | | 14.4 |
| 52.1 | 46.7 | 42.3 | 37.8 | 34.4 | 30 | 26.1 | 22.7 | | | |
| 55.5 | 49.8 | 45.1 | 40.4 | 36.7 | 32 | 27.9 | 24.5 | | | |
| 59 | 52.9 | 48 | 42.9 | 39 | 34 | 29.7 | 25.1 | | | |
| 52.4 | 56 | 50.8 | 45.4 | 41.3 | 36 | 31.4 | 27.2 | | | |

Weight of a Square Foot of Wrought and Cast Iron, Steel, Copper, Lead, Brass, and Zinc Plate
From .0625 to 1 Inch in Thickness.

| Thickness. | Wrought Iron. | Cast Iron. | Steel. | Copper. | Lead. | Brass. | Gun-metal. |
|------------|---------------|------------|--------|---------|--------|--------|------------|
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .0625 | 2.517 | 2.346 | 2.541 | 2.89 | 3.691 | 2.675 | 2.848 |
| .125 | 5.035 | 4.693 | 5.081 | 5.781 | 7.382 | 5.35 | 5.696 |
| .1875 | 7.552 | 7.039 | 7.622 | 8.672 | 11.074 | 8.025 | 8.545 |
| .25 | 10.07 | 9.386 | 10.163 | 11.562 | 14.765 | 10.7 | 11.393 |
| .3125 | 12.588 | 11.733 | 12.703 | 14.453 | 18.456 | 13.375 | 14.241 |
| .375 | 15.106 | 14.079 | 15.244 | 17.344 | 22.148 | 16.05 | 17.089 |
| .4375 | 17.623 | 16.426 | 17.785 | 20.234 | 25.839 | 18.725 | 19.938 |
| .5 | 20.141 | 18.773 | 20.326 | 23.125 | 29.53 | 21.4 | 22.786 |
| .5625 | 22.659 | 21.119 | 22.866 | 26.016 | 33.222 | 24.075 | 25.634 |
| .625 | 25.176 | 23.466 | 25.407 | 28.906 | 36.913 | 26.75 | 28.483 |
| .6875 | 27.694 | 25.812 | 27.948 | 31.797 | 40.604 | 29.425 | 31.331 |
| .75 | 30.211 | 28.159 | 30.488 | 34.688 | 44.296 | 32.1 | 34.179 |
| .8125 | 32.729 | 30.505 | 33.029 | 37.578 | 47.987 | 34.775 | 37.027 |
| .875 | 35.247 | 32.852 | 35.57 | 40.469 | 51.678 | 37.656 | 39.875 |
| .9375 | 37.764 | 35.199 | 38.11 | 43.359 | 55.37 | 39.331 | 42.723 |
| 1 | 40.282 | 37.545 | 40.651 | 46.25 | 59.061 | 42.8 | 45.572 |

From One Twentieth Inch to Two Inches in Thickness.

| Thickness. | Wrought Iron. | Cast Iron. | Steel. | Copper. | Lead. | Brass. | Gun-metal. |
|------------|---------------|------------|--------|---------|---------|--------|------------|
| Inch. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .05 | 2.014 | 1.877 | 2.033 | 2.312 | 2.593 | 2.14 | 2.279 |
| .1 | 4.028 | 3.754 | 4.065 | 4.625 | 5.906 | 4.28 | 4.557 |
| .15 | 6.042 | 5.632 | 6.098 | 6.938 | 8.859 | 6.42 | 6.836 |
| .2 | 8.056 | 7.509 | 8.13 | 9.25 | 11.812 | 8.56 | 9.114 |
| .25 | 10.071 | 9.386 | 10.163 | 11.562 | 14.765 | 10.7 | 11.393 |
| .3 | 12.085 | 11.264 | 12.195 | 13.875 | 17.718 | 12.84 | 13.672 |
| .35 | 14.099 | 13.141 | 14.228 | 16.187 | 20.671 | 14.98 | 15.95 |
| .4 | 16.113 | 15.018 | 16.26 | 18.5 | 23.624 | 17.12 | 18.229 |
| .45 | 18.127 | 16.895 | 18.293 | 20.812 | 26.577 | 19.26 | 20.507 |
| .5 | 20.141 | 18.773 | 20.325 | 23.125 | 29.53 | 21.4 | 22.786 |
| .55 | 22.155 | 20.65 | 22.358 | 25.437 | 32.484 | 23.54 | 25.065 |
| .6 | 24.169 | 22.527 | 24.391 | 27.75 | 35.437 | 25.68 | 27.343 |
| .65 | 26.183 | 24.409 | 26.423 | 30.063 | 38.39 | 27.82 | 29.622 |
| .7 | 28.197 | 26.281 | 28.456 | 32.375 | 41.343 | 29.96 | 31.9 |
| .75 | 30.211 | 28.154 | 30.488 | 34.687 | 44.296 | 32.1 | 34.179 |
| .8 | 32.226 | 30.035 | 32.521 | 37 | 47.249 | 34.24 | 36.458 |
| .85 | 34.24 | 31.912 | 34.553 | 39.312 | 50.202 | 36.38 | 38.736 |
| .9 | 36.254 | 33.79 | 36.586 | 41.625 | 53.154 | 38.52 | 41.015 |
| .95 | 38.268 | 35.668 | 38.628 | 43.937 | 56.108 | 40.66 | 43.293 |
| 1 | 40.282 | 37.545 | 40.651 | 46.25 | 59.061 | 42.8 | 45.572 |
| 1.125 | 45.317 | 42.238 | 45.732 | 52.031 | 66.443 | 48.15 | 51.268 |
| 1.25 | 50.352 | 46.931 | 50.814 | 57.813 | 73.826 | 53.5 | 56.965 |
| 1.5 | 52.87 | 49.278 | 53.354 | 60.703 | 77.517 | 56.17 | 59.813 |
| 1.75 | 55.387 | 51.624 | 55.895 | 63.594 | 81.209 | 58.85 | 62.661 |
| 2 | 57.905 | 53.971 | 58.436 | 66.484 | 84.9 | 61.53 | 65.51 |
| 2.25 | 60.422 | 56.317 | 60.976 | 69.375 | 88.591 | 64.2 | 68.358 |
| 2.5 | 62.94 | 58.663 | 63.517 | 72.266 | 92.283 | 66.88 | 71.206 |
| 2.75 | 65.458 | 61.011 | 66.058 | 75.156 | 95.974 | 69.55 | 74.054 |
| 3 | 67.975 | 63.359 | 68.509 | 78.047 | 99.665 | 72.22 | 76.902 |
| 3.25 | 70.492 | 65.706 | 70.966 | 80.938 | 103.356 | 74.9 | 79.751 |
| 3.5 | 73.009 | 68.053 | 73.483 | 83.829 | 107.047 | 77.57 | 82.600 |
| 3.75 | 75.526 | 70.400 | 75.990 | 86.719 | 110.739 | 80.25 | 85.447 |
| 4 | 78.043 | 72.747 | 78.507 | 89.610 | 114.430 | 82.93 | 88.295 |
| 4.25 | 80.560 | 75.094 | 81.024 | 92.5 | 118.122 | 85.6 | 91.144 |

Standard Cast Iron Water Pipes. (English.)
For a Head of 200 Feet.

| Thickness. | Depth of Socket. | Thickness of Socket. | Packing. | Weight per Yard.* | Lead Joint. | Diameter. | Thickness. | Depth of Socket. | Thickness of Socket. | Packing. | Weight per Yard.* | Lead Joint. |
|------------|------------------|----------------------|----------|-------------------|-------------|-----------|------------|------------------|----------------------|----------|-------------------|-------------|
| Inch. | Ins. | Inch. | Inch. | Lbs. | Lbs. | Ins. | Inch. | Ins. | Inch. | Inch. | Lbs. | Lbs. |
| .3125 | 3.5 | .625 | .25 | 36 | .8 | 8 | .4375 | 3.75 | .625 | .375 | 113 | 3.3 |
| .3125 | 3 | .625 | .25 | 51 | 1.2 | 9 | .4375 | 3.75 | .75 | .375 | 128 | 4.6 |
| .375 | 3 | .625 | .375 | 61 | 2 | 10 | .5 | 4 | .75 | .375 | 168 | 4.9 |
| .375 | 3.75 | .625 | .375 | 75 | 2.7 | 11 | .5 | 4 | .75 | .375 | 175 | 5.3 |
| .375 | 3.75 | .625 | .375 | 85 | 2.9 | 12 | .5625 | 4 | .875 | .375 | 213 | 5.7 |

* Measured as laid.

To Compute Weight of Metal Pipes.

² — d^2 C. D and d representing external and internal diameters in inches, C coefficient.

1st Iron 2.45. Wrought Iron 2.64. Brass 2.82. Copper 3.03. Lead 3.86.

Compute Weight of Metal Tubes and Pipes per Lineal Foot.

From .5 Inch to 6 Inches Internal Diameter.

| 1. | Area of Plate. | Diam. | Area of Plate. | Diam. | Area of Plate. | Diam. | Area of Plate. |
|----|----------------|--------|----------------|-------|----------------|-------|----------------|
| | Sq. Foot. | Ins. | Sq. Foot. | Ins. | Sq. Feet. | Ins. | Sq. Feet. |
| | .1309 | 1.3125 | .3436 | 2.75 | .7199 | 4.5 | 1.1781 |
| 5 | .1473 | 1.375 | .36 | 2.875 | .7526 | 4.625 | 1.2108 |
| | .1636 | 1.4375 | .3764 | 3 | .7854 | 4.75 | 1.2435 |
| 5 | .18 | 1.5 | .3927 | 3.125 | .8181 | 4.875 | 1.2763 |
| | .1964 | 1.625 | .4254 | 3.25 | .8508 | 5 | 1.309 |
| 5 | .2127 | 1.75 | .4581 | 3.375 | .8836 | 5.125 | 1.3417 |
| | .2291 | 1.875 | .4909 | 3.5 | .9163 | 5.25 | 1.3744 |
| 5 | .2454 | 2 | .5236 | 3.625 | .949 | 5.375 | 1.4072 |
| | .2618 | 2.125 | .5563 | 3.75 | .9818 | 5.5 | 1.4399 |
| 5 | .2782 | 2.25 | .587 | 4 | 1.0472 | 5.625 | 1.4726 |
| | .2945 | 2.375 | .6198 | 4.125 | 1.0799 | 5.75 | 1.5053 |
| 5 | .3105 | 2.5 | .6545 | 4.25 | 1.1126 | 5.875 | 1.5381 |
| | .3272 | 2.625 | .6872 | 4.375 | 1.1454 | 6 | 1.5708 |

Application of Table.

When Thickness of Metal is given in Divisions of an Inch.

internal diameter of tube or pipe add thickness of metal; take of the plate in square feet, from table for a diameter equal to of diameter and thickness of tube or pipe, and multiply it by of a square foot of metal for given thickness (see table, page and again by its length in feet.

EXAMPLE.—Required weight of 10 feet of copper tube 1 inch in diameter and an inch in thickness.

$\therefore .125 = 1.125 \times 3.1416 \div 12 = .2945$ square feet for 1 foot of length

of 1 square foot of copper .125th of an inch in thickness, 1.781 lbs.; then, .2945 (from table above) $\times 5.781 \times 10 = 17.02$

When Thickness of Metal is given in Numbers of a Wire

internal diameter of tube or pipe add thickness of 1 in. 120 or 121; multiply sum by 3.1416, divide product will give area of plate in square feet. Then proceed

148 WEIGHT OF IRON AND COPPER PIPES, BOLTS,

ILLUSTRATION.—Required weight of 10 feet of copper pipe 2 inches in diameter and No. 2 American wire-gauge in thickness.

$2 \div .25763 \times 3.1416 \div 12 = 2.25763 \times 3.1416 \div 12 = .591$ square feet; then $\times 11.6706$ (weight from table, page 118) = 6.897 lbs.

Weight of Riveted Iron and Copper Pipes

From 5 to 30 Inches in Diameter.

ONE FOOT IN LENGTH.

| Diameter. | Thickness. | Iron. | Copper. | Diameter. | Thickness. | Iron. |
|-----------|------------|-------|---------|-----------|------------|--------|
| Inch. | Inch. | Lbs. | Lbs. | Inch. | Inch. | Lbs. |
| 5 | .125 | 7.12 | 8.14 | 9 | .25 | 25.01 |
| | .1875 | 10.68 | 12.21 | | .25 | 26.33 |
| | .25 | 14.25 | 16.28 | 10 | .25 | 27.75 |
| 5.5 | .125 | 7.78 | 8.89 | 10.5 | .25 | 29.19 |
| | .1875 | 11.66 | 13.33 | 11 | .25 | 30.49 |
| | .25 | 15.56 | 17.78 | 12 | .25 | 33.13 |
| 6 | .125 | 8.44 | 9.64 | 13 | .25 | 35.88 |
| | .1875 | 12.65 | 14.46 | 14 | .25 | 38.52 |
| | .25 | 16.88 | 19.29 | 15 | .25 | 41.26 |
| 6.5 | .125 | 9.1 | 10.4 | | .3125 | 51.57 |
| | .1875 | 13.65 | 15.6 | 16 | .25 | 43.9 |
| | .25 | 18.2 | 20.8 | | .3125 | 54.87 |
| 7 | .125 | 9.78 | 11.18 | 17 | .25 | 46.53 |
| | .1875 | 14.68 | 16.78 | | .3125 | 58.17 |
| | .25 | 19.57 | 22.37 | 18 | .25 | 49.17 |
| 7.5 | .125 | 10.49 | 11.99 | | .3125 | 61.47 |
| | .1875 | 15.73 | 17.98 | 20 | .3125 | 68.07 |
| | .25 | 20.89 | 23.87 | 24 | .3125 | 81.33 |
| 8 | .1875 | 16.7 | 19.08 | 25 | .3125 | 84.57 |
| | .25 | 22.26 | 25.44 | 28 | .3125 | 94.56 |
| 8.5 | .25 | 23.59 | 26.96 | 30 | .3125 | 101.14 |

Above weights include laps of sheets for riveting and calking.

Weights of the rivets are not added, as number per lineal foot of pipe upon the distance they are placed apart, and their diameter and length upon thickness of metal of the pipe.

Weight of Copper Rods or Bolts,

From .125 Inch to 4 Inches in Diameter.

ONE FOOT IN LENGTH.

| Diameter. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diameter. | Weight. |
|-----------|---------|-----------|---------|-----------|---------|-----------|---------|
| Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. |
| .125 | .047 | .8125 | 1.998 | 1.5 | 6.811 | 2.75 | 2 |
| .1875 | .106 | .875 | 2.318 | .5625 | 7.39 | .875 | 2 |
| .25 | .189 | .9375 | 2.66 | .625 | 7.993 | 3 | 2 |
| .3125 | .296 | 1 | 3.03 | .75 | 9.27 | .125 | 2 |
| .375 | .426 | 1.0625 | 3.42 | .875 | 10.642 | .25 | 3 |
| .4375 | .579 | .125 | 3.831 | 2 | 12.108 | .375 | 3 |
| .5 | .757 | .1875 | 4.269 | .125 | 13.668 | .5 | 3 |
| .5625 | .958 | .25 | 4.723 | .25 | 15.325 | .625 | 3 |
| .625 | 1.182 | .3125 | 5.21 | .375 | 17.075 | .75 | 4 |
| .75 | 1.431 | .375 | 5.723 | .5 | 18.916 | .875 | 4 |
| | 1.703 | .4375 | 6.255 | .625 | 20.856 | 4 | |

Weight of Metals of a Given Sectional Area.

From .1 Square Inch to 10 Square Inches.

PER LINEAL FOOT. (D. K. Clark.)

| | Wrought Iron. 1. | Cast Iron. 1.025 | Steel. 1.02 | Brass. 1.052 | Gun- metal. 1.092 | Sect. Area. | Wrought Iron. 1. | Cast Iron. 1.025 | Steel. 1.02 | Brass. 1.052 | Gun- metal. 1.092 |
|------|------------------------|------------------------|----------------|-----------------|-------------------------|----------------|------------------------|------------------------|----------------|-----------------|-------------------------|
| | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Sq. Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| .33 | .31 | .34 | .35 | .36 | .36 | 5.1 | 17 | 15.9 | 17.3 | 17.9 | 18.6 |
| .67 | .62 | .68 | .7 | .73 | .73 | 5.2 | 17.3 | 16.3 | 17.7 | 18.2 | 18.9 |
| 1 | .94 | 1.02 | 1.05 | 1.09 | 1.09 | 5.3 | 17.7 | 16.6 | 18 | 18.6 | 19.3 |
| 1.33 | 1.25 | 1.36 | 1.43 | 1.46 | 1.46 | 5.4 | 18 | 16.9 | 18.4 | 18.9 | 19.7 |
| 1.67 | 1.56 | 1.7 | 1.75 | 1.82 | 1.82 | 5.5 | 18.3 | 17.2 | 18.7 | 19.3 | 20 |
| 2 | 1.88 | 2.04 | 2.11 | 2.18 | 2.18 | 5.6 | 18.7 | 17.5 | 19 | 19.6 | 20.4 |
| 2.33 | 2.19 | 2.38 | 2.46 | 2.55 | 2.55 | 5.7 | 19 | 17.8 | 19.4 | 20 | 20.8 |
| 2.67 | 2.5 | 2.72 | 2.81 | 2.91 | 2.91 | 5.8 | 19.3 | 18.1 | 19.7 | 20.3 | 21.1 |
| 3 | 2.81 | 3.06 | 3.16 | 3.28 | 3.28 | 5.9 | 19.7 | 18.4 | 20.1 | 20.7 | 21.5 |
| 3.33 | 3.15 | 3.4 | 3.51 | 3.64 | 3.64 | 6 | 20 | 18.8 | 20.4 | 21 | 21.8 |
| 3.67 | 3.44 | 3.74 | 3.86 | 4 | 4 | 6.1 | 20.3 | 19.1 | 20.7 | 21.4 | 22.2 |
| 4 | 3.75 | 4.08 | 4.21 | 4.37 | 4.37 | 6.2 | 20.7 | 19.4 | 21.1 | 21.7 | 22.6 |
| 4.33 | 4.06 | 4.42 | 4.56 | 4.73 | 4.73 | 6.3 | 21 | 19.7 | 21.4 | 22.1 | 22.9 |
| 4.67 | 4.38 | 4.76 | 4.91 | 5.1 | 5.1 | 6.4 | 21.3 | 20 | 21.8 | 22.4 | 23.3 |
| 5 | 4.69 | 5.1 | 5.26 | 5.46 | 5.46 | 6.5 | 21.7 | 20.3 | 22.1 | 22.8 | 23.7 |
| 5.33 | 5 | 5.44 | 5.61 | 5.82 | 5.82 | 6.6 | 22 | 20.6 | 22.4 | 23.1 | 24 |
| 5.67 | 5.31 | 5.78 | 5.96 | 6.19 | 6.19 | 6.7 | 22.3 | 20.9 | 22.8 | 23.5 | 24.4 |
| 6 | 5.63 | 6.12 | 6.31 | 6.55 | 6.55 | 6.8 | 22.7 | 21.3 | 23.1 | 23.9 | 24.8 |
| 6.33 | 5.94 | 6.46 | 6.66 | 6.92 | 6.92 | 6.9 | 23 | 21.6 | 23.5 | 24.2 | 25.1 |
| 6.67 | 6.25 | 6.8 | 7.01 | 7.28 | 7.28 | 7 | 23.3 | 21.9 | 23.8 | 24.6 | 25.5 |
| 7 | 6.56 | 7.14 | 7.36 | 7.64 | 7.64 | 7.1 | 23.7 | 22.2 | 24.1 | 24.9 | 25.8 |
| 7.33 | 6.88 | 7.48 | 7.72 | 8.01 | 8.01 | 7.2 | 24 | 22.5 | 24.5 | 25.3 | 26.2 |
| 7.67 | 7.19 | 7.82 | 8.07 | 8.37 | 8.37 | 7.3 | 24.3 | 22.8 | 24.8 | 25.6 | 26.6 |
| 8 | 7.5 | 8.16 | 8.42 | 8.74 | 8.74 | 7.4 | 24.7 | 23.1 | 25.2 | 26 | 26.9 |
| 8.33 | 7.81 | 8.5 | 8.77 | 9.1 | 9.1 | 7.5 | 25 | 23.4 | 25.5 | 26.3 | 27.3 |
| 8.67 | 8.13 | 8.84 | 9.12 | 9.46 | 9.46 | 7.6 | 25.3 | 23.8 | 25.9 | 26.7 | 27.7 |
| 9 | 8.44 | 9.18 | 9.47 | 9.83 | 9.83 | 7.7 | 25.7 | 24.1 | 26.2 | 27 | 28 |
| 9.33 | 8.75 | 9.52 | 9.82 | 10.2 | 10.2 | 7.8 | 26 | 24.4 | 26.5 | 27.4 | 28.4 |
| 9.67 | 9.06 | 9.86 | 10.2 | 10.6 | 10.6 | 7.9 | 26.3 | 24.7 | 26.9 | 27.7 | 28.8 |
| 10 | 9.38 | 10.2 | 10.5 | 10.9 | 10.9 | 8 | 26.7 | 25 | 27.2 | 28.1 | 29.1 |
| 10.3 | 9.69 | 10.5 | 10.9 | 11.3 | 11.3 | 8.1 | 27 | 25.3 | 27.5 | 28.4 | 29.5 |
| 10.7 | 10 | 10.9 | 11.2 | 11.7 | 11.7 | 8.2 | 27.3 | 25.6 | 27.9 | 28.8 | 29.9 |
| 11 | 10.3 | 11.2 | 11.6 | 12 | 12 | 8.3 | 27.7 | 25.9 | 28.2 | 29.1 | 30.2 |
| 11.3 | 10.6 | 11.6 | 11.9 | 12.4 | 12.4 | 8.4 | 28 | 26.3 | 28.6 | 29.5 | 30.6 |
| 11.7 | 10.9 | 11.9 | 12.3 | 12.7 | 12.7 | 8.5 | 28.3 | 26.6 | 28.9 | 29.8 | 30.9 |
| 12 | 11.3 | 12.2 | 12.6 | 13.1 | 13.1 | 8.6 | 28.7 | 26.9 | 29.2 | 30.2 | 31.3 |
| 12.3 | 11.6 | 12.6 | 13 | 13.5 | 13.5 | 8.7 | 29 | 27.2 | 29.6 | 30.5 | 31.7 |
| 12.7 | 11.9 | 12.9 | 13.3 | 13.8 | 13.8 | 8.8 | 29.3 | 27.5 | 29.9 | 30.9 | 32 |
| 13 | 12.2 | 13.3 | 13.7 | 14.2 | 14.2 | 8.9 | 29.7 | 27.8 | 30.3 | 31.2 | 32.4 |
| 13.3 | 12.5 | 13.6 | 14 | 14.6 | 14.6 | 9 | 30 | 28.1 | 30.6 | 31.6 | 32.8 |
| 13.7 | 12.8 | 13.9 | 14.4 | 14.9 | 14.9 | 9.1 | 30.3 | 28.4 | 30.9 | 31.9 | 33.1 |
| 14 | 13.1 | 14.3 | 14.7 | 15.3 | 15.3 | 9.2 | 30.7 | 28.8 | 31.3 | 32.3 | 33.5 |
| 14.3 | 13.4 | 14.6 | 15.1 | 15.7 | 15.7 | 9.3 | 31 | 29.1 | 31.6 | 32.6 | 33.9 |
| 14.7 | 13.8 | 15 | 15.4 | 16 | 16 | 9.4 | 31.3 | 29.4 | 32 | 33 | 34.2 |
| 15 | 14.1 | 15.3 | 15.8 | 16.4 | 16.4 | 9.5 | 31.7 | 29.7 | 32.3 | 33.3 | 34.6 |
| 15.3 | 14.4 | 15.6 | 16.1 | 16.7 | 16.7 | 9.6 | 32 | 30 | | | |
| 15.7 | 14.7 | 16 | 16.5 | 17.1 | 17.1 | 9.7 | 32.3 | 30.3 | | | |
| 16 | 15 | 16.3 | 16.8 | 17.5 | 17.5 | 9.8 | 32.7 | 30.6 | | | |
| 16.3 | 15.3 | 16.7 | 17.2 | 17.8 | 17.8 | 9.9 | 33 | 30.9 | | | |
| 16.7 | 15.6 | 17 | 17.5 | 18.2 | 18.2 | 10 | 33.3 | 31.2 | | | |

150 LEAD PIPES.—COPPER PIPES AND COCKS.

Weight of Lead Pipe. (*English.*)

ONE FOOT IN LENGTH.

| Diam. | Thick- ness. | Weight. | Diam. | Thick- ness. | Weight. | Diam. | Thick- ness. | Weight. | Diam. | Thick- ness. |
|-------|-----------------|---------|-------|-----------------|---------|-------|-----------------|---------|-------|-----------------|
| Inch. | Inch. | Lbs. | Inch. | Inch. | Lbs. | Inch. | Inch. | Lbs. | Inch. | Inch. |
| .5 | .097 | .93 | 1 | .136 | 2.4 | 1.75 | .166 | 5 | 3 | .275 |
| | .112 | 1.07 | | .156 | 2.8 | | .199 | 6 | 3.5 | .225 |
| | .124 | 1.2 | | .2 | 3.73 | | .228 | 7 | | .273 |
| | .146 | 1.47 | | .225 | 4.27 | | .256 | 8 | 4 | .257 |
| .625 | .089 | 1 | 1.25 | .139 | 3 | 2 | .178 | 6 | | .3125 |
| | .101 | 1.13 | | .16 | 3.5 | | .204 | 7 | | .327 |
| | .121 | 1.4 | | .18 | 4 | | .231 | 8 | 4.25 | .3125 |
| | .14 | 2 | | .193 | 4.33 | | .266 | 9.33 | 4.5 | .232 |
| .75 | .112 | 1.6 | 1.5 | .156 | 4 | 2.5 | .2 | 8.4 | | .295 |
| | .147 | 1.87 | | .179 | 4.67 | | .227 | 9.6 | | .3125 |
| | .181 | 2.13 | | .224 | 6 | | .261 | 11.2 | 4.75 | .3125 |
| | .215 | 2.4 | | .257 | 7 | 3 | .218 | 11.2 | 5 | .3125 |

Dimensions of Copper Pipes and Composite Cocks.

From 1 Inch to 23 Inches in Diameter.

| Diam. of Pipe and Cock. | Flange Diameter. | | Thick- ness. | Bolts. | | Diam. of Pipe and Cock. | Flange Diam. Pipe. | Thick- ness. | B No. |
|----------------------------------|------------------|-------|-----------------|--------|-------|----------------------------------|--------------------------|-----------------|----------|
| | Pipe. | Cock. | | No. | Diam. | | | | |
| Inch. | Inch. | Inch. | Inch. | No. | Inch. | Inch. | Inch. | Inch. | No. |
| 1 | 3.375 | 3.5 | .375 | 3 | .5 | 9 | 12.75 | .625 | 9 |
| 1.25 | 3.625 | 3.75 | .375 | 3 | .5 | 9.25 | 13.125 | .625 | 10 |
| 1.5 | 3.875 | 4.25 | .375 | 3 | .5 | 9.5 | 13.375 | .6875 | 10 |
| 1.75 | 4.125 | 4.375 | .4375 | 4 | .5 | 9.75 | 13.625 | .6875 | 10 |
| 2 | 4.375 | 4.75 | .4375 | 4 | .5 | 10 | 13.875 | .6875 | 10 |
| 2.25 | 4.625 | 5.25 | .4375 | 5 | .5 | 10.5 | 14.5 | .6875 | 10 |
| 2.5 | 4.875 | 5.5 | .4375 | 5 | .5 | 11 | 15 | .6875 | 10 |
| 2.75 | 5.25 | 5.75 | .4375 | 5 | .5 | 11.5 | 15.625 | .75 | 10 |
| 3 | 6 | 6.25 | .5 | 5 | .625 | 12 | 16.125 | .75 | 10 |
| 3.25 | 6.125 | 6.625 | .5 | 5 | .625 | 12.5 | 16.625 | .75 | 10 |
| 3.5 | 6.375 | 6.875 | .5 | 6 | .625 | 13 | 17.25 | .75 | 10 |
| 3.75 | 6.625 | 7.25 | .5 | 6 | .625 | 13.5 | 17.875 | .75 | 10 |
| 4 | 6.875 | 7.375 | .5 | 6 | .625 | 14 | 18.375 | .75 | 10 |
| 4.25 | 7.125 | 7.625 | .5 | 6 | .625 | 14.5 | 18.875 | .75 | 10 |
| 4.5 | 7.375 | 8.25 | .5 | 6 | .625 | 15 | 19.5 | .75 | 10 |
| 4.75 | 7.625 | 8.5 | .5 | 6 | .625 | 15.5 | 20 | .75 | 10 |
| 5 | 8 | 9 | .5 | 6 | .625 | 16 | 20.5 | .75 | 10 |
| 5.25 | 8.25 | 9.25 | .5 | 6 | .625 | 16.5 | 21.125 | .75 | 10 |
| 5.5 | 8.5 | 9.5 | .5 | 6 | .625 | 17 | 21.625 | .75 | 11 |
| 5.75 | 9 | 9.875 | .5 | 6 | .625 | 17.5 | 22.125 | .75 | 11 |
| 6 | 9.25 | | .625 | 8 | .625 | 18 | 22.75 | .75 | 11 |
| 6.25 | 9.75 | | .625 | 8 | .625 | 18.5 | 23.25 | .75 | 11 |
| 6.5 | 10 | | .625 | 8 | .625 | 19 | 23.75 | .75 | 12 |
| 6.75 | 10 | | .625 | 8 | .625 | 19.5 | 24.375 | .75 | 12 |
| 7 | 10.5 | | .625 | 8 | .625 | 20 | 24.875 | .75 | 12 |
| 7.25 | 10.75 | | .625 | 8 | .625 | 20.5 | 25.375 | .75 | 13 |
| | | | .625 | 8 | .625 | 21 | 26 | .75 | 13 |
| | | | .625 | 8 | .625 | 21.5 | 26.5 | .75 | 13 |
| | | | .625 | 9 | .625 | 22 | 27 | .75 | 13 |
| | | | .625 | 9 | .625 | 22.5 | 27.625 | .75 | 14 |
| | | | .625 | 9 | .625 | 23 | 28.125 | .75 | 14 |

Weight of Sheet Lead.

PER SQUARE FOOT.

| Thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. |
|------------|---------|------------|---------|------------|---------|------------|---------|
| Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. |
| .017 | 1 | .068 | 4 | .118 | 7 | .169 | 10 |
| .034 | 2 | .085 | 5 | .135 | 8 | .186 | 11 |
| .051 | 3 | .101 | 6 | .152 | 9 | .203 | 12 |

Weight of Tin Pipe.

ONE FOOT IN LENGTH.

| Inch. | THICKNESS. | | Diam. External. | THICKNESS. | | Diam. External. | THICKNESS. | Diam. External. | THICKNESS. |
|-------|----------------------|---------------------|-----------------|----------------------|---------------------|-----------------|----------------------|-----------------|------------|
| | $\frac{3}{32}$ Inch. | $\frac{1}{8}$ Inch. | | $\frac{3}{32}$ Inch. | $\frac{1}{8}$ Inch. | | $\frac{3}{32}$ Inch. | | |
| | Lbs. | Lbs. | Inch. | Lbs. | Lbs. | Inch. | Lbs. | Lbs. | Lbs. |
| | .148 | — | 1.25 | 1.095 | 1.417 | 2.25 | 5.04 | 3.25 | 7.56 |
| | .384 | .472 | 1.5 | 1.328 | 1.732 | 2.5 | 5.67 | 3.5 | 8.19 |
| | .62 | .787 | 1.75 | 1.564 | 2.047 | 2.75 | 6.3 | 3.75 | 8.82 |
| | .856 | 1.103 | 2 | 1.802 | 2.362 | 3 | 6.93 | 4 | 9.45 |

Weight of Lead Encased Tin Pipes.

| Inch. | Light Weights. | | | For Supply of Water Head.* | | |
|-------|----------------|------|------|----------------------------|-----------------|------------------|
| | Lbs. | Lbs. | Lbs. | 50 feet and under. | 51 to 250 feet. | 251 to 500 feet. |
| 5 | 1 | 1.5 | 2 | 2.5 to 4 | 3 to 4.5 | 3.5 to 5 |
| | 2 | 2.5 | 3 | 3.5 " 5 | 4 " 6 | 4.5 " 7 |
| 5 | 3 | 3.5 | 4 | 4.5 " 7 | 5.25 " 8 | 6 " 9 |
| | 3.5 | 4 | 4.5 | 5.5 " 8 | 6 " 9 | 7 " 10 |
| | 4.5 | 5 | 5.5 | 7.25 " 10 | 8 " 11 | 9 " 12 |
| | 6.5 | 7 | 8 | 9 " 12.5 | 10 " 14 | 12 " 16 |
| | 8 | 9 | 10 | 11 " 16 | 12.5 " 18 | 14 " 21 |
| | 11 | 13 | — | 16 " 23 | 18.5 " 26 | 21 " 30 |

* The extreme weights are for extra heavy pipe with less proportion of tin.

Dimensions and Weight of Sheet Zinc. (Vielles-Montagne.)

PER SQUARE FOOT.

| Thickness. | | 2X.5 metres ; area, 1 square metre. | | 2X.65 metres ; area, 1.3 sq. metres. | | 2X.8 metres ; area, 1.6 sq. metres. | | Weight. |
|------------|-------|--|-------|--|-------|---|-------|---------|
| | | 6.56X1.64 feet ; area, 10.76 square feet. | | 6.56X2.13 feet ; area, 13.99 square feet. | | 6.56X2.62 ft. ; area, 17.22 square feet. | | |
| Millim. | Inch. | Kilom. | Lbs. | Kilom. | Lbs. | Kilom. | Lbs. | Lbs. |
| .41 | .0161 | 2.9 | 6.39 | 3.7 | 8.16 | 4.6 | 10.14 | .589 |
| .51 | .0201 | 3.45 | 7.61 | 4.45 | 9.81 | 5.5 | 12.12 | .704 |
| .6 | .0236 | 4.05 | 8.93 | 5.3 | 11.68 | 6.5 | 14.33 | .832 |
| .69 | .0272 | 4.65 | 10.25 | 6.1 | 13.45 | 7.5 | 16.53 | .96 |
| .78 | .0307 | 5.3 | 11.68 | 6.9 | 15.21 | 8.5 | 18.74 | 1.088 |
| .87 | .0343 | 5.95 | 13.12 | 7.7 | 16.94 | 9.5 | 20.94 | 1.216 |
| .96 | .0378 | 6.55 | 14.44 | 8.55 | 18.85 | 10.5 | 23.15 | 1.344 |
| 1 | .0433 | 7.5 | 16.53 | 9.75 | 21.5 | 12 | 26.46 | 1.536 |
| .23 | .0485 | 8.45 | 18.63 | 10.95 | 24.14 | 13.5 | 29.97 | 1.74 |
| .36 | .0536 | 9.35 | 20.61 | 12.2 | 26.9 | 15 | 33.07 | 1.92 |
| .48 | .0583 | 10.3 | 22.71 | 13.4 | 29.54 | 16.5 | 36.38 | 2.112 |
| .6 | .0654 | 11.25 | 24.8 | 14.6 | 32.19 | 18 | 39.60 | 2.304 |
| .75 | .0729 | 12.5 | 27.56 | 16.25 | 35.82 | 20 | 44 | |
| .9 | .0795 | 13.75 | 30.31 | 17.9 | 39.46 | 22 | 48 | |
| 1.0 | .0862 | 15 | 33.07 | 19.5 | 42.99 | 24 | 52 | |
| 1.2 | .0933 | 16.25 | 35.82 | 21.1 | 46.52 | 26 | 56 | |
| 1.5 | .0992 | 17.5 | 38.58 | 22.75 | 50.15 | 28 | 60 | |
| 1.8 | .1047 | 18.8 | 41.44 | 24.4 | 53.79 | 31 | 66 | |

152 WEIGHT OF SHEET ZINC.—SPIKES, HORSESHOES.

Table—(Continued).
Special Sizes for Sheathing Ships.

| No. | Thickness. | | Dimensions of Sheets. | | | | | | Weight per Sq. Foot. |
|---------|------------|--------|---|--------|--|-------|-------|--|----------------------------|
| | | | 1.15 X .35 metres; area, .402 sq. metre. | | 1.3 X .4 metres; area, .52 sq. metre. | | | | |
| | | | 3.77 X 1.15 feet; area, 4.33 sq. feet. | | 4.26 X 1.31 feet; area, 5.6 sq. feet. | | | | |
| Millim. | Inch. | Kilom. | Lbs. | Kilom. | Lbs. | Lbs. | | | |
| 15 | .96 | .0378 | 2.65 | 5.84 | 3.4 | 7.5 | 1.344 | | |
| 16 | 1.1 | .0433 | 3 | 6.61 | 3.9 | 8.6 | 1.530 | | |
| 17 | 1.23 | .0485 | 3.4 | 7.5 | 4.4 | 9.7 | 1.74 | | |
| 18 | 1.36 | .0536 | 3.75 | 8.27 | 4.9 | 10.8 | 1.92 | | |
| 19 | 1.48 | .0583 | 4.15 | 9.15 | 5.35 | 11.79 | 2.112 | | |
| 20 | 1.66 | .0654 | 4.55 | 10.03 | 5.85 | 12.9 | 2.304 | | |

NOTE.—A deviation of 25 dekagrammes, or about half a pound, more or less, from the proper weight of each number of sheet, is allowed.

Nos. 1 to 9 are employed for perforated articles, as sleeves, and for articles de Paris. Nos. 10 to 12 are used in manufacture of lamps, lanterns, and tin-ware generally, and for stamped ornaments. The last numbers are used for lining reservoirs, and for baths and pumps.

Ship and Railroad Spikes.

DIMENSIONS AND NUMBER PER POUND. (F. C. Page, Mass.)

Ship Spikes.

| $\frac{1}{8}$ In. Sq. | $\frac{3}{16}$ In. Sq. | $\frac{1}{2}$ In. Sq. | $\frac{3}{4}$ In. Sq. | $\frac{1}{2}$ In. Sq. | $\frac{3}{4}$ In. Sq. | $\frac{1}{2}$ In. Sq. | $\frac{3}{4}$ In. Sq. |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. | Length. No. in Pound. |
| Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. |
| 3 19 | 3 10 | 4 5.4 | 5 3.4 | 6 2.2 | 8 1.4 | 10 .8 | |
| 3.5 15.8 | 3.5 9.6 | 4.5 4.5 | 5.5 3.1 | 6.5 2 | 9 1.2 | 15 .6 | |
| 4 13.2 | 4 8 | 5 4.6 | 6 3 | 7 1.9 | 10 1.1 | — | |
| 4.5 12.2 | 4.5 6 | 5.5 4.2 | 6.5 2.8 | 7.5 1.8 | 11 1 | — | |
| 5 10.2 | 5 5.8 | 6 4 | 7 2.6 | 8 1.7 | — | — | |
| — | 6 5.2 | 6.5 3.2 | 7.5 2.4 | 8.5 1.6 | — | — | |
| — | — | — | 8 2.2 | 9 1.5 | — | — | |
| — | — | — | — | 10 1.4 | — | — | |

Railroad Spikes 5 inch square X 5.5 ins. 2 per lb.
" " 5.625 " " X 5.5 " 1.6 "

Spikes and Horseshoes.

LENGTH AND NUMBER PER POUND. (H. Burden, Troy, N. Y.)

| Length. | Boat Spikes. | | | Ship Spikes. | | | | Hook Head. | | Horseshoe | |
|---------|---------------|---------|---------------|--------------|---------------|---------|---------------|-------------|---------------|-----------|---------------|
| | No. in Lb. | Length. | No. in Lb. | Length. | No. in Lb. | Length. | No. in Lb. | Length. | No. in Lb. | Length. | No. in Lb. |
| Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. |
| 3 | 17.5 | 6.5 | 4.78 | 4 | 8 | 7.5 | 2.5 | 4 X .375 | 5.55 | 1 | .8 |
| 3.5 | 14.68 | 7 | 3.62 | 4.5 | 6.5 | 8 | 1.74 | 4.5 X .4375 | 4.14 | 2 | .7 |
| 4 | 12.57 | 7.5 | 3.37 | 5 | 4.37 | 8.5 | 1.63 | 5 X .5 | 2.52 | 3 | .6 |
| | | 8 | 2.95 | 5.5 | 4.3 | 9 | 1.55 | 5.5 X .5 | 2.41 | 4 | .5 |
| | | 8.5 | 2.9 | 6 | 4.2 | 10 | 1.15 | 5.5 X .5625 | 1.87 | 5 | .3 |
| | | 9 | 2.1 | 6.5 | 3.77 | — | — | 6 X .5625 | 1.72 | — | — |
| | 9 | 1.98 | 7 | 2.75 | — | — | — | 6 X .625 | 1.38 | — | — |

Weight and Volume of Cast Iron and Lead Balls.

From 1 Inch to 20 Inches in Diameter.

| r. | Volume. | Cast Iron. | Lead. | Diameter. | Volume. | Cast Iron. | Lead. |
|----|-----------|------------|---------|-----------|-----------|------------|----------|
| | Cube Ins. | Lbs. | Lbs. | Ins. | Cube Ins. | Lbs. | Lbs. |
| | .523 | .136 | .215 | 9 | 381.703 | 99.51 | 156.553 |
| | 1.767 | .461 | .725 | 9.5 | 448.92 | 117.034 | 184.121 |
| | 4.189 | 1.092 | 1.718 | 10 | 523.599 | 136.502 | 214.749 |
| | 8.181 | 2.133 | 3.355 | 10.5 | 606.132 | 158.043 | 248.587 |
| | 14.137 | 3.685 | 5.798 | 11 | 696.91 | 181.765 | 285.832 |
| | 22.449 | 5.852 | 9.207 | 11.5 | 796.33 | 207.635 | 326.591 |
| | 33.51 | 8.736 | 13.744 | 12 | 904.778 | 235.876 | 371.096 |
| | 47.713 | 12.439 | 19.569 | 12.5 | 1022.656 | 266.647 | 419.512 |
| | 65.45 | 17.063 | 26.843 | 13 | 1150.346 | 299.623 | 471.806 |
| | 87.114 | 22.721 | 35.729 | 14 | 1436.754 | 374.563 | 589.273 |
| | 113.097 | 29.484 | 46.385 | 15 | 1767.145 | 460.696 | 724.781 |
| | 143.793 | 37.453 | 58.976 | 16 | 2144.66 | 559.114 | 879.616 |
| | 179.594 | 46.82 | 73.659 | 17 | 2572.44 | 670.717 | 1055.066 |
| | 220.893 | 57.587 | 90.598 | 18 | 3053.627 | 796.082 | 1252.422 |
| | 268.082 | 69.889 | 109.952 | 19 | 3591.363 | 936.271 | 1472.97 |
| | 321.555 | 83.84 | 131.883 | 20 | 4188.79 | 1092.02 | 1717.995 |

E.—To compute weight of balls of other metals, multiply weight given in following multipliers:

| | | | |
|-------------------|--------|----------------|--------|
| Wrought Iron..... | 1.067. | Brass..... | 1.12. |
| Steel..... | 1.088. | Gun-metal..... | 1.165. |

Weight and Diameter of Cast Iron Balls.

| Diameter. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diameter. |
|-----------|---------|-----------|---------|-----------|---------|-----------|---------|-----------|
| Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. |
| 1.94 | 12 | 4.45 | 50 | 7.16 | 224 | 11.8 | 1344 | 21.44 |
| 2.45 | 14 | 4.68 | 56 | 7.43 | 336 | 13.51 | 1568 | 22.57 |
| 2.8 | 16 | 4.89 | 60 | 7.6 | 448 | 14.87 | 1792 | 23.6 |
| 3.08 | 18 | 5.09 | 70 | 8.01 | 560 | 16.02 | 2016 | 24.54 |
| 3.32 | 20 | 5.27 | 80 | 8.37 | 672 | 17.02 | 2240 | 25.42 |
| 3.53 | 25 | 5.68 | 90 | 8.71 | 784 | 17.91 | 2800 | 27.38 |
| 3.72 | 28 | 5.9 | 100 | 9.02 | 896 | 18.73 | 3360 | 29.1 |
| 3.89 | 30 | 6.04 | 112 | 9.37 | 1008 | 19.48 | 3920 | 30.64 |
| 4.04 | 40 | 6.64 | 168 | 10.72 | 1120 | 20.17 | 4480 | 32.03 |

Length of Horseshoe Nails.

By Numbers.

| | | |
|----------------|-----------------------|----------------------|
| 1.5 Ins. | No. 7..... 1.875 Ins. | No. 9..... 2.25 Ins. |
| 1.75 " | " 8..... 2 " | " 10..... 2.5 " |

Weights of Iron Nails, and Number in a

| No. | Size. | L'gth. | No. | Size. | L'gth. | No. | Size. | L'gth. | No. | Size. |
|-----|-------|--------|-----|-------|--------|-----|-------|--------|-----|-------|
| | | Ins. | | | Ins. | | | Ins. | | |
| 420 | 5d. | 1.75 | 220 | 8d. | 2.5 | 100 | 12d. | 3.25 | 52 | 30c |
| 270 | 6 | 2 | 175 | 10 | 3 | 65 | 20 | 3.5 | 28 | 40 |

Wrought Iron Cut Nails, Tacks, Spike*(Cumberland Nail and Iron Co.)**Lengths and Number per Lb.*

| Ordinary. | | | Finishing. | | | Shin | |
|---------------------|---------|-------------|----------------|---------|-------------|----------------|---------|
| Size. | Length. | No. per Lb. | Size. | Length. | No. per Lb. | Size. | Length. |
| | In. | | | In. | | | In. |
| 2 ^d | .875 | 716 | 4 ^d | 1.375 | 384 | 5 ^d | 1.75 |
| 3 ^d fine | 1.0625 | 588 | 5 | 1.75 | 256 | 8 | 2.5 |
| 3 | 1.0625 | 448 | 6 | 2 | 204 | 9 | 2.75 |
| 4 | 1.375 | 336 | 8 | 2.5 | 102 | 10 | 3 |
| 5 | 1.75 | 216 | 10 | 3 | 80 | | Tac. |
| 6 | 2 | 166 | 12 | 3.625 | 65 | 1 02. | .125 |
| 7 | 2.25 | 118 | 20 | 3.875 | 46 | 1.5 | .187 |
| 8 | 2.5 | 94 | | Core. | | 2 | .25 |
| 10 | 2.75 | 72 | 6 ^d | 2 | 143 | 2.5 | .312 |
| 12 | 3.5 | 50 | 8 | 2.5 | 68 | 3 | .375 |
| 20 | 3.75 | 32 | 10 | 2.333 | 60 | 4 | .437 |
| 30 | 4.25 | 20 | 12 | 3.125 | 42 | 6 | .562 |
| 40 | 4.75 | 17 | 20 | 3.75 | 25 | 8 | .625 |
| 50 | 5 | 14 | 30 | 4.25 | 18 | 10 | .687 |
| 60 | 5.5 | 10 | 40 | 4.75 | 14 | 12 | .75 |
| | Light. | | WH | 2.5 | 69 | 14 | .812 |
| 4 ^d | 1.375 | 373 | WHL | 2.25 | 72 | 16 | .875 |
| 5 | 1.75 | 272 | | Clinoh. | | 18 | .937 |
| 6 | 2 | 196 | 6 ^d | 2 | 152 | 20 | 1 |
| | Brads. | | 7 | 2.25 | 133 | | Bo. |
| 6 ^d | 2 | 163 | 8 | 2.5 | 92 | | Size. |
| 8 | 2.5 | 96 | 10 | 2.75 | 72 | | In. |
| 10 | 2.75 | 74 | — | 3 | 60 | | 1.5 |
| 12 | 3.125 | 50 | — | 3.25 | 43 | | Spil |
| | Fence. | | | Slate. | | | 3.5 |
| 6 ^d | 2 | 96 | 3 ^d | 1.625 | 288 | | 4 |
| 7 | 2.25 | 66 | 4 | 1.4375 | 244 | | 4.5 |
| 8 | 2.5 | 56 | 5 | 1.75 | 187 | | 5 |
| 10 | 2.75 | 50 | 6 | 2 | 146 | | 5.5 |
| — | 3 | 40 | | | | | 6 |

Railroad Spikes.*Number in a Keg of 150 lbs.*

| Length. | No. | Length. | No. | Length. | No. | Len |
|------------|-----|-------------|-----|----------|-----|-------|
| No. | | In. | | In. | | In |
| 3 X .375 | 930 | 3.5 X .4375 | 675 | 4 X .5 | 450 | 5 X |
| 3.5 X .375 | 890 | 4 X .4375 | 540 | 4.5 X .5 | 400 | 5.5 X |
| 4 X .375 | 760 | 4.5 X .4375 | 510 | 5 X .5 | 340 | |

5.5 X .5625 standard for a gauge of 4 feet 8.5 ins.

Ship and Boat Spikes.*Number in a Keg of 150 lbs.*

| Length. | No. | Length. | No. | Length. | No. | Len |
|-----------|------|-----------|-----|-----------|-----|-------|
| In. | | In. | | In. | | In |
| 4 X .25 | 1650 | 5 X .3125 | 930 | 8 X .375 | 455 | 10 X. |
| 4.5 X .25 | 1464 | 6 X .3125 | 868 | 9 X .375 | 424 | 8 X. |
| 5 X .25 | 1380 | 7 X .3125 | 662 | 10 X .375 | 390 | 9 X. |
| 6 X .25 | 1292 | 6 X .375 | 570 | 8 X .4375 | 384 | 10 X |
| 7 X .25 | 1161 | 7 X .375 | 482 | 9 X .4375 | 300 | 11 |

Weight of Various Metals.

Per Cube Inch and Foot.

| TALS. | Spec. Gravity. | W'ght in an Inch. | Ins. in a Lb. | Weight in a Foot. | METALS. | Specific Gravity. | W'ght in an Inch. | Ins. in a Lb. | Weight in a Foot. |
|-----------------|----------------|-------------------|---------------|-------------------|----------------|-------------------|-------------------|---------------|-------------------|
| | | Lb. | | Lbs. | | | Lb. | | Lb. |
| ght-iron plates | 7734 | .2797 | 3.57 | 483.38 | Brass, rolled. | 8.217 | .2972 | 3.37 | 513.6 |
| wire. | 7774 | .2812 | 3.55 | 485.87 | " cast.. | 8.080 | .2922 | 3.42 | 505 |
| ron.... | 7209 | .2607 | 3.84 | 450.54 | Lead, rolled. | 11.340 | .4101 | 2.44 | 708.73 |
| plates.. | 7804 | .2823 | 3.54 | 487.8 | Tin, cast.... | 7.292 | .2673 | 3.74 | 462 |
| wire... | 7847 | .2838 | 3.52 | 490.45 | Zinc, rolled.. | 7.188 | .26 | 3.85 | 449.28 |
| r. {... | 8697 | .3146 | 3.19 | 543.6 | Alumini- } | 2.560 | .0926 | 10.8 | 160 |
| ed {... | 8880 | .3212 | 3.11 | 555 | um, cast } | 10.480 | .3791 | 2.64 | 655 |
| metal, } | 8750 | .3165 | 3.16 | 546.875 | Silver..... | 8.379 | .3031 | 3.299 | 523.69 |
| } | | | | | Tobin Bronze. | | | | |

English. (D. K. Clark.)

| | | | | | | | | | |
|----------|-------|------|------|-------|---------------|--------|------|------|-----|
| ght iron | 7.698 | .278 | 3.6 | 480 | Tin | 7.409 | .268 | 3.74 | 462 |
| ron.... | 7.217 | .26 | 3.84 | 450 | Zinc | 7.008 | .253 | 3.95 | 437 |
| | 7.852 | .283 | 3.53 | 489.6 | Lead..... | 11.418 | .412 | 2.43 | 712 |
| r plates | 8.805 | .318 | 3.15 | 549 | Brass, cast.. | 8.099 | .292 | 3.42 | 505 |
| metal... | 8.404 | .304 | 2.02 | 524 | " wire.. | 8.548 | .308 | 3.24 | 533 |

WROUGHT AND CAST IRON.

To Compute Weight of Wrought or Cast Iron.

1.—Ascertain number of cube inches in piece; multiply sum by .2816* for ht iron and .2607* for cast, and product will give weight in pounds.

or cast iron multiply weight of pattern, if of pine, by from 18 to 20, according to degree of dryness.

W.P.L.—What is weight of a cube of wrought iron 10 inches square by 15 in length?

$$10 \times 10 \times 15 \times .2816 = 422.4 \text{ lbs.}$$

COPPER.

To Compute Weight of Copper.

1.—Ascertain number of cube inches in piece; multiply sum by .32118,* product will give weight in pounds.

Sheathing and Braziers' Sheets.

Dimensions and weights see Measures and Weights, pages 118-121, 131, 142.

LEAD.

To Compute Weight of Lead.

—Ascertain number of cube inches in piece; multiply sum by .41015,* product will give weight in pounds.

P.L.E.—What is weight of a leaden pipe 12 feet long, 3.75 inches in diameter, 1/2 inch thick?

Rule in Mensuration of Surfaces, to ascertain Area of Cylindrical Rings.

$$\text{Area of } (3.75 + 1 + 1) = 25.967$$

$$\text{" " } 3.75 = 11.044$$

$$\text{Difference, } 14.923 \text{ (area of ring)} \times 144 \text{ (12 feet)} = 2148.9$$

$$; = 881.376 \text{ lbs.}$$

BRASS.

Compute Weight of Ordinary Brass Castings.

—Ascertain number of cube inches in piece; multiply sum by .2816* and product will give weight in pounds.

Weights of a cube inch as here given are for the ordinary metals the metal under consideration is accurately known, the weight for the units here given.

156 DIMENSIONS AND WEIGHTS OF BOLTS AND NUTS.

Dimensions and Weights of Wrought Iron Bolts and Nuts.

SQUARE AND HEXAGONAL HEADS AND NUTS.
 Rough, and from .25 Inch to 4 Inches in Diameter.
 Square Head and Nut.

| Diameter of Bolt. | Width. | | Diagonal. | | Depth. | | Weight. | | Threads per inch. |
|-------------------|--------|------|-----------|------|--------|-------|---------------|----------------|-------------------|
| | Head. | Nut. | Head. | Nut. | Head. | Nut. | Head and Nut. | Bolt per inch. | |
| In. | In. | In. | In. | In. | In. | In. | Lbs. | Lbs. | No. |
| .25 | .36 | .49 | .51 | .69 | .25 | .25 | .024 | .014 | 20 |
| .3125 | .45 | .58 | .64 | .82 | .3 | .3125 | .043 | .022 | 18 |
| .375 | .54 | .67 | .76 | .95 | .34 | .375 | .068 | .031 | 16 |
| .4375 | .63 | .76 | .89 | 1.07 | .4 | .4375 | .104 | .042 | 14 |
| .5 | .72 | .84 | 1.02 | 1.19 | .44 | .5 | .145 | .055 | 13 |
| .5625 | .82 | .94 | 1.16 | 1.33 | .48 | .5625 | .204 | .07 | 12 |
| .625 | .91 | 1.03 | 1.29 | 1.46 | .53 | .625 | .273 | .086 | 11 |
| .6875 | 1 | 1.12 | 1.41 | 1.58 | .58 | .6875 | .356 | .104 | 11 |
| .75 | 1.09 | 1.21 | 1.54 | 1.71 | .63 | .75 | .454 | .124 | 10 |
| .8125 | 1.18 | 1.3 | 1.67 | 1.84 | .67 | .8125 | .565 | .145 | 10 |
| .875 | 1.27 | 1.39 | 1.8 | 1.96 | .72 | .875 | .666 | .168 | 9 |
| 1 | 1.45 | 1.57 | 2.05 | 2.22 | .81 | 1 | 1.013 | .22 | 8 |
| 1.125 | 1.63 | 1.75 | 2.3 | 2.47 | .9 | 1.125 | 1.416 | .278 | 7 |
| 1.25 | 1.81 | 1.94 | 2.56 | 2.74 | 1 | 1.25 | 1.923 | .344 | 7 |
| 1.375 | 1.99 | 2.12 | 2.81 | 3 | 1.1 | 1.375 | 2.543 | .416 | 6 |
| 1.5 | 2.17 | 2.3 | 3.07 | 3.25 | 1.18 | 1.5 | 3.234 | .495 | 6 |
| 1.625 | 2.36 | 2.48 | 3.34 | 3.51 | 1.28 | 1.625 | 4.105 | .581 | 5.5 |
| 1.75 | 2.54 | 2.66 | 3.59 | 3.76 | 1.37 | 1.75 | 5.087 | .674 | 5 |
| 1.875 | 2.72 | 2.84 | 3.85 | 4.02 | 1.46 | 1.875 | 6.182 | .773 | 5 |
| 2 | 2.9 | 3.02 | 4.1 | 4.27 | 1.56 | 2 | 7.491 | .88 | 4.5 |
| 2.125 | 3.08 | 3.21 | 4.35 | 4.54 | 1.65 | 2.125 | 8.936 | .993 | 4.5 |
| 2.25 | 3.26 | 3.39 | 4.61 | 4.79 | 1.75 | 2.25 | 10.543 | 1.113 | 4.5 |
| 2.375 | 3.44 | 3.57 | 4.86 | 5.05 | 1.84 | 2.375 | 12.335 | 1.24 | 4.375 |
| 2.5 | 3.62 | 3.75 | 5.12 | 5.3 | 1.94 | 2.5 | 14.359 | 1.375 | 4.25 |
| 2.625 | 3.81 | 3.93 | 5.49 | 5.56 | 2.03 | 2.625 | 16.549 | 1.515 | 4 |
| 2.75 | 3.99 | 4.11 | 5.64 | 5.81 | 2.12 | 2.75 | 18.897 | 1.663 | 4 |
| 2.875 | 4.17 | 4.29 | 5.9 | 6.07 | 2.22 | 2.875 | 21.545 | 1.818 | 3.75 |
| 3 | 4.35 | 4.47 | 6.15 | 6.32 | 2.31 | 3 | 24.464 | 1.979 | 3.5 |
| 3.25 | 4.71 | 4.84 | 6.66 | 6.84 | 2.5 | 3.25 | 30.922 | 2.323 | 3.5 |
| 3.5 | 5.07 | 5.2 | 7.17 | 7.35 | 2.68 | 3.5 | 38.391 | 2.694 | 3.25 |
| 3.75 | 5.44 | 5.56 | 7.69 | 7.86 | 2.87 | 3.75 | 47.168 | 3.093 | 3 |
| 4 | 5.8 | 5.92 | 8.2 | 8.37 | 3.06 | 4 | 56.882 | 3.518 | 3 |

FINISHED.—Deduct .0625 from diameters of bolts and depths of all heads and nuts.

Screws with square threads have but one half number of threads of those with triangular threads.

NOTE.—The loss of tensile strength of a bolt by cutting of thread is, for one of 1.25 ins. diameter, 8 per cent. The safe stress or capacity of a wrought iron bolt and nut may be taken at 5000 lbs. per square inch.

Preceding width, depth, etc., are for work to exact dimensions, whether forged or finished.

To Compute Weight of a Bolt and Nut.

—Ascertain from table weight of head and nut for given diameter; and add thereto weight of bolt per inch of its length, multiplied by length of its body from inside of its head to end.

—Length of a bolt and nut for measurement, as such, is taken from inside of nut, or its greatest capacity when in position.

ILLUSTRATION.—A wrought iron bolt and nut with a square head and nut is 1 inch diameter and 10 inches in length; what is its weight?

Weight of head and nut.....1.013 lbs.
 " bolt per inch of length .22 $\times 10 = 2.2$ "
 3.213 "

Hexagonal Head and Nut.

| Diameter in. | Width. | | Diagonal. | | Depth. | | Weight. | | Threads per Inch. |
|-----------------|--------|--------|-----------|------|--------|--------|------------------|-------------------|----------------------|
| | Head. | Nut. | Head. | Nut. | Head. | Nut. | Head and Nut. | Bolt per Inch. | |
| | In. | In. | In. | In. | In. | In. | Lbs. | Lbs. | No. |
| 5 | .375 | .5 | .43 | .58 | .25 | .25 | .022 | .014 | 20 |
| 125 | .4375 | .5625 | .5 | .65 | .3 | .3125 | .037 | .022 | 18 |
| 15 | .5625 | .6875 | .65 | .79 | .34 | .375 | .062 | .031 | 16 |
| 175 | .625 | .75 | .72 | .87 | .4 | .4375 | .094 | .042 | 14 |
| 2 | .75 | .875 | .87 | 1 | .44 | .5 | .134 | .055 | 13 |
| 225 | .8125 | .9375 | .94 | 1.08 | .48 | .5625 | .18 | .07 | 12 |
| 25 | .9375 | 1.0625 | 1.08 | 1.23 | .53 | .625 | .249 | .086 | 11 |
| 275 | 1 | 1.125 | 1.16 | 1.3 | .58 | .6875 | .318 | .104 | 11 |
| 3 | 1.125 | 1.25 | 1.3 | 1.44 | .63 | .75 | .413 | .124 | 10 |
| 125 | 1.25 | 1.375 | 1.44 | 1.59 | .67 | .8125 | .522 | .145 | 10 |
| 375 | 1.3125 | 1.4375 | 1.52 | 1.66 | .72 | .875 | .639 | .168 | 9 |
| 4 | 1.5 | 1.625 | 1.73 | 1.88 | .81 | 1 | .931 | .22 | 8 |
| 25 | 1.6875 | 1.8125 | 1.95 | 2.09 | .9 | 1.125 | 1.299 | .278 | 7 |
| 5 | 1.875 | 2 | 2.17 | 2.31 | 1 | 1.25 | 1.759 | .344 | 7 |
| 75 | 2 | 2.1875 | 2.31 | 2.53 | 1.1 | 1.375 | 2.263 | .416 | 6 |
| | 2.25 | 2.375 | 2.6 | 2.74 | 1.18 | 1.5 | 2.958 | .495 | 6 |
| 15 | 2.4375 | 2.5625 | 2.81 | 2.96 | 1.28 | 1.625 | 3.741 | .581 | 5.5 |
| 1 | 2.625 | 2.75 | 3.03 | 3.18 | 1.37 | 1.75 | 4.654 | .674 | 5 |
| 3 | 2.8125 | 2.9375 | 3.25 | 3.39 | 1.46 | 1.875 | 5.675 | .773 | 5 |
| | 3 | 3.125 | 3.46 | 3.61 | 1.56 | 2 | 6.854 | .88 | 4.5 |
| 5 | 3.1875 | 3.3125 | 3.68 | 3.83 | 1.65 | 2.125 | 8.163 | .993 | 4.5 |
| | 3.375 | 3.5 | 3.9 | 4.04 | 1.75 | 2.25 | 9.658 | 1.113 | 4.5 |
| 5 | 3.5625 | 3.6875 | 4.11 | 4.26 | 1.84 | 2.375 | 11.263 | 1.24 | 4.375 |
| | 3.75 | 3.875 | 4.33 | 4.47 | 1.94 | 2.5 | 13.149 | 1.375 | 4.25 |
| 5 | 3.9375 | 4.0625 | 4.55 | 4.69 | 2.03 | 2.625 | 15.15 | 1.515 | 4 |
| | 4.125 | 4.25 | 4.77 | 4.91 | 2.12 | 2.75 | 17.285 | 1.663 | 4 |
| 5 | 4.3125 | 4.4375 | 4.99 | 5.12 | 2.22 | 2.875 | 19.751 | 1.818 | 3.75 |
| | 4.5 | 4.625 | 5.2 | 5.34 | 2.31 | 3 | 22.378 | 1.979 | 3.5 |
| | 4.875 | 5 | 5.63 | 5.77 | 2.5 | 3.25 | 28.258 | 2.323 | 3.5 |
| | 5.25 | 5.375 | 6.06 | 6.21 | 2.68 | 3.5 | 35.081 | 2.694 | 3.25 |
| | 5.625 | 5.75 | 6.5 | 6.64 | 2.87 | 3.75 | 43.178 | 3.093 | 3 |
| 6 | 6.125 | 6.93 | 7.07 | 3.06 | 4 | 51.942 | 3.518 | 3 | |

FINISHED.—Deduct .0625 from diameters of bolts and depths of all heads and nuts.

For Wood or Carpentry.

Head and Nut (Square), 1.75 diameter of bolt. *Depth of Head,* .75, 1 1/4, 1 1/2.

Washer.—Thickness, .35 to .4 of diameter of bolt, on *Pine* 3.5 diameter 2.5.

English.

Washworth gives following elements of Thread of Bolts:

Angle of thread, 55°. *Depth of thread* = Pitch of screw.

Number of threads per Inch.—*Square,* half number of inches.

Angle of thread.—*.64* pitch for angular and *.475* for a

158 DIMENSIONS AND WEIGHTS OF BOLTS AND NUTS.

French Standard Bolts and Nuts. (Armengaud's.)

HEXAGONAL HEADS AND NUTS.

| <i>Equilateral Triangular Thread.</i> | | | | | | | | <i>Square Thread.</i> | | | | | |
|---------------------------------------|--------------------|-------------------|-------|------------|-----------------------|----------------------|-------|-----------------------|------------------|-------------------|------|----------------------|-------|
| Diameter | | Thickness. | | Thickness. | | Safe Tensile Stress. | | Diameter of Bolt. | | Thickness of Nut. | | Safe Tensile Stress. | |
| of Bolt. | at Base of Thread. | Threads per Inch. | Head. | Nut. | Breadth across Flats. | | | | Depth of Thread. | Threads per Inch. | | | |
| Mm. | Ins. | Ins. | No. | Ins. | Ins. | Ins. | Lbs. | Mm. | Ins. | Ins. | No. | Ins. | Lbs. |
| 5 | .2 | .13 | 18.1 | .24 | .2 | .55 | 44 | 20 | .79 | .072 | 6.57 | 1.82 | 717 |
| 7.5 | .3 | .22 | 16 | .3 | .3 | .68 | 99 | 25 | .98 | .081 | 5.97 | 2.01 | 1142 |
| 10 | .39 | .31 | 14.1 | .38 | .39 | .88 | 178 | 30 | 1.18 | .093 | 5.4 | 2.22 | 1635 |
| 12.5 | .49 | .39 | 12.7 | .44 | .49 | 1.04 | 277 | 35 | 1.38 | .1 | 4.93 | 2.41 | 2218 |
| 15 | .59 | .48 | 11.5 | .52 | .59 | 1.2 | 400 | 40 | 1.57 | .106 | 4.53 | 2.63 | 2912 |
| 17.5 | .69 | .58 | 10.6 | .58 | .69 | 1.4 | 545 | 45 | 1.77 | .114 | 4.2 | 2.85 | 3674 |
| 20 | .79 | .66 | 9.8 | .66 | .79 | 1.5 | 713 | 50 | 1.97 | .128 | 3.91 | 3.07 | 4547 |
| 22.5 | .89 | .76 | 9.1 | .72 | .89 | 1.68 | 902 | 55 | 2.17 | .13 | 3.65 | 3.3 | 5288 |
| 25 | .98 | .84 | 8.5 | .8 | .98 | 1.84 | 1120 | 60 | 2.36 | .14 | 3.43 | 3.5 | 6540 |
| 30 | 1.18 | 1.02 | 7.5 | .94 | 1.18 | 2.16 | 1635 | 65 | 2.56 | .15 | 3.23 | 3.7 | 7660 |
| 35 | 1.38 | 1.2 | 6.7 | 1.08 | 1.38 | 2.48 | 2218 | 70 | 2.76 | .158 | 3.06 | 3.92 | 8893 |
| 40 | 1.58 | 1.4 | 6 | 1.22 | 1.58 | 2.8 | 2912 | 75 | 2.95 | .166 | 2.92 | 4.13 | 10214 |
| 45 | 1.77 | 1.56 | 5.5 | 1.36 | 1.77 | 3.2 | 3674 | 80 | 3.15 | .174 | 2.76 | 4.36 | 11603 |
| 50 | 1.97 | 1.74 | 5.1 | 1.5 | 1.97 | 3.44 | 4547 | 85 | 3.35 | .183 | 2.63 | 4.58 | 13100 |
| 55 | 2.17 | 1.92 | 4.7 | 1.64 | 2.17 | 3.76 | 5288 | 90 | 3.54 | .192 | 2.51 | 4.78 | 14794 |
| 60 | 2.36 | 2.08 | 4.4 | 1.74 | 2.36 | 4.08 | 6540 | 95 | 3.74 | .2 | 2.41 | 5 | 16352 |
| 65 | 2.56 | 2.26 | 4.1 | 1.92 | 2.56 | 4.4 | 7660 | 100 | 3.94 | .209 | 2.31 | 5.22 | 18144 |
| 70 | 2.76 | 2.44 | 3.8 | 2.06 | 2.76 | 4.7 | 8893 | 105 | 4.13 | .22 | 2.22 | 5.43 | 20000 |
| 75 | 2.95 | 2.6 | 3.5 | 2.2 | 2.95 | 5 | 10214 | 110 | 4.33 | .226 | 2.13 | 5.66 | 21950 |
| 80 | 3.15 | 2.78 | 3.4 | 2.34 | 3.15 | 5.35 | 11468 | 115 | 4.53 | .23 | 2.06 | 5.87 | 23990 |

English Bolts and Nuts. (Whitworth's.)

Hexagonal Heads and Nuts, and Triangular Threads.

| Diameter. | | | Depth. | | | Width of Head and Nut. | | | Diameter. | | | Depth. | | | Width of Head and Nut. | | |
|-----------|-----------------|-------------------|--------|-------|-------|------------------------|-----------------|-------------------|-----------|-------|-------|--------|-----------------|-------------------|------------------------|------|------|
| Bolt. | Base of Thread. | Threads per Inch. | Head. | Nut. | | Bolt. | Base of Thread. | Threads per Inch. | Head. | Nut. | | Bolt. | Base of Thread. | Threads per Inch. | Head. | Nut. | |
| Ins. | Inch. | No. | Inch. | Ins. | Ins. | Ins. | Ins. | No. | Ins. | Ins. | Ins. | Ins. | Ins. | No. | Ins. | Ins. | Ins. |
| .125 | .093 | 40 | .109 | .125 | .338 | 1.25 | 1.067 | 7 | 1.004 | 1.25 | 2.048 | | | | | | |
| .1875 | .134 | 24 | .164 | .1875 | .448 | 1.375 | 1.161 | 6 | 1.203 | 1.375 | 2.215 | | | | | | |
| .2187 | — | 24 | — | — | — | 1.5 | 1.286 | 6 | 1.312 | 1.5 | 2.413 | | | | | | |
| .25 | .186 | 20 | .219 | .25 | .525 | 1.625 | 1.369 | 5 | 1.422 | 1.625 | 2.576 | | | | | | |
| .3125 | .241 | 18 | .273 | .3125 | .601 | 1.75 | 1.494 | 5 | 1.531 | 1.75 | 2.758 | | | | | | |
| .375 | .295 | 16 | .328 | .375 | .709 | 1.875 | 1.59 | 4.5 | 1.641 | 1.875 | 3.018 | | | | | | |
| .4375 | .346 | 14 | .383 | .4375 | .82 | 2 | 1.715 | 4.5 | 1.75 | 2 | 3.149 | | | | | | |
| .5 | .393 | 12 | .437 | .5 | .919 | 2.125 | 1.84 | 4.5 | 1.859 | 2.125 | 3.337 | | | | | | |
| .5625 | .456 | 12 | .492 | .5625 | 1.011 | 2.25 | 1.93 | 4 | 1.969 | 2.25 | 3.546 | | | | | | |
| .625 | .508 | — | .547 | .625 | 1.101 | 2.375 | 2.055 | 4 | 2.078 | 2.375 | 3.75 | | | | | | |
| .6875 | .571 | — | .601 | .6875 | 1.201 | 2.5 | 2.18 | 4 | 2.187 | 2.5 | 3.894 | | | | | | |
| .75 | .622 | 10 | .656 | .75 | 1.301 | 2.625 | 2.305 | 4 | 2.297 | 2.625 | 4.049 | | | | | | |
| .875 | .684 | 10 | .711 | .875 | 1.39 | 2.75 | 2.384 | 3.5 | 2.406 | 2.75 | 4.181 | | | | | | |
| 1 | .73 | 9 | .766 | 1 | 1.479 | 2.875 | 2.509 | 3.5 | 2.516 | 2.875 | 4.346 | | | | | | |
| 1.125 | .783 | 8 | .82 | 1.125 | 1.574 | 3 | 2.634 | 3.5 | 2.625 | 3 | 4.531 | | | | | | |
| 1.25 | .833 | 7 | .866 | 1.25 | 1.67 | 3.25 | 2.84 | 3.25 | — | — | — | | | | | | |
| 1.375 | .883 | 6 | .916 | 1.375 | 1.86 | 3.5 | 3.06 | 3.25 | — | — | — | | | | | | |

Square Heads and Nuts. (Whitworth's.)

| Diameter. | | | Diameter. | | | Diameter. | | |
|-----------|-----------------|-------------------|-----------|-----------------|-------------------|-----------|-----------------|-------------------|
| lt. | Base of Thread. | Threads per Inch. | Bolt. | Base of Thread. | Threads per Inch. | Bolt. | Base of Thread. | Threads per Inch. |
| a. | Ins. | No. | Ins. | Ins. | No. | Ins. | Ins. | No. |
| 75 | 3.25 | 3 | 4.5 | 3.875 | 2.875 | 5.25 | 4.4375 | 2.625 |
| | 3.5 | 3 | 4.75 | 4.0625 | 2.75 | 5.5 | 4.625 | 2.625 |
| 15 | 3.75 | 2.875 | 5 | 4.25 | 2.75 | 6 | 4.875 | 2.5 |

Weight of Heads and Nuts in Lbs. (Molesworth.)

Hexagonal, $1.07 D^3$. Square, $1.35^3 D^3$. D representing diameter of bolt in inches.

tentiveness of Wrought Iron Spikes and Nails.

Deduced from Experiments of Johnson and Bean.

SPIKES.

| SPIKE. | WOOD. | Breadth. | Depth. | Depth of Insertion. | Force required to draw it. | Ratio of force to weight. | REMARKS. |
|-----------------|-------------|----------|--------|---------------------|----------------------------|---------------------------|-------------------|
| | | Ins. | Ins. | Ins. | Lbs. | | |
| re. | Hemlock† | .39 | .3 | 3.5 | 1297 | 1.58 | Seasoned in part. |
| * | Chestnut | .37 | .38 | 3.5 | 1873 | 2.16 | Unseasoned. |
| * | Yellow pine | .375 | .375 | 3.375 | 2052 | 2.37 | Seasoned. |
| * | White oak | .375 | .375 | 3.375 | 3910 | 4.52 | " |
| | Locust | .4 | .4 | 3.5 | 5967 | 6.33 | " |
| narrow.. | Chestnut | .39 | .25 | 3.5 | 2223 | 3.93 | Unseasoned. |
| " | White oak | .39 | .25 | 3.5 | 3990 | 7.05 | Seasoned. |
| " | Locust | .39 | .25 | 3.5 | 5673 | 9.32 | " |
| broad.. | Chestnut | .539 | .288 | 3.5 | 2394 | 2.66 | Unseasoned. |
| " | White oak | .539 | .288 | 3.5 | 5330 | 5.71 | Seasoned. |
| " | Locust | .539 | .288 | 3.5 | 7040 | 7.84 | " |
| e } Draw filed. | Hemlock† | .4 | .39 | 3.5 | 1638 | 1.75 | Seasoned in part. |
| | Chestnut† | .4 | .39 | 3.5 | 1790 | 1.81 | Unseasoned. |
| | Locust† | .4 | .39 | 3.5 | 3990 | 4.17 | Seasoned in part. |
| l and } rved.. | Ash | Diam. .5 | | 3.5 | 2052 | 2.21 | Seasoned. |
| " | " | " | .5 | 3.5 | 2451 | 2.41 | " |
| " | White oak | " | .48 | 3.5 | 3876 | 3.2 | " |

Burden's patent.

† Soaked in water after the spikes were driven.

NAILS.

| | | Force required to draw it. | | | | | | Pressure required to force them into Pine. |
|----|---------|----------------------------|-------|----------|------|------|--------|--|
| | Length. | Depth of Insertion. | Pine. | Hemlock. | Elm. | Oak. | Beech. | |
| | Ins. | Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| ny | 2 | 1 | 187 | 312 | 327 | 507 | 667 | ~ ~ ~ |
| | 2 | 1.5 | 327 | 539 | 571 | 675 | 88 | |
| | 2 | 2 | 530 | 857 | 899 | 1394 | 18 | |

General Remarks.

At a given breadth of face, a decrease of depth will soft woods, a blunt-pointed spike forces the fibres out so as to leave the fibres longitudinally in contact.

160 ANGLES AND DISTANCES.—DISTANCES AND ANGLES.

To obtain greatest effect, fibres of the wood should press faces of the spike in direction of their length; thus, a round blunt bolt, driven into a hole of a less diameter, has a retention equal to that of any other form, when wholly driven, as without boring.

The retention of a spike, whether square or flat, in unseasoned chestnut, from two to four inches in length of insertion, is about 800 lbs. per square inch of the two surfaces which laterally compress the faces of the spike.

When wood was soaked in water, after spikes were driven, order of their retentive power was Locust, White oak, Chestnut, Hemlock, and Yellow Pine.

Gas Pipe Threads.

| | | | | | | | | | | |
|------------------------|------|-----|------|----|-----|----|------|-----|------|----|
| Diameter in Inches. . | .125 | .25 | .375 | .5 | .75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| Threads per Inch . . . | 28 | 19 | 19 | 14 | 14 | 11 | 11 | 11 | 11 | 11 |

ANGLES AND DISTANCES.

Angles and Distances corresponding to Opening of a Rule of Two Feet.

| Angle. | Distance. | Angle. | Distance. | Angle. | Distance. | Angle. | Distance. | Angle. | Distance. |
|--------|-----------|--------|-----------|--------|-----------|--------|-----------|--------|-----------|
| 0 | Ins. | 0 | Ins. | 0 | Ins. | 0 | Ins. | 0 | Ins. |
| 1 | .2 | 19 | 3.96 | 37 | 7.61 | 55 | 11.08 | 73 | 14.28 |
| 2 | .42 | 20 | 4.17 | 38 | 7.81 | 56 | 11.27 | 74 | 14.44 |
| 3 | .63 | 21 | 4.37 | 39 | 8.01 | 57 | 11.45 | 75 | 14.61 |
| 4 | .84 | 22 | 4.58 | 40 | 8.2 | 58 | 11.64 | 76 | 14.78 |
| 5 | 1.05 | 23 | 4.78 | 41 | 8.4 | 59 | 11.82 | 77 | 14.94 |
| 6 | 1.26 | 24 | 4.99 | 42 | 8.6 | 60 | 12 | 78 | 15.11 |
| 7 | 1.47 | 25 | 5.19 | 43 | 8.8 | 61 | 12.18 | 79 | 15.27 |
| 8 | 1.67 | 26 | 5.4 | 44 | 8.99 | 62 | 12.36 | 80 | 15.43 |
| 9 | 1.88 | 27 | 5.6 | 45 | 9.18 | 63 | 12.54 | 81 | 15.59 |
| 10 | 2.09 | 28 | 5.81 | 46 | 9.38 | 64 | 12.72 | 82 | 15.75 |
| 11 | 2.3 | 29 | 6.01 | 47 | 9.57 | 65 | 12.9 | 83 | 15.9 |
| 12 | 2.51 | 30 | 6.21 | 48 | 9.76 | 66 | 13.07 | 84 | 16.06 |
| 13 | 2.72 | 31 | 6.41 | 49 | 9.95 | 67 | 13.25 | 85 | 16.21 |
| 14 | 2.92 | 32 | 6.62 | 50 | 10.14 | 68 | 13.42 | 86 | 16.37 |
| 15 | 3.13 | 33 | 6.82 | 51 | 10.33 | 69 | 13.59 | 87 | 16.52 |
| 16 | 3.34 | 34 | 7.02 | 52 | 10.52 | 70 | 13.77 | 88 | 16.67 |
| 17 | 3.55 | 35 | 7.22 | 53 | 10.71 | 71 | 13.94 | 89 | 16.82 |
| 18 | 3.75 | 36 | 7.42 | 54 | 10.9 | 72 | 14.11 | 90 | 16.97 |

Distances and Angles corresponding to Opening of a Rule of Two Feet.

| Distance. | Angle. | Distance. | Angle. | Distance. | Angle. | Distance. | Angle. | Distance. | Angle. |
|-----------|--------|-----------|--------|-----------|--------|-----------|--------|-----------|--------|
| Ins. | 0 | Ins. | 0 | Ins. | 0 | Ins. | 0 | Ins. | 0 |
| .25 | 1.12 | 3 | 14.22 | 6.5 | 31.26 | 10 | 49.14 | 13.5 | 68.28 |
| .375 | 1.48 | 3.25 | 15.34 | 6.75 | 32.4 | 10.25 | 50.34 | 13.75 | 69.54 |
| .5 | 2.24 | 3.5 | 16.46 | 7 | 33.54 | 10.5 | 51.54 | 14 | 71.22 |
| .625 | 2.59 | 3.75 | 17.58 | 7.25 | 35.09 | 10.75 | 53.14 | 14.25 | 72.5 |
| | 3.35 | 4 | 19.11 | 7.5 | 36.24 | 11 | 54.34 | 14.5 | 74.2 |
| | 4.12 | 4.25 | 20.24 | 7.75 | 37.4 | 11.25 | 55.54 | 14.75 | 75.5 |
| | 4.48 | 4.5 | 21.37 | 8 | 38.56 | 11.5 | 57.16 | 15 | 77.22 |
| | 5.58 | 4.75 | 22.5 | 8.25 | 40.12 | 11.75 | 58.38 | 15.25 | 78.5 |
| | .1 | 5 | 24.4 | 8.5 | 41.28 | 12 | 60 | 15.5 | 80.22 |
| | .22 | 5.25 | 25.16 | 8.75 | 42.46 | 12.25 | 61.23 | 15.75 | 81.5 |
| | | | 26.3 | 9 | 44.2 | 12.5 | 62.46 | 16 | 83.22 |
| | | | 27.44 | 9.25 | 45.2 | 12.75 | 64.1 | 16.25 | 84.5 |
| | | | 28.58 | 9.5 | 46.38 | 13 | 65.36 | 16.5 | 86.22 |
| | | | 29.72 | 9.75 | 47.56 | 13.25 | 67.02 | 16.75 | 87.5 |

WIRE ROPE.

Wire rope will run over sheaves of like diameter to *Hemp rope* of same length; but larger sheaves reduce wear. Adhesion is the same as that of *hemp rope*. Wear increases rapidly with speed. Short bends should be avoided. In substituting wire rope for hemp, allow same weight per foot. *Wire rope* materially damages and often destroys it.

For transmission of power, wire rope can be used up to distances of 3 miles. For distances less than 100 feet, it is not advised for long transmission; sheaves are placed at intervals, dividing it into a number of shorter ones of 250 to 300 feet.

Strength per square inch of section of rope is about 50 per cent. of an equal section of solid metal of same strength per square inch.

Stationary wire ropes should be kept well painted or tarred to prevent oxidation. Running ropes should always be well lubricated and protected from grit with linseed-oil, pine tar, graphite grease, or any similar acid substances.

Standard wire rope is made of 6 strands of 7, 12, or 19 wires each, with a wire centre. Wire centre adds 10 per cent. to strength and weight, but reduces its flexibility proportionally.

The working load for standing ropes is about one fourth ultimate strength, for running ropes it is from one fifth to one seventh.

Ropes for hoisting are composed of 6 strands of 19 wires each around a centre.

Ropes for transmission of power, for guys and rigging, are composed of strands of 7 or 12 wires each.

Ultimate strength of wires of which wire ropes are made are for: *Wire*, 70 000 to 90 000 lbs. per sq. inch; *Bessemer steel wire*, 100 000 to 120 000 lbs.; *Crucible cast-steel wire*, 150 000 to 180 000 lbs., and *Special cast-steel wire*, 210 000 to 300 000 lbs.

Flat ropes can be made of 4, 6, 8, etc., strands of varied construction. Ropes are also made flat, composed of several strands alternately laid to right and left, laid alongside each other, and sewed together with iron wire.

Hawsers of steel are made of 6 strands of 12 wires each with hemp around a common hemp centre, and are as flexible as hemp hawsers of equal strength.

Galvanized wire rope replaces hemp for rigging, because of its cheapness, and resistance to stretch. It is one fifth bulk for equal strength, and offers less surface to wind.

Ropes for vessel-steering gear are made of 6 small ropes, each small rope composed of 6 strands of 7 wires—252 wires in all in the rope, giving great flexibility.

Rigging of galvanized cast-steel rope is one third to one fourth the weight of *hemp rope* of equal strength.

Elements of Running and Standing Wire Rope.

John A. Roebling's Sons Co., New York.

HOISTING ROPE.

19 Wires in a Strand. Hemp Centre.

IRON.

| Diam. | Circum. | Weight per Foot. | Ultimate Strength. | Safe Load .2 | Circum. Equiv- alent to New Hemp Rope. | Min. Size Sheave. | Ultimate Strength. | Safe Load .2 | Circum. Equiv- alent to New Hemp Rope. | Min. Size Sheave. |
|-------|---------|------------------------|-----------------------|--------------------|---|-------------------------|-----------------------|--------------------|---|-------------------------|
| Inch. | Inch. | Lbs. | Lbs. | Lbs. | Inch. | Ft. | Lbs. | Lbs. | Inch. | Ft. |
| .375 | 1.178 | .26 | 5 000 | 1 000 | 2.5 | 1.5 | 9 000 | 1 800 | 3.5 | 1.5 |
| .5 | 1.708 | .35 | 6 960 | 1 392 | 3 | 2.25 | 14 000 | 2 800 | 4.5 | 2.25 |
| .625 | 1.964 | .6 | 10 260 | 2 052 | 3.75 | 3.5 | 24 000 | 4 800 | 5.75 | 3 |
| .75 | 2.256 | .88 | 17 280 | 3 456 | 4.75 | 4 | 36 000 | 7 200 | 7 | 3.5 |
| .875 | 2.749 | 1.2 | 23 000 | 4 600 | 5.5 | 4.5 | 50 000 | 10 000 | 8.5 | 4.5 |
| 1 | 3.142 | 1.58 | 32 000 | 6 400 | 6.5 | 5.25 | 66 000 | 13 200 | 9.5 | 5 |
| 1.125 | 3.534 | 2 | 40 000 | 8 000 | 7.5 | 6 | 84 000 | 16 800 | 11 | 5.5 |
| 1.25 | 3.927 | 2.5 | 54 000 | 10 800 | 8.5 | 6.5 | 104 000 | 20 800 | 12 | 6 |
| 1.375 | 4.32 | 3 | 66 000 | 13 200 | 9.5 | 7 | 126 000 | 25 200 | 13 | 6.5 |
| 1.5 | 4.712 | 3.65 | 78 000 | 15 600 | 10 | 7.5 | 154 000 | 30 800 | 14 | 7 |
| 1.625 | 5.105 | 4.1 | 88 000 | 17 600 | 11 | 8.5 | 176 000 | 35 200 | 15 | 7.5 |
| 1.75 | 5.498 | 5.25 | 108 000 | 21 600 | 12 | 10 | 212 000 | 42 400 | — | 8 |
| 2 | 6.283 | 6.3 | 130 000 | 26 000 | 13 | 12 | 250 000 | 50 000 | — | 8.5 |
| 2.25 | 7.069 | 8 | 148 000 | 29 600 | 14 | 13 | 310 000 | 62 000 | — | — |

Transmission and Standing Rope.

7 Wires in a Strand. Hemp Centre.

IRON.

| Diam. | Circum. | Weight per Foot. | Ultimate Strength. | Safe Load .2 | Circum. Equiv- alent to New Hemp Rope. | Min. Size Sheave. | Ultimate Strength. | Safe Load .2 | Circum. Equiv- alent to New Hemp Rope. | Min. Size Sheave. |
|-------|---------|------------------------|-----------------------|--------------------|---|-------------------------|-----------------------|--------------------|---|-------------------------|
| Inch. | Inch. | Lbs. | Lbs. | Lbs. | Inch. | Ft. | Lbs. | Lbs. | Inch. | Ft. |
| .375 | 1.178 | .19 | 3 300 | 660 | 2.25 | 2.75 | 8 000 | 1 600 | 3.25 | 2 |
| .5 | 1.571 | .31 | 5 660 | 1 132 | 2.75 | 4 | 12 000 | 2 400 | 4 | 2.5 |
| .625 | 1.964 | .57 | 11 600 | 2 320 | 4 | 5.25 | 22 000 | 4 400 | 5.5 | 3 |
| .75 | 2.356 | .88 | 17 600 | 3 520 | 4.75 | 6.75 | 34 000 | 6 800 | 7 | 3.5 |
| .875 | 2.749 | 1.12 | 24 600 | 4 920 | 5.75 | 7.5 | 44 000 | 8 800 | 8 | 4 |
| 1 | 3.142 | 1.5 | 32 000 | 6 400 | 6.5 | 8.5 | 60 000 | 12 000 | 9 | 4.5 |
| 1.125 | 3.534 | 1.82 | 40 000 | 8 000 | 7.5 | 9.5 | 72 000 | 14 400 | 10 | 5 |
| 1.25 | 3.927 | 2.28 | 50 000 | 10 000 | 8.5 | 10.75 | 88 000 | 17 600 | 11 | 5.5 |
| 1.375 | 4.320 | 2.77 | 60 000 | 12 000 | 9 | 12 | 104 000 | 20 800 | 12 | 6 |
| 1.5 | 4.712 | 3.37 | 72 000 | 14 400 | 10 | 13 | 124 000 | 24 800 | 13 | 6.5 |

NOTE.—Add 10 per cent. to weight for wire centre.

Galvanized Charcoal Iron Wire Rope.

Vessels' Rigging and Derrick Guys.

7 or 12 Wires in a Strand.

| Circum. | Weight per Fathom. | Ultimate Strength. | Safe Load .25 | Circum. Equiv- alent to New Hemp Rope. | Circum. | Weight per Fathom. | Ultimate Strength. | Safe Load .25 | Circum. Equiv- alent to New Hemp Rope. |
|---------|--------------------------|-----------------------|---------------------|---|---------|--------------------------|-----------------------|---------------------|---|
| Inch. | Lbs. | Lbs. | Lbs. | Inch. | Inch. | Lbs. | Lbs. | Lbs. | Inch. |
| — | — | — | — | — | 4.5 | 19 | 60 000 | 15 000 | 9 |
| — | — | — | — | — | 4.75 | 21 | 66 000 | 16 500 | 9.5 |
| — | — | — | — | — | 5 | 22 | 70 000 | 17 500 | 10 |
| — | — | — | — | — | 5.25 | 24.5 | 80 000 | 20 000 | 10.5 |
| — | — | — | — | — | 5.5 | 26.5 | 86 000 | 21 500 | 11 |
| — | — | — | — | — | 6 | 28.5 | 100 000 | 25 000 | 11.5 |

Galvanized Charcoal Iron Wire Rope.

Vessels' Rigging and Derrick Guys.

John A. Roebling's Sons Co., New York.

7 Wires in a Strand.

| Weight per Thom. | Ultimate Strength. | Safe Load .25 | Circum. Equiv. to New Hemp Rope. | Circum. | Weight per Fathom. | Ultimate Strength. | Safe Load .25 | Circum. Equiv. to New Hemp Rope. |
|------------------|--------------------|---------------|----------------------------------|---------|--------------------|--------------------|---------------|----------------------------------|
| Lbs. | Lbs. | Lbs. | Inch. | Inch. | Lbs. | Lbs. | Lbs. | Inch. |
| .25 | 750 | 187.5 | 1.125 | 1.5 | 2 | 7000 | 1750 | 3 |
| .375 | 1250 | 312.5 | 1.25 | 1.75 | 2.5 | 10000 | 2500 | 3.75 |
| .5 | 1500 | 375 | 1.5 | 2 | 3.5 | 14000 | 3500 | 4.5 |
| .75 | 2000 | 500 | 1.75 | 2.25 | 4.5 | 16000 | 4000 | 4.75 |
| .875 | 4000 | 1000 | 2.25 | 2.5 | 5.5 | 18000 | 4500 | 5 |
| .25 | 4500 | 1125 | 2.375 | 2.75 | 6.75 | 20000 | 5000 | 5.25 |
| .75 | 5000 | 1250 | 2.5 | — | — | — | — | — |

Galvanized Steel Hawasers.

For Sea and Lake Towing.

| Ultimate Strength. | Circum. equiv. to New Hemp Hawaser. | Circumference. | Ultimate Strength. | Circum. equiv. to New Hemp Hawaser. |
|--------------------|-------------------------------------|----------------|--------------------|-------------------------------------|
| Lbs. | Inch. | Inch. | Lbs. | Inch. |
| 30000 | 6.5 | 3.5 | 58000 | 9 |
| 36000 | 7 | 4 | 70000 | 10 |
| 44000 | 8.5 | — | — | — |

Galvanized Steel Cables for Suspension Bridges.

| Ultimate Strength. | Weight per Foot. | Diameter. | Ultimate Strength. | Weight per Foot. | Diameter. | Ultimate Strength. | Weight per Foot. |
|--------------------|------------------|-----------|--------------------|------------------|-----------|--------------------|------------------|
| Lbs. | Lbs. | Inch. | Lbs. | Lbs. | Inch. | Lbs. | Lbs. |
| 130000 | 3.7 | 1.875 | 200000 | 5.8 | 2.375 | 360000 | 10 |
| 150000 | 4.35 | 2 | 220000 | 6.5 | 2.5 | 400000 | 11.3 |
| 190000 | 5.6 | 2.25 | 310000 | 8.64 | 2.625 | 440000 | 13 |

Use, Weight, and Length of Iron Wire.

| Weight per 100 Feet. | Weight of one Mile. | 63 lbs. Bundle. | Area. | Gauge. | Diam. | Weight per 100 Feet. | Weight of one Mile. | 63 lbs. Bundle. | Area. |
|----------------------|---------------------|-----------------|-----------|--------|-------|----------------------|---------------------|-----------------|-----------|
| Lbs. | Lbs. | Feet. | Sq. Inch. | No. | Inch. | Lbs. | Lbs. | Feet. | Sq. Inch. |
| 56.1 | 2962 | 112 | .166 19 | 16 | .063 | 1.05 | 55 | 6000 | .003 117 |
| 49.01 | 2588 | 129 | .145 22 | 17 | .054 | .77 | 41 | 8 182 | .002 29 |
| 40.94 | 2162 | 154 | .121 304 | 18 | .047 | .58 | 31 | 10 862 | .001 734 |
| 34.73 | 1834 | 181 | .102 921 | 19 | .041 | .45 | 24 | 14 000 | .001 32 |
| 29.04 | 1533 | 217 | .086 049 | 20 | .035 | .32 | 17 | 19 687 | .000 962 |
| 27.66 | 1460 | 228 | .074 023 | 21 | .032 | .27 | 14 | 23 333 | .000 804 |
| 21.23 | 1121 | 296 | .062 901 | 22 | .028 | .21 | 11 | 30 000 | .000 615 |
| 18.34 | 968 | 343 | .054 325 | 23 | .025 | .175 | 9.24 | 36 000 | .000 40 |
| 15.78 | 833 | 399 | .046 759 | 24 | .023 | .14 | 7.39 | 45 000 | .000 |
| 13.39 | 707 | 470 | .039 76 | 25 | .02 | .116 | 6.124 | 54 310 | .000 |
| 11.35 | 599 | 555 | .033 653 | 26 | .018 | .093 | 4.91 | 67 742 | .000 |
| 9.73 | 514 | 647 | .028 952 | 27 | .017 | .083 | 4.382 | 75 903 | .000 |
| 8.03 | 439 | 759 | .024 605 | 28 | .016 | .074 | 3.907 | 85 135 | .000 |
| 6.66 | 367 | 905 | .020 612 | 29 | .015 | .061 | 3.22 | 103 278 | .000 |
| 5.68 | 306 | 1086 | .017 203 | 30 | .014 | — | 2.851 | 116 666 | .000 |
| 4.83 | 255 | 1304 | .014 313 | 31 | — | — | — | 126 000 | .000 |
| 3.82 | 202 | 1649 | .011 309 | 32 | — | — | — | 136 956 | .000 |
| 2.92 | 154 | 2158 | .008 659 | 33 | — | — | — | 150 270 | .000 |
| 2.24 | 118 | 2813 | .006 647 | 34 | — | — | — | 170 000 | .000 |
| 1.69 | 89 | 3728 | .005 026 | 35 | — | — | — | 190 000 | .000 |
| 1.37 | 72 | 4598 | .004 071 | 36 | — | — | — | 220 000 | .000 |

Weight and Strength of Single Strand and Cable laid Fence Wire. (F. Morton & Co.)

| Strands. | No. | Single Wire of equal Diameter. | | Length per 1000 lbs. | | Strands. | No. | Single Wire of equal Diameter. | | Length per 1000 lbs. | |
|----------|-----|--------------------------------|-------|----------------------|----------|----------|-----|--------------------------------|-------|----------------------|----------|
| | | | | Of a Strand. | Of Rope. | | | | | Of a Strand. | Of Rope. |
| No. | | No. | Inch. | Feet. | Feet. | No. | | No. | Inch. | Feet. | Feet. |
| 3 | 2A | 8 | .159 | 20 090 | 15 270 | 7 | 00 | 4 | .229 | 8300 | 7366 |
| 4 | 2 | 7 | .174 | 14 730 | 12 790 | 7 | 3/0 | 3 | .25 | 8036 | 6228 |
| 7 | 1 | 6 | .191 | 13 125 | 10 580 | 7 | 4/0 | 2 | .274 | 7500 | 5156 |
| 7 | 0 | 5 | .209 | 10 446 | 8 928 | 7 | 5/0 | 1 | .3 | 5090 | 4226 |

No. and diameter of wire is that of Ryland's Bros., pp. 122-4.

Hemp, Iron, and Steel. (R. S. Newall & Co.)

| ROUND. | | | | | | | |
|----------------|------------------|----------------|------------------|----------------|------------------|-------------------|--------------------|
| HEMP. | | IRON. | | STEEL. | | Tensile Strength. | |
| Circumference. | Weight per Foot. | Circumference. | Weight per Foot. | Circumference. | Weight per Foot. | Safe Load. | Ultimate Strength. |
| Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Lbs. | Lbs. |
| 2.75 | .33 | 1 | .16 | — | — | 672 | 4480 |
| | | 1.5 | .25 | 1 | .16 | 1008 | 6720 |
| 3.75 | .66 | 1.625 | .33 | — | — | 1344 | 8960 |
| | | 1.75 | .42 | 1.5 | .25 | 1680 | 11200 |
| 4.5 | .83 | 1.875 | .5 | — | — | 2016 | 13440 |
| | | 2 | .58 | 1.625 | .33 | 2352 | 15680 |
| 5.5 | 1.16 | 2.125 | .66 | 1.75 | .42 | 2688 | 17920 |
| | | 2.25 | .75 | — | — | 3024 | 20160 |
| 6 | 1.5 | 2.375 | .83 | 1.875 | .5 | 3360 | 22400 |
| | | 2.5 | .92 | — | — | 3696 | 24640 |
| 6.5 | 1.66 | 2.625 | 1 | 2 | .58 | 4032 | 26680 |
| | | 2.75 | 1.08 | 2.125 | .66 | 4368 | 29120 |
| 7 | 2 | 2.875 | 1.16 | 2.25 | .75 | 4704 | 31360 |
| | | 3 | 1.25 | — | — | 5040 | 33600 |
| 7.5 | 2.33 | 3.125 | 1.33 | 2.375 | .83 | 5376 | 36840 |
| | | 3.25 | 1.41 | — | — | 5712 | 38080 |
| 8 | 2.66 | 3.375 | 1.5 | 2.5 | .92 | 6048 | 40320 |
| | | 3.5 | 1.66 | 2.625 | 1 | 6720 | 44800 |
| 8.5 | 3 | 3.625 | 1.83 | 2.75 | 1.08 | 7392 | 49280 |
| | | 3.75 | 2 | — | — | 8064 | 53760 |
| 9.5 | 3.66 | 3.875 | 2.16 | 3.25 | 1.33 | 8736 | 58240 |
| 10 | 4.33 | 4 | 2.33 | — | — | 9408 | 62720 |
| | | 4.25 | 2.5 | 3.375 | 1.5 | 10080 | 67200 |
| 11 | 5 | 4.375 | 2.66 | — | — | 10752 | 71680 |
| | | 4.5 | 3 | 3.5 | 1.66 | 12096 | 80640 |
| 12 | 5.66 | 4.625 | 3.33 | 3.75 | 2 | 13440 | 89600 |

FLAT.

| Dimensions. | Dimensions. | Dimensions. | Dimensions. |
|--------------|-------------|-------------|-------------|
| 4 X .5 | 3.33 | 2.25 X .5 | 1.85 |
| 5 X 1.25 | 4 | 2.5 X .5 | 2.16 |
| 5.5 X 1.375 | 4.33 | 2.75 X .625 | 2.5 |
| 5.75 X 1.5 | 4.66 | 3 X .625 | 2.66 |
| 6 X 1.5 | 5 | 3.25 X .625 | 3 |
| 7 X 1.875 | 6 | 3.5 X .625 | 3.33 |
| 8.25 X 2.125 | 6.66 | 3.75 X .625 | 3.66 |
| | | 4 X .75 | 4.16 |
| | | 4.25 X .75 | 4.66 |
| | | 4.5 X .75 | 5.33 |
| | | 4.75 X .75 | 5.66 |
| | | 5 X .75 | 6 |
| | | 5.5 X .75 | 6.66 |
| | | 6 X .75 | 7.33 |
| | | 6.5 X .75 | 8 |
| | | 7 X .75 | 8.66 |
| | | 7.5 X .75 | 9.33 |
| | | 8 X .75 | 10 |
| | | 8.5 X .75 | 10.66 |
| | | 9 X .75 | 11.33 |
| | | 9.5 X .75 | 12 |
| | | 10 X .75 | 12.66 |
| | | 10.5 X .75 | 13.33 |
| | | 11 X .75 | 14 |
| | | 11.5 X .75 | 14.66 |
| | | 12 X .75 | 15.33 |
| | | 12.5 X .75 | 16 |
| | | 13 X .75 | 16.66 |
| | | 13.5 X .75 | 17.33 |
| | | 14 X .75 | 18 |
| | | 14.5 X .75 | 18.66 |
| | | 15 X .75 | 19.33 |
| | | 15.5 X .75 | 20 |
| | | 16 X .75 | 20.66 |
| | | 16.5 X .75 | 21.33 |
| | | 17 X .75 | 22 |
| | | 17.5 X .75 | 22.66 |
| | | 18 X .75 | 23.33 |
| | | 18.5 X .75 | 24 |
| | | 19 X .75 | 24.66 |
| | | 19.5 X .75 | 25.33 |
| | | 20 X .75 | 26 |
| | | 20.5 X .75 | 26.66 |
| | | 21 X .75 | 27.33 |
| | | 21.5 X .75 | 28 |
| | | 22 X .75 | 28.66 |
| | | 22.5 X .75 | 29.33 |
| | | 23 X .75 | 30 |
| | | 23.5 X .75 | 30.66 |
| | | 24 X .75 | 31.33 |
| | | 24.5 X .75 | 32 |
| | | 25 X .75 | 32.66 |
| | | 25.5 X .75 | 33.33 |
| | | 26 X .75 | 34 |
| | | 26.5 X .75 | 34.66 |
| | | 27 X .75 | 35.33 |
| | | 27.5 X .75 | 36 |
| | | 28 X .75 | 36.66 |
| | | 28.5 X .75 | 37.33 |
| | | 29 X .75 | 38 |
| | | 29.5 X .75 | 38.66 |
| | | 30 X .75 | 39.33 |
| | | 30.5 X .75 | 40 |
| | | 31 X .75 | 40.66 |
| | | 31.5 X .75 | 41.33 |
| | | 32 X .75 | 42 |
| | | 32.5 X .75 | 42.66 |
| | | 33 X .75 | 43.33 |
| | | 33.5 X .75 | 44 |
| | | 34 X .75 | 44.66 |
| | | 34.5 X .75 | 45.33 |
| | | 35 X .75 | 46 |
| | | 35.5 X .75 | 46.66 |
| | | 36 X .75 | 47.33 |
| | | 36.5 X .75 | 48 |
| | | 37 X .75 | 48.66 |
| | | 37.5 X .75 | 49.33 |
| | | 38 X .75 | 50 |
| | | 38.5 X .75 | 50.66 |
| | | 39 X .75 | 51.33 |
| | | 39.5 X .75 | 52 |
| | | 40 X .75 | 52.66 |
| | | 40.5 X .75 | 53.33 |
| | | 41 X .75 | 54 |
| | | 41.5 X .75 | 54.66 |
| | | 42 X .75 | 55.33 |
| | | 42.5 X .75 | 56 |
| | | 43 X .75 | 56.66 |
| | | 43.5 X .75 | 57.33 |
| | | 44 X .75 | 58 |
| | | 44.5 X .75 | 58.66 |
| | | 45 X .75 | 59.33 |
| | | 45.5 X .75 | 60 |
| | | 46 X .75 | 60.66 |
| | | 46.5 X .75 | 61.33 |
| | | 47 X .75 | 62 |
| | | 47.5 X .75 | 62.66 |
| | | 48 X .75 | 63.33 |
| | | 48.5 X .75 | 64 |
| | | 49 X .75 | 64.66 |
| | | 49.5 X .75 | 65.33 |
| | | 50 X .75 | 66 |
| | | 50.5 X .75 | 66.66 |
| | | 51 X .75 | 67.33 |
| | | 51.5 X .75 | 68 |
| | | 52 X .75 | 68.66 |
| | | 52.5 X .75 | 69.33 |
| | | 53 X .75 | 70 |
| | | 53.5 X .75 | 70.66 |
| | | 54 X .75 | 71.33 |
| | | 54.5 X .75 | 72 |
| | | 55 X .75 | 72.66 |
| | | 55.5 X .75 | 73.33 |
| | | 56 X .75 | 74 |
| | | 56.5 X .75 | 74.66 |
| | | 57 X .75 | 75.33 |
| | | 57.5 X .75 | 76 |
| | | 58 X .75 | 76.66 |
| | | 58.5 X .75 | 77.33 |
| | | 59 X .75 | 78 |
| | | 59.5 X .75 | 78.66 |
| | | 60 X .75 | 79.33 |
| | | 60.5 X .75 | 80 |
| | | 61 X .75 | 80.66 |
| | | 61.5 X .75 | 81.33 |
| | | 62 X .75 | 82 |
| | | 62.5 X .75 | 82.66 |
| | | 63 X .75 | 83.33 |
| | | 63.5 X .75 | 84 |
| | | 64 X .75 | 84.66 |
| | | 64.5 X .75 | 85.33 |
| | | 65 X .75 | 86 |
| | | 65.5 X .75 | 86.66 |
| | | 66 X .75 | 87.33 |
| | | 66.5 X .75 | 88 |
| | | 67 X .75 | 88.66 |
| | | 67.5 X .75 | 89.33 |
| | | 68 X .75 | 90 |
| | | 68.5 X .75 | 90.66 |
| | | 69 X .75 | 91.33 |
| | | 69.5 X .75 | 92 |
| | | 70 X .75 | 92.66 |
| | | 70.5 X .75 | 93.33 |
| | | 71 X .75 | 94 |
| | | 71.5 X .75 | 94.66 |
| | | 72 X .75 | 95.33 |
| | | 72.5 X .75 | 96 |
| | | 73 X .75 | 96.66 |
| | | 73.5 X .75 | 97.33 |
| | | 74 X .75 | 98 |
| | | 74.5 X .75 | 98.66 |
| | | 75 X .75 | 99.33 |
| | | 75.5 X .75 | 100 |
| | | 76 X .75 | 100.66 |
| | | 76.5 X .75 | 101.33 |
| | | 77 X .75 | 102 |
| | | 77.5 X .75 | 102.66 |
| | | 78 X .75 | 103.33 |
| | | 78.5 X .75 | 104 |
| | | 79 X .75 | 104.66 |
| | | 79.5 X .75 | 105.33 |
| | | 80 X .75 | 106 |
| | | 80.5 X .75 | 106.66 |
| | | 81 X .75 | 107.33 |
| | | 81.5 X .75 | 108 |
| | | 82 X .75 | 108.66 |
| | | 82.5 X .75 | 109.33 |
| | | 83 X .75 | 110 |
| | | 83.5 X .75 | 110.66 |
| | | 84 X .75 | 111.33 |
| | | 84.5 X .75 | 112 |
| | | 85 X .75 | 112.66 |
| | | 85.5 X .75 | 113.33 |
| | | 86 X .75 | 114 |
| | | 86.5 X .75 | 114.66 |
| | | 87 X .75 | 115.33 |
| | | 87.5 X .75 | 116 |
| | | 88 X .75 | 116.66 |
| | | 88.5 X .75 | 117.33 |
| | | 89 X .75 | 118 |
| | | 89.5 X .75 | 118.66 |
| | | 90 X .75 | 119.33 |
| | | 90.5 X .75 | 120 |
| | | 91 X .75 | 120.66 |
| | | 91.5 X .75 | 121.33 |
| | | 92 X .75 | 122 |
| | | 92.5 X .75 | 122.66 |
| | | 93 X .75 | 123.33 |
| | | 93.5 X .75 | 124 |
| | | 94 X .75 | 124.66 |
| | | 94.5 X .75 | 125.33 |
| | | 95 X .75 | 126 |
| | | 95.5 X .75 | 126.66 |
| | | 96 X .75 | 127.33 |
| | | 96.5 X .75 | 128 |
| | | 97 X .75 | 128.66 |
| | | 97.5 X .75 | 129.33 |
| | | 98 X .75 | 130 |
| | | 98.5 X .75 | 130.66 |
| | | 99 X .75 | 131.33 |
| | | 99.5 X .75 | 132 |
| | | 100 X .75 | 132.66 |

1 preceding tables following results are determined:

| | Ultimate Strength per Lb. Weight per Foot. | SAFE LOAD | |
|-------|--|-----------------------------|---|
| | | per Lb. Weight per Foot. | per Square of Circum- ference in Inches. |
| | Lbs. | Lbs. | Lbs. |
| | 15 000 | 4550 | 100 |
| | 22 000 | 4500 | 600 |
| | { 30 000 | { 6000 | { 1000 |
| | { 45 500 | { 8000 | { 1300 |

FLAT MINING ROPES.

John A. Roebling's Sons Co., New York.

| Thickness. | Weight per Foot. | Safe Load. | Width. | Thickness. | Weight per Foot. | Safe Load. |
|------------|---------------------|------------|--------|------------|---------------------|------------|
| Ina. | Lbs. | Lbs. | Ina. | Ina. | Lbs. | Lbs. |
| .375 | 1.19 | 63 000 | 5.5 | .375 | 3.9 | 156 000 |
| .375 | 1.86 | 74 000 | 5.5 | .5 | 4.8 | 193 000 |
| .375 | 2.32 | 93 000 | 6 | .375 | 4.34 | 173 000 |
| .5 | 2.97 | 118 000 | 6 | .4375 | 4.5 | 160 000 |
| .375 | 2.86 | 114 000 | 6 | .5 | 5.1 | 210 000 |
| .5 | 3.3 | 130 000 | 6.5 | .5 | 5.5 | 224 000 |
| .375 | 3.12 | 125 000 | 7 | .5 | 5.9 | 238 000 |
| .5 | 4 | 160 000 | 7.5 | .5 | 6.25 | 250 000 |
| .375 | 3.4 | 125 000 | 6 | .5 | 6.75 | 270 000 |
| .5 | 4.27 | 170 000 | | | | |

Ropes and Chains of Equal Strength.

| Size in. | CIRCUMFERENCE. | | | WEIGHT PER FOOT. | | | | Safe Load. |
|-------------|----------------|----------------------------|---------------------------|------------------|---------------|---------------|----------------|---------------|
| | Hemp Rope. | Crucible Steel Rope. | Charcoal Iron Rope. | Steel Rope. | Iron Rope. | Hemp Rope. | Iron Chain. | |
| | Ina. | Ina. | Ina. | Lbs. | Lbs. | Lbs. | Lbs. | Tons. |
| 75 | 2.75 | — | 1 | — | .14 | .34 | .5 | .3 |
| | 3 | — | 1.18 | — | .21 | .46 | .65 | .4 |
| 25 | 3.5 | 1 | 1.39 | .17 | .28 | .67 | .81 | .5 |
| 5 | 4.25 | 1.26 | 1.57 | .25 | .33 | .75 | .96 | .6 |
| | 4.5 | 1.45 | 1.77 | .3 | .45 | .83 | 1.38 | .8 |
| 5 | 5 | 1.57 | 1.97 | .35 | .57 | 1.16 | 1.76 | 1 |
| 75 | 5.5 | 1.77 | 2.19 | .45 | .7 | 1.2 | 2.2 | 1.3 |
| | 5.75 | 1.96 | 2.36 | .59 | .83 | 1.6 | 2.63 | 1.5 |
| | 6.75 | 2.36 | 2.75 | .85 | 1.08 | 2 | 4.21 | 2.3 |
| 5 | 7.75 | 2.75 | 3.14 | 1.1 | 1.43 | 2.65 | 4.83 | 3.1 |
| | 8.75 | 2.95 | 3.53 | 1.28 | 1.8 | 3.35 | 5.75 | 3.8 |
| | 9.75 | 3.14 | 3.93 | 1.45 | 2.3 | 4.6 | 7.5 | 4.8 |
| 5 | 10.5 | 3.53 | 4.32 | 1.83 | 2.94 | 4.92 | 9.33 | 5 |
| 5 | 11.75 | 3.93 | 4.71 | 2.33 | 3.56 | 5.83 | 10.6 | 5 |
| | 12.75 | 4.32 | 5.1 | 2.98 | 4 | 6.2 | 11.9 | 5 |
| | 14.75 | 4.71 | 5.5 | 3.58 | 4.8 | 8.7 | 14.5 | 5 |
| | 15.25 | 4.81 | 5.89 | 3.65 | 5.6 | 9 | 17.6 | 5 |
| | 15.75 | 5.1 | 6.28 | 4.04 | 6.3 | | 20 | 12 |
| | 17.75 | 5.8 | 7.07 | 5.65 | 7.9 | | 23 | 15 |
| | 19.5 | 6.35 | 7.85 | 6.5 | 9.8 | | | 19.6 |

experiments of U. S. Navy, hemp rope of this size 309 lbs., and a wire rope of 5.34 ins. has

—calculated

Weight of Hemp and Wire Rope. (Molinsworth)

In Lbs. per Fathom.

| Circum- ference. | Hemp. | | Wire. | | Circum- ference. | Hemp. | |
|---------------------|-------------|-------------|-------------|-------------|---------------------|-------------|-------------|
| | Common. | Good. | Iron. | Steel. | | Common. | Good. |
| Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Ins. | Lbs. | Lbs. |
| 1 | .18 | .24 | .87 | .89 | 5 | 4.5 | 6 |
| 1.5 | .41 | .54 | 1.96 | 2 | 5.5 | 5.45 | 7.26 |
| 1.75 | .55 | .74 | 2.66 | 2.73 | 6 | 6.48 | 8.64 |
| 2 | .72 | .96 | 3.48 | 3.56 | 6.5 | 7.61 | 10.14 |
| 2.25 | .91 | 1.22 | 4.4 | 4.51 | 7 | 8.82 | 11.76 |
| 2.5 | 1.13 | 1.5 | 5.44 | 5.56 | 7.5 | 10.13 | 13.5 |
| 2.75 | 1.36 | 1.82 | 6.58 | 6.73 | 8 | 11.52 | 15.36 |
| 3 | 1.62 | 2.16 | 7.83 | 8.01 | 8.5 | 13.05 | 17.34 |
| 3.25 | 1.9 | 2.54 | 9.19 | 9.4 | 9 | 14.58 | 19.44 |
| 3.5 | 2.21 | 2.94 | 10.66 | 10.9 | 10 | 18 | 24 |
| 3.75 | 2.53 | 3.38 | 12.23 | 12.52 | 12 | 26 | 34.56 |
| 4 | 2.88 | 3.84 | 13.92 | 14.24 | 15 | 40.52 | 54 |

To Compute Stress upon a Rope set at an Inclination.

RULE.—Multiply sine of angle of elevation by strain in lbs., add an allowance for rolling friction and weight of rope, and multiply by factor of safety.

Factor of safety.—For standing rope 4, for running 5, and for inclined planes from 5 to 7.

ILLUSTRATION.—Inclination of rope 92.5 feet in 100, velocity 1500 feet per minute, and strain 2000 lbs.; what should be diam. of iron rope, 7 wires to a strand?

Angle of 92.5 feet in 100 = 43° , and sine of 43° = .682. $.682 \times 2000 = 1364$, to which is to be added rolling friction and weight of rope, assumed to be 11; hence, $1364 + 11 = 1375$.

Factor of safety assumed at 6, consequently $1375 \times 6 = 8250$ lbs., capacity or breaking weight or stress of rope.

By table, page 162, 8200 lbs. is breaking weight of a wire rope of 7 strands, .653 inch in diam.

To Compute Tension of a Rope.

$\frac{HP}{v} = t$. v representing velocity of rope in feet per minute, HP horses' power, and t tension in lbs.

ILLUSTRATION.—Assume wheel 7 feet in diameter, revolution 140 per minute, and HP as per preceding table, 29.6.

$$\text{Then } \frac{29.6 \times 33000}{7 \times 3.1416 \times 140} = \frac{976800}{3079} = 317.2 \text{ lbs.}$$

To Compute Operative Deflection of a Rope.

$\frac{D^2 w}{10.7 t} = d$. D representing distance between centres of wheels or drums in feet, w weight of rope in feet per lb., t tension, or power required to produce required power or tension of rope when at rest, and d deflection in feet.

ILLUSTRATION.—Take elements of preceding case: diam. of wire rope of 7 strands = .5625 inch, and by table, page 162, $w = .41$ lb., and $D = 300$ feet.

$$\text{Then } \frac{300^2 \times .41}{10.7 \times 317.2} = 10.87 \text{ feet.}$$

of the river Rhine there is a wire rope in operation for a distance exceeding one mile

Endless Ropes.

Ropes, when practicable and proper for application, can be used for transmission of power at a less cost than belting or shafting.

Transmission of Power.

| Revolutions per Minute. | Diameter of Rope. | Horse Power. | Diameter of Wheel. | Revolutions per Minute. | Diameter of Rope. | Horse Power. | Diameter of Wheel. | Revolutions per Minute. | Diameter of Rope. | Horse Power. |
|-------------------------|-------------------|--------------|--------------------|-------------------------|-------------------|--------------|--------------------|-------------------------|-------------------|--------------|
| | Inch. | | Feet. | | Inch. | | Feet. | | Inch. | |
| 80 | .375 | 3.3 | 7 | 100 | .5625 | 21.1 | 11 | 140 | .6875 | 132.1 |
| 100 | .375 | 4.1 | 7 | 140 | .5625 | 29.6 | 12 | 80 | .75 | 99.3 |
| 120 | .375 | 5 | 8 | 80 | .625 | 22 | 12 | 100 | .75 | 124.1 |
| 140 | .375 | 5.8 | 8 | 100 | .625 | 27.5 | 12 | 140 | .75 | 173.7 |
| 80 | .4375 | 6.9 | 8 | 140 | .625 | 38.5 | 13 | 80 | .75 | 122.6 |
| 100 | .4375 | 8.6 | 9 | 80 | .625 | 41.5 | 13 | 100 | .75 | 153.2 |
| 120 | .4375 | 10.3 | 9 | 100 | .625 | 51.9 | 13 | 120 | .75 | 183.9 |
| 140 | .4375 | 12.1 | 9 | 140 | .625 | 72.6 | 14 | 80 | .875 | 148 |
| 80 | .5 | 10.7 | 10 | 80 | .6875 | 58.4 | 14 | 100 | .875 | 176 |
| 100 | .5 | 13.4 | 10 | 100 | .6875 | 73 | 14 | 120 | .875 | 222 |
| 120 | .5 | 16.1 | 10 | 140 | .6875 | 102.2 | 15 | 80 | .875 | 217 |
| 140 | .5 | 18.7 | 11 | 80 | .6875 | 75.5 | 15 | 100 | .875 | 259 |
| 80 | .5625 | 16.9 | 11 | 100 | .6875 | 94.4 | 15 | 120 | .875 | 300 |

Wire Rope and Equivalent Belt.

Substituting wire rope for an ordinary flat belt, the diameter is determined by rule in practice for estimating power transmitted by a belt—viz., horse power for every 70 square feet of running belt surface per minute. Thus, a belt 15 inches wide running at rate of 1400 feet per minute would be equal to $(1400 \times 15) \div (70 \times 12) = 25$ horses' power. The same result is obtained by the use of a wire rope .5625 inch in diameter running over a wheel 6 feet in diameter, making 130 revolutions per minute.

The average life of iron wire rope with good care is from 3 to 5 years, and steel rope is greater. Wear increases rapidly with velocity.

General Notes.—Hemp and Wire Ropes.

Hemp Rope, 2 inches in circumference, of different manufactures, parted at 4413 to 6160 lbs.

Wire Ropes, of Italian, Russian, and French manufacture parted with an stress of 5128 lbs. = 1633 lbs. per square inch of rope.

The carrying capacity of a hemp rope is proportional to its thickness, number of strands, slackness with which they are twisted, and quality of the rope.

and Wire Ropes.—Ultimate Strength is 2240 lbs. per lb. per fathom of hemp, 3300 lbs. for iron, 7000 lbs. for cast-steel, and 10000 lbs. for steel.

Working Load is 336 lbs. per lb. weight per fathom for round hemp, 660 lbs. for iron, 1400 lbs. for cast-steel, and 2000 lbs. for plough-steel.

The area is 3 times square of circumference in inches for round hemp, 5 times for iron, and 9 times square of circumference for steel.

(D. K. Clark.)

Ropes may be one half less in weight than iron or hemp ropes.

IRON WIRE AND UNITED STATES NAVY HEMP ROPE.

Wire 6 Strands, Hemp Core. Rope 4 Strands.

| WIRE. | | | | | HEMP. | | | | |
|----------------|----------|-------|--------|------------------|----------------|----------|--------|------------------|--|
| Circumference. | | Core. | Wires. | Breaking Weight. | Circumference. | | Yarns. | Breaking Weight. | |
| Actual. | Nominal. | | | | Actual. | Nominal. | | | |
| Inch. | Inch. | Inch. | No. | Lbs. | Inch. | Inch. | No. | Lbs. | |
| 7 | 7 | 2.35 | 108 | 187 400 | 12 | 13.25 | 1168 | 75 900 | |
| 6 | 6 | 2.25 | 108 | 104 050 | 11 | 12.25 | 1036 | 77 633 | |
| 4.937 | 4.9 | 1.57 | 114 | 65 409 | 10.5 | 11.875 | 928 | 76 933 | |
| 4.375 | 4.5 | 1.57 | 114 | 55 316 | 10 | 11.375 | 876 | 70 533 | |
| 3.5 | 3.36 | 1.27 | 114 | 34 480 | 9.5 | 10.5 | 800 | 58 766 | |
| 3.187 | 2.98 | 1.17 | 114 | 28 606 | 9 | 10.312 | 712 | 56 466 | |
| 2.75 | 2.68 | .78 | 114 | 21 846 | 8.5 | 9.437 | 640 | 42 866 | |
| 2.5 | 2.45 | .78 | 114 | 15 692 | 8 | 8.812 | 560 | 40 000 | |
| 2.375 | 2.4 | .78 | 42 | 15 718 | 7.5 | 8.437 | 484 | 35 500 | |
| 2 | 2.06 | .39 | 114 | 10 925 | 7 | 7.812 | 436 | 32 166 | |

Weight and Strength of Stud-link Chain Cable.
(English.)

| DIMENSIONS. | | | | | DIMENSIONS. | | | | |
|---------------------|-----------------|----------------|--------------------|--|---------------------|-----------------|----------------|--------------------|--|
| Diam. of each Link. | Length of Link. | Width of Link. | Weight per Fathom. | Admiralty Proof-stress (adopted by Lloyd's). | Diam. of each Link. | Length of Link. | Width of Link. | Weight per Fathom. | Admiralty Proof-stress (adopted by Lloyd's). |
| Inch. | Inch. | Inch. | Lbs. | Tons. | Inch. | Inch. | Inch. | Lbs. | Tons. |
| .4375 | 2.625 | 1.575 | 11.3 | 3.5 | 1.5 | 9 | 5.4 | 121 | 40.5 |
| .5 | 3 | 1.8 | 13.4 | 4.5 | 1.625 | 9.75 | 5.85 | 142 | 47.5 |
| .5625 | 3.375 | 2.025 | 17.2 | 5.5 | 1.75 | 10.5 | 6.3 | 164.6 | 55.125 |
| .625 | 3.75 | 2.25 | 21 | 7 | 1.875 | 11.25 | 6.75 | 189 | 63.25 |
| .6875 | 4.125 | 2.475 | 25.4 | 8.5 | 2 | 12 | 7.2 | 215 | 72 |
| .75 | 4.5 | 2.7 | 30.2 | 10.125 | 2.125 | 12.75 | 7.65 | 242.8 | 81.25 |
| .875 | 5.25 | 3.15 | 41.2 | 13.75 | 2.25 | 13.5 | 8.1 | 276.2 | 91.125 |
| 1 | 6 | 3.6 | 53.8 | 18 | 2.375 | 14.25 | 8.55 | 303.2 | 101.5 |
| 1.125 | 6.75 | 4.05 | 69 | 22.75 | 2.5 | 15 | 9 | 336 | 112.5 |
| 1.25 | 7.5 | 4.5 | 84 | 28.125 | 2.75 | 16.5 | 9.9 | 406.6 | 136.125 |
| 1.375 | 8.25 | 4.95 | 101.6 | 34 | | | | | |

NOTE 1.—*Safe Working-stress* is taken at half *Proof-stress*, 3.82 tons per sq. inch of section.2.—*Proof-stress* and *Safe Working-stress* for close-link chains are respectively two-thirds of those of stud-link chains.3.—*Proof-stress* averages 72 per cent. ultimate strength, and *Ultimate Strength* averages 8 tons per square inch of section of rod or one side of a link.

Weight of close-link chain is about three times weight of bar from which it is made, for equal lengths.

Karl von Ott, comparing weight, cost, and strength of the three materials, hemp, iron wire, and chain iron, concludes that the proportion between cost of hemp rope, wire rope, and chain is as 2 : 1 : 3, and that, therefore, for equal resistances, wire rope is only half the cost of hemp rope, and a third of cost of chains.

Safe Working Load of Chains. (Molesworth).

| Diameter of Iron. | Load. | Diameter of Iron. | Load. | Diameter of Iron. | Load. | Diameter of Iron. | Load. |
|-------------------|-------|-------------------|--------|-------------------|--------|-------------------|--------|
| Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. | Inch. | Lbs. |
| .5 | 2240 | .6875 | 7 390 | .9375 | 13 700 | 1.1875 | 22 400 |
| .75 | 3800 | .75 | 8 960 | 1 | 15 680 | 1.25 | 22 400 |
| .9 | 4900 | .8125 | 10 280 | 1.0625 | 17 920 | 1.3125 | 22 400 |
| 1.125 | 6270 | .875 | 12 320 | 1.125 | 20 160 | 1.375 | 22 400 |

Breaking Strain and Proof of Chain Cables.

| Mam. Chain. | Breaking Strain. | Diam. of Chain. | Breaking Strain. | Diam. of Chain. | Breaking Strain. | Diam. of Chain. | Breaking Strain. |
|-------------|------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|
| Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| | 67 700 | 1.1875 | 92 940 | 1.5 | 143 100 | 2 | 243 180 |
| 0625 | 75 640 | 1.25 | 102 160 | 1.625 | 165 920 | 2.125 | 272 580 |
| 125 | 84 100 | 1.375 | 121 840 | 1.75 | 216 120 | 2.25 | 303 280 |

Proof-stress is 50 per cent. of estimated strength of weakest link and 46 cent. of strongest.

Comparison of Wire Ropes and Tarred Hemp Rope, Hawseers, and Cables.

| COARSE LAID. | | | | | | FINE LAID. | | | | | |
|--------------|------------|----------------|---------------|----------------|----------------|------------|------------|---------------|----------------|----------------|----------------|
| Ropes. | | | Hawseers. | | | Ropes. | | | Hawseers. | | |
| Circum. | Safe Load. | Three Strands. | Four Strands. | Three Strands. | Three Strands. | Diameter. | Safe Load. | Four Strands. | Three Strands. | Three Strands. | Three Strands. |
| Ins. | Lbs. | Ins. | Ins. | Ins. | Ins. | Ins. | Lbs. | Ins. | Ins. | Ins. | Ins. |
| .78 | 425 | 1.25 | — | — | — | .5 | 1 875 | 3.12 | 2.87 | — | — |
| 1 | 690 | 2.43 | 2.25 | 3.32 | — | .5625 | 2 420 | 3.56 | 3.25 | 4.87 | — |
| 1.25 | 825 | 2.68 | 2.375 | 3.5 | — | .625 | 2 900 | 3.93 | 3.62 | 5.25 | — |
| 1.375 | 1 600 | 2.87 | 2.62 | 3.87 | — | .75 | 4 320 | 4.81 | 4.37 | 6.37 | — |
| 1.75 | 2 800 | 3.81 | 3.5 | 5.18 | — | .875 | 5 700 | 5.5 | 5 | 7.25 | — |
| 2.125 | 3 800 | 4.75 | 4.25 | 6.12 | — | 1 | 8 200 | 7.25 | 6.25 | 8.75 | — |
| 2.375 | 4 400 | 5.25 | 4.87 | 7 | — | 1.125 | 10 100 | 8.18 | 7 | 9.5 | — |
| 2.625 | 6 150 | 6.12 | 5.75 | 8 | 8 | 1.25 | 13 600 | 8.81 | 8.06 | 11 | — |
| 3 | 8 400 | 6.62 | 6.12 | 8.62 | 8.62 | 1.5 | 17 500 | 10 | 9.75 | 12.5 | — |
| 3.75 | 13 400 | 8.81 | 8.5 | 10.93 | 10.93 | 1.625 | 21 800 | 11.8 | 10.93 | — | — |
| 4.25 | 16 800 | 9.87 | 9.56 | 12.25 | 12.12 | 1.75 | 27 000 | 12.5 | 12.12 | — | — |
| 4.625 | 20 160 | 10.75 | 10.5 | 13 | 13.12 | 1.875 | 32 500 | — | — | — | — |
| 5 | 24 600 | — | 11.87 | 11.56 | 11.75 | 2 | 37 000 | — | — | — | — |

above table, determination of circumference of rope, etc., is based upon King Weight or Tensile resistance of wire being reduced by one fourth, ultimate resistances of rope, etc., are reduced one third.

Result of Experiments upon Wire Rope at U. S. Navy Yard, Washington. (J. A. Roebling's Sons.)

| Reference. | | Wire in each Strand. | | Diam. of Wire by W. G. | | Weight per Foot. | | Breaking Weight. | | Circumference. | | Wire in each Strand. | | Diam. of Wire by W. G. | | Weight per Foot. | | Breaking Weight. | |
|------------|----------|----------------------|-----|------------------------|--------|------------------|------|------------------|------|----------------|----------|----------------------|-----|------------------------|--|------------------|------|------------------|--|
| i. | Nominal. | | | | | | | | | Actual. | Nominal. | | | | | | | | |
| | Ins. | No. | No. | | Lbs. | Lbs. | | Ins. | Ins. | No. | No. | | No. | No. | | Lbs. | Lbs. | | |
| 5 | 4.9 | 19 | 11 | 3.14 | 65 409 | 2.375 | 2.4 | 7 | 13 | 14 | 15 718 | | | | | | | | |
| | 4.5 | 19 | 13 | 2.15 | 55 316 | 2.1875 | 2.12 | 7 | 14 | .1 | 14 478 | | | | | | | | |
| 5 | 3.91 | 19 | 14 | 2.0875 | 44 420 | 2 | 2.06 | 19 | 19 | 10 925 | | | | | | | | | |
| | 3.36 | 19 | 14 | 1.1525 | 34 840 | 1.9375 | 1.9 | 7 | 14 | 10 118 | | | | | | | | | |
| 5 | 2.98 | 19 | 15 | 1.09 | 28 606 | 1.75 | 1.85 | 7 | 17 | .07 | 7 880 | | | | | | | | |
| | 2.68 | 19 | 17 | 1.0275 | 21 846 | 1.4375 | 1.45 | 19 | 20 | .06 | 5 687 | | | | | | | | |
| 5 | 2.56 | 7 | 13 | 1.0225 | 18 810 | 1.3125 | 1.31 | 7 | 18 | .05 | 4 428 | | | | | | | | |
| | 2.45 | 19 | 18 | .14 | 15 692 | 1.125 | 1.11 | 7 | 19 | .035 | 3 729 | | | | | | | | |

Compute Circumference of Wire Rope with Hemp Core, of Corresponding Strength to Hemp Rope, and Hemp Rope to Circumference of Wire Rope.

RE 1.—Multiply square of circumference of hemp rope by .223 for iron and .12 for steel, and extract square root of product.

Multiply square of circumference of hemp-core wire rope by 4.5 for iron and 8.4 for steel wire.

EXAMPLE.—What are the circumferences of an iron and steel wire rope corresponding to one of hemp-core, having a circumference of 8 ins.?

$$\sqrt{8^2 \times .223} = 3.78 \text{ ins. iron, and } \sqrt{8^2 \times .12} = 2.77 \text{ ins. steel.}$$

ROPES, HAWSERS, AND CABLES.

Ropes of hemp fibres are laid with three or four strands of twisted fl and are made up to a circumference of 12 ins., and those of four strands to 8 ins. are fully 16 per cent. stronger than those of three strands.

Hawsers are laid with three or four strands of rope. Cables are laid but three strands of rope. Hawsers and Cables, from having a less proportionate number of fibres, and from the irregularity of the resistance of fibres in consequence of the twisting of them, have less strength than ropes, the difference varying from 35 to 45 per cent., being greatest with least circumference, and those of three strands up to 12 ins. are fully 10 per cent. stronger than those having four strands.

Tarred ropes, hawsers, etc., have 25 per cent. less strength than white ropes; this is in consequence of the injury fibres receive from the high temperature of the tar, viz. 290°.

Tarred hemp and Manila ropes are of about equal strength, and have 25 to 30 per cent. less strength than white ropes.

White ropes are more durable than tarred.

The greater degree of twisting given to fibres of a rope, etc., less strength, as exterior, alone resists greater portion of strain.

Ultimate strength of ropes varies from 7000 to 12000 lbs. per square of section, according as they are wetted, tarred, or dry. One sixth of ultimate strength is a safe working load = 1166 to 2000 lbs. per square inch.

Units for computing Safe Strain that may be borne
New Ropes, Hawsers, and Cables. (U. S. Navy.)

| DESCRIPTION. | Circumference. | ROPES. | | | | HAWSERS. | | CABLE. | |
|--------------|----------------|------------|------------|-----------|-----------|-----------|-----------|-----------|-----------|
| | | White. | | Tarred. | | White. | Tarred. | White. | Tarred. |
| | | 3 strands. | 4 strands. | 3 str'ds. | 4 str'ds. | 3 str'ds. | 3 str'ds. | 3 str'ds. | 3 str'ds. |
| | Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| White | 2.5 to 6 | 1140 | 1330 | — | — | 600 | — | — | — |
| " | 6 " 8 | 1090 | 1260 | — | — | 570 | — | 510 | — |
| " | 8 " 12 | 1045 | 880 | — | — | 530 | — | 530 | — |
| " | 12 " 18 | — | — | — | — | 550 | — | 550 | — |
| " | 18 " 26 | — | — | — | — | — | — | 560 | — |
| Tarred | 2.5 " 5 | — | — | 855 | 1005 | — | 460 | — | — |
| " | 5 " 8 | — | — | 825 | 940 | — | 480 | — | — |
| " | 8 " 12 | — | — | 780 | 820 | — | 505 | — | — |
| " | 12 " 18 | — | — | — | — | — | — | — | — |
| " | 18 " 26 | — | — | — | — | — | — | — | — |
| Manila | 2.5 " 6 | 810 | 950 | — | — | 440 | — | — | — |
| " | 6 " 12 | 760 | 835 | — | — | 465 | — | 510 | — |
| " | 12 " 18 | — | — | — | — | — | — | 535 | — |
| " | 18 " 26 | — | — | — | — | — | — | 560 | — |

ILLUSTRATION.—What weight can be borne with safety by a Manila rope strands, having a circumference of 6 inches? (See Rule, page 167.)

$$6^2 \times 760 = 27\,360 \text{ lbs.}$$

When it is required to ascertain weight or strain that can be borne ropes, etc., in general use, preceding Units should be reduced from one to two thirds, in order to meet their condition or reduction of their strength by chafing and exposure to weather. Molesworth's table is based upon reduction of three fourths.

ILLUSTRATION.—What weight can be borne by a tarred hawser of 3 strands having a circumference, in general use?

$$10^2 \times (505 - 505 \div 3) = 100 \times 336.67 = 33\,667 \text{ lbs.}$$

Destructive Strength of Tarred Hemp Ropes.

(D. K. Clark.)

| Circum. | Diam. | Register. | | Circum. | Diam. | Register. | |
|---------|-------|--------------|---------------|---------|-------|--------------|---------------|
| | | Common Cold. | Russian Warm. | | | Common Cold. | Russian Warm. |
| Ins. | Ins. | Lbs. | Lbs. | Ins. | Ins. | Lbs. | Lbs. |
| 3 | .95 | 7 390 | 8 620 | 5.5 | 1.75 | 24 800 | 29 120 |
| 3.5 | 1.11 | 11 200 | 11 760 | 6 | 1.91 | 28 985 | 33 150 |
| 4 | 1.27 | 13 100 | 15 340 | 6.5 | 2.07 | 34 030 | 40 550 |
| 4.5 | 1.43 | 16 330 | 19 440 | 7 | 2.24 | 40 320 | 47 041 |
| 5 | 1.59 | 19 580 | 23 990 | 8 | 2.54 | 52 480 | 61 420 |

specimens furnished by National Association of Rope and Twine Spinners, As tested by Mr. Kirkaldy.

| Rope. | Circumference. | Weight per Lb. | Extreme Strength. | Breaking Weight per lb. per Fathom. | Extension in 50 Ins. Length at Stress per lb. Weight per Fathom of | | |
|-------------------------------|----------------|----------------|-------------------|-------------------------------------|--|-----------|-----------|
| | | | | | 1000 lbs. | 2000 lbs. | 3000 lbs. |
| | Ins. | Lbs. | Lbs. | Lbs. | Ins. | Ins. | Ins. |
| Russian rope . . . 48 thr'ds. | 5.26 | .926 | 11 088 | 1933 | 5.29 | — | — |
| Chinese yarn . . . 50 " | 5.37 | .891 | 11 514 | 2152 | 4.53 | 6.56 | — |
| Ind-spun yarn, 51 " | 5.39 | 1.006 | 18 278 | 3024 | 4.46 | 5.91 | 6.63 |

Breaking Strength of Tarred Hemp Ropes. (Mr. Glynn.)

| Diam. | Old Method. | | By Register. | | Circum. | Diam. | Old Method. | | By Register. | |
|-------|--------------|---------------|--------------|--------|---------|-------|--------------|---------------|--------------|--------|
| | Common Hemp. | Best Russian. | Cold. | Warm. | | | Common Hemp. | Best Russian. | Cold. | Warm. |
| Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Ins. | Ins. | Lbs. | Lbs. | Lbs. | Lbs. |
| .95 | 5 056 | 6 248 | 7 392 | 8 624 | 5.5 | 1.75 | 15 456 | 18 414 | 24 797 | 29 120 |
| 1.11 | 7 466 | 8 668 | 11 200 | 11 760 | 6 | 1.91 | 18 144 | 21 610 | 28 986 | 33 150 |
| 1.27 | 8 780 | 10 460 | 13 104 | 17 810 | 6.5 | 2.07 | 20 518 | 23 610 | 34 630 | 40 544 |
| 1.43 | 10 300 | 12 432 | 16 330 | 19 443 | 7 | 2.24 | 22 938 | 27 462 | 40 320 | 47 040 |
| 1.59 | 13 328 | 15 859 | 20 496 | 23 990 | 8 | 2.54 | 26 680 | 32 032 | 52 483 | 61 420 |

Compute Strain that may be borne with safety by new Ropes, Hawasers, and Cables.

deduced from experiments of Russian Government upon relative strength of different Circumferences of Ropes, Hawasers, etc.

S. Navy test is 4200 lbs. for a White rope of three strands of best Riga 2, of 1.75 inches in circumference (= 17 000 lbs. per square inch of fibre), in preceding table (page 166) 14 000 lbs. is taken as unit of strain that be borne with safety.

NOTE.—Square circumference of rope, hawser, etc., and multiply it by 8 in table.

Compute Circumference of a Rope, Hawaser, or Cable for a Given Strain.

NOTE.—Divide strain in pounds by appropriate units in preceding table, square root of product will give circumference of rope, etc., in ins.

EXAMPLE 1.—Stress to be borne in safety is 165 550 lbs.; what should be circumference of a tarred cable to withstand it?

$$165\,550 \div 550 = 301, \text{ and } \sqrt{301} = 17.35$$

What should be circumference of a Manila cable at use, of 149 336 lbs.?

assuming circumference to exceed 18 ins., unit = 560.

$$149\,336 \div (560 - 560 \div 3) = 400, \text{ and } \sqrt{400}$$

a strain, in

To Compute Weight of Ropes, Hawasers, and Cables.

RULE.—Square circumference, and multiply it by appropriate unit in following table, and product will give weight per foot in lbs.:

| HAWSERS. | | | | CABLES. | | | |
|----------------------------|------|---------|------|----------------------------|------|---------|------|
| ROPES. | | CABLES. | | ROPES. | | CABLES. | |
| 3-strand Hemp..... | .032 | .031 | .031 | 4-strand Hemp..... | .033 | — | — |
| 3-strand tarred Hemp, .042 | .041 | .041 | | 4-strand tarred Hemp, .048 | — | — | — |
| 3-strand Manila..... | .032 | .031 | .031 | 4-strand Manila..... | .035 | .034 | .034 |

Units for Thread Ropes is same as that for Ropes of like material.

EXAMPLE.—What is weight of a coil of 10-inch Manila hawser of 4 strands of 2 fathoms?

$$10^2 \times .034 = 3.4, \text{ and } 120 \times 6 \times 3.4 = 2448 \text{ lbs.}$$

Weight and Strength of Hemp and Wire Ropes.
(Molesworth.)

$$C^2 y = W; \quad C^2 k = L; \quad C^2 x = S; \quad \text{and } \sqrt{\frac{L}{k}} = C.$$

C representing circumference in ins., *W* weight of rope in lbs. per fathom
L working load in tons, and *S* destructive stress in tons.

VALUES OF *y*, *x*, AND *k*.

| ROPES. | <i>y</i> | <i>x</i> | <i>k</i> | ROPES. | <i>y</i> | <i>x</i> | <i>k</i> |
|---------------------------|----------|----------|----------|---------------------|----------|----------|----------|
| Hawser, hemp..... | .131 | — | — | Warm register, hemp | — | .7 | .18 |
| Cable "..... | .117 | — | — | Manila hawser..... | .177 | .27 | .04 |
| Tarred hawser, hemp. .235 | .22 | .037 | | " cable..... | .155 | .19 | .03 |
| " cable, " .207 | .15 | .025 | | Iron rope..... | .87 | 1.8 | .20 |
| Cold register, " . | .6 | .1 | | Steel "..... | .89 | 2.8 | .45 |

To Compute Circumference of Hemp or Wire Rope for Fore or Main Standing Rigging. (U. S. Navy.)

RULE.—To length of mast between partners and deck, add half extreme breadth of beam of vessel and divide sum by half extreme breadth. Multiply quotient by half square root of tonnage (OM) and extract square root of product.

For Mizzen, take .74 of Fore and Main.

EXAMPLE.—Required circumference of hemp rope, for main-mast of a vessel having a breadth of beam of 45 feet and a burden of 3213 tons?

| | | |
|--|-------|--------|
| Extreme length of mast..... | 94.4 | feet. |
| Depth of hold, or total bury of mast, 21.4 | feet. | |
| Head..... | 15 | 36.4 " |
| Breadth of beam, 45 feet. | 58 | " |

$$58 + \frac{45}{2} \div \frac{45}{2} = 3.58, \text{ and } \sqrt{\left(3.58 \times \frac{\sqrt{3213}}{2}\right)} = \sqrt{101.46} = 10.11 \text{ ins.}$$

Then if circumference for a wire rope is required, see table, page 164.

Thus, a hemp rope 10 ins. in circumference has equivalent strength of an iron wire rope of 4 ins. and a steel rope of 3.25+ ins.

Galvanized Iron Wire.—Experiments at Navy Yard, Washington, gave for the ability a mean loss of 30 per cent., and for tensile strength a like loss of 13.5 per cent.

Relative Dimensions of Hemp Rope and Iron and Steel Wire Rope. (U. S. Navy.)

Circumference in Inches.

| | | | | | | | | | | |
|---|-----|------|-------|------|------|------|-----|-------|-------|-----|
| 4 | 4.5 | 5.25 | 6.5 | 7.75 | 8.5 | 9.5 | 11 | 11.75 | 13.5 | 16. |
| | | | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 7 |
| | | | 2.125 | 2.5 | 2.75 | 3.25 | 3.5 | 4 | 4.375 | 5 |

ANCHORS, CABLES, ETC.

or, Chains, etc., for a Given Tonnage.

(American Shipmasters' Association.)

SAILS.

| Anchors. | | | | CHAINS, CABLES, ETC. | | | | | | |
|----------|------------------|--------|--------------|----------------------|---------|------------------------|---------------------|----------------|--------------|--------------|
| No. | Including Stock. | | | Diameter. | Length. | Adm- ralty Test. | Weights per Fathom. | | | Eng- ine. |
| | Stream. | Kedge. | at Kedge. | | | | Small. | Short Line. | Eng- ine. | |
| 7 | 168 | 84 | — | .8125 | 90 | 11 | 40 | 42 | 35 | |
| 8 | 196 | 112 | — | .875 | 105 | 13 | 44 | 45 | — | |
| 9 | 224 | 112 | — | .9375 | 120 | 15 | 51 | 55 | 48 | |
| 10 | 280 | 140 | — | 1 | 120 | 17.5 | 59 | 63 | 54 | |
| 1 | 336 | 168 | — | 1.0625 | 120 | 20 | 66 | 70 | — | |
| 2 | 392 | 196 | — | 1.125 | 120 | 22.5 | 75 | 79 | 68 | |
| 3 | 448 | 224 | 112 | 1.1875 | 135 | 25 | 82 | 88 | — | |
| 4 | 504 | 252 | 126 | 1.25 | 135 | 28 | 91 | 98 | 84 | |
| 5-5 | 560 | 280 | 140 | 1.3125 | 150 | 31 | 100 | 106 | — | |
| 7 | 616 | 308 | 154 | 1.3125 | 150 | 31 | 100 | 106 | — | |
| 8-5 | 672 | 336 | 168 | 1.375 | 165 | 37 | 115 | 118 | 102 | |
| 10 | 784 | 392 | 196 | 1.4375 | 165 | 40 | 120 | — | — | |
| 2 | 896 | 448 | 224 | 1.5 | 180 | 44 | 132 | — | 122 | |
| 4 | 1008 | 504 | 252 | 1.5625 | 180 | 47 | 145 | — | — | |
| 5 | 1120 | 560 | 280 | 1.625 | 180 | 51 | 150 | — | 143 | |
| 8 | 1232 | 616 | 308 | 1.6875 | 180 | 55 | 162 | — | — | |
| 9-5 | 1344 | 672 | 336 | 1.75 | 180 | 59 | 175 | — | 166 | |
| 11 | 1456 | 738 | 364 | 1.875 | 180 | 63 | 189 | — | 191 | |
| 2-5 | 1568 | 784 | 392 | 1.9375 | 180 | 67 | 205 | — | — | |
| 4 | 1680 | 840 | 420 | 2 | 180 | 72 | 219 | — | — | |
| 5-5 | 1792 | 896 | 448 | 2 | 180 | 72 | 240 | — | 217 | |
| 7 | 1904 | 952 | 504 | 2.0625 | 180 | 81 | — | — | — | |
| 9 | 2128 | 1120 | 560 | 2.125 | 180 | 86 | — | — | 244 | |
| 11 | 2352 | 1232 | 616 | 2.1875 | 180 | 90 | — | — | — | |

† Brown, Lamont, & Co.

To Compute Tonnage.

Dimensions as follows. *Length*.—From after-side of stern to fore-
most post, measured on spar or upper deck in vessels having
1 under, and on main deck in vessels having three or more
decks.—*Extreme* at widest point. *Draft*.—At forward or main
deck, from top of ceiling at side of keelson to under side of keel.
Apply these dimensions together, divide product by 100, and
omit.

Have 2 bowers and 1 each stream and 1 kedge anchor, and for
adding 1400 a third bower is recommended.

Ways to be 60 fathoms in length.

Shrouds.

Size. *Hemp*.—5.75 ins. in diameter for a tonnage of 100 tons,
increasing up to 12.75 ins. for 3000 tons.

Size. *Wire*.—From .25 to 1 inch in diameter proportion-
ately.

Number of hemp, increasing very slightly as
tonnage increases. 12.75 ins. for hemp and 6.875 ins.
for wire.

(American Shipmasters' Association.)

STEAM.

| Tonnage computed as per Rule preceding. | ANCHORS. | | | | | | CHAIN CABLE.—STEEL. | | | | | |
|--|------------------------|----------------------|------------------|--------|---------------|--------|---------------------|---------|----------------------|------------------|------------------|--|
| | Bowers. | | Including Stock. | | | | Diam- eter. | Length. | Admiral- ty Test. | Diam. Stream. | Weight per Foot. | |
| | With- out Stock. | Admiral- ty Test. | Stream. | Kedge. | ad Kedges. | Stud. | | | | | Short Link. | |
| | | | | | | | | | | | | |
| Lbs. | Tons. | Lbs. | Lbs. | Lbs. | Ins. | Faths. | Tons. | Ins. | Lbs. | Lbs. | | |
| 100 | 336 | 4.9 | 112 | — | — | .6875 | 105 | 8.1 | .5 | — | — | |
| 150 | 448 | 6.4 | 196 | — | — | .8125 | 120 | 11.9 | .5625 | 40 | 42 | |
| 200 | 616 | 7.6 | 224 | — | — | .875 | 120 | 13.8 | .5625 | 44 | 48 | |
| 250 | 672 | 8.2 | 280 | — | — | .9375 | 120 | 15.8 | .625 | 51 | 55 | |
| 300 | 812 | 9.5 | 308 | — | — | 1 | 120 | 18 | .625 | 59 | 63 | |
| 350 | 924 | 10.4 | 336 | — | — | 1.0625 | 120 | 20.3 | .6875 | 66 | 70 | |
| 400 | 1120 | 12 | 532 | 252 | — | 1.125 | 135 | 22.8 | .6875 | 75 | 79 | |
| 450 | 1344 | 13.9 | 560 | 280 | — | 1.1875 | 135 | 25.4 | .75 | 82 | 88 | |
| 500 | 1512 | 15.2 | 672 | 336 | — | 1.25 | 150 | 28.1 | .75 | 91 | 98 | |
| 600 | 1708 | 16.7 | 738 | 364 | — | 1.3125 | 150 | 31 | .8125 | 100 | 106 | |
| 700 | 1876 | 18 | 784 | 392 | — | 1.375 | 165 | 34 | .8125 | 115 | 118 | |
| 800 | 2026 | 19 | 866 | 448 | 224 | 1.4375 | 165 | 37.2 | .875 | 120 | — | |
| 900 | 2352 | 21.6 | 1008 | 504 | 252 | 1.5 | 180 | 40.5 | .875 | 132 | — | |
| 1000 | 2632 | 23.5 | 1120 | 560 | 280 | 1.5625 | 180 | 44 | .9375 | 145 | — | |
| 1200 | 2856 | 25.2 | 1176 | 588 | 308 | 1.625 | 180 | 47.5 | .9375 | 156 | — | |
| 1400 | 3108 | 26.9 | 1232 | 616 | 308 | 1.6875 | 180 | 51.2 | 1 | 162 | — | |
| 1600 | 3360 | 28.6 | 1344 | 672 | 336 | 1.75 | 180 | 55.1 | 1 | 175 | — | |
| 1800 | 3584 | 30.1 | 1456 | 738 | 364 | 1.8125 | 180 | 59.1 | 1.0625 | 189 | — | |
| 2000 | 3808 | 31.6 | 1512 | 766 | 364 | 1.875 | 180 | 63.3 | 1.0625 | 205 | — | |
| 2300 | 4088 | 33.4 | 1568 | 784 | 392 | 1.9375 | 180 | 67.6 | 1.125 | 215 | — | |
| 2600 | 4256 | 34.5 | 1624 | 812 | 392 | 2 | 270 | 72 | 1.125 | 240 | — | |
| 3000 | 4480 | 35.7 | 1680 | 840 | 420 | 2.0625 | 270 | 76.6 | 1.1875 | — | — | |
| 3500 | 4592 | 37 | 1792 | 896 | 476 | 2.125 | 270 | 81.3 | 1.1875 | — | — | |
| 4000 | 4816 | 38 | 1960 | 952 | 504 | 2.1875 | 270 | 86.1 | 1.25 | — | — | |
| 4500 | 5040 | 39.2 | 2128 | 1064 | 532 | 2.25 | 270 | 91.1 | 1.25 | — | — | |
| 5000 | 5264 | 41 | 2352 | 1120 | 560 | 2.3125 | 270 | 96 | 1.3125 | — | — | |

* Brown, Lennox, & Co.

ANCHORS AND KEDGES.

(U. S. Navy.)

To Compute Weight of a Bower Anchor for a Vessel of a given Character and Rate.

RULE.—Multiply approximate displacement in tons, by unit in following table, and product will give weight in lbs., inclusive of stock.

Units to determine Weights and Number of Anchors or Kedges.

| Displacement of Vessel in Tons. | Unit. | Bower. | Sheet. | Stream. | Kedge. | Displacement of Vessel in Tons. | Unit. | Bower. | Sheet. |
|---------------------------------|-------|--------|--------|---------|--------|---------------------------------|-------|--------|--------|
| Over 3700 | 1.75 | 2 | 2 | 1 | 4 | Over 1500 .. | 2.5 | 2 | 2 |
| “ 2400 | 2 | 2 | 2 | 1 | 3 | “ 900 .. | 2.75 | 2 | 1 |
| “ 1900 | 2.25 | 2 | 2 | 1 | 3 | 900 and under | 3 | 2 | 1 |

EXAMPLE.—Tonnage of a bark-rigged steamer is 1500.

$$1500 \times 2.5 = 3750 \text{ lbs., weight of anchor.}$$

* and Sheet Anchors should be alike in weight.

Anchors and Kedges are proportional to weight of bowers. Anchor .25 weight. Kedges.—If 1, .125 weight; if 2, .06 weight; if 3, .16, .125, and .1 weight.

Compute Diameter of a Chain Cable corresponding to a Given Weight of Anchor.

(*U. S. Navy.*)

—Cut off the two right-hand figures of the anchor's weight in lbs., square root of remainder by 4, and result will give diameter of sixteenths of an inch.

EX.—The weight of an anchor is 2500 lbs.

$$\sqrt{25.00 \times 4} = 20 \text{ sixteenths} = 1.25 \text{ ins.}$$

—Diam. of a messenger should be .66 that of the cable to which it is applied.

Lengths of Chain Cables for each Anchor.

(*U. S. Navy.*)

| Anchor. | Bower. | Sheet. | Stream. | Weight of Anchor. | Bower. | Sheet. | Stream. |
|---------|----------|----------|----------|-------------------|----------|----------|----------|
| | Fathoms. | Fathoms. | Fathoms. | Lbs. | Fathoms. | Fathoms. | Fathoms. |
| 800 | 60 | 60 | 60 | Over 2000 | 120 | 120 | 90 |
| 800 | 90 | 90 | 60 | " 3000 | 120 | 120 | 90 |
| 1200 | 90 | 90 | 75 | " 5000 | 120 | 120 | 105 |
| 1600 | 105 | 105 | 75 | " 7500 | 135 | 135 | 105 |

ANCHORS.

Experiments of a Joint Committee of Representatives of Ship-owners and Admiralty of Great Britain.

Anchor of ordinary or Admiralty pattern, Trotman or Porter's (pivot fluke), Honiball, Porter's, Aylin's, Rodgers's, Mitcheson's, and 's, each weighing, inclusive of stock, 27 000 lbs., withstood without proof strain of 45 000 lbs.

—Comparing weights between a Porter and Admiralty anchor, as tested at the Dock-yard, were as 43 to 14.

Comparative Resistance to Dragging.

—Comparing the resistance of the following anchors: Trotman's dragged Aylin's, Honiball's Mitcheson's and Lennox's; Aylin's Mitcheson's dragged Rodgers's; and Rodgers's and Lennox's dragged Trotman's.

TONNAGE OF VESSELS.

To Compute Tonnage of Vessels.

—For the United States of America, with amendments of 1882 relative to tonnage, see *Mechanics' Tables*, with rule and illustrated diagrams, by H. Haswell, 3d edition, Harper & Bros., New York, 1878.

English Registered Tonnage. (*New Measurement.*)

—Divide length of upper deck between after-part of stem and fore-part of stern-post into 6 equal parts, and note foremost, middle, and aftermost points of division. Divide depths at these three points in feet and tenths of a foot; also depths from the top of upper deck to ceiling of limber-strake; or in case of a break in the deck, from a line stretched in continuation of the deck. For breadths, divide the length into 5 equal parts, and measure the inside breadths at following points, to wit: .2 and .8 from upper deck of foremost and aftermost depths; and from the top of upper deck of amidship depth. Take length at half amidship depth between fore-part of stem to fore-part of stern-post.

—To twice amidship depth add foremost and aftermost depths for *sum of depths*; and add together foremost upper and lower breadths, 3 times upper breadth of foremost breadth at amidship, and upper and twice lower breadth at after division of breadths.

—Multiply together sum of depths, sum of breadths, and length, and divide product which will give number of tons.

—If a vessel has a poop or half-deck, or a break in upper deck, measure the length, breadth, and height of such part thereof as may be included in the *sum of depths*; multiply these three measurements together, divide product by 100, and the result will give number of tons to be added to result as above ascertained.

For Open Vessels.—Depths are to be taken from upper edge of upper strake.

For Steam Vessels.—Tonnage due to engine-room is deducted from total tonnage computed by above rule. To determine this, measure inside of the engine-room from foremost to aftermost bulkhead; then multiply this length by amidship depth of vessel, and product by inside amidship breadth at .4 of depth from deck, and divide final product by 92.4.

The volume of the poop, deck-houses, and other permanently enclosed spaces, available for cargo or passengers, is to be measured and included in the tonnage, but following deductions are allowed, the remainder being the *Register tonnage*.

Deductions.—Houses for the shelter of passengers only; space allotted to crew (12 square feet in surface and 72 cube feet in volume for each person); and space occupied by propelling power.

Approximate Rule.

Gross Register.—Tonnage of a vessel expresses her entire cubical volume in tons of 100 cube feet each, and is ascertained by following formula :

$$\frac{L B D}{100} = \text{Gross tonnage, and } \frac{L B D}{100} c = \text{Register tonnage. } L \text{ representing length of keel between perpendiculars, } B \text{ breadth of vessel, and } D \text{ depth of hold, all in feet.}$$

Builders' Measurement.

$$\frac{(L - .6 B) \times B \times .5 B}{94} = \text{Tonnage.}$$

Fore-perpendicular is taken at fore-part of stem at height of upper deck.

Aft-perpendicular is taken at back of stern-post at height of upper deck.

In three-deckers, middle deck is taken instead of upper deck.

Breadth is taken as extreme breadth at height of the wales, subtracting difference between thickness of wales and bottom plank. Deductions to be made for rake of stem and stern.

$$\text{Iron Vessels. } \frac{18}{10000} \left(\frac{\text{Girth} + \text{Breadth}}{2} \right)^2 \times \text{length} = \text{Gross tonnage.}$$

Length measured on upper deck, between outside of outer plank at stem and the after-side of stern-post and rabbet of stern-post, at point where counter-plank crosses it. Girth measured by a chain passed under bottom from upper deck at extreme breadth, on one side, to corresponding point on the other.

$$\text{Register tonnage} = \frac{L \times B \times D}{100} \times C. \quad C \text{ representing a coefficient for vessels as follows:}$$

| | | | |
|---------------------------------------|-----|----------------------------|-----|
| Ships of usual form..... | .7 | Yachts above 60 tons..... | .5 |
| Clippers and Steamers { 2 decks... .. | .65 | Small vessels { sharp..... | .45 |
| { 3 " .68 | | { very sharp..... | .4 |

Units for Measurement and Dead-weight Cargoes.

(C. Mackrow, M. S. N. A.)

To Compute Approximately for an Average Length of Voyage the Measurement Cargo, at 40 feet per Ton, which a Vessel can carry.

RULE.—Multiply number of register tons by unit 1.875, and product will give approximate measurement cargo.

To Compute Approximately Dead-weight Cargo in Tons which a Vessel can carry on an Average Length of Voyage.

RULE.—Multiply number of register tons by 1.5, and product will give approximate dead-weight cargo required.

With regard to cargoes of coasters and colliers, as ascertained above, about 75 per cent. may be added to said results, while about 10 per cent. may be deducted in cases of larger vessels on longer voyages.

case of measurement cargoes of steam-vessels, spaces occupied by masonry, fuel, and passenger cabins under the deck must be deducted from the gross tonnage under deck before application of measurement unit thereto.

case of dead-weight cargoes, weight of machinery, water in boilers, and other items must be deducted from whole dead weight, as ascertained above by application of dead-weight unit.

The deductions necessary for provisions, stores, etc., are allowed for in the application of the two units.

Ascertain Weight of Cargo for an Average Length of Voyage. (Moorsom.)

Deduct tonnage of spaces of passenger accommodations from net register tonnage, and multiply remainder by 1.5.

Average space for each ton weight of cargo on such a voyage 67 cube feet.

Freight Tonnage or Measurement Cargo.

Freight Tonnage or Measurement Cargo is 40 cube feet of space for cargo, it is about 1.875 times net register tonnage less that for passenger space.

Royal Thames Yacht Club.

Measure length of yacht in a straight line at deck from fore-part of stem to after-part of stern-post, from which deduct extreme breadth (measured from outside of the planking), both in feet; remainder is length for tonnage. Multiply length by extreme breadth, that product by half extreme breadth, divide result by 94, and quotient will give tonnage.

Any part of stem or stern-post projects beyond length as taken above, such extension or projections shall, for purpose of computing tonnage, be added to length as before mentioned.

Fractional parts of a ton are to be considered as a ton.

Measurements to be taken either above or below main walcs.

$$\frac{L - B \times B \times .5 B}{94} = \text{Tons. } L \text{ representing length and } B \text{ breadth, in feet.}$$

Corinthian and New Thames Yacht Club.

Measure length and breadth as in foregoing rule, and depth to top of covering; multiply length, breadth, and depth together, divide result by 200, and quotient will give tonnage.

$$\frac{L \times B \times D}{200} = \text{Tons.}$$

Suez Canal Tonnage.

Net Tonnage.—Spaces under tonnage deck, below tonnage and uppermost deck, and enclosed or closed-in spaces, such as poop, fore-castle, officers' cabins, galley, hold, and wheel houses, and all inclosed or covered-in spaces for working

which are to be deducted berthing accommodations for crew, not including stewards and passengers' servants; berthing accommodations for officer captain; galleys, cook-houses, etc., used exclusively for crew, and incl. above uppermost deck, designed for working the vessel. In none of the above can passengers be berthed or cargo carried, and total deduction under all cases must not exceed 5 per cent. of gross tonnage. Owners with standing coal-bunkers, English rule may be followed, or owners may elect to have tonnage of his vessel computed by "Danube rule," which is a case of 50 per cent. above space allowed to machinery in side-wheel steamers and screw steamers.

case, however, except with tow-boats, must deduction for passengers 10 per cent. of gross tonnage.

WORKS OF MAGNITUDE.

American.

Aqueducts, Roads, and Railroads.

Croton Aqueduct, N. Y. — Has a section of 53.34 square feet and capacity of 100 000 000 to 118 000 000 gallons per day, and from Dam to Receiving Reservoir is 38.134 miles in length.

Aqueduct, Washington. — Cylinder of masonry 9 feet in diameter. Stone arch over Cabin John's Creek, 220 feet span, 57.25 feet rise.

National Road. — Over the Alleghany Mountains, Cumberland to Illinois Town, 650.625 miles in length, and 80 feet in width. Macadamized for a width of 30 feet.

Illinois Central Railroad. — Chicago to Cairo, length 365 miles, Centralia to Dan-leith 344 miles, total 709 miles.

Bridges.

Suspension Bridge, Niagara River. — Wire, Span 1042 feet 10 ins.

Suspension Bridge, New York and Brooklyn. — Length of river span 1595 feet 6 ins.; of each land span 930 feet; length of Brooklyn approach 971 feet; of N. Y. approach 1562 feet 6 ins.; total length of bridge 5989 feet; width 85 feet; number of cables 4; diameter of each cable 15.5 ins.; each consisting of 6300 parallel steel wires No. 7 gauge, closely laid and wrapped to a solid cylinder; ultimate strength of each cable 11 200 tons; depth of tower foundation below high water, Brooklyn, 45 feet — New York 78 feet; towers at high-water line 140 X 59 feet; towers at roof course 136 X 53 feet; total height of towers above high water 277 feet; clear height of bridge in centre of river span above high water, at 50°, 135 feet; height of floor at towers above high water 119 feet 3 ins.; grade of roadway 3 feet in 100; anchor-ages, at base 129 X 119 feet, at top 117 X 104 feet; weight of each anchor-plate 23 tons.

Iron Pipe Bridge over Rock Creek. — 200 feet span, 20 feet rise. Arch of 2 lateral courses of cast-iron pipe, 4 feet internal diameter, and 1 inch thick. These pipes conveying the water not only sustain themselves over the great span, but support a street road and railway.

Iron Bridge over Kentucky River near Shakers' Ferry, Md. — 3 spans, each 375 feet, and 275.5 feet above low water.

Bridge on line of New York, Erie, and Western Railroad across the Kinsua. — Of iron; length 2060 feet; central span 301 feet in height.

Iron Truss. — Cincinnati and Southern Railway, over Ohio River, 519 feet.

Foreign.

Pyramids, Statues, etc.

Pyramid of Cheops, Egypt. — Length of side at base 762 feet; height to present summit 453.3 feet; to original summit 485.2 feet; inclined length 568.25 feet; angle of side 51° 51' 14"; area of each face = square of height; weight 5 272 600 tons; built 2170 years B.C.

Peter the Great, St. Petersburg, Russia. — Bronze; height of horse 17 feet; of man 11 feet; base of rock 42 feet at bottom, 36 at top, 21 wide, and 17 high, weighing 1100 tons.

Liberty, New York Harbor. — Bronze; 110 feet in height from head to foot and 151.1 feet to flambeau; including base, 305.6 feet. Weight of statue 225 tons.

Daibutsu, of stone, Japan. — Sitting posture; height 44 feet, circumference 87 feet; face 8.5 feet; circumference of thumb 3 5 feet.

Colossus of Rhodes. — Height, 105 feet.

Bridge.

War Bridge. — Of iron, with a double line of Railway, 964 feet in approaches of 230 feet each. Weight 3658 tons.

Monoliths.

Obelisk at Karnak, Egypt.—Of granite, 108 feet 10 ins.; pedestal 13 feet 2 ins.; ght 400 tons.

Obelisk in Central Park, N. Y.—Of granite, 68 feet 11 ins.; weight 168 tons.

T. S. Treasury, Washington.—Some stones of, are heavier than any in the Pyramids of Egypt.

Steam Hammers.

at workshops of Herr Krupp, at Essen, there is a steam hammer weighing 50 tons, giving a fall of 3 metres; and at Creusot there is a hammer weighing between 75 and 80 tons having a fall of 5 metres.

Crane.

at Creusot there is a steam crane having a capacity to lift and revolve with 150 tons.

Chimneys.

Townsend's chemical works, Glasgow, diameter at foundation 50 feet; at top 8 feet 8 ins.; height from foundation 488 feet; from ground 474 feet.

New York Steam Heating Co., 220 feet in height.

Pillar.

at a gate near Delhi is a wrought-iron pillar having diameters of 16.4 ins. at 22 feet in its height above ground and 12 ins. at its top. It is estimated from the records of excavations at its base to be 60 feet in length or height and to weigh 17 tons. Its period of structure is assigned to the 3d or 4th century A.D.

Roofs.

and Railway Station, London. 240 ft. | *Union Railway Station, Glasgow.* 195 ft.
Imperial Riding-School, Moscow. 235 " | *Grand Central Station, N. Y.* 200 "

Diameters of Domes.

| DOME. | Feet. | DOME. | Feet. | DOME. | Feet. |
|---------------------|-------|----------------------|-------|-------|-------|
| St. Paul's, London. | 112 | Midland Rail'y. Lon. | 240 | | |
| St. Peter's, Rome. | 139 | Great North'n, Eng. | 210 | | |

Lengths of Tunnels.

| TUNNEL. | Feet. | TUNNEL. | Feet. | TUNNEL. | Feet. |
|----------------|-------|----------------|-------|--------------|-------|
| Great Eastern. | 11880 | Gunpowder, Md. | 36500 | Nerthe. | 15153 |
| Goathead. | 4280 | Sutro. | 20028 | Nochistongo. | 21659 |
| St. Lawrence. | 25031 | Semmering. | 5630 | Riquivel. | 18623 |

Thames and Medway, 11880 feet. Weehawken, 4000 feet.

at Genis 7.5 miles 242 yards, rises 1 in 45, and descends 1 in 2000.

Goathead Tunnels and Roads 9 miles 477 yards in length; tunnels 116 156.5 feet, rises 1 in 233 in whole length; 26.5 feet in width; 19 feet 10 ins. in height. Mum grade 2.7 feet per 100. Schemnitz, 10.27 miles in length, 9 feet 10 ins. light by 5.25 feet in width.

Miscellaneous.

Fortress Monroe, Old Point Comfort, Va.—Largest fortress.

Graph Wire.—Span over river Kistnah between Bozorah and Sectanag, set in length.

Frederik Park, Copenhagen.—4200 acres.

Trinity College, England.—Largest University; said to have been founded

Imperial St. Peter's, Rome.—Width of front 216 feet; of the cross 251 feet; height 469.5 feet.

Mersey Great Eastern.—Of iron, 680 feet in length; 83 feet width of beam, depth of hold; 22927 tons; built at Millwall, England, 1857.

Wessex Wall.—25 feet at base; 15 at top; height, with a parapet of 1250 miles.

Wells Well, Perth.—3050 feet in depth; temperature of water varies 18000 gallons per day.

Weights of Bells.

| BELLS. | Lbs. | BELLS. | Lbs. | BELLS. | Lbs. |
|--------------------|---------|---------------------|---------|---------------------|--------|
| Pekin..... | 120 000 | Oxford, "Great | | St. Peter's, Rome. | 18 000 |
| Lewiston, Me..... | 10 233 | Tom," Eng..... | 27 024 | Vienna..... | 40 000 |
| Montreal, Can..... | 28 560 | Olmütz, Bohemia. | 40 320 | Westm'ster, "Big | |
| Moscow, Russia.... | 443 772 | Rouen, France.... | 40 000 | Ben," England. | 35 600 |
| Erfurt, Saxony.... | 30 800 | St. Paul's, Eng.... | 42 000 | York..... | 24 000 |
| Notre Dame, Paris | 28 670 | St. Ivan's, Moscow | 127 830 | State House, Phila. | 13 000 |

Rangoon, Burmah, 201 600 lbs.

Capacity of Principal Churches and Opera Houses.

Estimating a person to occupy an Area of 19.7 Ins. Square.

Churches.

| | | | |
|-----------------------------------|--------|-----------------------------|--------|
| St. Peter's..... | 54 000 | St. John, Lateran..... | 23 000 |
| Milan Cathedral..... | 37 000 | Notre Dame, Paris..... | 21 300 |
| St. Paul's, Rome..... | 32 000 | Pisa Cathedral..... | 13 000 |
| St. Paul's, London..... | 25 600 | St. Stephen's, Vienna..... | 19 400 |
| St. Petronio, Bologna..... | 24 400 | St. Dominic's, Bologna..... | 13 000 |
| Florence Cathedral..... | 24 300 | Tabernacle, London..... | 7 000 |
| Antwerp Cathedral..... | 24 000 | " Brooklyn..... | 5 500 |
| St. Sophia's, Constantinople..... | 23 000 | St. Mark's, Venice..... | 7 000 |

Opera Houses and Theatres.

| | | | |
|--------------------------------|------|----------------------------------|------|
| Carlo Felice, Genoa..... | 2560 | Teatro del Liceo, Barcelona..... | 4200 |
| Opera House, Munich..... | 2370 | Covent Garden, London..... | 2800 |
| Alexander, St. Petersburg..... | 2332 | Opera House, Berlin..... | 1650 |
| San Carlos, Naples..... | 2240 | New York Academy..... | 1500 |
| Imperial, St. Petersburg..... | 2160 | Metropolitan Opera, N. Y..... | 3000 |
| La Scala, Milan..... | 2113 | Philadelphia Academy..... | 3200 |
| Academy of Paris..... | 2092 | Chicago..... | 3000 |

Heights of Columns, Towers, Domes, Spires, etc.

| LOCATIONS. | Feet. | LOCATIONS. | Feet. |
|----------------------------------|-------|------------------------------|-------|
| CHIMNEYS. | | TOWERS AND DOMES. | |
| Townsend's.....Glasgow.. | 474 | Cathedral.....Florence.. | 350.5 |
| St. Rollox....." | 455.5 | ".....Magdeburg | 339.9 |
| Musprat's.....Liverpool.. | 406 | ".....Milan..... | 430 |
| Gas Works.....Edinburgh | 341.5 | ".....Petersburg | 303 |
| New England Glass Co. Boston.... | 230 | Leaning.....Pisa..... | 186 |
| Steam Heating Co.....New York. | 220 | Porcelain.....China..... | 200 |
| COLUMNS. | | St. Mark's.....Venice..... | 306 |
| Alexander.....St. Peters'g | 175. | St. Paul's.....London... | 353.1 |
| Bunker Hill.....Mass..... | 221 | SPIRES. | |
| City.....London..... | 202 | Cathedral.....New York. | 395 |
| July.....Paris..... | 157 | ".....Strasbourg. | 409.9 |
| Napoleon....." | 132 | ".....Antwerp.. | 404.4 |
| Nelson's.....Dublin..... | 134 | Grace Church.....New York. | 216 |
| Nelson's.....London..... | 171 | Freiburg..... | 410 |
| Place Vendôme.....Paris.... | 136 | Salisbury..... | 430 |
| Pompey's Pillar.....Egypt.... | 114 | St. John's.....New York. | 220 |
| Trajan.....Rome..... | 145 | St. Paul's....." | 200 |
| Washington.....Wash'gton | 555 | St. Mary's.....Lübeck... | 404 |
| York.....London..... | 138 | Trinity Church.....New York. | 206 |
| TOWERS AND DOMES. | | Balustrade of Notre | |
| Babel..... | 680 | Dame.....Paris..... | 216 |
| Babel..... | 500 | Towers of ditto..... | 239.9 |
| Cambridge.....Wash'gton | 287.5 | Hôtel des Invalides....." | 344 |
|Rome..... | 469.5 | St. Nicholas.....Hamburg.. | 473 |
|Cologne..... | 524.9 | St. Stephen.....Vienna.... | 413 |
|Cremona..... | 392 | Strasbourg..... | 406 |
|Escorial... 200 | | Utrecht..... | 414 |
| | | Votive Church.....Vienna.... | 17 |

Areas of Lakes in Europe, Asia, and Africa.

| RS. | Sq. Miles. | LAKES. | Sq. Miles. | LAKES. | Sq. Miles. |
|----------|------------|--------------------|------------|---------------------|------------|
| | 400 | Dembia, Abyssinia. | 13 000 | Lough Neagh, Irel'd | 80 |
| ica..... | 11 600 | Loch Lomond..... | 27 | Tonting, China..... | 1200 |

Lengths of Bridges.

| GES. | Feet. | BRIDGES. | Feet. | BRIDGES. | Feet. |
|--------|-------|---------------------|-------|--------------------|-------|
| | 1710 | Lyons..... | 1560 | Potomac..... | 5300 |
| | 1874 | Menai..... | 1050 | Riga..... | 2600 |
| | 2500 | N. Y. and Brook- } | | St. Lawrence Riv'r | 9144 |
| S..... | 995 | lyn spans and } | 5989 | Strasburg..... | 3390 |
| | 3483 | approaches.... } | | Vauxhall..... | 860 |
| | 950 | Pont St. Esprit.. } | 3060 | Westminster..... | 1223 |

Lengths of Spans of Bridges.

| GES. | Feet. | BRIDGES. | Feet. | BRIDGES. | Feet. |
|-------|-------|---------------------|-------|-----------------|-------|
| | 460 | Niag'a at the Falls | 1268 | Schuylkill..... | 340 |
| | 400 | " at Queens- | | Southwark..... | 240 |
| | 580 | town..... | 1040 | Wheeling..... | 1010 |

Canals.

—Lake Erie to Albany 352 miles; Chesapeake and Ohio 307; Schuylkill ware and Hudson 109; Rideau 132; London to Liverpool 265; Caledonia 100 and Leeds 127.5; Rhone to Rhine 203.

7 of Locks of Erie 240 tons, and of Welland 1500.

26.77 miles. Lake Erie to Montreal via Canal 70.5; Lake and River

1 to Kingston.—Canal 120 miles; River 126.25. Suez, see page 183.

Breakwaters.

2.—Average depth of water 29.4 feet below low-water level; range of tide Outer slope 45°; Inner slopes 1.5, 5, 3, and 1.3 to 1; length of base 172.12

1.—Outer slopes 1.75 to 1 from bottom to 7 feet 6 ins. below low-water 1 to low-water line; 16 to 1 to 4 feet 6 ins. above low-water line; 5 to 1 ter; Inner slope 1.5 to 1 above low-water line; 2 to 1 below low-water line. f water at high tide 46.5 feet; at low tide 30 feet.

breakwater cased with large squared stones cramped together.

2.—Depth of high water 58 feet; of low water 51 feet. Outer slopes 1 to 1 om to 20 feet below low water; 2 to 1 to 12 feet below low water; 6 to 1 ter line; 4 to 1 to high-water line; Inner slope 1.25 to 1.

breakwater, rubble, with crest wall of ashlar.

3.—Depth of high-water line 61 feet; of low-water line 42 feet.

breakwater, concrete blocks faced with granite; batter 3 inches to the ed up in each course.

4.—Depth of water 33 feet; Outer casing of beton 25.5 tons each; average of casing from 14 to 20 feet; slope 1 to 1 from bottom to water line; 2.5 e water-line; all other slopes .33 to 1; Inner casing of first-class rubble 2 to 5 tons weight), about 12 feet thick; Hearting, second-class rubble 1.5 to 2 tons weight), about 6 feet thick; Nucleus, of quarry rubbish.

5.—Depth of water 50 feet; rubble base carried up to 33 feet from surface ; remainder composed of large beton blocks 25.5 tons each; slopes of rul ; Outer slope of beton blocks 1.25 to 1; Inner slope of beton blocks 1 t id (Suez Canal).—Concrete blocks, 10 cubic metres each, composed of 12 lbs lime to 13 of sand, mixed with sea water; 4 days in the mold and d this before being put in position. In some instances the composition s is .33 lime or cement to .66 sand and broken stone, about the size of

or Block Filling.—Proportion of interstices to volume of breakwater 1st-class rubble of 2 to 5 tons, .25; second-class rubble of .5 to 1 rubble, quarry chips, etc., .16; beton blocks, 15 to 25 tons, .1.

For force of water, see Waves of the Sea, page 853.

Areas, Depths, and Heights of Great Northern Lakes of United States.

| LAKES. | Length. | Breadth. | Mean Depth. | Height above Sea. | Area. |
|----------------|---------|----------|-------------|-------------------|------------|
| | Miles. | Miles. | Feet. | Feet. | Sq. Miles. |
| Erie..... | 250 | 80 | 200 | 564 | 9 900 |
| Huron..... | 200 | 160 | 120 | 574 | 23 800 |
| Michigan..... | 360 | 109 | 900 | 587 | 22 000 |
| Ontario..... | 180 | 65 | 500 | 234 | 7 200 |
| Superior*..... | 400 | 160 | 288 | 635 | 32 000 |

* Greatest depth 5400 feet.

Elevation Above Tide-water at Albany. — Lake Erie 570.6 feet; Hudson River 2.46 feet.

Mean Depths and Areas of the Oceans and Seas.

(Herr Krümmel.)

| | Fathoms. | Area Sq. Miles. | | Fathoms. | Area Sq. Miles. |
|------------------------|----------|-----------------|---------------------|----------|-----------------|
| Atlantic..... | 2013 | 29 514 275 | Gulf of Mexico..... | 1001 | 1 765 920 |
| Archipelago..... | 487 | 3 046 600 | “ “ St. Lawrence | 160 | 101 075 |
| Azof..... | — | 8 800 | Indian..... | 1829 | 28 369 595 |
| Baltic Sea..... | 36 | 159 690 | Japan..... | 1200 | 383 205 |
| Black Sea..... | — | 150 000 | Mediterranean..... | 729 | 1 109 236 |
| Behring's Straits..... | 550 | 864 555 | North Sea..... | 48 | 210 295 |
| Caspian Sea..... | — | 120 000 | North Ice Sea..... | 845 | 5 264 600 |
| China (East) Sea..... | 66 | 472 210 | Persian Gulf..... | 20 | 90 200 |
| Dead Sea..... | — | 370 | Pacific..... | 3887 | 60 343 690 |
| English Channel, etc. | 47 | 78 416 | Red Sea..... | 243 | 170 000 |

Mean depth of Ocean surrounding land 1877 fathoms = 2.19 miles.

In his subsequent computations he estimates ocean area at 143 703 000 square miles and determines area of land to water as 1 to 2.75, and that mean height of land = 1377 feet, or one eighth that of Ocean.

Heights of Mountains, Volcanoes, and Passes above Level of Sea.

| MOUNTAINS. | Feet. | MOUNTAINS. | Feet. | MOUNTAINS. | Feet. |
|---------------------|--------|--|--------|----------------------|--------|
| EUROPE. | | Mount Everest (Himalaya, highest)..... | 29 003 | Mount Pitt..... | 9 549 |
| Azores Pico..... | 7 613 | Mount Libanus.... | 9 523 | Mount Washington. | 6 420 |
| Barthélemy, France | 7 365 | Petcha..... | 15 000 | Nevado de Sorata... | 25 248 |
| Ben Lomond..... | 3 240 | Sinai..... | 7 496 | Orizaba..... | 18 879 |
| Ben Nevis..... | 4 380 | AFRICA. | | Potosi..... | 18 000 |
| Elbrus, Caucasus... | 17 776 | Atlas..... | 10 400 | Sierra Nevada..... | 15 700 |
| Guadarrama, Spain. | 8 520 | Compass, Cape of | | Tahiti..... | 10 895 |
| Hecla..... | 5 147 | Good Hope..... | 10 000 | White Mountains... | 6 230 |
| Ida..... | 4 960 | Dianai Peak, St Helena. | 2 700 | VOLCANOES. | |
| Jungfrau, Switz'd. | 13 725 | Kilimanjaro..... | 20 000 | Cotopaxi..... | 18 867 |
| Mont Blanc..... | 15 797 | Ruivo, Madeira.... | 5 160 | Etna..... | 10 874 |
| “ Cenis..... | 6 780 | Teneriffe Peak.... | 12 300 | Hecla..... | 5 000 |
| Mont d'Or, France. | 6 510 | AMERICA. | | Popocatepetl..... | 17 764 |
| Mulhassan, Gren'a. | 11 663 | Aconcagua (highest in America)..... | 23 910 | Sahama..... | 22 320 |
| Nepbin, Ireland.... | 2 634 | Blue Mount, Jam'a. | 8 000 | St. Helen's, Oregon. | 13 300 |
| Olympus..... | 6 510 | Catskill..... | 3 804 | Vesuvius..... | 3 920 |
| Parnassus..... | 6 000 | Chimborazo..... | 21 441 | PASSES. | |
| Plynlimmon, Wales. | 2 463 | Correde, Potosi.... | 16 036 | Cordilleras..... | 13 598 |
| The Cylindar, Pyr.. | 10 930 | Crows' Nest, Highlands, N. Y.... | 1 370 | Mont Cenis..... | 13 200 |
| Wetterhorn..... | 12 154 | Great Peak, New Mexico..... | 19 788 | “ Cervis..... | 6 776 |
| ASIA. | | Mauna Loa, Hawaii | 13 805 | Pont d'Or..... | 9 64 |
| | 17 100 | | | St. Bernard, Great. | 8 17 |
| | 16 433 | | | “ Little..... | 7 |
| | 28 077 | | | St. Gothard..... | 6 1 |
| | 8 500 | | | Simplon..... | 1 |
| | 12 000 | | | | |

Dimensions of Canal Locks.—(U. S.)

| CANAL. | Length. | Breadth. | Depth. | Length of Canal. | CANAL. | Length. | Breadth. | Depth. | Length of Canal. |
|--|---------|----------|--------|-------------------------------|-----------------------|---------|----------|--------|------------------|
| | Feet. | Feet. | Feet. | Miles. | | Feet. | Feet. | Feet. | Miles. |
| Chesapeake and Potomac River. | 220 | 40 | 6 | 14 | Champlain. | 110 | 18 | 5 | 66.75 |
| Brook'd Lake, Chenango, Chemung, and Genesee Valley. | 90 | 15 | 4 | 77 8 97 33 113.75 | Cayuga and Seneca. | 110 | 18 | 7 | 24.75 |
| Chesapeake and Delaware. | 220 | 24 | 9 | 14 | Delaware and Raritan. | 220 | 24 | 7 | 43 |
| | | | | | Dismal Swamp. | 90 | 17.5 | 5.5 | 44 |
| | | | | | Erie. | 110 | 18 | 7 | 35.2 |
| | | | | | Falls of Ohio, Ky. | 350 | 80 | 2-60 | — |
| | | | | | Oswego. | 110 | 18 | 4 | 38 |
| | | | | | Welland, Canada. | 270 | 45 | 14 | 28 |

Length of vessel that can be transported is somewhat less than lengths of locks.

Suez Canal. — Width 196 to 328 feet at surface, 72 at bottom, and 26 deep, length 99 miles.

Elevations of obtained Elevations, and various Places and Points above the Sea.

| LOCATIONS. | Feet. | LOCATIONS. | Feet. | LOCATIONS. | Feet. |
|---|--------|-------------------------------|--------|-----------------------|--------|
| Acagua, Chili. | 23 910 | Geneva city. | 1 220 | Mont Rosa, Alps. | 15 155 |
| Asisana, highest established elevation (Farmhouse). | 13 434 | Geneva Lake. | 1 096 | Mount Adams. | 5 930 |
| Don (Gay Lussac) (Green, 1837). | 27 000 | Gibraltar. | 1 439 | Mount Katahdin. | 5 360 |
| (Glaisher and Kewell). | 37 000 | Humboldt's highest elevation. | 19 400 | Mount Pitt. | 9 549 |
| Ill, Quito, and Mexico plains. | 6 000 | Isthmus of Darien. | 645 | Mount Washington. | 6 426 |
| Or's flight. | 8 000 | Jungfrau, Switz'd. | 13 725 | Paris, city. | 115 |
| Or's flight. | 29 500 | La Paz, Bolivia. | 12 225 | Pont d' Oro, Pyr's. | 9 843 |
| Or's flight. | 16 500 | Laguna, Teneriffe. | 2 000 | Posthouse, Ap., Peru. | 14 377 |
| Or's flight. | 16 500 | London, city. | 64 | Potosi, Bolivia. | 13 223 |
| Or's flight. | 29 003 | Madrid. | 2 200 | Quito. | 13 500 |
| Or's flight. | 29 003 | Mexico, city of. | 7 525 | St. Bernard's Mon'y. | 8 040 |
| Or's flight. | 29 003 | Mont Blanc, Alps. | 15 797 | Vegetation. | 17 000 |
| Or's flight. | 29 003 | | | White Mountain. | 6 230 |

Lengths of Rivers.

| RIVERS. | Miles. | RIVERS. | Miles. | RIVERS. | Miles. |
|---------|--------|----------------------|--------|---------------------|--------|
| EUROPE. | | Ganges. | 1514 | Kansas. | 1400 |
| Don. | 1800 | Houng Ho. | 3040 | La Platte. | 850 |
| Elbe. | 1243 | Indus. | 1800 | Mackenzie. | 2440 |
| Elbe. | 400 | Jordan. | 176 | Mississippi. | 3160 |
| Elbe. | 1035 | Lena. | 2762 | Missouri. | 3030 |
| Elbe. | 780 | Tigris. | 1160 | Ohio and Allegheny. | 1480 |
| Elbe. | 442 | Yenesei and Solenga. | 3580 | Potomac. | 420 |
| Elbe. | 545 | Yang-Tse. | 3314 | Red. | 1520 |
| Elbe. | 420 | | | Rio Bravo. | 220 |
| Elbe. | 760 | AFRICA. | | Rio Grande. | 180 |
| Elbe. | 510 | Gambia. | 700 | St. Lawrence. | 2 |
| Elbe. | 450 | Niger. | 2400 | Susquehanna. | |
| Elbe. | 250 | Nile. | 4000 | Tennessee. | |
| Elbe. | 510 | | | | |
| Elbe. | 220 | NORTH AMERICA. | | | |
| Elbe. | 190 | Arkansas. | 2070 | Amazon. | |
| Elbe. | 630 | Colorado. | 1050 | Essequibo. | |
| Russia. | 2400 | Columbia. | 1200 | Magdalena. | |
| ASIA. | | Connecticut. | 410 | Orinoco. | |
| Amur. | 2500 | Delaware. | 420 | Platte. | |
| Amur. | 1786 | Hudson and Mohawk. | 325 | Rio Made. | |
| | | | | Rio Negro. | |
| | | | | Uruguay. | |

Large Trees in California.

"*Keystone State*."—Calavera Grove, is 325 feet in height.

"*Father of the Forest*."—Felled, is 385 feet in length, and a man on horseback can ride erect 90 feet inside of its trunk.

"*Mother of the Forest*."—Is 315 feet in height, 84 feet in circumference (26.75 feet in diameter) inside of its bark, and is computed to contain 537 000 feet of sound 1 inch lumber.

Sea Depths.

| | Feet. | | Feet. | | Feet. |
|-----------------------|-------|---------------------|--------|---------------------|-------|
| Baltic Sea..... | 120 | Coast of Spain.... | 6 000 | Off Cape Canaveral. | 2400 |
| Adriatic..... | 130 | West of St. Helena. | 27 000 | " Charleston..... | 4200 |
| English Channel... | 300 | Tortugas to Cuba.. | 4 200 | " Cape Hatteras... | 3120 |
| Straits of Gibraltar. | 200 | Gulf of Florida.... | 3 720 | " Cape Henry..... | 4200 |
| Eastward of " | 3000 | Off Cape Florida... | 1 950 | " Sandy Hook.... | 2400 |

Estimated depth of Atlantic..... 26 000 feet.

" " Pacific..... 29 000 "

250 miles off Cape Cod, no bottom at 7800 feet.

Cascades and Waterfalls.

| LOCATION. | Feet. | LOCATION. | Feet. | LOCATION. | Feet. |
|------------------------|-------|----------------------|------------|---------------------|-------|
| Arve, Savoy..... | 1600 | Genesee, N. Y. | 100 | Niagara..... | 164 |
| Cascade, Alps..... | 2400 | Lidford, England... | 100 | Great Fall..... | 152 |
| | | Lulea, Sweden..... | 600 | Passaic..... | 74 |
| Cataracts of the Nile. | { 30 | Mohawk..... | 68 | Potomac..... | 74 |
| | { 34 | | | Ribbon, Yosemite } | |
| | { 40 | | | Valley..... | 3300 |
| Chachia, Asia..... | 362 | Missouri..... | { 80 | | |
| Foyers, Scotland... | 197 | | { 94 | Ruican, Norway... | 800 |
| Garisha, India..... | 1000 | Montmorenci..... | 250 | Staubach, Switz'd.. | 798 |
| Gavarny, Pyrenees.. | 1260 | Nant d'Apresias.... | 800 | Tendon, France.... | 125 |
| | | Yosemite Valley..... | 2600 feet. | | |

Expansion and Contraction of Building Stones for each Degree of Temperature. (*Lieut. W. H. C. Bartlett, U. S. E.*)

| | For One Inch. | | For One Inch. |
|--------------|---------------|----------------|---------------|
| Granite..... | .000 004 825 | Sandstone..... | .000 009 532 |
| Marble..... | .000 005 668 | Whitepine..... | .000 002 55 |

Resistance of Stones, etc., to the Effects of Freezing.

Various experiments show that the power of stones, etc., to resist effects of freezing is a fair exponent of that to resist compression.

Magnetic Bearings of New York.

The Avenues of the City of New York bear 28° 50' 30" East of North.

Filters for Waterworks.

1 square yard of filter for each 840 U. S. and 700 Imp'l gallons in 24 hours; formed of 2.5 feet of fine sand or gravel and 6 inches of common sand or shells.

Led off by perforated pipes laid in lowest stratum.

Distances between New York, Boston, Philadelphia, Baltimore, and Western Cities of U. S.

Assuming Boston as standard, New York averages 12 per cent. nearer to these cities, Philadelphia 18 per cent., and Baltimore 22 per cent.

Between New York and Chicago the line of the Pennsylvania Railroad is 47 miles shorter than that by the Erie and its connections, 50 miles shorter than that by the N. Y. Central and Hudson River and its connections, and 114 miles shorter than that by the Baltimore and Ohio and its connections.

For Distances between these and other cities of the U. S., see page 88.

Weather-foretelling Plants. (Hanneman.)

rain is imminent.—Chickweed,* *Stellaria media*; its flowers droop not open. Crowfoot anemone, *Anemone ranunculoides*; its blossoms Bladder Ketmia, *Hibiscus trionum*; its blossoms do not open. Thistle, *s acaulis*; its flowers close. Clover, *Trifolium pratense*, and its allied and Whitlow grass, *Draba verna*; all droop their leaves. Nipple-ampsana communis; its blossoms will not close for the night. Yel-dstraw, *Galium verum*; it swells, and exhales strongly; and Birch, *alba*, exhales and scents the air.

signs of Rain.—Marigold, *Calendula pluvialis*; when its flowers do not open by 7 A. M. Hog Thistle, *Sonchus arvensis* and *oleraceus*; when its leaves are open.

signs of short duration.—Chickweed, *Stellaria media*; if its leaves open not fully.

signs of Wind.—Wind-flower, or Wood Anemone, *Anemone memorasa*; its flowers droop.

signs of Rain.—Clover, *Trifolium pratense*; if it contracts its leaves Birdweed and Pimpernel, *Convolvulus* and *Anagallis arvensis*; if they spread their leaves.

signs of Storm Weather.—Marigold, *Calendula pluvialis*; if its flowers open early A. M. and remain open until 4 P. M.

signs of Wind Weather.—Wind-flower, or Wood Anemone, *Anemone memorasa*; when its flowers are erect. Hog Thistle, *Sonchus arvensis* and *oleraceus*; when its blossoms close at and remain closed during the night.

Antidotes to Severe Ordinary Poisons.

Antidotes in very small doses.

Antidote for Opium and Ether.—Cold affusions on head and neck, and ammonia fumes. *Antidote.*—Camphor, petroleum, sulphur.

Antidote for Mushrooms.—(Inedible mushroom). *Antidote.*—Same as for chloroform.

Antidote for Fly Powder.—Emetic; after free vomiting give calcined magnesia. If poison has passed out of stomach, give castor oil.

Antidote.—Camphor, nux vomica, ipecacuanha.

Antidote for Lead (Sugar of lead).—Mustard emetic, followed by salts, draughts of milk with white of eggs.

Antidote.—Alum, sulphuric acid alike to lemonade, belladonna, strychnine.

Antidote for Arsenic Sublimate (Bug poison).—White of eggs in 1 quart of cold water give cupful every two minutes. Induce vomiting without aid of medicine. Soapsuds and wheat flour is a substitute for white of eggs.

Antidote.—Nitric acid, camphor, opium, sulphate of zinc.

Antidote for Phosphorus Matches.—*Rat Paste.*—Two teaspoonfuls of calcined magnesia mixed by mucilaginous drinks. *Antidote.*—Camphor, coffee, nux vomica.

Antidote for Sulfuric Acid (Charcoal fumes), Chlorine, Nitrous Oxide, or Ordinary Gas.—Fresh air, artificial respiration, ammonia, ether, or vapor of hot water.

Antidote.—Camphor, coffee, nux vomica.

Antidote for Nightshade.—Emetic and stomach pump, morphine and coffee. *Antidote.*—Camphor.

Antidote for Opium.—Stomach pump or emetic of sulphate of zinc, 20 or 30 grains, or 1 or 2 salt. Keep patient in motion. Cold water to head and chest.

Antidote.—Strong coffee freely and by injection, camphor, ether, and nux vomica.

Antidote for Stramonium (Nux vomica).—Stomach pump or emetic, camphor, animal charcoal, lard, or fat.

Antidote.—Wine, coffee, camphor, opium freely, and alcohol.

Antidote for Mustard Poisons.—As a rule, an emetic of mustard is the best.

* Spreads its leaves about 9 A. M., and they remain open until 4 P. M.

Veterinary.

Horses.—Cathartic Ball.—Cape Aloe, 6 to 10 drs.; Castile Soap, 1 dr.; Syrup of Wine, 1 dr.; Sirup to form a ball. If Calomel is required, add from 20 to 30 grains. During its operation, feed upon mash and give plenty of water.

Cattle.—Cathartic.—Cape Aloe, 4 drs. to 1 oz.; Epsom Salts, 4 to 6 oz.; Syrup of Marshmallows, 3 drs. Mix, and give in a quart of gruel. For Calves, one third will be sufficient.

Horses and Cattle.—Tonic.—Sulphate of Copper, 1 oz. to 12 drs.; Syrup of Marshmallows, 5 oz. Mix, and divide into 8 powders, and give one or two daily in food.

Cordial.—Opium, 1 dr.; Ginger, 2 drs.; Allspice, 3 drs., and Caraway Seeds, 3 drs., all powdered. Make into a ball with sirup, or give as a drench in gruel.

Cordial Astringent Drench, for Diarrhoea, Purging, or Scouring.—Tincture of Opium, .5 oz.; Allspice, 2.5 drs.; powdered Caraway, .5 oz.; Catechu Powder, 2 drs.; strong Ale or Gruel, 1 pint. Give every morning till purging ceases. For Sheep, .25 this quantity.

Alterative.—Ethiop's Mineral, .5 oz.; Cream of Tartar, 1 oz.; Nitre, 2 drs. Divide into from 16 to 24 doses, one morning and evening in all cutaneous diseases.

Diuretic Ball.—Hard Soap and Turpentine, each 4 drs.; Oil of Juniper, 20 drops, and powdered Resin to form a ball.

For Dropsy, Water Farcy, Broken Wind, or Febrile Diseases, add to above, Allspice and Ginger, each 2 drs. Divide into 4 balls, and give one morning and evening.

Alterative or Condition Powder.—Resin and Nitre, each 2 oz.; levigated Ammoniac, 1 oz. Mix for 8 or 10 doses, and give one morning and evening. When given to Cattle, add Glauber Salts, 1 lb.

Fever Ball.—Cape Aloe, 2 oz.; Nitre, 4 oz.; Sirup to form a mass. Divide into 12 balls, and give one morning and evening until bowels are relaxed; then give an Alterative Powder or Worm Ball.

Hoof Ointment.—Tar and Tallow, each 1 lb.; Turpentine .5 lb. Melt and mix.

Dogs.—Cathartic.—Cape Aloe, .5 dr. to 1 oz.; Calomel, 2 to 3 grs.; Oil of Caraway, 6 drops; Sirup to form a ball. Repeat every 5 hours till it operates.

Emetic.—2 to 4 grs. of Tartar Emetic in a meat ball, or a teaspoonful or two of common salt. Give twice a week if required.

Distemper Powder.—Antimonial Powder, 2, 3, or 4 grs.; Nitre, 5, 10, or 15 grs.; powdered Ipecacuanha, 2, 3, or 4 grs. Make into a ball, and give two or three times a day. If there is much cough, add from .5 gr. to 1 gr. of Digitalis, and every 3 or 4 days give an Emetic.

Mange Ointment.—Powdered Aloe, 2 drs.; White Hellebore, 4 drs.; Sulphur, 4 oz.; Lard, 6 oz.—**Red Mange,** add 1 oz. of Mercurial Ointment, and apply a muslin.

NOTE.—Physic, except in urgent cases, should be given in morning, and upon an empty stomach; and, if required to be repeated, there should be an interval of several days between each dose.

Age of Horses.

To Ascertain a Horse's Age.

A foal of six months has six grinders in each jaw, three in each side, and also six nippers or front teeth, with a cavity in each.

At age of one year, cavities in front teeth begin to decrease, and he has four grinders upon each side, one of permanent and remainder of milk set.

At age of two years he loses the first milk grinders above and below, and front teeth have their cavities filled up alike to teeth of horses of eight years of age.

At age of three years, or two and a half, he casts his two front uppers, and in a short time after the two next.

At four, grinders are six upon each side; and, about four and a half, his nippers are permanent by replacing of remaining two corner teeth; tushes then appear, and he is no longer a colt.

At five, a horse has his tushes, and there is a black-colored cavity in centre of all his nippers.

Black cavity is obliterated in the two front lower nippers.

At six, the next two are filled up, and tushes blunted; and at eight, the next two are filled up, and tushes blunted. Horse may now be said to be aged. Cavities in front teeth are obliterated till horse is about ten years old, after which the nippers project and change their surface.

stances between Principal Cities of East and West. In Miles.

| Cities. | Boston. | New York. | Phila- delphia. | Balti- more. | Cities. | Boston. | New York. | Phila- delphia. | Balti- more. |
|-------------|---------|-----------|--------------------|-----------------|----------------|---------|-----------|--------------------|-----------------|
| ington, Ia. | 1216 | 1106 | 1030 | 995 | Louisville.... | 1161 | 870 | 794 | 706 |
| ago..... | 1009 | 900 | 823 | 802 | Memphis.... | 1438 | 1247 | 1171 | 1083 |
| nnati.... | 927 | 743 | 667 | 576 | Milwaukee.... | 998 | 947 | 908 | 887 |
| eland.... | 671 | 580 | 504 | 483 | Omaha..... | 1503 | 1393 | 1317 | 1294 |
| mbus, O.. | 807 | 623 | 547 | 512 | St. Joseph.... | 1478 | 1356 | 1280 | 1223 |
| oit..... | 724 | 673 | 682 | 661 | St. Louis.... | 1212 | 1050 | 973 | 917 |
| anapolis.. | 951 | 810 | 735 | 700 | St. Paul..... | 1418 | 1308 | 1234 | 1211 |
| sas City.. | 1487 | 1324 | 1248 | 1192 | Toledo..... | 784 | 693 | 617 | 596 |

Population of Principal Cities (1882).

| | | | | | |
|--------------|-----------|-----------------|---------|-----------------|---------|
| lon..... | 3 832 440 | Marseilles..... | 357 530 | Stockholm..... | 168 770 |
| ago..... | 2 225 910 | St. Louis..... | 350 518 | Brussels..... | 161 820 |
| nn..... | 1 222 500 | Warsaw..... | 339 400 | Cleveland..... | 160 146 |
| York..... | 1 206 299 | Baltimore..... | 332 313 | Pittsburgh..... | 156 389 |
| na..... | 1 103 110 | Milan..... | 321 440 | Buffalo..... | 155 134 |
| etersburg.. | 876 570 | Amsterdam..... | 317 010 | Antwerp..... | 150 650 |
| delphia.... | 847 170 | Rome..... | 300 470 | Washington.... | 147 293 |
| ow..... | 611 970 | Lisbon..... | 246 300 | Cologne..... | 144 770 |
| tantinople.. | 600 000 | Palermo..... | 244 990 | Frankfort..... | 136 820 |
| ago..... | 583 185 | Copenhagen.... | 234 850 | Newark..... | 136 508 |
| oklyn..... | 566 663 | San Francisco.. | 233 959 | Venice..... | 132 830 |
| burg..... | 410 120 | Munich..... | 230 200 | Louisville..... | 123 758 |
| sa..... | 403 110 | Cincinnati..... | 225 139 | Jersey City.... | 120 722 |
| s..... | 372 890 | Bucharest..... | 221 800 | Detroit..... | 116 340 |
| id..... | 367 280 | Dresden..... | 220 820 | Milwaukee..... | 115 587 |
| n..... | 362 839 | New Orleans.... | 216 190 | Providence..... | 104 857 |
| -Pesth..... | 360 580 | Florence..... | 169 000 | Rouen..... | 104 010 |

Treatment of Drowning Persons.

practice adopted by Board of Health, New York.

the patient face downward, with one of his wrists under his forehead. Cleanse mouth. If he does not breathe, turn him on his back with shoulders raised on port. Grasp tongue gently but firmly with fingers covered with end of a handkerchief or cloth, draw it out beyond lips, and retain it in this position.

Produce and Imitate Movements of Breathing.—Raise patient's extended arms up to sides of his head, pull them steadily, firmly, slowly, outwards. Turn elbows by patient's sides, and bring arms closely and firmly across pit of chest, and press them and sides and front of chest gently but strongly for a moment quickly begin to repeat first movement.

these two movements be made very deliberately and without ceasing until he breathes, and let the two movements be repeated about twelve or fifteen in a minute, but not more rapidly, bearing in mind that to thoroughly fill the with air is the object of first or upward and outward movement, and to expel air as practicable is object of second or downward motion and pressure. Artificial respiration should be maintained for forty minutes or more, when the patient appears not to breathe; and after natural breathing begins, let same motion be gently continued, and give proper stimulants in intervals.

What Else is to be Done, and What is Not to be Done, while the Movements are Made.—If help and blankets are at hand, have body stripped, wrapped in blankets, but not allow movements to be stopped. Briskly rub feet and legs, press firmly and rubbing upward, while the movements of the arms and chest progress. Apply hartshorn, or like stimulus, or a feather within the nostrils; finally, and sprinkle or lightly dash cold water upon face and neck. The head feet should be rubbed and wrapped in hot blankets, if blue or cold, or is cold.

What to do when Patient Begins to Breathe.—Give stimulants by teaspoonful every five minutes, until beating of pulse can be felt at wrist, but be careful not to give more of stimulant than is necessary. Warmth should be kept at the feet, and as soon as patient breathes naturally, let him be carefully covered, and placed in bed, under medical care.

MISCELLANEOUS ELEMENTS.

Earth.

Polar diameter 7899.3 miles. Mean density or specific gravity of mass 5.672. Mass 5 272 600 000 000 000 000 tons. Apparent diameter as seen from Sun 17 seconds.

Sun.

Heat of Sun equal to 322 794 thermal units per minute for each sq. foot of the sphere or solar surface.

Diameter of Sun 882 000 miles, tangential velocity 1.25 miles per second or 44 times greater than that of the Earth.

Distance from Earth 91.5 to 92 millions of miles.

Mason and Dixon's Line.

39° 43' 26.3" N. mean latitude. 68.895 miles.

Area and Population. (Behm and Wagner.)

| Divisions. | Area. | Population. | Divisions. | Area. | Population. |
|--------------|------------|-------------|---------------|------------|---------------|
| | Sq. Miles. | | | Sq. Miles. | |
| America..... | 14 491 000 | 95 495 500 | Oceanica..... | 4 500 000 | 4 031 000 |
| Europe..... | 3 760 000 | 315 929 000 | Greenland } | | 80 000 |
| Asia..... | 16 313 000 | 834 707 000 | Iceland } | | |
| Africa..... | 10 936 000 | 205 679 000 | Total..... | 50 000 000 | 1 455 923 500 |

Countries.

| | | | | | |
|-------------|----------------------|------------------|------------------|-------------------|-------------|
| Austria } | 38 000 000 | Germany..... | 43 900 000 | India, British .. | 240 298 000 |
| Hungary } | 37 000 000 | Great Britain.. | 34 000 000 | Canada | 3 839 000 |
| China..... | 434 626 000 | { Russia..... | 66 000 000 | Mexico | 9 485 000 |
| France..... | 37 000 000 | { Territories .. | 22 000 000 | Brazil | 11 206 000 |
| | { United States..... | 50 000 000 | { Turkey..... | 8 866 000 | |
| | { Indians | 300 000 | { " in Asia..... | 16 320 000 | |

About one thirtieth of whole population are born every year, and nearly an equal number die in same time; making about one birth and one death per second.

Earlier authority estimated population at 1 288 000 000, divided as follows:

| | | | | | |
|-----------------|-------------|-----------------|-------------|------------------|-------------|
| Caucasians..... | 360 000 000 | Malays and } | 177 000 000 | Mohammedans..... | 190 000 000 |
| Mongolians..... | 552 000 000 | Indo-Amer's } | 80 000 000 | Pagans..... | 300 000 000 |
| Ethiopians..... | 190 000 000 | Protestants.... | 5 000 000 | Catholics } | 250 000 000 |
| Asiatics..... | 60 000 000 | Israelites..... | | Rom. & Greek } | |

Descent of Western Rivers.

Slope of rivers flowing into Mississippi from East is about 3 inches per mile; and from West 6 inches.

Mean descent of Ohio River from Pittsburgh to Mississippi, 975 miles, is about 5.2 inches per mile; and that of Mississippi to Gulf of Mexico, 1180 miles, about 2.8 inches.

Transmission of Horse Power.

Largest, and perhaps most successful, wire rope transmission is one at Schaffhausen, at Falls of the Rhine. Here, power of a number of turbines, amounting to over 600 HP, is conveyed across the stream, and thence a mile to a town, where it is distributed and utilized.

At mines of Falun, Sweden, a power of over 100 horses is transmitted in like manner for a distance of three miles.

Acids.

Acetic Acid (Vinegar), acid of Malt beer, etc. Tartaric Acid, acid of Grape wine. Lactic Acid, acid of Milk, Millet beer, and Cider.

Manures.

Relative Fertilizing Properties of Various Manures.

| | | | | | |
|---|------|-------------|------|----------------|-------|
| Peruvian Guano | 1 | Horse | .048 | Farm-yard..... | .0298 |
| Human, mixed..... | .069 | Swine..... | .044 | Cow..... | .0259 |
| Or, 1 lb. guano = 14.5 human, 21 horse, 22.5 swine, 33.5 farm-yard, and 38.5 cow. | | | | | |

Relative Value, Covered and Uncovered, on an Acre of Ground.

| | | | | | |
|-----------------|----|-----------------------------|-----------------|----------|--|
| Covered | 11 | tons 1665 lbs. potatoes, 61 | lbs. wheat, 275 | lbs. oil | |
| Uncovered | 7 | " 1397 " | " 61.5 " | " 156 " | |

Yield of Oil of Several Seeds.

| Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
|----------------|-------------|----------------|----------------|-------------------|
| ppy.. 56 to 63 | Castor.. 25 | Sunflower.. 15 | Hemp. 14 to 25 | Linseed. 11 to 22 |

Thickness of Walls of Buildings. (English.) (Molesworth.)

| OUTER WALLS. | Maximum Height of Wall. | Width of Footings. | Minimum Width of Walls. | | | | | | | |
|-------------------|-------------------------|--------------------|-------------------------|------------|-----------|-----------|------------|------------|------------|--|
| | | | Ground Floor. | 1st Floor. | 2d Floor. | 3d Floor. | 4th Floor. | 5th Floor. | 6th Floor. | |
| t class dwelling. | 85 | 38.5 | 21.5 | 21.5 | 17.5 | 17.5 | 17.5 | 13 | 13 | |
| " " | 70 | 30.5 | 17.5 | 17.5 | 17.5 | 13 | 13 | 13 | — | |
| " " | 52 | 30.5 | 17.5 | 13 | 13 | 13 | 13 | — | — | |
| " " | 38 | 21.5 | 13 | 13 | 8.5 | 8.5 | — | — | — | |
| PARTY WALLS. | | | | | | | | | | |
| t class dwelling. | 85 | 38.5 | 21.5 | 21.5 | 17.5 | 17.5 | 17.5 | 13 | 13 | |
| " " | 70 | 30.5 | 17.5 | 17.5 | 17.5 | 13 | 13 | 13 | — | |
| " " | 52 | 30.5 | 17.5 | 13 | 13 | 13 | 8.5 | — | — | |
| " " | 38 | 21.5 | 13 | 8.5 | 8.5 | 8.5 | — | — | — | |

f walls are more than 70 feet in length, those of lower stories must be widened half a brick.

| Warehouses | Minimum Width of Wall. | Warehouses | Minimum Width of Wall. |
|--|------------------------|--|------------------------|
| 1st Class. | | 2d Class. | |
| a height of 36 feet from topmost ceiling..... | 17.5 | For a height of 22 feet below topmost ceiling..... | 13 |
| a height of 40 feet lower.. | 21.5 | For a height of 36 feet lower.. | 17.5 |
| " 24 feet lower.. | 26 | " 8 feet lower.. | 21.5 |
| footings..... | 43.5 | For footings..... | 34.5 |
| 3d Class. | | 4th Class. | |
| a height of 28 feet below topmost ceiling..... | 13 | For a height of 9 feet below topmost ceiling..... | 8.5 |
| a height of 16 feet lower.. | 17.5 | For a height of 13 feet below.. | 13 |
| footings..... | 30.5 | For footings..... | 21.5 |

Wooden Roofs. (English.)

| Principal Beam. | Tie Beam. | King Posts. | Queen Posts. | Small Queens. | Straining Beam. | Struts. |
|-----------------|-----------|-------------|--------------|---------------|-----------------|---------|
| 4 x 4 | 9 x 4 | 4 x 4 | — | — | — | 3 x 3 |
| 5 x 4 | 10 x 5 | 5 x 5 | — | — | — | 5 x 3 |
| 6 x 4 | 11 x 6 | 6 x 6 | — | — | — | 6 x 3 |
| 5 x 4 | 11 x 4 | — | 4 x 4 | — | 7 x 4 | 4 x 2 |
| 6 x 5 | 13 x 6 | — | 6 x 6 | — | 7 x 6 | 5 x 3 |
| 8 x 6 | 13 x 8 | — | 8 x 8 | 8 x 4 | 9 x 6 | 5 x 3 |
| 8 x 7 | 14 x 9 | — | 9 x 8 | 9 x 4 | 10 x 6 | 5.5 x 3 |
| 8 x 8 | 15 x 10 | — | 10 x 8 | 10 x 4 | 11 x 6 | 6 x 3 |

Mineral Constituents absorbed or removed from an Acre of Soil by several Crops. (Johnson.)

| CROPS. | Wheat, 25 bushels. | Barley, 40 bushels. | Turnips, 20 tons. | Hay, 15 tons. | CROPS. | Wheat, 25 bushels. | Barley, 40 bushels. | Turnips, 20 tons. | Hay, 15 tons. |
|---------------------|--------------------|---------------------|-------------------|---------------|---------------------|--------------------|---------------------|-------------------|---------------|
| Sulphuric Acid..... | 29.6 | 17.5 | 47.1 | 38.2 | Sulphuric Acid..... | 10.6 | 2 | 13.3 | 9.2 |
| Chlorine..... | 3 | 5.2 | 8.2 | 12 | Chlorine..... | 2 | 16 | 3.6 | 4.1 |
| Silica..... | 12.9 | 17 | 29.9 | 44 | Silica..... | 118.1 | — | — | — |
| of Iron..... | 10.6 | 9.2 | 19.7 | 7 | Alumina..... | — | — | — | — |
| phoric..... | 2.6 | 2.1 | 7.1 | .6 | Total..... | 210 | — | — | — |
| d..... | 20.6 | 25.8 | 46.3 | 15.1 | | | | | |

Average Quantity of Tannin in Several Substances.
(Morfil.)

| <i>Catechu.</i> | Per Cent. | <i>Oak.</i> | Per Cent. | <i>Sumac.</i> | Per Cent. |
|--------------------|-----------|------------------------------|--------------|---------------------|-----------|
| Bombay | 55 | Young, inner b'k | 15.2 | Sicily and Malaga | 16 |
| Bengal | 44 | " entire b'k. | 6 | Virginia | 10 |
| <i>Kino.</i> | 75 | " spring- | 22 | Carolina | 5 |
| <i>Nut-pals.</i> | | cut bark } | | <i>Willow.</i> | |
| Aleppo | 65 | " root bark } | 8.9 | Inner bark | 16 |
| Chinese | 69 | <i>Chestnut.</i> | | Weeping | 16 |
| <i>Oak.</i> | | Amer. rose, bark | 8 | Sycamore bark | 16 |
| Old, inner bark { | 14.2 | Horse, " | 2 | Tan shrub " | 13 |
| | 21 | <i>Sassafras</i> , root bark | 58 | Cherry-tree | 24 |
| | | <i>Alder bark</i> | 36 per cent. | | |

To Convert Chemical Formulæ into a Mathematical Expression.

RULE.—Multiply together equivalent and exponent of each substance, and product will give proportion in compound by weight. Divide 1000 by sum of their products, and multiply this quotient by each of these products, and products will give respective proportion of each part by weight in 1000.

EXAMPLE.—Chemical formula for alcohol is $C_4H_6O_2$. Required their proportional parts by weight in 1000?

$$\begin{array}{l}
 C_4 \text{ Carbon} = 6.1 \times 4 = 24.4 \\
 H_6 \text{ Hydrogen} = 1 \times 6 = 6 \\
 O_2 \text{ Oxygen} = 8 \times 2 = 16
 \end{array}
 \times 21.55 \begin{array}{l} \{ 525.82 \\ \{ 129.3 \\ \{ 344.8 \end{array} \text{ by weight.}$$

$$1000 \div 45.4 = 21.55 \quad 999.92$$

Elementary Bodies, with their Symbols and Equivalents.

| Body. | Symb. | Equiv. | Body. | Symb. | Equiv. | Body. | Symb. | Equiv. |
|--------------|-------|--------|-----------------|-------|--------------|---------------|-------|--------|
| Aluminium... | Al | 13.7 | Gold..... | Au | 195.6 | Platinum.... | Pt | 98.8 |
| Antimony.... | Sb | 64.6 | Hydrogen.... | H | 1 | Potassium.... | K | 39.2 |
| Arsenic..... | As | 37.7 | Iodine..... | I | 126.5 | Rhodium.... | R | 52.2 |
| Barium..... | Ba | 62.6 | Iridium..... | Ir | 98.5 | Selenium.... | Se | 40 |
| Bismuth.... | Bi | 71.5 | Iron..... | Fe | 28 | Silicon..... | Si | 22 |
| Boron..... | B | 11 | Lead..... | Pb | 103.7 | Silver..... | Ag | 108.3 |
| Bromine.... | Br | 73.4 | Lithium..... | Li | 7 | Sodium..... | Na | 23.5 |
| Cadmium.... | Cd | 55.8 | Magnesium... | Mg | 12.7 | Strontium.... | Sr | 43.8 |
| Calcium.... | Ca | 20.5 | Manganese... | Mn | 26 | Sulphur..... | S | 16.1 |
| Carbon..... | C | 6.1 | Mercury..... | Hg | 200 | Tellurium.... | Te | 64.2 |
| Chlorine.... | Cl | 35.5 | Molybdenum.. | Mo | 47.9 | Tin..... | Sn | 58.9 |
| Chromium.... | Cr | 26.2 | Nickel..... | Ni | 29.5 | Titanium.... | Ti | 24.5 |
| Cobalt..... | Co | 29.5 | Nitrogen..... | N | 14.2 | Tungsten.... | W | 92 |
| Columbium.. | Ta | 54.8 | Osmium..... | Os | 99.7 | Uranium.... | U | 60 |
| Copper..... | Cu | 31.7 | Oxygen..... | O | 8 | Yttrium..... | Y | 32 |
| Fluorine.... | F | 18.7 | Palladium.... | Pd | 53.3 | Zinc..... | Zn | 32.3 |
| Glucinum.... | G | 6.9 | Phosphorus... P | 15.9 | Zirconium... | Zr | 34 | |

Analysis of certain Organic Substances by Weight.

| BODY. | Car-bon. | Hydro-gen. | Oxy-gen. | Nitro-gen. | BODY. | Car-bon. | Hydro-gen. | Oxy-gen. | Nitro-gen. |
|------------------|----------|------------|----------|------------|-----------------|----------|------------|----------|------------|
| Albumen..... | 52.9 | 7.5 | 23.9 | 15.7 | Morphine..... | 72.3 | 6.4 | 16.3 | 5 |
| Alcohol..... | 52.7 | 12.9 | 34.4 | — | Narcotine..... | 65 | 5.5 | 27 | 2.5 |
| Atmospheric air | — | 77 | 23 | — | Oil, Castor.... | 74 | 10.3 | 15.7 | — |
| Camphor..... | 73.4 | 10.7 | 15.6 | .3 | Linseed..... | 76 | 11.3 | 12.7 | — |
| Cantharouc | 87.2 | 12.8 | — | — | Spermaceti.... | 78 | 11.8 | 10.2 | — |
| | 59.8 | 7.4 | 11.4 | 21.4 | Quinine..... | 75.8 | 7.5 | 8.6 | 8.1 |
| | 53.4 | 7 | 19.7 | 19.9 | Starch..... | 44.2 | 6 | 49.1 | — |
| | 47.0 | 7.9 | 27.2 | 17 | Strychnine.... | 76.4 | 6.7 | 11.1 | 5.8 |
| | | | 50.9 | — | Sugar..... | 42.2 | 6.6 | 51.2 | — |
| | | | 7.6 | 1.8 | Tannin..... | 52.6 | 3.8 | 13.6 | — |
| | | | 18 | — | Urea..... | 18.9 | 9.7 | 25.2 | 43.2 |

Dilution Per Cent. Necessary to Reduce Spirituous Liquors.

Water to be added to 100 volumes of spirit when of following strength:

| Strength. | 90 | 85 | 80 | 75 | 70 | 65 | 60 | 55 | 50 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. |
| 15 | 5.9 | — | — | — | — | — | — | — | — |
| 10 | 12.5 | 6.3 | — | — | — | — | — | — | — |
| 5 | 20 | 13.3 | 6.7 | — | — | — | — | — | — |
| 0 | 28.6 | 21.4 | 14.3 | 7.1 | — | — | — | — | — |
| 15 | 38.5 | 30.8 | 23.1 | 15.4 | 7.7 | — | — | — | — |
| 10 | 50 | 41.7 | 33.3 | 25 | 16.7 | 8.3 | — | — | — |
| 5 | 63.6 | 54.5 | 45.5 | 36.4 | 27.4 | 18.2 | 9.1 | — | — |
| 0 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 10 | — |
| 10 | 125 | 112.5 | 100 | 87.5 | 75 | 62.5 | 50 | 37.5 | 25 |
| 0 | 200 | 183.3 | 166.7 | 150 | 133.3 | 116.7 | 100 | 83.3 | 66.7 |

ILLUSTRATION.—100 volumes of spirituous liquor having 90 per cent. of spirit consists: alcohol 90, water 10, = 100.

To reduce it to 50 per cent. there is required 200 volumes of water.

$$\text{Hence } 200 + 10 = 210, \text{ and } \frac{90}{210} = \frac{30}{70} = \frac{30 \text{ spirit,}}{70 \text{ water,}} \text{ or } 30 \text{ per cent.}$$

Proportion of Alcohol Per Cent.

In 100 Parts of Spirit, by Weight or Volume, at 60°.

| Vol. | Specific Gravity. | Alcohol. | Specific Gravity. | Alcohol. | Specific Gravity. | Alcohol. | Specific Gravity. |
|------|-------------------|----------|-------------------|----------|-------------------|----------|-------------------|
| 0 | 1 | 20 | .972 | 50 | .918 | 80 | .848 |
| 5 | .991 | 30 | .958 | 60 | .896 | 90 | .823 |
| 0 | .984 | 40 | .94 | 70 | .872 | 100 | .794 |

In 100 Parts of Alcohol and Water, by Weight, at 60°.

| Vol. | Specific Gravity. | Alcohol. | Specific Gravity. | Alcohol. | Specific Gravity. | Alcohol. | Specific Gravity. |
|------|-------------------|----------|-------------------|----------|-------------------|----------|-------------------|
| 13 | 1 | 1.99 | .996 | 5.01 | .991 | 7.99 | .987 |
| 12 | .999 | 3.02 | .994 | 6.02 | .99 | 9.05 | .985 |
| 11 | .993 | 4.02 | .993 | 7.02 | .988 | 10.07 | .984 |

Widths of Atlantic and Pacific Oceans at Isthmus of Panama. (Totten.)

Atlantic, Navy Bay.—Highest tide 1.5 feet; lowest .63 feet.

Pacific, Panama Bay.—Highest tide 17.72 to 21.3 feet; lowest 9.7 feet.

Areas of U. S. Coal Fields.

| STATE. | Sq. Miles. | STATE. | Sq. Miles. | STATE. | Sq. Miles. |
|--------|------------|----------|------------|-----------|------------|
| Miss. | 44 000 | Ohio | 11 900 | Tennessee | 4300 |
| Ala. | 21 000 | Indiana | 7 700 | Alabama | 3400 |
| Penn. | 15 437 | Missouri | 6 000 | Maryland | 550 |
| W. Va. | 13 500 | Michigan | 5 000 | Georgia | 150 |

* Bituminous and Anthracite.

† Anthracite.

Extremes of Heat in Various Countries.

| | | | | | |
|---------|--------|--------|--------|--------|--------|
| Denmark | 96° | Greece | 105° | Egypt | 116.1° |
| Sweden | 106.5° | Italy | 104° | Africa | 133.4° |
| Norway | 102° | Spain | 102° | Asia | 120° |
| Russia | 102° | Tunis | 112.5° | Suez | 126.5° |

Germany.....103° | Manila.....113.5° | N. America.....102°

Extremes of temperature upon the Earth 240°.

Extremes of Cold in Various Countries.

| | | | | | |
|---------|------|---------|------|---------|---|
| Denmark | 5° | France | —24° | Italy | — |
| Sweden | —12° | Russia | —46° | Fort | — |
| Norway | —67° | Germany | —32° | Siberia | — |

Mean Temperatures of Various Localities.

| | | | | | | |
|-----------------|-----|---------------|-----|------------------|-------------|--------------------|
| London | 51° | Rome | 60° | Poles | -13° | Polar Regions. 36° |
| Edinburgh | 41° | Equator | 82° | Torrid Zone. 75° | Globe | 50° |

Line of Perpetual Congelation, or Snow Line.

| Latitude. | Height. | Latitude. | Height. | Latitude. | Height. | Latitude. | Height. |
|-----------|---------|-----------|---------|-----------|---------|-----------|---------|
| 0 | Feet. | 0 | Feet. | 0 | Feet. | 0 | Feet. |
| 10 | 14 764 | 30 | 11 484 | 50 | 6334 | 70 | 1278 |
| 15 | 14 760 | 35 | 10 287 | 55 | 5020 | 75 | 1016 |
| 20 | 13 478 | 40 | 9 000 | 60 | 3818 | 80 | 451 |
| 25 | 12 557 | 45 | 7 670 | 65 | 2230 | 85 | 327 |

At the Equator it is 15 260 feet; at the Alps 8120 feet; and in Iceland 3084 feet. At Polar Regions ice is constant at surface of the Earth.

Limits of Vegetation in Temperate Zone.

The Vine ceases to grow at about 2300 feet above level of the sea, Indian Corn at 2800, Oak at 3350, Walnut at 3600, Ash at 4800, Yellow Pine at 6200, and Fir at 6700.

Periods of Gestation and Number of Young.

| Weeks. | No. | Weeks. | No. | Weeks. | No. | Weeks. | No. |
|----------------|-----|--------------|-----|---------------|-----|--------------|-----|
| Elephant. 100 | 1 | Cow..... 41 | 1 | Sheep... 21 | 2 | Dog..... 9 | 6 |
| Horse.... { 43 | 1 | Buffalo.. 40 | 1 | Goat.... 22 | 2 | Fox..... 9 | 5 |
| { 50 | 1 | Stag..... 36 | 1 | Beaver.. 17 | 3 | Cat..... 8 | 6 |
| Camel.... 45 | 1 | Bear..... 30 | 2 | Pig..... 17 | 12 | Rat..... 5 | 8 |
| Ass..... 43 | 1 | Deer.... 24 | 2 | Wolf.... 10 | 5 | Squirrel.. 4 | 6 |
| | | Rabbit... 4 | 6 | Guinea Pig. 3 | 3 | | |

Periods of Incubation of Birds.

Swan, 42 days; Parrot, 40 days; Goose and Pheasant, 35 days; Duck, Turkey, and Peafowl, 28 days; Hens of all gallinaceous birds, 21 days; Pigeon and Canary, 14 days. Temperature of incubation is 104°.

Ages of Animals, etc.

Whale, estimated 1000 years; Elephant, 400; Swan, 300; Camel, 100; Eagle, 100; Raven, 100; Tortoise, 100 to —; Lion, 70; Dolphin, 30; Horse, 30; Porpoise, 30; Bear, 20; Cow, 20; Deer, 20; Rhinoceros, 20; Swine, 20; Wolf, 20; Cat, 15; Fox, 15; Dog, 15; Sheep, 10; Hare, Rabbit, and Squirrel, 7.

Relative Weights of Brain.

Man, 154.33; Mammifers, 29.88; Birds, 26.22; Reptiles, 4.2; Fish, 1.

Buoyancy of Casks.

Buoyancy of a cask in fresh water in lbs. = 11.97 times volume of it in U. S. gallons and 10 times in Imperial gallons, less weight of cask.

Transportation of Horses and Cattle.

Space required on board of a Marine Transport is: for Horses, 30 ins. by 9 feet; Beeves, 32 ins. by 9 feet. Provender required *per diem* is: for Horses, Hay, 15 lbs.; Oats, 6 quarts; Water, 4 gallons. Beeves, Hay, 18 lbs.; Water, 6 gallons.

Rock and Earth Excavation and Embankment.

Number of Cube Feet of various Earths in a Ton.

| | | | | | |
|--------------------|------|---------------------|------|-----------------------|------|
| Common Earth | 24 | Clay | 18.6 | Clay with Gravel | 14.4 |
| | 18.6 | Earth with Gravel.. | 17.8 | Common Soil..... | 15.6 |

Earth and Sand in embankment exceeds that in a primary average proportions:

| | | | |
|--------------------|-----|-------------|-----|
| Rock, ballast..... | 1.2 | Clay | 111 |
| and..... | 143 | Gravel..... | 29 |

out 12.

Hills or Plants in an Area of One Acre.

From 1 to 40 feet apart from centres.

| Part. | No. | Feet apart. | No. | Feet apart. | No. | Feet apart. | No. |
|-------|--------|-------------|------|-------------|-----|-------------|-----|
| | 43 560 | 5 | 1742 | 9 | 538 | 16 | 171 |
| | 19 360 | 5.5 | 1440 | 9.5 | 482 | 17 | 151 |
| | 10 800 | 6 | 1210 | 10 | 435 | 18 | 135 |
| | 6 960 | 6.5 | 1031 | 10.5 | 361 | 20 | 108 |
| | 4 840 | 7 | 889 | 12 | 302 | 25 | 69 |
| | 3 556 | 7.5 | 775 | 13 | 258 | 30 | 48 |
| | 2 722 | 8 | 680 | 14 | 223 | 35 | 35 |
| | 2 151 | 8.5 | 602 | 15 | 193 | 40 | 27 |

Number of several Seeds in a Bushel, and Number per Square Foot per Acre.

| | No. | Sq. Foot. | | No. | Sq. Foot. |
|-------------|------------|-----------|------------|---------|-----------|
| Barley..... | 41 823 360 | 960 | Rye..... | 888 390 | 20.4 |
| Rye..... | 16 400 960 | 376 | Wheat..... | 556 290 | 12.8 |

Volumes.

Permanent gases, as air, etc., are diminished in their volume in a ratio direct that of pressure applied to them. With vapor, as steam, etc., this rule is modified in consequence of presence of the temperature of vaporization.

Minerals.

Relative Hardness of some Minerals.

| | | | | | | |
|-----------|-------------------|-----|----------------|-----|--------------|----|
| 1 | Barytes..... | 3.5 | Opal..... | 6 | Emerald..... | 8 |
| 2 | Fluor-spar..... | 4 | Quartz..... | 7 | Topaz..... | 8 |
| 2.5 | Feldspar..... | 6 | Tourmalin..... | 7 | Ruby..... | 9 |
| 3 | Lapis Lazuli..... | 6 | Garnet..... | 7.5 | Diamond..... | 10 |

Weight of Diamonds.

| | Carats. | | Carats. | | Carats. |
|----------------------|-------------------------|----------------|-------------------------|-------------------|---------|
| 367 | Regent or Pitt..... | 136.75 | Dresden..... | 76.5 | |
| Mogul*..... 279.9 | Star of the South†..... | 125 | Sancy..... | 53.5 | |
| 194.25 | Koh-i-Noor†..... | 106.06 | Eugenie, brilliant..... | 51 | |
| 139.5 | Piggott..... | 82.25 | Hope (blue)..... | 48.5 | |
| of Portugal... 138.5 | Napac..... | 78.625 | Polar Star..... | 40.25 | |
| weigh 900. | | † Rough 254.5. | | ‡ Originally 793. | |

Heat of the Sun.

| | | | |
|-------------------|------------|----------------|-------------|
| at Newton..... | 3 138 740° | Waterston..... | 16 000 000° |
| ohn Ericsson..... | 4 909 860° | Soret..... | 10 443 323° |

by others ranging from 2520° to 183600°.

1.—Distance of Moon from Earth 237 000 miles.

Frigorific Mixture.

Lowest temperature yet procured. Faraday obtained 166° by evaporation of a mixture of solid carbonic acid and sulphuric ether.

Current of Rivers.

A fall of .1 of an inch in a mile will produce a current in rivers.

Sandstones.

Structures of sandstone erected in England in 12th century are yet in good preservation.

Canal Transportation.

Canal and Hudson River.—From Buffalo to New York, 495 miles, cost of transportation 2.46 mills per ton (inclusive of tolls) per mile. Transportation of coals when it reaches New York 4.72 cents per bushel, and .61 cents per ton for elevating and trimming.

1.—**Erie Canal.**—Four mules will tow 22 tons, involving a period of 30 days, at a cost

of freight down and 100 cents a mile for a course

Matter.

Unit of the Physicist is a molecule, and a mass of matter is composed of having same physical properties as parent mass.

It exists in three forms, known as solid, liquid, and gaseous. Solids have individuality of form, and they press downward alone. Liquids have not individuality of form, except in spherical form of a drop, and they press downward and sideways. Gases are wholly deficient in form, expanding in all directions, and consequently they press upward, downward, and sideward.

Liquids are compressible to a very moderate degree. Water has been pressed through pores of silver, and it may be compressed by a pressure of one pound square inch to the 330,000th part of its volume.

Gases may be liquefied by pressure or by reduction of their temperature.

Combustible matter (as coal) may be burned, a structure (as a house) destroyed as such, and the fluid (of an ink) may be evaporated, yet the matter which coal and house were composed, although dissipated, exists, and the coloring matter of the ink are yet in existence.

Spaces between the particles of a body are termed pores.

All matter is porous. Polished marble will absorb moisture, as evidenced by discoloration by presence of a colored fluid, as ink, etc.

Silica is the base of the mineral world, and *Carbon* of the organized.

Minuteness of Matter.

A piece of metal, stone, or earth, divided to a powder, a particle of it, 1 minute, is yet a piece of the original material from which it was separated into its identity, and is termed a molecule.

It is estimated there are 120,000,000 corpuscles in a drop of blood of the man. Thread of a spider's web is of a cable form, is but one sixth diameter of a silk, and 4 miles of it is estimated to have a weight of but 1 grain.

One imperial gallon (277.24 cube ins.) of water will be colored by mixture of a grain of carmine or indigo.

A grain of platinum can be drawn out the length of a mile.

Film of a soap-and-water bubble is estimated to be but the 300,000th part of an inch in thickness.

It is computed that it would require 12,000 of the insect known as the monad to fill up a line one inch in length.

A drop of water, or a minute volume of gas, however much expanded—the volume of the Earth—would present distinct molecules.

Gold leaf is the 280,000th part of an inch in thickness.

A thread of silk is 2500th of an inch in diameter.

A cube inch of chalk in some places in vicinity of Paris contains 100,000 of the foraminifera.

There are animalcules so small that it requires 75,000,000 of them to weigh

Velocity, Weight, and Volume of Molecules.

Velocity.—Collisions among the particles of *Hydrogen* are estimated to the rate of 17 million-million-million per second, and in *Oxygen* less than number.

Weight.—A million-million-million molecules of *Hydrogen* are equal to weight but 60 grains.

Volume.—10 million-million-million molecules of *Hydrogen* have a volume cube ins. **Diameter.**—Five millions in a line would measure but .1 inch.

Charcoal, Alcohol.

Charcoal as yet has not been liquefied, nor has Alcohol been solidified.

Metals.

Metals have five degrees of lustre—splendid, shining, glistening, glimmer

and can be vaporized, or exist as a gas, by application to them of the temperature of conversion.

Heating of a metal renders it brittle; reheating it restores its strength, and renders it harder, and up to twelfth time it

of all metals.

Impenetrability.

penetrability expresses the inability of two or more bodies to occupy same space at same time.

Mixture of two or more fluids may compose a less volume than that due to sum of their original volume, in consequence of a denser or closer occupation of their spaces. This is evident in the mixture of alcohol and water in the proportion of 5 volumes of former to 25 of latter, when there is a loss of one volume.

Elasticity.

Elasticity is the term for the capacity of a body to recover its former volume, after being subjected to compression by percussion or deflection.

Glass, ivory, and steel are the most elastic of all bodies, and clay and putty are among the least. Caoutchouc (India rubber) is but slightly elastic; it possesses contractility, however, in a great degree.

Momentum.

Momentum is quantity of motion, and is product of mass and its velocity. Thus, the momentum of a cannon-ball is product of its velocity in feet per second and its weight, and is denominated *foot-pounds*.

Foot-pound is the power that will raise one pound one foot.

Sound.

Velocity of sound is proportionate to its volume; thus, report of a blast with 2000 lbs. of powder passed 667 feet in one second, and one of 1200 lbs. 1210 feet. It passes later with a velocity of 4708 feet per second. Conversation in a low tone has been maintained through cast-iron water pipes for a distance of 3120 feet, and its velocity is from 4 to 16 times greater in metals and wood than air.

Light.

Solar rays have a velocity of 185,000 miles per second, equal to 7.5 times around Earth.

Color Blindness.

Blindness of elementary sensation corresponding to red.

Luminous Point.

To produce a visual circle, a luminous point must have a velocity of 10 feet in a second, the diameter not exceeding 15 ins.

Solid bodies become luminous at 800 degrees of heat.

Mirage.

When air near to surface of Earth becomes so highly heated, as upon a sandy plain, that its density within a defined distance from it increases upwards, a line of vision directed obliquely downwards will be rendered by refraction, gradually bending, more and more nearly horizontal as it advances, until its direction is so altered as to produce a total reflection, and the reflected ray then, by successive reflections, is gradually elevated until it meets the eye of the observer.

Superior mirage is inverted mirage, frequently seen over calm water, and is effect of difference of surface stratum of air being colder than that above it.

Snow Flakes.

Various forms of snow flakes have been observed.

Melted Snow.

Produces from .25 to .125 of its bulk in water.

Strength of Ice.

3 inches thick will support men in single file on planks 6 feet apart; 4 inches support cavalry, light guns, and carts; and 6 inches wagons drawn by horses.

Temperature.

Sulphuric acid and water produce a much greater proportionate contraction than alcohol and water. Both of these mixtures, however low their temperature, produce a heat which is in a direct proportion to their diminution in volume.

At the depth of 45 feet, the temperature of the Earth is uniform throughout the

Temperature of Earth increases about 1° for every 50 to 60 feet.
It is estimated at 30 miles.

At the Equator weighs two hundred and eighty-nine parts less

Ages of Animals, Fishes, etc.*(Additional to page 192.)*

Tiger, Leopard, Jaguar, and Hyena (in confinement), 25 years; Beaver, 50; Stag, under 50; Ox and Ass, 30; Chamois, 25; Llama, Monkey, and Baboon, 15 to 18; Parrot, 200; Tortoise, 100 to 200; Crocodile, 100; Carp, 70 to 150; Goose, 80; Pelican, 45; Hawk, 30 to 40; Crane, 24; Peacock, Goldfinch, Chaffinch, from 10 to 25; Domestic Fowls, Pigeons, Blackbird, Nightingale, and Linnet, 10 to 16; Thrush, Robin, and Starling, 8 to 12; Wren, 2 to 3; Salmon, 16; Eel, 10; Codfish, 4 to 17; Pike, 9 to 40; Queen Bee, 4; Bee, 6 months, and Drones, 4 months. (*Houghtaling.*)

Birds and Insects.—(M. De Lacy.)

Elements of Flight.—Resistance of air to a body in motion is in ratio of surface of body and as square of its velocity.

Wing Surface.—Extent or area of winged surface is in an inverse ratio to weight of bird or insect.

A Stag-beetle weighs 460 times more than a Gnat, and has but one fourteenth of its wing surface; 150 times more than a Lady Bird (bug), and has but one fifth. An Australian Crane weighs 339 times more than a sparrow, and has but one seventh; 3 000 000 times more than a Gnat, and has but one hundred and fortieth. A Stork weighs eight times more than a Pigeon, and has but one half. A Pigeon weighs ten times more than a Sparrow, and has but one half; 97 000 times more than a Gnat, and has but one fortieth.

A resisting surface of 30 sq. yards will enable a man of ordinary weight to descend safely from a great elevation.

Strength of Insects.—Insects are relatively strongest of all animals. A Cricket can leap 80 times its length, and a Flea 200 times.

Application for Stings and Burns.

Sting of Insects.—Ammonia, or Soda moistened with water, and applied as a paste.

Burns.—Hot alcohol or turpentine, and afterwards bathed with lime water and sweet oil. Cold water not to be applied.

To Preserve Meat.

Meat of any kind may be preserved in a temperature of from 80° to 100°, for a period of ten days, after it has been soaked in a solution of 1 pint of salt dissolved in 4 gallons of cold water and .5 gallon of a solution of bisulphate of calcium.

By repeating this process, preservation may be extended by addition of a solution of gelatin or white of an egg to the salt and water.

To Detect Starch in Milk.

Add a few drops of acetic acid to a small quantity of milk; boil it, and after it has cooled filter the whey. If starch is present, a drop of iodine solution will produce a blue tint.

This process is so delicate that it will show the presence of a milligram of starch in a cube centimeter of whey (1 grain of starch in 2.16 fluid-ounces).

Retaining Walls of Iron Piles.

Sheet Piles.—7 feet from centres, 18 ins. in width and 2 ins. in thickness, strengthened with 2 ribs 8 ins. in depth.

lating.—7 feet in length by 5 feet in width and 1 inch in thickness, with a central feather 1 by 6 ins. 6-rods 2 ins. in diameter.

Stone Sawing.

Marble Stone Sawing.—(Emerson.) Alabama marble 6 feet X 2.5 feet in 22 min. = 41 sq. feet per hour.

Wood Sawing.

Hard measure, from 9 round logs in 1 hour. Engine 12 hp.

Cost of Dredging.

il cost, if on an extended work, inclusive of Delivery, if dredging into or on a vessel alongside of dredger.—(Trautwine.)

Labor at \$1 per day and Repairs of Plant included.

| h. | Centa. | Depth. | Centa. | Depth. | Centa. | Depth. | Centa. |
|----|-------------|--------|-------------|--------|-------------|--------|-------------|
| 1. | Cube Yards. | Feet. | Cube Yards. | Feet. | Cube Yards. | Feet. | Cube Yards. |
| | 6 | 20 | 8 | 25 | 10 | 35 | 18 |
| | 7 | 22 | 9 | 30 | 13 | 40 | 25 |

charge of Scows or Camels.—Towing .25 mile 4 cents per cube yard, .5 mile 6 , .75 mile 8 cents, and 1 mile 10 cents.

RE.—A *Scow* is a flat-bottomed vessel or boat. A *Camel* is a shallow, flat-med and decked vessel, designed for the transportation of heavy freight or the ining of attached bodies, as a vessel, by its buoyancy.

Dredging.

team dredge will raise 6 cube yards, or 8.5 tons, per hour per HP.

Metal Boring and Turning.

ring.—*Cast iron.*—Divide 25 by the diameter of the cylinder in inches for the ntions per minute.

rought iron.—The speed is one fourth to one fifth greater than for cast iron.

ass.—The speed is about twice that for cast iron.

ring.—*Cast iron.*—The speed is twice that of boring.

rought iron.—The speed is one fourth to one fifth greater than that for cast iron.

ass.—The speed is twice that of boring.

ritical boring.—The speed may be twice that of horizontal boring.

e feed depends upon the stability of the machine and depth of the cut.

Well Boring.

Coventry, Eng., 750000 galls. of water per day are obtained by two borings of 18 ins., at depths of 200 and 300 feet.

Liverpool, Eng., 3000000 galls. of water per day are obtained by a bore 6 ins. diameter and 161 feet in depth.

is large yield is ascribed to the existence of a *fault* near to it, and extending to th of 484 feet.

Kentish Town, Eng., a well is bored to the depth of 1302 feet.

Passy, France, a well with a bore of 1 meter in diameter is sunk to a depth of feet, and for a diameter of 2 feet 4 ins. it is further sunk to a depth of 109 feet 1, or 1903 feet 10 ins., from which a yield of 5582000 galls. of water are obtained y.

Tempering Boring Instruments.

it the tool to a blood-red heat; hammer it until it is nearly cold; reheat it to id-red heat, and plunge it into a mixture of 2 oz. each of vitriol, soda, sal-am-uc, and spirits of nitre, 1 oz. of oil of vitriol, .5 oz. of saltpetre, and 3 galls. of , retaining it there until it is cool.

Circular Saws.

olutions per Minute.—8 ins. 4500, 10 ins. 3600, and 36 ins. 1000.

Masonry.

crete or Beton should be thrown, or let fall from a height of at least 10 feet, l beaten down.

average weight of brickwork in mortar is about 102 lbs. per cube foot.

Plastering.

measuring Plasterers' work all openings, as doors, winde
at one half of their areas, and cornices are measured u
including that cut off by mitring.

Glazing.

lasiors' work, oval and round windows are measured as sq

U. S. ENSIGN, PENNANTS, AND FLAGS.

(From July 4, 1890.)

—Head (Depth, or Hoist).—Ten nineteenthths of its length.

—Seven horizontal stripes of equal breadth, alternately red and white, with red.

Blue field in upper quarter, next the head, $\frac{1}{4}$ of length of field, $\frac{1}{10}$ of depth, with white stars ranged in equidistant, horizontal line, equal in number to number of States of the Union.

— (Narrow).—Head.—6.24 ins. to a length of 70 feet; 5.04 ins. to a length of 42 ins. to a length of 35 feet. *Night*, 3.6 ins. to a length of 20 feet; 2.52 ins. to a length of 6 feet.

Blue field at head, one fourth the length, with 13 white stars in a horizontal line.—A red and white stripe uniformly tapered to a point, red upper and white lower.—Union to have but 7 stars.

Jack.—Alike to the Union of an Ensign in dimensions and stars.

—President.—Rectangle, with arms of the U. S. in centre of which are 13 stars in an arc.

—Army of Navy.—Rectangle, with a vertical white fowl anchor on a blue field, with four white stars in a rectangle, set quadrilateral on the anchor.

—Rectangle, with 4 white stars in centre of a blue field, set as

Admiral.—Same as Admiral's, with 3 white stars set as an triangle.

Admiral.—Same as Admiral's, with 2 white stars set vertically. Where Rear-Admirals in command afloat should meet, their seniority is indicated respectively by a Blue flag, a Red with White stars, and a White flag with Blue stars.

—Commodore. (Broad Pennant).—Blue, Red, or White, according to the star in centre of field, being white in blue and red pennants, and blue in white.

—Rectangle, angle at tail, bisected by a line drawn at a right angle from centre of field, and at a distance from head of three fifths of length of pennant; the rectangular with head or hoist; upper side tapered, running the width of the tails to the hoist. *Head.—*6 length. *Fly* 1.66 hoist.

—Signal Marks.—Triangle, 1st Blue, 2d Red, 3d White, Blue Reserve Division.—Yellow, Red vertical. Division mark is worn on the end of a division of a squadron at mizzen, when not authorized on a Commodore or Flag of an Admiral. *Fly* .8 hoist.

Numbers.—*Fly* 1.25 hoist. *Signal Pennants, Fly* 4.6 hoist. *Reserve Pennants, Fly* .89 hoist.

—Active Pennants.—Of a Senior Officer Present, is the Division mark of the First Division of a fleet.

Signals.—Very's System.

—Signal Number, Square, and Signal Pennants. *Fly* .3 hoist.

Suspension Bridges. Length of Spans in Feet.

| | | | |
|-------------|-----|-------------------------------|-----------|
| Albany..... | 330 | Niagara..... | 822 |
| Albany..... | 342 | Lewistown and Queenstown..... | 1040 |
| Albany..... | 422 | Cincinnati..... | 1057 |
| Albany..... | 660 | Niagara Fall..... | 1280 |
| Albany..... | | Albany..... | 135 feet. |

Alimentary Principles.

Primary division of Food is into Organic and Inorganic.

Organic is subdivided into Nitrogenous and Non-Nitrogenous; Inorganic is composed of water and various saline principles. The former elements are destined for growth and maintenance of the body, and are termed "plastic elements of nutrition." The latter are designed for undergoing oxidation, and thus become source of heat, and are termed "elements of respiration," or "Calorificient."

Although Fat is non-nitrogenous, it is so mixed with nitrogenous matter that it becomes a nutrient as well as a calorificient.

Alimentary Principles. — 1. Water; 2. Sugar; 3. Gum; 4. Starch; 5. Pectine; 6. Acetic Acid; 7. Alcohol; 8. Oil or Fat. *Vegetable and Animal.* — 9. Albumen; 10. Fibrine; 11. Caseine; 12. Gluten; 13. Gelatine; 14. Chloride of Sodium.

These alimentary principles, by their mixture or union, form our ordinary food, which, by way of distinction, may be denominated *compound aliments*; thus, meat is composed of fibrine, albumen, gelatine, fat, etc.; wheat consists of starch, gluten, sugar, gum, etc.

Analysis of Meats, Fish, Vegetables, etc.

| Food. | Water. | Nitrogenous Matter. | Fat. | Saline Matter. | Non-Nitrogenous Matter. | Sugar. | Cellulose. | Ash, &c. |
|----------------------|--------|---------------------|-------|----------------|-------------------------|--------|------------|----------|
| Arrowroot..... | 18 | — | — | — | 82 | — | — | — |
| Barley Meal..... | 15 | 6.3 | 2.4 | 2 | 59.4 | — | — | — |
| Beans, White..... | 9.9 | 25.5 | 2.8 | — | 55.7 | 4.9 | — | — |
| Beef, roast..... | 54 | 27.6 | 15.45 | 2.95 | — | — | — | — |
| fat..... | 51 | 14.8 | 29.8 | — | — | — | — | — |
| lean..... | 72 | 19.3 | 3.6 | 5.1 | — | — | — | — |
| salt..... | 49.1 | 29.6 | .2 | 21.1 | — | — | — | — |
| Beer and Porter.... | 91 | .1 | — | .2 | 8.7 | — | — | — |
| Buckwheat..... | 13 | 13.1 | 3 | .4 | 64.5 | — | 3.5 | 2.5 |
| Butter and Fats.... | 15 | — | 83 | 2 | — | — | — | — |
| Cabbage..... | 91 | 2 | .5 | .7 | 5.8 | — | — | — |
| Carrots..... | 83 | 1.3 | .2 | 1 | 7.4 | 6.1 | — | 1 |
| Cheese..... | 36.8 | 33.5 | 24.3 | 5.4 | — | — | — | — |
| Corn Meal..... | 14 | 11.1 | 8.1 | 1.7 | 57.6 | .4 | 5.9 | 1.8 |
| Cream..... | 66 | 2.7 | 26.7 | 1.8 | — | 2.8 | — | — |
| Egg..... | 74 | 14 | 10.5 | 1.5 | — | — | — | — |
| yolk..... | 52 | 16 | 30.7 | 1.3 | — | — | — | — |
| Fish, white flesh... | 78 | 18.1 | 2.9 | 1 | — | — | — | — |
| Eels..... | 75 | 9.9 | 13.8 | 1.3 | — | — | — | — |
| Lobster, flesh..... | 76.6 | 19.17 | 1.17 | 1.8 | 1.26 | — | — | — |
| Oysters..... | 80.39 | 14.01 | 1.52 | 2.7 | 1.38 | — | — | — |
| Liver, Calf's..... | 72.33 | 20.55 | 5.58 | 1.54 | — | — | — | — |
| Milk, Cow's..... | 86 | 4.1 | 3.9 | .8 | — | 5.2 | — | — |
| Mutton, fat..... | 53 | 12.4 | 31.1 | 3.5 | — | — | — | — |
| Oatmeal..... | 15 | 12.6 | 5.6 | 3 | 58.4 | 5.4 | — | — |
| Onions..... | 21 | 14.4 | 5.5 | — | 48.2 | — | 7.6 | 3.3 |
| Parsnips..... | 82 | 1.1 | .5 | 1 | 9.6 | 5.8 | — | — |
| Peas..... | 15 | 23 | 2.1 | 2.5 | 50.2 | 2 | 3.1 | 2.1 |
| Pork, fat..... | 39 | 9.8 | 48.9 | 2.3 | — | — | — | — |
| Bacon, dry..... | 15 | 8.8 | 73.3 | 2.9 | — | — | — | — |
| Potatoes..... | 75 | 2.1 | .2 | .7 | 16.8 | 3.2 | 1 | 1 |
| Poultry..... | 74 | 21 | 3.8 | 1.2 | — | — | — | — |
| Rice..... | 13 | 6.3 | .7 | .5 | 78.1 | .4 | — | 1 |
| Rye Meal..... | 15 | 8 | 2 | 1.8 | 69.5 | 3.7 | — | — |
| Sugar..... | 5 | — | — | — | 95 | — | — | — |
| Turnips..... | 68 | 13.2 | 16.4 | 2.4 | — | — | — | — |
| Turnips..... | 91 | 1.2 | — | .6 | 4.3 | 2.1 | — | .8 |
| Veal..... | 63 | 16.5 | 15.8 | 4.7 | — | — | — | — |
| Wheat Flour..... | 15 | 10.8 | 2 | 1.7 | 61.1 | 4.2 | 3.5 | 1.7 |
| Bread*..... | 37 | 8.1 | 1.6 | 2.3 | 45.4 | 3.6 | — | 2 |
| Bran.... | 18 | 6 | — | — | 60 | — | — | 1 |

* Water absorbed most, 100 l

from 40 to 60 per cent. of weight of flour, the best quality of bread.

Analysis of Different Foods In their Natural Condition.

| | Ni- trates. | Carbon- ates. | Phos- phates. | Water. | | Ni- trates. | Carbon- ates. | Phos- phates. | Water. |
|----------|----------------|------------------|------------------|--------|---------------|----------------|------------------|------------------|--------|
| MS..... | 5 | 10 | 1 | 84 | Milk of cow.. | 5 | 8 | 1 | 86 |
| Y..... | 17 | 69.5 | 3.5 | 10 | Mutton..... | 12.5 | 40 | 4.5 | 43 |
| S..... | 24 | 57.7 | 3.5 | 14.8 | Oats..... | 17 | 66.4 | 3 | 13.6 |
| | 15 | 30 | 5 | 50 | Parsnips..... | 9.2 | 7 | 1 | 82.8 |
| wheat.. | 8.6 | 75.4 | 1.8 | 14.2 | Pork..... | 10 | 50 | 1.5 | 38.5 |
| age..... | 4 | 5 | 1 | 90 | Potatoes..... | 2.4 | 22.5 | .9 | 74.2 |
| ten..... | 19 | 3.5 | 4.5 | 73 | " sweet | 1.5 | 28.4 | 2.6 | 67.5 |
| North'n | 12 | 73 | 1 | 14 | Rice..... | 6.5 | 79.5 | .5 | 13.5 |
| South'n | 35 | 48 | 3 | 14 | Turnips..... | 5 | 4 | .5 | 90.5 |
| mbers.. | 1.5 | 1 | .5 | 97 | Veal..... | 16 | 10.5 | 4.5 | 63 |
| h..... | 11 | 35.5 | 3.5 | 50 | Wheat..... | 15 | 69.2 | 1.6 | 14.2 |

trates—Are that class which supplies waste of muscle.

rbonates—Are that class which supplies lungs with fuel, and thus furnishes heat system, and supplies fat or adipose substances.

osphates—Are that class which supplies bones, brains, and nerves, and gives power, both muscular and mental.

From above it appears, that Southern corn produces most muscle and least fat, contains enough of phosphates to give vital power to brain, and make bones strong. Mutton is the meat which should be eaten with Southern corn.

The nitrates in all the fine bread which a man can eat will not sustain life beyond days; but others, fed on unbolted flour bread, would continue to thrive for an indefinite period. It is immaterial whether the general quantity of food be reduced or, whether either of the muscle-making or heat-producing principles be drawn while the other is fully supplied. In either case the effect will be the same. A man will become weak, dwindle away and die, sooner or later, according to efficiency; and if food is eaten which is deficient in either principle, the appetite will demand it in quantity till the deficient element is supplied. All food, be the amount necessary to supply the principle that is not deficient, is not only good, but burdens the system with efforts to dispose of it.

Analysis of Fruits.

| FRUIT. | Water. | Sugar. | Acid. | Albumi- nous sub- stances. | Insoluble matter. | Pectous sub- stances. | Ash. |
|------------------|--------|--|-------|----------------------------------|----------------------|-----------------------------|------|
| , white..... | 85 | 7.6 | 1 | .22 | 1.83 | 3.88 | .47 |
| ot, average..... | 83.5 | 1.8 | 1.1 | .51 | 4.7 | 7.55 | .84 |
| berry..... | 86.4 | 4.44 | 1.19 | .51 | 5.26 | 1.72 | .48 |
| y, red..... | 75.4 | 13.1 | .35 | .9 | 5.83 | 3.73 | .69 |
| sour..... | 80.5 | 8.77 | 1.28 | .83 | 5.91 | 2.07 | .64 |
| black..... | 79.7 | 10.7 | .56 | 1 | 6.04 | 1.33 | .67 |
| it, red..... | 85.4 | 5.6 | 1.7 | .36 | 3.74 | 2.4 | .8 |
| berry, red..... | 85.6 | 8 | 1.35 | .44 | 2.92 | 1.26 | .43 |
| yellow..... | 85.4 | 7 | 1.2 | .46 | 3.17 | 2.4 | .37 |
| white..... | 80 | 13.78 | 1 | .83 | 2.48 | 1.44 | .47 |
| Dutch..... | 85 | 1.58 | .61 | .46 | 5.49 | 6.4 | .46 |
| ed..... | 83.5 | 7.5 | .07 | .25 | 3.54 | 4.8 | .34 |
| yellow gage.... | 80.8 | 2.96 | .96 | .48 | 3.98 | 10.48 | .34 |
| large..... | 79.7 | 3.4 | .87 | .4 | 3.01 | 11.3 | .42 |
| black blue..... | 88.7 | 2 | 1.27 | .4 | 6.86 | .23 | .54 |
| " red..... | 85.3 | 2.25 | 1.33 | .43 | 4.23 | 5.85 | .61 |
| Italian, sweet.. | 81.3 | 6.73 | .84 | .83 | 4.01 | 5.63 | .66 |
| rry, wild..... | 83.9 | 3.6 | 2 | .55 | 8.37 | 1.28 | .4 |
| erry, "..... | 87 | 4 | 1.5 | .6 | 5.5 | .4 | 1 |
| | 73.9 | Sugar, Pectin, Salt, Acid, etc., 26.1. | | | | | |

r and Water in Various Products not Included in the Table. (Per Cent.)

| | Sugar. | Water. | | Water. |
|------------|--------|-----------------|----|-------------|
| crude..... | 95 | Molasses..... | 23 | Cabbage.... |
| | 77 | Lean beef..... | 72 | Ale and Be |
| alk..... | 6.4 | Buttermilk..... | 88 | Coffee and |

Relative Values of Foods of Assimilating Quality to make an Equal Quantity of Flesh in Cattle or Sheep.
(Ewart.)

| ARTICLE. | Cattle. | Sheep. | ARTICLE. | Cattle. | Sheep. |
|----------------------------|---------|--------|-------------------------|---------|--------|
| Turnips | 800 | 400 | Wheat bran | 45 | 105 |
| Carrots | 630 | — | Corn and Barley meal .. | 35 | — |
| Beets | 600 | 300 | Oatmeal | 34 | — |
| Parsnips and Swedes ... | 600 | 200 | Beanmeal | 33 | — |
| Meadow grass in bloom .. | 400 | — | Peameal | 32 | — |
| Vetches, pods open | 360 | 90 | Cabbage | — | 300 |
| Potatoes at maturity | 280 | 200 | Pea straw | — | 200 |
| Oat straw, cut green | 125 | — | Rye bran | — | 109 |
| Bean or Vetch straw | — | 200 | Oats | — | 70 |
| Meadow hay | 100 | 100 | Buckwheat | — | 65 |
| Vetch " | 90 | — | Barley | — | 60 |
| Linseed cake | 50 | — | Pease or Beans | — | 54 |

NOTE. — When these values express weight in lbs., then such food will produce about 4 to 5 lbs. beef or mutton.

Nutritive Constituents and Values of Food in Grains per Pound.

| Food. | Carbon. | Nitrogen. | Food. | Carbon. | Nitrogen. |
|------------------------|---------|-----------|--------------------|---------|-----------|
| Bakers' Bread | 1975 | 88 | Mutton | 1900 | 189 |
| Barley Meal | 2563 | 68 | New Milk | 599 | 44 |
| Beef | 1854 | 184 | Oatmeal | 2831 | 136 |
| Beer and Porter | 274 | 1 | Parsnips | 554 | 12 |
| Bullock's Liver | 934 | 204 | Pearl Barley | 2660 | 91 |
| Buttermilk | 387 | 44 | Potatoes | 769 | 22 |
| Carrots | 508 | 14 | Red Herrings | 1435 | 217 |
| Cheddar Cheese | 3344 | 306 | Rice | 2732 | 68 |
| Cocoa | 3934 | 140 | Rye Meal | 2693 | 86 |
| Dry Bacon | 5987 | 95 | Salt Butter | 4585 | — |
| Fat Pork | 4113 | 106 | Skim Cheese | 1947 | 483 |
| Flour, Seconds | 2700 | 116 | Skimmed Milk | 438 | 43 |
| Fresh Butter | 6456 | — | Split Pease | 2698 | 248 |
| Green Bacon | 5426 | 76 | Suet | 4710 | — |
| Green Vegetables | 420 | 14 | Sugar | 2955 | — |
| Indian Meal | 3016 | 120 | Turnips | 263 | 13 |
| Lard | 4819 | — | Whey | 154 | 13 |
| Molasses | 2395 | — | Whitefish | 871 | 195 |

The *Full Daily Diet* of a man is held to be 12 oz. bread, 8 oz. potatoes 6 oz. meat, 4 oz. boiled rice with milk, .375 pint of broth or pea soup, 1 pint milk, and 1 pint of beer.

Nutritive Values and Constituents of Milk. — (Payson.)

| ANIMAL. | Nitrogenous Matter and Insoluble Salts. | Butter. | Lactic and Soluble Salts. | Water. | ANIMAL. | Nitrogenous Matter and Insoluble Salts. | Butter. | Lactic and Soluble Salts. | Water. |
|----------|---|---------|---------------------------|--------|-----------|---|---------|---------------------------|--------|
| Goat ... | 4.5 | 4.1 | 5.8 | 85.6 | Ass. | 1.7 | 1.4 | 6.4 | 90.5 |
| Cow | 4.55 | 3.7 | 5.35 | 86.4 | Mare ... | 1.62 | .2 | 8.75 | 89.43 |
| Woman. | 3.35 | 3.34 | 3.77 | 89.54 | Ewe | 4.68 | 4.2 | 5.5 | 85.62 |

Weight of some Different Foods required to furnish 1220 Grains of Nitrogenous Matter.

| | Lbs. | | Lbs. | | Lbs. | | Lbs. |
|------------------|------|----------------|------|----------------|------|---------------|------|
| Cheese..... | .4 | Meat, fat..... | 1.3 | Bacon, fat.... | 1.8 | Barley Meal.. | 2.1 |
| Pease..... | .7 | Oatmeal..... | 1.5 | Bread..... | 2.1 | Milk..... | 4.1 |
| Meat, lean.... | .9 | Corn Meal.... | 1.6 | Rye Meal..... | 2.3 | Potatoes..... | 8.1 |
| Fish, White... 1 | | Wheat Flour.. | 1.7 | Rice..... | 2.8 | Parsnips..... | 15.1 |

Turnips, 15.9 lbs.; Beer or Porter, 158.6 lbs.

Proportion of Sugar and Acid in Various Fruits.
(*Fresenius.*)

| FRUIT. | Sugar. | | Acid. | FRUIT. | Sugar. | | Acid. |
|-----------------|-----------|-----------|-------|-------------------|-----------|-----------|-------|
| | Per Cent. | Per Cent. | | | Per Cent. | Per Cent. | |
| Apple..... | 8.4 | .8 | | Plum..... | 2.1 | 1.3 | |
| Apricot..... | 1.8 | 1.1 | | Prune..... | 6.3 | .9 | |
| Blackberry..... | 4.4 | 1.2 | | Raspberry..... | 4 | 1.5 | |
| Cherries..... | 6.1 | 2 | | Red Pear..... | 7.5 | .1 | |
| Cranberry..... | 7.2 | 1.5 | | Sour Cherry..... | 8.8 | 1.3 | |
| Gooseberry..... | 14.9 | .7 | | Strawberry..... | 5.7 | 1.3 | |
| Raspberry..... | 9.2 | 1.9 | | Sweet Cherry..... | 10.8 | .6 | |
| Strawberry..... | 1.6 | .7 | | Whortleberry..... | 5.8 | 1.3 | |

Proportion of Oil in Various Air-dry Seeds. (*Berjot.*)

| | | | |
|-----------------|-----------------|----------------|----------------|
| Almond..... 24 | Mustard..... 30 | Almond..... 40 | Orange..... 40 |
| Linseed..... 28 | Flax..... 34 | Colza..... 40 | Poppy..... 50 |
| Peanut..... 36 | Peanut..... 38 | Colza..... 45 | |

Analysis of different Articles of Food, with Reference only to their Properties for giving Heat and Strength.

(*Payen.*) In 100 Parts.

| SUBSTANCES. | Carb. | Nitro- | SUBSTANCES. | Carb. | Nitro- | SUBSTANCES. | Carb. | Nitro- |
|---------------------|-------|--------|--------------------|-------|--------|------------------|-------|--------|
| | n. | gen. | | bon. | gen. | | bon. | gen. |
| Alcohol..... | 52 | — | Coffee..... | 9 | 1.1 | Oil, Olive..... | 98 | — |
| Barley..... | 40 | 1.9 | Corn..... | 44 | 1.7 | Oysters..... | 7.18 | 2.13 |
| Beans..... | 42 | 4.5 | Eels..... | 30.05 | 2 | Peanut..... | 44 | 3.66 |
| Beef, meat..... | 11 | 3 | Eggs..... | 13.5 | 1.9 | Potatoes..... | 11 | .33 |
| Butter, strong..... | 4.5 | .08 | Figs, dried..... | 34 | .92 | Rice..... | 41 | 1.8 |
| Butter, stale..... | 28 | 1.07 | Herring, salt..... | | | Rye Flour..... | 41 | 1.75 |
| Wheat..... | 42.5 | 2.2 | Edible..... | 23 | 3.11 | Salmon..... | 16 | 2.09 |
| Butter..... | 83 | .64 | Liver, Calf's..... | 15.68 | 3.93 | Sardines..... | 29 | 6 |
| Butter, oil..... | 5.5 | .31 | Lobster..... | 10.06 | 2.93 | Tea..... | 2.1 | .2 |
| Butter, are..... | 27.41 | 4.49 | Mackerel..... | 19.26 | 3.74 | Truffles..... | 9.45 | 1.35 |
| Case, Chest'r..... | 41.04 | 4.13 | Milk, Cow's..... | 8 | .66 | Wheat..... | 41 | 3 |
| Case, salt'd..... | 58 | 1.52 | Nuts..... | 10.65 | 1.4 | Wheat Flour..... | 38.5 | 1.64 |
| | 16 | 5.02 | Oatmeal..... | 44 | 1.95 | Wine..... | 4 | .015 |

NOTE.—Multiply figures representing nitrogen by 6.5, and equivalent amount of organic matter is obtained.

Human and Animal Sustenance.

Least Quantity of Food required to Sustain Life. (E. Smith, M.D.)

| | Carbon. | | Hydrogen. |
|------------------|---------|---------------|--------------|
| | Gr. | | Gr. |
| Adult Man..... | 4300 | } Mean, 4100. | 200 |
| Adult Woman..... | 3900 | | 180 |
| | | | } Mean, 190. |

1 adult man, for his daily sustenance, requires about 1220 grs. nitrogenous matter or 200 of nitrogen, and bread contains 8.1 per cent. of it.

NOTE, $\frac{1220}{.081} = 15062$ grains which $\div 7000$ in a lb. = 2 lbs. 2.43 oz. of bread.

These quantities and proportions are also contained in about 16 lbs. of rye.

18, by table of nutritive values, page 202, turnips have 263 grains of carbon and nitrogen.

NOTE, $\frac{4300}{263}$ and $\frac{200}{13} = 16.35$ lbs. for the necessary carbon and 15.4 lbs. for the nitrogen.

Relative Value of Foods compared with 100 lbs. of Good Hay.

| | Lbs. | | Lbs. | | Lbs. |
|-----------------|------|-----------------|------|--------------|------|
| Rye, green..... | 400 | Rye straw..... | 442 | Carrots..... | 276 |
| Green..... | 275 | Oat straw..... | 195 | Barley..... | 54 |
| Straw..... | 374 | Cornstalks..... | 400 | Oats..... | 57 |
| | | | | Corn..... | |
| | | | | Linseed..... | |
| | | | | Wheat..... | |

Weight of Articles of Food required to be consumed in the human system to develop a power equal to raising 140 lbs. to a height of 10 000 feet. (Frankland)

| SUBSTANCES. | Weight. | SUBSTANCES. | Weight. | SUBSTANCES. | Weight. |
|--------------------|-----------|---------------------|------------|--------------------|-----------|
| Cod-liver oil..... | Lbs. .553 | Rice..... | Lbs. 1.341 | Salt Beef..... | Lbs. 3.65 |
| Beef, fat..... | .555 | Isinglass..... | 1.379 | Veal, lean..... | 4.3 |
| Bacon..... | .67 | Sugar, lump..... | 1.505 | Porter..... | 4.65 |
| Butter..... | .693 | Cream..... | 2.062 | Potatoes..... | 5.68 |
| Cocoa..... | .797 | Egg, boiled..... | 2.209 | Fish..... | 6.56 |
| Fat of Pork..... | .97 | Bread..... | 2.345 | Apples..... | 7.85 |
| Cheese..... | 1.156 | Salt Pork..... | 2.826 | Milk..... | 8.05 |
| Oatmeal..... | 1.281 | Ham, lean, boiled.. | 3.001 | Egg, white of..... | 8.95 |
| Arrowroot..... | 1.287 | Mackerel..... | 3.124 | Carrots..... | 9.65 |
| Wheat flour..... | 1.311 | Alc, bottled..... | 3.461 | Cabbage..... | 12.01 |

Relative Value of Various Foods as Productive of Force when Oxidized in the Body.

| | | | |
|--------------------|---------------------|----------------------|-------------------|
| Cabbage..... 1 | Porter..... 2.6 | Egg, hard boil'd 5.4 | Oatmeal..... 93 |
| Carrots..... 1.2 | Veal, lean..... 2.8 | Cream..... 5.9 | Cheese..... 104 |
| Skimmed Milk. 1.2 | Salt Beef..... 3.3 | Egg, yolk..... 7.9 | Fat of Pork. 124 |
| White of Egg.. 1.4 | Poultry..... 3.3 | Sugar..... 8 | Cocoa..... 163 |
| Milk..... 1.5 | Lean Beef..... 3.4 | Isinglass..... 8.7 | Pemmican.. 169 |
| Apples..... 1.5 | Mackerel..... 3.8 | Rice..... 8.9 | Butter..... 171 |
| Alc..... 1.8 | Ham, lean..... 4 | Pea Meal..... 9 | Bacon..... 174 |
| Fish..... 1.9 | Salt Pork..... 4.3 | Wheat Flour.. 9.1 | Fat of Beef.. 211 |
| Potatoes..... 2.4 | Bread, crumb.. 5.1 | Arrowroot.... 9.3 | Cod-liver Oil 217 |

Nutritious Properties of different Vegetables and Oil-cake, compared with each other in Quantities.

| | | | |
|---------------------|--------------------|-------------------|------------------|
| Oil-cake..... 1 | Rye..... 2.5 | Clover hay.... 4 | Cabbage..... 18 |
| Pease and Beans 1.5 | Bran, wheat { 2.75 | Hay..... 5 | Wheat straw.. 26 |
| Wheat, flour... 2 | 3 | Potatoes..... 14 | Barley " 26 |
| " grain... 2.5 | Corn..... 3 | " old.... 20 | Oat " 27.5 |
| Oats..... 2.5 | Barley..... 3 | Carrots..... 17.5 | Turnips..... 30 |

ILLUSTRATION.—1 lb. of oil-cake is equal to 18 lbs. of cabbage.

Volume of Oxygen required to Oxidize 100 parts of following Foods as consumed in the Body.

Grape Sugar .. 106 | Starch..... 120 | Albumen..... 150 | Fat..... 263

Hence, assuming capacity for oxidation as a measure, albumen has half value of fat as a food-producing element, and a greater value than either starch or sugar.

Proportion of Alcohol in 100 Parts of following Liquors. (Brande.)

| | | |
|----------------------------|----------------------------|----------------------------|
| Small Beer... 1 and 1.03 | Hermitage, red..... 12.32 | Lisbon..... 18.94 |
| Porter..... 3.5 and 5.26 | Champagne..... 12.61 | Lachryma..... 19.7 |
| Cider..... 5.2 and 9.8 | Amontillado..... 12.63 | Teneriffe..... 19.78 |
| Brown Stout. 5.5 and 6.8 | Frontignac..... 12.89 | Curran Wine..... 20.55 |
| Alc..... 6.87 and 10 | Barsac..... 13.86 | Madeira..... 22.17 |
| Rhenish..... 7.58 | Sauterne..... 14.22 | Port..... 23 |
| Moselle..... 8.7 | Champagne Burg'dy, 14.57 | Sherry, old..... 23.86 |
| Johannisberger..... 8.71 | White Port..... 15 | Marsala..... 25.09 |
| Elder Wine..... 8.79 | Bordeaux..... 15.1 | Raisin Wine..... 25.15 |
| Claret ordinaire..... 8.99 | Malmsey..... 16.4 | Madeira, Sercial..... 27.4 |
| Tokay..... 9.33 | Sherry..... 17.17 | Cape Madeira..... 29.1 |
| Rudesheimer..... 10.72 | Malaga..... 17.2 | Gili..... 31.6 |
| Marobrunner..... 11.6 | Alba Flora..... 17.26 | Brandy..... 33.3 |
| Gooseberry Wine... 11.84 | Hermitage, white... 17.43 | Rum..... 33.8 |
| Hockheimer..... 12.03 | Cape Muscat..... 18.25 | Irish Whiskey..... 34 |
| Vin de Grave..... 12.08 | Constantia, red..... 18.92 | Scotch Whiskey..... 34.7 |

Proportion of Food Appropriated and Expended by following Animals.

| | Oxen. | Sheep. | Swine. |
|-------------------------------|-------|--------|--------|
| Proportion appropriated | 6.2 | 8 | 17.6 |
| “ in manure | 36.5 | 31.9 | 16.9 |
| “ respired | 57.3 | 60.1 | 65.5 |
| | 100 | 100 | 100 |

Specific Gravity of Milk and Percentage of Cream, etc.

| Milk. | Specific Gravity. | Volume of Cream. | Volume of Curd. | Specific Gravity when skimmed. |
|--------------------------|-------------------|------------------|-----------------|--------------------------------|
| 100 pure* | 1030 | 12 | 6.3 | 1032 |
| 10 per cent. water | 1027 | 10.5 | 5.6 | 1029 |
| 20 “ “ “ | 1024 | 8.5 | 4.9 | 1026 |
| 30 “ “ “ | 1021 | 6 | 4.2 | 1023 |

* For a method of testing the purity of milk, see Payson on Food (Philadelphia, 1874), page 196.

NOTE.—The average proportion of cream is 10, or 10 per cent.

Proportion Per cent. of Starch in sundry Vegetables.

| | | | |
|--------------------|----------------------|------------------|-------------------|
| Wheat flour ... 82 | Wheat flour ... 66.3 | Oatmeal ... 58.4 | Potatoes ... 18.8 |
| 79.1 | Corn meal ... 64.7 | Pease ... 55.4 | Turnips ... 5.1 |

Composition of Cheese of Different Countries.—(Payson)

| | Fat. | Nitrogen. | Salt. | Water. | | Fat. | Nitrogen. | Salt. | Water. |
|-------------|-------|-----------|-------|--------|---------------|-------|-----------|-------|--------|
| Cheddar ... | 18.74 | 2.28 | 4.25 | 61.87 | Chester ... | 25.41 | 5.56 | 4.78 | 50.39 |
| Swiss ... | 21.68 | 5.48 | 7.09 | 30.31 | Gruyères ... | 28.4 | 5.4 | 4.29 | 32.05 |
| | 24.83 | 2.39 | 5.63 | 53.99 | Marolles ... | 28.73 | 3.73 | 5.93 | 40.07 |
| and ... | 25.06 | 4.1 | 6.21 | 41.41 | Roquefort ... | 32.31 | 5.07 | 4.45 | 26.53 |

nutritive Equivalents. Computed from Amount of Nitrogen in Substances when Dried. Human Milk at 1.

| | | | | | | | |
|-----------|------|-----------------|------|---------------|------|---------------|------|
| | .81 | Bread, White. | 1.42 | Cheese | 3.31 | Lamb | 8.33 |
| Des | .84 | Milk, Cows' .. | 2.37 | Eel | 4.34 | Egg, White .. | 8.45 |
| | 1 | Pease | 2.39 | Mussel | 5.28 | Lobster | 8.59 |
| | 1.06 | Lentils | 2.76 | Liver, Ox ... | 5.7 | Veal | 8.73 |
| | 1.19 | Egg, Yolk | 3.05 | Pigeon | 7.56 | Beef | 8.8 |
| | 1.25 | Oysters | 3.05 | Mutton | 7.73 | Pork | 8.93 |
| | 1.38 | Beans | 3.2 | Salmon | 7.76 | Ham | 9.1 |

Herring, 9.14.

Thermometric Power and Mechanical Energy of 100 grains of Various Substances in their Natural Condition, when Oxidized in the Animal Body into Carbonic Acid, Water, and Urea.—(Frankland.)

| SUBSTANCE. | Water raised 1°. | | SUBSTANCE. | Water raised 1°. | | SUBSTANCE. | Water raised 1°. | |
|-------------|------------------|---------------------|----------------|------------------|---------------------|--------------|------------------|---------------------|
| | Lbs. | Lifted 1 foot high. | | Lbs. | Lifted 1 foot high. | | Lbs. | Lifted 1 foot high. |
| Wheat's ... | 1.99 | 1.54 | Cheese | 11.2 | 8.65 | Mackerel ... | 4.14 | 3.2 |
| | 1.48 | 1.29 | Cocoa-nibs ... | 17 | 7.3 | Milk | 1.64 | 1.25 |
| Root ... | 10.06 | 7.77 | Cod-liver oil. | 11 | 18.12 | Oatmeal ... | | 7.8 |
| Pea ... | 3.66 | 2.83 | Egg, h'd boil. | 5.86 | 4.53 | Pea m ... | | 49 |
| | 5.52 | 4.26 | “ yolk ... | 8.5 | 6.56 | Potato ... | | |
| | 18.68 | 14.42 | “ white ... | 1.48 | 1.14 | Porte ... | | |
| Be ... | 1.08 | .83 | Flour, wheat. | 9.87 | 7.62 | Rice ... | | |
| Ham ... | 1.33 | 1.03 | Ham, boiled. | 4.3 | 3.32 | Suga ... | | |

Digestion.

Time required for Digestion of several Articles of Food.
(Beaumont, M.D.)

| Food. | Time. | Food. | Time. |
|---------------------------------|-------|----------------------------------|-------|
| Apple, sweet and mellow . . . | 1 50 | Heart, Animal, fried | 4 |
| sour and mellow | 2 | Lamb, boiled | 2 30 |
| sour and hard | 2 50 | Liver, Beef's, boiled | 2 |
| Barley, boiled | 2 | Meat and Vegetables, hashed . . | 2 30 |
| Bean, boiled | 2 30 | Milk, boiled or fresh | 2 |
| Bean and Green Corn, boiled . | 3 45 | Mutton, roasted | 3 15 |
| Beef, roasted rare | 3 | broiled or boiled | 3 |
| roasted dry | 3 30 | Oyster | 2 5 |
| Steak, broiled | 3 | roasted | 3 15 |
| boiled | 2 45 | stewed | 3 30 |
| boiled, with mustard, etc. | 3 30 | Parsnip, boiled | 2 30 |
| Tendon, boiled | 5 30 | Pig, sucking, roasted | 2 30 |
| " fried | 4 | Feet, soured, boiled | 1 |
| old salted, boiled | 4 15 | Pork, fat and lean, roasted . . | 5 15 |
| Beet, boiled | 3 45 | recently salted, boiled . . | 4 30 |
| Bread, Corn, baked | 3 15 | " " fried | 4 15 |
| Wheat, baked, fresh . . . | 3 30 | " " broiled | 3 15 |
| Butter, melted | 3 30 | " " raw | 3 |
| Cabbage, crude | 2 30 | Potato, boiled | 3 30 |
| crude, vinegar | 2 | baked | 3 30 |
| crude, vin'r, boiled { | 4 30 | roasted | 2 30 |
| Carrot, boiled | 3 15 | Rice, boiled | 1 |
| Cartilage, boiled | 4 15 | Sago, boiled | 1 45 |
| Cheese, old and strong | 3 30 | Sausage, Pork, broiled | 3 30 |
| Chicken, fricasseed | 2 45 | Soup, Barley | 1 30 |
| Custard, baked | 2 45 | Beef and Vegetables . . . | 4 |
| Duck, roasted | 4 30 | Chicken | 3 |
| Dumpling, Apple, boiled . . . | 3 | Mutton or Oyster | 3 30 |
| Egg | 2 | Sponge-cake, baked | 2 30 |
| whipped | 1 30 | Suet, Beef, boiled | 5 30 |
| boiled hard | 3 30 | Mutton, boiled | 4 30 |
| " soft | 3 | Tapioca, boiled | 2 |
| fried | 3 30 | Tripe, soured | 1 |
| Fish, Cod or Flounder, fried . | 3 30 | Turkey, roasted { Wild | 2 15 |
| Cod, cured, boiled | 2 | boiled { Domestic | 2 30 |
| Salmon, salt'd and boil'd | 4 | Turnip, boiled | 3 30 |
| Trout, boiled or fried . . . | 1 30 | Veal, roasted | 4 |
| Fowl, boiled or roasted | 4 | fried | 4 30 |
| Geese, roasted | 3 | Brain, boiled | 1 45 |
| Gelatine, boiled | 2 30 | Venison Steak, broiled | 1 15 |

General Notes.

The per-centage of loss in the cooking of meats is as follows: Boiling 23; Baking 31; Roasting 34.

Potatoes possess anti-scorbutic power in a greater degree than any other of the succulent vegetables.

The average yearly consumption of wheat and wheat flour in Great Britain is 14 bushels per capita of its population.

The daily ration of an Esquimaux is 20 lbs. of flesh and blubber.—(Sir John Ross)

n adult healthy man, according to Dr. Edward Smith, requires daily of

| | |
|--|-------------------------------|
| Phosphoric acid from .. 32 to 79 grains. | Potash 27 to 107 grains. |
| { Chlorine..... 51 " 175 " | Soda..... 80 " 171 " |
| { Or of common salt... 85 " 291 " | Lime..... 2.3 " 6.3 " |
| and of Magnesia 2.5 to 3 grains. | |

a common fowl's egg contains 120 grains of Carbon and 17.75 of Nitrogen.

an ordinary working-man requires for his daily sustenance

| | | | |
|--------------------------|------|--------------|-------|
| Oxygen | 1.47 | Starch | .66 |
| Albuminous matter..... | .305 | Salts..... | .04 |
| Fat | .22 | Water..... | 4.535 |
| = 7.23 lbs. avoirdupois. | | | |

Milk.—If the milk of an animal is taken at three immediately successive periods, that which is first received will not be as rich in milk-fat as the last.

as a Devon cow, milked in this manner, the first milk gave but 1.166 per cent. of and the last, or that known as "strippings," 5.81 per cent.

Relative Richness of Milk of Several Animals.

Human Milk = 1.

| | Milk-fat. | Casein. | Sugar. | | Milk-fat. | Casein. | Sugar. |
|---------|-----------|---------|--------|------------|-----------|---------|--------|
| v..... | 1.66 | 1.38 | .69 | Ass..... | .5 | .39 | .94 |
| re..... | 1.19 | .75 | .94 | Sheep..... | 2.52 | 2.1 | .72 |
| it..... | 2 | 1.04 | .69 | Camel..... | 1.4 | — | .96 |

the condensation of milk reduces it to about one third of its original volume.

Farm of second-rate quality, properly cultivated, will sustain 100 head of cattle 100 acres, besides laboring stock (employed in cultivation of farm), and swine. (Swart.)

hus, calves 25; do. 1 year 25; do. 2 years 25; cows 25.

ane Sugar (Saccharose)—Is insoluble in absolute alcohol, and in diluted alcohol soluble only in proportion to its weakness. Loaf sugar, as a rule, is chemically a

et Root Sugar—Contains 85 to 96 per cent. of cane sugar, 1.6 to 5.1 of organic ter, and 2 to 4.3 of water.

oney—Contains 32 per cent. of sugar (levulose), 25.5 of water, 27.9 of dextrine, 14.6 of other matter, as mannite, wax, pollen, and insoluble matter.

olasses—Contains 47 per cent. of cane sugar, 20.4 of fruit sugar, 2.6 of salts, 2.7 active and coloring matter, and 27.3 of water.

lour.—Tests of flour, see A. W. Blyth, London, 1882, page 152.

read.—Wheat, water lost by drying after 1 day 7.71 per cent., 3 days 8.86, and 58 14.05 per cent.

igo.—2.5 lbs. per day will support a healthy man.

g—Contains nearly as much gluten as wheat bread (as 6 to 7), and in starch and it is 16 per cent. richer.

oseberry (dry)—Is as nutritious as wheat bread.

atermelon, Vegetable marrow, and Cucumber—Contain 94, 95, and 97 per cent. ater respectively.

ion (dry)—Contains 25 to 30 per cent. of gluten. Potato containing but 5.

bbage, Cauliflower, Broccoli, and Leaves are generally rich in gluten, while the is poor.

Ratio of Flesh-formers of Tubers.

Per Cent.

| TUBERS. | Flesh-formers. | Starch, etc. | Ratio to Heat-giv'rs. | TUBERS. | Flesh-formers. | Starch, etc. | Ratio to Heat-giv'rs. |
|-----------|----------------|--------------|-----------------------|---------------|----------------|--------------|-----------------------|
| root..... | .4 | 13.4 | 1:30 | Parsnip..... | 1.2 | 8.7 | |
| ip..... | .5 | 4 | 1:8 | Onion..... | 1.5 | 4.8 | |
| it..... | .5 | 5 | 1:10 | Sweet Potato. | 1.5 | 20. | |
| o..... | 1.2 | 18 | 1:16 | Yam..... | 2.2 | 16. | |

GRAVITY OF BODIES.

GRAVITY acts equally on all bodies at equal distances from Earth's centre; its force diminishes as distance increases, and increases as distance diminishes.

Gravitating forces of bodies are to each other,

1. Directly as their masses.
2. Inversely as squares of their distances.

Gravity of a body, or its weight above Earth's surface, decreases as square of its distance from Earth's centre in semi-diameters of Earth.

ILLUSTRATION 1.—If a body weighs 900 lbs. at surface of the Earth, what will it weigh 2000 miles above surface?—Earth's semi-diameter is 3963 miles (say 4000).

Then $2000 + 4000 = 6000 = 1.5$ semi-diam's, and $900 \div 1.5^2 = \frac{900}{2.25} = 400$ lbs.

Inversely, If a body weighs 400 lbs. at 2000 miles above Earth's surface, what will it weigh at surface?

$$400 \times 1.5^2 = 900 \text{ lbs.}$$

2.—A body at Earth's surface weighs 360 lbs.; how high must it be elevated to weigh 40 lbs.?

$\frac{360}{40} = 9$ semi-diameters, if gravity acted directly; but as it is inversely as square of the distance, then $\sqrt{9} = 3$ semi-diameters $= 3 \times 4000 = 12000$ miles.

3.—To what height must a body be raised to lose half its weight?

As $\sqrt{1} : \sqrt{2} :: 4000 : 5656$ = as square root of one semi-diameter is to square root of two semi-diameters, so is one semi-diameter to distance required.

Hence $5656 - 4000 = 1656$ = distance from Earth's surface.

Diameters of two Globes being equal, and their densities different, weight of a body on their surfaces will be as their densities.

Their densities being equal and their diameters different, weight of them will be as their diameters.

Diameters and densities being different, weight will be as their product.

ILLUSTRATION.—If a body weighs 10 lbs. at surface of Earth, what will it weigh at surface of Sun, densities being 392 and 100, and diameters 8000 and 883000 miles?

$883000 \times 100 \div 8000 \times 392 = 28.157$ = quotient of product of diameter of Sun and its density, and product of diameter of Earth and its density.

Then $28.157 \times 10 = 281.57$ lbs.

NOTE.—Gravity of a body is .00346 less at Equator than at Poles.

SPECIFIC GRAVITY AND WEIGHT.

Specific Gravity or Weight of a body is the proportion it bears to the weight of another body of known density or of equal volume, and which is adopted as a standard.

If a body float on a fluid, the part immersed is to whole body as specific gravity of body is to specific gravity of fluid.

When a body is immersed in a fluid, it loses such a portion of its own weight as is equal to that of the fluid it displaces.

An immersed body, ascending or descending in a fluid, has a force equal to difference between its own weight and weight of its bulk of the fluid, less resistance of the fluid to its passage.

Water is well adapted for standard of gravity; and as a cube foot of it weighs 997.68 ounces avoirdupois, its weight is taken as the unit, *viz.* 1000.

rench standard temperature for comparison of density of solid bodies determination of their specific gravities, is that of maximum density of r, at 4° C. or 39.1° F., and for gases and vapors under one atmosphere or centimeters of mercury is 32° F. or 0° C., and specific gravity of a body pressed by weight in kilogrammes of a cube decimeter of that body.

ensities of metals vary greatly.

stassium, Sodium, Barium, and Lithium are lighter than water. Mercuryaviest liquid and Platinum heaviest metal. Volcanic scorix is lighter water.

megranate and Lignum-vitæ are heaviest of woods. Pearl is heaviestimal substances, and Flax and Cotton are heaviest of vegetable substances, former weighing nearly twice as much as water.

reon is heaviest of precious stones, being 4.5 times heavier than water. et is 4 times heavier, Diamond 3.5 times, and Opal, lightest of all, is but e as heavy as water.

Ascertain Specific Gravity of a Solid Body heavier than Water.

JLE.—Weigh it both in and out of water, and note difference; then, as ht lost in water is to whole weight, so is 1000 to specific gravity of body.

$\frac{W \times 1000}{W - w} = G$, W and w representing weights out and in water, and G fic gravity.

AMPLE.—What is specific gravity of a stone which weighs in air 15 lbs., in r 10 lbs.?

15—10=5; then 5 : 15 :: 1000 : 3000 *Spec. Grav.*

Ascertain Specific Gravity of a Body lighter than Water.

JLE.—Annex to lighter body one that is heavier than water, or fluid; weigh piece added and compound mass separately, both in and out of r, or fluid; ascertain how much each loses, by subtracting its weight its weight in air, and subtract less of these differences from greater. en, as last remainder is to weight of light body in air, so is 1000 to fic gravity of body.

AMPLE.—What is specific gravity of a piece of wood that weighs 20 lbs. in air; red to it is a piece of metal that weighs 24 lbs. in air and 21 lbs. in water, and 70 pieces in water weigh 8 lbs.?

$20 + 24 - 8 = 44 - 8 = 36 = \text{loss of compound mass in water};$
 $24 - 21 = 3 = \text{loss of heavy body in water.}$
 33 : 20 :: 1000 : 606.06 *Spec. Grav.*

To Ascertain Specific Gravity of a Fluid.

JLE.—Take a body of known specific gravity, weigh it in and out of luid; then, as weight of body is to loss of weight, so is specific gravity dy to that of fluid.

AMPLE.—What is specific gravity of a fluid in which a piece of copper (*spec.* = 9000) weighs 70 lbs. in, and 80 lbs. out of it?

80 : 80—70=10 :: 9000 : 1125 *Spec. Grav.*

Ascertain Specific Gravity of a Solid Body which is soluble in Water.

LE.—Weigh it in a liquid in which it is not soluble, divide it f the liquid by loss of its weight in the liquid, and multiply ecific gravity of liquid; the product is specific gravity.

AMPLE.—What is specific gravity of a piece of clay, which weighs 15 lbs. in a liquid of a specific gravity of 1500, in which it is insoluble

$15 \div 10 \times 1500 = 2250$ *Spec. Grav.*

SOLIDS.

| SUBSTANCES. | Specific Gravity. | Weight of a Cube Inch. | SUBSTANCES. | Specific Gravity. |
|------------------------------|-------------------|------------------------|-----------------------------|-------------------|
| Metals. | | | Metals. | |
| Aluminum, cast..... | 2 560 | .0926 | Mercury 60°..... | 13 569 |
| " wrought..... | 2 670 | .0906 | " 212°..... | 13 370 |
| " Bronze..... | 7 700 | .2785 | Molybdenum..... | 8 600 |
| Antimony..... | 6 712 | .2428 | Nickel..... | 8 800 |
| Arsenic..... | 5 763 | .2084 | " cast..... | 8 279 |
| Barium..... | 4 70 | .017 | Osmium..... | 10 000 |
| Bismuth..... | 9 823 | .3553 | Palladium..... | 11 350 |
| Boron..... | 2 000 | .0723 | Platinum, hammered..... | 20 337 |
| Brass..... | | | " native..... | 16 000 |
| Sheet, cop. 75, zinc 25..... | 8 450 | .3056 | " rolled..... | 22 069 |
| Yellow " 66, " 34..... | 8 300 | .2997 | Potassium, 59°..... | 865 |
| Muntz " 60, " 40..... | 8 200 | .2966 | Red lead..... | 8 940 |
| Plate..... | 8 380 | .3026 | Rhodium..... | 10 650 |
| Cast..... | 8 100 | .2930 | Rubidium..... | 1 520 |
| Wire..... | 8 214 | .2972 | Ruthenium..... | 8 600 |
| Bromine..... | 3 000 | .1085 | Selenium..... | 4 500 |
| Bronze, gun metal..... | 8 750 | .3165 | Silver, pure, cast..... | 10 474 |
| " ordinary mean..... | 8 217 | .2972 | " " hammered..... | 10 511 |
| " cop. 84, tin 16..... | 8 832 | .3194 | Sodium..... | 970 |
| " " 81, " 19..... | 8 700 | .2929 | Steel, minimum..... | 7 700 |
| " Tobin..... | 8 379 | .3021 | " maximum..... | 7 900 |
| " 35, tin 65..... | 8 060 | .291 | " plates, mean..... | 7 806 |
| " 21, tin 74..... | 7 390 | .2668 | " soft..... | 7 833 |
| Cadmium..... | 8 650 | .3129 | " temper'd and hard..... | |
| Calcium..... | 1 580 | .057 | ened..... | 7 818 |
| Chromium..... | 5 900 | .2134 | " wire..... | 7 847 |
| Cinnabar..... | 8 098 | .2929 | " blistered..... | 7 823 |
| Cobalt..... | 8 600 | .3111 | " crucible..... | 7 842 |
| Columbium..... | 6 000 | .217 | " cast..... | 7 848 |
| Copper, cast..... | 8 608 | .3113 | " Bessemer..... | 7 852 |
| " plates..... | 8 698 | .3146 | " ordinary mean..... | 7 834 |
| " wire and bolts..... | 8 880 | .3212 | Strontium..... | 2 540 |
| " ordinary mean..... | 8 880 | .3212 | Tellurium..... | 6 110 |
| Gold, pure, cast..... | 19 258 | .6965 | Thallium..... | 11 850 |
| " hammered..... | 19 361 | .7003 | Tin, Cornish, hammered..... | 7 390 |
| " 22 carats fine..... | 17 486 | .6325 | " " pure..... | 7 291 |
| " 20 " "..... | 15 709 | .5682 | Titanium..... | 5 300 |
| Iridium..... | 18 680 | .6756 | Tungsten..... | 17 000 |
| " hammered..... | 23 000 | .8319 | Uranium..... | 18 330 |
| Iron, Cast, gun metal..... | 7 308 | .264 | Wolfram..... | 7 119 |
| " minimum..... | 6 900 | .2491 | Zinc, cast..... | 6 861 |
| " maximum..... | 7 500 | .2707 | " rolled..... | 7 191 |
| " ordinary mean..... | 7 207 | .2607 | | |
| " mean, Eng..... | 7 217 | .2609 | Woods (Dry). | |
| " cast, hot blast..... | 7 065 | .2555 | Alder..... | 800 |
| " " cold "..... | 7 218 | .2611 | Apple..... | 793 |
| " Wrought bars..... | 7 788 | .2817 | "..... | 845 |
| " " wire..... | 7 774 | .2811 | Ash..... | 690 |
| " " rolled plates..... | 7 794 | .2787 | Bamboo..... | 400 |
| " " average..... | 7 698 | .2779 | Baytree..... | 822 |
| " " Eng. rails..... | 7 540 | .2722 | "..... | 852 |
| " " Lowmoor..... | 7 808 | .2819 | Beech..... | 690 |
| " " pure..... | 8 140 | .2938 | "..... | 567 |
| " ordinary mean..... | 7 744 | .2801 | Birch..... | 720 |
| Lead, cast..... | 11 352 | .4106 | Blackwood, India..... | 898 |
| " rolled..... | 11 388 | .4119 | Boxwood, Brazil..... | 1 031 |
| Lithium..... | 590 | .0213 | " France..... | 1 328 |
| Magnesium..... | 1 750 | .0633 | " Holland..... | 912 |
| " manganese..... | 8 000 | .2894 | Bullet-wood..... | 928 |
| " dry - 40°..... | 15 632 | .5661 | Butternut..... | 376 |
| + 32°..... | 13 598 | .4918 | | |

| ANCES. | Specific Gravity. | Weight of a Cube Foot. | SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. |
|------------------|-------------------|------------------------|----------------------------|-------------------|------------------------|
| ds (Dry). | | Lbs. | Woods (Dry). | | Lbs. |
| Y..... | 913 | 57.062 | Oak, English..... | 858 | 53.625 |
| | 561 | 35.062 | “ green..... | 932 | 58.25 |
| in..... | 1315 | 82.157 | “ heart, 60 years..... | 1146 | 71.625 |
| pine..... | 441 | 27.562 | “ live, green..... | 1170 | 73.125 |
| resh burned..... | 380 | 23.75 | “ seasoned..... | 1260 | 78.75 |
| ak..... | 1573 | 98.312 | “ white..... | 1068 | 66.75 |
| soft wood..... | 280 | 17.5 | Olive..... | 860 | 53.75 |
| riturated..... | 1380 | 86.25 | Orange..... | 680 | 42.5 |
| | 715 | 44.687 | Pear..... | 705 | 44.062 |
| sweet..... | 610 | 38.125 | Persimmon..... | 661 | 41.312 |
| | 726 | 45.375 | Plum..... | 710 | 44.375 |
| | 1040 | 65 | Pine, pitch..... | 785 | 49.062 |
| | 240 | 15 | “ red..... | 660 | 41.25 |
| anish..... | 644 | 40.25 | “ white..... | 590 | 36.875 |
| | 756 | 47.25 | “ yellow..... | 554 | 34.625 |
| erican..... | 1331 | 83.187 | “ Norway..... | 461 | 28.812 |
| lian..... | 1209 | 75.562 | Pomegranate..... | 740 | 46.25 |
| | 695 | 43.437 | Poon..... | 1354 | 84.625 |
| | 570 | 35.625 | Poplar..... | 580 | 36.25 |
| | 671 | 41.937 | “ white..... | 383 | 23.937 |
| | 800 | 50 | Quince..... | 529 | 33.062 |
| lia..... | 1014 | 63.375 | Rosewood..... | 705 | 44.062 |
| | 600 | 37.5 | Sassafras..... | 728 | 45.5 |
| y Spruce..... | 512 | 32 | Satinwood..... | 482 | 30.125 |
| ic..... | 582 | 36.375 | Spruce..... | 885 | 55.312 |
| or Sipirl..... | 970 | 60.625 | Sycamore..... | 500 | 31.25 |
| | 1055 | 65.95 | Tamarack..... | 623 | 38.937 |
| | 843 | 52.687 | Tenk (African oak)..... | 383 | 23.937 |
| r..... | 1000 | 62.5 | Walnut..... | 657 | 41.062 |
| ak..... | 592 | 37 | “ black..... | 980 | 61.25 |
| | 910 | 56.875 | Willow..... | 671 | 41.937 |
| | 860 | 53.75 | Yew, Dutch..... | 500 | 31.25 |
| lg-nut..... | 368 | 23 | “ Spanish..... | 486 | 30.375 |
| ell-bark..... | 792 | 49.5 | (Well Seasoned. *) | 585 | 36.562 |
| | 690 | 43.125 | Ash..... | 788 | 49.25 |
| | 760 | 47.5 | Beech..... | 807 | 50.437 |
| | 990 | 61.875 | Cherry..... | 722 | 45.125 |
| | 770 | 48.125 | Cypress..... | 624 | 39 |
| | 566 | 35.375 | Hickory, red..... | 606 | 37.875 |
| ia..... | 1171 | 73.187 | Mahogany, St. Domingo..... | 441 | 27.562 |
| l, mean..... | 720 | 45 | Pine, white..... | 838 | 52.375 |
| | 544 | 34 | “ yellow..... | 720 | 45 |
| | 560 | 35 | Poplar..... | 473 | 29.562 |
| | 703 | 43.937 | White Oak, upland..... | 541 | 33.812 |
| | 650 | 40.625 | “ James River..... | 587 | 36.687 |
| | 1333 | 83.312 | Stones, Earths, etc. | 687 | 42.937 |
| | 804 | 50.25 | Alabaster, white..... | 759 | 42.437 |
| | 604 | 37.75 | “ yellow..... | 2730 | 170.625 |
| | 728 | 45.5 | Alum..... | 2699 | 168.687 |
| | 913 | 57.062 | Amber..... | 1714 | 107.125 |
| | 720 | 45 | Ambergris..... | | 67.375 |
| | 1063 | 66.437 | Asbestos, starry..... | | 2.062 |
| Honduras..... | 560 | 35 | Asphalte..... | | 25 |
| Spanish..... | 852 | 53.25 | Barytes, sulphate..... | | |
| | 750 | 46.875 | Beton, N Y, St. Col..... | | |
| eye..... | 576 | 36 | | | |
| | 849 | 53.062 | | | |
| | 561 | 35.062 | | | |
| | 897 | 56.062 | | | |
| | 823 | 51.437 | | | |
| | 872 | 54.5 | | | |
| | 759 | 47.437 | | | |

| SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. | SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. |
|--------------------------------|-------------------|------------------------|-----------------------------|-------------------|------------------------|
| | | Lbs. | | | Lbs. |
| Stones, Earths, etc. | | | Stones, Earths, etc. | | |
| Basalt..... | 2740 | 171.25 | Glass, green..... | 2642 | 165.125 |
| Bitumen, red..... | 2864 | 179 | “ optical..... | 3450 | 215.625 |
| “ brown..... | 1160 | 72.3 | “ white..... | 2892 | 180.75 |
| Borax..... | 830 | 51.7 | “ window..... | 2642 | 165.125 |
| Brick..... | 1714 | 107.125 | “ soluble..... | 1250 | 78.125 |
| “ pressed..... | 1367 | 85.437 | Gneiss, common..... | 270 | 15.875 |
| “ fire..... | 1900 | 118.75 | Granite, Egyptian red..... | 2654 | 165.675 |
| “ work in cement..... | 2400 | 150 | “ Pataspco..... | 2640 | 165 |
| “ “ mortar..... | 2201 | 137.562 | “ Quincy..... | 2652 | 165.75 |
| Carbon..... | 1800 | 112.5 | “ Scotch..... | 2625 | 164.062 |
| Cement, Portland..... | 1600 | 100 | “ Susquehanna..... | 2704 | 169 |
| “ Roman..... | 2000 | 125 | “ gray..... | 2800 | 175 |
| Chalk..... | 3500 | 218.75 | Graphite..... | 2200 | 137.5 |
| Cement, Portland..... | 1300 | 81.25 | Gravel, common..... | 1749 | 109.312 |
| “ Roman..... | 1560 | 97.25 | Grindstone..... | 2143 | 133.937 |
| Clay..... | 1520 | 95 | Gypsum, opaque..... | 2168 | 135.5 |
| “ with gravel..... | 2784 | 174 | Hone, white, razor..... | 2876 | 179.75 |
| Coal, Anthracite..... | 1030 | 120.625 | Hornblende..... | 3540 | 221.25 |
| “ “ “..... | 2480 | 155 | Iodine..... | 4940 | — |
| “ “ “..... | 1350 | 84.375 | Lava, Vesuvius..... | 1710 | 108.875 |
| “ “ “..... | 1436 | 89.75 | “ “ “..... | 2810 | 175.625 |
| “ “ “..... | 1640 | 102.5 | Lias..... | 1350 | 146.875 |
| “ “ “..... | 1090 | 80.625 | Lime, quick..... | 804 | 50.25 |
| “ “ “..... | 1238 | 77.375 | “ hydraulic..... | 2745 | 174.562 |
| “ “ “..... | 1318 | 82.375 | Limestone, white..... | 3156 | 197.25 |
| “ “ “..... | 1277 | 79.812 | “ green..... | 3180 | 198.75 |
| “ “ “..... | 1276 | 79.75 | Magnesia, carbonate..... | 2400 | 150 |
| “ “ “..... | 1290 | 80.625 | Magnetic ore..... | 5094 | 317.6 |
| “ “ “..... | 1292 | 80.75 | Marble, Adelaide..... | 2715 | 169.687 |
| “ “ “..... | 1273 | 79.562 | “ African..... | 2708 | 169.25 |
| “ “ “..... | 1355 | 84.687 | “ Biscayan, black..... | 2695 | 168.437 |
| “ “ “..... | 1270 | 79.375 | “ Carrara..... | 2716 | 169.75 |
| “ “ “..... | 1300 | 81.25 | “ common..... | 2686 | 167.875 |
| “ “ “..... | 1259 | 78.687 | “ Egyptian..... | 2668 | 166.75 |
| “ “ “..... | 1300 | 81.25 | “ French..... | 2640 | 165.875 |
| “ “ “..... | 1302 | 81.375 | “ Italian, white..... | 2708 | 169.25 |
| “ “ “..... | 1315 | 82.187 | “ Parian..... | 2838 | 177.375 |
| Coke..... | 1000 | 62.5 | “ Vermont, white..... | 2650 | 165.37 |
| “ Nat'l, Va..... | 746 | 46.64 | “ Silesian..... | 2730 | 170.625 |
| Concrete, in cement..... | 2200 | 137.5 | Marl, mean..... | 1750 | 109.375 |
| “ mean..... | 2000 | 125 | “ tough..... | 2340 | 140.25 |
| Earth, * common soil, dry..... | 1216 | 76 | Masonry, rubble..... | 2050 | 128.125 |
| “ loose..... | 1500 | 93.75 | “ Granite..... | 2640 | 165 |
| “ moist sand..... | 2050 | 128.125 | “ Limestone..... | 2640 | 165 |
| “ mold, fresh..... | 2050 | 128.125 | “ Sandstone..... | 2160 | 135 |
| “ rammed..... | 1600 | 100 | “ Brick..... | 2240 | 140 |
| “ rough sand..... | 1920 | 120 | “ “ rough work..... | 1600 | 100 |
| “ with gravel..... | 2020 | 126.25 | Mica..... | 2800 | 175 |
| “ Potters'..... | 1900 | 118.75 | Millstone..... | 2484 | 155.25 |
| “ light vegetable..... | 1400 | 87.5 | “ Quartz..... | 1260 | 78.75 |
| Emery..... | 4000 | 250 | Mortar..... | 1384 | 86.5 |
| Feldspar..... | 2600 | 162.5 | “ “ “..... | 1750 | 109.375 |
| Flint, black..... | 2582 | 161.375 | Mud..... | 1630 | 101.875 |
| “ white..... | 2594 | 162.125 | “ wet and fluid..... | 1782 | 112 |
| Fluorine..... | 1320 | 82.5 | “ “ pressed..... | 1920 | 120 |
| Fuel, Warlich's..... | 1150 | 71.875 | Nitre..... | 1900 | 118.75 |
| “ Lignite..... | 1300 | 81.25 | Oyster-shell..... | 2092 | 130.75 |
| Glass, bottle..... | 2732 | 170.75 | Paving-stone..... | 2416 | 151 |
| “ Crown..... | 2487 | 155.437 | Peat, Irish, light..... | 278 | 17.375 |
| “ Flint..... | 2933 | 183.312 | “ dense..... | 562 | 35.125 |
| “ “ “..... | 3200 | 190 | “ very..... | 675 | 42.187 |

* Specific gravity of earth is estimated at from 1520 to 2200.

| SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. | SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. |
|-------------------------------|-------------------|------------------------|--------------------------------------|-------------------|------------------------|
| ones, Earths, etc. | | Lbs. | Granite. | | Lbs. |
| Black, black | 1058 | 66.125 | (Gen'l Gillmore, U. S. A.) | | |
| phorus..... | 1329 | 83.062 | Duluth, Minn., dark.... | 2780 | 173.7 |
| ter of Paris..... | 1770 | 110.625 | Fall River, Mass., gray.. | 2635 | 164.7 |
| " " dry..... | 1176 | 73.5 | Garrison's, N. Y. | 2580 | 161.2 |
| " " dry..... | 3400 | 212.5 | Jersey City, N. J., soap. | 3030 | 189.3 |
| nbago..... | 1400 | 87.5 | Keene, N. H., bluish gray | 2656 | 166 |
| elaine, China..... | 2100 | 131.25 | Maine..... | 2635 | 164.7 |
| hyry, red..... | 2300 | 143.75 | Millstone Pt., Conn..... | 2700 | 169.1 |
| ice-stone..... | 2765 | 172.812 | New London, "..... | 2660 | 166.25 |
| rtz..... | 915 | 57.187 | Quincy, Mass., light.... | 2695 | 168.5 |
| lead..... | 2660 | 166.25 | Richmond, Va..... | 2727 | 170.5 |
| n..... | 8940 | 558.75 | " " gray..... | 2630 | 164.4 |
| k, crystal..... | 1089 | 68.062 | Staten Island, N. Y..... | 2861 | 178.8 |
| en-stone..... | 2735 | 170.937 | Westchester Co., N. Y.... | 2655 | 165.9 |
| common..... | 1981 | 123.812 | Westerly, R. I., gray.... | 2670 | 166.9 |
| rock..... | 2130 | 133.125 | Limestone. | | |
| petre..... | 2300 | 137.5 | (Gen'l Gillmore, U. S. A.) | | |
| coarse..... | 2090 | 130.625 | Bardstown, Ky., dark.. | 2670 | 166.9 |
| common..... | 1800 | 112.5 | Caen, France..... | 1900 | 118.8 |
| damp and loose.. | 1670 | 104.375 | Canajoharie, N. Y..... | 2685 | 167.8 |
| dried " ".... | 1392 | 87 | Cooper Co., Mo., d'k drab | 2320 | 141.3 |
| dry..... | 1500 | 97.5 | Erie Co., N. Y., blue.... | 2640 | 165 |
| mortar, Ft. Richm'd | 1420 | 88.75 | Garrison's, N. Y..... | 2635 | 164.7 |
| " Brooklyn.. | 1059 | 103.66 | Glens' Falls, "..... | 2700 | 168.7 |
| silicious..... | 1716 | 107.25 | Joliet, Ill., white..... | 2540 | 158.7 |
| stone, mean..... | 1701 | 106.33 | Kingston, N. Y..... | 2690 | 168.1 |
| " Sydney..... | 2200 | 137.5 | Lake Champlain, N. Y.... | 2750 | 171.9 |
| ori..... | 2237 | 139.81 | Lime Island, Mich., drab | 2500 | 156.3 |
| ria, volcanic..... | 3170 | 198.125 | Marblehead, Ohio, white | 2400 | 150 |
| er pipe, mean..... | 830 | 51.875 | Marquette, Mich., drab. | 2340 | 146.25 |
| le..... | 2250 | 140.625 | Sturgeon Bay, Wis., bluish drab..... | 2780 | 173.7 |
| e..... | 2600 | 162.5 | Marble. | | |
| purple..... | 2672 | 167 | (Gen'l Gillmore, U. S. A.) | | |
| ul..... | 2900 | 181.25 | Dorset, Vt..... | 2635 | 164.7 |
| stone..... | 2784 | 174 | East Chester, N. Y..... | 2875 | 179.7 |
| r, calcareous..... | 2440 | 152.5 | Italian, common..... | 2690 | 168.1 |
| Feld, blue..... | 2730 | 170.625 | Mill Creek, Ill., drab.. | 2570 | 171.9 |
| " green..... | 2735 | 170.937 | North Bay, Wis., ".... | 2800 | 175 |
| Fluor..... | 2693 | 168.312 | Sandstone. | | |
| ular ore..... | 2704 | 169 | (Gen'l Gillmore, U. S. A.) | | |
| ctite..... | 3400 | 212.5 | Albion, N. Y., brown.... | 2420 | 151.25 |
| e, Bath, Engl..... | 5251 | 328.187 | Bellefonte, N. J., gray.. | 2259 | 141.2 |
| Blue Hill..... | 2415 | 150.937 | Berea, Ohio, drab..... | 2110 | 131.9 |
| Bluestone (basalt) | 1961 | 122.562 | Cleveland, " olive green | 2240 | 140 |
| Breakneck, N. Y.... | 2640 | 165 | Edinb'h, Scotl., Craigleith | 2260 | 141.25 |
| Bristol, Engl..... | 2625 | 164.062 | Fond du Lac, Wis., purple | 2220 | 138.7 |
| Caen, Normandy..... | 2704 | 169 | Fontenac, Minn., J'g't bluff | 2325 | 145.31 |
| common..... | 2510 | 156.875 | Haverstraw, N. Y., red.. | 2130 | 133.1 |
| Craigleith, Scotl.... | 2076 | 129.75 | Kasota, Minn., pink..... | 2600 | 164.375 |
| Kentish rag, ".... | 2520 | 157.5 | Little Falls, N. Y., br | 2400 | 140.6 |
| Kip's Bay, N. Y.... | 2316 | 144.75 | Marquette, Mich., y | 242.5 | 142.5 |
| Norfolk (Parliament House)... | 2651 | 165.687 | Masillon, O., yellow | 247 | 147 |
| Portland, Engl.... | 2759 | 172 | Medina, N. Y., pink | | |
| Staten Isl'd, N. Y.... | 2304 | 144 | Middletown, Ct., bl | | |
| Sullivan Co., ".... | 2368 | 148 | Seneca, Ohio, red | | |
| ur, native..... | 2976 | 186 | Vermillion, Ohio, d | | |
| Cotta..... | 2688 | 168 | Warrensburg, Mo.... | | |
| | 2033 | 127.062 | | | |
| | 1952 | 122 | | | |
| | 1815 | 113.437 | | | |
| | 2720 | 170 | | | |

Precious Stones.

| | Spec. Grav. | | Spec. Grav. | | Spec. Grav. |
|-------------------------|-------------|--------------------|-------------|-----------------------|-------------|
| Agate | 2590 | Emerald, aqua ma- | | Onyx | 2700 |
| Amethyst | 3920 | rine | 2730 | Opal | 2590 |
| Carnelian | 2613 | Garnet | 4189 | Pearl, Oriental | 2590 |
| Chrysolite | 2782 | " black | 3750 | Ruby | 3750 |
| Diamond, Oriental | 3521 | Jasper | 2600 | Sapphire | 3594 |
| " Brazilian | 3444 | Jet | 1300 | Topaz | 3590 |
| " pure | 3520 | Lapis lazuli | 2960 | Tourmaline | 3750 |
| Emerald | 3950 | Malachite | 4020 | Turquoise | 3750 |

| SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. | SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. |
|------------------------|-------------------|------------------------|-----------------------------------|-------------------|------------------------|
| Miscellaneous. | | | Liquids. | | |
| Amber | 1090 | 68.125 | Acid, Acetic | 1066 | 66.375 |
| Atmospheric Air | .001292 | .080728 | " Benzole | 667 | 41.125 |
| Beeswax | 965 | 60.312 | " Citric | 1034 | 64.125 |
| Bone | 1900 | 118.75 | " Concentrated | 1521 | 95.062 |
| Butter | 942 | 58.875 | " Fluoric | 1500 | 93.75 |
| Camphor | 988 | 61.75 | " Muriatic | 1200 | 75 |
| Caoutchouc | 930 | 58.125 | " Nitric | 1217 | 76.062 |
| Cotton | 950 | 59.375 | " Nitrous | 1550 | 96.875 |
| Dynamite | 1650 | 103.125 | " Phosphoric | 1558 | 97.375 |
| Egg | 1090 | — | " solid | 2800 | 175 |
| Fat of Beef | 923 | 57.687 | " Sulphuric | 1849 | 115.937 |
| " Hogs | 936 | 58.5 | Alcohol, pure, 60° | 794 | 49.625 |
| " Mutton | 923 | 57.687 | " 95 per cent. | 816 | 51 |
| Flax | 1790 | 111.875 | " 80 " | 863 | 53.687 |
| Gamboge | 1222 | — | " 50 " | 934 | 58.375 |
| Glycerine, 60° | 1261 | 78.752 | " 40 " | 951 | 59.437 |
| Grain, Barley | 590 | 36.875 | " 25 " | 970 | 60.625 |
| " Wheat | 750 | 46.875 | " 10 " | 986 | 61.687 |
| " Oats | 500 | 31.25 | " 5 " | 992 | 62 |
| Gum Arabic | 1452 | 90.75 | " proof spirit, 50° | 800 | 50.062 |
| Gunpowder, loose | 900 | 56.25 | " per cent., 60° | 934 | 58.375 |
| " shaken | 1000 | 62.5 | " proof spirit, 50° | 875 | 54.687 |
| " solid | 1550 | 96.875 | " per cent., 80° | 891 | 55.687 |
| Gutta-percha | 1800 | 112.5 | Ammonia, 27.9 per cent. | 1300 | 81.25 |
| Hay, old compact | 980 | 61.25 | Aqua fortis, double | 1200 | 75 |
| Hay, old compact | 128.8 | 8.05 | " single | 1034 | 64.625 |
| Horn | 1689 | 105.562 | Beer | 850 | 53.125 |
| Human body | 1070 | 66.937 | Benzine | 848 | 53 |
| Ice, at 32° | 922 | 57.5 | Bitumen, liquid | 1054 | 65.625 |
| Indigo | 1009 | 63.062 | Blood (human) | 924 | 57.75 |
| Isinglass | 1111 | 69.437 | Brandy, .83 or .9 of spirit | 2966 | 185.375 |
| Ivory | 1825 | 114.062 | Bromine | 1018 | 63.625 |
| Lard | 947 | 59.187 | Cider | 866 | 54.125 |
| Leather | 960 | 60 | " Muriatic | 845 | 53.812 |
| Mastic | 1074 | 67.125 | " Nitric | 1110 | 69.375 |
| Myrrh | 1360 | 85 | " Sulphuric | 715 | 44.687 |
| Nitro-Glycerine | 1600 | 100 | Honey | 1450 | 90.625 |
| Opium | 1336 | 83.5 | Milk | 1032 | 64.5 |
| Potash | 2100 | 131.25 | Oil, Anise-seed | 986 | 61.625 |
| Resin | 1089 | 68.062 | " Codfish | 923 | 57.687 |
| Snow | .0833 | 5.2 | " Whale | 923 | 57.687 |
| Soap, Castile | 1071 | 66.937 | " Linseed | 940 | 58.75 |
| Spermaceti | 943 | 58.937 | " Naphtha | 850 | 53.125 |
| Starch | 950 | 59.375 | " Olive | 915 | 57.187 |
| Sugar | 1606 | 100.375 | " Palm | 909 | 60.625 |
| " .66 | 972 | 60.25 | " Petroleum | 880 | 54 |
| " .66 | 1326 | 82.875 | " Rape | 914 | 57.687 |
| Tallow | 941 | 58.812 | " Sunflower | 926 | 57.687 |
| " | 964 | 60.25 | " Turpentine | 870 | 54 |
| " | 970 | 60.625 | | | |

* gravity of proof spirit according to Ure's Table for Sykes's Hydrometer.

| SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. | SUBSTANCES. | Specific Gravity. | Weight of a Cube Foot. |
|--------------------------|-------------------|------------------------|-------------------------|-------------------|------------------------|
| Liquids. | | Lbs. | Liquids. | | Lbs. |
| rit, rectified..... | 824 | 51.5 | Water, Dead Sea..... | 1240 | 77.5 |
| am, at 212°..... | .00061 | .038* | " Mediterranean... 1029 | | 64.312 |
| | 1015 | 63.437 | " sea..... 1029 | | 64.312 |
| egar..... | 1080 | 67.5 | " Black Sea..... 1016 | | 63.5 |
| ter, at 32°..... | 998.7 | 62.418 | " rain..... 1000 | | 62.5 |
| " 39.1°..... | 998.8 | 62.425 | Wine, Burgundy..... 992 | | 62 |
| " 62°f..... | 997.7 | 62.355 | " Champagne..... 997 | | 62.312 |
| " 212°..... | 956.4 | 59.64 | " Madeira..... 1038 | | 64.375 |
| " distilled, at 39°..... | 998 | 62.379 | " Port..... 997 | | 62.312 |

* .03818.

† 1 cube inch at standard temperature = 252.5954 grains.

ompression of following fluids under a pressure of 15 lbs. per square inch:

ohol. .0000216 | Mercury. .00000265 | Water. .00004663 | Ether. .00006158

Elastic Fluids.

ube Foot of Atmospheric Air at 32° weighs .080728 lbs. Avoirdupois = 565.096 grains, and at 62° 532.679 grains.

Its assumed Gravity of 1 is Unit for Elastic Fluids.

| Spec. Grav. | Spec. Grav. | Spec. Grav. |
|-----------------------|---------------------------|-------------------------|
| tic Ether..... 3.04 | Nitric acid..... 1.217 | Vapor. |
| monia..... .589 | " oxide..... 1.094 | Alcohol..... 1.613 |
| os. air, at 32°.. 1 | Nitrogen..... .974 | Bisulphuret of |
| te..... .976 | Nitrous acid..... 2.638 | Carbon..... 2.64 |
| onic acid..... 1.53 | Nitrous oxide..... 1.527 | Bromine..... 5.4 |
| " oxide..... .972 | Olefant gas..... .9672 | Chloric Ether.... 3.44 |
| uret'd Hydrog. .559 | Oxygen..... 1.106 | Chloroform..... 4.2 |
| rine..... 2.421 | Phosphurett'd Hy- | Ether..... 2.586 |
| ro-carbonic.... 3.389 | drogen..... 1.77 | Hydrochlor. Ether 2.255 |
| roform..... 5.3 | Sulphuretted Hy- | Iodine..... 8.716 |
| rogen..... 1.815 | drogen..... 1.17 | Nitric acid..... 3.75 |
| coal..... { | Sulphurous acid.. 2.21 | Spirits of Turpen- |
| .752 | Steam, † at 212°.. .47295 | tine..... 5.013 |
| rochloric acid. 1.278 | Smoke. | Sulphuric acid... 2.7 |
| rocyanic " .942 | Bitum. Coal.... .102 | " Ether.. 2.586 |
| rogen..... .0692 | Coke..... .105 | Sulphur..... 2.214 |
| atic acid..... 1.247 | Wood..... .09 | Water..... .623 |

† weight of a cube foot 267.26 grains, and compared with water at 62° specific gravity = .0006123.

Weight of a Cube Foot of Gases at 32° F., and under Pressure of one Atmosphere, or 2116.4 lbs. per Square Foot.

| Lbs. | Lbs. | Lbs. |
|---------------------|--------------------------|------------------------|
| at 32°..... .080728 | Chlorine..... .197 | Hydrogen..... .005594 |
| " 62°..... .076097 | Chloroform..... .428 | Nitrogen..... .078596 |
| ol..... .1302 | Coal gas..... .03536 | Olefant gas..... .0795 |
| mic acid.... .12344 | Ether, Sulphuric.. .2093 | Oxygen..... .089256 |
| ret. Hydrog. .04462 | Gaseous steam.... .05022 | Steam..... .05022 |

Sulphurous acid..... .1814 lbs.

Compute Weight of a Body or Substance when Specific Gravity is given.**LE.**—Multiply specific gravity by unit or stand or sub-
e, and product is the weight.

Divide specific gravity of body or substance by weight of a cube foot of it in lbs.

EXAMPLE.—Specific gravity is 2250; what is weight of a cu

$$2250 \times 62.5 = 140.625 \text{ lbs.}$$

Weights and Volumes of various Substances in Ordinary Use.

| SUBSTANCES. | Cube Foot. | Cube Inch. | SUBSTANCES. | Cube Foot. | Cube Inch. |
|----------------------|------------------|------------------|-----------------------|-------------|-------------|
| Metals. | Lbs. | Lbs. | Woods. | Lbs. | Lbs. |
| Brass... {copper 67} | 488.75 | .2829 | Spruce..... | 31.25 | 72.68 |
| “ zinc 33 | | | Walnut, black, dry... | 31.25 | 71.68 |
| “ gun metal. | 543.75 | .3147 | Willow..... | 36.562 | 61.25 |
| “ sheets..... | 513.6 | .297 | “ dry..... | 30.375 | 73.74 |
| “ wire..... | 524.16 | .3033 | Miscellaneous. | | |
| Copper, cast..... | 547.25 | .3179 | Air..... | .075291 | — |
| “ plates..... | 543.625 | .3167 | Basalt, mean..... | 175 | 12.8 |
| Iron, cast..... | 450.437 | .2607 | Brick, fire..... | 137.562 | 16.84 |
| “ gun metal..... | 466.5 | .27 | “ mean..... | 102 | 21.96 |
| “ heavy forging.. | 479.5 | .2775 | Coal, anthracite.... | 89.75 | 24.68 |
| “ plates..... | 481.5 | .2787 | “ bitumin., mean. | 102.5 | 21.54 |
| “ wrought bars... | 486.75 | .2816 | “ Cannel..... | 80 | 28 |
| Lead, cast..... | 709.5 | .4106 | “ Cumberland..... | 94.875 | 23.69 |
| “ rolled..... | 711.75 | .4119 | “ Welsh, mean.... | 81.25 | 20.49 |
| Mercury, 60°..... | 848.7487 | .491174 | Coke..... | 62.5 | 27.39 |
| Steel, plates..... | 487.75 | .2823 | Cotton, bale, mean.. | 14.5 | 35.4 |
| “ soft..... | 489.562 | .2833 | “ pressed { | 20 | 154.6 |
| Tin..... | 455.687 | .2637 | Earth, clay..... | 25 | 114 |
| Zinc, cast..... | 428.812 | .2482 | “ common soil.... | 120.625 | 89.6 |
| “ rolled..... | 449.437 | .2601 | “ gravel..... | 137.125 | 18.59 |
| Woods. | Cube Feet | in a Ton. | “ dry, sand..... | 109.312 | 16.38 |
| Ash..... | 52.812 | 42.414 | “ loose..... | 93.75 | 20.49 |
| Bay..... | 51.375 | 43.601 | “ moist, sand.... | 128.125 | 18.84 |
| Blue Gum..... | 64.3 | 34.837 | “ mold..... | 128.125 | 17.46 |
| Cork..... | 15 | 149.333 | “ mud..... | 101.875 | 21.97 |
| Cedar..... | 35.062 | 63.886 | “ with gravel.... | 126.25 | 17.74 |
| Chestnut..... | 38.125 | 58.754 | Granite, Quincy.... | 165.75 | 13.54 |
| Hickory, pig-nut.. | 49.5 | 45.252 | “ Susquehanna... | 169 | 13.54 |
| “ shell-bark.... | 43.125 | 51.942 | Gypsum..... | 135.5 | 16.58 |
| Lignum-vitæ..... | 83.312 | 26.886 | Hay, bale..... | 12 | 186.66 |
| Logwood..... | 57.062 | 39.255 | “ hard pressed... | 25 | 89.6 |
| Mahoga'y, Hondur's | 35 | 64 | Ice, at 32°..... | 57.5 | 38.95 |
| Oak, Canadian..... | 66.437 | 33.714 | India rubber..... | 56.437 | 39.69 |
| “ English..... | 54.5 | 41.101 | “ vulcanized..... | — | — |
| “ live, seasoned.. | 58.25 | 38.455 | Limestone..... | 197.25 | 11.355 |
| “ white, dry..... | 66.75 | 33.558 | Marble, mean..... | 167.875 | 13.343 |
| “ upland..... | 42.937 | 52.169 | Mortar, dry, mean.. | 97.98 | 22.66 |
| Pine, pitch..... | 41.25 | 54.303 | Plaster of Paris.... | 73.5 | 30.476 |
| “ red..... | 36.875 | 60.745 | Water, rain..... | 62.5 | 35.4 |
| “ white..... | 34.625 | 64.693 | “ salt..... | 64.312 | 34.83 |
| “ well seasoned.. | 29.562 | 75.773 | “ at 62°..... | 62.355 | 35.955 |
| Pine, yellow..... | 33.812 | 66.248 | | | |

Metals.—Tobin Bronze..... 522.02 lbs. 3021 lbs.

To Compute Proportions of Two Ingredients in a Compound, or to Discover Adulteration in Metals.

RULE.—Take differences of each specific gravity of ingredients and specific gravity of compound, then multiply gravity of one by difference of the other; and, as sum of products is to respective products, so is specific gravity of body to proportions of the ingredients.

EXAMPLE.—A compound of gold (*spec. grav.* = 18.888) and silver (*spec. grav.* = 10.535) of gravity of 14; what is proportion of each metal?

$$10.535 = 51.495 \quad 14 - 10.535 = 3.465 \times 18.888 = 65.447$$

$$14 : 7.835 \text{ gold, } 65.447 : 51.495 :: 14 : 6.165 \text{ silver}$$

Weights of Various Substances per Cube Foot in Bulk.

| | Lbs. | | Lbs. | | Lbs. |
|--------------------------------|----------------------|------------------------------------|------------|------------------------|------|
| in pigs..... | 567 | Potters' clay..... | 130 | Coal, caking..... | 50 |
| "..... | 360 | Loam..... | 126 | Wheat..... | 48 |
| 8, in blocks } .. 172 | | Gravel..... | 109 | Barley..... | 38 |
| stone, " } .. 170 | | Sand..... | 95 | Fruit and vegetables.. | 22 |
| | 170 | Bricks, common.... | 93 | Cotton seeds..... | 12 |
| te, in blocks .. | 164 | Ice, at 32°..... | 57.5 | Cotton..... | 10 |
| stone..... | 141 | Oak, seasoned..... | 52 | Hay, old..... | 8 |
| lry, 100 feet BM.... | .175 ton. | Earth, loose..... | 93.75 lbs. | | |
| white, "..... | .141 " | Elm, dry, 100 feet BM | .13 ton. | | |
| nt, struck bushel and | | Gypsum, ground, str. bush. 70 lbs. | | | |
| packed* | 100 lbs. | " " well shaken 80 " | | | |
| nt, Portland, bushel. 110 lbs. | | Hemlock, dry, 100 feet BM. . | .093 ton. | | |
| y, dry, 100 BM..... | .156 ton. | Hickory, " " .. | .197 " | | |
| nut, dry, 100 BM ... | .153 " | Masonry, Granite, dressed.. | 165 lbs. | | |
| anthracite, 1 cub. yd. | | " " rough ... | 126 " | | |
| broken and loose ... | 1.75 yds. | " " Limestone, dres'd 165 " | | | |
| " " 1 ton.. | 41.5 cub. feet. | " " Sandstone..... | 135 " | | |
| ton = | .80 to 97 cub. feet. | " " Brick, pressed ... | 140 " | | |
| 1, common soil..... | 137.125 lbs. | " " com'n, rough. 100 " | | | |

* One packed bushel = 1.43 loose.

Comparative Weight of Green and Seasoned Timber.

| TIMBER. | Weight of a Cube Foot. | | TIMBER. | Weight of a Cube Foot. | |
|-------------------|------------------------|-----------|-------------------|------------------------|------------|
| | Green. | Seasoned. | | Green. | Seasoned. |
| Italian Pine..... | Lbs. 44.75 | Lbs. 30.7 | Cedar..... | Lbs. 32 | Lbs. 28.25 |
| | 58.18 | 50 | English Oak | 71.6 | 43.5 |
| 1..... | 60 | 53.37 | Riga Fir..... | 48.75 | 35.5 |

Application of the Tables.

When Weight of a Solid or Liquid Substance is required. RULE.—Ascertain volume of substance in cube feet; multiply it by unit in second column (its specific gravity), and divide product by 16; quotient will give it in lbs.

When Volume is given or ascertained in Inches. RULE.—Multiply it by 16 in third column of tables (weight of a cube inch), and product will give it in lbs.

EXAMPLE.—What is weight of a cube of Italian marble, sides being 3 feet?

$$3^3 \times 2708 = 73\ 116\ \text{oz.}, \text{ which } \div 16 = 4569.75\ \text{lbs.}$$

What is weight of a sphere of cast iron 2 inches in diameter?

$$2^3 \times .5236 \times .2607\ \text{weight of a cube inch} = 1.092\ \text{lbs.}$$

When Weight of an Elastic Fluid is required. RULE.—Multiply specific gravity of fluid by 532.679 (weight of a cube foot of air at 62° in grains), and product by 7000 (grains in a lb. Avoirdupois), and quotient will give it of a cube foot in lbs.

EXAMPLE.—What is weight of a cube foot of hydrogen?

Specific gravity of hydrogen .0692.

$$532.679 \times .0692 \div 7000 = .005\ 265\ 9\ \text{lbs.}$$

How to compute Weight of Cast Metal by Weight of Pattern is of White Pine.

RULE.—Multiply weight of pattern by following multipliers, and product will give weight of metal.

Multipliers: Iron, 14; Brass, 15; Lead, 22; Tin, 14; Zinc, 13.5.

When there are Circular Cores or Prints. Multiply square of pattern by its length in inches, the product by .01, and subtract pattern of core or print to be deducted from weight of metal.

GEOMETRY.

Definitions.

Point has position, but not magnitude.

Line is length without breadth, and is either *Right*, *Curved*, or *Mixed*.

Right Line is shortest distance between two points.

Curved Line is one that continually changes its direction.

Mixed Line is composed of a right and a curved line.

Superficies has length and breadth only, and is plane or curved.

Solid has length, breadth, and thickness, or depth.

Angle is opening of two lines having different directions, and is either *Acute*, *Right*, or *Obtuse*.

Right Angle is made by a line perpendicular to another falling upon it.

Acute Angle is less than a right angle.

Obtuse Angle is greater than a right angle.

Triangle is a figure of three sides.

Equilateral Triangle has all its sides equal.

Isoceles Triangle has two of its sides equal.

Scalene Triangle has all its sides unequal.

Right-angled Triangle has one right angle.

Obtuse-angled Triangle has one obtuse angle.

Acute-angled Triangle has all its angles acute.

Oblique-angled Triangle has no right angle.

Quadrangle or *Quadrilateral* is a figure of four sides, and has following particular designations—viz.,

Parallelogram, having its opposite sides parallel.

Square, having length and breadth equal.

Rectangle, a parallelogram having a right angle.

Rhombus or *Lozenge*, having equal sides, but its angles not right angles.

Rhomboid, a parallelogram, its angles not being right angles.

Trapezium, having unequal sides.

Trapezoid, having only one pair of opposite sides parallel.

Note.—*Triangle* is sometimes termed a *Trigon*, and a *Square* a *Tetragon*.

Common is space included between the lines forming two similar parallelograms, of which smaller is inscribed within larger, so as to have one angle each common to both.

Polygons are plane figures having more than four sides, and are either *Regular* or *Irregular*, according as their sides and angles are equal or unequal, and they are named from number of their sides or angles. Thus:

Pentagon has five sides.

Hexagon " six "

Heptagon " seven "

Octagon " eight "

Nonagon has nine sides.

Decagon " ten "

Undecagon " eleven "

Dodecagon " twelve "

Circle is a plane figure bounded by a curved line, termed *Circumference* or *Periphery*.

Diameter is a right line passing through centre of a circle or sphere, and terminated at each end by periphery or surface.

Arc is any part of circumference of a circle.

Chord is a right line joining extremities of an arc.

Segment of a circle is any part bounded by an arc and

Radius of a circle is a line drawn from centre to circum-

Sector is any part of a circle bounded by an arc and its

Semicircle is half a circle.

Quadrant is a quarter of a circle.

Annulus is a part of a circle included between two parallel

Segment is space between the intersecting arcs of two eccen-

Secant is line running from centre of circle to extremity of tangent of an arc.
Cosecant is secant of complement of an arc, or line running from centre of circle to extremity of cotangent of arc.

Sine of an arc is a line running from one extremity of an arc perpendicular to a diameter passing through other extremity, and sine of an angle is sine of arc that measures that angle.

Versed Sine of an arc or angle is part of diameter intercepted between sine and arc.

Cosine of an arc or angle is part of diameter intercepted between sine and centre.

Coversed Sine of an arc or angle is part of secondary radius intercepted between cosine and circumference.

Tangent is a right line that touches a circle without cutting it.

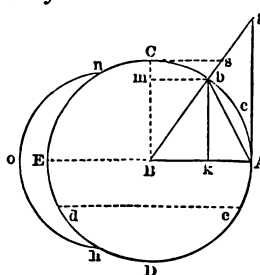
Cotangent is tangent of complement of arc.

Circumference of every circle is supposed to be divided into 360 equal parts, termed *Degrees*; each degree into 60 *Minutes*, and each minute into 60 *Seconds*, and so on.

Complement of an angle is what remains after subtracting angle from 90 degrees.

Supplement of an angle is what remains after subtracting angle from 180 degrees.

To exemplify these definitions, let Acb , in following Figure, be an assumed arc of a circle described with radius BA :



Acb , an Arc of circle $ACED$.

Ab , Chord of that arc.

BA , an Initial radius.

BC , a Secondary radius.

ed , a Segment of the circle.

ABb , a Sector.

ADE , a Semicircle.

CBE , a Quadrant.

$AedE$, a Zone.

noh , a Lune.

Bg , Secant of arc Acb ; written Sec .

bk , Sine of arc Acb ; written Sin .

ak , Versed Sine of arc Acb ; written $Versin$.

Bk or mb , Cosine of arc Acb .

Ag , Tangent of arc Acb .

CBb , Complement, and bBE , Supplement of arc Acb .

Cs , Cotangent of arc, written Cot . Bs , Cosecant of arc; written $Cosec$.

mC , Coversed sine of arc, or, by convention, of angle ABb ; written $Coversin$.

Vertex of a figure is its top or upper point. In Conic Sections it is point through which generating line of the conical surface always passes.

Altitude, or height of a figure, is a perpendicular let fall from its vertex to opposite side, termed base.

Measure of an angle is an arc of a circle contained between the two lines estimated by number of degrees in arc.

Plane, parallel to base.

After segment is cut off.

m of all its sides.

d to be done.

d.

o be demonstrated.

o render what follows more easy.

on a preceding demonstration.

ing going before it.

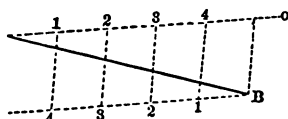
f Surfaces and Solids, and Conic Sections

Lengths of following Elements, Radius = 1.

| | Angle 45°. | Angle 60°. | | Angle 45°. | Angle 60°. |
|------------|------------|------------|---------------|------------|------------|
| | .707 107 | .866 025 | Cosecant..... | 1.414 214 | 1.154 7 |
| ne..... | .707 107 | .5 | Tangent..... | 1 | 1.732 05 |
| ed Sine.. | .292 893 | .5 | Cotangent... | 1 | .577 349 |
| ersed " .. | .292 893 | .133 975 | Chord..... | .765 366 | 1 |
| nt..... | 1.414 214 | 2 | Arc..... | .785 398 | 1.047 2 |

Scales.

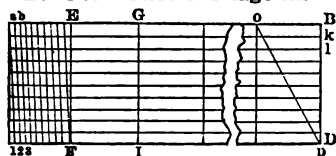
Divide a Line, as A B, with any required Number of Equal Parts.—Fig. 1.



From A and B draw two parallel lines, A o, B r, to an indefinite length, and upon them point off required number of equal parts, as A 1, 2, 3, 4, and B 1, 2, 3, 4; join o B, 4 1, etc.

Or, point off on A o, join o B, and draw the other lines parallel thereto.

To Construct a Diagonal Scale, as A B.—Fig. 2.



Divide a line into as many divisions as there are hundreds of feet, spaces of ten feet, feet, or inches required.

Draw perpendiculars from each division to a parallel line, C D. Divide one of divisions, A E, C F, into spaces of ten if for feet and hundredths, and twelve if for feet and inches; draw the lines A 1,

1 2, etc., and they will complete scale.

as: Line A B representing ten feet; A to E, E to G, etc., will measure one A to a, C to 1, 1 to 2, etc., will measure 1-10th of a foot. The several lines a 2, etc., will measure upon lines k, l, etc., 1-100th of a foot; and o p will are upon k, l, etc., divisions of 1-10th of a foot.

Lines.

Draw a Perpendicular to a Right Line, as o r, Fig. 3, c A, Fig. 4, or from a Point external to it, as A, Fig. 5, and from any two Points, as c d, Fig. 6.

With any radius as r A, r B, cut line at A and B; then with a longer radius, as A o, B o, describe arcs cutting each other at o, and connect o r. (Fig. 3.)

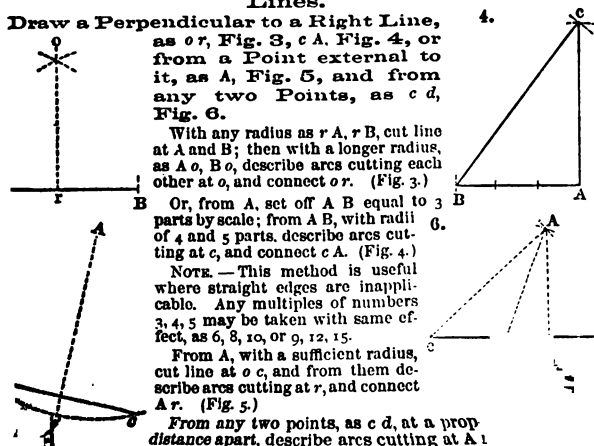
Or, from A, set off A B equal to 3 parts by scale; from A B, with radii of 4 and 5 parts, describe arcs cutting at c, and connect c A. (Fig. 4.)

NOTE.—This method is useful where straight edges are inapplicable. Any multiples of numbers 3, 4, 5 may be taken with same effect, as 6, 8, 10, or 9, 12, 15.

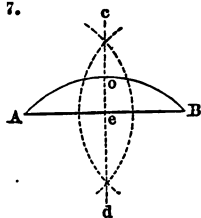
From A, with a sufficient radius, cut line at o c, and from them describe arcs cutting at r, and connect A r. (Fig. 5.)

From any two points, as c d, at a proper distance apart, describe arcs cutting at A 1 and connect them. (Fig. 6.)

T*



7.

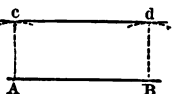


To Bisect a Right Line or an Arc of a Circle, and to Draw a Perpendicular to a Circular or Right Line, or a Radial Arc.—Fig. 7.

From A B as centres describe arcs cutting each other at c and d, and line and arc are bisected at e.

Line cd is also perpendicular to a right line as AB, and radial to a circular arc as A O B.

8.



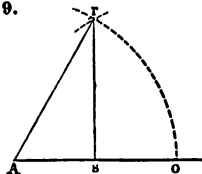
To Draw a Line Parallel to a Given Right Line, as c d, Fig. 8.

From A B describe arcs A c, B d, and draw a line parallel thereto, touching arcs c and d.

Angles.

To Describe Angles of 30° and 60° , Fig. 9, and 45° , Fig. 10.

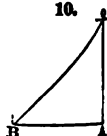
9.



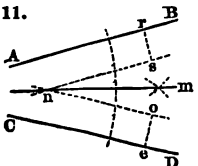
From A, with an r radius, A o, describe $o r$, and from o with a like radius cut it at r, let fall perpendicular $r s$; then $o A r = 60^\circ$, and $A r s = 30^\circ$. (Fig. 9.)

Set off any distance, as A B, erect perpendicular A o = A B, and connect o B. (Fig. 10.)

10.



11.



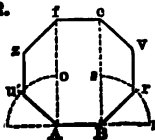
To Bisect Inclination of Two Lines, when Point of Intersection is Inaccessible.—Fig. 11.

Upon given lines, A B, C D, at any points draw perpendiculars e o, s r, of equal lengths, and from o and s draw parallels to their respective lines, cutting at n; bisect angle o n s, connect n m, and line will bisect lines as required.

Rectilineal Figures.

To Describe an Octagon upon a Line, as A B.—Fig. 12.

12.



From points A B erect indefinite perpendiculars A f, B e; produce A B to m and n, and bisect angles m A o and n B p with A u and B r.

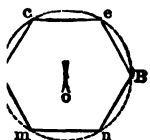
Make A u and B r equal to A B, and draw u z, r v parallel to A f, and equal to A B.

From z and v, as centres, with a radius equal to A B, describe arcs cutting A f, B e, in f and e. Connect z f, f e, and e v.

To Inscribe any Regular Polygon in a Circle, or to Divide Circumference into given Number of Equal Parts.—Fig. 13.

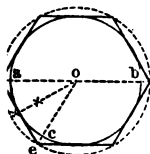
Circle is to contain a Heptagon.—Draw angle A o B as for $360^\circ \div 7 = 51^\circ 42' 51''$, or $51\frac{3}{7}$, then set off upon same distance A B or remaining angles A o B.

Inscribe a Hexagon in a Circle.—Fig. 14.



Draw a diameter, AoB . From A and B as centres, with Ao and Bo , cut circle at cm and en , and connect.

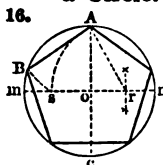
Describe a Hexagon about a Circle.—Fig. 15.



Describe circle, within which describe hexagon as above.

Draw a diameter as $ao b$; and with ao cut circle at c ; join ac , and bisect it with radius or , through r draw er parallel to ca , cutting diameter at m ; then with radius

To Inscribe a Pentagon in a Circle.—Fig. 16.

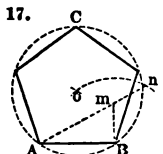


one side of a pentagon.

Draw diameters $A c$ and $m n$, at right angles to each other; bisect on in r , and with $r A$ describe $A s$; from A with $A s$ describe $s B$.

Connect AB , and distance is equal to

To Describe a Pentagon upon a Line, as AB .—Fig. 17.

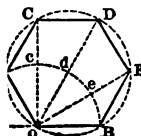


then from o , with radius $o B$, describe circle ACB , and line AB is equal to one side of a pentagon upon circle described.

Draw Bm perpendicular to AB , and equal to one half of it; extend Am until mn is equal to Bm .

From A and B , with radius Bn , describe arcs cutting each other in o ; then from o , with radius $o B$, describe circle ACB , and line AB is equal to one side of a pentagon upon circle described.

Describe a Regular Polygon of any required Number of Sides.—Fig. 18.



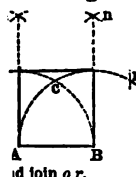
From point o , with distance $o B$, describe semicircle $B b A$, which divide into as many equal parts, $A a, a b, b c$, etc., as the polygon is to have sides.

Thus, let a Hexagon be required:

From o to second point b of six divisions draw ob , and through other points, c, d , and e , draw oc, od, oe .

Apply distance $o B$, from B to E , from E to D , from D to C , etc. Join these points, as $b C, C D$, etc.

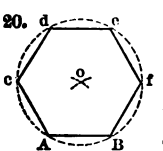
Construct a Square or Rectangle on a given line.—Fig. 19.



and join or .

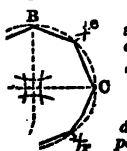
On AB as centres, with AB as radius, describe arcs cutting at c ; on c describe arcs cutting at or ; and on or describe others, cutting at $m n$; draw Am and

To Construct a Hexagon upon a given Line.—Fig. 20.



From ends of line, AB , describe arcs cutting each other at o , and from o as a centre, with radius $o A$, describe a circle, and with same radius set off $A c, c d, B f, f e$, and connect them.

To Inscribe an Octagon in a Circle.—Fig. 21.



Draw diameters, $A C, B D$, at right angles, bisect arcs, AB, BC , etc., at s, r, o, e , and join $A o, o B$, etc. (Fig. 21.)

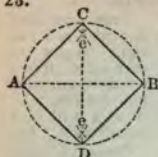
To Describe an Octagon about a Circle.—Fig. 22.

Describe a square about circle $A B$, draw diagonals $c f, e d$, draw $o i$, etc., perpendicular to diagonals and touching circle. (Fig. 22.)



To Inscribe a Square in a Circle.—Fig. 23.

23.



Draw line AB through centre of circle; take any radius, as Ae , and describe the arcs Aee , Bee ; connect ee , continuing line to C and D ; join AC , AD , etc. (Fig. 23.)

To Describe a Square about a Circle.—Fig. 24.

Draw line AB through centre of circle. Take any radius, as Ae ; describe arcs Aee , Bee ; connect ee , continuing line to C , D .



Describe Br and Dr ; draw and extend Br and Dr , and sides A and C parallel to them. (Fig. 24.)

To Describe an Octagon in a Square.—Fig. 25.

25.



Let $ABCD$ be given square.

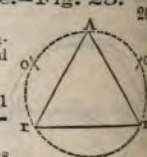
Describe $Aorr$, $Borr$, etc.; join intersections $rrrr$, etc., and figure formed is octagon required. (Fig. 25.)

To Inscribe an Equilateral Triangle in a Circle.—Fig. 26.

From point A , with Ao equal to radius

of circle, describe oo ; from o and o describe orr , orr ; join A , r , r , and r , A . (Fig. 26.)

NOTE.—All figures of 10 or 20 sides are readily determined from side of a pentagon, being halved or quartered; and in like manner, all figures of 6, 12, or 24 sides are readily determined from radius of a circle, being equal to the side of a hexagon.

**Circles.****To Describe an Arc of a Circle, through Two given Points, with a given Radius.—Fig. 27.**

27.



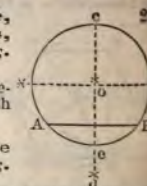
On A , B as centres, with given radius, describe arcs cutting at o , and from o with same radius describe arc A , B . (Fig. 27.)

To Ascertain Centre of a Circle or of an Arc of a Circle.—Fig. 28.

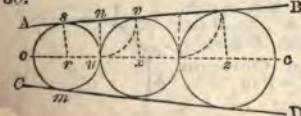
Draw chord AB , bisect it with perpendicular cd , then bisect cd for centre o . (Fig. 28.)

**To Describe a Circular Segment that will both fill the angle between two diverging lines and touch them.—Fig. 29.**

Bisect inclined lines, AB , DE , by line ef , and connect perpendicular thereto, BD , to define boundary of segment to be described. Bisect angles at B and D by lines cutting at o , and from o , with radius oe , describe arc m e n .

**To Draw a Series of Circles between Two Inclined Lines, touching them and each other.—Fig. 30.**

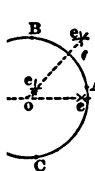
30.



Bisect given lines AB , CD , by line oe . From a point r in this line erect rs perpendicular to AB , and on r describe circle m , cutting centre line at u ; from u erect un perpendicular to centre line, cutting AB at n , and from n describe an arc n u v , cutting AB at v , erect zv parallel to rs , making z centre of next circle to be described, with radius zu , and so on.

r .—Largest circle may be described first.

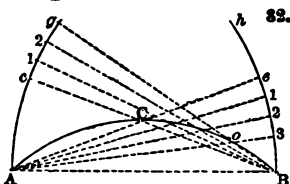
Describe a Circle that shall pass through any three given Points, as A B C.—Figs. 31 and 32.



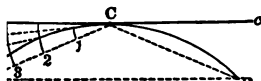
Upon points A and B, with any opening of a dividers, describe arcs cutting each other at *e*.

On points B C describe two more cutting each other in points *c*.

Draw lines *ee* and *cc*, and intersection of these lines, *o*, is centre of circle A B C. (Fig. 31.)



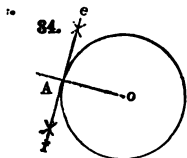
m Centre is not attainable. — From A B as centres, describe arcs A *g*, B *h*; from C draw A *e*, B *c*. Divide A *e* and B *c* into any number of equal parts, also divide A B into a like number. Draw A 1, 2, 3, etc., and B 1, 2, 3, etc., and intersect these lines as at *o* are points in the circle required. (Fig. 32.)



Or, let A B C be given points, connect A B, A C, C B, and draw *ec* parallel to A B. Divide C A into a number of equal parts, as at 1, 2, and 3, and from C describe arcs through these points to meet right lines from C to points 1, 2, and 3, on A *e*, and these are points in a circle, to be drawn as

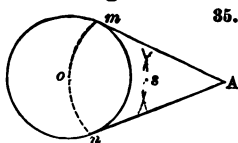
directed. (Fig. 33.)

Draw a Tangent to a circle from a given Point Circumference.—Fig.



From point A draw radial line A *o*, erect perpendicular *ef*. (Fig. 34.)

To Draw Tangents to a Circle from a Point without it.—Fig. 35.



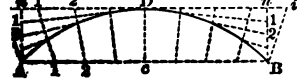
From A draw A *o*, and bisect it at *s*; describe arc through *o*, cutting circle at *m* *n*; join A *m* or A *n*.

Draw from or to Circumference of a Circle, Lines according to an Inaccessible Centre.—Fig. 36.



Draw end lines, as A *r*, F *r*. From *b* describe arc *o*, and with radius *b* 1, from *a* as centre, cut arcs A *r*, etc., and lines A *r*, F *r*, will lead to centre.

Describe an Arc, or Segment of a Circle, of a large Radius.—Fig. 37.



Draw chord A B; also line *h* D *i* parallel with equal to the chord in *c*; join A *c* D; join B *c* D; also line *h* D *i* at a distance; bisect perpendicular A and B at

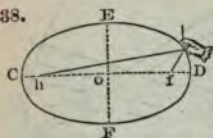
Equal to A D, B D; erect also perpendiculars A *n* and B *n*; number of equal parts; draw lines 1 1, 2 2, etc., number of equal parts in A B; draw lines *t* and at points of intersection with former line.

h t
B n
it

Ellipse.

To Describe an Ellipse to any Length and Breadth given.—Fig. 38.

38.

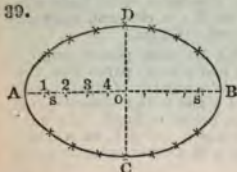


Let longest diameter be CD , and shortest EF distance Co or oD , and with it, from points F describe arcs h and f upon diameter CD .

Insert pins at h and at f , and loop a string them of such a length that when a pencil is introduced within it it will just reach to E or F . Be string, sweep it around centre o , and it will describe ellipse.

NOTE.—It is a property of Ellipse that sum of two lines drawn from foci to meet in any curve is equal to transverse diameter.

39.



Bisect transverse axis AB at o , and on erect perpendicular CD , making oD and oC equal to half conjugate axis. From C or radius AO , cut transverse axis at ss' for foci Ao into any number of equal parts, as 1. With radii $A1$, $B1$, on s and s' as centres, arcs, and repeat this operation for all others 1, 2, 3, etc., and these points of intersection give line of curve.

To Ascertain Centre and Two Diameters of an Ellipse.—Fig. 40.



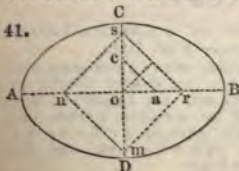
Let $ABcu$ be diameters of an Ellipse.

Draw at pleasure two lines, qq' , oo' , passing each other, and equidistant from A and B them in points hn , and draw line ur ; in s , and upon s , as a centre, describe a pleasure, as flr , cutting figure in points j .

Draw right line fv ; bisect it in t , and points ts draw greatest diameter AB , and centre, s , draw least diameter cu , parallel

To Describe an Ellipse approximately by Circular Arcs.—Fig. 41.

41.



Set off differences of axes from centre o , c on oA and oC ; draw ac and bisect it, and its half to r ; draw rs parallel to ac , set equal to or , connect ns , and draw parallel nm ; from m , with radii ms and sm , describe through C and D , and from n and r describe through A and B .

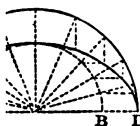
NOTE.—This method is not satisfactory when conjugate axis is less than two thirds of transverse.

With Arcs of Three Radii.—Fig. 42.



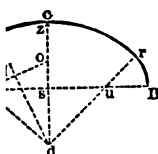
On transverse axis AB draw rectangle on height oe ; to diagonal Ae draw perpendicular dhO ; set off or equal to oe , describe a semi-circle on Ar , and produce Os to l ; set off om equal to ol , and on o describe an arc with radius Al , with radius ol , cut this arc at a . Thus centres, O , a , a' , h , h' , are found, from which arcs are described to form ellipse.

NOTE.—This process answers for nearly all portions of ellipses. It is used in striking stone bridges, etc.



To Construct an Ellipse from Two Circles.—Fig. 43.

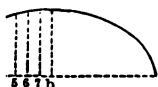
Describe two semicircles, as A B, C D, diameters of which are respectively lengths of major and minor axes. The intersection of the horizontal and vertical lines drawn from any radial line will give a point in the curve C D.



To Construct an Ellipse, when Two Diameters are Given.—Fig. 44.

Make co and Av equal to each other, but less than half breadth. Draw vo , and from its centre i draw and extend perpendicular at i to d , draw dvm , make $Bu = Av$, draw dur , from u and v describe Br and Am , from d describe mcr , extend cs to s , and it will be centre for other half of figure.

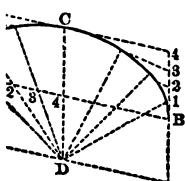
Construct an Ellipse by Ordinates.—Fig. 45.



Divide semi-transverse axis, as A b, into 8 or 10 divisions, as may be convenient, and erect ordinates, the lengths of which are equal to semi-conjugate, multiplied by the units for each division as follows:

| Eighths. | | Divisions. | | Tenths. | |
|----------|-------------|--------------|-------------|-------------|--|
| 4 12 | 5 — .927 03 | 1 — .435 385 | 5 — .866 02 | 9 — .994 99 | |
| 1 44 | 6 — .968 24 | 2 — .6 | 6 — .916 51 | 10 — 1 | |
| 10 63 | 7 — .992 16 | 3 — .714 14 | 7 — .953 94 | | |
| 16 03 | 8 — 1 | 4 — .8 | 8 — .979 79 | | |

Construct an Ellipse when Diameters do not Intersect at Right Angles.—Fig. 46.



Let A B and C D be given diameters.

Draw boundary lines parallel to diameters, divide longest diameter into any number of equal parts, and divide shortest boundary lines into same number of equal parts.

From one end of shortest diameter, D, draw radial lines through divisions of longest diameter, and from opposite end, C, draw radial lines to divisions on shortest boundary lines; the intersection of these lines will give points in the curve.

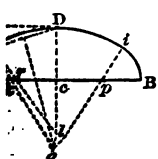
Arcs.

To Describe a Gothic Arc.—Fig. 47.

Take line A B. At points A and B draw arcs B a and A c, and it will describe arc required.



Describe an Elliptic Arc, Chord and Height being given.—Fig. 48.



Bisect A B at c; erect perpendicular A g, and draw line q D equal and parallel to A c.

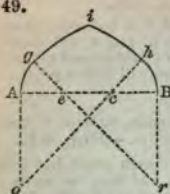
Bisect A c and A g in e and f; make e l equal to e f; bisect c D, and draw line l r g; bisect e f with a line at r q; bisect f g with a line at p q; draw line o q; draw line o p i.

Then, from o as a centre, describe arc s D i; and from k as a centre, describe arcs A s a l.

scribe arc s D i

To Describe a Gothic Arc.—Figs. 49 and 50.

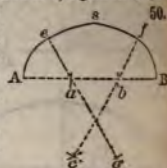
49.



Divide line AB into three equal parts, ec ; from points A and B let fall perpendiculars Ao and Br , equal in length to two of divisions of line AB ; draw lines oh and rg from points e, c ; with length of ec , describe arcs Ag and Bh , and from points o and r describe arcs gi and ih . (Fig. 49.)

Or, divide line AB into three equal parts at a and b , and on points A, a, b , and B , with distance of two divisions, make four arcs intersecting at c and o .

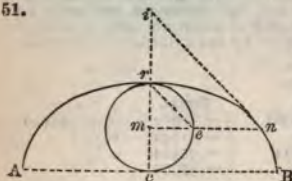
Through points c, o , and divisions a, b , draw lines cf and oe , on points a and b describe arcs Ae and Bf , and on points co arcs fs and es . (Fig. 50.)



Cycloid and Epicycloid.

To Describe a Cycloid.—Fig. 51.

51.



n is to curve at point n . The following are some of properties of Cycloid:

Horizontal line en = arc of circle er .
Half-base Ac = half-circumference cer .
Arc of Cycloid rn = twice chord re .
Half-arc of cycloid Ar = twice diameter of circle re .

When a circle, as a wheel, rolls over a straight right line, beginning as at A and ending at B , it completes one revolution and measures a straight line, AB , exactly equal to circumference of circle cer , which is termed the *generating circle*, and a point or pencil fixed at point r in circumference traces out a curvilinear path, ArB , termed a *cycloid*. AB is its base and cr its axis.

Place generating circle in middle of Cycloid, as in figure; draw a line, mn , parallel to base, cutting circle at e ; and tangent

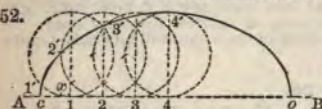
Or, whole arc of Cycloid ArB = four times axis cr .

Area of Cycloid ArB = three times area of generating circle re .

Tangent ni is parallel to chord er .

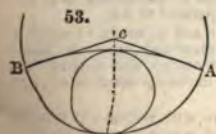
To Describe Curve of a Cycloid.—Fig. 52.

52.



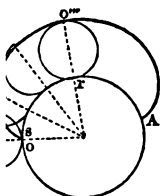
On an indefinite line, AB , set off co = circumference of generating circle, divide this line into any number of equal parts (8 in figure), and at points of division erect perpendiculars thereto. Upon each of these lines describe a circle = generating circle. On cx take $1x = .25cx$, and with x as a centre, with radius $zc = .75cx$, describe an arc cutting circle at $1'$; from 2 on next circle, with two distances of $1x$, measured as chords, cut circle at $2'$; from 3 on next circle, with three distances of $1x$, cut circle at $3'$, and proceed in like manner from each side until figure is complete.

To Describe an Interior Epicycloid or Hypocycloid.—Fig. 53.



explanation, Fig. 54.

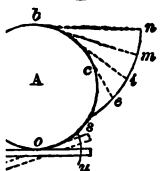
If generating circle is rolled on inside of fundamental circle, as in Fig. 53, it forms an *interior epicycloid*, or *hypocycloid*, AcB , which becomes in this case nearly a straight line. Other points of reference in figure correspond to those in Fig. 51. When diameter of generating circle is equal to half that of fundamental circle epicycloid becomes a straight line, being diameter of the larger circle.



To Describe an Exterior Epicycloid.—Fig. 54.

An Epicycloid differs from a Cycloid in this, that it is generated by a point, o''' , in one circle, $o r$, rolling upon circumference of another, $A r s$, instead of upon a right line or horizontal surface, former being *generating circle* and latter *fundamental circle*.

Generating circle is shown in four positions, in which its generating point is indicated by $o o' o'' o'''$. $A o''' s$ is an Epicycloid.



Involute.

To Describe an Involute.—Fig. 55.

Assume A as centre of a circle, $b c o$; a cord laid partly upon its circumference, as $b c$; then the curve $e i m n$, described by a tracer at end of cord, when unwound from a circle, is an involute.

This curve can also be defined by a batten, x , rolling on a circle, as $s u$.

Parabola.

To Construct a Parabola by Ordinates or Abscissa.—Figs. 56 and 57.

By Ordinates.

Divide ordinate $a b$ into 10 equal parts, and erect perpendiculars, length of which will be determined by multiplying abscissa $a c$ by respective units for each perpendicular, as follows: 57.



Divisions.

| | | | | |
|-------|-------|-------|-------|-------|
| 1—.19 | 3—.51 | 5—.75 | 7—.91 | 9—.99 |
| 2—.36 | 4—.64 | 6—.84 | 8—.96 | 10—1 |

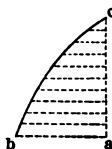
By Abscissa.

Divide abscissa $a c$ into 8 or 10 equal parts, as may be convenient, raw ordinates thereto, the lengths of which will be determined by multiplying half ordinate $a b$ by respective units for ordinate, as follows:

Divisions.

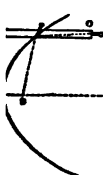
Tenths.

| <i>Eighths.</i> | | | |
|-----------------|-----------|-----------|-----------|
| — .353 5 | 5—.790 57 | 1—.316 23 | 6—.774 6 |
| — .5 | 6—.866 02 | 2—.447 21 | 7—.836 66 |
| — .612 37 | 7—.935 41 | 3—.547 72 | 8—.894 43 |
| — .707 11 | 8—1 | 4—.632 45 | 9—.948 68 |
| | | 5—.707 11 | 10—1 |



58.

With a Square and Cord.—Fig. 58.



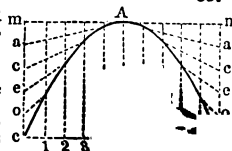
Place a straight edge to directrix $A B$, and apply to it a square, $c o$.

Attach to end o end of a cord equal to $o A$, and attach other end to focus e ; slide square along straight edge, maintaining cord taut against edge of square, by a point or pencil, and curve will be traced. (Fig. 58.)

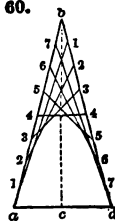
59.

When Height and Base are given.—Fig. 59.

Draw $A B$ axis and $c d$ a double ordinate or base. Then draw $m n$ parallel to $c d$, and through c draw $e m, d n$, parallel to axis $A B$. Divide $c m$, in any number of equal parts, as at $a c e o$, also $d n$ into a like number of parts. Through points a, b, c, d draw lines parallel to axis, and through e, f, g, h lines to vertex A , cutting these perpendiculars, and the curve will be traced. (Fig. 59.)



60.



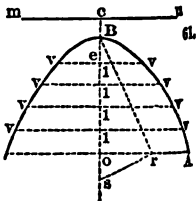
To Describe Curve of a Parabola, Base and Height being given.—Fig. 60.

Draw an isosceles triangle, as $a b d$, base of which shall be equal to, and its height, $c b$, twice that of proposed parabola. Divide each side, $a b$, $d b$, into any number of equal parts; then draw lines $1, 2, 3, 4, 5, 6, 7$, etc., and their intersection will define curve. (Fig. 60.)

To Describe a Parabola, any Ordinate to Axis and its Abscissa being given.—Fig. 61.

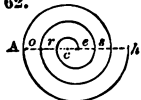
Bisect ordinate, as $A o$ in r ; join $B r$, and draw $r s$ perpendicular to it, meeting axis continued to s . Set off $B c$, $B e$, each equal to $o s$; draw $m c u$ perpendicular to $B s$, then $m u$ is directrix and $B e$ focus; through e and any number of points, $1, 2, 3, 4, 5, 6, 7$, etc., in axis, draw double ordinates $v x v$, and on centre e , with radii $e c$, $1 c$, etc., cut respective ordinates at v , etc., and trace curve through these points.

NOTE.—Line $v e v$ passing through focus is parameter.



Spiral.

62.

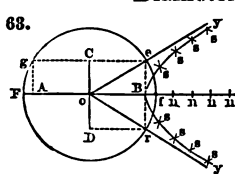


To Draw a Spiral about a given Point.—Fig. 62.

Assume c the centre. Draw $A h$, divide it into twice number of parts that there are to be revolutions of line. Upon c describe $r e$, $o s$, $A h$, and upon e describe $r s$, $o s$, etc.

Hyperbola.

To Describe a Hyperbola, Transverse and Conjugate Diameters being given.—Fig. 63.



Let $A B$ represent transverse diameter, and $C D$ conjugate.

Draw $C e$ parallel to $A B$, and $e r$ parallel to $C D$; draw $o e$, and with radius $o e$, with o as a centre, describe circle $F e r$, cutting transverse axis produced in F and f ; then will F and f be foci of figure.

In $o B$ produced take any number of points, $1, 2, 3, 4, 5, 6, 7$, etc., and from F and f as centres, with $A 1$ and $B 1$ as radii, describe arcs cutting each other in s, s' , etc. Through s, s' , etc., draw curve $s s s' s'' B s s' s''$.

NOTE.—If straight lines, as $o e y$ and $o r y$, are drawn from centre o through extremities e, r , they will be asymptotes of hyperbola, property of which is to approach continually to curve, and yet never to touch it.

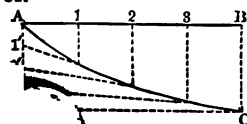
When *Foci and Conjugate Axis are given*.—Let F and f be foci, and $C D$ conjugate axis, as in preceding figure.

Through C draw $g C e$ parallel to F and f ; then, with o as a centre and $o F$ as a radius, describe an arc cutting $g C e$ at g and e ; from these points let fall perpendiculars upon line connecting F and f , and part intercepted between them, as $A B$, will be transverse axis.

Catenary.

To Delineate a Catenary, Span and Versed Sine being given.—Fig. 64. (W. Hildenbrand.)

64.



Divide half span, as $A B$, into any required number of equal parts, as $1, 2, 3$, and let fall $B C$ and $A o$, each equal to versed sine of curve; divide $A o$ into like number of parts, $1', 2', 3'$, as $A B$. Connect $C 1'$, $C 2'$, and $C 3'$, and points of intersection of perpendiculars let fall from $A B$ will give points through which curve is to be drawn.

Or, suspend a finely linked chain against a vertical height, or of given width or length of arc.

For other methods see D. R. Clark's Manual, pp. 18, 19.

Areas of Circles, from $\frac{1}{4}$ to 150.

| AREA. | DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. |
|----------|------------------|---------|------------------|----------|------------------|---------|
| .000 192 | 3 $\frac{1}{16}$ | 7.0686 | 7 $\frac{1}{8}$ | 38.4846 | 14 $\frac{1}{8}$ | 153.938 |
| .000 767 | 3 $\frac{1}{8}$ | 7.3662 | 7 $\frac{1}{4}$ | 39.8713 | 14 $\frac{1}{4}$ | 156.7 |
| .003 068 | 3 $\frac{1}{4}$ | 7.6699 | 7 $\frac{1}{2}$ | 41.2826 | 14 $\frac{1}{2}$ | 159.485 |
| .012 272 | 3 $\frac{1}{2}$ | 7.9798 | 8 | 42.7184 | 15 | 162.256 |
| .027 612 | 3 $\frac{3}{8}$ | 8.2958 | 8 $\frac{1}{8}$ | 44.1787 | 15 $\frac{1}{8}$ | 165.13 |
| .049 087 | 3 $\frac{1}{2}$ | 8.618 | 8 $\frac{1}{4}$ | 45.6636 | 15 $\frac{1}{4}$ | 167.99 |
| .076 699 | 3 $\frac{5}{8}$ | 8.9462 | 8 $\frac{1}{2}$ | 47.1731 | 15 $\frac{1}{2}$ | 170.874 |
| .110 447 | 4 | 9.2807 | 9 | 48.7071 | 16 | 173.782 |
| .150 33 | 4 $\frac{1}{8}$ | 9.6211 | 9 $\frac{1}{8}$ | 50.2656 | 16 $\frac{1}{8}$ | 176.715 |
| .196 35 | 4 $\frac{1}{4}$ | 9.968 | 9 $\frac{1}{4}$ | 51.8487 | 16 $\frac{1}{4}$ | 179.673 |
| .248 505 | 4 $\frac{1}{2}$ | 10.3206 | 9 $\frac{1}{2}$ | 53.4563 | 16 $\frac{1}{2}$ | 182.655 |
| .306 796 | 4 $\frac{3}{8}$ | 10.679 | 9 $\frac{3}{8}$ | 55.0884 | 16 $\frac{3}{8}$ | 185.661 |
| .371 224 | 4 $\frac{1}{2}$ | 11.0447 | 9 $\frac{1}{2}$ | 56.7451 | 17 | 188.692 |
| .441 787 | 4 $\frac{5}{8}$ | 11.416 | 9 $\frac{5}{8}$ | 58.4264 | 17 $\frac{1}{8}$ | 191.748 |
| .518 487 | 5 | 11.7933 | 10 | 60.1322 | 17 $\frac{1}{4}$ | 194.828 |
| .601 322 | 5 $\frac{1}{8}$ | 12.177 | 10 $\frac{1}{8}$ | 61.8625 | 17 $\frac{1}{2}$ | 197.933 |
| .690 292 | 5 $\frac{1}{4}$ | 12.5664 | 10 $\frac{1}{4}$ | 63.6174 | 17 $\frac{3}{8}$ | 201.062 |
| .785 4 | 5 $\frac{1}{2}$ | 12.962 | 10 $\frac{1}{2}$ | 65.3968 | 18 | 204.216 |
| .886 6 | 5 $\frac{3}{8}$ | 13.3641 | 10 $\frac{3}{8}$ | 67.2008 | 18 $\frac{1}{8}$ | 207.395 |
| .994 02 | 5 $\frac{1}{2}$ | 13.772 | 10 $\frac{1}{2}$ | 69.0293 | 18 $\frac{1}{4}$ | 210.598 |
| 1.107 5 | 5 $\frac{5}{8}$ | 14.1863 | 10 $\frac{5}{8}$ | 70.8823 | 18 $\frac{1}{2}$ | 213.825 |
| 1.227 2 | 6 | 14.606 | 11 | 72.7599 | 18 $\frac{3}{8}$ | 217.077 |
| 1.353 | 6 $\frac{1}{8}$ | 15.033 | 11 $\frac{1}{8}$ | 74.6621 | 18 $\frac{1}{2}$ | 220.354 |
| 1.484 9 | 6 $\frac{1}{4}$ | 15.465 | 11 $\frac{1}{4}$ | 76.5888 | 19 | 223.655 |
| 1.622 9 | 6 $\frac{1}{2}$ | 15.9043 | 11 $\frac{1}{2}$ | 78.54 | 19 $\frac{1}{8}$ | 226.981 |
| 1.767 1 | 6 $\frac{3}{8}$ | 16.349 | 11 $\frac{3}{8}$ | 80.5158 | 19 $\frac{1}{4}$ | 230.331 |
| 1.917 5 | 6 $\frac{1}{2}$ | 16.8002 | 11 $\frac{1}{2}$ | 82.5161 | 19 $\frac{1}{2}$ | 233.706 |
| 2.073 9 | 6 $\frac{5}{8}$ | 17.257 | 12 | 84.5409 | 19 $\frac{3}{8}$ | 237.105 |
| 2.236 5 | 7 | 17.7206 | 12 $\frac{1}{8}$ | 86.5903 | 19 $\frac{1}{2}$ | 240.529 |
| 2.405 3 | 7 $\frac{1}{8}$ | 18.19 | 12 $\frac{1}{4}$ | 88.6643 | 19 $\frac{1}{2}$ | 243.977 |
| 2.58 | 7 $\frac{1}{4}$ | 18.6655 | 12 $\frac{1}{2}$ | 90.7628 | 20 | 247.45 |
| 2.761 2 | 7 $\frac{3}{8}$ | 19.147 | 12 $\frac{3}{8}$ | 92.8858 | 20 $\frac{1}{8}$ | 250.948 |
| 2.948 3 | 7 $\frac{1}{2}$ | 19.635 | 13 | 95.0334 | 20 $\frac{1}{4}$ | 254.47 |
| 3.141 6 | 7 $\frac{1}{2}$ | 20.129 | 13 $\frac{1}{8}$ | 97.2055 | 20 $\frac{1}{2}$ | 258.016 |
| 3.338 | 7 $\frac{5}{8}$ | 20.629 | 13 $\frac{1}{4}$ | 99.4022 | 20 $\frac{3}{8}$ | 261.587 |
| 3.546 6 | 8 | 21.135 | 13 $\frac{1}{2}$ | 101.6234 | 20 $\frac{1}{2}$ | 265.183 |
| 3.758 4 | 8 $\frac{1}{8}$ | 21.6476 | 13 $\frac{3}{8}$ | 103.8691 | 20 $\frac{3}{4}$ | 268.803 |
| 3.976 1 | 8 $\frac{1}{4}$ | 22.166 | 13 $\frac{1}{2}$ | 106.1394 | 21 | 272.448 |
| 4.2 | 8 $\frac{3}{8}$ | 22.6907 | 14 | 108.4343 | 21 $\frac{1}{8}$ | 276.117 |
| 4.430 1 | 8 $\frac{1}{2}$ | 23.221 | 14 $\frac{1}{8}$ | 110.7537 | 21 $\frac{1}{4}$ | 279.811 |
| 4.666 4 | 8 $\frac{1}{2}$ | 23.7583 | 14 $\frac{1}{4}$ | 113.098 | 21 $\frac{1}{2}$ | 283.529 |
| 4.908 7 | 8 $\frac{5}{8}$ | 24.301 | 14 $\frac{1}{2}$ | 115.466 | 21 $\frac{3}{8}$ | 287.272 |
| 5.157 3 | 8 $\frac{3}{4}$ | 24.8505 | 15 | 117.859 | 21 $\frac{1}{2}$ | 291.04 |
| 5.411 9 | 8 $\frac{7}{8}$ | 25.406 | 15 $\frac{1}{8}$ | 120.277 | 21 $\frac{3}{4}$ | 294.832 |
| 5.672 3 | 9 | 25.9673 | 15 $\frac{1}{4}$ | 122.719 | 21 $\frac{1}{2}$ | 298.648 |
| 5.939 6 | 9 $\frac{1}{8}$ | 26.535 | 15 $\frac{3}{8}$ | 125.185 | 22 | 302.489 |
| 6.212 6 | 9 $\frac{1}{4}$ | 27.1086 | 15 $\frac{1}{2}$ | 127.677 | 22 $\frac{1}{8}$ | 306.355 |
| 6.491 8 | 9 $\frac{1}{2}$ | 27.688 | 16 | 130.192 | 22 $\frac{1}{4}$ | 310.245 |
| 6.777 2 | 9 $\frac{3}{8}$ | 28.2744 | 16 $\frac{1}{8}$ | 132.733 | 22 $\frac{1}{2}$ | 314.16 |
| | 9 $\frac{1}{2}$ | 29.4648 | 16 $\frac{1}{4}$ | 135.297 | 22 $\frac{3}{8}$ | 318.099 |
| | 9 $\frac{1}{2}$ | 30.6797 | 16 $\frac{1}{2}$ | 137.887 | 22 $\frac{1}{2}$ | 322.063 |
| | 9 $\frac{5}{8}$ | 31.9191 | 16 $\frac{3}{8}$ | 140.501 | 22 $\frac{3}{4}$ | 326.051 |
| | 9 $\frac{3}{4}$ | 33.1831 | 17 | 143.139 | 23 | 330.064 |
| | 9 $\frac{7}{8}$ | 34.4717 | 17 $\frac{1}{8}$ | 145.802 | | |
| | 10 | 35.7848 | 17 $\frac{1}{4}$ | 148.49 | | |
| | 10 $\frac{1}{8}$ | 37.1224 | 17 $\frac{1}{2}$ | 151.202 | | |

| DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. |
|---------------|---------|---------------|---------|---------------|----------|---------------|---------|
| 21 | 346.361 | 28 | 615.754 | 35 | 962.115 | 42 | 1385.45 |
| $\frac{1}{8}$ | 350.497 | $\frac{1}{8}$ | 621.264 | $\frac{1}{8}$ | 969 | $\frac{1}{8}$ | 1393.7 |
| $\frac{1}{4}$ | 354.657 | $\frac{1}{4}$ | 626.798 | $\frac{1}{4}$ | 975.909 | $\frac{1}{4}$ | 1401.99 |
| $\frac{3}{8}$ | 358.842 | $\frac{3}{8}$ | 632.357 | $\frac{3}{8}$ | 982.842 | $\frac{3}{8}$ | 1410.3 |
| $\frac{1}{2}$ | 363.051 | $\frac{1}{2}$ | 637.941 | $\frac{1}{2}$ | 989.8 | $\frac{1}{2}$ | 1418.63 |
| $\frac{5}{8}$ | 367.285 | $\frac{5}{8}$ | 643.549 | $\frac{5}{8}$ | 996.783 | $\frac{5}{8}$ | 1426.99 |
| $\frac{3}{4}$ | 371.543 | $\frac{3}{4}$ | 649.182 | $\frac{3}{4}$ | 1003.79 | $\frac{3}{4}$ | 1435.37 |
| $\frac{7}{8}$ | 375.826 | $\frac{7}{8}$ | 654.84 | $\frac{7}{8}$ | 1010.822 | $\frac{7}{8}$ | 1443.77 |
| 22 | 380.134 | 29 | 660.521 | 36 | 1017.878 | 43 | 1452.2 |
| $\frac{1}{8}$ | 384.466 | $\frac{1}{8}$ | 666.228 | $\frac{1}{8}$ | 1024.96 | $\frac{1}{8}$ | 1460.66 |
| $\frac{1}{4}$ | 388.822 | $\frac{1}{4}$ | 671.959 | $\frac{1}{4}$ | 1032.065 | $\frac{1}{4}$ | 1469.14 |
| $\frac{3}{8}$ | 393.203 | $\frac{3}{8}$ | 677.714 | $\frac{3}{8}$ | 1039.195 | $\frac{3}{8}$ | 1477.64 |
| $\frac{1}{2}$ | 397.609 | $\frac{1}{2}$ | 683.494 | $\frac{1}{2}$ | 1046.349 | $\frac{1}{2}$ | 1486.17 |
| $\frac{5}{8}$ | 402.038 | $\frac{5}{8}$ | 689.299 | $\frac{5}{8}$ | 1053.528 | $\frac{5}{8}$ | 1494.73 |
| $\frac{3}{4}$ | 406.494 | $\frac{3}{4}$ | 695.128 | $\frac{3}{4}$ | 1060.732 | $\frac{3}{4}$ | 1503.3 |
| $\frac{7}{8}$ | 410.973 | $\frac{7}{8}$ | 700.982 | $\frac{7}{8}$ | 1067.96 | $\frac{7}{8}$ | 1511.91 |
| 23 | 415.477 | 30 | 706.86 | 37 | 1075.213 | 44 | 1520.53 |
| $\frac{1}{8}$ | 420.004 | $\frac{1}{8}$ | 712.763 | $\frac{1}{8}$ | 1082.49 | $\frac{1}{8}$ | 1529.19 |
| $\frac{1}{4}$ | 424.558 | $\frac{1}{4}$ | 718.69 | $\frac{1}{4}$ | 1089.792 | $\frac{1}{4}$ | 1537.86 |
| $\frac{3}{8}$ | 429.135 | $\frac{3}{8}$ | 724.642 | $\frac{3}{8}$ | 1097.118 | $\frac{3}{8}$ | 1546.56 |
| $\frac{1}{2}$ | 433.737 | $\frac{1}{2}$ | 730.618 | $\frac{1}{2}$ | 1104.469 | $\frac{1}{2}$ | 1555.29 |
| $\frac{5}{8}$ | 438.364 | $\frac{5}{8}$ | 736.619 | $\frac{5}{8}$ | 1111.844 | $\frac{5}{8}$ | 1564.04 |
| $\frac{3}{4}$ | 443.015 | $\frac{3}{4}$ | 742.645 | $\frac{3}{4}$ | 1119.244 | $\frac{3}{4}$ | 1572.81 |
| $\frac{7}{8}$ | 447.69 | $\frac{7}{8}$ | 748.695 | $\frac{7}{8}$ | 1126.669 | $\frac{7}{8}$ | 1581.61 |
| 24 | 452.39 | 31 | 754.769 | 38 | 1134.118 | 45 | 1590.43 |
| $\frac{1}{8}$ | 457.115 | $\frac{1}{8}$ | 760.869 | $\frac{1}{8}$ | 1141.591 | $\frac{1}{8}$ | 1599.28 |
| $\frac{1}{4}$ | 461.864 | $\frac{1}{4}$ | 766.992 | $\frac{1}{4}$ | 1149.089 | $\frac{1}{4}$ | 1608.16 |
| $\frac{3}{8}$ | 466.638 | $\frac{3}{8}$ | 773.14 | $\frac{3}{8}$ | 1156.612 | $\frac{3}{8}$ | 1617.05 |
| $\frac{1}{2}$ | 471.436 | $\frac{1}{2}$ | 779.313 | $\frac{1}{2}$ | 1164.159 | $\frac{1}{2}$ | 1625.97 |
| $\frac{5}{8}$ | 476.259 | $\frac{5}{8}$ | 785.51 | $\frac{5}{8}$ | 1171.731 | $\frac{5}{8}$ | 1634.92 |
| $\frac{3}{4}$ | 481.107 | $\frac{3}{4}$ | 791.732 | $\frac{3}{4}$ | 1179.327 | $\frac{3}{4}$ | 1643.89 |
| $\frac{7}{8}$ | 485.979 | $\frac{7}{8}$ | 797.979 | $\frac{7}{8}$ | 1186.948 | $\frac{7}{8}$ | 1652.89 |
| 25 | 490.875 | 32 | 804.25 | 39 | 1194.593 | 46 | 1661.91 |
| $\frac{1}{8}$ | 495.796 | $\frac{1}{8}$ | 810.545 | $\frac{1}{8}$ | 1202.263 | $\frac{1}{8}$ | 1670.95 |
| $\frac{1}{4}$ | 500.742 | $\frac{1}{4}$ | 816.865 | $\frac{1}{4}$ | 1209.958 | $\frac{1}{4}$ | 1680.02 |
| $\frac{3}{8}$ | 505.712 | $\frac{3}{8}$ | 823.21 | $\frac{3}{8}$ | 1217.677 | $\frac{3}{8}$ | 1689.11 |
| $\frac{1}{2}$ | 510.706 | $\frac{1}{2}$ | 829.579 | $\frac{1}{2}$ | 1225.42 | $\frac{1}{2}$ | 1698.23 |
| $\frac{5}{8}$ | 515.726 | $\frac{5}{8}$ | 835.972 | $\frac{5}{8}$ | 1233.188 | $\frac{5}{8}$ | 1707.37 |
| $\frac{3}{4}$ | 520.769 | $\frac{3}{4}$ | 842.391 | $\frac{3}{4}$ | 1240.981 | $\frac{3}{4}$ | 1716.54 |
| $\frac{7}{8}$ | 525.838 | $\frac{7}{8}$ | 848.833 | $\frac{7}{8}$ | 1248.798 | $\frac{7}{8}$ | 1725.73 |
| 26 | 530.93 | 33 | 855.301 | 40 | 1256.64 | 47 | 1734.95 |
| $\frac{1}{8}$ | 536.048 | $\frac{1}{8}$ | 861.792 | $\frac{1}{8}$ | 1264.506 | $\frac{1}{8}$ | 1744.19 |
| $\frac{1}{4}$ | 541.19 | $\frac{1}{4}$ | 868.309 | $\frac{1}{4}$ | 1272.397 | $\frac{1}{4}$ | 1753.45 |
| $\frac{3}{8}$ | 546.356 | $\frac{3}{8}$ | 874.85 | $\frac{3}{8}$ | 1280.312 | $\frac{3}{8}$ | 1762.74 |
| $\frac{1}{2}$ | 551.547 | $\frac{1}{2}$ | 881.415 | $\frac{1}{2}$ | 1288.252 | $\frac{1}{2}$ | 1772.06 |
| $\frac{5}{8}$ | 556.763 | $\frac{5}{8}$ | 888.005 | $\frac{5}{8}$ | 1296.217 | $\frac{5}{8}$ | 1781.4 |
| $\frac{3}{4}$ | 562.003 | $\frac{3}{4}$ | 894.62 | $\frac{3}{4}$ | 1304.206 | $\frac{3}{4}$ | 1790.76 |
| $\frac{7}{8}$ | 567.267 | $\frac{7}{8}$ | 901.259 | $\frac{7}{8}$ | 1312.219 | $\frac{7}{8}$ | 1800.15 |
| | 572.557 | 34 | 907.922 | 41 | 1320.257 | 48 | 1809.56 |
| | 577.87 | $\frac{1}{8}$ | 914.611 | $\frac{1}{8}$ | 1328.32 | $\frac{1}{8}$ | 1819 |
| | 583.209 | $\frac{1}{4}$ | 921.323 | $\frac{1}{4}$ | 1336.407 | $\frac{1}{4}$ | 1828.46 |
| | 588.571 | $\frac{3}{8}$ | 928.061 | $\frac{3}{8}$ | 1344.519 | $\frac{3}{8}$ | 1837.95 |
| | 593.959 | $\frac{1}{2}$ | 934.822 | $\frac{1}{2}$ | 1352.655 | $\frac{1}{2}$ | 1847.46 |
| | 599.371 | $\frac{5}{8}$ | 941.609 | $\frac{5}{8}$ | 1360.816 | $\frac{5}{8}$ | 1856.99 |
| | 604.807 | $\frac{3}{4}$ | 948.42 | $\frac{3}{4}$ | 1369.001 | $\frac{3}{4}$ | 1866.55 |
| | 610.268 | $\frac{7}{8}$ | 955.255 | $\frac{7}{8}$ | 1377.211 | $\frac{7}{8}$ | 1876.14 |

| M. | AREA. | DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. |
|---------------|---------|---------------|---------|---------------|---------|---------------|---------|
| | 1885.75 | 56 | 2463.01 | 63 | 3117.25 | 70 | 3848.46 |
| $\frac{1}{8}$ | 1895.38 | $\frac{1}{8}$ | 2474.02 | $\frac{1}{8}$ | 3129.64 | $\frac{1}{8}$ | 3862.22 |
| $\frac{1}{4}$ | 1905.04 | $\frac{1}{4}$ | 2485.05 | $\frac{1}{4}$ | 3142.04 | $\frac{1}{4}$ | 3876 |
| $\frac{3}{8}$ | 1914.72 | $\frac{3}{8}$ | 2496.11 | $\frac{3}{8}$ | 3154.47 | $\frac{3}{8}$ | 3889.8 |
| $\frac{1}{2}$ | 1924.43 | $\frac{1}{2}$ | 2507.19 | $\frac{1}{2}$ | 3166.93 | $\frac{1}{2}$ | 3903.63 |
| $\frac{5}{8}$ | 1934.16 | $\frac{5}{8}$ | 2518.3 | $\frac{5}{8}$ | 3179.41 | $\frac{5}{8}$ | 3917.49 |
| $\frac{3}{4}$ | 1943.91 | $\frac{3}{4}$ | 2529.43 | $\frac{3}{4}$ | 3191.91 | $\frac{3}{4}$ | 3931.37 |
| $\frac{7}{8}$ | 1953.69 | $\frac{7}{8}$ | 2540.58 | $\frac{7}{8}$ | 3204.44 | $\frac{7}{8}$ | 3945.27 |
| | 1963.5 | 57 | 2551.76 | 64 | 3217 | 71 | 3959.2 |
| $\frac{1}{8}$ | 1973.33 | $\frac{1}{8}$ | 2562.97 | $\frac{1}{8}$ | 3229.58 | $\frac{1}{8}$ | 3973.15 |
| $\frac{1}{4}$ | 1983.18 | $\frac{1}{4}$ | 2574.2 | $\frac{1}{4}$ | 3242.18 | $\frac{1}{4}$ | 3987.13 |
| $\frac{3}{8}$ | 1993.06 | $\frac{3}{8}$ | 2585.45 | $\frac{3}{8}$ | 3254.81 | $\frac{3}{8}$ | 4001.13 |
| $\frac{1}{2}$ | 2002.97 | $\frac{1}{2}$ | 2596.73 | $\frac{1}{2}$ | 3267.46 | $\frac{1}{2}$ | 4015.16 |
| $\frac{5}{8}$ | 2012.89 | $\frac{5}{8}$ | 2608.03 | $\frac{5}{8}$ | 3280.14 | $\frac{5}{8}$ | 4029.21 |
| $\frac{3}{4}$ | 2022.85 | $\frac{3}{4}$ | 2619.36 | $\frac{3}{4}$ | 3292.84 | $\frac{3}{4}$ | 4043.29 |
| $\frac{7}{8}$ | 2032.82 | $\frac{7}{8}$ | 2630.71 | $\frac{7}{8}$ | 3305.56 | $\frac{7}{8}$ | 4057.39 |
| | 2042.83 | 58 | 2642.09 | 65 | 3318.31 | 72 | 4071.51 |
| $\frac{1}{8}$ | 2052.85 | $\frac{1}{8}$ | 2653.49 | $\frac{1}{8}$ | 3331.09 | $\frac{1}{8}$ | 4085.66 |
| $\frac{1}{4}$ | 2062.9 | $\frac{1}{4}$ | 2664.91 | $\frac{1}{4}$ | 3343.89 | $\frac{1}{4}$ | 4099.84 |
| $\frac{3}{8}$ | 2072.98 | $\frac{3}{8}$ | 2676.36 | $\frac{3}{8}$ | 3356.71 | $\frac{3}{8}$ | 4114.04 |
| $\frac{1}{2}$ | 2083.08 | $\frac{1}{2}$ | 2687.84 | $\frac{1}{2}$ | 3369.56 | $\frac{1}{2}$ | 4128.26 |
| $\frac{5}{8}$ | 2093.2 | $\frac{5}{8}$ | 2699.33 | $\frac{5}{8}$ | 3382.44 | $\frac{5}{8}$ | 4142.51 |
| $\frac{3}{4}$ | 2103.35 | $\frac{3}{4}$ | 2710.86 | $\frac{3}{4}$ | 3395.33 | $\frac{3}{4}$ | 4156.78 |
| $\frac{7}{8}$ | 2113.52 | $\frac{7}{8}$ | 2722.41 | $\frac{7}{8}$ | 3408.26 | $\frac{7}{8}$ | 4171.08 |
| | 2123.72 | 59 | 2733.98 | 66 | 3421.2 | 73 | 4185.4 |
| $\frac{1}{8}$ | 2133.94 | $\frac{1}{8}$ | 2745.57 | $\frac{1}{8}$ | 3434.17 | $\frac{1}{8}$ | 4199.74 |
| $\frac{1}{4}$ | 2144.19 | $\frac{1}{4}$ | 2757.2 | $\frac{1}{4}$ | 3447.17 | $\frac{1}{4}$ | 4214.11 |
| $\frac{3}{8}$ | 2154.46 | $\frac{3}{8}$ | 2768.84 | $\frac{3}{8}$ | 3460.19 | $\frac{3}{8}$ | 4228.51 |
| $\frac{1}{2}$ | 2164.76 | $\frac{1}{2}$ | 2780.51 | $\frac{1}{2}$ | 3473.24 | $\frac{1}{2}$ | 4242.93 |
| $\frac{5}{8}$ | 2175.08 | $\frac{5}{8}$ | 2792.21 | $\frac{5}{8}$ | 3486.3 | $\frac{5}{8}$ | 4257.37 |
| $\frac{3}{4}$ | 2185.42 | $\frac{3}{4}$ | 2803.93 | $\frac{3}{4}$ | 3499.4 | $\frac{3}{4}$ | 4271.84 |
| $\frac{7}{8}$ | 2195.79 | $\frac{7}{8}$ | 2815.67 | $\frac{7}{8}$ | 3512.52 | $\frac{7}{8}$ | 4286.33 |
| | 2206.19 | 60 | 2827.44 | 67 | 3525.66 | 74 | 4300.85 |
| $\frac{1}{8}$ | 2216.61 | $\frac{1}{8}$ | 2839.23 | $\frac{1}{8}$ | 3538.83 | $\frac{1}{8}$ | 4315.39 |
| $\frac{1}{4}$ | 2227.05 | $\frac{1}{4}$ | 2851.05 | $\frac{1}{4}$ | 3552.02 | $\frac{1}{4}$ | 4329.96 |
| $\frac{3}{8}$ | 2237.52 | $\frac{3}{8}$ | 2862.89 | $\frac{3}{8}$ | 3565.24 | $\frac{3}{8}$ | 4344.55 |
| $\frac{1}{2}$ | 2248.01 | $\frac{1}{2}$ | 2874.76 | $\frac{1}{2}$ | 3578.48 | $\frac{1}{2}$ | 4359.17 |
| $\frac{5}{8}$ | 2258.53 | $\frac{5}{8}$ | 2886.65 | $\frac{5}{8}$ | 3591.74 | $\frac{5}{8}$ | 4373.81 |
| $\frac{3}{4}$ | 2269.07 | $\frac{3}{4}$ | 2898.57 | $\frac{3}{4}$ | 3605.04 | $\frac{3}{4}$ | 4388.47 |
| $\frac{7}{8}$ | 2279.64 | $\frac{7}{8}$ | 2910.51 | $\frac{7}{8}$ | 3618.35 | $\frac{7}{8}$ | 4403.16 |
| | 2290.23 | 61 | 2922.47 | 68 | 3631.69 | 75 | 4417.87 |
| $\frac{1}{8}$ | 2300.84 | $\frac{1}{8}$ | 2934.46 | $\frac{1}{8}$ | 3645.05 | $\frac{1}{8}$ | 4432.61 |
| $\frac{1}{4}$ | 2311.48 | $\frac{1}{4}$ | 2946.48 | $\frac{1}{4}$ | 3658.44 | $\frac{1}{4}$ | 4447.38 |
| $\frac{3}{8}$ | 2322.15 | $\frac{3}{8}$ | 2958.52 | $\frac{3}{8}$ | 3671.86 | $\frac{3}{8}$ | 4462.16 |
| $\frac{1}{2}$ | 2332.83 | $\frac{1}{2}$ | 2970.58 | $\frac{1}{2}$ | 3685.29 | $\frac{1}{2}$ | 4476.98 |
| $\frac{5}{8}$ | 2343.55 | $\frac{5}{8}$ | 2982.67 | $\frac{5}{8}$ | 3698.76 | $\frac{5}{8}$ | 4491.81 |
| $\frac{3}{4}$ | 2354.29 | $\frac{3}{4}$ | 2994.78 | $\frac{3}{4}$ | 3712.24 | $\frac{3}{4}$ | 4506.67 |
| $\frac{7}{8}$ | 2365.05 | $\frac{7}{8}$ | 3006.92 | $\frac{7}{8}$ | 3725.75 | $\frac{7}{8}$ | 4521.56 |
| | 2375.83 | 62 | 3019.08 | 69 | 3739.29 | 76 | 4536.47 |
| $\frac{1}{8}$ | 2386.65 | $\frac{1}{8}$ | 3031.26 | $\frac{1}{8}$ | 3752.85 | $\frac{1}{8}$ | 4551.41 |
| $\frac{1}{4}$ | 2397.48 | $\frac{1}{4}$ | 3043.47 | $\frac{1}{4}$ | 3766.43 | $\frac{1}{4}$ | 4566.36 |
| $\frac{3}{8}$ | 2408.34 | $\frac{3}{8}$ | 3055.71 | $\frac{3}{8}$ | 3780.04 | $\frac{3}{8}$ | 4581.32 |
| $\frac{1}{2}$ | 2419.23 | $\frac{1}{2}$ | 3067.97 | $\frac{1}{2}$ | 3793.68 | $\frac{1}{2}$ | 4596.31 |
| $\frac{5}{8}$ | 2430.14 | $\frac{5}{8}$ | 3080.25 | $\frac{5}{8}$ | 3807.34 | $\frac{5}{8}$ | 4611.32 |
| $\frac{3}{4}$ | 2441.07 | $\frac{3}{4}$ | 3092.56 | $\frac{3}{4}$ | 3821.02 | $\frac{3}{4}$ | 4626.36 |
| $\frac{7}{8}$ | 2452.03 | $\frac{7}{8}$ | 3104.89 | $\frac{7}{8}$ | 3834.73 | $\frac{7}{8}$ | 4641.42 |

| DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. |
|---------------|---------|---------------|---------|---------------|---------|---------------|---------|
| 77 | 4656.64 | 84 | 5541.78 | 91 | 6503.9 | 98 | 7542.98 |
| $\frac{1}{8}$ | 4671.77 | $\frac{1}{8}$ | 5558.29 | $\frac{1}{8}$ | 6521.78 | $\frac{1}{8}$ | 7562.24 |
| $\frac{1}{4}$ | 4686.92 | $\frac{1}{4}$ | 5574.82 | $\frac{1}{4}$ | 6539.68 | $\frac{1}{4}$ | 7581.52 |
| $\frac{3}{8}$ | 4702.1 | $\frac{3}{8}$ | 5591.37 | $\frac{3}{8}$ | 6557.61 | $\frac{3}{8}$ | 7600.82 |
| $\frac{1}{2}$ | 4717.31 | $\frac{1}{2}$ | 5607.95 | $\frac{1}{2}$ | 6575.56 | $\frac{1}{2}$ | 7620.15 |
| $\frac{5}{8}$ | 4732.54 | $\frac{5}{8}$ | 5624.56 | $\frac{5}{8}$ | 6593.54 | $\frac{5}{8}$ | 7639.5 |
| $\frac{3}{4}$ | 4747.79 | $\frac{3}{4}$ | 5641.18 | $\frac{3}{4}$ | 6611.55 | $\frac{3}{4}$ | 7658.88 |
| $\frac{7}{8}$ | 4763.07 | $\frac{7}{8}$ | 5657.84 | $\frac{7}{8}$ | 6629.57 | $\frac{7}{8}$ | 7678.28 |
| 78 | 4778.37 | 85 | 5674.51 | 92 | 6647.63 | 99 | 7697.71 |
| $\frac{1}{8}$ | 4793.7 | $\frac{1}{8}$ | 5691.22 | $\frac{1}{8}$ | 6665.7 | $\frac{1}{8}$ | 7717.16 |
| $\frac{1}{4}$ | 4809.05 | $\frac{1}{4}$ | 5707.94 | $\frac{1}{4}$ | 6683.8 | $\frac{1}{4}$ | 7736.63 |
| $\frac{3}{8}$ | 4824.43 | $\frac{3}{8}$ | 5724.69 | $\frac{3}{8}$ | 6701.93 | $\frac{3}{8}$ | 7756.13 |
| $\frac{1}{2}$ | 4839.83 | $\frac{1}{2}$ | 5741.47 | $\frac{1}{2}$ | 6720.08 | $\frac{1}{2}$ | 7775.66 |
| $\frac{5}{8}$ | 4855.26 | $\frac{5}{8}$ | 5758.27 | $\frac{5}{8}$ | 6738.25 | $\frac{5}{8}$ | 7795.21 |
| $\frac{3}{4}$ | 4870.71 | $\frac{3}{4}$ | 5775.1 | $\frac{3}{4}$ | 6756.45 | $\frac{3}{4}$ | 7814.78 |
| $\frac{7}{8}$ | 4886.18 | $\frac{7}{8}$ | 5791.94 | $\frac{7}{8}$ | 6774.68 | $\frac{7}{8}$ | 7834.38 |
| 79 | 4901.68 | 86 | 5808.82 | 93 | 6792.92 | 100 | 7854 |
| $\frac{1}{8}$ | 4917.21 | $\frac{1}{8}$ | 5825.72 | $\frac{1}{8}$ | 6811.2 | $\frac{1}{8}$ | 7893.32 |
| $\frac{1}{4}$ | 4932.75 | $\frac{1}{4}$ | 5842.64 | $\frac{1}{4}$ | 6829.49 | $\frac{1}{4}$ | 7932.74 |
| $\frac{3}{8}$ | 4948.33 | $\frac{3}{8}$ | 5859.59 | $\frac{3}{8}$ | 6847.82 | $\frac{3}{8}$ | 7972.25 |
| $\frac{1}{2}$ | 4963.92 | $\frac{1}{2}$ | 5876.56 | $\frac{1}{2}$ | 6866.16 | 101 | 8011.87 |
| $\frac{5}{8}$ | 4979.55 | $\frac{5}{8}$ | 5893.55 | $\frac{5}{8}$ | 6884.53 | $\frac{1}{4}$ | 8051.58 |
| $\frac{3}{4}$ | 4995.19 | $\frac{3}{4}$ | 5910.58 | $\frac{3}{4}$ | 6902.93 | $\frac{1}{2}$ | 8091.39 |
| $\frac{7}{8}$ | 5010.86 | $\frac{7}{8}$ | 5927.62 | $\frac{7}{8}$ | 6921.35 | $\frac{3}{4}$ | 8131.3 |
| 80 | 5026.56 | 87 | 5944.69 | 94 | 6939.79 | 102 | 8171.3 |
| $\frac{1}{8}$ | 5042.28 | $\frac{1}{8}$ | 5961.79 | $\frac{1}{8}$ | 6958.26 | $\frac{1}{4}$ | 8211.41 |
| $\frac{1}{4}$ | 5058.03 | $\frac{1}{4}$ | 5978.91 | $\frac{1}{4}$ | 6976.76 | $\frac{1}{2}$ | 8251.61 |
| $\frac{3}{8}$ | 5073.79 | $\frac{3}{8}$ | 5996.05 | $\frac{3}{8}$ | 6995.28 | $\frac{3}{4}$ | 8291.91 |
| $\frac{1}{2}$ | 5089.59 | $\frac{1}{2}$ | 6013.22 | $\frac{1}{2}$ | 7013.82 | 103 | 8332.31 |
| $\frac{5}{8}$ | 5105.41 | $\frac{5}{8}$ | 6030.41 | $\frac{5}{8}$ | 7032.39 | $\frac{1}{4}$ | 8372.81 |
| $\frac{3}{4}$ | 5121.25 | $\frac{3}{4}$ | 6047.63 | $\frac{3}{4}$ | 7050.98 | $\frac{1}{2}$ | 8413.4 |
| $\frac{7}{8}$ | 5137.12 | $\frac{7}{8}$ | 6064.87 | $\frac{7}{8}$ | 7069.59 | $\frac{3}{4}$ | 8454.09 |
| 81 | 5153.01 | 88 | 6082.14 | 95 | 7088.23 | 104 | 8494.89 |
| $\frac{1}{8}$ | 5168.93 | $\frac{1}{8}$ | 6099.43 | $\frac{1}{8}$ | 7106.9 | $\frac{1}{4}$ | 8535.78 |
| $\frac{1}{4}$ | 5184.87 | $\frac{1}{4}$ | 6116.74 | $\frac{1}{4}$ | 7125.59 | $\frac{1}{2}$ | 8576.76 |
| $\frac{3}{8}$ | 5200.83 | $\frac{3}{8}$ | 6134.08 | $\frac{3}{8}$ | 7144.31 | $\frac{3}{4}$ | 8617.85 |
| $\frac{1}{2}$ | 5216.82 | $\frac{1}{2}$ | 6151.45 | $\frac{1}{2}$ | 7163.04 | 105 | 8659.03 |
| $\frac{5}{8}$ | 5232.84 | $\frac{5}{8}$ | 6168.84 | $\frac{5}{8}$ | 7181.81 | $\frac{1}{4}$ | 8700.32 |
| $\frac{3}{4}$ | 5248.88 | $\frac{3}{4}$ | 6186.25 | $\frac{3}{4}$ | 7200.6 | $\frac{1}{2}$ | 8741.7 |
| $\frac{7}{8}$ | 5264.94 | $\frac{7}{8}$ | 6203.69 | $\frac{7}{8}$ | 7219.41 | $\frac{3}{4}$ | 8783.18 |
| 82 | 5281.03 | 89 | 6221.15 | 96 | 7238.25 | 106 | 8824.75 |
| $\frac{1}{8}$ | 5297.14 | $\frac{1}{8}$ | 6238.64 | $\frac{1}{8}$ | 7257.11 | $\frac{1}{4}$ | 8866.43 |

| IN. | AREA. | DIAM. | AREA. | DIAM. | AREA. | DIAM. | AREA. |
|---------------|----------|---------------|----------|---------------|----------|---------------|----------|
| 1 | 9503.34 | 120 | 11309.76 | 130 | 13273.26 | 140 | 15393.84 |
| $\frac{1}{4}$ | 9546.59 | $\frac{1}{4}$ | 11356.93 | $\frac{1}{4}$ | 13324.36 | $\frac{1}{4}$ | 15448.87 |
| $\frac{1}{2}$ | 9589.93 | $\frac{1}{2}$ | 11404.2 | $\frac{1}{2}$ | 13375.56 | $\frac{1}{2}$ | 15503.99 |
| $\frac{3}{4}$ | 9633.37 | $\frac{3}{4}$ | 11451.57 | $\frac{3}{4}$ | 13426.85 | $\frac{3}{4}$ | 15559.22 |
| 2 | 9676.91 | 121 | 11499.04 | 131 | 13478.25 | 141 | 15614.54 |
| $\frac{1}{4}$ | 9720.55 | $\frac{1}{4}$ | 11546.61 | $\frac{1}{4}$ | 13529.74 | $\frac{1}{4}$ | 15669.96 |
| $\frac{1}{2}$ | 9764.29 | $\frac{1}{2}$ | 11594.27 | $\frac{1}{2}$ | 13581.33 | $\frac{1}{2}$ | 15725.48 |
| $\frac{3}{4}$ | 9808.12 | $\frac{3}{4}$ | 11642.03 | $\frac{3}{4}$ | 13633.02 | $\frac{3}{4}$ | 15781.09 |
| 3 | 9852.06 | 122 | 11689.89 | 132 | 13684.81 | 142 | 15836.81 |
| $\frac{1}{4}$ | 9896.09 | $\frac{1}{4}$ | 11737.85 | $\frac{1}{4}$ | 13736.69 | $\frac{1}{4}$ | 15892.62 |
| $\frac{1}{2}$ | 9940.22 | $\frac{1}{2}$ | 11785.91 | $\frac{1}{2}$ | 13788.68 | $\frac{1}{2}$ | 15948.53 |
| $\frac{3}{4}$ | 9984.45 | $\frac{3}{4}$ | 11834.06 | $\frac{3}{4}$ | 13840.76 | $\frac{3}{4}$ | 16004.54 |
| 4 | 10028.77 | 123 | 11882.32 | 133 | 13892.94 | 143 | 16060.64 |
| $\frac{1}{4}$ | 10073.2 | $\frac{1}{4}$ | 11930.67 | $\frac{1}{4}$ | 13945.22 | $\frac{1}{4}$ | 16116.85 |
| $\frac{1}{2}$ | 10117.72 | $\frac{1}{2}$ | 11979.12 | $\frac{1}{2}$ | 13997.6 | $\frac{1}{2}$ | 16173.15 |
| $\frac{3}{4}$ | 10162.34 | $\frac{3}{4}$ | 12027.66 | $\frac{3}{4}$ | 14050.07 | $\frac{3}{4}$ | 16229.55 |
| 5 | 10207.06 | 124 | 12076.31 | 134 | 14102.64 | 144 | 16286.05 |
| $\frac{1}{4}$ | 10251.88 | $\frac{1}{4}$ | 12125.05 | $\frac{1}{4}$ | 14155.31 | $\frac{1}{4}$ | 16342.65 |
| $\frac{1}{2}$ | 10296.79 | $\frac{1}{2}$ | 12173.9 | $\frac{1}{2}$ | 14208.08 | $\frac{1}{2}$ | 16399.35 |
| $\frac{3}{4}$ | 10341.8 | $\frac{3}{4}$ | 12222.84 | $\frac{3}{4}$ | 14253.09 | $\frac{3}{4}$ | 16456.14 |
| 6 | 10386.91 | 125 | 12271.87 | 135 | 14313.91 | 145 | 16513.03 |
| $\frac{1}{4}$ | 10432.12 | $\frac{1}{4}$ | 12321.01 | $\frac{1}{4}$ | 14366.98 | $\frac{1}{4}$ | 16570.02 |
| $\frac{1}{2}$ | 10477.43 | $\frac{1}{2}$ | 12370.25 | $\frac{1}{2}$ | 14420.14 | $\frac{1}{2}$ | 16627.11 |
| $\frac{3}{4}$ | 10522.84 | $\frac{3}{4}$ | 12419.58 | $\frac{3}{4}$ | 14473.4 | $\frac{3}{4}$ | 16684.3 |
| 7 | 10568.34 | 126 | 12469.01 | 136 | 14526.76 | 146 | 16741.59 |
| $\frac{1}{4}$ | 10613.94 | $\frac{1}{4}$ | 12518.54 | $\frac{1}{4}$ | 14580.21 | $\frac{1}{4}$ | 16798.97 |
| $\frac{1}{2}$ | 10659.65 | $\frac{1}{2}$ | 12568.17 | $\frac{1}{2}$ | 14633.77 | $\frac{1}{2}$ | 16856.45 |
| $\frac{3}{4}$ | 10705.44 | $\frac{3}{4}$ | 12618.09 | $\frac{3}{4}$ | 14687.42 | $\frac{3}{4}$ | 16914.03 |
| 8 | 10751.34 | 127 | 12667.72 | 137 | 14741.17 | 147 | 16971.71 |
| $\frac{1}{4}$ | 10797.34 | $\frac{1}{4}$ | 12717.64 | $\frac{1}{4}$ | 14795.02 | $\frac{1}{4}$ | 17029.48 |
| $\frac{1}{2}$ | 10843.43 | $\frac{1}{2}$ | 12767.66 | $\frac{1}{2}$ | 14848.97 | $\frac{1}{2}$ | 17087.36 |
| $\frac{3}{4}$ | 10889.62 | $\frac{3}{4}$ | 12817.78 | $\frac{3}{4}$ | 14903.01 | $\frac{3}{4}$ | 17145.33 |
| 9 | 10935.91 | 128 | 12867.99 | 138 | 14957.16 | 148 | 17203.4 |
| $\frac{1}{4}$ | 10982.3 | $\frac{1}{4}$ | 12918.31 | $\frac{1}{4}$ | 15011.4 | $\frac{1}{4}$ | 17261.57 |
| $\frac{1}{2}$ | 11028.78 | $\frac{1}{2}$ | 12968.72 | $\frac{1}{2}$ | 15065.74 | $\frac{1}{2}$ | 17319.84 |
| $\frac{3}{4}$ | 11075.37 | $\frac{3}{4}$ | 13019.23 | $\frac{3}{4}$ | 15120.18 | $\frac{3}{4}$ | 17378.2 |
| 10 | 11122.05 | 129 | 13069.84 | 139 | 15174.71 | 149 | 17436.67 |
| $\frac{1}{4}$ | 11168.83 | $\frac{1}{4}$ | 13120.55 | $\frac{1}{4}$ | 15229.35 | $\frac{1}{4}$ | 17495.23 |
| $\frac{1}{2}$ | 11215.71 | $\frac{1}{2}$ | 13171.35 | $\frac{1}{2}$ | 15284.08 | $\frac{1}{2}$ | 17553.89 |
| $\frac{3}{4}$ | 11262.69 | $\frac{3}{4}$ | 13222.26 | $\frac{3}{4}$ | 15338.91 | $\frac{3}{4}$ | 17611.5 |

Compute Area of a Circle greater than any in Table.

RULE.—Divide dimension by two, three, four, etc., if practicable to do so, if it is reduced to a diameter to be found in table.

Take tabular area for this diameter, multiply it by square of divisor, and duct will give area required.

EXAMPLE.—What is area for a diameter of 1050?

$1050 \div 7 = 150$; tab. area, 150 = 17671.5, which $\times 7^2 = 865903.5$, area.

Compute Area of a Circle in Feet and Inches, etc., by preceding Table.

NOTE.—Reduce dimension to inches or eighths, as the case may be, area in that term from table for that number.

Divide this number by 64 (square of 8) if it is in eighths, and quotient will give area in inches, and divide again by 144 (square of 12) if it is in inches, and quotient will give area in feet.

EXAMPLE.—What is area of 1 foot 6.375 ins.?

1 foot 6.375 ins. = 18.375 ins. = 147 eighths. Area of 147 = 16971.71, which $\div 64$ = 265.18125 ins.; and by 144 = 1.84125 feet.

To Compute Area of a Circle Composed of an Integer and a Fraction.

RULE.—Double, treble, or quadruple dimension given, until fraction is increased to a whole number, or to one of those in the table, as $\frac{1}{2}$, $\frac{1}{4}$, etc., provided it is practicable to do so.

Take area for this diameter; and if it is double of that for which area is required, take one fourth of it; if treble, take one sixteenth of it, etc.

EXAMPLE.—Required area for a circle of 2.1875 ins.

$2.1875 \times 2 = 4.375$, area for which = 15.0331, which $\div 4 = 3.758$ ins.

When Diameter is composed of Integers and Fractions contained in Table.

RULE.—Point off a decimal to a diameter from table, and add twice as many figures or ciphers to the right of the area as there are figures cut off from the diameter.

EXAMPLE 1.—What is area of 9675 feet diameter?

Area of 96.75 = 7351.79; hence, area = 73 517 900 feet

2.—What is area of 24 375 feet diameter?

Area of 2.4375 = 4.6664; hence, area = 466 640 000 feet.

To Ascertain Area of a Circle as 300, 3000, etc., not contained in Table.

RULE.—Take area of 3 or 30, and add twice the excess of ciphers to the result.

EXAMPLE.—What is area of a circle 3000 feet in diameter?

Area of 30 = 706.86, hence area of 3000 = 7 068 600 feet.

To Compute Area of a Circle by Logarithms.

RULE.—To twice log. of diameter add $\bar{1}.895091$ (log. of .7854), and sum is log. of area, for which take number.

EXAMPLE.—What is area of a circle 1200 feet in diameter?

Log. $1200 \times 2 + \bar{1}.895091 = 6.158362 + \bar{1}.895091 = 6.053453$, and number for which = 1 130 976 feet.

Areas of Birmingham Wire Gauge.

| Diam. | Area. | Diam. | Area. | Diam. | Area. | Diam. | Area. |
|-------|-----------|-------|-----------|-------|-----------|-------|-----------|
| No. | Sq. Inch. | No. | Sq. Inch. | No. | Sq. Inch. | No. | Sq. Inch. |
| 1 | .070686 | 10 | .014103 | 19 | .001385 | 28 | .000154 |
| 2 | .063347 | 11 | .011309 | 20 | .000962 | 29 | .000133 |
| 3 | .052685 | 12 | .009331 | 21 | .000804 | 30 | .000113 |
| 4 | .044488 | 13 | .007088 | 22 | .000616 | 31 | .000078 |
| 5 | .038013 | 14 | .005411 | 23 | .000491 | 32 | .000064 |
| 6 | .032365 | 15 | .004071 | 24 | .00038 | 33 | .00005 |
| 7 | .025447 | 16 | .003318 | 25 | .000314 | 34 | .000038 |
| 8 | .021382 | 17 | .002642 | 26 | .000254 | 35 | .00002 |
| 9 | .017203 | 18 | .001886 | 27 | .000201 | 36 | .000013 |

Circumferences of Circles, from $\frac{1}{4}$ to 150.

| Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|----------|-----------------|---------|-----------------|---------|-------|----------|
| .049 09 | 3 | 9.4248 | 8 | 25.1328 | 15 | 47.124 |
| .098 18 | | 9.6211 | $8\frac{1}{4}$ | 25.5255 | | 47.5167 |
| .196 35 | | 9.8175 | $8\frac{1}{2}$ | 25.9182 | | 47.9094 |
| .392 7 | | 10.014 | $8\frac{3}{4}$ | 26.3109 | | 48.3021 |
| .589 | | 10.2102 | 9 | 26.7036 | | 48.6948 |
| .785 4 | | 10.406 | $9\frac{1}{4}$ | 27.0963 | | 49.0875 |
| .981 75 | | 10.6029 | $9\frac{1}{2}$ | 27.4890 | | 49.4802 |
| 1.178 1 | | 10.799 | $9\frac{3}{4}$ | 27.8817 | | 49.8729 |
| 1.374 45 | | 10.9956 | 9 | 28.2744 | 10 | 50.2656 |
| 1.570 8 | | 11.191 | $9\frac{1}{4}$ | 28.6671 | | 50.6583 |
| 1.767 15 | | 11.3883 | $9\frac{1}{2}$ | 29.0598 | | 51.051 |
| 1.963 5 | | 11.584 | $9\frac{3}{4}$ | 29.4525 | | 51.4437 |
| 2.159 85 | 4 | 11.781 | 10 | 29.8452 | | 51.8364 |
| 2.356 2 | $4\frac{1}{4}$ | 11.977 | $10\frac{1}{4}$ | 30.2379 | | 52.2291 |
| 2.552 55 | $4\frac{1}{2}$ | 12.1737 | $10\frac{1}{2}$ | 30.6306 | | 52.6218 |
| 2.748 9 | $4\frac{3}{4}$ | 12.369 | $10\frac{3}{4}$ | 31.0233 | | 53.0145 |
| 2.945 25 | 5 | 12.5664 | 10 | 31.416 | 17 | 53.4072 |
| 3.141 6 | $5\frac{1}{4}$ | 12.762 | $10\frac{1}{4}$ | 31.8087 | | 53.7999 |
| 3.337 9 | $5\frac{1}{2}$ | 12.9591 | $10\frac{1}{2}$ | 32.2014 | | 54.1926 |
| 3.534 3 | $5\frac{3}{4}$ | 13.155 | $10\frac{3}{4}$ | 32.5941 | | 54.5853 |
| 3.730 6 | 6 | 13.3518 | 11 | 32.9868 | | 54.978 |
| 3.927 | $6\frac{1}{4}$ | 13.547 | $11\frac{1}{4}$ | 33.3795 | | 55.3707 |
| 4.123 3 | $6\frac{1}{2}$ | 13.7445 | $11\frac{1}{2}$ | 33.7722 | | 55.7634 |
| 4.319 7 | $6\frac{3}{4}$ | 13.94 | $11\frac{3}{4}$ | 34.1649 | | 56.1561 |
| 4.516 | 7 | 14.1372 | 12 | 34.5576 | 18 | 56.5488 |
| 4.712 4 | $7\frac{1}{4}$ | 14.333 | $12\frac{1}{4}$ | 34.9503 | | 56.9415 |
| 4.908 7 | $7\frac{1}{2}$ | 14.5299 | $12\frac{1}{2}$ | 35.343 | | 57.3342 |
| 5.105 1 | $7\frac{3}{4}$ | 14.725 | $12\frac{3}{4}$ | 35.7357 | | 57.7269 |
| 5.301 4 | 8 | 14.9226 | 13 | 36.1284 | | 58.1196 |
| 5.497 8 | $8\frac{1}{4}$ | 15.119 | $13\frac{1}{4}$ | 36.5211 | | 58.5123 |
| 5.694 1 | $8\frac{1}{2}$ | 15.3153 | $13\frac{1}{2}$ | 36.9138 | | 58.905 |
| 5.890 5 | $8\frac{3}{4}$ | 15.511 | $13\frac{3}{4}$ | 37.3065 | | 59.2977 |
| 6.086 8 | 9 | 15.708 | 14 | 37.6992 | 19 | 59.6904 |
| 6.283 2 | $9\frac{1}{4}$ | 16.1007 | $14\frac{1}{4}$ | 38.0919 | | 60.0831 |
| 6.479 5 | $9\frac{1}{2}$ | 16.4934 | $14\frac{1}{2}$ | 38.4846 | | 60.4758 |
| 6.675 9 | $9\frac{3}{4}$ | 16.8861 | $14\frac{3}{4}$ | 38.8773 | | 60.8685 |
| 6.872 2 | 10 | 17.2788 | 15 | 39.27 | | 61.2612 |
| 7.068 5 | $10\frac{1}{4}$ | 17.6715 | $15\frac{1}{4}$ | 39.6627 | | 61.6539 |
| 7.264 9 | $10\frac{1}{2}$ | 18.0642 | $15\frac{1}{2}$ | 40.0554 | | 62.0466 |
| 7.461 3 | $10\frac{3}{4}$ | 18.4569 | $15\frac{3}{4}$ | 40.4481 | | 62.4393 |
| 7.657 7 | 11 | 18.8496 | 16 | 40.8408 | 20 | 62.832 |
| 7.854 1 | $11\frac{1}{4}$ | 19.2423 | $16\frac{1}{4}$ | 41.2335 | | 63.2247 |
| 8.050 5 | $11\frac{1}{2}$ | 19.635 | $16\frac{1}{2}$ | 41.6262 | | 63.6174 |
| 8.246 9 | $11\frac{3}{4}$ | 20.0277 | $16\frac{3}{4}$ | 42.0189 | | 64.0101 |
| 8.443 3 | 12 | 20.4204 | 17 | 42.4116 | | 64.4028 |
| 8.639 7 | $12\frac{1}{4}$ | 20.8131 | $17\frac{1}{4}$ | 42.8043 | | 64.7955 |
| 8.836 1 | $12\frac{1}{2}$ | 21.2058 | $17\frac{1}{2}$ | 43.197 | | 65.1882 |
| 9.032 5 | $12\frac{3}{4}$ | 21.5985 | $17\frac{3}{4}$ | 43.5897 | | 65.5809 |
| 9.228 9 | 13 | 21.9912 | 18 | 43.9824 | 21 | 65.9736 |
| 9.425 3 | $13\frac{1}{4}$ | 22.3839 | $18\frac{1}{4}$ | 44.3751 | | 66.3663 |
| 9.621 7 | $13\frac{1}{2}$ | 22.7766 | $18\frac{1}{2}$ | 44.7678 | | 66.759 |
| 9.818 1 | $13\frac{3}{4}$ | 23.1693 | $18\frac{3}{4}$ | 45.1605 | | 67.1517 |
| 10.014 5 | 14 | 23.562 | 19 | 45.5532 | | 67.5444 |
| 10.210 9 | $14\frac{1}{4}$ | 23.9547 | $19\frac{1}{4}$ | 45.9459 | | 67.9371 |
| 10.406 3 | $14\frac{1}{2}$ | 24.3474 | $19\frac{1}{2}$ | 46.3386 | | 68.3298 |
| 10.602 7 | $14\frac{3}{4}$ | 24.7401 | $19\frac{3}{4}$ | 46.7313 | | 68.7225 |
| 10.799 1 | 15 | 25.1328 | 20 | 47.124 | | 69.1152 |
| 10.995 5 | $15\frac{1}{4}$ | 25.5255 | $20\frac{1}{4}$ | 47.5167 | | 69.5079 |
| 11.191 9 | $15\frac{1}{2}$ | 25.9182 | $20\frac{1}{2}$ | 47.9094 | | 69.9006 |
| 11.388 3 | $15\frac{3}{4}$ | 26.3109 | $20\frac{3}{4}$ | 48.3021 | | 70.2933 |
| 11.584 7 | 16 | 26.7036 | 21 | 48.6948 | | 70.686 |
| 11.781 1 | $16\frac{1}{4}$ | 27.0963 | $21\frac{1}{4}$ | 49.0875 | | 71.0787 |
| 11.977 5 | $16\frac{1}{2}$ | 27.4890 | $21\frac{1}{2}$ | 49.4802 | | 71.4714 |
| 12.173 9 | $16\frac{3}{4}$ | 27.8817 | $21\frac{3}{4}$ | 49.8729 | | 71.8641 |
| 12.369 3 | 17 | 28.2744 | 22 | 50.2656 | | 72.2568 |
| 12.566 7 | $17\frac{1}{4}$ | 28.6671 | $22\frac{1}{4}$ | 50.6583 | | 72.6495 |
| 12.762 1 | $17\frac{1}{2}$ | 29.0598 | $22\frac{1}{2}$ | 51.051 | | 73.0422 |
| 12.959 5 | $17\frac{3}{4}$ | 29.4525 | $22\frac{3}{4}$ | 51.4437 | | 73.4349 |
| 13.155 9 | 18 | 29.8452 | 23 | 51.8364 | | 73.8276 |
| 13.351 3 | $18\frac{1}{4}$ | 30.2379 | $23\frac{1}{4}$ | 52.2291 | | 74.2203 |
| 13.547 7 | $18\frac{1}{2}$ | 30.6306 | $23\frac{1}{2}$ | 52.6218 | | 74.613 |
| 13.744 1 | $18\frac{3}{4}$ | 31.0233 | $23\frac{3}{4}$ | 53.0145 | | 75.0057 |
| 13.94 5 | 19 | 31.416 | 24 | 53.4072 | | 75.3984 |
| 14.137 9 | $19\frac{1}{4}$ | 31.8087 | $24\frac{1}{4}$ | 53.7999 | | 75.7911 |
| 14.333 3 | $19\frac{1}{2}$ | 32.2014 | $24\frac{1}{2}$ | 54.1926 | | 76.1838 |
| 14.529 7 | $19\frac{3}{4}$ | 32.5941 | $24\frac{3}{4}$ | 54.5853 | | 76.5765 |
| 14.725 1 | 20 | 32.9868 | 25 | 54.978 | | 76.9692 |
| 14.922 5 | $20\frac{1}{4}$ | 33.3795 | $25\frac{1}{4}$ | 55.3707 | | 77.3619 |
| 15.119 9 | $20\frac{1}{2}$ | 33.7722 | $25\frac{1}{2}$ | 55.7634 | | 77.7546 |
| 15.315 3 | $20\frac{3}{4}$ | 34.1649 | $25\frac{3}{4}$ | 56.1561 | | 78.1473 |
| 15.511 7 | 21 | 34.5576 | 26 | 56.5488 | | 78.54 |
| 15.708 1 | $21\frac{1}{4}$ | 34.9503 | $26\frac{1}{4}$ | 56.9415 | | 78.9327 |
| 15.904 5 | $21\frac{1}{2}$ | 35.343 | $26\frac{1}{2}$ | 57.3342 | | 79.3254 |
| 16.100 9 | $21\frac{3}{4}$ | 35.7357 | $26\frac{3}{4}$ | 57.7269 | | 79.7181 |
| 16.296 3 | 22 | 36.1284 | 27 | 58.1196 | | 80.1108 |
| 16.492 7 | $22\frac{1}{4}$ | 36.5211 | $27\frac{1}{4}$ | 58.5123 | | 80.5035 |
| 16.688 1 | $22\frac{1}{2}$ | 36.9138 | $27\frac{1}{2}$ | 58.905 | | 80.8962 |
| 16.884 5 | $22\frac{3}{4}$ | 37.3065 | $27\frac{3}{4}$ | 59.2977 | | 81.2889 |
| 17.080 9 | 23 | 37.6992 | 28 | 59.6904 | | 81.6816 |
| 17.276 3 | $23\frac{1}{4}$ | 38.0919 | $28\frac{1}{4}$ | 60.0831 | | 82.0743 |
| 17.472 7 | $23\frac{1}{2}$ | 38.4846 | $28\frac{1}{2}$ | 60.4758 | | 82.467 |
| 17.668 1 | $23\frac{3}{4}$ | 38.8773 | $28\frac{3}{4}$ | 60.8685 | | 82.8597 |
| 17.864 5 | 24 | 39.27 | 29 | 61.2612 | | 83.2524 |
| 18.060 9 | $24\frac{1}{4}$ | 39.6627 | $29\frac{1}{4}$ | 61.6539 | | 83.6451 |
| 18.256 3 | $24\frac{1}{2}$ | 40.0554 | $29\frac{1}{2}$ | 62.0466 | | 84.0378 |
| 18.452 7 | $24\frac{3}{4}$ | 40.4481 | $29\frac{3}{4}$ | 62.4393 | | 84.4305 |
| 18.648 1 | 25 | 40.8408 | 30 | 62.832 | | 84.8232 |
| 18.844 5 | $25\frac{1}{4}$ | 41.2335 | $30\frac{1}{4}$ | 63.2247 | | 85.2159 |
| 19.040 9 | $25\frac{1}{2}$ | 41.6262 | $30\frac{1}{2}$ | 63.6174 | | 85.6086 |
| 19.236 3 | $25\frac{3}{4}$ | 42.0189 | $30\frac{3}{4}$ | 64.0101 | | 85.9999 |
| 19.432 7 | 26 | 42.4116 | 31 | 64.4028 | | 86.3926 |
| 19.628 1 | $26\frac{1}{4}$ | 42.8043 | $31\frac{1}{4}$ | 64.7955 | | 86.7853 |
| 19.824 5 | $26\frac{1}{2}$ | 43.197 | $31\frac{1}{2}$ | 65.1882 | | 87.178 |
| 20.020 9 | $26\frac{3}{4}$ | 43.5897 | $31\frac{3}{4}$ | 65.5809 | | 87.5707 |
| 20.216 3 | 27 | 43.9824 | 32 | 65.9736 | | 87.9634 |
| 20.412 7 | $27\frac{1}{4}$ | 44.3751 | $32\frac{1}{4}$ | 66.3663 | | 88.3561 |
| 20.608 1 | $27\frac{1}{2}$ | 44.7678 | $32\frac{1}{2}$ | 66.7588 | | 88.7488 |
| 20.804 5 | $27\frac{3}{4}$ | 45.1605 | $32\frac{3}{4}$ | 67.1515 | | 89.1415 |
| 21.000 9 | 28 | 45.5532 | 33 | 67.5442 | | 89.5342 |
| 21.196 3 | $28\frac{1}{4}$ | 45.9459 | $33\frac{1}{4}$ | 67.9369 | | 89.9269 |
| 21.392 7 | $28\frac{1}{2}$ | 46.3386 | $33\frac{1}{2}$ | 68.3296 | | 90.3196 |
| 21.588 1 | $28\frac{3}{4}$ | 46.7313 | $33\frac{3}{4}$ | 68.7223 | | 90.7123 |
| 21.784 5 | 29 | 47.124 | 34 | 69.115 | | 91.105 |
| 21.980 9 | $29\frac{1}{4}$ | 47.5167 | $34\frac{1}{4}$ | 69.5077 | | 91.4977 |
| 22.176 3 | $29\frac{1}{2}$ | 47.9094 | $34\frac{1}{2}$ | 69.9004 | | 91.8904 |
| 22.372 7 | $29\frac{3}{4}$ | 48.3021 | $34\frac{3}{4}$ | 70.2931 | | 92.2831 |
| 22.568 1 | 30 | 48.6948 | 35 | 70.6858 | | 92.6758 |
| 22.764 5 | $30\frac{1}{4}$ | 49.0875 | $35\frac{1}{4}$ | 71.0785 | | 93.0685 |
| 22.960 9 | $30\frac{1}{2}$ | 49.4802 | $35\frac{1}{2}$ | 71.4712 | | 93.4612 |
| 23.156 3 | $30\frac{3}{4}$ | 49.8729 | $35\frac{3}{4}$ | 71.8639 | | 93.8539 |
| 23.352 7 | 31 | 50.2656 | 36 | 72.2566 | | 94.2466 |
| 23.548 1 | $31\frac{1}{4}$ | 50.6583 | $36\frac{1}{4}$ | 72.6493 | | 94.6393 |
| 23.744 5 | $31\frac{1}{2}$ | 51.051 | $36\frac{1}{2}$ | 73.042 | | 95.032 |
| 23.940 9 | $31\frac{3}{4}$ | 51.4437 | $36\frac{3}{4}$ | 73.4347 | | 95.4247 |
| 24.136 3 | 32 | 51.8364 | 37 | 73.8274 | | 95.8174 |
| 24.332 7 | $32\frac{1}{4}$ | 52.2291 | $37\frac{1}{4}$ | 74.2201 | | 96.2101 |
| 24.528 1 | $32\frac{1}{2}$ | 52.6218 | $37\frac{1}{2}$ | 74.6128 | | 96.6028 |
| 24.724 5 | $32\frac{3}{4}$ | 53.0145 | $37\frac{3}{4}$ | 75.0055 | | 96.9955 |
| 24.920 9 | 33 | 53.4072 | 38 | 75.3982 | | 97.3882 |
| 25.116 3 | $33\frac{1}{4}$ | 53.7999 | $38\frac{1}{4}$ | 75.7909 | | 97.7809 |
| 25.312 7 | $33\frac{1}{2}$ | 54.1926 | $38\frac{1}{2}$ | 76.1836 | | 98.1736 |
| 25.508 1 | $33\frac{3}{4}$ | 54.5853 | $38\frac{3}{4}$ | 76.5763 | | 98.5663 |
| 25.704 5 | 34 | 54.978 | 39 | 76.969 | | 98.959 |
| 25.900 9 | $34\frac{1}{4}$ | 55.3707 | $39\frac{1}{4}$ | 77.3617 | | 99.3517 |
| 26.096 3 | $34\frac{1}{2}$ | 55.7634 | $39\frac{1}{2}$ | 77.7544 | | 99.7444 |
| 26.292 7 | $34\frac{3}{4}$ | 56.1561 | $39\frac{3}{4}$ | 78.1471 | | 100.1371 |
| 26.488 1 | 35 | 56.5488 | 40 | 78.5398 | | 100.5298 |
| 26.684 5 | $35\frac{1}{4}$ | 56.9415 | $40\frac{1}{4}$ | 78.9325 | | 100.9225 |
| 26.880 9 | $35\frac{1}{2}$ | 57.3342 | $40\frac{1}{2}$ | 79.3252 | | 101.3152 |
| 27.076 3 | $35\frac{3}{4}$ | 57.7269 | $40\frac{3}{4}$ | 79.7179 | | 101.7079 |
| 27.272 7 | 36 | 58.1196 | 41 | 80.1106 | | 102.1006 |
| 27.468 1 | $36\frac{1}{4}$ | 58 | | | | |

| DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | C |
|---------------|---------|---------------|----------|---------------|---------|---------------|----|
| 22 | 69.1152 | 29 | 91.1064 | 36 | 113.098 | 43 | 13 |
| $\frac{1}{8}$ | 69.5079 | $\frac{1}{8}$ | 91.4991 | $\frac{1}{8}$ | 113.49 | $\frac{1}{8}$ | 13 |
| $\frac{1}{4}$ | 69.9006 | $\frac{1}{4}$ | 91.8918 | $\frac{1}{4}$ | 113.883 | $\frac{1}{4}$ | 13 |
| $\frac{3}{8}$ | 70.2933 | $\frac{3}{8}$ | 92.2845 | $\frac{3}{8}$ | 114.276 | $\frac{3}{8}$ | 13 |
| $\frac{1}{2}$ | 70.686 | $\frac{1}{2}$ | 92.6772 | $\frac{1}{2}$ | 114.668 | $\frac{1}{2}$ | 13 |
| $\frac{5}{8}$ | 71.0787 | $\frac{5}{8}$ | 93.0699 | $\frac{5}{8}$ | 115.061 | $\frac{5}{8}$ | 13 |
| $\frac{3}{4}$ | 71.4714 | $\frac{3}{4}$ | 93.4626 | $\frac{3}{4}$ | 115.454 | $\frac{3}{4}$ | 13 |
| $\frac{7}{8}$ | 71.8641 | $\frac{7}{8}$ | 93.8553 | $\frac{7}{8}$ | 115.846 | $\frac{7}{8}$ | 13 |
| 23 | 72.2568 | 30 | 94.248 | 37 | 116.239 | 44 | 13 |
| $\frac{1}{8}$ | 72.6495 | $\frac{1}{8}$ | 94.6407 | $\frac{1}{8}$ | 116.632 | $\frac{1}{8}$ | 13 |
| $\frac{1}{4}$ | 73.0422 | $\frac{1}{4}$ | 95.0334 | $\frac{1}{4}$ | 117.025 | $\frac{1}{4}$ | 13 |
| $\frac{3}{8}$ | 73.4349 | $\frac{3}{8}$ | 95.4261 | $\frac{3}{8}$ | 117.417 | $\frac{3}{8}$ | 13 |
| $\frac{1}{2}$ | 73.8276 | $\frac{1}{2}$ | 95.8188 | $\frac{1}{2}$ | 117.81 | $\frac{1}{2}$ | 13 |
| $\frac{5}{8}$ | 74.2203 | $\frac{5}{8}$ | 96.2115 | $\frac{5}{8}$ | 118.203 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | 74.613 | $\frac{3}{4}$ | 96.6042 | $\frac{3}{4}$ | 118.595 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | 75.0057 | $\frac{7}{8}$ | 96.9969 | $\frac{7}{8}$ | 118.988 | $\frac{7}{8}$ | 14 |
| 24 | 75.3984 | 31 | 97.3896 | 38 | 119.381 | 45 | 14 |
| $\frac{1}{8}$ | 75.7911 | $\frac{1}{8}$ | 97.7823 | $\frac{1}{8}$ | 119.773 | $\frac{1}{8}$ | 14 |
| $\frac{1}{4}$ | 76.1838 | $\frac{1}{4}$ | 98.175 | $\frac{1}{4}$ | 120.166 | $\frac{1}{4}$ | 14 |
| $\frac{3}{8}$ | 76.5765 | $\frac{3}{8}$ | 98.5677 | $\frac{3}{8}$ | 120.559 | $\frac{3}{8}$ | 14 |
| $\frac{1}{2}$ | 76.9692 | $\frac{1}{2}$ | 98.9604 | $\frac{1}{2}$ | 120.952 | $\frac{1}{2}$ | 14 |
| $\frac{5}{8}$ | 77.3619 | $\frac{5}{8}$ | 99.3531 | $\frac{5}{8}$ | 121.344 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | 77.7546 | $\frac{3}{4}$ | 99.7458 | $\frac{3}{4}$ | 121.737 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | 78.1473 | $\frac{7}{8}$ | 100.1385 | $\frac{7}{8}$ | 122.13 | $\frac{7}{8}$ | 14 |
| 25 | 78.54 | 32 | 100.5312 | 39 | 122.522 | 46 | 14 |
| $\frac{1}{8}$ | 78.9327 | $\frac{1}{8}$ | 100.9239 | $\frac{1}{8}$ | 122.915 | $\frac{1}{8}$ | 14 |
| $\frac{1}{4}$ | 79.3254 | $\frac{1}{4}$ | 101.3166 | $\frac{1}{4}$ | 123.308 | $\frac{1}{4}$ | 14 |
| $\frac{3}{8}$ | 79.7181 | $\frac{3}{8}$ | 101.7093 | $\frac{3}{8}$ | 123.7 | $\frac{3}{8}$ | 14 |
| $\frac{1}{2}$ | 80.1108 | $\frac{1}{2}$ | 102.102 | $\frac{1}{2}$ | 124.093 | $\frac{1}{2}$ | 14 |
| $\frac{5}{8}$ | 80.5035 | $\frac{5}{8}$ | 102.4947 | $\frac{5}{8}$ | 124.486 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | 80.8962 | $\frac{3}{4}$ | 102.8874 | $\frac{3}{4}$ | 124.879 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | 81.2889 | $\frac{7}{8}$ | 103.2801 | $\frac{7}{8}$ | 125.271 | $\frac{7}{8}$ | 14 |
| 26 | 81.6816 | 33 | 103.673 | 40 | 125.664 | 47 | 14 |
| $\frac{1}{8}$ | 82.0743 | $\frac{1}{8}$ | 104.065 | $\frac{1}{8}$ | 126.057 | $\frac{1}{8}$ | 14 |
| $\frac{1}{4}$ | 82.467 | $\frac{1}{4}$ | 104.458 | $\frac{1}{4}$ | 126.449 | $\frac{1}{4}$ | 14 |
| $\frac{3}{8}$ | 82.8597 | $\frac{3}{8}$ | 104.851 | $\frac{3}{8}$ | 126.842 | $\frac{3}{8}$ | 14 |
| $\frac{1}{2}$ | 83.2524 | $\frac{1}{2}$ | 105.244 | $\frac{1}{2}$ | 127.235 | $\frac{1}{2}$ | 14 |
| $\frac{5}{8}$ | 83.6451 | $\frac{5}{8}$ | 105.636 | $\frac{5}{8}$ | 127.627 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | 84.0378 | $\frac{3}{4}$ | 106.029 | $\frac{3}{4}$ | 128.02 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | 84.4305 | $\frac{7}{8}$ | 106.422 | $\frac{7}{8}$ | 128.413 | $\frac{7}{8}$ | 14 |
| 27 | 84.8232 | 34 | 106.814 | 41 | 128.806 | 48 | 14 |
| $\frac{1}{8}$ | 85.2159 | $\frac{1}{8}$ | 107.207 | $\frac{1}{8}$ | 129.198 | $\frac{1}{8}$ | 14 |
| $\frac{1}{4}$ | 85.6086 | $\frac{1}{4}$ | 107.6 | $\frac{1}{4}$ | 129.591 | $\frac{1}{4}$ | 14 |
| $\frac{3}{8}$ | 86.0013 | $\frac{3}{8}$ | 107.992 | $\frac{3}{8}$ | 129.984 | $\frac{3}{8}$ | 14 |
| $\frac{1}{2}$ | 86.394 | $\frac{1}{2}$ | 108.385 | $\frac{1}{2}$ | 130.376 | $\frac{1}{2}$ | 14 |
| $\frac{5}{8}$ | 86.7867 | $\frac{5}{8}$ | 108.778 | $\frac{5}{8}$ | 130.769 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | 87.1794 | $\frac{3}{4}$ | 109.171 | $\frac{3}{4}$ | 131.162 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | 87.5721 | $\frac{7}{8}$ | 109.563 | $\frac{7}{8}$ | 131.554 | $\frac{7}{8}$ | 14 |
| 28 | 87.9648 | 35 | 109.956 | 42 | 131.947 | 49 | 14 |
| $\frac{1}{8}$ | 88.3575 | $\frac{1}{8}$ | 110.349 | $\frac{1}{8}$ | 132.34 | $\frac{1}{8}$ | 14 |
| $\frac{1}{4}$ | 88.7502 | $\frac{1}{4}$ | 110.741 | $\frac{1}{4}$ | 132.733 | $\frac{1}{4}$ | 14 |
| $\frac{3}{8}$ | 89.1429 | $\frac{3}{8}$ | 111.134 | $\frac{3}{8}$ | 133.125 | $\frac{3}{8}$ | 14 |
| $\frac{1}{2}$ | 89.5356 | $\frac{1}{2}$ | 111.527 | $\frac{1}{2}$ | 133.518 | $\frac{1}{2}$ | 14 |
| $\frac{5}{8}$ | 89.9283 | $\frac{5}{8}$ | 111.919 | $\frac{5}{8}$ | 133.911 | $\frac{5}{8}$ | 14 |
| $\frac{3}{4}$ | | $\frac{3}{4}$ | 112.312 | $\frac{3}{4}$ | 134.303 | $\frac{3}{4}$ | 14 |
| $\frac{7}{8}$ | | $\frac{7}{8}$ | 112.705 | $\frac{7}{8}$ | 134.696 | $\frac{7}{8}$ | 14 |

| CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. |
|---------|---------------|---------|---------------|---------|---------------|---------|
| 157.08 | 57 | 179.071 | 64 | 201.062 | 71 | 223.054 |
| 157.473 | $\frac{1}{8}$ | 179.464 | $\frac{1}{8}$ | 201.455 | $\frac{1}{8}$ | 223.446 |
| 157.865 | $\frac{1}{4}$ | 179.857 | $\frac{1}{4}$ | 201.848 | $\frac{1}{4}$ | 223.839 |
| 158.258 | $\frac{3}{8}$ | 180.249 | $\frac{3}{8}$ | 202.24 | $\frac{3}{8}$ | 224.232 |
| 158.651 | $\frac{1}{2}$ | 180.642 | $\frac{1}{2}$ | 202.633 | $\frac{1}{2}$ | 224.624 |
| 159.043 | $\frac{5}{8}$ | 181.035 | $\frac{5}{8}$ | 203.026 | $\frac{5}{8}$ | 225.017 |
| 159.436 | $\frac{3}{4}$ | 181.427 | $\frac{3}{4}$ | 203.419 | $\frac{3}{4}$ | 225.41 |
| 159.829 | $\frac{7}{8}$ | 181.82 | $\frac{7}{8}$ | 203.811 | $\frac{7}{8}$ | 225.802 |
| 160.222 | 58 | 182.213 | 65 | 204.204 | 72 | 226.195 |
| 160.614 | $\frac{1}{8}$ | 182.605 | $\frac{1}{8}$ | 204.597 | $\frac{1}{8}$ | 226.588 |
| 161.007 | $\frac{1}{4}$ | 182.998 | $\frac{1}{4}$ | 204.989 | $\frac{1}{4}$ | 226.981 |
| 161.4 | $\frac{3}{8}$ | 183.391 | $\frac{3}{8}$ | 205.382 | $\frac{3}{8}$ | 227.373 |
| 161.792 | $\frac{1}{2}$ | 183.784 | $\frac{1}{2}$ | 205.775 | $\frac{1}{2}$ | 227.766 |
| 162.185 | $\frac{5}{8}$ | 184.176 | $\frac{5}{8}$ | 206.167 | $\frac{5}{8}$ | 228.159 |
| 162.578 | $\frac{3}{4}$ | 184.569 | $\frac{3}{4}$ | 206.56 | $\frac{3}{4}$ | 228.551 |
| 162.97 | $\frac{7}{8}$ | 184.962 | $\frac{7}{8}$ | 206.953 | $\frac{7}{8}$ | 228.944 |
| 163.363 | 59 | 185.354 | 66 | 207.346 | 73 | 229.337 |
| 163.756 | $\frac{1}{8}$ | 185.747 | $\frac{1}{8}$ | 207.738 | $\frac{1}{8}$ | 229.729 |
| 164.149 | $\frac{1}{4}$ | 186.14 | $\frac{1}{4}$ | 208.131 | $\frac{1}{4}$ | 230.122 |
| 164.541 | $\frac{3}{8}$ | 186.532 | $\frac{3}{8}$ | 208.524 | $\frac{3}{8}$ | 230.515 |
| 164.934 | $\frac{1}{2}$ | 186.925 | $\frac{1}{2}$ | 208.916 | $\frac{1}{2}$ | 230.908 |
| 165.327 | $\frac{5}{8}$ | 187.318 | $\frac{5}{8}$ | 209.309 | $\frac{5}{8}$ | 231.3 |
| 165.719 | $\frac{3}{4}$ | 187.711 | $\frac{3}{4}$ | 209.702 | $\frac{3}{4}$ | 231.693 |
| 166.112 | $\frac{7}{8}$ | 188.103 | $\frac{7}{8}$ | 210.094 | $\frac{7}{8}$ | 232.086 |
| 166.505 | 60 | 188.496 | 67 | 210.487 | 74 | 232.478 |
| 166.897 | $\frac{1}{8}$ | 188.889 | $\frac{1}{8}$ | 210.88 | $\frac{1}{8}$ | 232.871 |
| 167.29 | $\frac{1}{4}$ | 189.281 | $\frac{1}{4}$ | 211.273 | $\frac{1}{4}$ | 233.264 |
| 167.683 | $\frac{3}{8}$ | 189.674 | $\frac{3}{8}$ | 211.665 | $\frac{3}{8}$ | 233.656 |
| 168.076 | $\frac{1}{2}$ | 190.067 | $\frac{1}{2}$ | 212.058 | $\frac{1}{2}$ | 234.049 |
| 168.468 | $\frac{5}{8}$ | 190.459 | $\frac{5}{8}$ | 212.451 | $\frac{5}{8}$ | 234.442 |
| 168.861 | $\frac{3}{4}$ | 190.852 | $\frac{3}{4}$ | 212.843 | $\frac{3}{4}$ | 234.835 |
| 169.254 | $\frac{7}{8}$ | 191.245 | $\frac{7}{8}$ | 213.236 | $\frac{7}{8}$ | 235.227 |
| 169.646 | 61 | 191.638 | 68 | 213.629 | 75 | 235.62 |
| 170.039 | $\frac{1}{8}$ | 192.03 | $\frac{1}{8}$ | 214.021 | $\frac{1}{8}$ | 236.013 |
| 170.432 | $\frac{1}{4}$ | 192.423 | $\frac{1}{4}$ | 214.414 | $\frac{1}{4}$ | 236.405 |
| 170.824 | $\frac{3}{8}$ | 192.816 | $\frac{3}{8}$ | 214.807 | $\frac{3}{8}$ | 236.798 |
| 171.217 | $\frac{1}{2}$ | 193.208 | $\frac{1}{2}$ | 215.2 | $\frac{1}{2}$ | 237.191 |
| 171.61 | $\frac{5}{8}$ | 193.601 | $\frac{5}{8}$ | 215.592 | $\frac{5}{8}$ | 237.583 |
| 172.003 | $\frac{3}{4}$ | 193.994 | $\frac{3}{4}$ | 215.985 | $\frac{3}{4}$ | 237.976 |
| 172.395 | $\frac{7}{8}$ | 194.386 | $\frac{7}{8}$ | 216.378 | $\frac{7}{8}$ | 238.369 |
| 172.788 | 62 | 194.779 | 69 | 216.77 | 76 | 238.762 |
| 173.181 | $\frac{1}{8}$ | 195.172 | $\frac{1}{8}$ | 217.163 | $\frac{1}{8}$ | 239.154 |
| 173.573 | $\frac{1}{4}$ | 195.565 | $\frac{1}{4}$ | 217.556 | $\frac{1}{4}$ | 239.547 |
| 173.966 | $\frac{3}{8}$ | 195.957 | $\frac{3}{8}$ | 217.948 | $\frac{3}{8}$ | 239.94 |
| 174.359 | $\frac{1}{2}$ | 196.35 | $\frac{1}{2}$ | 218.341 | $\frac{1}{2}$ | 240.332 |
| 174.751 | $\frac{5}{8}$ | 196.743 | $\frac{5}{8}$ | 218.734 | $\frac{5}{8}$ | 240.725 |
| 175.144 | $\frac{3}{4}$ | 197.135 | $\frac{3}{4}$ | 219.127 | $\frac{3}{4}$ | 241.118 |
| 175.537 | $\frac{7}{8}$ | 197.528 | $\frac{7}{8}$ | 219.519 | $\frac{7}{8}$ | 241.51 |
| 175.93 | 63 | 197.921 | 70 | 219.912 | 77 | 241.903 |
| 176.322 | $\frac{1}{8}$ | 198.313 | $\frac{1}{8}$ | 220.305 | | |
| 176.715 | $\frac{1}{4}$ | 198.706 | $\frac{1}{4}$ | 220.697 | | |
| 177.108 | $\frac{3}{8}$ | 199.099 | $\frac{3}{8}$ | 221.09 | | |
| 177.5 | $\frac{1}{2}$ | 199.492 | $\frac{1}{2}$ | 221.483 | | |
| 177.893 | $\frac{5}{8}$ | 199.884 | $\frac{5}{8}$ | 221.875 | | |
| 178.286 | $\frac{3}{4}$ | 200.277 | $\frac{3}{4}$ | 222.268 | | |
| 178.678 | $\frac{7}{8}$ | 200.67 | $\frac{7}{8}$ | 222.661 | | |

| DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. |
|---------------|---------|---------------|---------|---------------|---------|---------------|---------|
| 78 | 245.045 | 85 | 267.036 | 92 | 289.027 | 99 | 311.018 |
| $\frac{1}{8}$ | 245.437 | $\frac{1}{8}$ | 267.429 | $\frac{1}{8}$ | 289.42 | $\frac{1}{8}$ | 311.411 |
| $\frac{1}{4}$ | 245.83 | $\frac{1}{4}$ | 267.821 | $\frac{1}{4}$ | 289.813 | $\frac{1}{4}$ | 311.804 |
| $\frac{3}{8}$ | 246.223 | $\frac{3}{8}$ | 268.214 | $\frac{3}{8}$ | 290.205 | $\frac{3}{8}$ | 312.196 |
| $\frac{1}{2}$ | 246.616 | $\frac{1}{2}$ | 268.607 | $\frac{1}{2}$ | 290.598 | $\frac{1}{2}$ | 312.589 |
| $\frac{5}{8}$ | 247.008 | $\frac{5}{8}$ | 268.999 | $\frac{5}{8}$ | 290.991 | $\frac{5}{8}$ | 312.982 |
| $\frac{3}{4}$ | 247.401 | $\frac{3}{4}$ | 269.392 | $\frac{3}{4}$ | 291.383 | $\frac{3}{4}$ | 313.375 |
| $\frac{7}{8}$ | 247.794 | $\frac{7}{8}$ | 269.785 | $\frac{7}{8}$ | 291.776 | $\frac{7}{8}$ | 313.767 |
| 79 | 248.186 | 86 | 270.178 | 93 | 292.169 | 100 | 314.16 |
| $\frac{1}{8}$ | 248.579 | $\frac{1}{8}$ | 270.57 | $\frac{1}{8}$ | 292.562 | $\frac{1}{8}$ | 314.945 |
| $\frac{1}{4}$ | 248.972 | $\frac{1}{4}$ | 270.963 | $\frac{1}{4}$ | 292.954 | $\frac{1}{4}$ | 315.731 |
| $\frac{3}{8}$ | 249.364 | $\frac{3}{8}$ | 271.356 | $\frac{3}{8}$ | 293.347 | $\frac{3}{8}$ | 316.516 |
| $\frac{1}{2}$ | 249.757 | $\frac{1}{2}$ | 271.748 | $\frac{1}{2}$ | 293.74 | 101 | 317.302 |
| $\frac{5}{8}$ | 250.15 | $\frac{5}{8}$ | 272.141 | $\frac{5}{8}$ | 294.132 | $\frac{1}{4}$ | 318.087 |
| $\frac{3}{4}$ | 250.543 | $\frac{3}{4}$ | 272.534 | $\frac{3}{4}$ | 294.525 | $\frac{1}{2}$ | 318.872 |
| $\frac{7}{8}$ | 250.935 | $\frac{7}{8}$ | 272.926 | $\frac{7}{8}$ | 294.918 | $\frac{3}{4}$ | 319.658 |
| 80 | 251.328 | 87 | 273.319 | 94 | 295.31 | 102 | 320.443 |
| $\frac{1}{8}$ | 251.721 | $\frac{1}{8}$ | 273.712 | $\frac{1}{8}$ | 295.703 | $\frac{1}{8}$ | 321.229 |
| $\frac{1}{4}$ | 252.113 | $\frac{1}{4}$ | 274.105 | $\frac{1}{4}$ | 296.096 | $\frac{1}{4}$ | 322.014 |
| $\frac{3}{8}$ | 252.506 | $\frac{3}{8}$ | 274.497 | $\frac{3}{8}$ | 296.488 | $\frac{3}{8}$ | 322.799 |
| $\frac{1}{2}$ | 252.899 | $\frac{1}{2}$ | 274.89 | $\frac{1}{2}$ | 296.881 | 103 | 323.585 |
| $\frac{5}{8}$ | 253.291 | $\frac{5}{8}$ | 275.283 | $\frac{5}{8}$ | 297.274 | $\frac{1}{4}$ | 324.37 |
| $\frac{3}{4}$ | 253.684 | $\frac{3}{4}$ | 275.675 | $\frac{3}{4}$ | 297.667 | $\frac{1}{2}$ | 325.156 |
| $\frac{7}{8}$ | 254.077 | $\frac{7}{8}$ | 276.068 | $\frac{7}{8}$ | 298.059 | $\frac{3}{4}$ | 325.941 |
| 81 | 254.47 | 88 | 276.461 | 95 | 298.452 | 104 | 326.726 |
| $\frac{1}{8}$ | 254.862 | $\frac{1}{8}$ | 276.853 | $\frac{1}{8}$ | 298.845 | $\frac{1}{8}$ | 327.512 |
| $\frac{1}{4}$ | 255.255 | $\frac{1}{4}$ | 277.246 | $\frac{1}{4}$ | 299.237 | $\frac{1}{4}$ | 328.297 |
| $\frac{3}{8}$ | 255.648 | $\frac{3}{8}$ | 277.629 | $\frac{3}{8}$ | 299.63 | $\frac{3}{8}$ | 329.083 |
| $\frac{1}{2}$ | 256.04 | $\frac{1}{2}$ | 278.032 | $\frac{1}{2}$ | 300.023 | 105 | 329.868 |
| $\frac{5}{8}$ | 256.433 | $\frac{5}{8}$ | 278.424 | $\frac{5}{8}$ | 300.415 | $\frac{1}{4}$ | 330.653 |
| $\frac{3}{4}$ | 256.826 | $\frac{3}{4}$ | 278.817 | $\frac{3}{4}$ | 300.808 | $\frac{1}{2}$ | 331.439 |
| $\frac{7}{8}$ | 257.218 | $\frac{7}{8}$ | 279.21 | $\frac{7}{8}$ | 301.201 | $\frac{3}{4}$ | 332.224 |
| 82 | 257.611 | 89 | 279.602 | 96 | 301.594 | 106 | 333.01 |
| $\frac{1}{8}$ | 258.004 | $\frac{1}{8}$ | 279.995 | $\frac{1}{8}$ | 301.986 | $\frac{1}{8}$ | 333.795 |
| $\frac{1}{4}$ | 258.397 | $\frac{1}{4}$ | 280.388 | $\frac{1}{4}$ | 302.379 | $\frac{1}{4}$ | 334.58 |
| $\frac{3}{8}$ | 258.789 | $\frac{3}{8}$ | 280.78 | $\frac{3}{8}$ | 302.772 | $\frac{3}{8}$ | 335.366 |
| $\frac{1}{2}$ | 259.182 | $\frac{1}{2}$ | 281.173 | $\frac{1}{2}$ | 303.164 | 107 | 336.151 |
| $\frac{5}{8}$ | 259.575 | $\frac{5}{8}$ | 281.566 | $\frac{5}{8}$ | 303.557 | $\frac{1}{4}$ | 336.937 |
| $\frac{3}{4}$ | 259.967 | $\frac{3}{4}$ | 281.959 | $\frac{3}{4}$ | 303.95 | $\frac{1}{2}$ | 337.722 |
| $\frac{7}{8}$ | 260.36 | $\frac{7}{8}$ | 282.351 | $\frac{7}{8}$ | 304.342 | $\frac{3}{4}$ | 338.507 |
| 83 | 260.753 | 90 | 282.744 | 97 | 304.735 | 108 | 339.293 |
| $\frac{1}{8}$ | 261.145 | $\frac{1}{8}$ | 283.137 | $\frac{1}{8}$ | 305.128 | $\frac{1}{8}$ | 340.078 |
| $\frac{1}{4}$ | 261.538 | $\frac{1}{4}$ | 283.529 | $\frac{1}{4}$ | 305.521 | $\frac{1}{4}$ | 340.864 |
| $\frac{3}{8}$ | 261.931 | $\frac{3}{8}$ | 283.922 | $\frac{3}{8}$ | 305.913 | $\frac{3}{8}$ | 341.649 |
| $\frac{1}{2}$ | 262.324 | $\frac{1}{2}$ | 284.315 | $\frac{1}{2}$ | 306.306 | 109 | 342.434 |
| $\frac{5}{8}$ | 262.716 | $\frac{5}{8}$ | 284.707 | $\frac{5}{8}$ | 306.699 | $\frac{1}{4}$ | 343.22 |
| $\frac{3}{4}$ | 263.109 | $\frac{3}{4}$ | 285.1 | $\frac{3}{4}$ | 307.091 | $\frac{1}{2}$ | 344.005 |
| $\frac{7}{8}$ | 263.502 | $\frac{7}{8}$ | 285.493 | $\frac{7}{8}$ | 307.484 | $\frac{3}{4}$ | 344.791 |
| 84 | 263.894 | 91 | 285.886 | 98 | 307.877 | 110 | 345.576 |
| $\frac{1}{8}$ | 264.287 | $\frac{1}{8}$ | 286.278 | $\frac{1}{8}$ | 308.27 | $\frac{1}{8}$ | 346.361 |
| $\frac{1}{4}$ | 264.68 | $\frac{1}{4}$ | 286.671 | $\frac{1}{4}$ | 308.662 | $\frac{1}{4}$ | 347.147 |
| $\frac{3}{8}$ | 265.072 | $\frac{3}{8}$ | 287.064 | $\frac{3}{8}$ | 309.055 | $\frac{3}{8}$ | 347.932 |
| $\frac{1}{2}$ | 265.465 | $\frac{1}{2}$ | 287.456 | $\frac{1}{2}$ | 309.448 | 111 | 348.718 |
| $\frac{5}{8}$ | 265.858 | $\frac{5}{8}$ | 287.849 | $\frac{5}{8}$ | 309.84 | $\frac{1}{4}$ | 349.503 |
| $\frac{3}{4}$ | 266.251 | $\frac{3}{4}$ | 288.242 | $\frac{3}{4}$ | 310.233 | $\frac{1}{2}$ | 350.288 |
| $\frac{7}{8}$ | 266.643 | $\frac{7}{8}$ | 288.634 | $\frac{7}{8}$ | 310.626 | $\frac{3}{4}$ | 351.074 |

| CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. | DIAM. | CIRCUM. |
|---------|---------------|---------|---------------|---------|---------------|---------|
| 351.859 | 121 | 380.134 | 130 | 408.408 | 139 | 436.682 |
| 352.645 | $\frac{1}{4}$ | 380.919 | $\frac{1}{4}$ | 409.192 | $\frac{1}{4}$ | 437.467 |
| 353.43 | $\frac{1}{2}$ | 381.704 | $\frac{1}{2}$ | 409.979 | $\frac{1}{2}$ | 438.253 |
| 354.215 | $\frac{3}{4}$ | 382.49 | $\frac{3}{4}$ | 410.763 | $\frac{3}{4}$ | 439.037 |
| 355.001 | 122 | 383.275 | 131 | 411.55 | 140 | 439.824 |
| 355.786 | $\frac{1}{4}$ | 384.061 | $\frac{1}{4}$ | 412.334 | $\frac{1}{4}$ | 440.608 |
| 356.572 | $\frac{1}{2}$ | 384.846 | $\frac{1}{2}$ | 413.12 | $\frac{1}{2}$ | 441.395 |
| 357.357 | $\frac{3}{4}$ | 385.631 | $\frac{3}{4}$ | 413.905 | $\frac{3}{4}$ | 442.179 |
| 358.142 | 123 | 386.417 | 132 | 414.691 | 141 | 442.966 |
| 358.928 | $\frac{1}{4}$ | 387.202 | $\frac{1}{4}$ | 415.476 | $\frac{1}{4}$ | 443.75 |
| 359.713 | $\frac{1}{2}$ | 387.988 | $\frac{1}{2}$ | 416.262 | $\frac{1}{2}$ | 444.536 |
| 360.499 | $\frac{3}{4}$ | 388.773 | $\frac{3}{4}$ | 417.046 | $\frac{3}{4}$ | 445.321 |
| 361.284 | 124 | 389.558 | 133 | 417.833 | 142 | 446.107 |
| 362.069 | $\frac{1}{4}$ | 390.344 | $\frac{1}{4}$ | 418.617 | $\frac{1}{4}$ | 446.891 |
| 362.855 | $\frac{1}{2}$ | 391.129 | $\frac{1}{2}$ | 419.404 | $\frac{1}{2}$ | 447.678 |
| 363.64 | $\frac{3}{4}$ | 391.915 | $\frac{3}{4}$ | 420.188 | $\frac{3}{4}$ | 448.462 |
| 364.426 | 125 | 392.7 | 134 | 420.974 | 143 | 449.249 |
| 365.211 | $\frac{1}{4}$ | 393.484 | $\frac{1}{4}$ | 421.759 | $\frac{1}{4}$ | 450.033 |
| 365.996 | $\frac{1}{2}$ | 394.271 | $\frac{1}{2}$ | 422.545 | $\frac{1}{2}$ | 450.82 |
| 366.782 | $\frac{3}{4}$ | 395.055 | $\frac{3}{4}$ | 423.33 | $\frac{3}{4}$ | 451.604 |
| 367.567 | 126 | 395.842 | 135 | 424.116 | 144 | 452.39 |
| 368.353 | $\frac{1}{4}$ | 396.626 | $\frac{1}{4}$ | 424.9 | $\frac{1}{4}$ | 453.175 |
| 369.138 | $\frac{1}{2}$ | 397.412 | $\frac{1}{2}$ | 425.687 | $\frac{1}{2}$ | 453.961 |
| 369.923 | $\frac{3}{4}$ | 398.197 | $\frac{3}{4}$ | 426.471 | $\frac{3}{4}$ | 454.745 |
| 370.709 | 127 | 398.983 | 136 | 427.258 | 145 | 455.532 |
| 371.494 | $\frac{1}{4}$ | 399.768 | $\frac{1}{4}$ | 428.042 | $\frac{1}{4}$ | 456.316 |
| 372.28 | $\frac{1}{2}$ | 400.554 | $\frac{1}{2}$ | 428.828 | $\frac{1}{2}$ | 457.103 |
| 373.065 | $\frac{3}{4}$ | 401.338 | $\frac{3}{4}$ | 429.613 | 146 | 458.674 |
| 373.85 | 128 | 402.125 | 137 | 430.399 | $\frac{1}{2}$ | 460.244 |
| 374.636 | $\frac{1}{4}$ | 402.909 | $\frac{1}{4}$ | 431.183 | 147 | 461.815 |
| 375.421 | $\frac{1}{2}$ | 403.696 | $\frac{1}{2}$ | 431.97 | $\frac{1}{2}$ | 463.386 |
| 376.207 | $\frac{3}{4}$ | 404.48 | $\frac{3}{4}$ | 432.754 | 148 | 464.957 |
| 376.992 | 129 | 405.266 | 138 | 433.541 | $\frac{1}{2}$ | 466.528 |
| 377.777 | $\frac{1}{4}$ | 406.051 | $\frac{1}{4}$ | 434.325 | 149 | 468.098 |
| 378.563 | $\frac{1}{2}$ | 406.837 | $\frac{1}{2}$ | 435.112 | $\frac{1}{2}$ | 469.669 |
| 379.348 | $\frac{3}{4}$ | 407.622 | $\frac{3}{4}$ | 435.896 | 150 | 471.24 |

note Circumference of a Diameter greater than any in preceding Table.

—Divide dimension by two, three, four, etc., if practicable to do so, reduced to a diameter in table.

ular circumference for this dimension, multiply it by divisor, as it was divided, and product will give circumference required.

2.—What is circumference for a diameter of 1050?

1 = 150; **tab. circum.**, 150 = 471.24, which $\times 7 = 3298.68$, **circumf**

note Circumference of a Diameter in Feet, Inches, etc., by preceding Table.

—Reduce dimension to inches or eighths, as the case may require in that term from table for that number.

this number by 8 if it is in eighths, and by 12 if in inch. will give circumference in feet.

EXAMPLE.—Required circumference of a circle of 1 foot 6.375 ins.

1 foot 6.375 ins. = 18.375 ins. = 147 eighths. Circum. of 147 = 461.815, which $\div 8$ = 57.727 ins.; and by 12 = 4.8106 feet.

To Compute Circumference for a Diameter composed of an Integer and a Fraction.

RULE.—Double, treble, or quadruple dimension given, until fraction is increased to a whole number or to one of those in the table, as $\frac{1}{8}$, $\frac{1}{4}$, etc., provided it is practicable to do so.

Take circumference for this diameter; and if it is double of that for which circumference is required, take one half of it; if treble, take one third of it; and if quadruple, one fourth of it.

EXAMPLE.—Required circumference of 2.218 75 ins.

2.218 75 $\times 2$ = 4.4375, which $\times 2$ = 8.875; circum. for which = 27.8817, which $\div 4$ = 6.9704 ins.

When Diameter consists of Integers and Fractions contained in Table.

RULE.—Point a decimal to a diameter in table, take circumference from table, and add as many figures to the right as there are figures cut off.

EXAMPLE.—What is circumference of a circle 9675 feet in diameter?

Circumference of 96.75 = 303.95; hence, circumference of 9675 = 30 395 feet.

To Ascertain Circumference for a Diameter, as 500, 5000, etc., not contained in Table.

Rule.—Take circumference of 5 or 50 from table, and add the excess of ciphers to the result.

EXAMPLE.—What is circumference of a circle 8000 feet in diameter?

Circumference of 80 = 251.38; hence, circumference of 8000 = 25 138 feet.

To Compute Circumference of a Circle by Logarithms.

RULE.—To log. of diameter add .497 15 (log. of 3.1416), and sum is log. of circumference, from which take number.

EXAMPLE.—What is circumference of a circle 1200 feet in diameter?

Log. 1200 = 3.079 18 + .497 15 = 3 576 33, and number for which = 3769.92 feet.

Circumferences of Birmingham Wire Gauge.

| Diam. | Circum. | Diam. | Circum. | Diam. | Circum. | Diam. | Circum. |
|-------|---------|-------|---------|-------|---------|-------|---------|
| No. | Ins. | No. | Ins. | No. | Ins. | No. | Ins. |
| 1 | .942 48 | 10 | .420 97 | 19 | .131 95 | 28 | .043 98 |
| 2 | .892 21 | 11 | .376 99 | 20 | .109 95 | 29 | .040 84 |
| 3 | .813 67 | 12 | .342 43 | 21 | .100 53 | 30 | .037 7 |
| 4 | .747 7 | 13 | .298 45 | 22 | .087 96 | 31 | .031 41 |
| 5 | .691 15 | 14 | .260 75 | 23 | .078 54 | 32 | .028 27 |
| 6 | .637 74 | 15 | .226 19 | 24 | .069 11 | 33 | .025 13 |
| 7 | .585 49 | 16 | .204 2 | 25 | .062 83 | 34 | .021 99 |
| 8 | .536 36 | 17 | .182 21 | 26 | .056 55 | 35 | .015 71 |
| 9 | .491 7 | 18 | .153 94 | 27 | .050 26 | 36 | .012 57 |

AREAS AND CIRCUMFERENCES OF CIRCLES. 24.

Areas and Circumferences. (Advancing by Tenths.)

| AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|----------|----------|-------|---------|---------|
| .007854 | .314 16 | .6 | 24.6301 | 17.593 |
| .031 416 | .628 32 | .7 | 25.5176 | 17.9071 |
| .070 686 | .942 48 | .8 | 26.4209 | 18.2213 |
| .125 664 | 1.256 6 | .9 | 27.3398 | 18.5354 |
| .196 35 | 1.570 8 | 6 | 28.2744 | 18.8496 |
| .282 744 | 1.885 | .1 | 29.2247 | 19.1638 |
| .384 846 | 2.199 1 | .2 | 30.1908 | 19.4779 |
| .502 656 | 2.513 3 | .3 | 31.1725 | 19.7921 |
| .636 174 | 2.827 4 | .4 | 32.17 | 20.1062 |
| .785 4 | 3.141 6 | .5 | 33.1831 | 20.4204 |
| .950 3 | 3.455 8 | .6 | 34.212 | 20.7346 |
| 1.131 | 3.769 9 | .7 | 35.2566 | 21.0487 |
| 1.327 3 | 4.084 1 | .8 | 36.3169 | 21.3629 |
| 1.539 4 | 4.398 2 | .9 | 37.3929 | 21.677 |
| 1.767 1 | 4.712 4 | 7 | 38.4846 | 21.9912 |
| 2.010 6 | 5.026 6 | .1 | 39.592 | 22.3054 |
| 2.269 8 | 5.340 7 | .2 | 40.7151 | 22.6195 |
| 2.544 7 | 5.654 9 | .3 | 41.854 | 22.9337 |
| 2.835 3 | 5.969 | .4 | 43.0085 | 23.2478 |
| 3.141 6 | 6.283 2 | .5 | 44.1787 | 23.562 |
| 3.463 6 | 6.597 4 | .6 | 45.3647 | 23.8762 |
| 3.801 3 | 6.911 5 | .7 | 46.5664 | 24.1903 |
| 4.154 8 | 7.225 7 | .8 | 47.7837 | 24.5045 |
| 4.523 9 | 7.539 8 | .9 | 49.0168 | 24.8186 |
| 4.908 7 | 7.854 | 8 | 50.2656 | 25.1328 |
| 5.309 3 | 8.168 2 | .1 | 51.5301 | 25.447 |
| 5.725 6 | 8.482 3 | .2 | 52.8103 | 25.7611 |
| 6.157 5 | 8.796 5 | .3 | 54.1062 | 26.0753 |
| 6.605 2 | 9.110 6 | .4 | 55.4178 | 26.3894 |
| 7.068 6 | 9.424 8 | .5 | 56.7451 | 26.7036 |
| 7.547 7 | 9.739 | .6 | 58.0882 | 27.0178 |
| 8.042 5 | 10.053 1 | .7 | 59.4469 | 27.3319 |
| 8.553 | 10.367 3 | .8 | 60.8214 | 27.6461 |
| 9.079 2 | 10.681 4 | .9 | 62.2115 | 27.9602 |
| 9.621 1 | 10.995 6 | 9 | 63.6174 | 28.2744 |
| 10.178 8 | 11.309 8 | .1 | 65.039 | 28.5886 |
| 10.752 1 | 11.623 9 | .2 | 66.4763 | 28.9027 |
| 11.341 2 | 11.938 1 | .3 | 67.9292 | 29.2169 |
| 11.945 9 | 12.252 2 | .4 | 69.3979 | 29.531 |
| 12.566 4 | 12.566 4 | .5 | 70.8823 | 29.8452 |
| 13.202 6 | 12.880 6 | .6 | 72.3825 | 30.1594 |
| 13.854 5 | 13.194 7 | .7 | 73.8983 | 30.4735 |
| 14.522 | 13.508 9 | .8 | 75.4298 | 30.7877 |
| 15.205 3 | 13.823 | .9 | 76.9771 | 31.1018 |
| 15.904 3 | 14.137 2 | 10 | 78.54 | 31 |
| 16.619 1 | 14.451 4 | .1 | 80.1187 | |
| 17.349 5 | 14.765 5 | .2 | 81.713 | |
| 18.095 6 | 15.079 7 | .3 | 83.3231 | |
| 18.857 5 | 15.393 8 | .4 | 84.9489 | |
| 19.635 | 15.708 | .5 | 86.5903 | |
| 20.428 3 | 16.022 2 | .6 | 88.2475 | |
| 21.237 2 | 16.336 3 | .7 | 89.9204 | |
| 22.061 9 | 16.650 5 | .8 | 91.6091 | |
| 22.902 3 | 16.964 6 | .9 | 93.3134 | |
| 23.758 3 | 17.278 8 | | | |

246 AREAS AND CIRCUMFERENCES OF CIRCLES.

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|-------|-----------|----------|-------|-----------|----------|
| 33 | 855.3006 | 103.6728 | .5 | 1164.1591 | 120.9516 |
| .1 | 860.4921 | 103.987 | .6 | 1170.2146 | 121.2658 |
| .2 | 865.6993 | 104.3011 | .7 | 1176.2857 | 121.5799 |
| .3 | 870.9222 | 104.6153 | .8 | 1182.3726 | 121.8941 |
| .4 | 876.1608 | 104.9294 | .9 | 1188.4751 | 122.2082 |
| .5 | 881.4151 | 105.2436 | 39 | 1194.5934 | 122.5224 |
| .6 | 886.6852 | 105.5578 | .1 | 1200.7274 | 122.8366 |
| .7 | 891.9709 | 105.8719 | .2 | 1206.8771 | 123.1507 |
| .8 | 897.2724 | 106.1861 | .3 | 1213.0424 | 123.4649 |
| .9 | 902.5895 | 106.5002 | .4 | 1219.2235 | 123.779 |
| 34 | 907.9224 | 106.8144 | .5 | 1225.4203 | 124.0932 |
| .1 | 913.271 | 107.1286 | .6 | 1231.6329 | 124.4074 |
| .2 | 918.6353 | 107.4427 | .7 | 1237.8611 | 124.7215 |
| .3 | 924.0152 | 107.7569 | .8 | 1244.105 | 125.0357 |
| .4 | 929.4109 | 108.071 | .9 | 1250.3647 | 125.3498 |
| .5 | 934.8223 | 108.3852 | 40 | 1256.64 | 125.664 |
| .6 | 940.2495 | 108.6994 | .1 | 1262.9311 | 125.9782 |
| .7 | 945.6923 | 109.0135 | .2 | 1269.2378 | 126.2923 |
| .8 | 951.1508 | 109.3277 | .3 | 1275.5603 | 126.6065 |
| .9 | 956.6251 | 109.6418 | .4 | 1281.8985 | 126.9206 |
| 35 | 962.115 | 109.956 | .5 | 1288.2523 | 127.2348 |
| .1 | 967.6207 | 110.2702 | .6 | 1294.6219 | 127.549 |
| .2 | 973.142 | 110.5843 | .7 | 1301.0072 | 127.8631 |
| .3 | 978.6791 | 110.8985 | .8 | 1307.4083 | 128.1773 |
| .4 | 984.2319 | 111.2126 | .9 | 1313.825 | 128.4914 |
| .5 | 989.8003 | 111.5268 | 41 | 1320.2574 | 128.8056 |
| .6 | 995.3845 | 111.841 | .1 | 1326.7055 | 129.1198 |
| .7 | 1000.9844 | 112.1551 | .2 | 1333.1694 | 129.4339 |
| .8 | 1006.6001 | 112.4693 | .3 | 1339.6489 | 129.7481 |
| .9 | 1012.2314 | 112.7834 | .4 | 1346.1442 | 130.0622 |
| 36 | 1017.8784 | 113.0976 | .5 | 1352.6551 | 130.3764 |
| .1 | 1023.5411 | 113.4118 | .6 | 1359.1818 | 130.6906 |
| .2 | 1029.2196 | 113.7259 | .7 | 1365.7242 | 131.0047 |
| .3 | 1034.9137 | 114.0401 | .8 | 1372.2823 | 131.3189 |
| .4 | 1040.6236 | 114.3542 | .9 | 1378.8561 | 131.633 |
| .5 | 1046.3491 | 114.6684 | 42 | 1385.4456 | 131.9472 |
| .6 | 1052.0904 | 114.9826 | .1 | 1392.0508 | 132.2614 |
| .7 | 1057.8474 | 115.2967 | .2 | 1398.6717 | 132.5755 |
| .8 | 1063.6201 | 115.6109 | .3 | 1405.3084 | 132.8897 |
| .9 | 1069.4085 | 115.925 | .4 | 1411.9607 | 133.2038 |
| 37 | 1075.2126 | 116.2392 | .5 | 1418.6287 | 133.518 |
| .1 | 1081.0324 | 116.5534 | .6 | 1425.3125 | 133.8322 |
| .2 | 1086.8679 | 116.8675 | .7 | 1432.012 | 134.1463 |
| .3 | 1092.7192 | 117.1817 | .8 | 1438.7271 | 134.4605 |
| .4 | 1098.5861 | 117.4958 | .9 | 1445.458 | 134.7746 |
| .5 | 1104.4687 | 117.81 | 43 | 1452.2046 | 135.0888 |
| .6 | 1110.3671 | 118.1242 | .1 | 1458.9669 | 135.403 |
| .7 | 1116.2812 | 118.4383 | .2 | 1465.7449 | 135.7171 |
| .8 | 1122.2109 | 118.7525 | .3 | 1472.5386 | 136.0313 |
| .9 | 1128.1564 | 119.0666 | .4 | 1479.348 | 136.3454 |
| 38 | 1134.1176 | 119.3808 | .5 | 1486.1731 | 136.6596 |
| | 1140.0945 | 119.695 | .6 | 1493.014 | 136.9738 |
| | 1146.0871 | 120.0091 | .7 | 1499.8705 | 137.2879 |
| | 1152.0954 | 120.3233 | .8 | 1506.7428 | 137.6021 |
| | 1158.1194 | 120.6374 | .9 | 1513.6307 | 137.9162 |

AREAS AND CIRCUMFERENCES OF CIRCLES. 247

| | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|---|-----------|----------|-------|-----------|----------|
| | 1520.5344 | 138.2304 | .5 | 1924.4263 | 155.5092 |
| 1 | 1527.4538 | 138.5446 | .6 | 1932.2097 | 155.8234 |
| 2 | 1534.3889 | 138.8587 | .7 | 1940.0087 | 156.1375 |
| 3 | 1541.3396 | 139.1729 | .8 | 1947.8234 | 156.4517 |
| 4 | 1548.3061 | 139.487 | .9 | 1955.6539 | 156.7658 |
| 5 | 1555.2883 | 139.8012 | 50 | 1963.5 | 157.08 |
| 5 | 1562.2863 | 140.1154 | .1 | 1971.3619 | 157.3942 |
| 7 | 1569.2999 | 140.4295 | .2 | 1979.2394 | 157.7083 |
| 3 | 1576.3292 | 140.7437 | .3 | 1987.1327 | 158.0225 |
| 9 | 1583.3743 | 141.0578 | .4 | 1995.0417 | 158.3366 |
| | 1590.435 | 141.372 | .5 | 2002.9663 | 158.6509 |
| 1 | 1597.5115 | 141.6862 | .6 | 2010.9067 | 158.965 |
| 2 | 1604.6036 | 142.0003 | .7 | 2018.8628 | 159.2791 |
| 3 | 1611.7115 | 142.3145 | .8 | 2026.8347 | 159.5933 |
| 4 | 1618.8351 | 142.6286 | .9 | 2034.8222 | 159.9074 |
| 5 | 1625.9743 | 142.9428 | 51 | 2042.8254 | 160.2216 |
| 5 | 1633.1293 | 143.257 | .1 | 2050.8443 | 160.5358 |
| 7 | 1640.3 | 143.5711 | .2 | 2058.879 | 160.8499 |
| 3 | 1647.4865 | 143.8853 | .3 | 2066.9293 | 161.1641 |
| 9 | 1654.6886 | 144.1994 | .4 | 2074.9954 | 161.4782 |
| | 1661.9064 | 144.5136 | .5 | 2083.0771 | 161.7924 |
| 1 | 1669.1399 | 144.8278 | .6 | 2091.1746 | 162.1066 |
| 1 | 1676.3892 | 145.1419 | .7 | 2099.2878 | 162.4207 |
| 3 | 1683.6541 | 145.4561 | .8 | 2107.4167 | 162.7349 |
| 4 | 1690.9348 | 145.7702 | .9 | 2115.5613 | 163.049 |
| 5 | 1698.2311 | 146.0844 | 52 | 2123.7216 | 163.3632 |
| 5 | 1705.5432 | 146.3986 | .1 | 2131.8976 | 163.6774 |
| 7 | 1712.871 | 146.7127 | .2 | 2140.0893 | 163.9915 |
| 3 | 1720.2145 | 147.0269 | .3 | 2148.2968 | 164.3057 |
| 9 | 1727.5737 | 147.341 | .4 | 2156.5199 | 164.6198 |
| | 1734.9486 | 147.6552 | .5 | 2164.7587 | 164.934 |
| 1 | 1742.3392 | 147.9694 | .6 | 2173.0133 | 165.2482 |
| 1 | 1749.7455 | 148.2835 | .7 | 2181.2836 | 165.5623 |
| 3 | 1757.1676 | 148.5977 | .8 | 2189.5695 | 165.8765 |
| 4 | 1764.6053 | 148.9118 | .9 | 2197.8712 | 166.1906 |
| 5 | 1772.0587 | 149.226 | 53 | 2206.1886 | 166.5048 |
| 5 | 1779.5279 | 149.5402 | .1 | 2214.5217 | 166.819 |
| 7 | 1787.0128 | 149.8543 | .2 | 2222.8705 | 167.1331 |
| 3 | 1794.5133 | 150.1685 | .3 | 2231.235 | 167.4473 |
| 4 | 1802.0296 | 150.4826 | .4 | 2239.6152 | 167.7614 |
| 5 | 1809.5616 | 150.7968 | .5 | 2248.0111 | 168.0756 |
| 5 | 1817.1093 | 151.111 | .6 | 2256.4228 | 168.3898 |
| 7 | 1824.6727 | 151.4251 | .7 | 2264.8501 | 168.7039 |
| 3 | 1832.2518 | 151.7393 | .8 | 2273.2932 | 169.0181 |
| 9 | 1839.8466 | 152.0534 | .9 | 2281.7519 | 169.3322 |
| | 1847.4571 | 152.3676 | 54 | 2290.2264 | 169.6464 |
| 1 | 1855.0834 | 152.6818 | .1 | 2298.7166 | 169.9606 |
| 1 | 1862.7253 | 152.9959 | .2 | 2307.2225 | 170.2748 |
| 3 | 1870.383 | 153.3101 | .3 | 2315.744 | 170.589 |
| 4 | 1878.0563 | 153.6242 | .4 | 2324.2813 | 170.9032 |
| 5 | 1885.7454 | 153.9384 | .5 | 2332.8343 | 171.2174 |
| 5 | 1893.4502 | 154.2526 | .6 | 2341.4031 | 171.5316 |
| 7 | 1901.1707 | 154.5667 | .7 | 2349.9875 | 171.8458 |
| 3 | 1908.9068 | 154.8809 | .8 | 2358.5876 | 172.16 |
| 9 | 1916.6587 | 155.195 | .9 | 2367.2035 | 172.4742 |

AREAS AND CIRCUMFERENCES OF CIRCLES. 251

| R.A.N. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|--------|-----------|----------|-------|-----------|----------|
| 8 | 6082.1376 | 276.4608 | .5 | 6866.1631 | 293.7396 |
| .1 | 6095.9685 | 276.775 | .6 | 6880.858 | 294.0538 |
| .2 | 6109.8151 | 277.0891 | .7 | 6895.5685 | 294.3679 |
| .3 | 6123.6774 | 277.4033 | .8 | 6910.2948 | 294.6821 |
| .4 | 6137.5554 | 277.7174 | .9 | 6925.0367 | 294.9962 |
| .5 | 6151.4491 | 278.0316 | 94 | 6939.7944 | 295.3104 |
| .6 | 6165.3586 | 278.3458 | .1 | 6954.5678 | 295.6246 |
| .7 | 6179.2837 | 278.6599 | .2 | 6969.3569 | 295.9387 |
| .8 | 6193.2246 | 278.9741 | .3 | 6984.1616 | 296.2529 |
| .9 | 6207.1811 | 279.2882 | .4 | 6998.9821 | 296.567 |
| 9 | 6221.1534 | 279.6024 | .5 | 7013.8183 | 296.8812 |
| .1 | 6235.1414 | 279.9166 | .6 | 7028.6703 | 297.1954 |
| .2 | 6249.1451 | 280.2307 | .7 | 7043.5379 | 297.5095 |
| .3 | 6263.1644 | 280.5449 | .8 | 7058.4212 | 297.8237 |
| .4 | 6277.1995 | 280.859 | .9 | 7073.3203 | 298.1378 |
| .5 | 6291.2503 | 281.1732 | 95 | 7088.235 | 298.452 |
| .6 | 6305.3169 | 281.4874 | .1 | 7103.1655 | 298.7662 |
| .7 | 6319.3991 | 281.8015 | .2 | 7118.1116 | 299.0803 |
| .8 | 6333.497 | 282.1157 | .3 | 7133.0735 | 299.3945 |
| .9 | 6347.6107 | 282.4298 | .4 | 7148.0511 | 299.7086 |
| 0 | 6361.74 | 282.744 | .5 | 7163.0443 | 300.0228 |
| .1 | 6375.8851 | 283.0582 | .6 | 7178.0533 | 300.337 |
| .2 | 6390.0458 | 283.3723 | .7 | 7193.078 | 300.6511 |
| .3 | 6404.2223 | 283.6865 | .8 | 7208.1185 | 300.9653 |
| .4 | 6418.4144 | 284.0006 | .9 | 7223.1746 | 301.2794 |
| .5 | 6432.6223 | 284.3148 | 96 | 7238.2464 | 301.5936 |
| .6 | 6446.8459 | 284.629 | .1 | 7253.3339 | 301.9078 |
| .7 | 6461.0852 | 284.9431 | .2 | 7268.4372 | 302.2219 |
| .8 | 6475.3403 | 285.2573 | .3 | 7283.5561 | 302.5361 |
| .9 | 6489.611 | 285.5714 | .4 | 7298.6908 | 302.8502 |
| 1 | 6503.8974 | 285.8856 | .5 | 7313.8411 | 303.1644 |
| .1 | 6518.1995 | 286.1998 | .6 | 7329.0072 | 303.4786 |
| .2 | 6532.5174 | 286.5139 | .7 | 7344.189 | 303.7927 |
| .3 | 6546.8509 | 286.8281 | .8 | 7359.3865 | 304.1069 |
| .4 | 6561.2002 | 287.1422 | .9 | 7374.5997 | 304.421 |
| .5 | 6575.5651 | 287.4564 | 97 | 7389.8286 | 304.7352 |
| .6 | 6589.9458 | 287.7706 | .1 | 7405.0732 | 305.0494 |
| .7 | 6604.3422 | 288.0847 | .2 | 7420.3335 | 305.3635 |
| .8 | 6618.7543 | 288.3989 | .3 | 7435.6096 | 305.6777 |
| .9 | 6633.1821 | 288.713 | .4 | 7450.9013 | 305.9918 |
| 1 | 6647.6256 | 289.0272 | .5 | 7466.2087 | 306.306 |
| .1 | 6662.0848 | 289.3414 | .6 | 7481.5319 | 306.6202 |
| .2 | 6676.5598 | 289.6555 | .7 | 7496.8708 | 306.9343 |
| .3 | 6691.0504 | 289.9697 | .8 | 7512.2253 | 307.2485 |
| .4 | 6705.5567 | 290.2838 | .9 | 7527.5956 | 307.5626 |
| .5 | 6720.0787 | 290.598 | 98 | 7542.9816 | 307.8768 |
| .6 | 6734.6165 | 290.9121 | .1 | 7558.3833 | 308.191 |
| .7 | 6749.17 | 291.2263 | .2 | 7573.8007 | 308.5052 |
| .8 | 6763.7391 | 291.5405 | .3 | 7589.2338 | 308.8194 |
| .9 | 6778.324 | 291.8546 | .4 | 7604.6826 | 309.1336 |
| 1 | 6792.9246 | 292.1688 | .5 | 7620.1471 | 309.4478 |
| .1 | 6807.5409 | 292.483 | .6 | 7635.6274 | 309.762 |
| .2 | 6822.1729 | 292.7971 | .7 | 7651.1233 | 310.0762 |
| .3 | 6836.8206 | 293.1113 | .8 | 7666.635 | 310.3904 |
| .4 | 6851.484 | 293.4254 | .9 | 7682.1623 | 310.7046 |

48 AREAS AND CIRCUMFERENCES OF CIRCLES.

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|-------|-----------|----------|-------|-----------|----------|
| 55 | 2375.835 | 172.788 | .5 | 2874.7603 | 190.0668 |
| .1 | 2384.4823 | 173.1022 | .6 | 2884.2715 | 190.381 |
| .2 | 2393.1452 | 173.4163 | .7 | 2893.7984 | 190.6951 |
| .3 | 2401.8239 | 173.7305 | .8 | 2903.3411 | 191.0093 |
| .4 | 2410.5183 | 174.0446 | .9 | 2912.8994 | 191.3234 |
| .5 | 2419.2283 | 174.3588 | 61 | 2922.4734 | 191.6376 |
| .6 | 2427.9541 | 174.673 | .1 | 2932.0631 | 191.9518 |
| .7 | 2436.6957 | 174.9871 | .2 | 2941.6686 | 192.2659 |
| .8 | 2445.4529 | 175.3013 | .3 | 2951.2897 | 192.5801 |
| .9 | 2454.2258 | 175.6154 | .4 | 2960.9266 | 192.8942 |
| 56 | 2463.0144 | 175.9296 | .5 | 2970.5791 | 193.2084 |
| .1 | 2471.8187 | 176.2438 | .6 | 2980.2474 | 193.5226 |
| .2 | 2480.6388 | 176.5579 | .7 | 2989.9314 | 193.8367 |
| .3 | 2489.4745 | 176.8721 | .8 | 2999.6311 | 194.1509 |
| .4 | 2498.326 | 177.1862 | .9 | 3009.3465 | 194.465 |
| .5 | 2507.1931 | 177.5004 | 62 | 3019.0776 | 194.7792 |
| .6 | 2516.076 | 177.8146 | .1 | 3028.8244 | 195.0934 |
| .7 | 2524.9736 | 178.1287 | .2 | 3038.5869 | 195.4075 |
| .8 | 2533.8889 | 178.4429 | .3 | 3048.3652 | 195.7217 |
| .9 | 2542.8189 | 178.757 | .4 | 3058.1591 | 196.0358 |
| 57 | 2551.7646 | 179.0712 | .5 | 3067.9687 | 196.35 |
| .1 | 2560.726 | 179.3854 | .6 | 3077.7941 | 196.6642 |
| .2 | 2569.7031 | 179.6995 | .7 | 3087.6341 | 196.9783 |
| .3 | 2578.696 | 180.0137 | .8 | 3097.4919 | 197.2925 |
| .4 | 2587.7045 | 180.3278 | .9 | 3107.3644 | 197.6066 |
| .5 | 2596.7287 | 180.642 | 63 | 3117.2526 | 197.9208 |
| .6 | 2605.7687 | 180.9562 | .1 | 3127.1565 | 198.235 |
| .7 | 2614.8244 | 181.2703 | .2 | 3137.0761 | 198.5491 |
| .8 | 2623.8957 | 181.5845 | .3 | 3147.0114 | 198.8633 |
| .9 | 2632.9828 | 181.8986 | .4 | 3156.9624 | 199.1774 |
| 58 | 2642.0856 | 182.2128 | .5 | 3166.9291 | 199.4916 |
| .1 | 2651.2041 | 182.527 | .6 | 3176.9116 | 199.8058 |
| .2 | 2660.3383 | 182.8411 | .7 | 3186.9097 | 200.1199 |
| .3 | 2669.4882 | 183.1553 | .8 | 3196.9236 | 200.4341 |
| .4 | 2678.6538 | 183.4694 | .9 | 3206.9531 | 200.7482 |
| .5 | 2687.8351 | 183.7836 | 64 | 3216.9984 | 201.0624 |
| .6 | 2697.0322 | 184.0978 | .1 | 3227.0594 | 201.3766 |
| .7 | 2706.2449 | 184.4119 | .2 | 3237.1361 | 201.6907 |
| .8 | 2715.4734 | 184.7261 | .3 | 3247.2284 | 202.0049 |
| .9 | 2724.7175 | 185.0402 | .4 | 3257.3365 | 202.319 |
| 59 | 2733.9774 | 185.3544 | .5 | 3267.4603 | 202.6332 |
| .1 | 2743.253 | 185.6686 | .6 | 3277.5999 | 202.9474 |
| .2 | 2752.5443 | 185.9827 | .7 | 3287.7551 | 203.2615 |
| .3 | 2761.8512 | 186.2969 | .8 | 3297.9261 | 203.5757 |
| .4 | 2771.1739 | 186.611 | .9 | 3308.1127 | 203.8898 |
| 5 | 2780.5123 | 186.9252 | 65 | 3318.315 | 204.204 |
| | 2789.8665 | 187.2394 | .1 | 3328.5331 | 204.5182 |
| | 2799.2363 | 187.5535 | .2 | 3338.7668 | 204.8323 |
| | 2808.6218 | 187.8677 | .3 | 3349.0163 | 205.1465 |
| | 2818.0231 | 188.1818 | .4 | 3359.2815 | 205.4606 |
| | 2827.44 | 188.496 | .5 | 3369.5623 | 205.7748 |
| | 2836.8727 | 188.8102 | .6 | 3379.8589 | 206.089 |
| | 2846.321 | 189.1243 | .7 | 3390.1712 | 206.4031 |
| | 2855.7851 | 189.4385 | .8 | 3400.4993 | 206.7173 |
| | 2865.2649 | 189.7526 | .9 | 3410.843 | 207.0314 |

| M. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|----|-----------|----------|-------|-----------|----------|
| | 3421.2024 | 207.3456 | .5 | 4015.1611 | 224.6244 |
| .1 | 3431.5775 | 207.6598 | .6 | 4026.4002 | 224.9386 |
| .2 | 3441.9684 | 207.9739 | .7 | 4037.655 | 225.2527 |
| .3 | 3452.3749 | 208.2881 | .8 | 4048.9255 | 225.5669 |
| .4 | 3462.7972 | 208.6022 | .9 | 4060.2117 | 225.881 |
| .5 | 3473.2351 | 208.9164 | 72 | 4071.5136 | 226.1952 |
| .6 | 3483.6888 | 209.2306 | .1 | 4082.8312 | 226.5094 |
| .7 | 3494.1582 | 209.5447 | .2 | 4094.1645 | 226.8235 |
| .8 | 3504.6433 | 209.8589 | .3 | 4105.5136 | 227.1377 |
| .9 | 3515.1441 | 210.173 | .4 | 4116.8783 | 227.4518 |
| | 3525.6606 | 210.4872 | .5 | 4128.2587 | 227.766 |
| .1 | 3536.1928 | 210.8014 | .6 | 4139.655 | 228.0802 |
| .2 | 3546.7407 | 211.1155 | .7 | 4151.0668 | 228.3943 |
| .3 | 3557.3044 | 211.4297 | .8 | 4162.4943 | 228.7085 |
| .4 | 3567.8837 | 211.7438 | .9 | 4173.9376 | 229.0226 |
| .5 | 3578.4787 | 212.058 | 73 | 4185.3966 | 229.3368 |
| .6 | 3589.0895 | 212.3722 | .1 | 4196.8713 | 229.651 |
| .7 | 3599.716 | 212.6863 | .2 | 4208.3617 | 229.9651 |
| .8 | 3610.3581 | 213.0005 | .3 | 4219.8678 | 230.2793 |
| .9 | 3621.016 | 213.3146 | .4 | 4231.3896 | 230.5934 |
| | 3631.6896 | 213.6288 | .5 | 4242.9271 | 230.9076 |
| .1 | 3642.3789 | 213.943 | .6 | 4254.4804 | 231.2218 |
| .2 | 3653.0839 | 214.2571 | .7 | 4266.0493 | 231.5359 |
| .3 | 3663.805 | 214.5713 | .8 | 4277.634 | 231.8501 |
| .4 | 3674.541 | 214.8854 | .9 | 4289.2343 | 232.1642 |
| .5 | 3685.2931 | 215.1996 | 74 | 4300.8504 | 232.4784 |
| .6 | 3696.061 | 215.5138 | .1 | 4312.4822 | 232.7926 |
| .7 | 3706.8445 | 215.8279 | .2 | 4324.1297 | 233.1067 |
| .8 | 3717.6438 | 216.1421 | .3 | 4335.7928 | 233.4209 |
| .9 | 3728.4587 | 216.4562 | .4 | 4347.4717 | 233.735 |
| | 3739.2894 | 216.7704 | .5 | 4359.1663 | 234.0492 |
| | 3750.1358 | 217.0846 | .6 | 4370.8767 | 234.3634 |
| | 3760.9979 | 217.3987 | .7 | 4382.6027 | 234.6775 |
| | 3771.8756 | 217.7129 | .8 | 4394.3444 | 234.9917 |
| | 3782.7691 | 218.027 | .9 | 4406.1019 | 235.3058 |
| | 3793.6783 | 218.3412 | 75 | 4417.875 | 235.62 |
| | 3804.6033 | 218.6554 | .1 | 4429.6639 | 235.9342 |
| | 3815.5439 | 218.9695 | .2 | 4441.4684 | 236.2483 |
| | 3826.5002 | 219.2837 | .3 | 4453.2887 | 236.5625 |
| | 3837.4722 | 219.5978 | .4 | 4465.1247 | 236.8766 |
| | 3848.46 | 219.912 | .5 | 4476.9763 | 237.1908 |
| | 3859.4635 | 220.2262 | .6 | 4488.8437 | 237.505 |
| | 3870.4826 | 220.5403 | .7 | 4500.7268 | 237.8191 |
| | 3881.5175 | 220.8545 | .8 | 4512.6257 | 238.1333 |
| | 3892.5681 | 221.1686 | .9 | 4524.5402 | 238.4474 |
| | 3903.6343 | 221.4828 | 76 | 4536.4704 | 238.7616 |
| | 3914.7163 | 221.797 | .1 | 4548.4163 | 239.0758 |
| | 3925.814 | 222.1111 | .2 | 4560.378 | 399 |
| | 3936.9275 | 222.4253 | .3 | 4572.3553 | 399 |
| | 3948.9566 | 222.7394 | .4 | 4584.348 | 399 |
| | 3959.2014 | 223.0536 | .5 | 4596.3571 | 399 |
| | 3970.3619 | 223.3678 | .6 | 4608.3811 | 399 |
| | 3981.5382 | 223.6819 | .7 | 4620.4211 | 399 |
| | 3992.7301 | 223.9961 | .8 | 4632.4771 | 399 |
| | 4003.9378 | 224.3102 | .9 | 4644.5493 | 399 |

250 AREAS AND CIRCUMFERENCES OF CIRCLES.

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|-------|-----------|----------|-------|-----------|----------|
| 77 | 4656.6366 | 241.9032 | .5 | 5345.6287 | 259.182 |
| .1 | 4668.7396 | 242.2174 | .6 | 5358.5957 | 259.4962 |
| .2 | 4680.8583 | 242.5315 | .7 | 5371.5784 | 259.8103 |
| .3 | 4692.9928 | 242.8457 | .8 | 5384.5767 | 260.1245 |
| .4 | 4705.1429 | 243.1598 | .9 | 5397.5908 | 260.4386 |
| .5 | 4717.3087 | 243.474 | 83 | 5410.6206 | 260.7528 |
| .6 | 4729.4903 | 243.7882 | .1 | 5423.6661 | 261.067 |
| .7 | 4741.6876 | 244.1023 | .2 | 5436.7273 | 261.3811 |
| .8 | 4753.9005 | 244.4165 | .3 | 5449.8042 | 261.6953 |
| .9 | 4766.1292 | 244.7306 | .4 | 5462.8968 | 262.0094 |
| 78 | 4778.3736 | 245.0448 | .5 | 5476.0051 | 262.3236 |
| .1 | 4790.6337 | 245.359 | .6 | 5489.1292 | 262.6378 |
| .2 | 4802.9095 | 245.6731 | .7 | 5502.2689 | 262.9519 |
| .3 | 4815.201 | 245.9873 | .8 | 5515.4244 | 263.2661 |
| .4 | 4827.5082 | 246.3014 | .9 | 5528.5955 | 263.5802 |
| .5 | 4839.8311 | 246.6156 | 84 | 5541.7824 | 263.8944 |
| .6 | 4852.1698 | 246.9298 | .1 | 5554.985 | 264.2086 |
| .7 | 4864.5241 | 247.2439 | .2 | 5568.2033 | 264.5227 |
| .8 | 4876.8942 | 247.5581 | .3 | 5581.4372 | 264.8369 |
| .9 | 4889.2799 | 247.8722 | .4 | 5594.6869 | 265.151 |
| 79 | 4901.6814 | 248.1864 | .5 | 5607.9523 | 265.4652 |
| .1 | 4914.0986 | 248.5006 | .6 | 5621.2335 | 265.7794 |
| .2 | 4926.5315 | 248.8147 | .7 | 5634.5303 | 266.0935 |
| .3 | 4938.98 | 249.1289 | .8 | 5647.8428 | 266.4077 |
| .4 | 4951.4443 | 249.443 | .9 | 5661.1711 | 266.7218 |
| .5 | 4963.9243 | 249.7572 | 85 | 5674.515 | 267.036 |
| .6 | 4976.4201 | 250.0714 | .1 | 5687.8747 | 267.3502 |
| .7 | 4988.9315 | 250.3855 | .2 | 5701.25 | 267.6643 |
| .8 | 5001.4586 | 250.6997 | .3 | 5714.6411 | 267.9785 |
| .9 | 5014.0015 | 251.0138 | .4 | 5728.0479 | 268.2926 |
| 80 | 5026.56 | 251.328 | .5 | 5741.4703 | 268.6068 |
| .1 | 5039.1343 | 251.6422 | .6 | 5754.9085 | 268.921 |
| .2 | 5051.7242 | 251.9563 | .7 | 5768.3624 | 269.2351 |
| .3 | 5064.3299 | 252.2705 | .8 | 5781.8321 | 269.5493 |
| .4 | 5076.9513 | 252.5846 | .9 | 5795.3174 | 269.8634 |
| .5 | 5089.5883 | 252.8988 | 86 | 5808.8184 | 270.1776 |
| .6 | 5102.2411 | 253.213 | .1 | 5822.3351 | 270.4918 |
| .7 | 5114.9096 | 253.5271 | .2 | 5835.8676 | 270.8059 |
| .8 | 5127.5939 | 253.8413 | .3 | 5849.4157 | 271.1201 |
| .9 | 5140.2938 | 254.1554 | .4 | 5862.9796 | 271.4342 |
| 81 | 5153.0094 | 254.4696 | .5 | 5876.5591 | 271.7484 |
| .1 | 5165.7407 | 254.7838 | .6 | 5890.1544 | 272.0626 |
| .2 | 5178.4878 | 255.0979 | .7 | 5903.7654 | 272.3767 |
| .3 | 5191.2505 | 255.4121 | .8 | 5917.3921 | 272.6909 |
| .4 | 5204.0289 | 255.7262 | .9 | 5931.0345 | 273.005 |
| .5 | 5216.8231 | 256.0404 | 87 | 5944.6926 | 273.3192 |
| .6 | 5229.633 | 256.3546 | .1 | 5958.3644 | 273.6334 |
| .7 | 5242.4586 | 256.6687 | .2 | 5972.0559 | 273.9475 |
| .8 | 5255.2999 | 256.9829 | .3 | 5985.7612 | 274.2617 |
| .9 | 5268.1569 | 257.297 | .4 | 5999.4821 | 274.5758 |
| 82 | 5281.0296 | 257.6112 | .5 | 6013.2187 | 274.89 |
| | 5293.918 | 257.9254 | .6 | 6026.9711 | 275.2042 |
| | 5221 | 258.2395 | .7 | 6040.7392 | 275.5183 |
| | 42 | 258.5537 | .8 | 6054.5229 | 275.8325 |
| | 775 | 258.8678 | .9 | 6068.3224 | 276.1466 |

AREAS AND CIRCUMFERENCES OF CIRCLES. 251

| AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|-----------|----------|-------|-----------|----------|
| 6082.1376 | 276.4608 | .5 | 6866.1631 | 293.7396 |
| 6095.9685 | 276.775 | .6 | 6880.858 | 294.0538 |
| 6109.8151 | 277.0891 | .7 | 6895.5685 | 294.3679 |
| 6123.6774 | 277.4033 | .8 | 6910.2948 | 294.6821 |
| 6137.5554 | 277.7174 | .9 | 6925.0367 | 294.9962 |
| 6151.4491 | 278.0316 | 94 | 6939.7944 | 295.3104 |
| 6165.3586 | 278.3458 | .1 | 6954.5678 | 295.6246 |
| 6179.2837 | 278.6599 | .2 | 6969.3569 | 295.9387 |
| 6193.2246 | 278.9741 | .3 | 6984.1616 | 296.2529 |
| 6207.1811 | 279.2882 | .4 | 6998.9821 | 296.567 |
| 6221.1534 | 279.6024 | .5 | 7013.8183 | 296.8812 |
| 6235.1414 | 279.9166 | .6 | 7028.6703 | 297.1954 |
| 6249.1451 | 280.2307 | .7 | 7043.5379 | 297.5095 |
| 6263.1644 | 280.5449 | .8 | 7058.4212 | 297.8237 |
| 6277.1995 | 280.859 | .9 | 7073.3203 | 298.1378 |
| 6291.2503 | 281.1732 | 95 | 7088.235 | 298.452 |
| 6305.3169 | 281.4874 | .1 | 7103.1655 | 298.7662 |
| 6319.3991 | 281.8015 | .2 | 7118.1116 | 299.0803 |
| 6333.497 | 282.1157 | .3 | 7133.0735 | 299.3945 |
| 6347.6107 | 282.4298 | .4 | 7148.0511 | 299.7086 |
| 6361.74 | 282.744 | .5 | 7163.0443 | 300.0228 |
| 6375.8851 | 283.0582 | .6 | 7178.0533 | 300.337 |
| 6390.0458 | 283.3723 | .7 | 7193.078 | 300.6511 |
| 6404.2223 | 283.6865 | .8 | 7208.1185 | 300.9653 |
| 6418.4144 | 284.0006 | .9 | 7223.1746 | 301.2794 |
| 6432.6223 | 284.3148 | 96 | 7238.2464 | 301.5936 |
| 6446.8459 | 284.629 | .1 | 7253.3339 | 301.9078 |
| 6461.0852 | 284.9431 | .2 | 7268.4372 | 302.2219 |
| 6475.3403 | 285.2573 | .3 | 7283.5561 | 302.5361 |
| 6489.611 | 285.5714 | .4 | 7298.6908 | 302.8502 |
| 6503.8974 | 285.8856 | .5 | 7313.8411 | 303.1644 |
| 6518.1995 | 286.1998 | .6 | 7329.0072 | 303.4786 |
| 6532.5174 | 286.5139 | .7 | 7344.189 | 303.7927 |
| 6546.8509 | 286.8281 | .8 | 7359.3865 | 304.1069 |
| 6561.2002 | 287.1422 | .9 | 7374.5997 | 304.421 |
| 6575.5651 | 287.4564 | 97 | 7389.8286 | 304.7352 |
| 6589.9458 | 287.7706 | .1 | 7405.0732 | 305.0494 |
| 6604.3422 | 288.0847 | .2 | 7420.3335 | 305.3635 |
| 6618.7543 | 288.3989 | .3 | 7435.6096 | 305.6777 |
| 6633.1821 | 288.713 | .4 | 7450.9013 | 305.9918 |
| 6647.6256 | 289.0272 | .5 | 7466.2087 | 306.306 |
| 6662.0848 | 289.3414 | .6 | 7481.5319 | 306.6202 |
| 6676.5598 | 289.6555 | .7 | 7496.8708 | 306.9343 |
| 6691.0504 | 289.9697 | .8 | 7512.2253 | 307.2485 |
| 6705.5567 | 290.2838 | .9 | 7527.5956 | 307.5626 |
| 6720.0787 | 290.598 | 98 | 7542.9816 | 307.8768 |
| 6734.6165 | 290.9121 | .1 | 7558.3833 | 308.191 |
| 6749.17 | 291.2263 | .2 | 7573.8007 | 308.5051 |
| 6763.7391 | 291.5405 | .3 | 7589.2338 | |
| 6778.324 | 291.8546 | .4 | 7604.6826 | |
| 6792.9246 | 292.1688 | .5 | 7620.1471 | |
| 6807.5409 | 292.483 | .6 | 7635.6274 | |
| 6822.1729 | 292.7971 | .7 | 7651.1233 | |
| 6836.8206 | 293.1113 | .8 | 7666.635 | |
| 6851.484 | 293.4254 | .9 | 7682.1623 | |

AREAS AND CIRCUMFERENCES OF CIRCLES.

| AREA. | CIRCUM. | DIAM. | AREA. | C |
|-----------|----------|-------|-----------|----|
| 757.7054 | 311.0184 | .5 | 7775.6563 | 31 |
| 7713.2642 | 311.3326 | .6 | 7791.2937 | 31 |
| 7728.8337 | 311.6467 | .7 | 7806.9467 | 31 |
| 7744.4288 | 311.9609 | .8 | 7822.6154 | 31 |
| 7760.0347 | 312.275 | .9 | 7838.2999 | 31 |

pute Area or Circumference of a Diameter
than any in preceding Table.

s. pages 235-6 and 241-2.

Diameter exceeds 100 and is less than 1001.

imal point, and take out area or circumference as for
removing decimal point, if for an area, two places to rig
umference, one place.

—What is area and what circumference of a circle 967 feet

is 7344.189; hence, for 967 it is 734 418.9; and circumferen
and for 967 it is 3037.927.

pute Area and Circumference of a Circle b:
arithms.

s. pages 236, 242.

Areas and Circumferences of Circles.

FROM 1 TO 50 FEET (advancing by an Inch).

OR, FROM 1 TO 50 INCHES (advancing by a Twelfth).

| AREA. | CIRCUM. | DIAM. | AREA. | C |
|--------|---------|-------|---------|----|
| Feet. | Feet. | | Feet. | |
| .7854 | 3.1416 | 3 ft. | 7.0686 | 9 |
| .9217 | 3.4034 | 1 | 7.4668 | 9 |
| 1.069 | 3.6652 | 2 | 7.8758 | 9 |
| 1.2272 | 3.927 | 3 | 8.2958 | 10 |
| 1.3963 | 4.1888 | 4 | 8.7267 | 10 |
| 1.5763 | 4.4506 | 5 | 9.1685 | 10 |
| 1.7671 | 4.7124 | 6 | 9.6211 | 10 |
| 1.969 | 4.9742 | 7 | 10.0848 | 11 |
| 2.1817 | 5.236 | 8 | 10.5593 | 11 |
| 2.4053 | 5.4978 | 9 | 11.0447 | 11 |
| 2.6398 | 5.7596 | 10 | 11.541 | 12 |
| 2.8853 | 6.0214 | 11 | 12.0483 | 12 |
| 3.1416 | 6.2832 | 4 ft. | 12.5664 | 12 |
| 3.4088 | 6.545 | 1 | 13.0955 | 12 |
| 3.687 | 6.8068 | 2 | 13.6354 | 13 |
| 3.9761 | 7.0686 | 3 | 14.1863 | 13 |
| | 7.3304 | 4 | 14.7481 | 13 |
| | 7.5922 | 5 | 15.3208 | 13 |
| | 7.854 | 6 | 15.9043 | 14 |
| | 8.1158 | 7 | 16.4989 | 14 |
| | 8.3776 | 8 | 17.1043 | 14 |
| | 8.6394 | 9 | 17.7206 | 14 |
| | 8.9012 | 10 | 18.3478 | 15 |
| | 9.163 | 11 | 18.9859 | 15 |

| AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|---------|---------|--------|----------|---------|
| Feet. | Feet. | | Feet. | Feet. |
| 19.635 | 15.708 | 6 | 70.8823 | 29.8452 |
| 20.2949 | 15.9698 | 7 | 72.1314 | 30.107 |
| 20.9658 | 16.2316 | 8 | 73.3913 | 30.3688 |
| 21.6476 | 16.4934 | 9 | 74.6621 | 30.6306 |
| 22.3403 | 16.7552 | 10 | 75.9439 | 30.8924 |
| 23.0439 | 17.017 | 11 | 77.2365 | 31.1542 |
| 23.7583 | 17.2788 | 10 ft. | 78.54 | 31.416 |
| 24.4837 | 17.5406 | 1 | 79.8545 | 31.6778 |
| 25.22 | 17.8024 | 2 | 81.1798 | 31.9396 |
| 25.9673 | 18.0642 | 3 | 82.5161 | 32.2014 |
| 26.7254 | 18.326 | 4 | 83.8633 | 32.4632 |
| 27.4944 | 18.5878 | 5 | 85.2214 | 32.725 |
| 28.2744 | 18.8496 | 6 | 86.5903 | 32.9868 |
| 29.0653 | 19.1114 | 7 | 87.9703 | 33.2486 |
| 29.867 | 19.3732 | 8 | 89.3611 | 33.5104 |
| 30.6797 | 19.635 | 9 | 90.7628 | 33.7722 |
| 31.5033 | 19.8968 | 10 | 92.1754 | 34.034 |
| 32.3378 | 20.1586 | 11 | 93.599 | 34.2958 |
| 33.1831 | 20.4204 | 11 ft. | 95.0334 | 34.5576 |
| 34.0394 | 20.6822 | 1 | 96.4787 | 34.8194 |
| 34.9067 | 20.944 | 2 | 97.935 | 35.0812 |
| 35.7848 | 21.2058 | 3 | 99.4022 | 35.343 |
| 36.6738 | 21.4676 | 4 | 100.8803 | 35.6048 |
| 37.5738 | 21.7294 | 5 | 102.3693 | 35.8666 |
| 38.4846 | 21.9912 | 6 | 103.8691 | 36.1284 |
| 39.4064 | 22.253 | 7 | 105.38 | 36.3902 |
| 40.339 | 22.5148 | 8 | 106.9017 | 36.652 |
| 41.2826 | 22.7766 | 9 | 108.4343 | 36.9138 |
| 42.2371 | 23.0384 | 10 | 109.9778 | 37.1756 |
| 43.2025 | 23.3002 | 11 | 111.5323 | 37.4374 |
| 44.1787 | 23.562 | 12 ft. | 113.0976 | 37.6992 |
| 45.1659 | 23.8238 | 1 | 114.6739 | 37.961 |
| 46.1641 | 24.0856 | 2 | 116.261 | 38.2228 |
| 47.1731 | 24.3474 | 3 | 117.8591 | 38.4846 |
| 48.193 | 24.6092 | 4 | 119.468 | 38.7464 |
| 49.2238 | 24.871 | 5 | 121.088 | 39.0082 |
| 50.2656 | 25.1328 | 6 | 122.7187 | 39.27 |
| 51.3183 | 25.3946 | 7 | 124.3605 | 39.5318 |
| 52.3818 | 25.6564 | 8 | 126.0131 | 39.7936 |
| 53.4563 | 25.9182 | 9 | 127.6766 | 40.0554 |
| 54.5417 | 26.18 | 10 | 129.351 | 40.3172 |
| 55.638 | 26.4418 | 11 | 131.0366 | 40.579 |
| 56.7451 | 26.7036 | 13 ft. | 132.7326 | 40.8408 |
| 57.8632 | 26.9654 | 1 | 134.4398 | 41.1026 |
| 58.9923 | 27.2272 | 2 | 136.1578 | 41.3644 |
| 60.1322 | 27.489 | 3 | 137.8868 | |
| 61.283 | 27.7508 | 4 | 139.6267 | |
| 62.4448 | 28.0126 | 5 | 141.3774 | |
| 63.6174 | 28.2744 | 6 | 143.1391 | |
| 64.801 | 28.5362 | 7 | 144.9117 | |
| 65.9954 | 28.798 | 8 | 146.6953 | |
| 67.2008 | 29.0598 | 9 | 148.4897 | |
| 68.417 | 29.3216 | 10 | 150.295 | |
| 69.6442 | 29.5834 | 11 | 152.1113 | |

254 AREAS AND CIRCUMFERENCES OF CIRCLES.

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|--------|----------|---------|--------|----------|---------|
| | Feet. | Feet. | | Feet. | Feet. |
| 14 ft. | 153.9384 | 43.9824 | 6 | 268.8031 | 58.1196 |
| 1 | 155.7704 | 44.2442 | 7 | 271.2302 | 58.3814 |
| 2 | 157.6254 | 44.506 | 8 | 273.6683 | 58.6432 |
| 3 | 159.4853 | 44.7678 | 9 | 276.1172 | 58.905 |
| 4 | 161.3501 | 45.0296 | 10 | 278.577 | 59.1668 |
| 5 | 163.2378 | 45.2914 | 11 | 281.0477 | 59.4286 |
| 6 | 165.1303 | 45.5532 | 19 ft. | 283.5294 | 59.6904 |
| 7 | 167.0338 | 45.815 | 1 | 286.0219 | 59.9522 |
| 8 | 168.9483 | 46.0768 | 2 | 288.5255 | 60.214 |
| 9 | 170.8736 | 46.3386 | 3 | 291.0398 | 60.4758 |
| 10 | 172.8098 | 46.6004 | 4 | 293.5651 | 60.7376 |
| 11 | 174.7569 | 46.8622 | 5 | 296.1012 | 60.9994 |
| 15 ft. | 176.715 | 47.124 | 6 | 298.6483 | 61.2612 |
| 1 | 178.684 | 47.3858 | 7 | 301.2064 | 61.523 |
| 2 | 180.6638 | 47.6476 | 8 | 303.7753 | 61.7848 |
| 3 | 182.6546 | 47.9094 | 9 | 306.3551 | 62.0466 |
| 4 | 184.6563 | 48.1712 | 10 | 308.9458 | 62.3084 |
| 5 | 186.6689 | 48.433 | 11 | 311.5475 | 62.5702 |
| 6 | 188.6924 | 48.6948 | 20 ft. | 314.16 | 62.832 |
| 7 | 190.7267 | 48.9566 | 1 | 316.7834 | 63.0938 |
| 8 | 192.7721 | 49.2184 | 2 | 319.4178 | 63.3556 |
| 9 | 194.8283 | 49.4802 | 3 | 322.0631 | 63.6174 |
| 10 | 196.8954 | 49.742 | 4 | 324.7193 | 63.8792 |
| 11 | 198.9734 | 50.0038 | 5 | 327.3864 | 64.141 |
| 16 ft. | 201.0624 | 50.2656 | 6 | 330.0643 | 64.4028 |
| 1 | 203.1622 | 50.5274 | 7 | 332.7532 | 64.6646 |
| 2 | 205.273 | 50.7892 | 8 | 335.4531 | 64.9264 |
| 3 | 207.3947 | 51.051 | 9 | 338.1638 | 65.1882 |
| 4 | 209.5273 | 51.3128 | 10 | 340.8854 | 65.45 |
| 5 | 211.6707 | 51.5746 | 11 | 343.618 | 65.7118 |
| 6 | 213.8252 | 51.8364 | 21 ft. | 346.3614 | 65.9736 |
| 7 | 215.9904 | 52.0982 | 1 | 349.1157 | 66.2354 |
| 8 | 218.1667 | 52.36 | 2 | 351.881 | 66.4972 |
| 9 | 220.3538 | 52.6218 | 3 | 354.6572 | 66.759 |
| 10 | 222.5518 | 52.8836 | 4 | 357.4442 | 67.0208 |
| 11 | 224.7607 | 53.1454 | 5 | 360.2422 | 67.2826 |
| 17 ft. | 226.9806 | 53.4072 | 6 | 363.0511 | 67.5444 |
| 1 | 229.2113 | 53.669 | 7 | 365.8709 | 67.8062 |
| 2 | 231.453 | 53.9308 | 8 | 368.7017 | 68.068 |
| 3 | 233.7056 | 54.1926 | 9 | 371.5433 | 68.3298 |
| 4 | 235.9691 | 54.4544 | 10 | 374.3958 | 68.5916 |
| 5 | 238.2434 | 54.7162 | 11 | 377.2592 | 68.8534 |
| 6 | 240.5287 | 54.978 | 22 ft. | 380.1336 | 69.1152 |
| 7 | 242.8249 | 55.2398 | 1 | 383.0188 | 69.377 |
| 8 | 245.1321 | 55.5016 | 2 | 385.915 | 69.6388 |
| 9 | 247.4501 | 55.7634 | 3 | 388.8221 | 69.9006 |
| 10 | 249.779 | 56.0252 | 4 | 391.74 | 70.1624 |
| 11 | 252.1188 | 56.287 | 5 | 394.6689 | 70.4242 |
| | 254 | 56.5488 | 6 | 397.6087 | 70.686 |
| | 2 | 56.8106 | 7 | 400.5594 | 70.9478 |
| | 1 | 57.0724 | 8 | 403.5211 | 71.2096 |
| | 1 | 57.3342 | 9 | 406.4936 | 71.4714 |
| | 7 | 57.596 | 10 | 409.477 | 71.7332 |
| | 9 | 57.8578 | 11 | 412.4713 | 71.995 |

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|--------|----------|---------|--------|----------|---------|
| | Feet. | Feet. | | Feet. | Feet. |
| 13 ft. | 415.4766 | 72.2568 | 6 | 593.9587 | 86.394 |
| 1 | 418.4927 | 72.5186 | 7 | 597.5639 | 86.6558 |
| 2 | 421.5198 | 72.7804 | 8 | 601.18 | 86.9176 |
| 3 | 424.5578 | 73.0422 | 9 | 604.8071 | 87.1794 |
| 4 | 427.6067 | 73.304 | 10 | 608.445 | 87.4412 |
| 5 | 430.6664 | 73.5658 | 11 | 612.0938 | 87.703 |
| 6 | 433.7371 | 73.8276 | 28 ft. | 615.7536 | 87.9648 |
| 7 | 436.8187 | 74.0894 | 1 | 619.4242 | 88.2266 |
| 8 | 439.91 | 74.3512 | 2 | 623.1058 | 88.4884 |
| 9 | 443.0147 | 74.613 | 3 | 626.7983 | 88.7502 |
| 10 | 446.129 | 74.8748 | 4 | 630.5016 | 89.012 |
| 11 | 449.2542 | 75.1366 | 5 | 634.2159 | 89.2738 |
| 14 ft. | 452.3904 | 75.3984 | 6 | 637.9411 | 89.5356 |
| 1 | 455.5374 | 75.6602 | 7 | 641.6772 | 89.7974 |
| 2 | 458.6954 | 75.922 | 8 | 645.4243 | 90.0592 |
| 3 | 461.8643 | 76.1838 | 9 | 649.1822 | 90.321 |
| 4 | 465.044 | 76.4456 | 10 | 652.951 | 90.5828 |
| 5 | 468.2347 | 76.7074 | 11 | 656.7307 | 90.8446 |
| 6 | 471.4363 | 76.9692 | 29 ft. | 660.5214 | 91.1064 |
| 7 | 474.6488 | 77.231 | 1 | 664.3229 | 91.3682 |
| 8 | 477.8723 | 77.4928 | 2 | 668.1354 | 91.63 |
| 9 | 481.1066 | 77.7546 | 3 | 671.9588 | 91.8918 |
| 10 | 484.3518 | 78.0164 | 4 | 675.7931 | 92.1536 |
| 11 | 487.6076 | 78.2782 | 5 | 679.6382 | 92.4154 |
| ft. | 490.875 | 78.54 | 6 | 683.4943 | 92.6772 |
| 1 | 494.1529 | 78.8018 | 7 | 687.3613 | 92.939 |
| 2 | 497.4418 | 79.0636 | 8 | 691.2393 | 93.2008 |
| 3 | 500.7416 | 79.3254 | 9 | 695.1281 | 93.4626 |
| 4 | 504.0523 | 79.5872 | 10 | 699.0278 | 93.7244 |
| 5 | 507.3738 | 79.849 | 11 | 702.9384 | 93.9862 |
| 6 | 510.7063 | 80.1108 | 30 ft. | 706.86 | 94.248 |
| 7 | 514.0485 | 80.3726 | 1 | 710.7924 | 94.5098 |
| 8 | 517.404 | 80.6344 | 2 | 714.7358 | 94.7716 |
| 9 | 520.7693 | 80.8962 | 3 | 718.6901 | 95.0334 |
| 10 | 524.1454 | 81.158 | 4 | 722.6553 | 95.2952 |
| 11 | 527.5324 | 81.4198 | 5 | 726.6313 | 95.557 |
| ft. | 530.9304 | 81.6816 | 6 | 730.6183 | 95.8188 |
| 1 | 534.3397 | 81.9434 | 7 | 734.6162 | 96.0806 |
| 2 | 537.759 | 82.2052 | 8 | 738.6251 | 96.3424 |
| 3 | 541.1897 | 82.467 | 9 | 742.6448 | 96.6042 |
| 4 | 544.6313 | 82.7288 | 10 | 746.6754 | 96.866 |
| 5 | 548.0837 | 82.9906 | 11 | 750.7164 | 97.1278 |
| 6 | 551.5471 | 83.2524 | 31 ft. | 754.7694 | 97.3896 |
| 7 | 555.0214 | 83.5142 | 1 | 758.8327 | 97.6514 |
| 8 | 558.5066 | 83.776 | 2 | 762.907 | 97.9132 |
| 9 | 562.0028 | 84.0378 | 3 | 766.9922 | 98.175 |
| 10 | 565.5098 | 84.2996 | 4 | 771.0883 | 98.4368 |
| 11 | 569.0277 | 84.5614 | 5 | 775.1952 | |
| ft. | 572.5566 | 84.8232 | 6 | 779.3131 | |
| 1 | 576.0963 | 85.085 | 7 | 783.4419 | |
| 2 | 579.6467 | 85.3468 | 8 | 787.5817 | |
| 3 | 583.2086 | 85.6086 | 9 | 791.7323 | |
| 4 | 586.781 | 85.8704 | 10 | 795.8938 | |
| 5 | 590.3644 | 86.1322 | 11 | 800.0662 | |

256 AREAS AND CIRCUMFERENCES OF CIRC

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. |
|--------|-----------|----------|--------|-----------|
| | Feet. | Feet. | | Feet. |
| 32 ft. | 804.2496 | 100.5312 | 6 | 1046.3491 |
| 1 | 808.4439 | 100.793 | 7 | 1051.1324 |
| 2 | 812.649 | 101.0548 | 8 | 1055.9266 |
| 3 | 816.8651 | 101.3166 | 9 | 1060.7318 |
| 4 | 821.092 | 101.5784 | 10 | 1065.5478 |
| 5 | 825.3299 | 101.8402 | 11 | 1070.3747 |
| 6 | 829.5787 | 102.102 | 37 ft. | 1075.2126 |
| 7 | 833.8384 | 102.3638 | 1 | 1080.0613 |
| 8 | 838.1091 | 102.6256 | 2 | 1084.921 |
| 9 | 842.3906 | 102.8874 | 3 | 1089.7916 |
| 10 | 846.683 | 103.1492 | 4 | 1094.6731 |
| 11 | 850.9863 | 103.411 | 5 | 1099.5654 |
| 33 ft. | 855.3006 | 103.6728 | 6 | 1104.4687 |
| 1 | 859.6257 | 103.9346 | 7 | 1109.3829 |
| 2 | 863.9618 | 104.1964 | 8 | 1114.308 |
| 3 | 868.3088 | 104.4582 | 9 | 1119.2441 |
| 4 | 872.6667 | 104.72 | 10 | 1124.191 |
| 5 | 877.0354 | 104.9818 | 11 | 1129.1489 |
| 6 | 881.4151 | 105.2436 | 38 ft. | 1134.1176 |
| 7 | 885.8057 | 105.5054 | 1 | 1139.0972 |
| 8 | 890.2073 | 105.7672 | 2 | 1144.0878 |
| 9 | 894.6197 | 106.029 | 3 | 1149.0893 |
| 10 | 899.043 | 106.2908 | 4 | 1154.1017 |
| 11 | 903.4772 | 106.5526 | 5 | 1159.1249 |
| 34 ft. | 907.9224 | 106.8144 | 6 | 1164.1591 |
| 1 | 912.3784 | 107.0762 | 7 | 1169.2042 |
| 2 | 916.8454 | 107.338 | 8 | 1174.2603 |
| 3 | 921.3233 | 107.5998 | 9 | 1179.3272 |
| 4 | 925.812 | 107.8616 | 10 | 1184.405 |
| 5 | 930.3117 | 108.1234 | 11 | 1189.4937 |
| 6 | 934.8223 | 108.3852 | 39 ft. | 1194.5934 |
| 7 | 939.3439 | 108.647 | 1 | 1199.7039 |
| 8 | 943.8763 | 108.9088 | 2 | 1204.8254 |
| 9 | 948.4196 | 109.1706 | 3 | 1209.9578 |
| 10 | 952.9738 | 109.4324 | 4 | 1215.101 |
| 11 | 957.5392 | 109.6942 | 5 | 1220.2552 |
| 35 ft. | 962.115 | 109.956 | 6 | 1225.4203 |
| 1 | 966.7019 | 110.2178 | 7 | 1230.5963 |
| 2 | 971.2998 | 110.4796 | 8 | 1235.7833 |
| 3 | 975.9086 | 110.7414 | 9 | 1240.9811 |
| 4 | 980.5287 | 111.0032 | 10 | 1246.1898 |
| 5 | 985.1588 | 111.265 | 11 | 1251.4094 |
| 6 | 989.8005 | 111.5268 | 40 ft. | 1256.64 |
| 7 | 994.4527 | 111.7886 | 1 | 1261.8814 |
| 8 | 999.116 | 112.0504 | 2 | 1267.1338 |
| 9 | 1003.7903 | 112.3122 | 3 | 1272.3971 |
| 10 | 1008.4754 | 112.574 | 4 | 1277.6712 |
| | 1013.1714 | 112.8358 | 5 | 1282.9563 |
| | 1017.8784 | 113.0976 | 6 | 1288.2523 |
| | 1022.5962 | 113.3594 | 7 | 1293.5592 |
| | 1027.325 | 113.6212 | 8 | 1298.877 |
| | 1032.0647 | 113.883 | 9 | 1304.2058 |
| | 1036.8153 | 114.1448 | 10 | 1309.5454 |
| | 1041.5767 | 114.4066 | 11 | 1314.8959 |

AREAS AND CIRCUMFERENCES OF CIRCLES. 257

| DIAM. | AREA. | CIRCUM. | DIAM. | AREA. | CIRCUM. |
|-------|-----------|----------|--------|-----------|----------|
| | Feet. | Feet. | | Feet. | Feet. |
| 1 ft. | 1320.2574 | 128.8056 | 6 | 1625.9743 | 142.9428 |
| 1 | 1325.6297 | 129.0674 | 7 | 1631.9357 | 143.2046 |
| 2 | 1331.013 | 129.3292 | 8 | 1637.9081 | 143.4664 |
| 3 | 1336.4072 | 129.591 | 9 | 1643.8913 | 143.7282 |
| 4 | 1341.8123 | 129.8528 | 10 | 1649.8854 | 143.99 |
| 5 | 1347.2282 | 130.1146 | 11 | 1655.8904 | 144.2518 |
| 6 | 1352.6551 | 130.3764 | 46 ft. | 1661.9004 | 144.5136 |
| 7 | 1358.0929 | 130.6382 | 1 | 1667.9332 | 144.7754 |
| 8 | 1363.5416 | 130.9 | 2 | 1673.971 | 145.0372 |
| 9 | 1369.0013 | 131.1618 | 3 | 1680.0197 | 145.299 |
| 10 | 1374.4718 | 131.4236 | 4 | 1686.0792 | 145.5608 |
| 11 | 1379.9532 | 131.6854 | 5 | 1692.1497 | 145.8226 |
| 1 ft. | 1385.4456 | 131.9472 | 6 | 1698.2311 | 146.0844 |
| 1 | 1390.9488 | 132.209 | 7 | 1704.3195 | 146.3462 |
| 2 | 1396.463 | 132.4708 | 8 | 1710.4267 | 146.608 |
| 3 | 1401.9881 | 132.7326 | 9 | 1716.5408 | 146.8698 |
| 4 | 1407.5241 | 132.9944 | 10 | 1722.6658 | 147.1316 |
| 5 | 1413.0709 | 133.2562 | 11 | 1728.8017 | 147.3934 |
| 6 | 1418.6287 | 133.518 | 47 ft. | 1734.9486 | 147.6552 |
| 7 | 1424.1974 | 133.7798 | 1 | 1741.1063 | 147.917 |
| 8 | 1429.777 | 134.0416 | 2 | 1747.275 | 148.1788 |
| 9 | 1435.3676 | 134.3034 | 3 | 1753.4546 | 148.4406 |
| 10 | 1440.969 | 134.5652 | 4 | 1759.6451 | 148.7024 |
| 11 | 1446.5813 | 134.827 | 5 | 1765.8464 | 148.9642 |
| ft. | 1452.2046 | 135.0888 | 6 | 1772.0587 | 149.226 |
| 1 | 1457.8387 | 135.3506 | 7 | 1778.2819 | 149.4878 |
| 2 | 1463.4838 | 135.6124 | 8 | 1784.516 | 149.7496 |
| 3 | 1469.1398 | 135.8742 | 9 | 1790.7611 | 150.0114 |
| 4 | 1474.8066 | 136.136 | 10 | 1797.017 | 150.2732 |
| 5 | 1480.4844 | 136.3978 | 11 | 1803.2838 | 150.535 |
| 6 | 1486.1731 | 136.6596 | 48 ft. | 1809.5616 | 150.7968 |
| 7 | 1491.8717 | 136.9214 | 1 | 1815.8502 | 151.0586 |
| 8 | 1497.5833 | 137.1832 | 2 | 1822.1498 | 151.3204 |
| 9 | 1503.3047 | 137.445 | 3 | 1828.4603 | 151.5822 |
| 10 | 1509.037 | 137.7068 | 4 | 1834.7817 | 151.844 |
| 11 | 1514.7802 | 137.9686 | 5 | 1841.1139 | 152.1058 |
| ft. | 1520.5344 | 138.2304 | 6 | 1847.4571 | 152.3676 |
| 1 | 1526.2994 | 138.4922 | 7 | 1853.8112 | 152.6294 |
| 2 | 1532.0754 | 138.754 | 8 | 1860.1763 | 152.8912 |
| 3 | 1537.8623 | 139.0158 | 9 | 1866.5522 | 153.153 |
| 4 | 1543.66 | 139.2776 | 10 | 1872.939 | 153.4148 |
| 5 | 1549.4687 | 139.5394 | 11 | 1879.3367 | 153.6766 |
| 6 | 1555.2883 | 139.8012 | 49 ft. | 1885.7454 | 153.9384 |
| 7 | 1561.1188 | 140.063 | 1 | 1892.1649 | 154.2002 |
| 8 | 1566.9603 | 140.3248 | 2 | 1898.5954 | 154.462 |
| 9 | 1572.8126 | 140.5866 | 3 | 1905.0368 | 154.7238 |
| 10 | 1578.6756 | 140.8484 | 4 | 1911.4807 | 154.9856 |
| 11 | 1584.5499 | 141.1102 | 5 | 1917.97 | 155.2474 |
| ft. | 1590.435 | 141.372 | 6 | 1924.47 | 155.5092 |
| 1 | 1596.3309 | 141.6338 | 7 | 1930.97 | |
| 2 | 1602.2378 | 141.8956 | 8 | 1937.47 | |
| 3 | 1608.1556 | 142.1574 | 9 | 1943.97 | |
| 4 | 1614.0843 | 142.4192 | 10 | 1950.47 | |
| 5 | 1620.0238 | 142.681 | 11 | 1956.97 | |
| | | | 50 ft. | 1963.5 | |

Sides of Squares—equal in Area to a Circle

Diameter from 1 to 100.

| Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. |
|---------------|-------------|---------------|-------------|---------------|-------------|---------------|-------------|
| 1 | .8862 | 14 | 12.4072 | 27 | 23.9281 | 40 | 31.4138 |
| $\frac{1}{2}$ | 1.1078 | $\frac{1}{4}$ | 12.6287 | $\frac{1}{4}$ | 24.1497 | $\frac{1}{4}$ | 31.6353 |
| $\frac{1}{3}$ | 1.3293 | $\frac{1}{3}$ | 12.8503 | $\frac{1}{3}$ | 24.3712 | $\frac{1}{3}$ | 31.8568 |
| $\frac{1}{4}$ | 1.5509 | $\frac{1}{2}$ | 13.0718 | $\frac{1}{2}$ | 24.5928 | $\frac{1}{2}$ | 32.0784 |
| 2 | 1.7724 | 15 | 13.2934 | 28 | 24.8144 | 41 | 32.2999 |
| $\frac{1}{2}$ | 1.994 | $\frac{1}{4}$ | 13.515 | $\frac{1}{4}$ | 25.0359 | $\frac{1}{4}$ | 32.5215 |
| $\frac{1}{3}$ | 2.2156 | $\frac{1}{3}$ | 13.7365 | $\frac{1}{3}$ | 25.2575 | $\frac{1}{3}$ | 32.7430 |
| $\frac{1}{4}$ | 2.4371 | $\frac{1}{2}$ | 13.9581 | $\frac{1}{2}$ | 25.479 | $\frac{1}{2}$ | 32.9646 |
| 3 | 2.6587 | 16 | 14.1796 | 29 | 25.7006 | 42 | 33.1861 |
| $\frac{1}{2}$ | 2.8802 | $\frac{1}{4}$ | 14.4012 | $\frac{1}{4}$ | 25.9221 | $\frac{1}{4}$ | 33.4077 |
| $\frac{1}{3}$ | 3.1018 | $\frac{1}{3}$ | 14.6227 | $\frac{1}{3}$ | 26.1437 | $\frac{1}{3}$ | 33.6292 |
| $\frac{1}{4}$ | 3.3233 | $\frac{1}{2}$ | 14.8443 | $\frac{1}{2}$ | 26.3653 | $\frac{1}{2}$ | 33.8508 |
| 4 | 3.5449 | 17 | 15.0659 | 30 | 26.5868 | 43 | 34.0723 |
| $\frac{1}{2}$ | 3.7665 | $\frac{1}{4}$ | 15.2874 | $\frac{1}{4}$ | 26.8084 | $\frac{1}{4}$ | 34.2939 |
| $\frac{1}{3}$ | 3.988 | $\frac{1}{3}$ | 15.509 | $\frac{1}{3}$ | 27.0299 | $\frac{1}{3}$ | 34.5154 |
| $\frac{1}{4}$ | 4.2096 | $\frac{1}{2}$ | 15.7305 | $\frac{1}{2}$ | 27.2515 | $\frac{1}{2}$ | 34.7370 |
| 5 | 4.4311 | 18 | 15.9521 | 31 | 27.473 | 44 | 34.9585 |
| $\frac{1}{2}$ | 4.6527 | $\frac{1}{4}$ | 16.1736 | $\frac{1}{4}$ | 27.6946 | $\frac{1}{4}$ | 35.1801 |
| $\frac{1}{3}$ | 4.8742 | $\frac{1}{3}$ | 16.3952 | $\frac{1}{3}$ | 27.9161 | $\frac{1}{3}$ | 35.4016 |
| $\frac{1}{4}$ | 5.0958 | $\frac{1}{2}$ | 16.6168 | $\frac{1}{2}$ | 28.1377 | $\frac{1}{2}$ | 35.6232 |
| 6 | 5.3174 | 19 | 16.8383 | 32 | 28.3593 | 45 | 35.8447 |
| $\frac{1}{2}$ | 5.5389 | $\frac{1}{4}$ | 17.0599 | $\frac{1}{4}$ | 28.5808 | $\frac{1}{4}$ | 36.0663 |
| $\frac{1}{3}$ | 5.7605 | $\frac{1}{3}$ | 17.2814 | $\frac{1}{3}$ | 28.8024 | $\frac{1}{3}$ | 36.2878 |
| $\frac{1}{4}$ | 5.982 | $\frac{1}{2}$ | 17.503 | $\frac{1}{2}$ | 29.0239 | $\frac{1}{2}$ | 36.5094 |
| 7 | 6.2036 | 20 | 17.7245 | 33 | 29.2455 | 46 | 36.7309 |
| $\frac{1}{2}$ | 6.4251 | $\frac{1}{4}$ | 17.9461 | $\frac{1}{4}$ | 29.467 | $\frac{1}{4}$ | 36.9525 |
| $\frac{1}{3}$ | 6.6467 | $\frac{1}{3}$ | 18.1677 | $\frac{1}{3}$ | 29.6886 | $\frac{1}{3}$ | 37.1740 |
| $\frac{1}{4}$ | 6.8683 | $\frac{1}{2}$ | 18.3892 | $\frac{1}{2}$ | 29.9102 | $\frac{1}{2}$ | 37.3956 |
| 8 | 7.0898 | 21 | 18.6108 | 34 | 30.1317 | 47 | 37.6171 |
| $\frac{1}{2}$ | 7.3114 | $\frac{1}{4}$ | 18.8323 | $\frac{1}{4}$ | 30.3533 | $\frac{1}{4}$ | 37.8387 |
| $\frac{1}{3}$ | 7.5329 | $\frac{1}{3}$ | 19.0539 | $\frac{1}{3}$ | 30.5748 | $\frac{1}{3}$ | 38.0602 |
| $\frac{1}{4}$ | 7.7545 | $\frac{1}{2}$ | 19.2754 | $\frac{1}{2}$ | 30.7964 | $\frac{1}{2}$ | 38.2818 |
| 9 | 7.976 | 22 | 19.497 | 35 | 31.0179 | 48 | 38.5033 |
| $\frac{1}{2}$ | 8.1976 | $\frac{1}{4}$ | 19.7185 | $\frac{1}{4}$ | 31.2395 | $\frac{1}{4}$ | 38.7249 |
| $\frac{1}{3}$ | 8.4192 | $\frac{1}{3}$ | 19.9401 | $\frac{1}{3}$ | 31.4611 | $\frac{1}{3}$ | 38.9464 |
| $\frac{1}{4}$ | 8.6407 | $\frac{1}{2}$ | 20.1617 | $\frac{1}{2}$ | 31.6826 | $\frac{1}{2}$ | 39.1680 |
| 10 | 8.8623 | 23 | 20.3832 | 36 | 31.9042 | 49 | 39.3895 |
| $\frac{1}{2}$ | 9.0838 | $\frac{1}{4}$ | 20.6048 | $\frac{1}{4}$ | 32.1257 | $\frac{1}{4}$ | 39.6111 |
| $\frac{1}{3}$ | 9.3054 | $\frac{1}{3}$ | 20.8263 | $\frac{1}{3}$ | 32.3473 | $\frac{1}{3}$ | 39.8326 |
| $\frac{1}{4}$ | 9.5269 | $\frac{1}{2}$ | 21.0479 | $\frac{1}{2}$ | 32.5688 | $\frac{1}{2}$ | 40.0542 |
| 11 | 9.7485 | 24 | 21.2694 | 37 | 32.7904 | 50 | 40.2757 |
| $\frac{1}{2}$ | 9.97 | $\frac{1}{4}$ | 21.491 | $\frac{1}{4}$ | 33.0112 | $\frac{1}{4}$ | 40.4973 |
| $\frac{1}{3}$ | 10.1916 | $\frac{1}{3}$ | 21.7126 | $\frac{1}{3}$ | 33.2335 | $\frac{1}{3}$ | 40.7188 |
| $\frac{1}{4}$ | 10.4132 | $\frac{1}{2}$ | 21.9341 | $\frac{1}{2}$ | 33.4551 | $\frac{1}{2}$ | 40.9404 |
| 12 | 10.6347 | 25 | 22.1557 | 38 | 33.6766 | 51 | 41.1619 |
| $\frac{1}{2}$ | 10.8563 | $\frac{1}{4}$ | 22.3772 | $\frac{1}{4}$ | 33.8982 | $\frac{1}{4}$ | 41.3835 |
| $\frac{1}{3}$ | 11.0778 | $\frac{1}{3}$ | 22.5988 | $\frac{1}{3}$ | 34.1197 | $\frac{1}{3}$ | 41.6050 |
| $\frac{1}{4}$ | 11.2994 | $\frac{1}{2}$ | 22.8203 | $\frac{1}{2}$ | 34.3413 | $\frac{1}{2}$ | 41.8266 |
| 13 | 11.5209 | 26 | 23.0419 | 39 | 34.5628 | 52 | 42.0481 |
| $\frac{1}{2}$ | 11.7425 | $\frac{1}{4}$ | 23.2634 | $\frac{1}{4}$ | 34.7844 | $\frac{1}{4}$ | 42.2697 |
| $\frac{1}{3}$ | 11.9641 | $\frac{1}{3}$ | 23.485 | $\frac{1}{3}$ | 35.006 | $\frac{1}{3}$ | 42.4912 |
| $\frac{1}{4}$ | 12.1856 | $\frac{1}{2}$ | 23.7066 | $\frac{1}{2}$ | 35.2275 | $\frac{1}{2}$ | 42.7128 |

| Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. |
|---------------|-------------|---------------|-------------|---------------|-------------|
| 65 | 57.6047 | 77 | 68.2395 | 89 | 78.8742 |
| $\frac{1}{4}$ | 57.8263 | $\frac{1}{4}$ | 68.461 | $\frac{1}{4}$ | 79.0957 |
| $\frac{1}{2}$ | 58.0479 | $\frac{1}{2}$ | 68.6826 | $\frac{1}{2}$ | 79.3173 |
| $\frac{3}{4}$ | 58.2694 | $\frac{3}{4}$ | 68.9041 | $\frac{3}{4}$ | 79.5389 |
| 66 | 58.491 | 78 | 69.1257 | 90 | 79.7604 |
| $\frac{1}{4}$ | 58.7125 | $\frac{1}{4}$ | 69.3473 | $\frac{1}{4}$ | 79.982 |
| $\frac{1}{2}$ | 58.9341 | $\frac{1}{2}$ | 69.5688 | $\frac{1}{2}$ | 80.2035 |
| $\frac{3}{4}$ | 59.1556 | $\frac{3}{4}$ | 69.7904 | $\frac{3}{4}$ | 80.4251 |
| 67 | 59.3772 | 79 | 70.0119 | 91 | 80.6467 |
| $\frac{1}{4}$ | 59.5988 | $\frac{1}{4}$ | 70.2335 | $\frac{1}{4}$ | 80.8682 |
| $\frac{1}{2}$ | 59.8203 | $\frac{1}{2}$ | 70.455 | $\frac{1}{2}$ | 81.0898 |
| $\frac{3}{4}$ | 60.0419 | $\frac{3}{4}$ | 70.6766 | $\frac{3}{4}$ | 81.3113 |
| 68 | 60.2634 | 80 | 70.8981 | 92 | 81.5329 |
| $\frac{1}{4}$ | 60.485 | $\frac{1}{4}$ | 71.1197 | $\frac{1}{4}$ | 81.7544 |
| $\frac{1}{2}$ | 60.7065 | $\frac{1}{2}$ | 71.3413 | $\frac{1}{2}$ | 81.976 |
| $\frac{3}{4}$ | 60.9281 | $\frac{3}{4}$ | 71.5628 | $\frac{3}{4}$ | 82.1975 |
| 69 | 61.1497 | 81 | 71.7844 | 93 | 82.4191 |
| $\frac{1}{4}$ | 61.3712 | $\frac{1}{4}$ | 72.0059 | $\frac{1}{4}$ | 82.6407 |
| $\frac{1}{2}$ | 61.5928 | $\frac{1}{2}$ | 72.2275 | $\frac{1}{2}$ | 82.8622 |
| $\frac{3}{4}$ | 61.8143 | $\frac{3}{4}$ | 72.4491 | $\frac{3}{4}$ | 83.0838 |
| 70 | 62.0359 | 82 | 72.6706 | 94 | 83.3053 |
| $\frac{1}{4}$ | 62.2574 | $\frac{1}{4}$ | 72.8921 | $\frac{1}{4}$ | 83.5269 |
| $\frac{1}{2}$ | 62.479 | $\frac{1}{2}$ | 73.1137 | $\frac{1}{2}$ | 83.7484 |
| $\frac{3}{4}$ | 62.7006 | $\frac{3}{4}$ | 73.3353 | $\frac{3}{4}$ | 83.97 |
| 71 | 62.9221 | 83 | 73.5568 | 95 | 84.1916 |
| $\frac{1}{4}$ | 63.1437 | $\frac{1}{4}$ | 73.7784 | $\frac{1}{4}$ | 84.4131 |
| $\frac{1}{2}$ | 63.3652 | $\frac{1}{2}$ | 73.9999 | $\frac{1}{2}$ | 84.6347 |
| $\frac{3}{4}$ | 63.5868 | $\frac{3}{4}$ | 74.2215 | $\frac{3}{4}$ | 84.8562 |
| 72 | 63.8083 | 84 | 74.4431 | 96 | 85.0778 |
| $\frac{1}{4}$ | 64.0299 | $\frac{1}{4}$ | 74.6647 | $\frac{1}{4}$ | 85.2993 |
| $\frac{1}{2}$ | 64.2514 | $\frac{1}{2}$ | 74.8862 | $\frac{1}{2}$ | 85.5209 |
| $\frac{3}{4}$ | 64.4730 | $\frac{3}{4}$ | 75.1077 | $\frac{3}{4}$ | 85.7425 |
| 73 | 64.6946 | 85 | 75.3293 | 97 | 85.9646 |
| $\frac{1}{4}$ | 64.9161 | $\frac{1}{4}$ | 75.5508 | $\frac{1}{4}$ | 86.185 |
| $\frac{1}{2}$ | 65.1377 | $\frac{1}{2}$ | 75.7724 | $\frac{1}{2}$ | 86.4071 |
| $\frac{3}{4}$ | 65.3592 | $\frac{3}{4}$ | 75.9934 | $\frac{3}{4}$ | 86.6289 |
| 74 | 65.5808 | 86 | 76.2155 | 98 | 86.8502 |
| $\frac{1}{4}$ | 65.8023 | $\frac{1}{4}$ | 76.4371 | $\frac{1}{4}$ | 87.0718 |
| $\frac{1}{2}$ | 66.0239 | $\frac{1}{2}$ | 76.6586 | $\frac{1}{2}$ | 87.2933 |
| $\frac{3}{4}$ | 66.2455 | $\frac{3}{4}$ | 76.8802 | $\frac{3}{4}$ | 87.5149 |
| 75 | 66.467 | 87 | 77.1017 | 99 | 87.7364 |
| $\frac{1}{4}$ | 66.6886 | $\frac{1}{4}$ | 77.3233 | $\frac{1}{4}$ | 87.958 |
| $\frac{1}{2}$ | 66.9104 | $\frac{1}{2}$ | 77.5449 | $\frac{1}{2}$ | 88.1796 |
| $\frac{3}{4}$ | 67.1317 | $\frac{3}{4}$ | 77.7664 | $\frac{3}{4}$ | 88.4011 |
| 76 | 67.3532 | 88 | 77.988 | 100 | 88.6227 |
| $\frac{1}{4}$ | 67.5748 | $\frac{1}{4}$ | 78.2095 | | |
| $\frac{1}{2}$ | 67.7964 | $\frac{1}{2}$ | 78.4316 | | |
| $\frac{3}{4}$ | 68.0179 | $\frac{3}{4}$ | 78.6526 | | |

Application of Table.

n a Square that has same Area as a Given Circle.

of a square that has same area as a circle of 73.25 ins. is required
 as, page 233, opposite to 73.25 is 4214.11; and in this table
 side of a square having same area as a circle of that diameter

Lengths of Circular Arcs, up to a Semicircle.

Diameter of a Circle = 1, and divided into 1000 equal Parts.

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|--------|---------|--------|---------|--------|---------|--------|---------|--------|---------|
| .1 | 1.02645 | .15 | 1.05896 | .2 | 1.10348 | .25 | 1.15912 | .3 | 1.22495 |
| .101 | 1.02698 | .151 | 1.05973 | .201 | 1.10447 | .251 | 1.16033 | .301 | 1.22638 |
| .102 | 1.02752 | .152 | 1.06051 | .202 | 1.10548 | .252 | 1.16157 | .302 | 1.22777 |
| .103 | 1.02806 | .153 | 1.0613 | .203 | 1.1065 | .253 | 1.16279 | .303 | 1.22911 |
| .104 | 1.0286 | .154 | 1.06209 | .204 | 1.10752 | .254 | 1.16402 | .304 | 1.23046 |
| .105 | 1.02914 | .155 | 1.06288 | .205 | 1.10855 | .255 | 1.16526 | .305 | 1.2318 |
| .106 | 1.0297 | .156 | 1.06368 | .206 | 1.10958 | .256 | 1.16649 | .306 | 1.23314 |
| .107 | 1.03026 | .157 | 1.06449 | .207 | 1.11062 | .257 | 1.16774 | .307 | 1.23449 |
| .108 | 1.03082 | .158 | 1.0653 | .208 | 1.11165 | .258 | 1.16899 | .308 | 1.23583 |
| .109 | 1.03139 | .159 | 1.06611 | .209 | 1.11269 | .259 | 1.17024 | .309 | 1.23718 |
| .11 | 1.03196 | .16 | 1.06693 | .21 | 1.11374 | .26 | 1.1715 | .31 | 1.23852 |
| .111 | 1.03254 | .161 | 1.06775 | .211 | 1.11479 | .261 | 1.17275 | .311 | 1.2407 |
| .112 | 1.03312 | .162 | 1.06858 | .212 | 1.11584 | .262 | 1.17401 | .312 | 1.24211 |
| .113 | 1.03371 | .163 | 1.06941 | .213 | 1.11692 | .263 | 1.17527 | .313 | 1.24346 |
| .114 | 1.0343 | .164 | 1.07025 | .214 | 1.11796 | .264 | 1.17655 | .314 | 1.2448 |
| .115 | 1.0349 | .165 | 1.07109 | .215 | 1.11904 | .265 | 1.17784 | .315 | 1.24615 |
| .116 | 1.03551 | .166 | 1.07194 | .216 | 1.12011 | .266 | 1.17912 | .316 | 1.2475 |
| .117 | 1.03611 | .167 | 1.07279 | .217 | 1.12118 | .267 | 1.1804 | .317 | 1.24894 |
| .118 | 1.03672 | .168 | 1.07365 | .218 | 1.12225 | .268 | 1.18162 | .318 | 1.25028 |
| .119 | 1.03734 | .169 | 1.07451 | .219 | 1.12334 | .269 | 1.18294 | .319 | 1.25163 |
| .12 | 1.03797 | .17 | 1.07537 | .22 | 1.12445 | .27 | 1.18428 | .32 | 1.253 |
| .121 | 1.0386 | .171 | 1.07624 | .221 | 1.12556 | .271 | 1.18557 | .321 | 1.255 |
| .122 | 1.03923 | .172 | 1.07711 | .222 | 1.12663 | .272 | 1.18688 | .322 | 1.2564 |
| .123 | 1.03987 | .173 | 1.07799 | .223 | 1.12774 | .273 | 1.18819 | .323 | 1.25781 |
| .124 | 1.04051 | .174 | 1.07888 | .224 | 1.12885 | .274 | 1.18969 | .324 | 1.25918 |
| .125 | 1.04116 | .175 | 1.07977 | .225 | 1.12997 | .275 | 1.19082 | .325 | 1.26055 |
| .126 | 1.04181 | .176 | 1.08066 | .226 | 1.13108 | .276 | 1.19214 | .326 | 1.26192 |
| .127 | 1.04247 | .177 | 1.08156 | .227 | 1.13219 | .277 | 1.19345 | .327 | 1.26329 |
| .128 | 1.04313 | .178 | 1.08246 | .228 | 1.13331 | .278 | 1.19477 | .328 | 1.26465 |
| .129 | 1.0438 | .179 | 1.08337 | .229 | 1.13444 | .279 | 1.1961 | .329 | 1.26601 |
| .13 | 1.04447 | .18 | 1.08428 | .23 | 1.13557 | .28 | 1.19743 | .33 | 1.26738 |
| .131 | 1.04515 | .181 | 1.08519 | .231 | 1.13671 | .281 | 1.19887 | .331 | 1.26874 |
| .132 | 1.04584 | .182 | 1.08611 | .232 | 1.13786 | .282 | 1.20011 | .332 | 1.27011 |
| .133 | 1.04652 | .183 | 1.08704 | .233 | 1.13903 | .283 | 1.20146 | .333 | 1.27147 |
| .134 | 1.04722 | .184 | 1.08797 | .234 | 1.1402 | .284 | 1.20282 | .334 | 1.27283 |
| .135 | 1.04792 | .185 | 1.0889 | .235 | 1.14136 | .285 | 1.20419 | .335 | 1.27419 |
| .136 | 1.04862 | .186 | 1.08984 | .236 | 1.14247 | .286 | 1.20558 | .336 | 1.27555 |
| .137 | 1.04932 | .187 | 1.09079 | .237 | 1.14363 | .287 | 1.20696 | .337 | 1.27691 |
| .138 | 1.05003 | .188 | 1.09174 | .238 | 1.1448 | .288 | 1.20828 | .338 | 1.27827 |
| .139 | 1.05075 | .189 | 1.09269 | .239 | 1.14597 | .289 | 1.20967 | .339 | 1.27963 |
| .14 | 1.05147 | .19 | 1.09365 | .24 | 1.14714 | .29 | 1.21202 | .34 | 1.281 |
| .141 | 1.0522 | .191 | 1.09461 | .241 | 1.14831 | .291 | 1.21239 | .341 | 1.28236 |
| .142 | 1.05293 | .192 | 1.09557 | .242 | 1.14949 | .292 | 1.21381 | .342 | 1.28372 |
| .143 | 1.05367 | .193 | 1.09654 | .243 | 1.15067 | .293 | 1.2152 | .343 | 1.28508 |
| .144 | 1.05441 | .194 | 1.09752 | .244 | 1.15186 | .294 | 1.21658 | .344 | 1.28644 |
| .145 | 1.05516 | .195 | 1.0985 | .245 | 1.15308 | .295 | 1.21794 | .345 | 1.2878 |
| .146 | 1.05591 | .196 | 1.09949 | .246 | 1.15429 | .296 | 1.21926 | .346 | 1.28916 |
| .147 | 1.05667 | .197 | 1.10048 | .247 | 1.15549 | .297 | 1.22061 | .347 | 1.29052 |
| .148 | 1.05743 | .198 | 1.10147 | .248 | 1.1567 | .298 | 1.22203 | .348 | 1.29188 |
| .149 | 1.05819 | .199 | 1.10247 | .249 | 1.15791 | .299 | 1.22347 | .349 | 1.29324 |

| ht. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|-----|----------|--------|----------|--------|----------|--------|----------|--------|----------|
| 1 | 1.299 97 | .38 | 1.348 99 | .41 | 1.400 77 | .44 | 1.455 12 | .47 | 1.511 85 |
| 2 | 1.301 56 | .381 | 1.350 68 | .411 | 1.402 54 | .441 | 1.456 97 | .471 | 1.513 78 |
| 3 | 1.303 15 | .382 | 1.352 37 | .412 | 1.404 32 | .442 | 1.458 83 | .472 | 1.515 71 |
| 4 | 1.304 74 | .383 | 1.354 06 | .413 | 1.406 1 | .443 | 1.460 69 | .473 | 1.517 64 |
| 5 | 1.306 34 | .384 | 1.355 75 | .414 | 1.407 88 | .444 | 1.462 55 | .474 | 1.519 58 |
| 6 | 1.307 94 | .385 | 1.357 44 | .415 | 1.409 66 | .445 | 1.464 41 | .475 | 1.521 52 |
| 7 | 1.309 54 | .386 | 1.359 14 | .416 | 1.411 45 | .446 | 1.466 28 | .476 | 1.523 46 |
| 8 | 1.311 15 | .387 | 1.360 84 | .417 | 1.413 24 | .447 | 1.468 15 | .477 | 1.525 41 |
| 9 | 1.312 76 | .388 | 1.362 54 | .418 | 1.415 03 | .448 | 1.470 02 | .478 | 1.527 36 |
| 10 | 1.314 37 | .389 | 1.364 25 | .419 | 1.416 82 | .449 | 1.471 89 | .479 | 1.529 31 |
| 11 | 1.315 99 | .39 | 1.365 96 | .42 | 1.418 61 | .45 | 1.473 77 | .48 | 1.531 26 |
| 12 | 1.317 61 | .391 | 1.367 67 | .421 | 1.420 41 | .451 | 1.475 65 | .481 | 1.533 22 |
| 13 | 1.319 23 | .392 | 1.369 39 | .422 | 1.422 22 | .452 | 1.477 53 | .482 | 1.535 18 |
| 14 | 1.320 86 | .393 | 1.371 11 | .423 | 1.424 02 | .453 | 1.479 42 | .483 | 1.537 14 |
| 15 | 1.322 49 | .394 | 1.372 83 | .424 | 1.425 83 | .454 | 1.481 31 | .484 | 1.539 1 |
| 16 | 1.324 13 | .395 | 1.374 55 | .425 | 1.427 64 | .455 | 1.483 2 | .485 | 1.541 06 |
| 17 | 1.325 77 | .396 | 1.376 28 | .426 | 1.429 45 | .456 | 1.485 09 | .486 | 1.543 02 |
| 18 | 1.327 41 | .397 | 1.378 01 | .427 | 1.431 27 | .457 | 1.486 99 | .487 | 1.544 99 |
| 19 | 1.329 05 | .398 | 1.379 74 | .428 | 1.433 09 | .458 | 1.488 89 | .488 | 1.546 96 |
| 20 | 1.330 69 | .399 | 1.381 48 | .429 | 1.434 91 | .459 | 1.490 79 | .489 | 1.548 93 |
| 21 | 1.332 34 | .4 | 1.383 22 | .43 | 1.436 73 | .46 | 1.492 69 | .49 | 1.550 9 |
| 22 | 1.333 99 | .401 | 1.384 96 | .431 | 1.438 56 | .461 | 1.494 5 | .491 | 1.552 88 |
| 23 | 1.335 64 | .402 | 1.386 71 | .432 | 1.440 39 | .462 | 1.496 51 | .492 | 1.554 86 |
| 24 | 1.337 3 | .403 | 1.388 46 | .433 | 1.442 22 | .463 | 1.498 42 | .493 | 1.556 85 |
| 25 | 1.338 96 | .404 | 1.390 21 | .434 | 1.444 05 | .464 | 1.500 33 | .494 | 1.558 84 |
| 26 | 1.340 63 | .405 | 1.391 96 | .435 | 1.445 89 | .465 | 1.502 24 | .495 | 1.560 83 |
| 27 | 1.342 29 | .406 | 1.393 72 | .436 | 1.447 73 | .466 | 1.504 16 | .496 | 1.562 82 |
| 28 | 1.343 96 | .407 | 1.395 48 | .437 | 1.449 57 | .467 | 1.506 08 | .497 | 1.564 81 |
| 29 | 1.345 63 | .408 | 1.397 24 | .438 | 1.451 42 | .468 | 1.508 | .498 | 1.566 8 |
| 30 | 1.347 31 | .409 | 1.399 | .439 | 1.453 27 | .469 | 1.509 92 | .499 | 1.568 79 |

to Ascertain Length of an Arc of a Circle by preceding Table.

RULE.—Divide height by base, find quotient in column of heights, take length for that height opposite to it in next column on the right hand. Multiply length thus obtained by base of arc, and product will give length.

EXAMPLE.—What is length of an arc of a circle, base or span of it being 100 feet, and height 25?

$25 \div 100 = .25$; and .25, per table, = 1.159 12, length of base, which, multiplied by 100 = 115.912 feet.

When, in division of a height by base, the quotient has a remainder after third place of decimals, and great accuracy is required.

RULE.—Take length for first three figures, subtract it from next following length; multiply remainder by this fractional remainder, add product to first length, and sum will give length for whole quotient.

EXAMPLE.—What is length of an arc of a circle, base of which is 35 feet, and height or versed sine 8 feet?

$8 \div 35 = .228 571 4$; tabular length for .228 = 1.133 31, and for .229 = 1.134 44, the difference between which is .001 13. Then $.5714 \times .001 13 = .000 645 682$.

$$\begin{array}{rcl} \text{Hence } .228 & = & 1.133 31, \\ \text{and } .000 571 4 & = & .000 645 682 \end{array}$$

1.133 955 682, the sum by which to be multiplied; and $1.133 955 682 \times 35 = 39.688 45$ feet.

| Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. | Diam. | Side of Sq. |
|-------------|---------------|-------------|---------------|-------------|---------------|-------------|
| 46.97 | 65 | 57.6047 | 77 | 68.2395 | 89 | 78.8742 |
| 47.1916 | $\frac{1}{8}$ | 57.8263 | $\frac{1}{8}$ | 68.461 | $\frac{1}{8}$ | 79.0957 |
| 47.4131 | $\frac{1}{4}$ | 58.0479 | $\frac{1}{4}$ | 68.6826 | $\frac{1}{4}$ | 79.3173 |
| 47.6347 | $\frac{3}{8}$ | 58.2694 | $\frac{3}{8}$ | 68.9041 | $\frac{3}{8}$ | 79.5389 |
| 47.8562 | 66 | 58.491 | 78 | 69.1257 | 90 | 79.7604 |
| 48.0778 | $\frac{1}{8}$ | 58.7125 | $\frac{1}{8}$ | 69.3473 | $\frac{1}{8}$ | 79.982 |
| 48.2994 | $\frac{1}{4}$ | 58.9341 | $\frac{1}{4}$ | 69.5688 | $\frac{1}{4}$ | 80.2035 |
| 48.5209 | $\frac{3}{8}$ | 59.1556 | $\frac{3}{8}$ | 69.7904 | $\frac{3}{8}$ | 80.4251 |
| 48.7425 | 67 | 59.3772 | 79 | 70.0119 | 91 | 80.6467 |
| 48.964 | $\frac{1}{8}$ | 59.5988 | $\frac{1}{8}$ | 70.2335 | $\frac{1}{8}$ | 80.8682 |
| 49.1856 | $\frac{1}{4}$ | 59.8203 | $\frac{1}{4}$ | 70.455 | $\frac{1}{4}$ | 81.0898 |
| 49.4071 | $\frac{3}{8}$ | 60.0419 | $\frac{3}{8}$ | 70.6766 | $\frac{3}{8}$ | 81.3113 |
| 49.6287 | 68 | 60.2634 | 80 | 70.8981 | 92 | 81.5329 |
| 49.8503 | $\frac{1}{8}$ | 60.485 | $\frac{1}{8}$ | 71.1197 | $\frac{1}{8}$ | 81.7544 |
| 50.0718 | $\frac{1}{4}$ | 60.7065 | $\frac{1}{4}$ | 71.3413 | $\frac{1}{4}$ | 81.976 |
| 50.2934 | $\frac{3}{8}$ | 60.9281 | $\frac{3}{8}$ | 71.5628 | $\frac{3}{8}$ | 82.1975 |
| 50.5149 | 69 | 61.1497 | 81 | 71.7844 | 93 | 82.4191 |
| 50.7365 | $\frac{1}{8}$ | 61.3712 | $\frac{1}{8}$ | 72.0059 | $\frac{1}{8}$ | 82.6407 |
| 50.958 | $\frac{1}{4}$ | 61.5928 | $\frac{1}{4}$ | 72.2275 | $\frac{1}{4}$ | 82.8622 |
| 51.1796 | $\frac{3}{8}$ | 61.8143 | $\frac{3}{8}$ | 72.4491 | $\frac{3}{8}$ | 83.0838 |
| 51.4012 | 70 | 62.0359 | 82 | 72.6706 | 94 | 83.3053 |
| 51.6227 | $\frac{1}{8}$ | 62.2574 | $\frac{1}{8}$ | 72.8921 | $\frac{1}{8}$ | 83.5269 |
| 51.8443 | $\frac{1}{4}$ | 62.479 | $\frac{1}{4}$ | 73.1137 | $\frac{1}{4}$ | 83.7484 |
| 52.0658 | $\frac{3}{8}$ | 62.7006 | $\frac{3}{8}$ | 73.3353 | $\frac{3}{8}$ | 83.97 |
| 52.2874 | 71 | 62.9221 | 83 | 73.5568 | 95 | 84.1916 |
| 52.5089 | $\frac{1}{8}$ | 63.1437 | $\frac{1}{8}$ | 73.7784 | $\frac{1}{8}$ | 84.4131 |
| 52.7305 | $\frac{1}{4}$ | 63.3652 | $\frac{1}{4}$ | 73.9999 | $\frac{1}{4}$ | 84.6347 |
| 52.9521 | $\frac{3}{8}$ | 63.5868 | $\frac{3}{8}$ | 74.2215 | $\frac{3}{8}$ | 84.8562 |
| 53.1736 | 72 | 63.8083 | 84 | 74.4431 | 96 | 85.0778 |
| 53.3952 | $\frac{1}{8}$ | 64.0299 | $\frac{1}{8}$ | 74.6647 | $\frac{1}{8}$ | 85.2993 |
| 53.6167 | $\frac{1}{4}$ | 64.2514 | $\frac{1}{4}$ | 74.8862 | $\frac{1}{4}$ | 85.5209 |
| 53.8383 | $\frac{3}{8}$ | 64.4730 | $\frac{3}{8}$ | 75.1077 | $\frac{3}{8}$ | 85.7425 |
| 54.0598 | 73 | 64.6946 | 85 | 75.3293 | 97 | 85.9646 |
| 54.2814 | $\frac{1}{8}$ | 64.9161 | $\frac{1}{8}$ | 75.5508 | $\frac{1}{8}$ | 86.185 |
| 54.503 | $\frac{1}{4}$ | 65.1377 | $\frac{1}{4}$ | 75.7724 | $\frac{1}{4}$ | 86.4071 |
| 54.7245 | $\frac{3}{8}$ | 65.3592 | $\frac{3}{8}$ | 75.9934 | $\frac{3}{8}$ | 86.6289 |
| 54.9461 | 74 | 65.5808 | 86 | 76.2155 | 98 | 86.8502 |
| 55.1676 | $\frac{1}{8}$ | 65.8023 | $\frac{1}{8}$ | 76.4371 | $\frac{1}{8}$ | 87.0718 |
| 55.3892 | $\frac{1}{4}$ | 66.0239 | $\frac{1}{4}$ | 76.6586 | $\frac{1}{4}$ | 87.2933 |
| 55.6107 | $\frac{3}{8}$ | 66.2455 | $\frac{3}{8}$ | 76.8802 | $\frac{3}{8}$ | 87.5149 |
| 55.8323 | 75 | 66.467 | 87 | 77.1017 | 99 | 87.7364 |
| 56.0538 | $\frac{1}{8}$ | 66.6886 | $\frac{1}{8}$ | 77.3233 | $\frac{1}{8}$ | 87.958 |
| 56.2754 | $\frac{1}{4}$ | 66.9104 | $\frac{1}{4}$ | 77.5449 | $\frac{1}{4}$ | 88.1796 |
| 56.497 | $\frac{3}{8}$ | 67.1317 | $\frac{3}{8}$ | 77.7664 | $\frac{3}{8}$ | 3.4011 |
| 56.7185 | 76 | 67.3532 | 88 | 77.988 | | 6227 |
| 56.9401 | $\frac{1}{8}$ | 67.5748 | $\frac{1}{8}$ | 78.2095 | | |
| 57.1616 | $\frac{1}{4}$ | 67.7964 | $\frac{1}{4}$ | 78.4316 | | |
| 57.3832 | $\frac{3}{8}$ | 68.0179 | $\frac{3}{8}$ | 78.6526 | | |

Application of Table.

Ascertain a Square that has same Area as Circle.

a.—If side of a square that has same area as a circle of 7; Table of Areas, page 233, opposite to 73.25 is 4214.11; 1; which is side of a square having same area as a circle of

Lengths of Circular Arcs, up to a Semicircle.

Diameter of a Circle = 1, and divided into 1000 equal Parts.

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|--------|---------|--------|---------|--------|---------|--------|---------|--------|---------|
| .1 | 1.02645 | .15 | 1.05896 | .2 | 1.10348 | .25 | 1.15912 | .3 | 1.22495 |
| .101 | 1.02698 | .151 | 1.05973 | .201 | 1.10447 | .251 | 1.16033 | .301 | 1.22635 |
| .102 | 1.02752 | .152 | 1.06051 | .202 | 1.10548 | .252 | 1.16157 | .302 | 1.22776 |
| .103 | 1.02806 | .153 | 1.0613 | .203 | 1.1065 | .253 | 1.16279 | .303 | 1.22918 |
| .104 | 1.0286 | .154 | 1.06209 | .204 | 1.10752 | .254 | 1.16402 | .304 | 1.23061 |
| .105 | 1.02914 | .155 | 1.06288 | .205 | 1.10855 | .255 | 1.16526 | .305 | 1.23205 |
| .106 | 1.0297 | .156 | 1.06368 | .206 | 1.10958 | .256 | 1.16649 | .306 | 1.23349 |
| .107 | 1.03026 | .157 | 1.06449 | .207 | 1.11062 | .257 | 1.16774 | .307 | 1.23494 |
| .108 | 1.03082 | .158 | 1.0653 | .208 | 1.11165 | .258 | 1.16899 | .308 | 1.23636 |
| .109 | 1.03139 | .159 | 1.06611 | .209 | 1.11269 | .259 | 1.17024 | .309 | 1.2378 |
| .11 | 1.03196 | .16 | 1.06693 | .21 | 1.11374 | .26 | 1.1715 | .31 | 1.23925 |
| .111 | 1.03254 | .161 | 1.06775 | .211 | 1.11479 | .261 | 1.17275 | .311 | 1.2407 |
| .112 | 1.03312 | .162 | 1.06858 | .212 | 1.11584 | .262 | 1.17401 | .312 | 1.24216 |
| .113 | 1.03371 | .163 | 1.06941 | .213 | 1.11692 | .263 | 1.17527 | .313 | 1.2436 |
| .114 | 1.0343 | .164 | 1.07025 | .214 | 1.11796 | .264 | 1.17655 | .314 | 1.24506 |
| .115 | 1.0349 | .165 | 1.07109 | .215 | 1.11904 | .265 | 1.17784 | .315 | 1.24654 |
| .116 | 1.03551 | .166 | 1.07194 | .216 | 1.12011 | .266 | 1.17912 | .316 | 1.24801 |
| .117 | 1.03611 | .167 | 1.07279 | .217 | 1.12118 | .267 | 1.1804 | .317 | 1.24949 |
| .118 | 1.03672 | .168 | 1.07365 | .218 | 1.12225 | .268 | 1.18162 | .318 | 1.25095 |
| .119 | 1.03734 | .169 | 1.07451 | .219 | 1.12334 | .269 | 1.18294 | .319 | 1.25243 |
| .12 | 1.03797 | .17 | 1.07537 | .22 | 1.12445 | .27 | 1.18428 | .32 | 1.25391 |
| .121 | 1.0386 | .171 | 1.07624 | .221 | 1.12556 | .271 | 1.18557 | .321 | 1.25539 |
| .122 | 1.03923 | .172 | 1.07711 | .222 | 1.12663 | .272 | 1.18688 | .322 | 1.25686 |
| .123 | 1.03987 | .173 | 1.07799 | .223 | 1.12774 | .273 | 1.18819 | .323 | 1.25836 |
| .124 | 1.04051 | .174 | 1.07888 | .224 | 1.12885 | .274 | 1.18960 | .324 | 1.25987 |
| .125 | 1.04116 | .175 | 1.07977 | .225 | 1.12997 | .275 | 1.19082 | .325 | 1.26137 |
| .126 | 1.04181 | .176 | 1.08066 | .226 | 1.13108 | .276 | 1.19214 | .326 | 1.26286 |
| .127 | 1.04247 | .177 | 1.08156 | .227 | 1.13219 | .277 | 1.19345 | .327 | 1.26437 |
| .128 | 1.04313 | .178 | 1.08246 | .228 | 1.13331 | .278 | 1.19477 | .328 | 1.26588 |
| .129 | 1.0438 | .179 | 1.08337 | .229 | 1.13444 | .279 | 1.1961 | .329 | 1.2674 |
| .13 | 1.04447 | .18 | 1.08428 | .23 | 1.13557 | .28 | 1.19743 | .33 | 1.26892 |
| .131 | 1.04515 | .181 | 1.08519 | .231 | 1.13671 | .281 | 1.19887 | .331 | 1.27044 |
| .132 | 1.04584 | .182 | 1.08611 | .232 | 1.13786 | .282 | 1.20011 | .332 | 1.27196 |
| .133 | 1.04652 | .183 | 1.08704 | .233 | 1.13903 | .283 | 1.20146 | .333 | 1.27349 |
| .134 | 1.04722 | .184 | 1.08797 | .234 | 1.1402 | .284 | 1.20282 | .334 | 1.27502 |
| .135 | 1.04792 | .185 | 1.0889 | .235 | 1.14136 | .285 | 1.20419 | .335 | 1.27656 |
| .136 | 1.04862 | .186 | 1.08984 | .236 | 1.14247 | .286 | 1.20558 | .336 | 1.2781 |
| .137 | 1.04932 | .187 | 1.09079 | .237 | 1.14363 | .287 | 1.20696 | .337 | 1.27964 |
| .138 | 1.05003 | .188 | 1.09174 | .238 | 1.1448 | .288 | 1.20828 | .338 | 1.28118 |
| .139 | 1.05075 | .189 | 1.09269 | .239 | 1.14597 | .289 | 1.20967 | .339 | 1.28273 |
| .14 | 1.05147 | .19 | 1.09365 | .24 | 1.14714 | .29 | 1.21202 | .34 | 1.28428 |
| .141 | 1.0522 | .191 | 1.09461 | .241 | 1.14831 | .291 | 1.21239 | .341 | 1.28583 |
| .142 | 1.05293 | .192 | 1.09557 | .242 | 1.14949 | .292 | 1.21381 | .342 | 1.28739 |
| .143 | 1.05367 | .193 | 1.09654 | .243 | 1.15067 | .293 | 1.2152 | .343 | 1.28895 |
| .144 | 1.05441 | .194 | 1.09752 | .244 | 1.15186 | .294 | 1.21658 | .344 | 1.29052 |
| .145 | 1.05516 | .195 | 1.0985 | .245 | 1.15308 | .295 | 1.21794 | .345 | 1.29209 |
| | 75591 | .196 | 1.09949 | .246 | 1.15429 | .296 | 1.21926 | .346 | 1.29366 |
| | | .197 | 1.10048 | .247 | 1.15549 | .297 | 1.22061 | .347 | 1.29523 |
| | | .198 | 1.10147 | .248 | 1.1567 | .298 | 1.22203 | .348 | 1.29681 |
| | | .199 | 1.10247 | .249 | 1.15791 | .299 | 1.22347 | .349 | 1.29839 |

| ht. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|-----|----------|--------|----------|--------|----------|--------|----------|--------|----------|
| 5 | 1.299 97 | .38 | 1.348 99 | .41 | 1.400 77 | .44 | 1.455 12 | .47 | 1.511 85 |
| 51 | 1.301 56 | .381 | 1.350 68 | .411 | 1.402 54 | .441 | 1.456 97 | .471 | 1.513 78 |
| 52 | 1.303 15 | .382 | 1.352 37 | .412 | 1.404 32 | .442 | 1.458 83 | .472 | 1.515 71 |
| 53 | 1.304 74 | .383 | 1.354 06 | .413 | 1.406 1 | .443 | 1.460 69 | .473 | 1.517 64 |
| 54 | 1.306 34 | .384 | 1.355 75 | .414 | 1.407 88 | .444 | 1.462 55 | .474 | 1.519 58 |
| 55 | 1.307 94 | .385 | 1.357 44 | .415 | 1.409 66 | .445 | 1.464 41 | .475 | 1.521 52 |
| 56 | 1.309 54 | .386 | 1.359 14 | .416 | 1.411 45 | .446 | 1.466 28 | .476 | 1.523 46 |
| 57 | 1.311 15 | .387 | 1.360 84 | .417 | 1.413 24 | .447 | 1.468 15 | .477 | 1.525 41 |
| 58 | 1.312 76 | .388 | 1.362 54 | .418 | 1.415 03 | .448 | 1.470 02 | .478 | 1.527 36 |
| 59 | 1.314 37 | .389 | 1.364 25 | .419 | 1.416 82 | .449 | 1.471 89 | .479 | 1.529 31 |
| 6 | 1.315 99 | .39 | 1.365 96 | .42 | 1.418 61 | .45 | 1.473 77 | .48 | 1.531 26 |
| 61 | 1.317 61 | .391 | 1.367 67 | .421 | 1.420 41 | .451 | 1.475 65 | .481 | 1.533 22 |
| 62 | 1.319 23 | .392 | 1.369 39 | .422 | 1.422 22 | .452 | 1.477 53 | .482 | 1.535 18 |
| 63 | 1.320 86 | .393 | 1.371 11 | .423 | 1.424 02 | .453 | 1.479 42 | .483 | 1.537 14 |
| 64 | 1.322 49 | .394 | 1.372 83 | .424 | 1.425 83 | .454 | 1.481 31 | .484 | 1.539 1 |
| 65 | 1.324 13 | .395 | 1.374 55 | .425 | 1.427 64 | .455 | 1.483 2 | .485 | 1.541 06 |
| 66 | 1.325 77 | .396 | 1.376 28 | .426 | 1.429 45 | .456 | 1.485 09 | .486 | 1.543 02 |
| 67 | 1.327 41 | .397 | 1.378 01 | .427 | 1.431 27 | .457 | 1.486 99 | .487 | 1.544 99 |
| 68 | 1.329 05 | .398 | 1.379 74 | .428 | 1.433 09 | .458 | 1.488 89 | .488 | 1.546 96 |
| 69 | 1.330 69 | .399 | 1.381 48 | .429 | 1.434 91 | .459 | 1.490 79 | .489 | 1.548 93 |
| 7 | 1.332 34 | .4 | 1.383 22 | .43 | 1.436 73 | .46 | 1.492 69 | .49 | 1.550 9 |
| 71 | 1.333 99 | .401 | 1.384 96 | .431 | 1.438 56 | .461 | 1.494 5 | .491 | 1.552 88 |
| 72 | 1.335 64 | .402 | 1.386 71 | .432 | 1.440 39 | .462 | 1.496 51 | .492 | 1.554 86 |
| 73 | 1.337 3 | .403 | 1.388 46 | .433 | 1.442 22 | .463 | 1.498 42 | .493 | 1.556 85 |
| 74 | 1.338 96 | .404 | 1.390 21 | .434 | 1.444 05 | .464 | 1.500 33 | .494 | 1.558 84 |
| 75 | 1.340 63 | .405 | 1.391 96 | .435 | 1.445 89 | .465 | 1.502 24 | .495 | 1.560 83 |
| 76 | 1.342 29 | .406 | 1.393 72 | .436 | 1.447 73 | .466 | 1.504 16 | .496 | 1.562 82 |
| 77 | 1.343 96 | .407 | 1.395 48 | .437 | 1.449 57 | .467 | 1.506 08 | .497 | 1.564 81 |
| 78 | 1.345 63 | .408 | 1.397 24 | .438 | 1.451 42 | .468 | 1.508 | .498 | 1.566 8 |
| 79 | 1.347 31 | .409 | 1.399 | .439 | 1.453 27 | .469 | 1.509 92 | .499 | 1.568 79 |
| 8 | | | | | | | | .5 | 1.570 79 |

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$100 \div 25 = .25$; and .25, per table, = 1.159 12, length of base, which, multiplied by = 115.912 feet.

When, in division of a height by base, the quotient has a remainder after a place of decimals, and great accuracy is required.

RULE.—Take length for first three figures, subtract it from next following *yth*; multiply remainder by this fractional remainder, add product to *length*, and sum will give length for whole quotient.

EXAMPLE.—What is length of an arc of a circle, base of which is 35 feet, and ht or versed sine 8 feet?

$35 \div 8 = .228$ 571 4; tabular length for .228 = 1.133 31, 229 = 1.134 44,
 difference between which is .001 13. Then .5714 X .001, = .682.

Hence .228 = 1.133 31,
 and .000 571 4 = .000 645 682

1.133 955 682
 is to be multiplied; and 1.133 955 682 X 35 = 39.688 45

Lengths of Circular Arcs from 1° to 180° .

(Radius = 1.)

| Degrees. | Length. | Degrees. | Length. | Degrees. | Length. | Degrees. | Length. |
|----------|---------|----------|---------|----------|---------|----------|---------|
| 1 | .0174 | 46 | .8028 | 91 | 1.5882 | 136 | 2.3736 |
| 2 | .0349 | 47 | .8203 | 92 | 1.6057 | 137 | 2.3911 |
| 3 | .0524 | 48 | .8377 | 93 | 1.6231 | 138 | 2.4085 |
| 4 | .0698 | 49 | .8552 | 94 | 1.6406 | 139 | 2.426 |
| 5 | .0873 | 50 | .8727 | 95 | 1.6581 | 140 | 2.4435 |
| 6 | .1147 | 51 | .8901 | 96 | 1.6755 | 141 | 2.4609 |
| 7 | .1222 | 52 | .9076 | 97 | 1.693 | 142 | 2.4784 |
| 8 | .1396 | 53 | .925 | 98 | 1.7104 | 143 | 2.4958 |
| 9 | .1571 | 54 | .9424 | 99 | 1.7279 | 144 | 2.5133 |
| 10 | .1745 | 55 | .9599 | 100 | 1.7453 | 145 | 2.5307 |
| 11 | .192 | 56 | .9774 | 101 | 1.7628 | 146 | 2.5482 |
| 12 | .2094 | 57 | .9948 | 102 | 1.7802 | 147 | 2.5656 |
| 13 | .2269 | 58 | 1.0123 | 103 | 1.7977 | 148 | 2.5831 |
| 14 | .2443 | 59 | 1.0297 | 104 | 1.8151 | 149 | 2.6005 |
| 15 | .2618 | 60 | 1.0472 | 105 | 1.8326 | 150 | 2.618 |
| 16 | .2792 | 61 | 1.0646 | 106 | 1.85 | 151 | 2.6354 |
| 17 | .2967 | 62 | 1.0821 | 107 | 1.8675 | 152 | 2.6529 |
| 18 | .3141 | 63 | 1.0995 | 108 | 1.8849 | 153 | 2.6703 |
| 19 | .3316 | 64 | 1.117 | 109 | 1.9024 | 154 | 2.6878 |
| 20 | .3491 | 65 | 1.1345 | 110 | 1.9199 | 155 | 2.7053 |
| 21 | .3665 | 66 | 1.1519 | 111 | 1.9373 | 156 | 2.7227 |
| 22 | .384 | 67 | 1.1694 | 112 | 1.9548 | 157 | 2.7402 |
| 23 | .4014 | 68 | 1.1868 | 113 | 1.9722 | 158 | 2.7576 |
| 24 | .4189 | 69 | 1.2043 | 114 | 1.9897 | 159 | 2.7751 |
| 25 | .4363 | 70 | 1.2217 | 115 | 2.0071 | 160 | 2.7925 |
| 26 | .4538 | 71 | 1.2392 | 116 | 2.0246 | 161 | 2.81 |
| 27 | .4712 | 72 | 1.2566 | 117 | 2.042 | 162 | 2.8274 |
| 28 | .4887 | 73 | 1.2741 | 118 | 2.0595 | 163 | 2.8449 |
| 29 | .5061 | 74 | 1.2915 | 119 | 2.0769 | 164 | 2.8623 |
| 30 | .5236 | 75 | 1.309 | 120 | 2.0944 | 165 | 2.8798 |
| 31 | .541 | 76 | 1.3264 | 121 | 2.1118 | 166 | 2.8972 |
| 32 | .5585 | 77 | 1.3439 | 122 | 2.1293 | 167 | 2.9147 |
| 33 | .5759 | 78 | 1.3613 | 123 | 2.1467 | 168 | 2.9321 |
| 34 | .5934 | 79 | 1.3788 | 124 | 2.1642 | 169 | 2.9496 |
| 35 | .6109 | 80 | 1.3963 | 125 | 2.1817 | 170 | 2.967 |
| 36 | .6283 | 81 | 1.4137 | 126 | 2.1991 | 171 | 2.9845 |
| 37 | .6458 | 82 | 1.4312 | 127 | 2.2166 | 172 | 3.002 |
| 38 | .6632 | 83 | 1.4486 | 128 | 2.2304 | 173 | 3.0194 |
| 39 | .6807 | 84 | 1.4661 | 129 | 2.2515 | 174 | 3.0369 |
| 40 | .6981 | 85 | 1.4835 | 130 | 2.2689 | 175 | 3.0543 |
| 41 | .7156 | 86 | 1.501 | 131 | 2.2864 | 176 | 3.0718 |
| 42 | .733 | 87 | 1.5184 | 132 | 2.3038 | 177 | 3.0892 |
| 43 | .7505 | 88 | 1.5359 | 133 | 2.3213 | 178 | 3.1067 |
| 44 | .7679 | 89 | 1.5533 | 134 | 2.3387 | 179 | 3.1241 |
| 45 | .7854 | 90 | 1.5708 | 135 | 2.3562 | 180 | 3.1416 |

Ascertain Length of a Circular Arc by Table.

From column opposite to degrees of arc, take length, and multiply by circumference of circle.

Degrees in an arc are 45° , and diameter of circle 5 feet.

= 1.9635 feet.

Lengths of Elliptic Arcs.

Up to a Semi-ellipse.

Transverse Diameter = 1, and divided into 1000 equal Parts.

| ht. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|-----|----------|--------|----------|--------|----------|--------|----------|--------|----------|
| | 1.041 62 | .15 | 1.093 33 | .2 | 1.150 14 | .25 | 1.211 36 | .3 | 1.276 69 |
| 31 | 1.042 62 | .151 | 1.094 48 | .201 | 1.151 31 | .251 | 1.212 63 | .301 | 1.278 03 |
| 32 | 1.043 62 | .152 | 1.095 58 | .202 | 1.152 48 | .252 | 1.213 9 | .302 | 1.279 37 |
| 33 | 1.044 62 | .153 | 1.096 69 | .203 | 1.153 66 | .253 | 1.215 17 | .303 | 1.280 71 |
| 34 | 1.045 62 | .154 | 1.097 8 | .204 | 1.154 84 | .254 | 1.216 44 | .304 | 1.282 05 |
| 35 | 1.046 62 | .155 | 1.098 91 | .205 | 1.156 02 | .255 | 1.217 72 | .305 | 1.283 39 |
| 36 | 1.047 62 | .156 | 1.100 02 | .206 | 1.157 2 | .256 | 1.219 | .306 | 1.284 74 |
| 37 | 1.048 62 | .157 | 1.101 13 | .207 | 1.158 38 | .257 | 1.220 28 | .307 | 1.286 09 |
| 38 | 1.049 62 | .158 | 1.102 24 | .208 | 1.159 57 | .258 | 1.221 56 | .308 | 1.287 44 |
| 39 | 1.050 63 | .159 | 1.103 35 | .209 | 1.160 76 | .259 | 1.222 84 | .309 | 1.288 79 |
| | 1.051 64 | .16 | 1.104 47 | .21 | 1.161 96 | .26 | 1.224 12 | .31 | 1.290 14 |
| 1 | 1.052 65 | .161 | 1.105 6 | .211 | 1.163 16 | .261 | 1.225 41 | .311 | 1.291 49 |
| 2 | 1.053 66 | .162 | 1.106 72 | .212 | 1.164 36 | .262 | 1.226 7 | .312 | 1.292 85 |
| 3 | 1.054 67 | .163 | 1.107 84 | .213 | 1.165 57 | .263 | 1.227 99 | .313 | 1.294 21 |
| 4 | 1.055 68 | .164 | 1.108 96 | .214 | 1.166 78 | .264 | 1.229 28 | .314 | 1.295 57 |
| 5 | 1.056 69 | .165 | 1.110 08 | .215 | 1.167 99 | .265 | 1.230 57 | .315 | 1.296 93 |
| 6 | 1.057 7 | .166 | 1.111 2 | .216 | 1.169 2 | .266 | 1.231 86 | .316 | 1.298 29 |
| 7 | 1.058 72 | .167 | 1.112 32 | .217 | 1.170 41 | .267 | 1.233 15 | .317 | 1.299 65 |
| 8 | 1.059 74 | .168 | 1.113 44 | .218 | 1.171 63 | .268 | 1.234 45 | .318 | 1.301 02 |
| 9 | 1.060 76 | .169 | 1.114 56 | .219 | 1.172 85 | .269 | 1.235 75 | .319 | 1.302 39 |
| | 1.061 78 | .17 | 1.115 69 | .22 | 1.174 07 | .27 | 1.237 05 | .32 | 1.303 76 |
| 1 | 1.062 8 | .171 | 1.116 82 | .221 | 1.175 29 | .271 | 1.238 35 | .321 | 1.305 13 |
| 2 | 1.063 82 | .172 | 1.117 95 | .222 | 1.176 51 | .272 | 1.239 66 | .322 | 1.306 5 |
| 3 | 1.064 84 | .173 | 1.119 08 | .223 | 1.177 74 | .273 | 1.240 97 | .323 | 1.307 87 |
| 4 | 1.065 86 | .174 | 1.120 21 | .224 | 1.178 97 | .274 | 1.242 28 | .324 | 1.309 24 |
| 5 | 1.066 89 | .175 | 1.121 34 | .225 | 1.180 2 | .275 | 1.243 59 | .325 | 1.310 61 |
| 5 | 1.067 92 | .176 | 1.122 47 | .226 | 1.181 43 | .276 | 1.244 8 | .326 | 1.311 98 |
| 7 | 1.068 95 | .177 | 1.123 6 | .227 | 1.182 66 | .277 | 1.246 12 | .327 | 1.313 35 |
| 3 | 1.069 98 | .178 | 1.124 73 | .228 | 1.183 9 | .278 | 1.247 44 | .328 | 1.314 72 |
| 9 | 1.070 01 | .179 | 1.125 86 | .229 | 1.185 14 | .279 | 1.248 76 | .329 | 1.316 1 |
| | 1.072 04 | .18 | 1.126 99 | .23 | 1.186 38 | .28 | 1.250 1 | .33 | 1.317 48 |
| 1 | 1.073 08 | .181 | 1.128 13 | .231 | 1.187 62 | .281 | 1.251 42 | .331 | 1.318 86 |
| 1 | 1.074 12 | .182 | 1.129 27 | .232 | 1.188 86 | .282 | 1.252 74 | .332 | 1.320 24 |
| 3 | 1.075 16 | .183 | 1.130 41 | .233 | 1.190 1 | .283 | 1.254 06 | .333 | 1.321 62 |
| 5 | 1.076 21 | .184 | 1.131 55 | .234 | 1.191 34 | .284 | 1.255 38 | .334 | 1.323 |
| 5 | 1.077 26 | .185 | 1.132 69 | .235 | 1.192 58 | .285 | 1.256 7 | .335 | 1.324 38 |
| 7 | 1.078 31 | .186 | 1.133 83 | .236 | 1.193 82 | .286 | 1.258 03 | .336 | 1.325 76 |
| 9 | 1.079 37 | .187 | 1.134 97 | .237 | 1.195 06 | .287 | 1.259 36 | .337 | 1.327 15 |
| 1 | 1.080 43 | .188 | 1.136 11 | .238 | 1.196 3 | .288 | 1.260 69 | .338 | 1.328 54 |
| 1 | 1.081 49 | .189 | 1.137 26 | .239 | 1.197 55 | .289 | 1.262 02 | .339 | 1.329 93 |
| | 1.082 55 | .19 | 1.138 41 | .24 | 1.198 8 | .29 | 1.263 35 | .34 | 1.331 32 |
| 1 | 1.083 62 | .191 | 1.139 56 | .241 | 1.200 05 | .291 | 1.264 68 | .341 | 1.332 72 |
| 1 | 1.084 69 | .192 | 1.140 71 | .242 | 1.201 3 | .292 | 1.266 | .342 | 1.334 12 |
| 3 | 1.085 76 | .193 | 1.141 86 | .243 | 1.202 55 | .293 | 1. | .343 | 1.335 52 |
| 5 | 1.086 84 | .194 | 1.143 01 | .244 | 1.203 8 | .294 | 1. | .344 | 1.336 92 |
| 5 | 1.087 92 | .195 | 1.144 16 | .245 | 1.205 06 | .295 | 1. | .345 | 1.338 33 |
| 7 | 1.089 01 | .196 | 1.145 31 | .246 | 1.206 32 | .296 | 1. | .346 | 1.339 74 |
| 9 | 1.090 1 | .197 | 1.146 46 | .247 | 1.207 58 | .297 | 1. | .347 | |
| 1 | 1.091 19 | .198 | 1.147 62 | .248 | 1.208 84 | .298 | 1. | .348 | |
| 1 | 1.092 28 | .199 | 1.148 88 | .249 | 1.210 1 | .299 | 1. | .349 | |

| H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|--------|----------|--------|----------|--------|----------|--------|----------|--------|----------|
| .35 | 1.345 39 | .405 | 1.425 33 | .46 | 1.508 42 | .515 | 1.594 08 | .57 | 1.680 36 |
| .351 | 1.346 81 | .406 | 1.426 81 | .461 | 1.509 96 | .516 | 1.595 64 | .571 | 1.681 95 |
| .352 | 1.348 23 | .407 | 1.428 29 | .462 | 1.511 5 | .517 | 1.597 2 | .572 | 1.683 54 |
| .353 | 1.349 65 | .408 | 1.429 77 | .463 | 1.513 04 | .518 | 1.598 76 | .573 | 1.685 13 |
| .354 | 1.351 08 | .409 | 1.431 25 | .464 | 1.514 58 | .519 | 1.600 32 | .574 | 1.686 72 |
| .355 | 1.352 51 | .41 | 1.432 73 | .465 | 1.516 12 | .52 | 1.601 88 | .575 | 1.688 31 |
| .356 | 1.353 94 | .411 | 1.434 21 | .466 | 1.517 66 | .521 | 1.603 44 | .576 | 1.689 9 |
| .357 | 1.355 37 | .412 | 1.435 69 | .467 | 1.519 2 | .522 | 1.605 | .577 | 1.691 49 |
| .358 | 1.356 8 | .413 | 1.437 18 | .468 | 1.520 74 | .523 | 1.606 56 | .578 | 1.693 08 |
| .359 | 1.358 23 | .414 | 1.438 67 | .469 | 1.522 29 | .524 | 1.608 12 | .579 | 1.694 67 |
| .36 | 1.359 67 | .415 | 1.440 16 | .47 | 1.523 84 | .525 | 1.609 68 | .58 | 1.696 26 |
| .361 | 1.361 11 | .416 | 1.441 65 | .471 | 1.525 39 | .526 | 1.611 24 | .581 | 1.697 85 |
| .362 | 1.362 55 | .417 | 1.443 14 | .472 | 1.526 91 | .527 | 1.612 8 | .582 | 1.699 45 |
| .363 | 1.363 99 | .418 | 1.444 63 | .473 | 1.528 49 | .528 | 1.614 36 | .583 | 1.701 05 |
| .364 | 1.365 43 | .419 | 1.446 13 | .474 | 1.530 04 | .529 | 1.615 92 | .584 | 1.702 64 |
| .365 | 1.366 88 | .42 | 1.447 63 | .475 | 1.531 59 | .53 | 1.617 48 | .585 | 1.704 24 |
| .366 | 1.368 33 | .421 | 1.449 13 | .476 | 1.533 14 | .531 | 1.619 04 | .586 | 1.705 84 |
| .367 | 1.369 78 | .422 | 1.450 64 | .477 | 1.534 69 | .532 | 1.620 6 | .587 | 1.707 45 |
| .368 | 1.371 23 | .423 | 1.452 14 | .478 | 1.536 25 | .533 | 1.622 16 | .588 | 1.709 05 |
| .369 | 1.372 68 | .424 | 1.453 64 | .479 | 1.537 81 | .534 | 1.623 72 | .589 | 1.710 65 |
| .37 | 1.374 14 | .425 | 1.455 15 | .48 | 1.539 37 | .535 | 1.625 28 | .59 | 1.712 25 |
| .371 | 1.376 62 | .426 | 1.456 65 | .481 | 1.540 93 | .536 | 1.626 84 | .591 | 1.713 86 |
| .372 | 1.377 08 | .427 | 1.458 15 | .482 | 1.542 49 | .537 | 1.628 4 | .592 | 1.715 46 |
| .373 | 1.378 54 | .428 | 1.459 66 | .483 | 1.544 05 | .538 | 1.629 96 | .593 | 1.717 07 |
| .374 | 1.38 | .429 | 1.461 67 | .484 | 1.545 61 | .539 | 1.631 52 | .594 | 1.718 68 |
| .375 | 1.381 46 | .43 | 1.462 68 | .485 | 1.547 18 | .54 | 1.633 09 | .595 | 1.720 29 |
| .376 | 1.382 92 | .431 | 1.464 19 | .486 | 1.548 75 | .541 | 1.634 65 | .596 | 1.721 9 |
| .377 | 1.384 39 | .432 | 1.465 7 | .487 | 1.550 32 | .542 | 1.636 23 | .597 | 1.723 5 |
| .378 | 1.385 85 | .433 | 1.467 21 | .488 | 1.551 89 | .543 | 1.637 8 | .598 | 1.725 11 |
| .379 | 1.387 32 | .434 | 1.468 72 | .489 | 1.553 46 | .544 | 1.639 37 | .599 | 1.726 72 |
| .38 | 1.388 79 | .435 | 1.470 23 | .49 | 1.555 03 | .545 | 1.640 94 | .6 | 1.728 33 |
| .381 | 1.390 24 | .436 | 1.471 74 | .491 | 1.556 6 | .546 | 1.642 51 | .601 | 1.729 94 |
| .382 | 1.391 69 | .437 | 1.473 26 | .492 | 1.558 17 | .547 | 1.644 08 | .602 | 1.731 55 |
| .383 | 1.393 14 | .438 | 1.474 78 | .493 | 1.559 74 | .548 | 1.645 65 | .603 | 1.733 16 |
| .384 | 1.394 59 | .439 | 1.476 3 | .494 | 1.561 31 | .549 | 1.647 22 | .604 | 1.734 77 |
| .385 | 1.396 05 | .44 | 1.477 82 | .495 | 1.562 89 | .55 | 1.648 79 | .605 | 1.736 38 |
| .386 | 1.397 51 | .441 | 1.479 34 | .496 | 1.564 47 | .551 | 1.650 36 | .606 | 1.737 99 |
| .387 | 1.398 97 | .442 | 1.480 86 | .497 | 1.566 05 | .552 | 1.651 93 | .607 | 1.739 6 |
| .388 | 1.400 43 | .443 | 1.482 38 | .498 | 1.567 63 | .553 | 1.653 5 | .608 | 1.741 21 |
| .389 | 1.401 89 | .444 | 1.483 91 | .499 | 1.569 21 | .554 | 1.655 07 | .609 | 1.742 83 |
| .39 | 1.403 35 | .445 | 1.485 44 | .5 | 1.570 89 | .555 | 1.656 65 | .61 | 1.744 44 |
| .391 | 1.404 81 | .446 | 1.486 97 | .501 | 1.572 34 | .556 | 1.658 23 | .611 | 1.746 05 |
| .392 | 1.406 27 | .447 | 1.488 5 | .502 | 1.573 89 | .557 | 1.659 81 | .612 | 1.747 67 |
| .393 | 1.407 73 | .448 | 1.490 03 | .503 | 1.575 44 | .558 | 1.661 39 | .613 | 1.749 29 |
| .394 | 1.409 19 | .449 | 1.491 57 | .504 | 1.576 99 | .559 | 1.662 97 | .614 | 1.750 91 |
| .395 | 1.410 65 | .45 | 1.493 11 | .505 | 1.578 54 | .56 | 1.664 55 | .615 | 1.752 52 |
| .396 | 1.412 11 | .451 | 1.494 65 | .506 | 1.580 09 | .561 | 1.666 13 | .616 | 1.754 14 |
| .397 | 1.413 57 | .452 | 1.496 18 | .507 | 1.581 64 | .562 | 1.667 71 | .617 | 1.755 76 |
| .398 | 1.415 04 | .453 | 1.497 71 | .508 | 1.583 19 | .563 | 1.669 29 | .618 | 1.757 38 |
| .399 | 1.416 51 | .454 | 1.499 24 | .509 | 1.584 74 | .564 | 1.670 87 | .619 | 1.759 |
| | | | | | | .565 | 1.672 45 | .620 | 1.760 62 |
| | | | | | | .566 | 1.674 03 | .621 | 1.762 24 |
| | | | | | | .567 | 1.675 61 | .622 | 1.763 86 |
| | | | | | | .568 | 1.677 19 | .623 | 1.765 48 |
| | | | | | | .569 | 1.678 77 | .624 | 1.767 1 |

| th. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. | H'ght. | Length. |
|------|--------|----------|--------|----------|--------|----------|--------|----------|
| 3 72 | .68 | 1.858 74 | .735 | 1.950 59 | .79 | 2.044 62 | .845 | 2.141 55 |
| 3 34 | .681 | 1.860 39 | .736 | 1.952 28 | .791 | 2.046 35 | .846 | 2.143 34 |
| 1 97 | .682 | 1.862 05 | .737 | 1.953 97 | .792 | 2.048 09 | .847 | 2.145 13 |
| 3 59 | .683 | 1.863 7 | .738 | 1.955 66 | .793 | 2.049 83 | .848 | 2.146 92 |
| 5 21 | .684 | 1.865 35 | .739 | 1.957 35 | .794 | 2.051 57 | .849 | 2.148 71 |
| 5 84 | .685 | 1.867 | .74 | 1.959 04 | .795 | 2.053 31 | .85 | 2.150 5 |
| 3 47 | .686 | 1.868 66 | .741 | 1.960 74 | .796 | 2.055 05 | .851 | 2.152 29 |
| 2 09 | .687 | 1.870 31 | .742 | 1.962 44 | .797 | 2.056 79 | .852 | 2.154 09 |
| 1 72 | .688 | 1.871 96 | .743 | 1.964 14 | .798 | 2.058 53 | .853 | 2.155 89 |
| 3 35 | .689 | 1.873 62 | .744 | 1.965 83 | .799 | 2.060 27 | .854 | 2.157 7 |
| 4 98 | .69 | 1.875 27 | .745 | 1.967 53 | .8 | 2.062 02 | .855 | 2.159 5 |
| 5 6 | .691 | 1.876 93 | .746 | 1.969 23 | .801 | 2.063 77 | .856 | 2.161 3 |
| 8 23 | .692 | 1.878 59 | .747 | 1.970 93 | .802 | 2.065 52 | .857 | 2.163 09 |
| 9 86 | .693 | 1.880 24 | .748 | 1.972 62 | .803 | 2.067 27 | .858 | 2.164 89 |
| 1 49 | .694 | 1.881 9 | .749 | 1.974 32 | .804 | 2.069 01 | .859 | 2.166 68 |
| 3 12 | .695 | 1.883 56 | .75 | 1.976 02 | .805 | 2.070 76 | .86 | 2.168 48 |
| 4 75 | .696 | 1.885 22 | .751 | 1.977 72 | .806 | 2.072 51 | .861 | 2.170 28 |
| 5 38 | .697 | 1.886 88 | .752 | 1.979 43 | .807 | 2.074 27 | .862 | 2.172 09 |
| 8 01 | .698 | 1.888 54 | .753 | 1.981 13 | .808 | 2.076 02 | .863 | 2.173 89 |
| 9 64 | .699 | 1.890 2 | .754 | 1.982 83 | .809 | 2.077 77 | .864 | 2.175 7 |
| 1 27 | .7 | 1.891 86 | .755 | 1.984 53 | .81 | 2.079 53 | .865 | 2.177 51 |
| 2 9 | .701 | 1.893 52 | .756 | 1.986 23 | .811 | 2.081 28 | .866 | 2.179 32 |
| 4 54 | .702 | 1.895 19 | .757 | 1.987 94 | .812 | 2.083 04 | .867 | 2.181 13 |
| 5 17 | .703 | 1.896 85 | .758 | 1.989 64 | .813 | 2.084 8 | .868 | 2.182 94 |
| 7 8 | .704 | 1.898 51 | .759 | 1.991 34 | .814 | 2.086 56 | .869 | 2.184 75 |
| 9 43 | .705 | 1.900 17 | .76 | 1.993 05 | .815 | 2.088 32 | .87 | 2.186 56 |
| 1 07 | .706 | 1.901 84 | .761 | 1.994 76 | .816 | 2.090 08 | .871 | 2.188 37 |
| 2 71 | .707 | 1.903 5 | .762 | 1.996 47 | .817 | 2.091 98 | .872 | 2.190 18 |
| 4 35 | .708 | 1.905 17 | .763 | 1.998 18 | .818 | 2.093 6 | .873 | 2.192 |
| 5 99 | .709 | 1.906 84 | .764 | 1.999 89 | .819 | 2.095 36 | .874 | 2.193 82 |
| 7 63 | .71 | 1.908 52 | .765 | 2.001 6 | .82 | 2.097 12 | .875 | 2.195 64 |
| 9 28 | .711 | 1.910 19 | .766 | 2.003 31 | .821 | 2.098 88 | .876 | 2.197 46 |
| 2 91 | .712 | 1.911 87 | .767 | 2.005 02 | .822 | 2.100 65 | .877 | 2.199 28 |
| 4 55 | .713 | 1.913 55 | .768 | 2.006 73 | .823 | 2.102 42 | .878 | 2.201 1 |
| 6 19 | .714 | 1.915 23 | .769 | 2.008 44 | .824 | 2.104 19 | .879 | 2.202 92 |
| 8 83 | .715 | 1.916 91 | .77 | 2.010 16 | .825 | 2.105 96 | .88 | 2.204 74 |
| 1 47 | .716 | 1.918 59 | .771 | 2.011 87 | .826 | 2.107 73 | .881 | 2.206 56 |
| 3 11 | .717 | 1.920 27 | .772 | 2.013 59 | .827 | 2.109 5 | .882 | 2.208 39 |
| 4 75 | .718 | 1.921 95 | .773 | 2.015 31 | .828 | 2.111 27 | .883 | 2.210 22 |
| 6 4 | .719 | 1.923 63 | .774 | 2.017 02 | .829 | 2.113 04 | .884 | 2.212 05 |
| 8 04 | .72 | 1.925 31 | .775 | 2.018 74 | .83 | 2.114 81 | .885 | 2.213 88 |
| 9 68 | .721 | 1.927 | .776 | 2.020 45 | .831 | 2.116 59 | .886 | 2.215 71 |
| 1 33 | .722 | 1.928 68 | .777 | 2.022 17 | .832 | 2.118 37 | .887 | 2.217 54 |
| 3 97 | .723 | 1.930 36 | .778 | 2.023 89 | .833 | 2.120 15 | .888 | 2.219 37 |
| 5 61 | .724 | 1.932 04 | .779 | 2.025 61 | .834 | 2.121 93 | .889 | 2.221 2 |
| 7 26 | .725 | 1.933 73 | .78 | 2.027 33 | .835 | 2.123 71 | .89 | 2.223 03 |
| 9 91 | .726 | 1.935 41 | .781 | 2.029 07 | .836 | 2.125 49 | .891 | 2.224 86 |
| 1 55 | .727 | 1.937 1 | .782 | 2.030 8 | .837 | 2.127 27 | .892 | 2.226 7 |
| 3 19 | .728 | 1.938 78 | .783 | 2.032 52 | .838 | 2.129 05 | | |
| 4 85 | .729 | 1.940 46 | .784 | 2.034 25 | .839 | 2.130 8 | | |
| 6 5 | .73 | 1.942 15 | .785 | 2.035 98 | .84 | 2.132 6 | | |
| 8 15 | .731 | 1.943 83 | .786 | 2.037 71 | .841 | 2.134 3 | | |
| 9 79 | .732 | 1.945 52 | .787 | 2.039 44 | .842 | 2.136 1 | | |
| 1 44 | .733 | 1.947 21 | .788 | 2.041 17 | .843 | 2.137 9 | | |
| 3 09 | .734 | 1.948 9 | .789 | 2.042 9 | .844 | 2.139 76 | | |

| Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. Area. | Versed Sine. | Seg. |
|--------------|------------|--------------|------------|--------------|------------|--------------|------------|--------------|------|
| .256 | .158 76 | .305 | .202 76 | .354 | .248 8 | .403 | .296 31 | .452 | .3 |
| .257 | .159 64 | .306 | .203 68 | .355 | .249 76 | .404 | .297 29 | .453 | .3 |
| .258 | .160 51 | .307 | .204 6 | .356 | .250 71 | .405 | .298 27 | .454 | .3 |
| .259 | .161 39 | .308 | .205 53 | .357 | .251 67 | .406 | .299 25 | .455 | .3 |
| .26 | .162 26 | .309 | .206 45 | .358 | .252 63 | .407 | .300 24 | .456 | .3 |
| .261 | .163 14 | .31 | .207 38 | .359 | .253 59 | .408 | .301 22 | .457 | .3 |
| .262 | .164 02 | .311 | .208 3 | .36 | .254 55 | .409 | .302 2 | .458 | .3 |
| .263 | .164 9 | .312 | .209 23 | .361 | .255 51 | .41 | .303 19 | .459 | .3 |
| .264 | .165 78 | .313 | .210 15 | .362 | .256 47 | .411 | .304 17 | .46 | .3 |
| .265 | .166 66 | .314 | .211 08 | .363 | .257 43 | .412 | .305 15 | .461 | .3 |
| .266 | .167 55 | .315 | .212 01 | .364 | .258 39 | .413 | .306 14 | .462 | .3 |
| .267 | .168 44 | .316 | .212 94 | .365 | .259 36 | .414 | .307 12 | .463 | .3 |
| .268 | .169 31 | .317 | .213 87 | .366 | .260 32 | .415 | .308 11 | .464 | .3 |
| .269 | .170 2 | .318 | .214 8 | .367 | .261 28 | .416 | .309 09 | .465 | .3 |
| .27 | .171 09 | .319 | .215 73 | .368 | .262 25 | .417 | .310 08 | .466 | .3 |
| .271 | .171 97 | .32 | .216 67 | .369 | .263 21 | .418 | .311 07 | .467 | .3 |
| .272 | .172 87 | .321 | .217 6 | .37 | .264 18 | .419 | .312 05 | .468 | .3 |
| .273 | .173 76 | .322 | .218 53 | .371 | .265 14 | .42 | .313 04 | .469 | .3 |
| .274 | .174 65 | .323 | .219 47 | .372 | .266 11 | .421 | .314 03 | .47 | .3 |
| .275 | .175 54 | .324 | .220 4 | .373 | .267 08 | .422 | .315 02 | .471 | .3 |
| .276 | .176 43 | .325 | .221 34 | .374 | .268 04 | .423 | .316 | .472 | .3 |
| .277 | .177 33 | .326 | .222 28 | .375 | .269 01 | .424 | .316 99 | .473 | .3 |
| .278 | .178 22 | .327 | .223 21 | .376 | .269 98 | .425 | .317 98 | .474 | .3 |
| .279 | .179 12 | .328 | .224 15 | .377 | .270 95 | .426 | .318 97 | .475 | .3 |
| .28 | .180 02 | .329 | .225 09 | .378 | .271 92 | .427 | .319 96 | .476 | .3 |
| .281 | .180 92 | .33 | .226 03 | .379 | .272 89 | .428 | .320 95 | .477 | .3 |
| .282 | .181 82 | .331 | .226 97 | .38 | .273 86 | .429 | .321 94 | .478 | .3 |
| .283 | .182 72 | .332 | .227 91 | .381 | .274 83 | .43 | .322 93 | .479 | .3 |
| .284 | .183 61 | .333 | .228 86 | .382 | .275 80 | .431 | .323 91 | .48 | .3 |
| .285 | .184 52 | .334 | .229 8 | .383 | .276 77 | .432 | .324 9 | .481 | .3 |
| .286 | .185 42 | .335 | .230 74 | .384 | .277 75 | .433 | .325 9 | .482 | .3 |
| .287 | .186 33 | .336 | .231 69 | .385 | .278 72 | .434 | .326 89 | .483 | .3 |
| .288 | .187 23 | .337 | .232 63 | .386 | .279 69 | .435 | .327 88 | .484 | .3 |
| .289 | .188 14 | .338 | .233 58 | .387 | .280 67 | .436 | .328 87 | .485 | .3 |
| .29 | .189 05 | .339 | .234 53 | .388 | .281 64 | .437 | .329 87 | .486 | .3 |
| .291 | .189 95 | .34 | .235 47 | .389 | .282 62 | .438 | .330 86 | .487 | .3 |
| .292 | .190 86 | .341 | .236 42 | .39 | .283 59 | .439 | .331 85 | .488 | .3 |
| .293 | .191 77 | .342 | .237 37 | .391 | .284 57 | .44 | .332 84 | .489 | .3 |
| .294 | .192 68 | .343 | .238 32 | .392 | .285 54 | .441 | .333 84 | .49 | .3 |
| .295 | .193 6 | .344 | .239 27 | .393 | .286 52 | .442 | .334 83 | .491 | .3 |
| .296 | .194 51 | .345 | .240 22 | .394 | .287 5 | .443 | .335 82 | .492 | .3 |
| .297 | .195 42 | .346 | .241 17 | .395 | .288 48 | .444 | .336 82 | .493 | .3 |
| .298 | .196 34 | .347 | .242 12 | .396 | .289 45 | .445 | .337 81 | .494 | .3 |
| .299 | .197 25 | .348 | .243 07 | .397 | .290 43 | .446 | .338 8 | .495 | .3 |
| .3 | .198 17 | .349 | .244 03 | .398 | .291 41 | .447 | .339 8 | .496 | .3 |
| .301 | .199 08 | .35 | .244 98 | .399 | .292 39 | .448 | .340 79 | .497 | .3 |
| .302 | .2 | .351 | .245 93 | .4 | .293 37 | .449 | .341 79 | .498 | .3 |
| .303 | .200 92 | .352 | .246 89 | .401 | .294 35 | .45 | .342 78 | .499 | .3 |
| .304 | .201 84 | .353 | .247 84 | .402 | .295 33 | .451 | .343 78 | .5 | .3 |

To Compute Area of a Segment of a Circle by preceding Table.

RULE.—Divide height or versed sine by diameter of circle; find quotient of versed sines. Take area for versed sine opposite to it in next right hand, multiply it by square of diameter, and it will give

—Required area of a segment of a circle, its height being 10 feet and circle 50.

$0 = .2$, and $.2$, per table, $= .11182$; then $.11182 \times 50^2 = 279.55$ feet.

Division of a Height by Base, Quotient has a Remainder after third Place of Decimals, and great Accuracy is required.

Take area for first three figures, subtract it from next following, multiply remainder by said fraction, add product to first area, and give area for whole quotient.

—What is area of a segment of a circle, diameter of which is 10 feet, and $h = .575$?

$h = .1575$; tabular area for $.157 = .07892$, and for $.158 = .07965$, the difference which is $.00073$.

$.00073 = .000365$.

$.157 = .07892$

$.0005 = .000365$

$.079285$, sum by which square of diameter to be multiplied; and $.079285 \times 10^2 = 7.9285$ feet.

Areas of Zones of a Circle.

Diameter of a Circle = 1, and divided into 1000 equal Parts.

| h. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|----|--------|---------|--------|---------|--------|---------|--------|---------|
| | .035 | .034 97 | .069 | .068 78 | .103 | .102 27 | .137 | .135 27 |
| | .036 | .035 97 | .07 | .069 77 | .104 | .103 25 | .138 | .136 23 |
| | .037 | .036 97 | .071 | .070 76 | .105 | .104 22 | .139 | .137 19 |
| | .038 | .037 96 | .072 | .071 75 | .106 | .105 2 | .14 | .138 15 |
| | .039 | .038 96 | .073 | .072 74 | .107 | .106 18 | .141 | .139 11 |
| | .04 | .039 96 | .074 | .073 73 | .108 | .107 15 | .142 | .140 07 |
| | .041 | .040 95 | .075 | .074 72 | .109 | .108 13 | .143 | .141 03 |
| | .042 | .041 95 | .076 | .075 5 | .11 | .109 11 | .144 | .141 98 |
| | .043 | .042 95 | .077 | .076 69 | .111 | .110 08 | .145 | .142 94 |
| | .044 | .043 94 | .078 | .077 68 | .112 | .111 06 | .146 | .143 9 |
| | .045 | .044 94 | .079 | .078 67 | .113 | .112 03 | .147 | .144 85 |
| | .046 | .045 93 | .08 | .079 66 | .114 | .113 | .148 | .145 81 |
| | .047 | .046 93 | .081 | .080 64 | .115 | .113 98 | .149 | .146 77 |
| | .048 | .047 93 | .082 | .081 63 | .116 | .114 95 | .15 | .147 72 |
| | .049 | .048 92 | .083 | .082 62 | .117 | .115 92 | .151 | .148 67 |
| | .05 | .049 92 | .084 | .083 6 | .118 | .116 9 | .152 | .149 62 |
| | .051 | .050 91 | .085 | .084 59 | .119 | .117 87 | .153 | .150 58 |
| | .052 | .051 9 | .086 | .085 57 | .12 | .118 84 | .154 | .151 53 |
| | .053 | .052 9 | .087 | .086 56 | .121 | .119 81 | .155 | .152 48 |
| | .054 | .053 89 | .088 | .087 54 | .122 | .120 78 | .156 | .153 |
| | .055 | .054 89 | .089 | .088 53 | .123 | .121 75 | .157 | .154 |
| | .056 | .055 88 | .09 | .089 51 | .124 | .122 72 | .158 | .155 |
| | .057 | .056 88 | .091 | .090 5 | .125 | .123 69 | .159 | .156 |
| | .058 | .057 87 | .092 | .091 48 | .126 | .124 69 | .16 | .157 |
| | .059 | .058 86 | .093 | .092 46 | .127 | .125 62 | .161 | .158 |
| | .06 | .059 86 | .094 | .093 44 | .128 | .126 59 | .162 | .159 |
| | .061 | .060 85 | .095 | .094 43 | .129 | .127 55 | .163 | .160 |
| | .062 | .061 84 | .096 | .095 4 | .13 | .128 52 | .164 | .161 |
| | .063 | .062 83 | .097 | .096 39 | .131 | .129 49 | .165 | .162 |
| | .064 | .063 82 | .098 | .097 37 | .132 | .130 45 | .166 | .163 |
| | .065 | .064 82 | .099 | .098 35 | .133 | .131 41 | .167 | .164 |
| | .066 | .065 8 | .1 | .099 33 | .134 | .132 38 | .168 | .165 |
| | .067 | .066 8 | .101 | .100 31 | .135 | .133 34 | .169 | .166 |
| | .068 | .067 8 | .102 | .101 29 | .136 | .134 3 | .17 | .167 |

| H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|--------|---------|--------|---------|--------|---------|--------|---------|--------|---------|
| .171 | .167 61 | .226 | .218 05 | .281 | .265 41 | .336 | .308 64 | .391 | .346 32 |
| .172 | .168 55 | .227 | .218 94 | .282 | .266 24 | .337 | .309 38 | .392 | .346 94 |
| .173 | .169 48 | .228 | .219 83 | .283 | .267 06 | .338 | .310 12 | .393 | .347 56 |
| .174 | .170 42 | .229 | .220 72 | .284 | .267 89 | .339 | .310 85 | .394 | .348 18 |
| .175 | .171 36 | .23 | .221 61 | .285 | .268 71 | .34 | .311 59 | .395 | .348 79 |
| .176 | .172 3 | .231 | .222 5 | .286 | .269 53 | .341 | .312 32 | .396 | .349 4 |
| .177 | .173 23 | .232 | .223 35 | .287 | .270 35 | .342 | .313 05 | .397 | .350 01 |
| .178 | .174 17 | .233 | .224 27 | .288 | .271 17 | .343 | .313 78 | .398 | .350 62 |
| .179 | .175 1 | .234 | .225 15 | .289 | .271 99 | .344 | .314 5 | .399 | .351 22 |
| .18 | .176 03 | .235 | .226 04 | .29 | .272 8 | .345 | .315 23 | .4 | .351 82 |
| .181 | .176 97 | .236 | .226 92 | .291 | .273 62 | .346 | .315 95 | .401 | .352 42 |
| .182 | .177 9 | .237 | .227 8 | .292 | .274 43 | .347 | .316 67 | .402 | .353 02 |
| .183 | .178 83 | .238 | .228 68 | .293 | .275 24 | .348 | .317 39 | .403 | .353 61 |
| .184 | .179 76 | .239 | .229 56 | .294 | .276 05 | .349 | .318 11 | .404 | .354 2 |
| .185 | .180 69 | .24 | .230 44 | .295 | .276 86 | .35 | .318 82 | .405 | .354 79 |
| .186 | .181 62 | .241 | .231 31 | .296 | .277 66 | .351 | .319 54 | .406 | .355 38 |
| .187 | .182 54 | .242 | .232 19 | .297 | .278 47 | .352 | .320 25 | .407 | .355 96 |
| .188 | .183 47 | .243 | .233 06 | .298 | .279 27 | .353 | .320 96 | .408 | .356 54 |
| .189 | .184 4 | .244 | .233 94 | .299 | .280 07 | .354 | .321 67 | .409 | .357 11 |
| .19 | .185 32 | .245 | .234 81 | .3 | .280 88 | .355 | .322 37 | .41 | .357 69 |
| .191 | .186 25 | .246 | .235 68 | .301 | .281 67 | .356 | .323 07 | .411 | .358 26 |
| .192 | .187 17 | .247 | .236 55 | .302 | .282 47 | .357 | .323 77 | .412 | .358 83 |
| .193 | .188 09 | .248 | .237 42 | .303 | .283 27 | .358 | .324 47 | .413 | .359 39 |
| .194 | .189 02 | .249 | .238 29 | .304 | .284 06 | .359 | .325 17 | .414 | .359 95 |
| .195 | .189 94 | .25 | .239 15 | .305 | .284 86 | .36 | .325 87 | .415 | .360 51 |
| .196 | .190 86 | .251 | .240 02 | .306 | .285 65 | .361 | .326 56 | .416 | .361 07 |
| .197 | .191 78 | .252 | .240 89 | .307 | .286 44 | .362 | .327 25 | .417 | .361 62 |
| .198 | .192 7 | .253 | .241 75 | .308 | .287 23 | .363 | .327 94 | .418 | .362 17 |
| .199 | .193 61 | .254 | .242 61 | .309 | .288 01 | .364 | .328 62 | .419 | .362 72 |
| .2 | .194 53 | .255 | .243 47 | .31 | .288 8 | .365 | .329 31 | .42 | .363 26 |
| .201 | .195 45 | .256 | .244 33 | .311 | .289 58 | .366 | .329 99 | .421 | .363 8 |
| .202 | .196 36 | .257 | .245 19 | .312 | .290 36 | .367 | .330 67 | .422 | .364 34 |
| .203 | .197 28 | .258 | .246 04 | .313 | .291 15 | .368 | .331 35 | .423 | .364 88 |
| .204 | .198 19 | .259 | .246 9 | .314 | .291 92 | .369 | .332 03 | .424 | .365 41 |
| .205 | .199 1 | .26 | .247 75 | .315 | .292 7 | .37 | .332 7 | .425 | .365 94 |
| .206 | .200 01 | .261 | .248 61 | .316 | .293 48 | .371 | .333 37 | .426 | .366 46 |
| .207 | .200 92 | .262 | .249 46 | .317 | .294 25 | .372 | .334 04 | .427 | .366 98 |
| .208 | .201 83 | .263 | .250 21 | .318 | .295 02 | .373 | .334 71 | .428 | .367 5 |
| .209 | .202 74 | .264 | .251 16 | .319 | .295 8 | .374 | .335 37 | .429 | .368 06 |
| .210 | .203 65 | .265 | .252 01 | .32 | .296 56 | .375 | .336 04 | .43 | .368 57 |
| .211 | .204 56 | .266 | .252 85 | .321 | .297 33 | .376 | .336 7 | .431 | .369 08 |
| .212 | .205 46 | .267 | .253 7 | .322 | .298 1 | .377 | .337 35 | .432 | .369 59 |
| .213 | .206 37 | .268 | .254 55 | .323 | .298 86 | .378 | .338 01 | .433 | .370 10 |
| .214 | .207 27 | .269 | .255 39 | .324 | .299 62 | .379 | .338 66 | .434 | .370 61 |
| .215 | .208 18 | .27 | .256 23 | .325 | .300 39 | .38 | .339 31 | .435 | .371 12 |
| .216 | .209 09 | .271 | .257 07 | .326 | .301 14 | .381 | .339 96 | .436 | .371 63 |
| .217 | .209 99 | .272 | .257 91 | .327 | .301 9 | .382 | .340 61 | .437 | .372 14 |
| .218 | .210 9 | .273 | .258 75 | .328 | .302 66 | .383 | .341 25 | .438 | .372 65 |
| .219 | .211 9 | .274 | .259 59 | .329 | .303 41 | .384 | .341 9 | .439 | .373 16 |
| .220 | .212 9 | .275 | .260 43 | .33 | .304 16 | .385 | .342 53 | .44 | .373 67 |
| .221 | .213 9 | .276 | .261 27 | .331 | .304 91 | .386 | .343 17 | .441 | .374 18 |
| .222 | .214 9 | .277 | .262 11 | .332 | .305 66 | .387 | .343 8 | .442 | .374 69 |
| .223 | .215 9 | .278 | .262 95 | .333 | .306 41 | .388 | .344 44 | .443 | .375 20 |
| .224 | .216 9 | .279 | .263 79 | .334 | .307 15 | .389 | .345 07 | .444 | .375 71 |
| .225 | .217 9 | .28 | .264 63 | .335 | .307 9 | .39 | .345 69 | .445 | .376 22 |

| Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. | H'ght. | Area. |
|---------|--------|---------|--------|---------|--------|---------|--------|---------|
| .376 24 | .457 | .380 96 | .468 | .385 14 | .479 | .388 67 | .49 | .391 37 |
| .376 69 | .458 | .381 37 | .469 | .385 49 | .48 | .388 95 | .491 | .391 56 |
| .377 14 | .459 | .381 77 | .47 | .385 83 | .481 | .389 23 | .492 | .391 75 |
| .377 58 | .46 | .382 16 | .471 | .386 17 | .482 | .389 5 | .493 | .391 92 |
| .378 02 | .461 | .382 55 | .472 | .386 5 | .483 | .389 76 | .494 | .392 08 |
| .378 45 | .462 | .382 94 | .473 | .386 83 | .484 | .390 01 | .495 | .392 23 |
| .378 88 | .463 | .383 32 | .474 | .387 15 | .485 | .390 26 | .496 | .392 36 |
| .379 31 | .464 | .383 69 | .475 | .387 47 | .486 | .390 5 | .497 | .392 48 |
| .379 73 | .465 | .384 06 | .476 | .387 78 | .487 | .390 73 | .498 | .392 58 |
| .380 14 | .466 | .384 43 | .477 | .388 08 | .488 | .390 95 | .499 | .392 66 |
| .380 56 | .467 | .384 79 | .478 | .388 38 | .489 | .391 17 | .5 | .392 7 |

is Table is computed only for Zones, longest Chord of which is Diam-

to Compute Area of a Zone by preceding Table.

When Zone is Less than a Semicircle.

EX.—Divide height by diameter, find quotient in column of heights, area for height opposite to it in next column on right hand, multiply square of diameter, and product will give area of zone.

EXPL.—Required area of a Zone, diameter of which is 50, and its height 15.

$15 \div 50 = .3$; and $.3$, as per table, = .280 88.

$\therefore .280 88 \times 50^2 = 702.2$ area.

When Zone is Greater than a Semicircle.

EX.—Take height on each side of diameter of circle, and ascertain, by long Rule, their respective areas; add areas of these two portions to and sum will give area.

EXPL.—Required area of a zone, diameter of circle being 50, and heights of each side of diameter of circle 20 and 15.

$20 \div 50 = .4$; $.4$, as per table, = .351 82; and $.351 82 \times 50^2 = 879.55$.

$15 \div 50 = .3$; $.3$, as per table, = .280 88; and $.280 88 \times 50^2 = 702.2$.

$879.55 + 702.2 = 1581.75$ area.

is Division of a Height by Chord, Quotient has a Remainder after Third Place of Decimals, and great Accuracy is required.

—Take area for first three figures, subtract it from the next following remainder by said fraction, and add product to first area; give area for whole quotient.

EX.—What is area of a zone of a circle, greater chord being 100 feet, r of 14 feet 3 ins?

3 ins. = 14.25, and $14.25 \div 100 = .1425$; tabular length for $.142 = .14$ 43 = .141 03, difference between which is .000 96.

$\therefore .000 96 = .000 48$. Hence $.142 = .140 97$

$.0005 = .000 48$

$.140 55, 8$

is multiplied; and $.140 55 \times 100^2 = 1405.51$

of area.

Squares, Cubes, and Square and Cube Root

From 1 to 1600.

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|---------|--------|--------------|------------|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 1.414 213 6 | 1.259 |
| 3 | 9 | 27 | 1.732 050 8 | 1.442 |
| 4 | 16 | 64 | 2 | 1.587 |
| 5 | 25 | 125 | 2.236 068 | 1.709 |
| 6 | 36 | 216 | 2.449 489 7 | 1.817 |
| 7 | 49 | 343 | 2.645 751 3 | 1.912 |
| 8 | 64 | 512 | 2.828 427 1 | 2 |
| 9 | 81 | 729 | 3 | 2.080 |
| 10 | 100 | 1000 | 3.162 277 7 | 2.154 |
| 11 | 121 | 1331 | 3.316 624 8 | 2.223 |
| 12 | 144 | 1728 | 3.464 101 6 | 2.289 |
| 13 | 169 | 2197 | 3.605 551 3 | 2.351 |
| 14 | 196 | 2744 | 3.741 657 4 | 2.410 |
| 15 | 225 | 3375 | 3.872 983 3 | 2.466 |
| 16 | 256 | 4096 | 4 | 2.519 |
| 17 | 289 | 4913 | 4.123 105 6 | 2.571 |
| 18 | 324 | 5832 | 4.242 640 7 | 2.622 |
| 19 | 361 | 6859 | 4.358 598 9 | 2.668 |
| 20 | 400 | 8000 | 4.472 136 | 2.714 |
| 21 | 441 | 9261 | 4.582 575 7 | 2.758 |
| 22 | 484 | 10648 | 4.690 415 8 | 2.802 |
| 23 | 529 | 12167 | 4.795 831 5 | 2.843 |
| 24 | 576 | 13824 | 4.898 979 5 | 2.884 |
| 25 | 625 | 15625 | 5 | 2.924 |
| 26 | 676 | 17576 | 5.099 019 5 | 2.962 |
| 27 | 729 | 19683 | 5.196 152 4 | 3 |
| 28 | 784 | 21952 | 5.291 502 6 | 3.036 |
| 29 | 841 | 24389 | 5.385 164 8 | 3.072 |
| 30 | 900 | 27000 | 5.477 225 6 | 3.107 |
| 31 | 961 | 29791 | 5.567 764 4 | 3.141 |
| 32 | 1024 | 32768 | 5.656 854 2 | 3.174 |
| 33 | 1089 | 35937 | 5.744 562 6 | 3.207 |
| 34 | 1156 | 39304 | 5.830 951 9 | 3.239 |
| 35 | 1225 | 42875 | 5.916 079 8 | 3.271 |
| 36 | 1296 | 46656 | 6 | 3.301 |
| 37 | 1369 | 50653 | 6.082 762 5 | 3.332 |
| 38 | 1444 | 54872 | 6.164 414 | 3.361 |
| 39 | 1521 | 59319 | 6.244 998 | 3.391 |
| 40 | 1600 | 64000 | 6.324 555 3 | 3.419 |
| 41 | 1681 | 68921 | 6.403 124 2 | 3.448 |
| 42 | 1764 | 74088 | 6.480 740 7 | 3.476 |
| 43 | 1849 | 79507 | 6.557 438 5 | 3.503 |
| 44 | 1936 | 85184 | 6.633 249 6 | 3.530 |
| 45 | 2025 | 91125 | 6.708 203 9 | 3.556 |
| 46 | 2116 | 97336 | 6.782 33 | 3.583 |
| 47 | 2209 | 103823 | 6.855 654 6 | 3.608 |
| 48 | 2304 | 110592 | 6.928 203 2 | 3.634 |
| 49 | 2401 | 117649 | 7 | 3.659 |
| 50 | 2500 | 125000 | 7.071 067 8 | 3.684 |
| 51 | 2601 | 132651 | 7.141 428 4 | 3.708 |
| 52 | 2704 | 140608 | 7.211 102 6 | 3.732 |
| 53 | 2809 | 148877 | 7.280 109 9 | 3.755 |
| | 2916 | 157464 | 7.348 469 2 | 3.777 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|-----------|--------------|-------------|
| 30 25 | 166 375 | 7.416 198 5 | 3.802 952 5 |
| 31 36 | 175 616 | 7.483 314 8 | 3.825 862 4 |
| 32 49 | 185 193 | 7.549 834 4 | 3.848 501 1 |
| 33 64 | 195 112 | 7.615 773 1 | 3.870 876 6 |
| 34 81 | 205 379 | 7.681 145 7 | 3.892 996 5 |
| 35 00 | 216 000 | 7.745 966 7 | 3.914 867 6 |
| 37 21 | 226 981 | 7.810 249 7 | 3.936 497 2 |
| 38 44 | 238 328 | 7.874 007 9 | 3.957 891 5 |
| 39 69 | 250 047 | 7.937 253 9 | 3.979 057 1 |
| 40 96 | 262 144 | 8 | 4 |
| 42 25 | 274 625 | 8.062 257 7 | 4.020 725 6 |
| 43 56 | 287 496 | 8.124 038 4 | 4.041 240 1 |
| 44 89 | 300 763 | 8.185 352 8 | 4.061 548 |
| 46 24 | 314 432 | 8.246 211 3 | 4.081 655 1 |
| 47 61 | 328 509 | 8.306 623 9 | 4.101 566 1 |
| 49 00 | 343 000 | 8.366 600 3 | 4.121 285 3 |
| 50 41 | 357 911 | 8.426 149 8 | 4.140 817 8 |
| 51 84 | 373 248 | 8.485 281 4 | 4.160 167 6 |
| 53 29 | 389 017 | 8.544 003 7 | 4.179 339 |
| 54 76 | 405 224 | 8.602 325 3 | 4.198 336 4 |
| 56 25 | 421 875 | 8.660 254 | 4.217 163 3 |
| 57 76 | 438 976 | 8.717 797 9 | 4.235 823 6 |
| 59 29 | 456 533 | 8.774 964 4 | 4.254 321 |
| 60 84 | 474 552 | 8.831 760 9 | 4.272 658 6 |
| 62 41 | 493 039 | 8.888 194 4 | 4.290 840 4 |
| 64 00 | 512 000 | 8.944 271 9 | 4.308 869 5 |
| 65 61 | 531 441 | 9 | 4.326 748 7 |
| 67 24 | 551 368 | 9.055 385 1 | 4.344 481 5 |
| 68 89 | 571 787 | 9.110 433 6 | 4.362 070 7 |
| 70 56 | 592 704 | 9.165 151 4 | 4.379 519 1 |
| 72 25 | 614 125 | 9.219 544 5 | 4.396 829 6 |
| 73 96 | 636 056 | 9.273 618 5 | 4.414 004 9 |
| 75 69 | 658 503 | 9.327 379 1 | 4.431 047 6 |
| 77 44 | 681 472 | 9.380 831 5 | 4.447 960 2 |
| 79 21 | 704 969 | 9.433 981 1 | 4.464 745 1 |
| 81 00 | 729 000 | 9.486 833 | 4.481 404 7 |
| 82 81 | 753 571 | 9.539 392 | 4.497 941 4 |
| 84 64 | 778 688 | 9.591 663 | 4.514 357 4 |
| 86 49 | 804 357 | 9.643 650 8 | 4.530 654 9 |
| 88 36 | 830 584 | 9.695 359 7 | 4.546 835 9 |
| 90 25 | 857 375 | 9.746 794 3 | 4.562 902 6 |
| 92 16 | 884 736 | 9.797 959 | 4.578 857 |
| 94 09 | 912 673 | 9.848 857 8 | 4.594 700 9 |
| 96 04 | 941 192 | 9.899 494 9 | 4.610 436 3 |
| 98 01 | 970 299 | 9.949 874 4 | 4.626 065 |
| 1 00 00 | 1 000 000 | 10 | 4.641 588 8 |
| 1 02 01 | 1 030 301 | 10.049 875 6 | 4.657 009 5 |
| 1 04 04 | 1 061 208 | 10.099 504 9 | 4.672 328 7 |
| 1 06 09 | 1 092 727 | 10.148 891 6 | 4.687 548 2 |
| 1 08 16 | 1 124 864 | 10.198 039 | 4.702 669 4 |
| 1 10 25 | 1 157 625 | 10.246 950 8 | 4.717 694 |
| 1 12 36 | 1 191 016 | 10.295 630 1 | 4.732 623 5 |
| 1 14 49 | 1 225 043 | 10.344 080 4 | 4.747 459 4 |
| 1 16 64 | 1 259 712 | 10.392 304 8 | 4.762 203 2 |
| 1 18 81 | 1 295 029 | 10.440 306 5 | 4.776 856 2 |
| 1 21 00 | 1 331 000 | 10.488 088 5 | 4.791 419 9 |

Squares, Cubes, and Square and Cube Root
From 1 to 1600.

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|---------|--------|--------------|-------------|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 1.414 213 6 | 1.259 921 0 |
| 3 | 9 | 27 | 1.732 050 8 | 1.442 249 5 |
| 4 | 16 | 64 | 2 | 1.587 401 1 |
| 5 | 25 | 125 | 2.236 068 | 1.709 975 9 |
| 6 | 36 | 216 | 2.449 489 7 | 1.817 128 4 |
| 7 | 49 | 343 | 2.645 751 3 | 1.913 285 4 |
| 8 | 64 | 512 | 2.828 427 1 | 2 |
| 9 | 81 | 729 | 3 | 2.080 083 8 |
| 10 | 100 | 1000 | 3.162 277 7 | 2.154 434 7 |
| 11 | 121 | 1331 | 3.316 624 8 | 2.223 980 1 |
| 12 | 144 | 1728 | 3.464 101 6 | 2.289 428 1 |
| 13 | 169 | 2197 | 3.605 551 3 | 2.351 356 0 |
| 14 | 196 | 2744 | 3.741 657 4 | 2.410 141 9 |
| 15 | 225 | 3375 | 3.872 983 3 | 2.466 212 1 |
| 16 | 256 | 4096 | 4 | 2.519 842 1 |
| 17 | 289 | 4913 | 4.123 105 6 | 2.571 233 5 |
| 18 | 324 | 5832 | 4.242 640 7 | 2.620 741 9 |
| 19 | 361 | 6859 | 4.358 598 9 | 2.668 353 7 |
| 20 | 400 | 8000 | 4.472 136 | 2.714 417 6 |
| 21 | 441 | 9261 | 4.582 575 7 | 2.759 921 0 |
| 22 | 484 | 10648 | 4.690 415 8 | 2.808 398 2 |
| 23 | 529 | 12167 | 4.795 831 5 | 2.849 380 8 |
| 24 | 576 | 13824 | 4.898 979 5 | 2.892 353 7 |
| 25 | 625 | 15625 | 5 | 2.927 001 9 |
| 26 | 676 | 17576 | 5.099 019 5 | 2.963 469 1 |
| 27 | 729 | 19683 | 5.196 152 4 | 3 |
| 28 | 784 | 21952 | 5.291 502 6 | 3.030 113 8 |
| 29 | 841 | 24389 | 5.385 164 8 | 3.070 176 6 |
| 30 | 900 | 27000 | 5.477 225 6 | 3.107 178 5 |
| 31 | 961 | 29791 | 5.567 764 4 | 3.141 286 5 |
| 32 | 1024 | 32768 | 5.656 854 2 | 3.171 660 9 |
| 33 | 1089 | 35937 | 5.744 562 6 | 3.200 000 0 |
| 34 | 1156 | 39304 | 5.830 951 9 | 3.226 388 6 |
| 35 | 1225 | 42875 | 5.916 079 8 | 3.250 000 0 |
| 36 | 1296 | 46656 | 6 | 3.300 000 0 |
| 37 | 1369 | 50653 | 6.082 762 5 | 3.333 333 3 |
| 38 | 1444 | 54872 | 6.164 414 | 3.363 718 8 |
| 39 | 1521 | 59319 | 6.244 998 | 3.391 250 0 |
| 40 | 1600 | 64000 | 6.324 555 3 | 3.416 407 9 |
| 41 | 1681 | 68921 | 6.403 124 2 | 3.440 353 7 |
| 42 | 1764 | 74088 | 6.480 740 7 | 3.462 500 0 |
| 43 | 1849 | 79507 | 6.557 438 5 | 3.500 000 0 |
| 44 | 1936 | 85184 | 6.633 249 6 | 3.533 333 3 |
| 45 | 2025 | 91125 | 6.708 203 9 | 3.563 333 3 |
| 46 | 2116 | 97336 | 6.782 33 | 3.590 000 0 |
| 47 | 2209 | 103823 | 6.855 654 6 | 3.614 457 8 |
| 48 | | 110592 | 6.928 203 2 | 3.636 363 6 |
| 49 | | 117649 | 7 | 3.655 544 4 |
| 50 | | | 7.071 067 8 | 3.680 000 0 |
| 51 | | | 7.141 428 4 | 3.703 703 7 |
| 52 | | | 7.211 102 6 | 3.725 000 0 |
| 53 | | | 7.280 109 9 | 3.744 444 4 |
| 54 | | | 7.348 469 2 | 3.761 111 1 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|-----------|--------------|-------------|
| 30 25 | 166 375 | 7.416 198 5 | 3.802 952 5 |
| 31 36 | 175 616 | 7.483 314 8 | 3.825 862 4 |
| 32 49 | 185 193 | 7.549 834 4 | 3.848 501 1 |
| 33 64 | 195 112 | 7.615 773 1 | 3.870 876 6 |
| 34 81 | 205 379 | 7.681 145 7 | 3.892 996 5 |
| 36 00 | 216 000 | 7.745 966 7 | 3.914 867 6 |
| 37 21 | 226 981 | 7.810 249 7 | 3.936 497 2 |
| 38 44 | 238 328 | 7.874 007 9 | 3.957 891 5 |
| 39 69 | 250 047 | 7.937 253 9 | 3.979 057 1 |
| 40 96 | 262 144 | 8 | 4 |
| 42 25 | 274 625 | 8.062 257 7 | 4.020 725 6 |
| 43 56 | 287 496 | 8.124 038 4 | 4.041 240 1 |
| 44 89 | 300 763 | 8.185 352 8 | 4.061 548 |
| 46 24 | 314 432 | 8.246 211 3 | 4.081 655 1 |
| 47 61 | 328 509 | 8.306 623 9 | 4.101 566 1 |
| 49 00 | 343 000 | 8.366 600 3 | 4.121 285 3 |
| 50 41 | 357 911 | 8.426 149 8 | 4.140 817 8 |
| 51 84 | 373 248 | 8.485 281 4 | 4.160 167 6 |
| 53 29 | 389 017 | 8.544 003 7 | 4.179 339 |
| 54 76 | 405 224 | 8.602 325 3 | 4.198 336 4 |
| 56 25 | 421 875 | 8.660 254 | 4.217 163 3 |
| 57 76 | 438 976 | 8.717 797 9 | 4.235 823 6 |
| 59 29 | 456 533 | 8.774 964 4 | 4.254 321 |
| 60 84 | 474 552 | 8.831 760 9 | 4.272 658 6 |
| 62 41 | 493 039 | 8.888 194 4 | 4.290 840 4 |
| 64 00 | 512 000 | 8.944 271 9 | 4.308 869 5 |
| 65 61 | 531 441 | 9 | 4.326 748 7 |
| 67 24 | 551 368 | 9.055 385 1 | 4.344 481 5 |
| 68 89 | 571 787 | 9.110 433 6 | 4.362 070 7 |
| 70 56 | 592 704 | 9.165 151 4 | 4.379 519 1 |
| 72 25 | 614 125 | 9.219 544 5 | 4.396 829 6 |
| 73 96 | 636 056 | 9.273 618 5 | 4.414 004 9 |
| 75 69 | 658 503 | 9.327 379 1 | 4.431 047 6 |
| 77 44 | 681 472 | 9.380 831 5 | 4.447 960 2 |
| 79 21 | 704 969 | 9.433 981 1 | 4.464 745 1 |
| 81 00 | 729 000 | 9.486 833 | 4.481 404 7 |
| 82 81 | 753 571 | 9.539 392 | 4.497 941 4 |
| 84 64 | 778 688 | 9.591 663 | 4.514 357 4 |
| 86 49 | 804 357 | 9.643 650 8 | 4.530 654 9 |
| 88 36 | 830 584 | 9.695 359 7 | 4.546 835 9 |
| 90 25 | 857 375 | 9.746 794 3 | 4.562 902 6 |
| 92 16 | 884 736 | 9.797 959 | 4.578 857 |
| 94 09 | 912 673 | 9.848 857 8 | 4.594 700 9 |
| 96 04 | 941 192 | 9.899 494 9 | 4.610 436 3 |
| 98 01 | 970 299 | 9.949 874 4 | 4.626 065 |
| 1 00 00 | 1 000 000 | 10 | 4.641 588 8 |
| 1 02 01 | 1 030 301 | 10.049 875 6 | 4.657 009 5 |
| 1 04 04 | 1 061 208 | 10.099 504 9 | 4.672 328 7 |
| 1 06 09 | 1 092 727 | 10.148 891 6 | 4.687 547 2 |
| 1 08 16 | 1 124 864 | 10.198 039 | 4 |
| 1 10 25 | 1 157 625 | 10.246 950 8 | |
| 1 12 36 | 1 191 016 | 10.295 630 1 | |
| 1 14 49 | 1 225 043 | 10.344 080 4 | |
| 1 16 64 | 1 259 712 | 10.392 304 8 | |
| 1 18 81 | 1 295 029 | 10.440 306 5 | |
| 1 21 00 | 1 331 000 | 10.488 088 5 | |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | Cu |
|---------|---------|---------|--------------|----|
| 111 | 12321 | 1367631 | 10.5356538 | 4. |
| 112 | 12544 | 1404928 | 10.5830052 | 4. |
| 113 | 12769 | 1442897 | 10.6301458 | 4. |
| 114 | 12996 | 1481544 | 10.6770783 | 4. |
| 115 | 13225 | 1520875 | 10.7238053 | 4. |
| 116 | 13456 | 1560896 | 10.7703296 | 4. |
| 117 | 13689 | 1601613 | 10.8166538 | 4. |
| 118 | 13924 | 1643032 | 10.8627805 | 4. |
| 119 | 14161 | 1685159 | 10.9087121 | 4. |
| 120 | 14400 | 1728000 | 10.9544512 | 4. |
| 121 | 14641 | 1771561 | 11 | 4. |
| 122 | 14884 | 1815848 | 11.045361 | 4. |
| 123 | 15129 | 1860867 | 11.0905365 | 4. |
| 124 | 15376 | 1906624 | 11.1355287 | 4. |
| 125 | 15625 | 1953125 | 11.1803399 | 5 |
| 126 | 15876 | 2000376 | 11.2249722 | 5. |
| 127 | 16129 | 2048383 | 11.2694277 | 5. |
| 128 | 16384 | 2097152 | 11.3137085 | 5. |
| 129 | 16641 | 2146689 | 11.3578167 | 5. |
| 130 | 16900 | 2197000 | 11.4017543 | 5. |
| 131 | 17161 | 2248091 | 11.4455231 | 5. |
| 132 | 17424 | 2299968 | 11.4891253 | 5. |
| 133 | 17689 | 2352637 | 11.5325626 | 5. |
| 134 | 17956 | 2406104 | 11.5758369 | 5. |
| 135 | 18225 | 2460375 | 11.61895 | 5. |
| 136 | 18496 | 2515456 | 11.6619038 | 5. |
| 137 | 18769 | 2571353 | 11.7046999 | 5. |
| 138 | 19044 | 2628072 | 11.7473401 | 5. |
| 139 | 19321 | 2685619 | 11.7898261 | 5. |
| 140 | 19600 | 2744000 | 11.8321596 | 5. |
| 141 | 19881 | 2803221 | 11.8743421 | 5. |
| 142 | 20164 | 2863288 | 11.9163753 | 5. |
| 143 | 20449 | 2924207 | 11.9582607 | 5. |
| 144 | 20736 | 2985984 | 12 | 5. |
| 145 | 21025 | 3048625 | 12.0415946 | 5. |
| 146 | 21316 | 3112136 | 12.083046 | 5. |
| 147 | 21609 | 3176523 | 12.1243557 | 5. |
| 148 | 21904 | 3241792 | 12.1655251 | 5. |
| 149 | 22201 | 3307949 | 12.2065556 | 5. |
| 150 | 22500 | 3375000 | 12.2474487 | 5. |
| 151 | 22801 | 3442951 | 12.2882057 | 5. |
| 152 | 23104 | 3511008 | 12.328828 | 5. |
| 153 | 23409 | 3581577 | 12.3693169 | 5. |
| 154 | 23716 | 3652264 | 12.4096736 | 5. |
| 155 | 24025 | 3723875 | 12.4498996 | 5. |
| 156 | 24336 | 3796416 | 12.489996 | 5. |
| 157 | 24649 | 3869893 | 12.5299041 | 5. |
| 158 | 24964 | 3944312 | 12.5698051 | 5. |
| | 25281 | 4019679 | 12.6095202 | 5. |
| | 25600 | 4096000 | 12.6491106 | 5. |
| | 25921 | 4173281 | 12.6885775 | 5. |
| | 26244 | 4251528 | 12.7279221 | 5. |
| | | 4330747 | 12.7671453 | 5. |
| | | 4410944 | 12.8062485 | 5. |
| | | 4492125 | 12.8452326 | 5. |
| | | 4574296 | 12.8840987 | 5 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|------------|--------------|-------------|
| 2 78 89 | 4 657 463 | 12.922 848 | 5.506 878 4 |
| 2 82 24 | 4 741 632 | 12.961 481 4 | 5.517 848 4 |
| 2 85 61 | 4 826 809 | 13 | 5.528 774 8 |
| 2 89 00 | 4 913 000 | 13.038 404 8 | 5.539 658 3 |
| 2 92 41 | 5 000 211 | 13.076 696 8 | 5.550 499 1 |
| 2 95 84 | 5 088 448 | 13.114 877 | 5.561 297 8 |
| 2 99 29 | 5 177 717 | 13.152 946 4 | 5.572 054 6 |
| 3 02 76 | 5 268 024 | 13.190 906 | 5.582 770 2 |
| 3 06 25 | 5 359 375 | 13.228 756 6 | 5.593 444 7 |
| 3 09 76 | 5 451 776 | 13.266 499 2 | 5.604 078 7 |
| 3 13 29 | 5 545 233 | 13.304 134 7 | 5.614 672 4 |
| 3 16 84 | 5 639 752 | 13.341 664 1 | 5.625 226 3 |
| 3 20 41 | 5 735 339 | 13.379 088 2 | 5.635 740 8 |
| 3 24 00 | 5 832 000 | 13.416 407 9 | 5.646 216 2 |
| 3 27 61 | 5 929 741 | 13.453 624 | 5.656 652 8 |
| 3 31 24 | 6 028 568 | 13.490 737 6 | 5.667 051 1 |
| 3 34 89 | 6 128 487 | 13.527 749 3 | 5.677 411 4 |
| 3 38 56 | 6 229 504 | 13 564 66 | 5.687 734 |
| 3 42 25 | 6 331 625 | 13.601 470 5 | 5.698 019 2 |
| 3 45 96 | 6 434 856 | 13.638 181 7 | 5.708 267 5 |
| 3 49 69 | 6 539 203 | 13.674 794 3 | 5.718 479 1 |
| 3 53 44 | 6 644 672 | 13.711 309 2 | 5.728 654 3 |
| 3 57 21 | 6 751 269 | 13.747 727 1 | 5.738 793 6 |
| 3 61 00 | 6 859 000 | 13.784 048 8 | 5.748 897 1 |
| 3 64 81 | 6 967 871 | 13.820 275 | 5.758 965 2 |
| 3 68 64 | 7 077 888 | 13.856 406 5 | 5.768 998 2 |
| 3 72 49 | 7 189 057 | 13.892 44 | 5.778 996 6 |
| 3 76 36 | 7 301 384 | 13.928 388 3 | 5.788 960 4 |
| 3 80 25 | 7 414 875 | 13.964 24 | 5.798 89 |
| 3 84 16 | 7 529 536 | 14 | 5.808 785 7 |
| 3 88 09 | 7 645 373 | 14.035 668 8 | 5.818 647 9 |
| 3 92 04 | 7 762 392 | 14.071 247 3 | 5.828 476 7 |
| 3 96 01 | 7 880 599 | 14.106 736 | 5.838 272 5 |
| 4 00 00 | 8 000 000 | 14.142 135 6 | 5.848 035 5 |
| 4 04 01 | 8 120 601 | 14.177 446 9 | 5.857 766 |
| 4 08 04 | 8 242 408 | 14.212 670 4 | 5.867 464 3 |
| 4 12 09 | 8 365 427 | 14.247 806 8 | 5.877 130 7 |
| 4 16 16 | 8 489 664 | 14.282 856 9 | 5.886 765 3 |
| 4 20 25 | 8 615 125 | 14.317 821 1 | 5.896 368 5 |
| 4 24 36 | 8 741 816 | 14.352 700 1 | 5.905 940 6 |
| 4 28 49 | 8 869 743 | 14.387 494 6 | 5.915 481 7 |
| 4 32 64 | 8 998 912 | 14.422 205 1 | 5.924 992 1 |
| 4 36 81 | 9 129 329 | 14.456 832 3 | 5.934 472 1 |
| 4 41 00 | 9 261 000 | 14.491 376 7 | 5.943 922 |
| 4 45 21 | 9 393 931 | 14.525 839 | 5.953 341 8 |
| 4 49 44 | 9 528 128 | 14.560 219 8 | 5.962 732 |
| 4 53 69 | 9 663 597 | 14.594 519 5 | 5.972 092 6 |
| 4 57 96 | 9 800 344 | 14.628 738 8 | 5.981 424 |
| 4 62 25 | 9 938 375 | 14.662 878 3 | 5.990 726 4 |
| 4 66 56 | 10 077 696 | 14.696 938 5 | 6 |
| 4 70 89 | 10 218 313 | 14.730 919 9 | 6.0 |
| 4 75 24 | 10 360 232 | 14.764 823 1 | 6.0 |
| 4 79 61 | 10 503 459 | 14.798 648 6 | 6.0 |
| 4 84 00 | 10 648 000 | 14.832 397 | 6.0 |
| 4 88 41 | 10 793 861 | 14.866 068 7 | 6. |
| 4 92 84 | 10 941 048 | 14.899 664 4 | 6.. |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|---------|------------|--------------|-------------|
| 223 | 4 97 29 | 11 089 567 | 14.933 184 5 | 6.064 127 |
| 224 | 5 01 76 | 11 239 424 | 14.966 629 5 | 6.073 177 9 |
| 225 | 5 06 25 | 11 390 625 | 15 | 6.082 202 |
| 226 | 5 10 76 | 11 543 176 | 15.033 296 4 | 6.091 199 4 |
| 227 | 5 15 29 | 11 697 083 | 15.066 519 2 | 6.100 170 2 |
| 228 | 5 19 84 | 11 852 352 | 15.099 668 9 | 6.109 114 7 |
| 229 | 5 24 41 | 12 008 989 | 15.132 746 | 6.118 033 2 |
| 230 | 5 29 00 | 12 167 000 | 15.165 750 9 | 6.126 925 7 |
| 231 | 5 33 61 | 12 326 391 | 15.198 684 2 | 6.135 792 4 |
| 232 | 5 38 24 | 12 487 168 | 15.231 546 2 | 6.144 633 7 |
| 233 | 5 42 89 | 12 649 337 | 15.264 337 5 | 6.153 445 5 |
| 234 | 5 47 56 | 12 812 904 | 15.297 058 5 | 6.162 240 1 |
| 235 | 5 52 25 | 12 977 875 | 15.329 709 7 | 6.171 005 8 |
| 236 | 5 56 96 | 13 144 256 | 15.362 291 5 | 6.179 746 6 |
| 237 | 5 61 69 | 13 312 053 | 15.394 804 3 | 6.188 462 8 |
| 238 | 5 66 44 | 13 481 272 | 15.427 248 6 | 6.197 154 4 |
| 239 | 5 71 21 | 13 651 919 | 15.459 624 8 | 6.205 821 8 |
| 240 | 5 76 00 | 13 824 000 | 15.491 933 4 | 6.214 465 |
| 241 | 5 80 81 | 13 997 521 | 15.524 174 7 | 6.223 084 3 |
| 242 | 5 85 64 | 14 172 488 | 15.556 349 2 | 6.231 679 7 |
| 243 | 5 90 49 | 14 348 907 | 15.588 457 3 | 6.240 251 5 |
| 244 | 5 95 36 | 14 526 784 | 15.620 499 4 | 6.248 799 8 |
| 245 | 6 00 25 | 14 706 125 | 15.652 475 8 | 6.257 324 8 |
| 246 | 6 05 16 | 14 886 936 | 15.684 387 1 | 6.265 826 6 |
| 247 | 6 10 09 | 15 069 223 | 15.716 233 6 | 6.274 305 4 |
| 248 | 6 15 04 | 15 252 992 | 15.748 015 7 | 6.282 761 3 |
| 249 | 6 20 01 | 15 438 249 | 15.779 733 8 | 6.291 194 6 |
| 250 | 6 25 00 | 15 625 000 | 15.811 388 3 | 6.299 605 3 |
| 251 | 6 30 01 | 15 813 251 | 15.842 979 5 | 6.307 993 5 |
| 252 | 6 35 04 | 16 003 008 | 15.874 507 9 | 6.316 359 6 |
| 253 | 6 40 09 | 16 194 277 | 15.905 973 7 | 6.324 703 5 |
| 254 | 6 45 16 | 16 387 064 | 15.937 377 5 | 6.333 025 6 |
| 255 | 6 50 25 | 16 581 375 | 15.968 719 4 | 6.341 325 7 |
| 256 | 6 55 36 | 16 777 216 | 16 | 6.349 604 2 |
| 257 | 6 60 49 | 16 974 593 | 16.031 219 5 | 6.357 861 1 |
| 258 | 6 65 64 | 17 173 512 | 16.062 378 4 | 6.366 096 8 |
| 259 | 6 70 81 | 17 373 979 | 16.093 476 9 | 6.374 311 1 |
| 260 | 6 76 00 | 17 576 000 | 16.124 515 5 | 6.382 504 3 |
| 261 | 6 81 21 | 17 779 581 | 16.155 494 4 | 6.390 676 5 |
| 262 | 6 86 44 | 17 984 728 | 16.186 414 1 | 6.398 827 9 |
| 263 | 6 91 69 | 18 191 447 | 16.217 274 7 | 6.406 958 5 |
| 264 | 6 96 96 | 18 399 744 | 16.248 076 8 | 6.415 068 7 |
| 265 | 7 02 25 | 18 609 625 | 16.278 820 6 | 6.423 158 3 |
| | 7 07 56 | 18 821 096 | 16.309 506 4 | 6.431 227 6 |
| | 7 12 89 | 19 034 163 | 16.340 134 6 | 6.439 276 7 |
| | 7 18 24 | 19 248 832 | 16.370 705 5 | 6.447 305 7 |
| | 7 23 61 | 19 465 109 | 16.401 219 5 | 6.455 314 8 |
| | 7 29 00 | 19 683 000 | 16.431 676 7 | 6.463 304 1 |
| | 7 34 41 | 19 902 511 | 16.462 077 6 | 6.471 273 6 |
| | 7 39 84 | 20 123 648 | 16.492 422 5 | 6.479 223 6 |
| | 7 45 29 | 20 346 417 | 16.522 711 6 | 6.487 154 1 |
| | 7 50 76 | 20 570 824 | 16.552 945 4 | 6.495 065 3 |
| | 7 56 25 | 20 796 875 | 16.583 124 | 6.502 957 2 |
| | 7 61 76 | 21 024 576 | 16.613 247 7 | 6.510 83 |
| | 7 67 29 | 21 253 933 | 16.643 317 | 6.518 683 9 |
| | 7 72 84 | 21 484 952 | 16.678 332 | 6.526 518 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|----------|------------|--------------|-------------|
| 7 78 41 | 21 717 639 | 16.703 293 1 | 6.534 335 1 |
| 7 84 00 | 21 952 000 | 16.733 200 5 | 6.542 132 6 |
| 7 89 61 | 22 188 041 | 16.763 054 6 | 6.549 911 6 |
| 7 95 24 | 22 425 768 | 16.792 855 6 | 6.557 672 2 |
| 8 00 89 | 22 665 187 | 16.822 603 8 | 6.565 414 4 |
| 8 06 56 | 22 906 304 | 16.852 299 5 | 6.573 138 5 |
| 8 12 25 | 23 149 125 | 16.881 943 | 6.580 844 3 |
| 8 17 06 | 23 393 656 | 16.911 534 5 | 6.588 532 3 |
| 8 23 69 | 23 639 903 | 16.941 074 3 | 6.596 202 3 |
| 8 29 44 | 23 887 872 | 16.970 562 7 | 6.603 854 5 |
| 8 35 21 | 24 137 569 | 17 | 6.611 489 |
| 8 41 00 | 24 389 000 | 17.029 386 4 | 6.619 106 |
| 8 46 81 | 24 642 171 | 17.058 722 1 | 6.626 705 4 |
| 8 52 64 | 24 897 088 | 17.088 007 5 | 6.634 287 4 |
| 8 58 49 | 25 153 757 | 17.117 242 8 | 6.641 852 2 |
| 8 64 36 | 25 412 184 | 17.146 428 2 | 6.649 399 8 |
| 8 70 25 | 25 672 375 | 17.175 564 | 6.656 930 2 |
| 8 76 16 | 25 934 336 | 17.204 650 5 | 6.664 443 7 |
| 8 82 09 | 26 198 073 | 17.233 687 9 | 6.671 940 3 |
| 8 88 04 | 26 463 592 | 17.262 676 5 | 6.679 42 |
| 8 94 01 | 26 730 899 | 17.291 616 5 | 6.686 883 1 |
| 9 00 00 | 27 000 000 | 17.320 508 1 | 6.694 329 5 |
| 9 06 01 | 27 270 901 | 17.349 351 6 | 6.701 759 3 |
| 9 12 04 | 27 543 608 | 17.378 147 2 | 6.709 172 9 |
| 9 18 09 | 27 818 127 | 17.406 895 2 | 6.716 57 |
| 9 24 16 | 28 094 464 | 17.435 595 8 | 6.723 950 8 |
| 9 30 25 | 28 372 625 | 17.464 249 2 | 6.731 315 5 |
| 9 36 36 | 28 652 616 | 17.492 855 7 | 6.738 664 1 |
| 9 42 49 | 28 934 443 | 17.521 415 5 | 6.745 996 7 |
| 9 48 64 | 29 218 112 | 17.549 928 8 | 6.753 313 4 |
| 9 54 81 | 29 503 629 | 17.578 395 8 | 6.760 614 3 |
| 9 61 00 | 29 791 000 | 17.606 816 9 | 6.767 899 5 |
| 9 67 21 | 30 080 231 | 17.635 192 1 | 6.775 169 |
| 9 73 44 | 30 371 328 | 17.663 521 7 | 6.782 422 9 |
| 9 79 69 | 30 664 297 | 17.691 806 | 6.789 661 3 |
| 9 85 96 | 30 959 144 | 17.720 045 1 | 6.796 884 4 |
| 9 92 25 | 31 255 875 | 17.748 239 3 | 6.804 092 1 |
| 9 98 56 | 31 554 496 | 17.776 388 8 | 6.811 284 7 |
| 10 04 89 | 31 855 013 | 17.804 493 8 | 6.818 462 |
| 10 11 24 | 32 157 432 | 17.832 554 5 | 6.825 624 2 |
| 10 17 61 | 32 461 759 | 17.860 571 1 | 6.832 771 4 |
| 10 24 00 | 32 768 000 | 17.888 543 8 | 6.839 903 7 |
| 10 30 41 | 33 076 161 | 17.916 472 9 | 6.847 021 3 |
| 10 36 84 | 33 386 248 | 17.944 358 4 | 6.854 124 |
| 10 43 29 | 33 698 267 | 17.972 200 8 | 6.861 212 |
| 10 49 76 | 34 012 224 | 18 | 6.868 285 5 |
| 10 56 25 | 34 328 125 | 18.027 756 4 | 6.875 344 3 |
| 10 62 76 | 34 645 976 | 18.055 470 1 | 6.882 388 8 |
| 10 69 29 | 34 965 783 | 18.083 141 3 | 6.889 418 8 |
| 10 75 84 | 35 287 552 | 18.110 770 3 | 6.896 |
| 10 82 41 | 35 611 289 | 18.138 357 1 | 6.9 |
| 10 89 00 | 35 937 000 | 18.165 902 1 | 6.9 |
| 10 95 61 | 36 264 691 | 18.193 405 4 | 6. |
| 11 02 24 | 36 594 368 | 18.220 867 2 | 6. |
| 11 08 89 | 36 926 037 | 18.248 287 6 | 6. |
| 11 15 56 | 37 259 704 | 18.275 666 9 | 6. |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE |
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| 335 | 11 22 25 | 37 595 375 | 18.303 005 2 | 6.94 |
| 336 | 11 28 96 | 37 933 056 | 18.330 302 8 | 6.95 |
| 337 | 11 35 69 | 38 272 753 | 18.357 559 8 | 6.95 |
| 338 | 11 42 44 | 38 614 472 | 18.384 776 3 | 6.96 |
| 339 | 11 49 21 | 38 958 219 | 18.411 952 6 | 6.97 |
| 340 | 11 56 00 | 39 304 000 | 18.439 088 9 | 6.97 |
| 341 | 11 62 81 | 39 651 821 | 18.466 185 3 | 6.98 |
| 342 | 11 69 64 | 40 001 688 | 18.493 242 | 6.99 |
| 343 | 11 76 49 | 40 353 607 | 18.520 259 2 | 7 |
| 344 | 11 83 36 | 40 707 584 | 18.547 237 | 7.00 |
| 345 | 11 90 25 | 41 063 625 | 18.574 175 6 | 7.01 |
| 346 | 11 97 16 | 41 421 736 | 18.601 075 2 | 7.02 |
| 347 | 12 04 09 | 41 781 923 | 18.627 936 | 7.02 |
| 348 | 12 11 04 | 42 144 192 | 18.654 758 1 | 7.03 |
| 349 | 12 18 01 | 42 508 549 | 18.681 541 7 | 7.04 |
| 350 | 12 25 00 | 42 875 000 | 18.708 286 9 | 7.04 |
| 351 | 12 32 01 | 43 243 551 | 18.734 994 | 7.05 |
| 352 | 12 39 04 | 43 614 208 | 18.761 663 | 7.06 |
| 353 | 12 46 09 | 43 986 977 | 18.788 294 2 | 7.06 |
| 354 | 12 53 16 | 44 361 864 | 18.814 887 7 | 7.07 |
| 355 | 12 60 25 | 44 738 875 | 18.841 443 7 | 7.08 |
| 356 | 12 67 36 | 45 118 016 | 18.867 962 3 | 7.08 |
| 357 | 12 74 49 | 45 499 293 | 18.894 443 6 | 7.09 |
| 358 | 12 81 64 | 45 882 712 | 18.920 887 9 | 7.10 |
| 359 | 12 88 81 | 46 268 279 | 18.947 295 3 | 7.10 |
| 360 | 12 96 00 | 46 656 000 | 18.973 666 | 7.11 |
| 361 | 13 03 21 | 47 045 831 | 19 | 7.12 |
| 362 | 13 10 44 | 47 437 928 | 19.026 297 6 | 7.12 |
| 363 | 13 17 69 | 47 832 147 | 19.052 558 9 | 7.13 |
| 364 | 13 24 96 | 48 228 544 | 19.078 784 | 7.14 |
| 365 | 13 32 25 | 48 627 125 | 19.104 973 2 | 7.14 |
| 366 | 13 39 56 | 49 027 896 | 19.131 126 5 | 7.15 |
| 367 | 13 46 89 | 49 430 863 | 19.157 244 1 | 7.15 |
| 368 | 13 54 24 | 49 836 032 | 19.183 326 1 | 7.16 |
| 369 | 13 61 61 | 50 243 409 | 19.209 372 7 | 7.17 |
| 370 | 13 69 00 | 50 653 000 | 19.235 384 1 | 7.17 |
| 371 | 13 76 41 | 51 064 811 | 19.261 360 3 | 7.18 |
| 372 | 13 83 84 | 51 478 848 | 19.287 301 5 | 7.19 |
| 373 | 13 91 29 | 51 895 117 | 19.313 207 9 | 7.19 |
| 374 | 13 98 76 | 52 313 624 | 19.339 079 6 | 7.20 |
| 375 | 14 06 25 | 52 734 375 | 19.364 916 7 | 7.21 |
| 376 | 14 13 76 | 53 157 376 | 19.390 719 4 | 7.21 |
| 377 | 14 21 29 | 53 582 633 | 19.416 487 8 | 7.22 |
| 378 | 14 28 84 | 54 010 152 | 19.442 222 1 | 7.23 |
| | 14 36 41 | 54 439 939 | 19.467 922 3 | 7.23 |
| | 14 44 00 | 54 872 000 | 19.493 588 7 | 7.24 |
| | 14 51 61 | 55 306 341 | 19.519 221 3 | 7.24 |
| | 14 59 24 | 55 742 968 | 19.544 820 3 | 7.25 |
| | 14 66 89 | 56 181 887 | 19.570 385 8 | 7.26 |
| | 14 74 56 | 56 623 104 | 19.595 917 9 | 7.26 |
| | 82 25 | 57 066 625 | 19.621 416 9 | 7.27 |
| | 9 96 | 57 512 456 | 19.646 882 7 | 7.28 |
| | 7 69 | 57 960 603 | 19.672 315 6 | 7.2 |
| | 5 44 | 58 411 072 | 19.697 715 6 | 7. |
| | 3 21 | 58 863 869 | 19.723 082 9 | |
| | 1 00 | 59 319 000 | 19.748 417 7 | |

| NUM. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|------|----------|------------|--------------|-------------|
| 1 | 15 28 81 | 59 776 471 | 19.773 719 9 | 7.312 382 8 |
| 2 | 15 36 64 | 60 236 288 | 19.798 989 9 | 7.318 611 4 |
| 3 | 15 44 49 | 60 698 457 | 19.824 227 6 | 7.324 829 5 |
| 4 | 15 52 36 | 61 162 984 | 19.849 433 2 | 7.331 036 9 |
| 5 | 15 60 25 | 61 629 875 | 19.874 600 9 | 7.337 233 9 |
| 6 | 15 68 16 | 62 099 136 | 19.899 748 7 | 7.343 420 5 |
| 7 | 15 76 09 | 62 570 773 | 19.924 858 8 | 7.349 596 6 |
| 8 | 15 84 04 | 63 044 792 | 19.949 937 3 | 7.355 762 4 |
| 9 | 15 92 01 | 63 521 199 | 19.974 984 4 | 7.361 917 8 |
| 0 | 16 00 00 | 64 000 000 | 20 | 7.368 063 |
| 1 | 16 08 01 | 64 481 201 | 20.024 984 4 | 7.374 197 9 |
| 2 | 16 16 04 | 64 964 808 | 20.049 937 7 | 7.380 322 7 |
| 3 | 16 24 09 | 65 450 827 | 20.074 859 9 | 7.386 437 3 |
| 4 | 16 32 16 | 65 939 264 | 20.099 751 2 | 7.392 541 8 |
| 5 | 16 40 25 | 66 430 125 | 20.124 611 8 | 7.398 636 3 |
| 6 | 16 48 36 | 66 923 416 | 20.149 441 7 | 7.404 720 6 |
| 7 | 16 56 49 | 67 419 143 | 20.174 241 | 7.410 795 |
| 8 | 16 64 64 | 67 917 312 | 20.199 009 9 | 7.416 859 5 |
| 9 | 16 72 81 | 68 417 929 | 20.223 748 4 | 7.422 914 2 |
| 0 | 16 81 00 | 68 921 000 | 20.248 456 7 | 7.428 958 9 |
| 1 | 16 89 21 | 69 426 531 | 20.273 134 9 | 7.434 993 8 |
| 2 | 16 97 44 | 69 934 528 | 20.297 783 1 | 7.441 018 9 |
| 3 | 17 05 69 | 70 444 997 | 20.322 401 4 | 7.447 034 2 |
| 4 | 17 13 96 | 70 957 944 | 20.346 989 9 | 7.453 039 9 |
| 5 | 17 22 25 | 71 473 375 | 20.371 548 8 | 7.459 035 9 |
| 6 | 17 30 56 | 71 991 296 | 20.396 078 1 | 7.465 022 3 |
| 7 | 17 38 89 | 72 511 713 | 20.420 577 9 | 7.470 999 1 |
| 8 | 17 47 24 | 73 034 632 | 20.445 048 3 | 7.476 966 4 |
| 9 | 17 55 61 | 73 560 059 | 20.469 489 5 | 7.482 924 2 |
| 0 | 17 64 00 | 74 088 000 | 20.493 901 5 | 7.488 872 4 |
| 1 | 17 72 41 | 74 618 461 | 20.518 284 5 | 7.494 811 3 |
| 2 | 17 80 84 | 75 151 448 | 20.542 638 6 | 7.500 740 6 |
| 3 | 17 89 29 | 75 686 967 | 20.566 963 8 | 7.506 660 7 |
| 4 | 17 97 76 | 76 225 024 | 20.591 260 3 | 7.512 571 5 |
| 5 | 18 06 25 | 76 765 625 | 20.615 528 1 | 7.518 473 |
| 6 | 18 14 76 | 77 308 776 | 20.639 767 4 | 7.524 365 2 |
| 7 | 18 23 29 | 77 854 483 | 20.663 978 3 | 7.530 248 2 |
| 8 | 18 31 84 | 78 402 752 | 20.688 160 9 | 7.536 122 1 |
| 9 | 18 40 41 | 78 953 589 | 20.712 315 2 | 7.541 986 7 |
| 0 | 18 49 00 | 79 507 000 | 20.736 441 4 | 7.547 842 3 |
| 1 | 18 57 61 | 80 062 991 | 20.760 539 5 | 7.553 688 8 |
| 2 | 18 66 24 | 80 621 568 | 20.784 609 7 | 7.559 526 3 |
| 3 | 18 74 89 | 81 182 737 | 20.808 652 | 7.565 354 8 |
| 4 | 18 83 56 | 81 746 504 | 20.832 666 7 | 7.571 174 3 |
| 5 | 18 92 25 | 82 312 875 | 20.856 653 6 | 7.576 984 9 |
| 6 | 19 00 96 | 82 881 856 | 20.880 613 | 7.582 786 5 |
| 7 | 19 09 69 | 83 453 453 | 20.904 545 | 7.588 579 3 |
| 8 | 19 18 44 | 84 027 672 | 20.928 449 5 | 7.594 363 3 |
| 9 | 19 27 21 | 84 604 519 | 20.952 326 8 | 7.600 138 5 |
| 0 | 19 36 00 | 85 184 000 | 20.976 177 | 7.605 001 0 |
| 1 | 19 44 81 | 85 766 121 | 21 | 7.6 |
| 2 | 19 53 64 | 86 350 888 | 21.023 796 | 7.6 |
| 3 | 19 62 49 | 86 938 307 | 21.047 565 2 | 7.6 |
| 4 | 19 71 36 | 87 528 384 | 21.071 307 5 | 7.6 |
| 5 | 19 80 25 | 88 121 125 | 21.095 023 1 | 7.6 |
| 6 | 19 89 16 | 88 716 536 | 21.118 712 1 | 7.6 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|----------|-------------|--------------|------------|
| 559 | 31 24 81 | 174 676 879 | 23.643 180 8 | 8.237 1 |
| 560 | 31 36 00 | 175 616 000 | 23.664 319 1 | 8.242 1 |
| 561 | 31 47 21 | 176 558 481 | 23.685 438 6 | 8.247 1 |
| 562 | 31 58 44 | 177 504 328 | 23.706 539 2 | 8.252 1 |
| 563 | 31 69 69 | 178 453 547 | 23.727 621 | 8.257 1 |
| 564 | 31 80 96 | 179 406 144 | 23.748 684 2 | 8.262 1 |
| 565 | 31 92 25 | 180 362 125 | 23.769 728 6 | 8.267 1 |
| 566 | 32 03 56 | 181 321 496 | 23.790 754 5 | 8.271 1 |
| 567 | 32 14 89 | 182 284 263 | 23.811 761 8 | 8.276 1 |
| 568 | 32 26 24 | 183 250 432 | 23.832 750 6 | 8.281 1 |
| 569 | 32 37 61 | 184 220 009 | 23.853 720 9 | 8.286 1 |
| 570 | 32 49 00 | 185 193 000 | 23.874 672 8 | 8.291 1 |
| 571 | 32 60 41 | 186 169 411 | 23.895 606 3 | 8.296 1 |
| 572 | 32 71 84 | 187 149 248 | 23.916 521 5 | 8.301 1 |
| 573 | 32 83 29 | 188 132 517 | 23.937 418 4 | 8.305 1 |
| 574 | 32 94 76 | 189 119 224 | 23.958 297 1 | 8.310 1 |
| 575 | 33 06 25 | 190 109 375 | 23.979 157 6 | 8.315 1 |
| 576 | 33 17 76 | 191 102 976 | 24 | 8.320 1 |
| 577 | 33 29 29 | 192 100 033 | 24.020 824 3 | 8.325 1 |
| 578 | 33 40 84 | 193 100 552 | 24.041 630 6 | 8.329 1 |
| 579 | 33 52 41 | 194 104 539 | 24.062 418 8 | 8.334 1 |
| 580 | 33 64 00 | 195 112 000 | 24.083 189 1 | 8.339 1 |
| 581 | 33 75 61 | 196 122 941 | 24.103 941 6 | 8.344 1 |
| 582 | 33 87 24 | 197 137 368 | 24.124 676 2 | 8.349 1 |
| 583 | 33 98 89 | 198 155 287 | 24.145 392 9 | 8.353 1 |
| 584 | 34 10 56 | 199 176 704 | 24.166 091 9 | 8.358 1 |
| 585 | 34 22 25 | 200 201 625 | 24.186 773 2 | 8.363 1 |
| 586 | 34 33 96 | 201 230 056 | 24.207 436 9 | 8.368 1 |
| 587 | 34 45 69 | 202 262 003 | 24.228 082 9 | 8.372 1 |
| 588 | 34 57 44 | 203 297 472 | 24.248 711 3 | 8.377 1 |
| 589 | 34 69 21 | 204 336 469 | 24.269 322 2 | 8.382 1 |
| 590 | 34 81 00 | 205 379 000 | 24.289 915 6 | 8.387 1 |
| 591 | 34 92 81 | 206 425 071 | 24.310 491 6 | 8.391 1 |
| 592 | 35 04 64 | 207 474 688 | 24.331 050 1 | 8.396 1 |
| 593 | 35 16 49 | 208 527 857 | 24.351 591 3 | 8.401 1 |
| 594 | 35 28 36 | 209 584 584 | 24.372 115 2 | 8.406 1 |
| 595 | 35 40 25 | 210 644 875 | 24.392 621 8 | 8.410 1 |
| 596 | 35 52 16 | 211 708 736 | 24.413 111 2 | 8.415 1 |
| 597 | 35 64 09 | 212 776 173 | 24.433 583 4 | 8.420 1 |
| 598 | 35 76 04 | 213 847 192 | 24.454 038 5 | 8.424 1 |
| 599 | 35 88 01 | 214 921 799 | 24.474 476 5 | 8.429 1 |
| 600 | 36 00 00 | 216 000 000 | 24.494 897 4 | 8.434 1 |
| 601 | 36 12 01 | 217 081 801 | 24.515 301 3 | 8.439 1 |
| 602 | 36 24 04 | 218 167 208 | 24.535 688 3 | 8.443 1 |
| 603 | 36 36 09 | 219 256 227 | 24.556 058 3 | 8.448 1 |
| 604 | 36 48 16 | 220 348 864 | 24.576 411 5 | 8.453 1 |
| 605 | 36 60 25 | 221 445 125 | 24.596 747 8 | 8.457 1 |
| 606 | 36 72 36 | 222 545 016 | 24.617 067 3 | 8.462 1 |
| 497 | 36 84 49 | 223 648 543 | 24.637 37 | 8.467 1 |
| 498 | 36 96 64 | 224 755 712 | 24.657 656 | 8.471 1 |
| 499 | 37 08 81 | 225 866 529 | 24.677 925 4 | 8.476 1 |
| 610 | 37 21 00 | 226 981 000 | 24.698 178 1 | 8.480 1 |
| 611 | 37 33 21 | 228 099 131 | 24.718 414 2 | 8.485 1 |
| 612 | 37 45 44 | 229 220 928 | 24.738 633 8 | 8.490 1 |
| 613 | 37 57 69 | 230 346 397 | 24.758 836 8 | 8.494 1 |
| 614 | 37 69 96 | 231 475 544 | 24.779 023 4 | 8.498 1 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 25 30 09 | 127 263 527 | 22.427 661 5 | 7.952 847 7 |
| 25 40 16 | 128 024 064 | 22.449 944 3 | 7.958 114 4 |
| 25 50 25 | 128 787 625 | 22.472 205 1 | 7.963 374 3 |
| 25 60 36 | 129 554 216 | 22.494 443 8 | 7.968 627 1 |
| 25 70 49 | 130 323 843 | 22.516 660 5 | 7.973 873 1 |
| 25 80 64 | 131 096 512 | 22.538 855 3 | 7.979 112 2 |
| 25 90 81 | 131 872 229 | 22.561 028 3 | 7.984 344 4 |
| 26 01 00 | 132 651 000 | 22.583 179 6 | 7.989 569 7 |
| 26 11 21 | 133 432 831 | 22.605 309 1 | 7.994 788 3 |
| 26 21 44 | 134 217 728 | 22.627 417 | 8 |
| 26 31 69 | 135 005 697 | 22.649 503 3 | 8.005 204 9 |
| 26 41 96 | 135 796 744 | 22.671 568 1 | 8.010 403 2 |
| 26 52 25 | 136 590 875 | 22.693 611 4 | 8.015 594 6 |
| 26 62 56 | 137 388 096 | 22.715 633 4 | 8.020 779 4 |
| 26 72 89 | 138 188 413 | 22.737 634 | 8.025 957 4 |
| 26 83 24 | 138 991 832 | 22.759 613 4 | 8.031 128 7 |
| 26 93 61 | 139 798 359 | 22.781 571 5 | 8.036 293 5 |
| 27 04 00 | 140 608 000 | 22.803 508 5 | 8.041 451 5 |
| 27 14 41 | 141 420 761 | 22.825 424 4 | 8.046 603 |
| 27 24 84 | 142 236 648 | 22.847 319 3 | 8.051 747 9 |
| 27 35 29 | 143 055 667 | 22.869 193 3 | 8.056 886 2 |
| 27 45 76 | 143 877 824 | 22.891 046 3 | 8.062 018 |
| 27 56 25 | 144 703 125 | 22.912 878 5 | 8.067 143 2 |
| 27 66 76 | 145 531 576 | 22.934 689 9 | 8.072 262 |
| 27 77 29 | 146 363 183 | 22.956 480 6 | 8.077 374 3 |
| 27 87 84 | 147 197 952 | 22.978 250 6 | 8.082 48 |
| 27 98 41 | 148 035 889 | 23 | 8.087 579 4 |
| 28 09 00 | 148 877 000 | 23.021 728 9 | 8.092 672 3 |
| 28 19 61 | 149 721 291 | 23.043 437 2 | 8.097 758 9 |
| 28 30 24 | 150 568 768 | 23.065 125 2 | 8.102 839 |
| 28 40 89 | 151 419 437 | 23.086 792 8 | 8.107 912 8 |
| 28 51 56 | 152 273 304 | 23.108 44 | 8.112 980 3 |
| 28 62 25 | 153 130 375 | 23.130 067 | 8.118 041 4 |
| 28 72 96 | 153 990 656 | 23.151 673 8 | 8.123 096 2 |
| 28 83 69 | 154 854 153 | 23.173 260 5 | 8.128 144 7 |
| 28 94 44 | 155 720 872 | 23.194 827 | 8.133 187 |
| 29 05 21 | 156 590 819 | 23.216 373 5 | 8.138 223 |
| 29 16 00 | 157 464 000 | 23.237 900 1 | 8.143 252 9 |
| 29 26 81 | 158 340 421 | 23.259 406 7 | 8.148 276 5 |
| 29 37 64 | 159 220 088 | 23.280 893 5 | 8.153 293 9 |
| 29 48 49 | 160 103 097 | 23.302 360 4 | 8.158 305 1 |
| 29 59 36 | 160 989 184 | 23.323 807 6 | 8.163 310 2 |
| 29 70 25 | 161 878 625 | 23.345 235 1 | 8.168 312 2 |
| 29 81 16 | 162 771 336 | 23.366 642 9 | 8.173 312 2 |
| 29 92 09 | 163 667 323 | 23.388 031 1 | 8.178 312 2 |
| 30 03 04 | 164 566 592 | 23.409 399 8 | 8.183 312 2 |
| 30 14 01 | 165 469 149 | 23.430 749 | 8.188 312 2 |
| 30 25 00 | 166 375 000 | 23.452 078 8 | 8.193 312 2 |
| 30 36 01 | 167 284 151 | 23.473 399 2 | 8.198 312 2 |
| 30 47 04 | 168 196 608 | 23.494 690 2 | 8.203 312 2 |
| 30 58 09 | 169 112 377 | 23.515 952 | 8.208 312 2 |
| 30 69 16 | 170 031 464 | 23.537 204 6 | 8.213 312 2 |
| 30 80 25 | 170 953 875 | 23.558 43 | 8.218 312 2 |
| 30 91 36 | 171 879 616 | 23.579 65 | 8.223 312 2 |
| 31 02 49 | 172 808 693 | 23.600 84 | 8.228 312 2 |
| 31 13 64 | 173 741 112 | 23.622 04 | 8.233 312 2 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE |
|---------|----------|-------------|--------------|------|
| 559 | 31 24 81 | 174 676 879 | 23.643 180 8 | 8.23 |
| 560 | 31 36 00 | 175 616 000 | 23.664 319 1 | 8.24 |
| 561 | 31 47 21 | 176 558 481 | 23.685 438 6 | 8.24 |
| 562 | 31 58 44 | 177 504 328 | 23.706 539 2 | 8.25 |
| 563 | 31 69 69 | 178 453 547 | 23.727 621 | 8.25 |
| 564 | 31 80 96 | 179 406 144 | 23.748 684 2 | 8.26 |
| 565 | 31 92 25 | 180 362 125 | 23.769 728 6 | 8.26 |
| 566 | 32 03 56 | 181 321 496 | 23.790 754 5 | 8.27 |
| 567 | 32 14 89 | 182 284 263 | 23.811 761 8 | 8.27 |
| 568 | 32 26 24 | 183 250 432 | 23.832 750 6 | 8.28 |
| 569 | 32 37 61 | 184 220 009 | 23.853 720 9 | 8.28 |
| 570 | 32 49 00 | 185 193 000 | 23.874 672 8 | 8.29 |
| 571 | 32 60 41 | 186 169 411 | 23.895 606 3 | 8.29 |
| 572 | 32 71 84 | 187 149 248 | 23.916 521 5 | 8.30 |
| 573 | 32 83 29 | 188 132 517 | 23.937 418 4 | 8.30 |
| 574 | 32 94 76 | 189 119 224 | 23.958 297 1 | 8.31 |
| 575 | 33 06 25 | 190 109 375 | 23.979 157 6 | 8.31 |
| 576 | 33 17 76 | 191 102 976 | 24 | 8.32 |
| 577 | 33 29 29 | 192 100 033 | 24.020 824 3 | 8.32 |
| 578 | 33 40 84 | 193 100 552 | 24.041 630 6 | 8.32 |
| 579 | 33 52 41 | 194 104 539 | 24.062 418 8 | 8.33 |
| 580 | 33 64 00 | 195 112 000 | 24.083 189 1 | 8.33 |
| 581 | 33 75 61 | 196 122 941 | 24.103 941 6 | 8.34 |
| 582 | 33 87 24 | 197 137 368 | 24.124 676 2 | 8.34 |
| 583 | 33 98 89 | 198 155 287 | 24.145 392 9 | 8.35 |
| 584 | 34 10 56 | 199 176 704 | 24.166 091 9 | 8.35 |
| 585 | 34 22 25 | 200 201 625 | 24.186 773 2 | 8.36 |
| 586 | 34 33 96 | 201 230 056 | 24.207 436 9 | 8.36 |
| 587 | 34 45 69 | 202 262 003 | 24.228 082 9 | 8.37 |
| 588 | 34 57 44 | 203 297 472 | 24.248 711 3 | 8.37 |
| 589 | 34 69 21 | 204 336 469 | 24.269 322 2 | 8.38 |
| 590 | 34 81 00 | 205 379 000 | 24.289 915 6 | 8.38 |
| 591 | 34 92 81 | 206 425 071 | 24.310 491 6 | 8.39 |
| 592 | 35 04 64 | 207 474 688 | 24.331 050 1 | 8.39 |
| 593 | 35 16 49 | 208 527 857 | 24.351 591 3 | 8.40 |
| 594 | 35 28 36 | 209 584 584 | 24.372 115 2 | 8.40 |
| 595 | 35 40 25 | 210 644 875 | 24.392 621 8 | 8.41 |
| 596 | 35 52 16 | 211 708 736 | 24.413 111 2 | 8.41 |
| 597 | 35 64 09 | 212 776 173 | 24.433 583 4 | 8.42 |
| 598 | 35 76 04 | 213 847 192 | 24.454 038 5 | 8.42 |
| 599 | 35 88 01 | 214 921 799 | 24.474 476 5 | 8.42 |
| 600 | 36 00 00 | 216 000 000 | 24.494 897 4 | 8.43 |
| 601 | 36 12 01 | 217 081 801 | 24.515 301 3 | 8.43 |
| 602 | 36 24 04 | 218 167 208 | 24.535 688 3 | 8.44 |
| 603 | 36 36 09 | 219 256 227 | 24.556 058 3 | 8.44 |
| 604 | 36 48 16 | 220 348 864 | 24.576 411 5 | 8.45 |
| 605 | 36 60 25 | 221 445 125 | 24.596 747 8 | 8.45 |
| 606 | 36 72 36 | 222 545 016 | 24.617 067 3 | 8.46 |
| 607 | 36 84 49 | 223 648 543 | 24.637 37 | 8.46 |
| 608 | | 224 755 712 | 24.657 656 | 8.47 |
| 609 | | 225 866 529 | 24.677 925 4 | 8.47 |
| 610 | | 226 981 000 | 24.698 178 1 | 8.48 |
| | | 228 099 131 | 24.718 414 2 | 8.48 |
| | | 229 220 928 | 24.738 633 8 | 8.49 |
| | | 230 346 397 | 24.758 836 8 | 8.49 |
| | | 231 475 544 | 24.779 023 4 | 8.49 |

| NUM. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|------|----------|-------------|--------------|-------------|
| 15 | 37 82 25 | 232 608 375 | 24.799 193 5 | 8.504 035 |
| 16 | 37 94 56 | 233 744 896 | 24.819 347 3 | 8.508 641 7 |
| 17 | 38 06 89 | 234 885 113 | 24.839 484 7 | 8.513 243 5 |
| 18 | 38 19 24 | 236 029 032 | 24.859 605 8 | 8.517 840 3 |
| 19 | 38 31 61 | 237 176 659 | 24.879 710 6 | 8.522 432 1 |
| 20 | 38 44 00 | 238 328 000 | 24.899 799 2 | 8.527 018 9 |
| 21 | 38 56 41 | 239 483 061 | 24.919 871 6 | 8.531 600 9 |
| 22 | 38 68 84 | 240 641 848 | 24.939 927 8 | 8.536 178 |
| 23 | 38 81 29 | 241 804 367 | 24.959 967 9 | 8.540 750 1 |
| 24 | 38 93 76 | 242 970 624 | 24.979 992 | 8.545 317 3 |
| 25 | 39 06 25 | 244 140 625 | 25 | 8.549 879 7 |
| 26 | 39 18 76 | 245 134 376 | 25.019 992 | 8.554 437 2 |
| 27 | 39 31 29 | 246 491 883 | 25.039 968 1 | 8.558 989 9 |
| 28 | 39 43 84 | 247 673 152 | 25.059 928 2 | 8.563 537 7 |
| 29 | 39 56 41 | 248 858 189 | 25.079 872 4 | 8.568 080 7 |
| 30 | 39 69 00 | 250 047 000 | 25.099 800 8 | 8.572 618 9 |
| 31 | 39 81 61 | 251 239 591 | 25.119 713 4 | 8.577 152 3 |
| 32 | 39 94 24 | 252 435 968 | 25.139 610 2 | 8.581 680 9 |
| 33 | 40 06 89 | 253 636 137 | 25.159 491 3 | 8.586 204 7 |
| 34 | 40 19 56 | 254 840 104 | 25.179 356 6 | 8.590 723 8 |
| 35 | 40 32 25 | 256 047 875 | 25.199 206 3 | 8.595 238 |
| 36 | 40 44 96 | 257 259 456 | 25.219 040 4 | 8.599 747 6 |
| 37 | 40 57 69 | 258 474 853 | 25.238 858 9 | 8.604 252 5 |
| 38 | 40 70 44 | 259 694 072 | 25.258 661 9 | 8.608 752 6 |
| 39 | 40 83 21 | 260 917 119 | 25.278 449 3 | 8.613 248 |
| 40 | 40 96 00 | 262 144 000 | 25.298 221 3 | 8.617 738 8 |
| 41 | 41 08 81 | 263 374 721 | 25.317 977 8 | 8.622 224 8 |
| 42 | 41 21 64 | 264 609 288 | 25.337 718 9 | 8.626 706 3 |
| 43 | 41 34 49 | 265 847 707 | 25.357 444 7 | 8.631 183 |
| 44 | 41 47 36 | 267 089 984 | 25.377 155 1 | 8.635 655 1 |
| 45 | 41 60 25 | 268 336 125 | 25.396 850 2 | 8.640 122 6 |
| 46 | 41 73 16 | 269 585 136 | 25.416 530 1 | 8.644 585 5 |
| 47 | 41 86 09 | 270 840 023 | 25.436 194 7 | 8.649 043 7 |
| 48 | 41 99 04 | 272 097 792 | 25.455 844 1 | 8.653 497 4 |
| 49 | 42 12 01 | 273 359 549 | 25.475 478 4 | 8.657 946 5 |
| 50 | 42 25 00 | 274 625 000 | 25.495 097 6 | 8.662 391 1 |
| 51 | 42 38 01 | 275 894 451 | 25.514 701 6 | 8.666 831 |
| 52 | 42 51 04 | 277 167 808 | 25.534 290 7 | 8.671 266 5 |
| 53 | 42 64 09 | 278 445 077 | 25.553 864 7 | 8.675 697 4 |
| 54 | 42 77 16 | 279 726 264 | 25.573 423 7 | 8.680 123 7 |
| 55 | 42 90 25 | 281 011 375 | 25.592 967 8 | 8.684 545 6 |
| 56 | 43 03 36 | 282 300 416 | 25.612 496 9 | 8.688 963 |
| 57 | 43 16 49 | 283 593 393 | 25.632 011 2 | 8.693 375 9 |
| 58 | 43 29 64 | 284 890 312 | 25.651 510 7 | 8.697 784 3 |
| 59 | 43 42 81 | 286 191 179 | 25.670 995 3 | 8.702 188 2 |
| 60 | 43 56 00 | 287 496 000 | 25.690 465 2 | 8.706 587 7 |
| 61 | 43 69 21 | 288 804 781 | 25.709 920 3 | 8.710 982 7 |
| 62 | 43 82 44 | 290 117 528 | 25.729 360 7 | 8.715 373 4 |
| 63 | 43 95 69 | 291 434 247 | 25.748 786 4 | 8.719 759 6 |
| 64 | 44 08 96 | 292 754 944 | 25.768 197 5 | 8.724 141 4 |
| 65 | 44 22 25 | 294 079 625 | 25.787 593 9 | 8.728 51 |
| 66 | 44 35 56 | 295 408 296 | 25.806 975 8 | 8.732 81 |
| 67 | 44 48 89 | 296 740 063 | 25.826 343 1 | 8.737 21 |
| 68 | 44 62 24 | 298 077 632 | 25.845 696 | 8.741 6 |
| 69 | 44 75 61 | 299 418 309 | 25.865 034 3 | 8.745 1 |
| 70 | 44 89 00 | 300 763 000 | 25.884 358 2 | 8.750 1 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE R |
|---------|----------|-------------|--------------|--------|
| 783 | 61 30 89 | 480 048 687 | 27.982 137 2 | 9.216 |
| 784 | 61 46 56 | 481 890 304 | 28 | 9.220 |
| 785 | 61 62 25 | 483 736 625 | 28.017 851 5 | 9.224 |
| 786 | 61 77 96 | 485 587 656 | 28.035 691 5 | 9.228 |
| 787 | 61 93 69 | 487 443 403 | 28.053 520 3 | 9.232 |
| 788 | 62 09 44 | 489 303 872 | 28.071 337 7 | 9.236 |
| 789 | 62 25 21 | 491 169 069 | 28.089 143 8 | 9.240 |
| 790 | 62 41 00 | 493 039 000 | 28.106 938 6 | 9.244 |
| 791 | 62 56 81 | 494 913 671 | 28.124 722 2 | 9.248 |
| 792 | 62 72 64 | 496 793 088 | 28.142 494 6 | 9.252 |
| 793 | 62 88 49 | 498 677 257 | 28.160 255 7 | 9.256 |
| 794 | 63 04 36 | 500 566 184 | 28.178 005 6 | 9.259 |
| 795 | 63 20 25 | 502 459 875 | 28.195 744 4 | 9.263 |
| 796 | 63 36 16 | 504 358 336 | 28.213 472 | 9.267 |
| 797 | 63 52 09 | 506 261 573 | 28.231 188 4 | 9.271 |
| 798 | 63 68 04 | 508 169 592 | 28.248 893 8 | 9.275 |
| 799 | 63 84 01 | 510 082 399 | 28.266 588 1 | 9.279 |
| 800 | 64 00 00 | 512 000 000 | 28.284 271 2 | 9.283 |
| 801 | 64 16 01 | 513 922 401 | 28.301 943 4 | 9.287 |
| 802 | 64 32 04 | 515 849 608 | 28.319 604 5 | 9.290 |
| 803 | 64 48 09 | 517 781 627 | 28.337 254 6 | 9.294 |
| 804 | 64 64 16 | 519 718 464 | 28.354 893 8 | 9.298 |
| 805 | 64 80 25 | 521 660 125 | 28.372 521 9 | 9.302 |
| 806 | 64 96 36 | 523 606 616 | 28.390 139 1 | 9.306 |
| 807 | 65 12 49 | 525 557 943 | 28.407 745 4 | 9.310 |
| 808 | 65 28 64 | 527 514 112 | 28.425 340 8 | 9.314 |
| 809 | 65 44 81 | 529 475 129 | 28.442 925 3 | 9.317 |
| 810 | 65 61 00 | 531 441 000 | 28.460 498 9 | 9.321 |
| 811 | 65 77 21 | 533 411 731 | 28.478 061 7 | 9.325 |
| 812 | 65 93 44 | 535 387 328 | 28.495 613 7 | 9.329 |
| 813 | 66 09 69 | 537 367 797 | 28.513 154 9 | 9.333 |
| 814 | 66 25 96 | 539 353 144 | 28.530 685 2 | 9.337 |
| 815 | 66 42 25 | 541 343 375 | 28.548 204 8 | 9.340 |
| 816 | 66 58 56 | 543 338 496 | 28.565 713 7 | 9.344 |
| 817 | 66 74 89 | 545 338 513 | 28.583 211 9 | 9.348 |
| 818 | 66 91 24 | 547 343 432 | 28.600 699 3 | 9.352 |
| 819 | 67 07 61 | 549 353 259 | 28.618 176 | 9.356 |
| 820 | 67 24 00 | 551 368 000 | 28.635 642 1 | 9.359 |
| 821 | 67 40 41 | 553 387 661 | 28.653 097 6 | 9.363 |
| 822 | 67 56 84 | 555 412 248 | 28.670 542 4 | 9.367 |
| 823 | 67 73 29 | 557 441 767 | 28.687 976 6 | 9.371 |
| 824 | 67 89 76 | 559 476 224 | 28.705 400 2 | 9.375 |
| 825 | 68 06 25 | 561 515 625 | 28.722 813 2 | 9.378 |
| 826 | 68 22 76 | 563 559 976 | 28.740 215 7 | 9.382 |
| 827 | 68 39 29 | 565 609 283 | 28.757 607 7 | 9.386 |
| 828 | 68 55 84 | 567 663 552 | 28.774 989 1 | 9.390 |
| 829 | 68 72 41 | 569 722 789 | 28.792 360 1 | 9.394 |
| 830 | 68 89 00 | 571 787 000 | 28.809 720 6 | 9.397 |
| 831 | 69 05 61 | 573 856 191 | 28.827 070 6 | 9.401 |
| 832 | 69 22 24 | 575 930 368 | 28.844 410 2 | 9.405 |
| 833 | 69 38 89 | 578 009 537 | 28.861 739 4 | 9.409 |
| 834 | 69 55 56 | 580 093 704 | 28.879 058 2 | 9.412 |
| | 69 72 25 | 582 182 875 | 28.896 366 6 | 9.416 |
| | 69 88 96 | 584 277 056 | 28.913 664 6 | 9.420 |
| | 70 05 69 | 586 376 253 | 28.930 952 3 | 9.424 |
| | 70 22 44 | 588 480 472 | 28.948 229 7 | 9.428 |

| L. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|----|----------|-------------|--------------|-------------|
| | 70 39 21 | 590 589 719 | 28.965 496 7 | 9.431 642 3 |
| | 70 56 00 | 592 704 000 | 28.982 753 5 | 9.435 38 |
| | 70 72 81 | 594 823 321 | 29 | 9.439 130 7 |
| | 70 89 64 | 596 947 688 | 29.017 236 3 | 9.442 870 4 |
| | 71 06 49 | 599 077 107 | 29.034 462 3 | 9.446 607 2 |
| | 71 23 36 | 601 211 584 | 29.051 678 1 | 9.450 341 |
| | 71 40 25 | 603 351 125 | 29.068 883 7 | 9.454 071 9 |
| | 71 57 16 | 605 495 736 | 29.086 079 1 | 9.457 799 9 |
| | 71 74 09 | 607 645 423 | 29.103 264 4 | 9.461 524 9 |
| | 71 91 04 | 609 800 192 | 29.120 439 6 | 9.465 247 |
| | 72 08 01 | 611 960 049 | 29.137 604 6 | 9.468 966 1 |
| | 72 25 00 | 614 125 000 | 29.154 759 5 | 9.472 682 4 |
| | 72 42 01 | 616 295 051 | 29.171 904 3 | 9.476 395 7 |
| | 72 59 04 | 618 470 208 | 29.189 039 | 9.480 106 1 |
| | 72 76 09 | 620 650 477 | 29.206 163 7 | 9.483 813 6 |
| | 72 93 16 | 622 835 864 | 29.223 278 4 | 9.487 518 2 |
| | 73 10 25 | 625 026 375 | 29.240 383 | 9.491 22 |
| | 73 27 36 | 627 222 016 | 29.257 477 7 | 9.494 918 8 |
| | 73 44 49 | 629 422 793 | 29.274 562 3 | 9.498 614 7 |
| | 73 61 64 | 631 628 712 | 29.291 637 | 9.502 307 8 |
| | 73 78 81 | 633 839 779 | 29.308 701 8 | 9.505 998 |
| | 73 96 00 | 636 056 000 | 29.325 756 6 | 9.509 685 4 |
| | 74 13 21 | 638 277 381 | 29.342 801 5 | 9.513 369 9 |
| | 74 30 44 | 640 503 928 | 29.359 836 5 | 9.517 051 5 |
| | 74 47 69 | 642 735 647 | 29.376 861 6 | 9.520 730 3 |
| | 74 64 96 | 644 972 544 | 29.393 876 9 | 9.524 406 3 |
| | 74 82 25 | 647 214 625 | 29.410 882 3 | 9.528 079 4 |
| | 74 99 56 | 649 461 896 | 29.427 877 9 | 9.531 749 7 |
| | 75 16 89 | 651 714 363 | 29.444 863 7 | 9.535 417 2 |
| | 75 34 24 | 653 972 032 | 29.461 839 7 | 9.539 081 8 |
| | 75 51 61 | 656 234 909 | 29.478 805 9 | 9.542 743 7 |
| | 75 69 00 | 658 503 000 | 29.495 762 4 | 9.546 402 7 |
| | 75 86 41 | 660 776 311 | 29.512 709 1 | 9.550 058 9 |
| | 76 03 84 | 663 054 848 | 29.529 646 1 | 9.553 712 3 |
| | 76 21 29 | 665 338 617 | 29.546 573 4 | 9.557 363 |
| | 76 38 76 | 667 627 624 | 29.563 491 | 9.561 010 8 |
| | 76 56 25 | 669 921 875 | 29.580 398 9 | 9.564 655 9 |
| | 76 73 76 | 672 221 376 | 29.597 297 2 | 9.568 298 2 |
| | 76 91 29 | 674 526 133 | 29.614 185 8 | 9.571 937 7 |
| | 77 08 84 | 676 836 152 | 29.631 064 8 | 9.575 574 5 |
| | 77 26 41 | 679 151 439 | 29.647 934 2 | 9.579 208 5 |
| | 77 44 00 | 681 472 000 | 29.664 793 9 | 9.582 839 7 |
| | 77 61 61 | 683 797 841 | 29.681 644 2 | 9.586 468 2 |
| | 77 79 24 | 686 128 968 | 29.698 484 8 | 9.590 093 7 |
| | 77 96 89 | 688 465 387 | 29.715 315 9 | 9.593 716 9 |
| | 78 14 56 | 690 807 104 | 29.732 137 5 | 9.597 337 3 |
| | 78 32 25 | 693 154 125 | 29.748 949 6 | 9.600 954 8 |
| | 78 49 96 | 695 506 456 | 29.765 752 1 | 9.604 569 6 |
| | 78 67 69 | 697 864 103 | 29.782 545 2 | 9.608 181 7 |
| | 78 85 44 | 700 227 072 | 29.799 328 9 | 9.611 791 1 |
| | 79 03 21 | 702 595 369 | 29.816 103 | |
| | 79 21 00 | 704 969 000 | 29.832 867 8 | |
| | 79 38 81 | 707 347 971 | 29.849 623 1 | |
| | 79 56 64 | 709 732 288 | 29.866 369 | |
| | 79 74 49 | 712 121 957 | 29.883 105 6 | |
| | 79 92 36 | 714 516 984 | 29.899 832 8 | |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE F |
|---------|-----------|-------------|--------------|--------|
| 895 | 80 10 25 | 716 917 375 | 29.916 550 6 | 9.636 |
| 896 | 80 28 16 | 719 323 136 | 29.933 259 1 | 9.640 |
| 897 | 80 46 09 | 721 734 273 | 29.949 958 3 | 9.644 |
| 898 | 80 64 04 | 724 150 792 | 29.966 648 1 | 9.647 |
| 899 | 80 82 01 | 726 572 699 | 29.983 328 7 | 9.651 |
| 900 | 81 00 00 | 729 000 000 | 30 | 9.654 |
| 901 | 81 18 01 | 731 432 701 | 30.016 662 | 9.658 |
| 902 | 81 36 04 | 733 870 808 | 30.033 314 8 | 9.662 |
| 903 | 81 54 09 | 736 314 327 | 30.049 958 4 | 9.665 |
| 904 | 81 72 16 | 738 763 264 | 30.066 592 8 | 9.669 |
| 905 | 81 90 25 | 741 217 625 | 30.083 217 9 | 9.672 |
| 906 | 82 08 36 | 743 677 416 | 30.099 833 9 | 9.676 |
| 907 | 82 26 49 | 746 142 643 | 30.116 440 7 | 9.679 |
| 908 | 82 44 64 | 748 613 312 | 30.133 038 3 | 9.682 |
| 909 | 82 62 81 | 751 089 429 | 30.149 626 9 | 9.686 |
| 910 | 82 81 00 | 753 571 000 | 30.166 266 3 | 9.690 |
| 911 | 82 99 21 | 756 058 031 | 30.182 776 5 | 9.694 |
| 912 | 83 17 44 | 758 550 528 | 30.199 337 7 | 9.697 |
| 913 | 83 35 69 | 761 048 497 | 30.215 889 9 | 9.701 |
| 914 | 83 53 96 | 763 551 944 | 30.232 432 9 | 9.704 |
| 915 | 83 72 25 | 766 060 875 | 30.248 966 9 | 9.708 |
| 916 | 83 90 56 | 768 575 296 | 30.265 491 9 | 9.711 |
| 917 | 84 08 89 | 771 095 213 | 30.282 007 9 | 9.715 |
| 918 | 84 27 24 | 773 620 632 | 30.298 514 8 | 9.718 |
| 919 | 84 45 61 | 776 151 559 | 30.315 012 8 | 9.722 |
| 920 | 84 64 00 | 778 688 000 | 30.331 501 8 | 9.725 |
| 921 | 84 82 41 | 781 229 961 | 30.347 981 8 | 9.729 |
| 922 | 85 00 84 | 783 777 448 | 30.364 452 9 | 9.732 |
| 923 | 85 19 29 | 786 330 467 | 30.380 915 1 | 9.736 |
| 924 | 85 37 76 | 788 889 024 | 30.397 368 3 | 9.739 |
| 925 | 85 56 25 | 791 453 125 | 30.413 812 7 | 9.742 |
| 926 | 85 74 76 | 794 022 776 | 30.430 248 1 | 9.746 |
| 927 | 85 93 29 | 796 597 983 | 30.446 674 7 | 9.750 |
| 928 | 86 11 84 | 799 178 752 | 30.463 092 4 | 9.753 |
| 929 | 86 30 41 | 801 765 089 | 30.479 501 3 | 9.757 |
| 930 | 86 49 00 | 804 357 000 | 30.495 901 4 | 9.761 |
| 931 | 86 67 61 | 806 954 491 | 30.512 292 6 | 9.764 |
| 932 | 86 86 24 | 809 557 568 | 30.528 675 | 9.768 |
| 933 | 87 04 89 | 812 166 237 | 30.545 048 7 | 9.771 |
| 934 | 87 23 56 | 814 780 504 | 30.561 413 6 | 9.774 |
| 935 | 87 42 25 | 817 400 375 | 30.577 769 7 | 9.778 |
| 936 | 87 60 96 | 820 025 856 | 30.594 117 1 | 9.781 |
| 937 | 87 79 69 | 822 656 953 | 30.610 455 7 | 9.785 |
| 938 | 87 98 44 | 825 293 672 | 30.626 785 7 | 9.788 |
| 939 | 88 17 21 | 827 936 019 | 30.643 106 9 | 9.792 |
| 940 | 88 36 00 | 830 584 000 | 30.659 419 4 | 9.795 |
| 941 | 88 54 81 | 833 237 621 | 30.675 723 3 | 9.799 |
| 942 | 88 73 64 | 835 896 888 | 30.692 018 5 | 9.802 |
| 943 | 88 92 49 | 838 561 807 | 30.708 305 1 | 9.806 |
| 944 | 89 11 36 | 841 232 384 | 30.724 583 | 9.809 |
| 945 | 89 30 25 | 843 908 625 | 30.740 852 3 | 9.813 |
| 946 | 89 49 16 | 846 590 536 | 30.757 113 | 9.816 |
| 947 | 89 68 09 | 849 278 123 | 30.773 365 1 | 9.820 |
| 948 | 89 87 04 | 851 971 392 | 30.789 608 6 | 9.823 |
| 949 | 89 106 01 | 854 670 349 | 30.805 843 6 | 9.827 |
| 950 | 89 29 50 | 857 375 000 | 30.822 07 | 9.830 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 90 44 01 | 860 085 351 | 30.838 287 9 | 9.833 923 8 |
| 90 63 04 | 862 801 408 | 30.854 497 2 | 9.837 369 5 |
| 90 82 09 | 865 523 177 | 30.870 698 1 | 9.840 812 7 |
| 91 01 16 | 868 250 664 | 30.886 890 4 | 9.844 253 6 |
| 91 20 25 | 870 983 875 | 30.903 074 3 | 9.847 692 |
| 91 39 36 | 873 722 816 | 30.919 247 7 | 9.851 128 |
| 91 58 49 | 876 467 493 | 30.935 416 6 | 9.854 561 7 |
| 91 77 64 | 879 217 912 | 30.951 575 1 | 9.857 992 9 |
| 91 96 81 | 881 974 079 | 30.967 725 1 | 9.861 421 8 |
| 92 16 00 | 884 736 000 | 30.983 866 8 | 9.864 848 3 |
| 92 35 21 | 887 503 681 | 31 | 9.868 272 4 |
| 92 54 44 | 890 277 128 | 31.016 124 8 | 9.871 694 1 |
| 92 73 69 | 893 056 347 | 31.032 241 3 | 9.875 113 5 |
| 92 92 96 | 895 841 344 | 31.048 349 4 | 9.878 530 5 |
| 93 12 25 | 898 632 125 | 31.064 449 1 | 9.881 945 1 |
| 93 31 56 | 901 428 696 | 31.080 540 5 | 9.885 357 4 |
| 93 50 89 | 904 231 063 | 31.096 623 6 | 9.888 767 3 |
| 93 70 24 | 907 039 232 | 31.112 698 4 | 9.892 174 9 |
| 93 89 61 | 909 853 209 | 31.128 764 8 | 9.895 580 1 |
| 94 09 00 | 912 673 000 | 31.144 823 | 9.898 983 |
| 94 28 41 | 915 498 611 | 31.160 872 9 | 9.902 383 5 |
| 94 47 84 | 918 330 048 | 31.176 914 5 | 9.905 781 7 |
| 94 67 29 | 921 167 317 | 31.192 947 9 | 9.909 177 6 |
| 94 86 76 | 924 010 424 | 31.208 973 1 | 9.912 571 2 |
| 95 06 25 | 926 859 375 | 31.224 99 | 9.915 962 4 |
| 95 25 76 | 929 714 176 | 31.240 998 7 | 9.919 351 3 |
| 95 45 29 | 932 574 833 | 31.256 999 2 | 9.922 737 9 |
| 95 64 84 | 935 441 352 | 31.272 991 5 | 9.926 122 2 |
| 95 84 41 | 938 313 739 | 31.288 975 7 | 9.929 504 2 |
| 96 04 00 | 941 192 000 | 31.304 951 7 | 9.932 883 9 |
| 96 23 61 | 944 076 141 | 31.320 919 5 | 9.936 261 3 |
| 96 43 24 | 946 966 168 | 31.336 879 2 | 9.939 636 3 |
| 96 62 89 | 949 862 087 | 31.352 830 8 | 9.943 009 2 |
| 96 82 56 | 952 763 904 | 31.368 774 3 | 9.946 379 7 |
| 97 02 25 | 955 671 625 | 31.384 709 7 | 9.949 747 9 |
| 97 21 96 | 958 585 256 | 31.400 636 9 | 9.953 113 8 |
| 97 41 69 | 961 504 803 | 31.416 556 1 | 9.956 477 5 |
| 97 61 44 | 964 430 272 | 31.432 467 3 | 9.959 838 9 |
| 97 81 21 | 967 361 669 | 31.448 370 4 | 9.963 198 1 |
| 98 01 00 | 970 299 000 | 31.464 265 4 | 9.966 554 9 |
| 98 20 81 | 973 242 271 | 31.480 152 5 | 9.969 909 5 |
| 98 40 64 | 976 191 488 | 31.496 031 5 | 9.973 261 9 |
| 98 60 49 | 979 146 657 | 31.511 902 5 | 9.976 612 |
| 98 80 36 | 982 107 784 | 31.527 765 5 | 9.979 959 9 |
| 99 00 25 | 985 074 875 | 31.543 620 6 | 9.983 305 5 |
| 99 20 16 | 988 047 936 | 31.559 467 7 | 9.986 648 8 |
| 99 40 09 | 991 026 973 | 31.575 306 8 | 9.989 99 |
| 99 60 04 | 994 011 992 | 31.591 138 | 9.993 328 9 |
| 99 80 01 | 997 002 999 | 31.606 961 3 | 9.996 665 6 |
| 1 00 00 00 | 1 000 000 000 | 31.622 776 6 | 10 |
| 1 00 20 01 | 1 003 003 001 | 31.638 584 | 10 |
| 1 00 40 04 | 1 006 012 008 | 31.654 383 6 | 10 |
| 1 00 60 09 | 1 009 027 027 | 31.670 175 2 | 10 |
| 1 00 80 16 | 1 012 048 064 | 31.685 959 | 10 |
| 1 01 00 25 | 1 015 075 125 | 31.701 734 9 | 10 |
| 1 01 20 36 | 1 018 108 216 | 31.717 503 | 10 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|------------|---------------|--------------|--------------|
| 1007 | 1 01 40 49 | 1 021 147 343 | 31.733 263 3 | 10.023 279 1 |
| 1008 | 1 01 60 64 | 1 024 192 512 | 31.749 015 7 | 10.026 595 8 |
| 1009 | 1 01 80 81 | 1 027 243 729 | 31.764 760 3 | 10.029 910 4 |
| 1010 | 1 02 01 00 | 1 030 301 000 | 31.780 497 2 | 10.033 222 8 |
| 1011 | 1 02 21 21 | 1 033 364 331 | 31.796 226 2 | 10.036 533 |
| 1012 | 1 02 41 44 | 1 036 433 728 | 31.811 947 4 | 10.039 841 |
| 1013 | 1 02 61 69 | 1 039 509 197 | 31.827 660 9 | 10.043 146 9 |
| 1014 | 1 02 81 96 | 1 042 590 744 | 31.843 366 6 | 10.046 450 6 |
| 1015 | 1 03 02 25 | 1 045 678 375 | 31.859 064 6 | 10.049 752 1 |
| 1016 | 1 03 22 56 | 1 048 772 096 | 31.874 754 9 | 10.053 051 4 |
| 1017 | 1 03 42 89 | 1 051 871 913 | 31.890 437 4 | 10.056 348 5 |
| 1018 | 1 03 63 24 | 1 054 977 832 | 31.906 112 3 | 10.059 643 5 |
| 1019 | 1 03 83 61 | 1 058 089 859 | 31.921 779 4 | 10.062 936 4 |
| 1020 | 1 04 04 00 | 1 061 208 000 | 31.937 438 8 | 10.066 227 1 |
| 1021 | 1 04 24 41 | 1 064 332 261 | 31.953 090 6 | 10.069 515 6 |
| 1022 | 1 04 44 84 | 1 067 462 648 | 31.968 734 7 | 10.072 802 |
| 1023 | 1 04 65 29 | 1 070 599 167 | 31.984 371 2 | 10.076 086 3 |
| 1024 | 1 04 85 76 | 1 073 741 824 | 32 | 10.079 368 4 |
| 1025 | 1 05 06 25 | 1 076 890 625 | 32.015 621 2 | 10.082 648 4 |
| 1026 | 1 05 26 76 | 1 080 045 576 | 32.031 234 8 | 10.085 926 2 |
| 1027 | 1 05 47 29 | 1 083 206 683 | 32.046 840 7 | 10.089 201 9 |
| 1028 | 1 05 67 84 | 1 086 373 952 | 32.062 439 1 | 10.092 475 5 |
| 1029 | 1 05 88 41 | 1 089 547 389 | 32.078 029 8 | 10.095 746 9 |
| 1030 | 1 06 09 00 | 1 092 727 000 | 32.093 613 1 | 10.099 016 3 |
| 1031 | 1 06 29 61 | 1 095 912 791 | 32.109 188 7 | 10.102 283 5 |
| 1032 | 1 06 50 24 | 1 099 104 768 | 32.124 756 8 | 10.105 548 7 |
| 1033 | 1 06 70 89 | 1 102 302 937 | 32.140 317 3 | 10.108 811 7 |
| 1034 | 1 06 91 56 | 1 105 507 304 | 32.155 870 4 | 10.112 072 6 |
| 1035 | 1 07 12 25 | 1 108 717 875 | 32.171 415 9 | 10.115 531 4 |
| 1036 | 1 07 32 96 | 1 111 934 656 | 32.186 953 9 | 10.118 588 2 |
| 1037 | 1 07 53 69 | 1 115 157 653 | 32.202 484 4 | 10.121 842 8 |
| 1038 | 1 07 74 44 | 1 118 386 872 | 32.218 007 4 | 10.126 095 3 |
| 1039 | 1 07 95 21 | 1 121 622 319 | 32.233 522 9 | 10.128 345 7 |
| 1040 | 1 08 16 00 | 1 124 864 000 | 32.249 031 | 10.131 594 1 |
| 1041 | 36 81 | 1 128 111 921 | 32.264 531 6 | 10.134 840 3 |
| 1042 | 08 57 64 | 1 131 366 088 | 32.280 024 8 | 10.138 084 5 |
| 1043 | 1 08 78 49 | 1 134 626 507 | 32.295 510 5 | 10.141 326 6 |
| 1044 | 1 08 99 36 | 1 137 893 184 | 32.310 988 8 | 10.144 566 7 |
| 1045 | 1 09 20 25 | 1 141 166 125 | 32.326 459 8 | 10.147 804 7 |
| 1046 | 1 09 41 16 | 1 144 445 336 | 32.341 923 3 | 10.151 040 6 |
| 1047 | 1 09 62 09 | 1 147 730 823 | 32.357 379 4 | 10.154 274 4 |
| 1048 | 1 09 83 04 | 1 151 022 592 | 32.372 828 1 | 10.157 506 2 |
| 1049 | 1 10 04 01 | 1 154 320 649 | 32.388 269 5 | 10.160 735 9 |
| 1050 | 1 10 25 00 | 1 157 625 000 | 32.403 703 5 | 10.163 963 6 |
| 1051 | 1 10 46 01 | 1 160 935 651 | 32.419 130 1 | 10.167 189 3 |
| 1052 | 1 10 67 04 | 1 164 252 608 | 32.434 549 5 | 10.170 412 9 |
| 1053 | 1 10 88 09 | 1 167 575 877 | 32.449 901 5 | 10.173 634 4 |
| 1054 | 1 11 09 16 | 1 170 905 464 | 32.465 366 2 | 10.176 853 9 |
| 1055 | 1 11 30 25 | 1 174 241 375 | 32.480 763 5 | 10.180 071 4 |
| 1056 | 1 11 51 36 | 1 177 583 616 | 32.496 153 6 | 10.183 286 8 |
| 1057 | 1 11 72 49 | 1 180 932 193 | 32.511 536 4 | 10.186 500 2 |
| 1058 | 1 11 93 64 | 1 184 287 112 | 32.526 911 9 | 10.189 711 6 |
| 1059 | 1 12 14 81 | 1 187 648 379 | 32.542 280 2 | 10.192 920 9 |
| 1060 | 1 12 36 00 | 1 191 016 000 | 32.557 641 2 | 10.196 128 3 |
| 1061 | 1 12 57 21 | 1 194 389 981 | 32.572 994 9 | 10.199 333 6 |
| 1062 | 1 12 78 44 | 1 197 770 328 | 32.588 341 5 | 10.202 539 9 |

| S. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| I 12 | 99 69 | I 201 157 047 | 32.603 680 7 | 10.205 738 2 |
| I 13 | 20 96 | I 204 550 144 | 32.619 012 9 | 10.208 937 5 |
| I 13 | 42 25 | I 207 949 625 | 32.634 337 7 | 10.212 134 7 |
| I 13 | 63 56 | I 211 355 496 | 32.649 655 4 | 10.215 33 |
| I 13 | 84 89 | I 214 767 763 | 32.664 965 9 | 10.218 523 3 |
| I 14 | 06 24 | I 218 186 432 | 32.680 269 3 | 10.221 714 6 |
| I 14 | 27 61 | I 221 611 509 | 32.695 565 4 | 10.224 903 9 |
| I 14 | 49 00 | I 225 043 000 | 32.710 854 4 | 10.228 091 2 |
| I 14 | 70 41 | I 228 480 911 | 32.726 136 3 | 10.231 276 6 |
| I 14 | 91 84 | I 231 925 248 | 32.741 411 1 | 10.234 459 9 |
| I 15 | 13 29 | I 235 376 017 | 32.756 678 7 | 10.237 641 3 |
| I 15 | 34 76 | I 238 833 224 | 32.771 939 2 | 10.240 820 7 |
| I 15 | 56 25 | I 242 296 875 | 32.787 192 6 | 10.243 998 1 |
| I 15 | 77 76 | I 245 766 976 | 32.802 438 9 | 10.247 173 5 |
| I 15 | 99 29 | I 249 243 533 | 32.817 678 2 | 10.250 347 |
| I 16 | 20 84 | I 252 726 552 | 32.832 910 3 | 10.253 518 6 |
| I 16 | 42 41 | I 256 216 039 | 32.848 135 4 | 10.256 688 1 |
| I 16 | 64 00 | I 259 712 000 | 32.863 353 5 | 10.259 855 7 |
| I 16 | 85 61 | I 263 214 441 | 32.878 564 4 | 10.263 021 3 |
| I 17 | 07 24 | I 266 723 368 | 32.893 768 4 | 10.266 185 |
| I 17 | 28 89 | I 270 238 787 | 32.908 965 3 | 10.269 346 7 |
| I 17 | 50 56 | I 273 760 704 | 32.924 155 3 | 10.272 506 5 |
| I 17 | 72 25 | I 277 289 125 | 32.939 338 2 | 10.275 664 4 |
| I 17 | 93 96 | I 280 824 056 | 32.954 514 1 | 10.278 820 3 |
| I 18 | 15 69 | I 284 365 503 | 32.969 683 | 10.281 974 3 |
| I 18 | 37 44 | I 287 913 472 | 32.984 845 | 10.285 126 4 |
| I 18 | 59 21 | I 291 467 969 | 33 | 10.288 276 5 |
| I 18 | 81 00 | I 295 029 000 | 33.015 148 | 10.291 424 7 |
| I 19 | 02 81 | I 298 596 571 | 33.030 289 1 | 10.294 570 9 |
| I 19 | 24 64 | I 302 170 688 | 33.045 423 3 | 10.297 715 3 |
| I 19 | 46 49 | I 305 751 357 | 33.060 550 5 | 10.300 857 7 |
| I 19 | 68 36 | I 309 338 584 | 33.075 670 8 | 10.303 998 2 |
| I 19 | 90 25 | I 312 932 375 | 33.090 784 2 | 10.307 136 8 |
| I 20 | 12 16 | I 316 532 736 | 33.105 890 7 | 10.310 273 5 |
| I 20 | 34 09 | I 320 139 673 | 33.120 990 3 | 10.313 408 3 |
| I 20 | 56 04 | I 323 753 192 | 33.136 083 | 10.316 541 ¹ 2 |
| I 20 | 78 01 | I 327 373 299 | 33.151 168 9 | 10.319 675 7 |
| I 21 | 00 00 | I 331 000 000 | 33.166 247 9 | 10.322 809 2 |
| I 21 | 22 01 | I 334 633 301 | 33.181 32 | 10.325 921 ¹ 2 |
| I 21 | 44 04 | I 338 273 208 | 33.196 385 3 | 10.329 055 ¹ 7 |
| I 21 | 66 09 | I 341 919 727 | 33.211 443 8 | 10.332 177 |
| I 21 | 88 16 | I 345 572 864 | 33.226 695 5 | 10.335 298 5 |
| I 22 | 10 25 | I 349 232 625 | 33.241 540 3 | 10.338 418 1 |
| I 22 | 32 36 | I 352 899 016 | 33.256 578 3 | 10.341 53 ¹ 2 |
| I 22 | 54 49 | I 356 572 043 | 33.271 609 5 | 10.344 6 |
| I 22 | 76 64 | I 360 251 712 | 33.286 633 9 | 10.347 |
| I 22 | 98 81 | I 363 938 029 | 33.301 651 6 | 10.350 |
| I 23 | 21 00 | I 367 631 000 | 33.316 662 5 | 10.353 |
| I 23 | 43 21 | I 371 330 631 | 33.331 666 6 | 10.357 |
| I 23 | 65 44 | I 375 036 928 | 33.346 664 | 10.360 |
| I 23 | 87 69 | I 378 749 897 | 33.361 654 6 | 10.363 |
| I 24 | 09 96 | I 382 469 544 | 33.376 638 5 | 10.366 |
| I 24 | 32 25 | I 386 195 875 | 33.391 615 7 | 10.369 5 |
| I 24 | 54 56 | I 389 928 896 | 33.406 586 2 | 10.372 6 |
| I 24 | 76 89 | I 393 668 613 | 33.421 549 9 | 10.375 ¹ 7 |
| I 24 | 99 24 | I 397 415 032 | 33.436 507 | 10.378 803 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 1119 | 1 25 21 61 | 1 401 168 159 | 33.451 457 3 | 10.381 896 5 |
| 1120 | 1 25 44 00 | 1 404 928 000 | 33.466 401 1 | 10.384 988 2 |
| 1121 | 1 25 66 41 | 1 408 694 561 | 33.481 338 1 | 10.388 078 1 |
| 1122 | 1 25 88 84 | 1 412 467 848 | 33.496 268 4 | 10.391 166 1 |
| 1123 | 1 26 11 29 | 1 416 247 867 | 33.511 192 1 | 10.394 252 3 |
| 1124 | 1 26 33 76 | 1 420 034 624 | 33.526 109 2 | 10.397 336 6 |
| 1125 | 1 26 56 25 | 1 423 828 125 | 33.541 019 6 | 10.400 419 2 |
| 1126 | 1 26 78 76 | 1 427 628 376 | 33.555 923 4 | 10.403 499 9 |
| 1127 | 1 27 01 29 | 1 431 435 383 | 33.570 820 6 | 10.406 578 7 |
| 1128 | 1 27 23 84 | 1 435 249 152 | 33.585 711 2 | 10.409 655 7 |
| 1129 | 1 27 46 41 | 1 439 069 689 | 33.600 595 2 | 10.412 731 |
| 1130 | 1 27 69 00 | 1 442 897 000 | 33.615 472 6 | 10.415 804 4 |
| 1131 | 1 27 91 61 | 1 446 731 091 | 33.630 343 4 | 10.418 876 |
| 1132 | 1 28 14 24 | 1 450 571 968 | 33.645 207 7 | 10.421 945 8 |
| 1133 | 1 28 36 89 | 1 454 419 637 | 33.660 065 3 | 10.425 013 8 |
| 1134 | 1 28 59 56 | 1 458 274 104 | 33.674 916 5 | 10.428 08 |
| 1135 | 1 28 82 25 | 1 462 135 375 | 33.689 761 | 10.431 144 3 |
| 1136 | 1 29 04 96 | 1 466 003 456 | 33.704 599 1 | 10.434 206 9 |
| 1137 | 1 29 27 69 | 1 469 878 353 | 33.719 430 6 | 10.437 267 7 |
| 1138 | 1 29 50 44 | 1 473 760 072 | 33.734 255 6 | 10.440 326 7 |
| 1139 | 1 29 73 21 | 1 477 648 619 | 33.749 074 1 | 10.443 383 9 |
| 1140 | 1 29 96 00 | 1 481 544 000 | 33.763 886 0 | 10.446 439 3 |
| 1141 | 1 30 18 81 | 1 485 446 221 | 33.778 691 5 | 10.449 492 9 |
| 1142 | 1 30 41 64 | 1 489 355 288 | 33.793 490 5 | 10.452 544 8 |
| 1143 | 1 30 64 49 | 1 493 271 207 | 33.808 283 | 10.455 594 8 |
| 1144 | 1 30 87 36 | 1 497 193 984 | 33.823 069 1 | 10.458 643 1 |
| 1145 | 1 31 10 25 | 1 501 123 625 | 33.837 848 6 | 10.461 689 6 |
| 1146 | 1 31 33 16 | 1 505 060 136 | 33.852 621 8 | 10.464 734 3 |
| 1147 | 1 31 56 09 | 1 509 003 523 | 33.867 388 4 | 10.467 777 3 |
| 1148 | 1 31 79 04 | 1 512 953 792 | 33.882 148 7 | 10.470 818 5 |
| 1149 | 1 32 02 01 | 1 516 910 949 | 33.896 902 5 | 10.473 857 9 |
| 1150 | 1 32 25 00 | 1 520 875 000 | 33.911 649 9 | 10.476 895 5 |
| 1151 | 1 32 48 01 | 1 524 845 951 | 33.926 390 9 | 10.479 931 4 |
| 1152 | 1 32 71 04 | 1 528 823 808 | 33.941 125 5 | 10.482 965 6 |
| 1153 | 1 32 94 09 | 1 532 808 577 | 33.955 853 7 | 10.485 998 |
| 1154 | 1 33 17 16 | 1 536 800 264 | 33.970 575 5 | 10.489 028 6 |
| 1155 | 1 33 40 25 | 1 540 798 875 | 33.985 291 | 10.492 057 5 |
| 1156 | 1 33 63 36 | 1 544 804 416 | 34 | 10.495 084 7 |
| 1157 | 1 33 86 49 | 1 548 816 893 | 34.014 702 7 | 10.498 110 1 |
| 1158 | 1 34 09 64 | 1 552 836 312 | 34.029 399 | 10.501 133 7 |
| 1159 | 1 34 32 81 | 1 556 862 679 | 34.044 089 | 10.504 155 6 |
| 1160 | 1 34 56 00 | 1 560 896 000 | 34.058 772 7 | 10.507 175 7 |
| 1161 | 1 34 79 21 | 1 564 936 281 | 34.073 450 1 | 10.510 194 2 |
| 1162 | 1 35 02 44 | 1 568 983 528 | 34.088 121 1 | 10.513 210 9 |
| 1163 | 1 35 25 69 | 1 573 037 747 | 34.102 785 8 | 10.516 225 9 |
| 1164 | 1 35 48 96 | 1 577 098 944 | 34.117 444 2 | 10.519 239 1 |
| 1165 | 1 35 72 25 | 1 581 167 125 | 34.132 096 3 | 10.522 250 6 |
| 1166 | 1 35 95 56 | 1 585 242 296 | 34.146 742 2 | 10.525 260 4 |
| 1167 | 1 36 18 89 | 1 589 324 463 | 34.161 381 7 | 10.528 268 5 |
| 1168 | 1 36 42 24 | 1 593 413 632 | 34.176 015 | 10.531 274 9 |
| 1169 | 1 36 65 61 | 1 597 509 809 | 34.190 642 | 10.534 279 5 |
| 1170 | 1 36 89 00 | 1 601 613 000 | 34.205 262 7 | 10.537 282 5 |
| 1171 | 1 37 12 41 | 1 605 723 211 | 34.219 877 3 | 10.540 283 7 |
| 1172 | 1 37 35 84 | 1 609 840 448 | 34.234 485 5 | 10.543 283 7 |
| 1173 | 1 37 59 29 | 1 613 964 717 | 34.249 087 5 | 10.546 281 |
| 1174 | 1 38 22 76 | 1 618 096 024 | 34.263 683 4 | 10.549 277 |

| SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 1 38 06 25 | 1 622 234 375 | 34.278 273 | 10.552 271 5 |
| 1 38 29 76 | 1 626 379 776 | 34.292 856 4 | 10.555 264 2 |
| 1 38 53 29 | 1 630 532 233 | 34.307 433 6 | 10.558 255 2 |
| 1 38 76 84 | 1 634 691 752 | 34.322 004 6 | 10.561 244 5 |
| 1 39 00 41 | 1 638 858 339 | 34.336 569 4 | 10.564 232 2 |
| 1 39 24 00 | 1 643 032 000 | 34.351 128 1 | 10.567 218 1 |
| 1 39 47 61 | 1 647 212 741 | 34.365 680 5 | 10.570 202 4 |
| 1 39 71 24 | 1 651 400 568 | 34.380 226 8 | 10.573 184 9 |
| 1 39 94 89 | 1 655 595 487 | 34.394 767 | 10.576 165 8 |
| 1 40 18 56 | 1 659 797 504 | 34.409 301 1 | 10.579 144 9 |
| 1 40 42 25 | 1 664 006 625 | 34.423 828 9 | 10.582 122 5 |
| 1 40 65 96 | 1 668 222 856 | 34.438 350 7 | 10.585 098 3 |
| 1 40 89 69 | 1 672 446 203 | 34.452 866 3 | 10.588 072 5 |
| 1 41 13 44 | 1 676 676 672 | 34.467 375 9 | 10.591 045 |
| 1 41 37 21 | 1 680 914 269 | 34.481 879 3 | 10.594 015 8 |
| 1 41 61 00 | 1 685 159 000 | 34.496 376 6 | 10.596 985 |
| 1 41 84 81 | 1 689 410 871 | 34.510 867 8 | 10.599 952 5 |
| 1 42 08 64 | 1 693 669 888 | 34.525 353 | 10.602 918 4 |
| 1 42 32 49 | 1 697 936 057 | 34.539 832 1 | 10.605 882 6 |
| 1 42 56 36 | 1 702 209 384 | 34.554 305 1 | 10.608 845 1 |
| 1 42 80 25 | 1 706 489 875 | 34.568 772 | 10.611 806 |
| 1 43 04 16 | 1 710 777 536 | 34.583 232 9 | 10.614 765 2 |
| 1 43 28 09 | 1 715 072 373 | 34.597 687 9 | 10.617 722 8 |
| 1 43 52 04 | 1 719 374 392 | 34.612 136 6 | 10.620 678 8 |
| 1 43 76 01 | 1 723 683 599 | 34.626 579 4 | 10.623 633 1 |
| 1 44 00 00 | 1 728 000 000 | 34.641 016 2 | 10.626 585 7 |
| 1 44 24 01 | 1 732 323 601 | 34.655 446 9 | 10.629 536 7 |
| 1 44 48 04 | 1 736 654 408 | 34.669 871 6 | 10.632 486 |
| 1 44 72 09 | 1 740 992 427 | 34.684 290 4 | 10.635 433 8 |
| 1 44 96 16 | 1 745 337 664 | 34.698 703 1 | 10.638 379 9 |
| 1 45 20 25 | 1 749 690 125 | 34.713 109 9 | 10.641 324 4 |
| 1 45 44 36 | 1 754 049 816 | 34.727 510 7 | 10.644 267 2 |
| 1 45 68 49 | 1 758 416 743 | 34.741 905 5 | 10.647 208 5 |
| 1 45 92 64 | 1 762 790 912 | 34.756 294 4 | 10.650 148 |
| 1 46 16 81 | 1 767 172 329 | 34.770 677 3 | 10.653 086 |
| 1 46 41 00 | 1 771 561 000 | 34.785 054 3 | 10.656 022 3 |
| 1 46 65 21 | 1 775 956 931 | 34.799 425 3 | 10.658 957 |
| 1 46 89 44 | 1 780 360 128 | 34.813 790 4 | 10.661 890 2 |
| 1 47 13 69 | 1 784 770 597 | 34.828 149 5 | 10.664 821 7 |
| 1 47 37 96 | 1 789 188 344 | 34.842 502 8 | 10.667 751 6 |
| 1 47 62 25 | 1 793 613 375 | 34.856 850 1 | 10.670 679 9 |
| 1 47 86 56 | 1 798 045 696 | 34.871 191 5 | 10.673 606 6 |
| 1 48 10 89 | 1 802 485 313 | 34.885 527 1 | 10.676 527 |
| 1 48 35 24 | 1 806 932 232 | 34.899 856 7 | 10.679 |
| 1 48 59 61 | 1 811 386 459 | 34.914 180 5 | 10.682 |
| 1 48 84 00 | 1 815 848 000 | 34.928 498 4 | 10.685 |
| 1 49 08 41 | 1 820 316 861 | 34.942 810 4 | 10.688 |
| 1 49 32 84 | 1 824 793 048 | 34.957 116 6 | 10.691 |
| 1 49 57 29 | 1 829 276 567 | 34.971 416 9 | 10.694 |
| 1 49 81 76 | 1 833 767 424 | 34.985 711 4 | 10.697 |
| 1 50 06 25 | 1 838 265 625 | 35 | 10.700 |
| 1 50 30 76 | 1 842 771 176 | 35.014 282 8 | 10.703 |
| 1 50 55 29 | 1 847 284 083 | 35.028 559 8 | 10.706 |
| 1 50 79 84 | 1 851 804 352 | 35.042 830 9 | 10.709 |
| 1 51 04 41 | 1 856 331 989 | 35.057 096 3 | 10.712 |
| 1 51 29 00 | 1 860 867 000 | 35.071 355 8 | 10.715 |

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 1231 | 1 51 53 61 | 1 865 409 391 | 35.085 609 6 | 10.717 315 5 |
| 1232 | 1 51 78 24 | 1 869 959 168 | 35.099 857 5 | 10.720 216 8 |
| 1233 | 1 52 02 89 | 1 874 516 337 | 35.114 099 7 | 10.723 116 5 |
| 1234 | 1 52 27 56 | 1 879 080 904 | 35.128 336 1 | 10.726 014 6 |
| 1235 | 1 52 52 25 | 1 883 652 875 | 35.142 556 8 | 10.728 911 2 |
| 1236 | 1 52 76 96 | 1 888 232 256 | 35.156 791 7 | 10.731 806 2 |
| 1237 | 1 53 01 69 | 1 892 819 053 | 35.171 010 8 | 10.734 699 7 |
| 1238 | 1 53 26 44 | 1 897 413 272 | 35.185 224 2 | 10.737 591 6 |
| 1239 | 1 53 51 21 | 1 902 014 919 | 35.199 431 8 | 10.740 481 9 |
| 1240 | 1 53 76 00 | 1 906 624 000 | 35.213 633 7 | 10.743 370 7 |
| 1241 | 1 54 00 81 | 1 911 240 521 | 35.227 829 9 | 10.746 257 9 |
| 1242 | 1 54 25 64 | 1 915 864 488 | 35.242 020 4 | 10.749 143 6 |
| 1243 | 1 54 50 49 | 1 920 495 907 | 35.256 205 1 | 10.752 027 7 |
| 1244 | 1 54 75 36 | 1 925 134 784 | 35.270 384 2 | 10.754 910 3 |
| 1245 | 1 55 00 25 | 1 929 781 125 | 35.284 557 5 | 10.757 791 3 |
| 1246 | 1 55 25 16 | 1 934 434 936 | 35.298 725 2 | 10.760 670 8 |
| 1247 | 1 55 50 09 | 1 939 096 223 | 35.312 887 2 | 10.763 548 8 |
| 1248 | 1 55 75 04 | 1 943 764 992 | 35.327 043 5 | 10.766 425 2 |
| 1249 | 1 56 00 01 | 1 948 441 249 | 35.341 194 1 | 10.769 300 1 |
| 1250 | 1 56 25 00 | 1 953 125 000 | 35.355 339 1 | 10.772 173 5 |
| 1251 | 1 56 50 01 | 1 957 816 251 | 35.369 478 4 | 10.775 045 3 |
| 1252 | 1 56 75 04 | 1 962 515 008 | 35.383 612 | 10.777 915 6 |
| 1253 | 1 57 00 09 | 1 967 221 277 | 35.397 74 | 10.780 784 3 |
| 1254 | 1 57 25 16 | 1 971 935 064 | 35.411 862 4 | 10.783 651 6 |
| 1255 | 1 57 50 25 | 1 976 656 375 | 35.425 979 2 | 10.786 517 3 |
| 1256 | 1 57 75 36 | 1 981 385 216 | 35.440 090 3 | 10.789 381 5 |
| 1257 | 1 58 00 49 | 1 986 121 593 | 35.454 195 8 | 10.792 244 1 |
| 1258 | 1 58 25 64 | 1 990 865 512 | 35.468 295 7 | 10.795 105 3 |
| 1259 | 1 58 50 81 | 1 995 616 979 | 35.482 39 | 10.797 964 9 |
| 1260 | 1 58 76 00 | 2 000 376 000 | 35.496 478 7 | 10.800 823 |
| 1261 | 1 59 01 21 | 2 005 142 581 | 35.510 561 8 | 10.803 679 7 |
| 1262 | 1 59 26 44 | 2 009 916 728 | 35.524 639 3 | 10.806 534 8 |
| 1263 | 1 59 51 69 | 2 014 698 447 | 35.538 711 3 | 10.809 388 4 |
| 1264 | 1 59 76 96 | 2 019 487 744 | 35.552 777 7 | 10.812 240 4 |
| 1265 | 1 60 02 25 | 2 024 284 625 | 35.566 838 5 | 10.815 090 9 |
| 1266 | 1 60 27 56 | 2 029 089 096 | 35.580 893 7 | 10.817 94 |
| 1267 | 1 60 52 89 | 2 033 901 163 | 35.594 943 4 | 10.820 787 6 |
| 1268 | 1 60 78 24 | 2 038 720 832 | 35.608 987 6 | 10.823 633 6 |
| 1269 | 1 61 03 61 | 2 043 548 109 | 35.623 026 2 | 10.826 478 2 |
| 1270 | 1 61 29 00 | 2 048 383 000 | 35.637 059 3 | 10.829 321 3 |
| 1271 | 1 61 54 41 | 2 053 225 511 | 35.651 086 9 | 10.832 162 9 |
| 1272 | 1 61 79 84 | 2 058 075 648 | 35.665 109 | 10.835 003 |
| 1273 | 1 62 05 29 | 2 062 933 417 | 35.679 125 5 | 10.837 841 6 |
| 1274 | 1 62 30 76 | 2 067 798 824 | 35.693 136 6 | 10.840 678 8 |
| 1275 | 1 62 56 25 | 2 072 671 875 | 35.707 142 1 | 10.843 514 4 |
| 1276 | 1 62 81 76 | 2 077 552 576 | 35.721 142 2 | 10.846 348 5 |
| 1277 | 1 63 07 29 | 2 082 440 933 | 35.735 136 7 | 10.849 181 2 |
| 1278 | 1 63 32 84 | 2 087 336 952 | 35.749 125 8 | 10.852 012 5 |
| 1279 | 1 63 58 41 | 2 092 240 639 | 35.763 109 5 | 10.854 842 2 |
| | 1 63 84 00 | 2 097 152 000 | 35.777 087 6 | 10.857 670 4 |
| | 1 64 09 61 | 2 102 071 041 | 35.791 060 3 | 10.860 497 2 |
| | 24 | 2 106 997 768 | 35.805 027 6 | 10.863 322 5 |
| | 49 | 2 111 932 187 | 35.818 989 4 | 10.866 146 4 |
| | 6 | 2 116 874 304 | 35.832 945 7 | 10.868 968 7 |
| | 1 | 2 121 824 125 | 35.846 896 6 | 10.871 789 7 |
| | | 2 126 781 650 | 35.860 842 1 | 10.874 609 |

| N. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
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| 165 | 63 69 | 2 131 746 903 | 35.874 782 2 | 10.877 427 1 |
| 165 | 89 44 | 2 136 719 872 | 35.888 716 9 | 10.880 243 6 |
| 166 | 15 21 | 2 141 700 569 | 35.902 646 1 | 10.883 058 7 |
| 166 | 41 00 | 2 146 689 000 | 35.916 569 9 | 10.885 872 3 |
| 166 | 66 81 | 2 151 685 171 | 35.930 488 4 | 10.888 684 5 |
| 166 | 92 64 | 2 156 689 088 | 35.944 401 5 | 10.891 495 2 |
| 167 | 18 49 | 2 161 700 757 | 35.958 309 2 | 10.894 304 4 |
| 167 | 44 36 | 2 166 720 184 | 35.972 211 5 | 10.897 112 3 |
| 167 | 70 25 | 2 171 747 375 | 35.986 108 4 | 10.899 918 6 |
| 167 | 96 16 | 2 176 782 336 | 36 | 10.902 723 5 |
| 168 | 22 09 | 2 181 825 073 | 36.013 886 2 | 10.905 526 9 |
| 168 | 48 04 | 2 186 875 592 | 36.027 767 1 | 10.908 329 |
| 168 | 74 01 | 2 191 933 899 | 36.041 642 6 | 10.911 129 6 |
| 169 | 00 00 | 2 197 000 000 | 36.055 512 8 | 10.913 928 7 |
| 169 | 26 01 | 2 202 073 901 | 36.069 377 6 | 10.916 726 5 |
| 169 | 52 04 | 2 207 155 608 | 36.083 237 1 | 10.919 522 8 |
| 169 | 78 09 | 2 212 245 127 | 36.097 091 3 | 10.922 317 7 |
| 170 | 04 16 | 2 217 342 464 | 36.110 940 2 | 10.925 111 1 |
| 170 | 30 25 | 2 222 447 625 | 36.124 783 7 | 10.927 903 1 |
| 170 | 56 36 | 2 227 560 616 | 36.138 622 | 10.930 693 7 |
| 170 | 82 49 | 2 232 681 443 | 36.152 455 | 10.933 482 9 |
| 171 | 08 64 | 2 237 810 112 | 36.166 282 6 | 10.936 270 6 |
| 171 | 34 81 | 2 242 946 629 | 36.180 105 | 10.939 056 9 |
| 171 | 61 00 | 2 248 091 000 | 36.193 922 1 | 10.941 841 8 |
| 171 | 87 21 | 2 253 243 231 | 36.207 734 | 10.944 625 3 |
| 172 | 13 44 | 2 258 403 328 | 36.221 540 6 | 10.947 507 4 |
| 172 | 39 69 | 2 263 571 297 | 36.235 341 9 | 10.950 388 |
| 172 | 65 96 | 2 268 747 144 | 36.249 137 9 | 10.952 967 3 |
| 172 | 92 25 | 2 273 930 875 | 36.262 928 7 | 10.955 745 1 |
| 173 | 18 56 | 2 279 122 496 | 36.276 714 3 | 10.958 521 5 |
| 173 | 44 89 | 2 284 322 013 | 36.290 494 6 | 10.961 296 5 |
| 173 | 71 24 | 2 289 529 432 | 36.304 269 7 | 10.964 070 1 |
| 173 | 97 61 | 2 294 744 759 | 36.318 039 6 | 10.966 842 3 |
| 174 | 24 00 | 2 299 968 000 | 36.331 804 2 | 10.969 613 1 |
| 174 | 50 41 | 2 305 199 161 | 36.345 563 7 | 10.972 382 5 |
| 174 | 76 84 | 2 310 438 248 | 36.359 317 9 | 10.975 150 5 |
| 175 | 03 29 | 2 315 685 267 | 36.373 067 | 10.977 917 1 |
| 175 | 29 76 | 2 320 940 224 | 36.386 810 8 | 10.980 682 3 |
| 175 | 56 25 | 2 326 203 125 | 36.400 549 4 | 10.983 446 2 |
| 175 | 82 76 | 2 331 473 976 | 36.414 282 9 | 10.986 208 6 |
| 176 | 09 29 | 2 336 752 783 | 36.428 011 2 | 10.988 969 6 |
| 176 | 35 84 | 2 342 039 552 | 36.441 734 3 | 10.991 729 3 |
| 176 | 62 41 | 2 347 334 289 | 36.455 452 3 | 10.994 487 6 |
| 176 | 89 00 | 2 352 637 000 | 36.469 165 | 10 |
| 177 | 15 61 | 2 357 947 691 | 36.482 872 7 | |
| 177 | 42 24 | 2 363 266 368 | 36.496 575 2 | |
| 177 | 68 89 | 2 368 593 037 | 36.510 272 5 | |
| 177 | 95 56 | 2 373 927 704 | 36.523 964 7 | |
| 178 | 22 25 | 2 379 270 375 | 36.537 651 8 | |
| 178 | 48 96 | 2 384 621 056 | 36.551 333 8 | |
| 178 | 75 69 | 2 389 979 753 | 36.565 010 6 | |
| 179 | 02 44 | 2 395 346 472 | 36.578 682 3 | |
| 179 | 29 21 | 2 400 721 219 | 36.592 348 9 | |
| 179 | 56 00 | 2 406 104 000 | 36.606 010 4 | |
| 179 | 82 81 | 2 411 494 821 | 36.619 666 8 | |
| 180 | 09 64 | 2 416 893 688 | 36.633 318 1 | |

1
11.
11.0.
11.027 7.
11.030 2

| NUMBER. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|---------|------------|---------------|--------------|--------------|
| 1343 | 1 80 36 49 | 2 422 300 607 | 36.646 964 4 | 11.032 959 |
| 1344 | 1 80 63 36 | 2 427 715 584 | 36.660 605 6 | 11.035 696 7 |
| 1345 | 1 80 90 25 | 2 433 138 625 | 36.674 241 6 | 11.038 433 |
| 1346 | 1 81 17 16 | 2 438 569 736 | 36.687 872 6 | 11.041 168 |
| 1347 | 1 81 44 09 | 2 444 008 923 | 36.701 498 6 | 11.043 901 7 |
| 1348 | 1 81 71 04 | 2 449 456 192 | 36.715 119 5 | 11.046 633 9 |
| 1349 | 1 81 98 01 | 2 454 911 549 | 36.728 735 3 | 11.049 364 9 |
| 1350 | 1 82 25 00 | 2 460 375 000 | 36.742 346 1 | 11.052 094 5 |
| 1351 | 1 82 52 01 | 2 465 846 551 | 36.755 951 9 | 11.054 822 7 |
| 1352 | 1 82 79 04 | 2 471 326 208 | 36.769 552 6 | 11.057 549 7 |
| 1353 | 1 83 06 09 | 2 476 813 977 | 36.783 148 3 | 11.060 275 2 |
| 1354 | 1 83 33 16 | 2 482 309 864 | 36.796 739 | 11.062 999 4 |
| 1355 | 1 83 60 25 | 2 487 813 875 | 36.810 324 6 | 11.065 722 2 |
| 1356 | 1 83 87 36 | 2 493 326 016 | 36.823 905 3 | 11.068 443 7 |
| 1357 | 1 84 14 49 | 2 498 846 293 | 36.837 480 9 | 11.071 163 9 |
| 1358 | 1 84 41 64 | 2 504 374 712 | 36.851 051 5 | 11.073 882 8 |
| 1359 | 1 84 68 81 | 2 509 911 279 | 36.864 617 2 | 11.076 600 3 |
| 1360 | 1 84 96 00 | 2 515 456 000 | 36.878 177 8 | 11.079 316 5 |
| 1361 | 1 85 23 21 | 2 521 008 881 | 36.891 733 5 | 11.082 031 4 |
| 1362 | 1 85 50 44 | 2 526 569 928 | 36.905 284 2 | 11.084 744 9 |
| 1363 | 1 85 77 69 | 2 532 139 147 | 36.918 829 9 | 11.087 457 1 |
| 1364 | 1 86 04 96 | 2 537 716 544 | 36.932 370 6 | 11.090 167 9 |
| 1365 | 1 86 32 25 | 2 543 302 125 | 36.945 906 4 | 11.092 877 5 |
| 1366 | 1 86 59 56 | 2 548 895 896 | 36.959 437 2 | 11.095 585 7 |
| 1367 | 1 86 86 89 | 2 554 497 863 | 36.972 963 1 | 11.098 292 6 |
| 1368 | 1 87 14 24 | 2 560 108 032 | 36.986 484 | 11.100 998 2 |
| 1369 | 1 87 41 61 | 2 565 726 409 | 37 | 11.103 702 5 |
| 1370 | 1 87 69 00 | 2 571 353 000 | 37.013 511 | 11.106 405 4 |
| 1371 | 1 87 96 41 | 2 576 987 811 | 37.027 017 2 | 11.109 107 |
| 1372 | 1 88 23 84 | 2 582 630 848 | 37.040 518 4 | 11.111 807 3 |
| 1373 | 1 88 51 29 | 2 588 282 117 | 37.054 014 6 | 11.114 506 4 |
| 1374 | 1 88 78 76 | 2 593 941 624 | 37.067 506 | 11.117 204 1 |
| 1375 | 1 89 06 25 | 2 599 609 375 | 37.080 992 4 | 11.119 900 4 |
| 1376 | 1 89 33 76 | 2 605 285 376 | 37.094 474 | 11.122 595 5 |
| 1377 | 1 89 61 29 | 2 610 969 633 | 37.107 950 6 | 11.125 289 3 |
| 1378 | 1 89 88 84 | 2 616 662 152 | 37.121 422 4 | 11.127 981 7 |
| 1379 | 1 90 16 41 | 2 622 362 939 | 37.134 889 3 | 11.130 672 9 |
| 1380 | 1 90 44 00 | 2 628 072 000 | 37.148 351 2 | 11.133 362 8 |
| 1381 | 1 90 71 61 | 2 633 789 341 | 37.161 808 4 | 11.136 051 4 |
| 1382 | 1 90 99 24 | 2 639 514 968 | 37.175 260 6 | 11.138 738 6 |
| 1383 | 1 91 26 89 | 2 645 248 887 | 37.188 707 9 | 11.141 424 6 |
| 1384 | 1 91 54 56 | 2 650 991 104 | 37.202 150 5 | 11.144 109 3 |
| 1385 | 1 91 82 25 | 2 656 741 625 | 37.215 588 1 | 11.146 792 6 |
| 1386 | 1 92 09 96 | 2 662 500 456 | 37.229 020 9 | 11.149 474 7 |
| 1387 | 1 92 37 69 | 2 668 267 603 | 37.242 448 9 | 11.152 155 5 |
| 1388 | 1 92 65 44 | 2 674 043 072 | 37.255 872 | 11.154 835 |
| 1389 | 1 92 93 21 | 2 679 826 869 | 37.269 290 3 | 11.157 513 3 |
| 1390 | 1 93 21 00 | 2 685 619 000 | 37.282 703 7 | 11.160 190 3 |
| 1391 | 1 93 48 81 | 2 691 419 471 | 37.296 112 4 | 11.162 865 9 |
| 1392 | 1 93 76 64 | 2 697 228 288 | 37.309 516 2 | 11.165 540 3 |
| 1393 | 1 94 04 49 | 2 703 045 457 | 37.322 915 2 | 11.168 213 4 |
| 1394 | 1 94 32 36 | 2 708 870 984 | 37.336 309 4 | 11.170 885 2 |
| 1395 | 1 94 60 25 | 2 714 704 875 | 37.349 698 8 | 11.173 555 8 |
| 1396 | | 2 720 547 136 | 37.363 083 4 | 11.176 225 |
| 1397 | | 26 397 773 | 37.376 463 2 | 11.178 893 |
| 1398 | | 2 256 792 | 37.389 838 2 | 11.181 559 |

| NR. | SQUARE. | CUBE. | SQUARE ROOT. | CUBE ROOT. |
|-----|------------|---------------|--------------|--------------|
| 1 | 195 72 01 | 2 738 124 199 | 37.403 208 4 | 11.184 225 2 |
| 2 | 196 00 00 | 2 744 000 000 | 37.416 573 8 | 11.186 889 4 |
| 3 | 196 28 01 | 2 749 884 201 | 37.429 934 5 | 11.189 552 3 |
| 4 | 196 56 04 | 2 755 776 808 | 37.443 290 4 | 11.192 213 9 |
| 5 | 196 84 09 | 2 761 677 827 | 37.456 641 6 | 11.194 874 3 |
| 6 | 197 12 16 | 2 767 587 264 | 37.469 988 | 11.197 533 4 |
| 7 | 197 40 25 | 2 773 505 125 | 37.483 329 6 | 11.200 191 3 |
| 8 | 197 68 36 | 2 779 431 416 | 37.496 666 5 | 11.202 847 9 |
| 9 | 197 96 49 | 2 785 366 143 | 37.509 998 7 | 11.205 503 2 |
| 10 | 198 24 64 | 2 791 309 312 | 37.523 326 1 | 11.208 157 3 |
| 11 | 198 52 81 | 2 797 260 929 | 37.536 648 7 | 11.210 810 1 |
| 12 | 198 81 00 | 2 803 221 000 | 37.549 966 7 | 11.213 461 7 |
| 13 | 199 09 21 | 2 809 189 531 | 37.563 279 9 | 11.216 112 |
| 14 | 199 37 44 | 2 815 166 528 | 37.576 583 5 | 11.218 761 1 |
| 15 | 199 65 69 | 2 821 151 997 | 37.589 892 2 | 11.221 408 9 |
| 16 | 199 93 95 | 2 827 145 944 | 37.603 191 3 | 11.224 005 4 |
| 17 | 200 22 25 | 2 833 148 375 | 37.616 485 7 | 11.226 700 7 |
| 18 | 200 50 56 | 2 839 159 296 | 37.629 775 4 | 11.229 344 8 |
| 19 | 200 78 89 | 2 845 178 713 | 37.643 060 4 | 11.231 987 6 |
| 20 | 201 07 24 | 2 851 206 632 | 37.656 340 7 | 11.234 629 2 |
| 21 | 201 35 61 | 2 857 243 059 | 37.669 616 4 | 11.237 269 6 |
| 22 | 201 64 00 | 2 863 288 000 | 37.682 887 4 | 11.239 908 7 |
| 23 | 201 92 41 | 2 869 341 461 | 37.696 153 6 | 11.242 546 5 |
| 24 | 202 20 84 | 2 875 403 448 | 37.709 415 3 | 11.245 183 1 |
| 25 | 202 49 29 | 2 881 473 967 | 37.722 672 2 | 11.247 818 5 |
| 26 | 202 77 76 | 2 887 553 024 | 37.735 924 5 | 11.250 452 7 |
| 27 | 203 06 25 | 2 893 640 625 | 37.749 172 2 | 11.253 085 6 |
| 28 | 203 34 76 | 2 899 736 776 | 37.762 415 2 | 11.255 717 3 |
| 29 | 203 63 29 | 2 905 841 483 | 37.775 653 5 | 11.258 347 8 |
| 30 | 203 91 84 | 2 911 954 752 | 37.788 887 3 | 11.260 977 |
| 31 | 204 20 41 | 2 918 076 589 | 37.802 116 3 | 11.263 605 |
| 32 | 204 49 00 | 2 924 207 000 | 37.815 340 8 | 11.266 231 8 |
| 33 | 204 77 61 | 2 930 345 991 | 37.828 560 6 | 11.268 857 3 |
| 34 | 205 06 24 | 2 936 493 568 | 37.841 775 9 | 11.271 481 6 |
| 35 | 205 34 89 | 2 942 649 737 | 37.854 986 4 | 11.274 104 7 |
| 36 | 205 63 56 | 2 948 814 504 | 37.868 192 4 | 11.276 726 6 |
| 37 | 205 92 25 | 2 954 987 875 | 37.881 393 8 | 11.279 347 2 |
| 38 | 206 20 96 | 2 961 169 856 | 37.894 590 6 | 11.281 966 6 |
| 39 | 206 49 69 | 2 967 360 453 | 37.907 782 8 | 11.284 584 9 |
| 40 | 206 78 44 | 2 973 559 672 | 37.920 970 4 | 11.287 201 9 |
| 41 | 207 07 21 | 2 979 767 519 | 37.934 153 5 | 11.289 817 7 |
| 42 | 207 36 00 | 2 985 984 000 | 37.947 331 9 | 11.292 432 3 |
| 43 | 207 64 81 | 2 992 209 121 | 37.960 505 8 | 11.295 045 7 |
| 44 | 207 93 64 | 2 998 442 888 | 37.973 675 1 | 11.297 657 9 |
| 45 | 208 22 49 | 3 004 685 307 | 37.986 839 8 | 11.300 269 9 |
| 46 | 208 51 36 | 3 010 936 384 | 38 | |
| 47 | 208 80 25 | 3 017 196 125 | 38.013 155 6 | |
| 48 | 209 09 16 | 3 023 464 536 | 38.026 306 7 | |
| 49 | 209 38 09 | 3 029 741 623 | 38.039 453 2 | |
| 50 | 209 67 04 | 3 036 027 392 | 38.052 595 2 | |
| 51 | 209 96 01 | 3 042 321 849 | 38.065 732 6 | |
| 52 | 2 10 25 00 | 3 048 625 000 | 38.078 865 5 | |
| 53 | 2 10 54 01 | 3 054 936 851 | 38.091 993 9 | |
| 54 | 2 10 83 04 | 3 061 257 408 | 38.105 117 8 | |
| 55 | 2 11 12 09 | 3 067 586 677 | 38.118 237 1 | |
| 56 | 2 11 41 16 | 3 073 924 664 | 38.131 351 9 | |

EXAMPLE.—What is square root of 53.75, and cube root of 843.75?

$$\begin{array}{r} \sqrt{54} = 7.3484 \\ \sqrt{53} = 7.2801 \\ \quad .0683 \\ \quad .75 \\ \quad .051225 \\ \sqrt{53} = 7.2801 \\ \sqrt{53.75} = 7.331325 \end{array}$$

$$\begin{array}{r} \sqrt[3]{844} = 9.4503 \\ \sqrt[3]{843} = 9.4466 \\ \quad .0037 \\ \quad .75 \\ \quad .002775 \\ \sqrt[3]{843} = 9.4466 \\ \sqrt[3]{843.75} = 9.449375 \end{array}$$

When the Square or Cube Root is required for Numbers not exceeding Roots given in Table.

Numbers in table are squares and cubes of roots.

RULE.—Find, by table, in column of numbers that number representing figures of integer and decimals for which root is required, and point it off decimally by places of 2 or 3 figures as square or cube root is required; and opposite to it, in column of roots, take root and point off 1 or 2 additional places of decimals to those in root, as square or cube root is required, and result is root required.

EXAMPLE 1.—What are square roots of .15, 1.50, and 15.00?

In table, 15 has for its root 3.87298; hence .387298 = square root for .15.
150 has for its root 12.24745; hence 1.224745 = square root for 1.50.
1500 has for its root 38.7298; hence 3.87298 = square root for 15.

2.—What are cube roots of .15, 1.50, and 15.00?

Add a cipher to each, to give the numbers three places of figures, as .150, 1.500, and 15.000.

In table 150 has for its root 5.3133; hence .53133 = cube root of .15.

1500 has for its root 11.447; hence 1.1447 = cube root of 1.50.

15 has for its root 2.4662; and 15.000, by addition of 3 places of figures, has 24.662; hence 2.4662 = cube root of 15.00.

To Ascertain Square or Cube Roots of Decimals alone.

RULE.—Point off number from decimal point into periods of two or three figures each, as square or cube root is required. Ascertain from table or by calculation root of number corresponding to decimal given, the same being read off by removing the decimal point one place to left for every period of 2 figures if square root is required, and one place for every period of 3 figures if cube root is required.

EXAMPLE.—What are square and cube roots of .810, .081, and .0081?

$$.810, \text{ when pointed off} = .81, \text{ and } \sqrt{.81} = .9.$$

$$.081, \text{ " " " " } = .081, \text{ " } \sqrt{.081} = .2846.$$

$$.0081, \text{ " " " " } = .0081, \text{ " } \sqrt{.0081} = .09.$$

$$.810, \text{ when pointed off} = .810, \text{ and } \sqrt[3]{.810} = 93217.$$

$$.081, \text{ " " " " } = .081, \text{ " } \sqrt[3]{.081} = .43267.$$

$$.0081, \text{ " " " " } = .0081, \text{ " } \sqrt[3]{.0081} = .20083.$$

To Compute 4th Root of a Number.

RULE.—Take square root of its square root.

EXAMPLE.—What is the 4th of 1600?

$$\sqrt{1600} = 40, \text{ and } \sqrt{40} = 6.3245553.$$

To Compute 6th Root of a Number.

Take cube root of its square root.

—What is the 6th of 441?

$$\sqrt{441} = 21, \text{ and } \sqrt[3]{21} = 2.7589243.$$

4th and 5th Powers of Numbers.

From 1 to 150.

| Number. | 4th Power. | 5th Power. | Number. | 4th Power. | 5th Power. |
|---------|------------|-------------|---------|-------------|----------------|
| 1 | 1 | 1 | 64 | 16 777 216 | 1 073 741 824 |
| 2 | 16 | 32 | 65 | 17 850 625 | 1 160 290 625 |
| 3 | 81 | 243 | 66 | 18 974 736 | 1 252 332 576 |
| 4 | 256 | 1 024 | 67 | 20 151 121 | 1 350 125 107 |
| 5 | 625 | 3 125 | 68 | 21 381 376 | 1 453 933 568 |
| 6 | 1 296 | 7 776 | 69 | 22 667 121 | 1 564 031 349 |
| 7 | 2 401 | 16 807 | 70 | 24 010 000 | 1 680 700 000 |
| 8 | 4 096 | 32 768 | 71 | 25 411 681 | 1 804 229 351 |
| 9 | 6 561 | 59 049 | 72 | 26 873 856 | 1 934 917 632 |
| 10 | 10 000 | 100 000 | 73 | 28 398 241 | 2 073 071 593 |
| 11 | 14 641 | 161 051 | 74 | 29 986 576 | 2 219 006 624 |
| 12 | 20 736 | 248 832 | 75 | 31 640 625 | 2 373 046 875 |
| 13 | 28 561 | 371 293 | 76 | 33 362 176 | 2 535 525 376 |
| 14 | 38 416 | 537 824 | 77 | 35 153 041 | 2 706 784 157 |
| 15 | 50 625 | 759 375 | 78 | 37 015 056 | 2 887 174 368 |
| 16 | 65 536 | 1 048 576 | 79 | 38 950 081 | 3 077 056 399 |
| 17 | 83 521 | 1 419 857 | 80 | 40 960 000 | 3 276 800 000 |
| 18 | 104 976 | 1 889 568 | 81 | 43 046 721 | 3 486 784 401 |
| 19 | 130 321 | 2 476 099 | 82 | 45 212 176 | 3 707 398 432 |
| 20 | 160 000 | 3 200 000 | 83 | 47 458 321 | 3 939 040 643 |
| 21 | 194 481 | 4 084 101 | 84 | 49 787 136 | 4 182 119 424 |
| 22 | 234 256 | 5 153 632 | 85 | 52 200 625 | 4 437 053 125 |
| 23 | 279 841 | 6 436 343 | 86 | 54 708 016 | 4 704 270 176 |
| 24 | 331 776 | 7 962 624 | 87 | 57 289 761 | 4 984 209 207 |
| 25 | 390 625 | 9 765 625 | 88 | 59 949 536 | 5 277 319 168 |
| 26 | 456 976 | 11 881 376 | 89 | 62 742 241 | 5 584 059 449 |
| 27 | 531 441 | 14 348 907 | 90 | 65 610 000 | 5 904 900 000 |
| 28 | 614 656 | 17 210 368 | 91 | 68 574 961 | 6 240 321 451 |
| 29 | 707 281 | 20 511 149 | 92 | 71 639 296 | 6 590 815 232 |
| 30 | 810 000 | 24 300 000 | 93 | 74 805 201 | 6 956 883 693 |
| 31 | 923 521 | 28 629 151 | 94 | 78 074 896 | 7 339 040 224 |
| 32 | 1 048 576 | 33 554 432 | 95 | 81 450 625 | 7 737 809 375 |
| 33 | 1 185 921 | 39 135 393 | 96 | 84 934 656 | 8 153 726 976 |
| 34 | 1 336 336 | 45 435 424 | 97 | 88 529 281 | 8 587 340 257 |
| 35 | 1 500 625 | 52 521 875 | 98 | 92 236 816 | 9 039 207 968 |
| 36 | 1 679 616 | 60 466 176 | 99 | 96 059 601 | 9 509 900 499 |
| 37 | 1 874 161 | 69 343 957 | 100 | 100 000 000 | 10 000 000 000 |
| 38 | 2 085 136 | 79 235 168 | 101 | 104 060 401 | 10 510 100 501 |
| 39 | 2 313 441 | 90 224 199 | 102 | 108 243 216 | 11 040 808 032 |
| 40 | 2 560 000 | 102 400 000 | 103 | 112 550 881 | 11 592 740 743 |
| 41 | 2 825 761 | 115 856 201 | 104 | 116 985 856 | 12 166 529 024 |
| 42 | 3 111 696 | 130 691 232 | 105 | 121 550 625 | 12 762 815 625 |
| 43 | 3 418 801 | 147 008 443 | 106 | 126 247 696 | 13 382 255 776 |
| 44 | 3 748 096 | 164 916 224 | 107 | 131 079 601 | 14 025 517 307 |
| 45 | 4 100 625 | 184 528 125 | 108 | 136 048 896 | 14 693 280 768 |
| 46 | 4 477 576 | 205 962 976 | 109 | 141 158 161 | 15 386 239 549 |
| 47 | 4 879 681 | 229 345 007 | 110 | 146 410 000 | 16 105 100 000 |
| 48 | 5 308 416 | 254 803 968 | 111 | 151 807 041 | 16 850 581 441 |
| 49 | 5 764 801 | 282 475 249 | 112 | 157 351 936 | 17 622 441 456 |
| 50 | 6 250 000 | 312 500 000 | 113 | 163 047 361 | 18 422 441 456 |
| 51 | 6 765 201 | 345 025 251 | 114 | 168 896 016 | 19 250 441 456 |
| 52 | 7 311 616 | 380 204 032 | 115 | 174 900 625 | 20 106 441 456 |
| 53 | 7 890 481 | 418 195 493 | 116 | 181 063 936 | 21 000 441 456 |
| 54 | 8 503 056 | 459 165 024 | 117 | 187 388 721 | 21 922 441 456 |
| 55 | 9 150 625 | 503 284 375 | 118 | 193 877 776 | 22 874 441 456 |
| 56 | 9 834 496 | 550 731 776 | 119 | 200 533 921 | 23 856 441 456 |
| 57 | 10 556 001 | 601 692 057 | 120 | 207 360 000 | 24 868 441 456 |
| 58 | 11 316 496 | 656 356 768 | 121 | 214 358 881 | 25 900 441 456 |
| 59 | 12 117 361 | 714 924 299 | 122 | 221 533 456 | 26 952 441 456 |
| 60 | 12 960 000 | 777 600 000 | 123 | 228 886 641 | 28 024 441 456 |
| 61 | 13 845 841 | 844 596 301 | 124 | 236 421 376 | 29 116 441 456 |
| 62 | 14 776 336 | 916 132 832 | 125 | 244 140 625 | 30 228 441 456 |
| 63 | 15 752 961 | 992 436 543 | 126 | 252 047 376 | 31 360 441 456 |

To Subtract a Positive Index. Change its sign to negative, and then add as in addition. As $\bar{2} - 2 = \bar{2} + 2 = \bar{4}$.

To Multiply a Negative Index. Multiply the fractional parts by the ordinary rule, then multiply the negative index, which will give a negative product, and when an excess over 10 is to be carried, subtract the less index from the greater, and the remainder gives the positive or negative index, according as the positive or negative index is the greater. As $2 \times 5 = 10$, and 1 to be carried = 9.

ILLUSTRATION.—Multiply $\bar{2}.3681$ by 2, and $\bar{3}.7856$ by 6.

$$\begin{array}{r} \bar{2}.3681 \\ \times 2 \\ \hline 4.7362 \end{array} \qquad \begin{array}{r} \bar{3}.7856 \\ \times 6 \\ \hline 14.7136 \end{array}$$

Here $\bar{2} \times 2 = \bar{4}$, also $\bar{3} \times 6 = \bar{18}$, with a positive excess of 4 = $\bar{14}$.

To Divide a Negative Index. If index is divisible by divisor, without a remainder, put quotient with a negative sign. If negative exponent is not divisible by divisor, add such a negative number to it as will make it divisible, and prefix an equal positive integer to fractional part of logarithm; then divide increased negative exponent and the other part of logarithm separately by ordinary rules, and former quotient, taken negatively, will be index to fractional part of quotient. As $\bar{6} \div 3 = \bar{2}$. $10 \div 3$ requires 2 to be added or 2 to be subtracted, to make it divisible without a remainder, then $10 + 2 = 12$, $12 \div 3 = 4$, and 2 (the sum subtracted) $\div 3 = .66$, the quotient therefore is $\bar{4}.66$.

ILLUSTRATION I.—Divide $\bar{6}.324282$ by 3.

$$\bar{6}.324282 \div 3 = \bar{2}.108094.$$

2.—Divide $\bar{14}.326745$ by 9.

$$\bar{14}.326745 \div 9 = \bar{18} + 4.326745 \div 9 = \bar{2}.480749+.$$

Here 4 is added to $\bar{14}$, that the sum $\bar{18}$ may be divided by 9, and as 4 is added, 4 must be prefixed to the fractional part of the logarithm, and thus the value of the logarithm is unchanged, for there is added 4, and 4 = 0, or 4 is subtracted and 4 added.

To Ascertain Logarithm of a Number by Table.

When the Number is less than 101.

Look into first page of table, and opposite to number is its logarithm with its index prefixed.

ILLUSTRATION.—Opposite 7 is .845098, its logarithm; hence $70 = 1.845098$, $.7 = 1.845098$, and $.07 = 2.845098$.

When the Number is between 100 and 1000.

3 given number in left-hand column of table headed No., and un-
mn is decimal part of its logarithm, to which is to be prefixed a
an index, of 1 or 2, according as the number consists of 2 or 3

t is logarithm of 450, and what of .45?

$$\text{Log. } 450 = 2.653213, \text{ and of } .45 = 1.653213.$$

When the Number is between 1000 and 10000.

1000-hand figures of the number in the left-hand column
under the 4th figure at top of table is the four last
of logarithm, to which is to be prefixed the proper

n of 4505, and what of .04505?

$$553695, \text{ and of } .04505 = 2.653695.$$

When the Number consists of Five Figures.

RULE.—Find the logarithm of the number composed of the first four figures as preceding, then take the tabular difference from the right-hand column under D and multiply it by the fifth figure; reject the right-hand figure of the product and add the other figures, which are, and are termed, a proportional part to the logarithm found as above, observing that the right-hand figure of the proportional part is to be added to that of the logarithm, and the rest in order.

EXAMPLE.—Required logarithm of 83407?

NOTE.—When the number consists of less than 4 figures conceive a cipher annexed to make it four.

$$\begin{array}{r} \text{Log. of } 8340 \text{ (83407)} = 4.921\ 166 \\ \text{Tabular difference } 52, \text{ which } \times 7 \text{ (5th figure)} = 364 = \underline{\hspace{1cm} 364 \hspace{1cm}} \\ 4.921\ 202\ 4 \text{ logarithm.} \end{array}$$

The difference of the numbers is nearly proportionate to the difference of their logarithms.

Thus, difference between the numbers 8340 and 8341, the next in order, is 1, and is difference between their logarithms or tabular difference is 52.

The log. of this 1 in the 4th place is therefore 52. The correction then, for the 7th place, which is .7 of 1 in the 4th place, is ascertained by the proportion : 52 :: .7 : 36.4.

The correction is obtained by multiplying the tabular difference by 7, rejecting the right hand figure of the product, if the log. is to be confined to six decimal places.

When the Number consists of any Number over Four Figures.

RULE.—Proceed as for four figures for the first four, multiplying the tabular difference by the excess of figures over 4 and rejecting one right-hand figure of the product for a number of five figures, and two for one of six, and so on.

EXAMPLE 1.—Required logarithm of 834079?

$$\begin{array}{r} \text{Log. of } 8340 \text{ (834079)} = 5.921\ 166 \\ \text{Tabular difference } 52, \text{ which } \times 79 = \underline{\hspace{1cm} 4108 \hspace{1cm}} \\ 5.921\ 207\ 08 \text{ logarithm.} \end{array}$$

2.—Required logarithm of 8340794?

$$\begin{array}{r} \text{Log. of } 8340 \text{ (8340794)} = 6.921\ 166 \\ \text{b. diff. } 52, \text{ which } \times 794 \text{ (5th, 6th, and 7th figures)} = \underline{\hspace{1cm} 41\ 288 \hspace{1cm}} \\ 6.921\ 207\ 288 \text{ logarithm.} \end{array}$$

$$\begin{array}{r} \text{Or, Mantissa of } 8340 = 921\ 166 \\ \text{" " } 7 \text{ (5th figure)} \times 52 \text{ tab. diff.} = 364 \\ \text{" " } 9 \text{ (6th ")} \times 52 \text{ " " } = 468 \\ \text{" " } 4 \text{ (7th ")} \times 52 \text{ " " } = 208 \\ \text{Log. with index for 7 figures} \dots\dots\dots 6.921\ 207\ 288 \end{array}$$

To Ascertain Logarithm of a Mixed Number.

RULE.—Take out logarithm of the number as if it were an integer or whole number; to which prefix the *index* of the integral part of the number.

EXAMPLE.—What is logarithm of 834.0794?

Mantissa of log. of 834.0794 = 9212 073; hence log. of 834.0794 = 2.921 207 3.

To Ascertain Logarithm of a Decimal Fraction.

RULE.—Take logarithm from table as if the figures were all integers, and prefix *les* as by previous rules.

EXAMPLE.—Logarithm of .1234 = 1.091 305.

To Ascertain Logarithm of a Vulg

RULE.—Reduce the fraction to a decimal, and proceed as tract logarithm of denominator from that of numerator, a logarithm required.

EXAMPLE.—Logarithm of $\frac{3}{5}$?

$$\begin{array}{r} \frac{3}{5} = .6 \text{ Log. } .6875 = 1.273\ 001 \text{ logarithm} \\ \text{Or, Log. } 3 = .477\ 121 \\ \text{" } 5 = 1.204\ 12 \\ \hline 1.273\ 001 \text{ logarithm.} \end{array}$$

To Ascertain the Number Corresponding to a Given Logarithm.

When the given or exact Logarithm is in the Table.

OPERATION.—Opposite to first two figures of logarithm, neglecting the *index*, in column *o*, look for the remaining figures of the log. in that column or in any of the nine at the right thereof; the first three figures of the number will be found at the left in column under No., and the fourth at top directly over the log.

The number is to be made to correspond to *index* of logarithm, by pointing off decimals or prefixing ciphers.

ILLUSTRATION.—What is number corresponding to log. 3.963 977 ?

Opposite to 963 977, in page 329, is 920, and at top of column is 4; hence, number = 9204.

When the given or exact Logarithm is not in the Table.

OPERATION.—Take the number for the next less logarithm from table, which will give first four figures of required number.

To ascertain the other figures, subtract the logarithm in table from the given logarithm, add ciphers, and divide by the difference in column *D* opposite the logarithm. Annex quotient to the four figures already ascertained, and place decimal point.

ILLUSTRATION 1.—What is number corresponding to log. 5.921 207 ?

| | | |
|--------------------|--------------------|---------|
| Given log. = | 5.921 207 | |
| Next less in table | 5.921 166 | 834 0 |
| | D = 52) 4100 (78 + | 78 |
| | <u>364</u> | 834 078 |
| | 460 | |
| | <u>416</u> | |
| | 44 | |

Hence, number = 834 078.

2.—What is number corresponding to log. 3.922 853 ?

| | | |
|--------------------|--------------------|---------|
| Given log. = | 3.922 853 | |
| Next less in table | 3.922 829 | 837 2 |
| | D = 52) 2400 (46 + | 46 |
| | <u>208</u> | 837 246 |
| | 320 | |
| | <u>312</u> | |
| | 8 | |

Hence, number = 8372.46.

Multiplication.

RULE.—Add together the logarithms of the numbers and the sum will give the logarithm of the product.

EXAMPLE 1.—Multiply 345.7 by 2.581.

| | | |
|--------------|----------------------------|-------------------|
| Log. 345.7 = | 2.538 699 | |
| " 2.581 = | .411 788 | |
| | 2.950 487 log. of product. | Number = 892.251. |

Multiply .039 02, 59.71, and .003 147.

| | | |
|-----------|----------------------------|-----------------------|
| .039 02 = | 2.591 287 | |
| | = 1.776 047 | |
| 147 = | 3.497 897 | |
| | 3.865 231 log. of product. | Number = .007 332 15. |

Division.

From dividend subtract that of divisor, and remainder will be *divident*.

| | | |
|--|--|--------------------|
| | | Number = 7.091 85. |
|--|--|--------------------|

Rule of Three, or Proportion.

RULE.—Add together the logarithms of the second and third terms, from their sum subtract logarithm of the first, and the remainder will give logarithm of the fourth term.

Or, instead of subtracting logarithm of first term, add its *Arithmetical Complement*, and subtract 10 from its index.

EXAMPLE 1.—What is fourth proportional to 723.4, .025 19, and 3574?

| | | | |
|------------|---------|--------|---|
| As | 723.4 | log. = | 2.859 379 |
| Is to | .025 19 | " = | 2.401 228 |
| So is | 3574 | " = | 3.553 155 |
| | | | <hr/> |
| First term | | " | 1.954 383 |
| | | | 2.859 379 |
| | | | <hr/> |
| | | | 1.095 004 <i>log. of 4th term.</i> Number = .124 453. |

By Arithmetical Complement.

| | | | | |
|--------------------------|---------|--------|-----------------------|-----------------------------------|
| ILLUSTRATION. —As | 723.4 | log. = | 2.859 379, Ar. com. = | 7.140 621 |
| Is to | .025 19 | " = | | 2.401 228 |
| So is | 3574 | " = | | 3.553 155 |
| | | | | <hr/> |
| | | | | 1.095 004 <i>log. of 4th term</i> |

Number = .124 453.

2.—If an engine of 67 HP can raise 57 600 cube feet of water in a given time, what HP is required to raise 8 575 000 cube feet in like time?

| | | |
|------|-------------|--|
| Log. | 8 575 000 = | 6.933 234 |
| " | 67 = | 1.826 075 |
| | | <hr/> |
| | | 8.759 309 |
| " | 57 600 = | 4.760 422 |
| | | <hr/> |
| | | 3.998 877 <i>log. of 4th term.</i> Number = 9974.4 HP. |

3.—If 14 men in 47 days excavate 5631 cube yards, what time will it require to excavate 47 280 at same rate of excavation?

394.626 days.

Involution.

RULE.—Multiply logarithm of given number by exponent of the power to which it is to be raised, and the product will give the logarithm of the required power.

EXAMPLE.—What is cube of 30.71?

| | | |
|------|---------|--|
| Log. | 30.71 = | 1.487 28 |
| | | <hr/> |
| | | 4.461 84 <i>log. of power.</i> Number = 28 962.73. |

Evolution.

RULE.—Divide logarithm of given number by exponent of the root which is to be extracted, and quotient will give logarithm of required root.

EXAMPLE 1.—What is cube root of 1234?

| | | |
|---------------|--------------------------------|---------------------|
| Log. | 1234 = | 3.091 315 |
| Divide by 3 = | 1.030 438 <i>log. of root.</i> | Number = 10.726 01. |

2.—What is 4th root of .007 654?

| | | |
|---|--------------------------------|-------------------|
| Log. | .007 654 = | 3.883 888 |
| Divide by 4 (here $\frac{3}{4} + 1 + 1$) = | 1.470 972 <i>log. of root.</i> | Number = .295 78. |

To Ascertain Reciprocal of a Number.

RULE.—Subtract decimal of logarithm of the number from .000 000; add 1 to its index of logarithm and change its sign. The result is logarithm of the reciprocal.

EXAMPLE.—Required reciprocal of 230?

| | | |
|------|-------|---|
| Log. | 230 = | 2.361 728 |
| | | <hr/> |
| | | 3.638 272 = <i>log. of .004 348 reciprocal.</i> |

Simple Interest.

RULE.—Add together logarithm of principal, rate per cent., and time in years, from the sum subtract 2, and the remainder will give logarithm of the interest.

EXAMPLE.—What is interest on \$500, @ 6 per cent., for 3 years?

$$\begin{array}{r} \text{Log. } 500 = 2.69897 \\ 6 = .778151 \\ 3 = .477121 \\ \hline 3.954242 \\ 2 \end{array}$$

1.954242 *log. of interest.* Number = 90 dollars.

Compound Interest.

RULE.—Compute amount of \$1 or £1, etc., at the given rate of interest for one year for the first term, which is termed the *ratio*.

Multiply logarithm of *ratio* by the time, add to product logarithm of the principal, and sum is logarithm of the amount.

Logarithms of Ratios at given Rates Per Cent.

| Rate. | Log. of Ratio. | Rate. | Log. of Ratio. | Rate. | Log. of Ratio. | Rate. | Log. of Ratio. |
|-------|----------------|-------|----------------|-------|----------------|-------|----------------|
| 1 | .0043214 | 3.25 | .0138901 | 5.5 | .0232525 | 7.75 | .0324733 |
| 1.25 | .005395 | 3.5 | .0149403 | 5.75 | .0242804 | 8 | .0334238 |
| 1.5 | .006466 | 3.75 | .0159881 | 6 | .0253059 | 8.25 | .0344279 |
| 1.75 | .0075344 | 4 | .0170333 | 6.25 | .0263289 | 8.5 | .0354297 |
| 2 | .0086002 | 4.25 | .0180761 | 6.5 | .0273496 | 8.75 | .0364293 |
| 2.25 | .0096633 | 4.5 | .0191163 | 6.75 | .0283763 | 9 | .0374265 |
| 2.5 | .0107239 | 4.75 | .020154 | 7 | .0293838 | 9.25 | .0384214 |
| 2.75 | .0117818 | 5 | .0211893 | 7.25 | .0303973 | 9.5 | .0394141 |
| 3 | .0128372 | 5.25 | .0222221 | 7.5 | .0314085 | 9.75 | .0404045 |

EXAMPLE.—What will \$364, at 6 per cent. per annum, compounded yearly, amount to in 23 years?

$$\begin{array}{r} \text{Log. of ratio from above table } .0253059 \\ 23 \end{array}$$

$$\begin{array}{r} .5820357 \\ 2.561101 \end{array}$$

" " 364

3.1431367 *log. of amount.* Number = 1390.39 doll

Miscellaneous Illustrations.

z. What is area and circumference of a circle of 21.72 feet in diameter?

$$\begin{array}{r} \text{Log. of } 21.72 \quad 1.336860 \\ 2 \end{array}$$

$$\text{Log. of } 21.72^2 = 2.673720$$

$$\text{" " } .7854 = 1.895091$$

$$\text{" " } 2.568811 = 370.54 \text{ feet area.}$$

$$\text{Log. of } 21.72 = 1.33686$$

$$\text{" " } 3.1416 = .49715$$

$$\text{" " } 1.83401 = 68.236 \text{ feet circum.}$$

gale are 564, 373, and 747 feet; what is its area?

$$\begin{array}{r} 564 + 373 + 747 \\ 2 \end{array} = 2.925312$$

$$a = 842 - 564 = 2.444045$$

$$b = 842 - 373 = 2.671173$$

$$c = 842 - 747 = 1.977724$$

$$2)10.018254$$

$$= \text{Number of } 5.009127 = 102120.4 \text{ feet.}$$

6,

$$B = 3.6 \times .90309 = 3.251124. \text{ Number} = 1782.89.$$

Logarithms of Numbers.

From 1 to 10 000.

| Logarithm. | No. | Logarithm. | No. | Logarithm. | No. | Logarithm. |
|------------|-----|------------|-----|------------|-----|------------|
| .0 | 26 | 1.414 973 | 51 | 1.707 57 | 76 | 1.880 814 |
| .301 03 | 27 | 1.431 364 | 52 | 1.716 003 | 77 | 1.886 491 |
| .477 121 | 28 | 1.447 158 | 53 | 1.724 276 | 78 | 1.892 095 |
| .602 06 | 29 | 1.462 398 | 54 | 1.732 394 | 79 | 1.897 627 |
| .698 97 | 30 | 1.477 121 | 55 | 1.740 363 | 80 | 1.903 09 |
| .778 151 | 31 | 1.491 362 | 56 | 1.748 188 | 81 | 1.908 485 |
| .845 098 | 32 | 1.505 15 | 57 | 1.755 875 | 82 | 1.913 814 |
| .903 09 | 33 | 1.518 514 | 58 | 1.763 428 | 83 | 1.919 078 |
| .954 243 | 34 | 1.531 479 | 59 | 1.770 852 | 84 | 1.924 279 |
| 1 | 35 | 1.544 068 | 60 | 1.778 151 | 85 | 1.929 419 |
| 1.041 393 | 36 | 1.556 303 | 61 | 1.785 33 | 86 | 1.934 498 |
| 1.079 181 | 37 | 1.568 202 | 62 | 1.792 392 | 87 | 1.939 519 |
| 1.113 943 | 38 | 1.579 784 | 63 | 1.799 341 | 88 | 1.944 483 |
| 1.146 128 | 39 | 1.591 065 | 64 | 1.806 18 | 89 | 1.949 39 |
| 1.176 091 | 40 | 1.602 06 | 65 | 1.812 913 | 90 | 1.954 243 |
| 1.204 12 | 41 | 1.612 784 | 66 | 1.819 544 | 91 | 1.959 041 |
| 1.230 449 | 42 | 1.623 249 | 67 | 1.826 075 | 92 | 1.963 788 |
| 1.255 273 | 43 | 1.633 468 | 68 | 1.832 509 | 93 | 1.968 483 |
| 1.278 754 | 44 | 1.643 453 | 69 | 1.838 849 | 94 | 1.973 128 |
| 1.301 03 | 45 | 1.653 213 | 70 | 1.845 098 | 95 | 1.977 724 |
| 1.322 219 | 46 | 1.662 758 | 71 | 1.851 258 | 96 | 1.982 271 |
| 1.342 423 | 47 | 1.672 098 | 72 | 1.857 332 | 97 | 1.986 772 |
| 1.361 728 | 48 | 1.681 241 | 73 | 1.863 323 | 98 | 1.991 226 |
| 1.380 211 | 49 | 1.690 196 | 74 | 1.869 232 | 99 | 1.995 635 |
| 1.397 94 | 50 | 1.698 97 | 75 | 1.875 061 | 100 | 2 |

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|----------|------|------|------|------|------|------|------|------|------|-----|
| 00- 0000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 | 432 |
| 00- 4321 | 4751 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 | 428 |
| 00- 86 | 9026 | 9451 | 9876 | — | — | — | — | — | — | 425 |
| 01- — | — | — | — | 03 | 0724 | 1147 | 157 | 1993 | 2415 | |
| 01- 2837 | 3259 | 368 | 41 | 4521 | 494 | 536 | 5779 | 6197 | 6616 | |
| 01- 7033 | 7451 | 7868 | 8284 | 87 | 9116 | 9532 | 9947 | — | — | |
| 02- — | — | — | — | — | — | — | — | 0361 | 0771 | |
| 02- 1189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 | |
| 02- 5306 | 5715 | 6125 | 6533 | 6942 | 735 | 7757 | 8164 | 8571 | 8978 | |
| 02- 9384 | 9789 | — | — | — | — | — | — | — | — | |
| 03- — | — | 0195 | 06 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 | |
| 03- 3424 | 3826 | 4227 | 4628 | 5029 | 543 | 583 | 623 | 6629 | 7028 | |
| 03- 7426 | 7825 | 8223 | 862 | 9017 | 9414 | 9811 | — | — | — | |
| 04- — | — | — | — | — | — | — | 0207 | — | — | |
| 04- 1393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | — | — | — | |
| 04- 5323 | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | — | — | — | |
| 04- 9218 | 9606 | 9993 | — | — | — | — | — | — | — | |
| 05- — | — | — | 038 | 0766 | 1153 | 1538 | — | — | — | |
| 05- 3078 | 3463 | 3846 | 423 | 4613 | 4996 | 5378 | — | — | — | |
| 05- 6905 | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | — | — | — | |
| 06- — | — | — | — | — | — | — | — | — | — | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | | | | |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|----------|------|------|------|------|------|------|------|------|------|
| 115 | 06- 0698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 |
| 116 | 06- 4458 | 4832 | 5206 | 558 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 |
| 117 | 06- 8186 | 8557 | 8928 | 9298 | 9668 | — | — | — | — | — |
| 117 | 07- — | — | — | — | — | 0038 | 0407 | 0776 | 1145 | 1514 |
| 118 | 07- 1882 | 225 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 |
| 119 | 07- 5547 | 5912 | 6276 | 664 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 |
| 120 | 07- 9181 | 9543 | 9904 | — | — | — | — | — | — | — |
| 120 | 08- — | — | — | 0266 | 0626 | 0987 | 1347 | 1707 | 2067 | 2426 |
| 121 | 08- 2785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 |
| 122 | 08- 636 | 6716 | 7071 | 7426 | 7781 | 8136 | 849 | 8845 | 9198 | 9552 |
| 123 | 08- 9905 | — | — | — | — | — | — | — | — | — |
| 123 | 09- — | 0258 | 0611 | 0963 | 1315 | 1667 | 2018 | 237 | 2721 | 3071 |
| 124 | 09- 3422 | 3772 | 4122 | 4471 | 482 | 5169 | 5518 | 5866 | 6215 | 6562 |
| 125 | 09- 691 | 7257 | 7604 | 7951 | 8298 | 8644 | 899 | 9335 | 9681 | — |
| 125 | 10- — | — | — | — | — | — | — | — | — | 0026 |
| 126 | 10- 0371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 |
| 127 | 10- 3804 | 4146 | 4487 | 4828 | 5169 | 551 | 5851 | 6191 | 6531 | 6871 |
| 128 | 10- 721 | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | — |
| 128 | 11- — | — | — | — | — | — | — | — | — | 0253 |
| 129 | 11- 059 | 0926 | 1263 | 1599 | 1934 | 227 | 2605 | 294 | 3275 | 3609 |
| 130 | 11- 3943 | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 694 |
| 131 | 11- 7271 | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | — |
| 131 | 12- — | — | — | — | — | — | — | — | — | 0245 |
| 132 | 12- 0574 | 0903 | 1231 | 156 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 |
| 133 | 12- 3852 | 4178 | 4504 | 483 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 |
| 134 | 12- 7105 | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 969 | — |
| 134 | 13- — | — | — | — | — | — | — | — | — | 0012 |
| 135 | 13- 0334 | 0655 | 0977 | 1298 | 1619 | 1939 | 226 | 258 | 29 | 3219 |
| 136 | 13- 3539 | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 |
| 137 | 13- 6721 | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 |
| 138 | 13- 9879 | — | — | — | — | — | — | — | — | — |
| 138 | 14- — | 0194 | 0508 | 0822 | 1136 | 145 | 1763 | 2076 | 2389 | 2702 |
| 139 | 14- 3015 | 3327 | 3639 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 |
| 140 | 14- 6128 | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 |
| 141 | 14- 9219 | 9527 | 9835 | — | — | — | — | — | — | — |
| 141 | 15- — | — | — | 0142 | 0449 | 0756 | 1063 | 137 | 1676 | 1982 |
| 142 | 15- 2288 | 2594 | 29 | 3205 | 351 | 3815 | 412 | 4424 | 4728 | 5032 |
| 143 | 15- 5336 | 564 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 |
| 144 | 15- 8362 | 8664 | 8965 | 9266 | 9567 | 9868 | — | — | — | — |
| 144 | 16- — | — | — | — | — | — | 0168 | 0469 | 0769 | 1068 |
| 145 | 16- 1368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 346 | 3758 | 4055 |
| 146 | 16- 4353 | 465 | 4947 | 5244 | 5541 | 5838 | 6134 | 643 | 6726 | 7022 |
| 147 | 16- 7317 | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 938 | 9674 | 9968 |
| 148 | 17- 0262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 |
| 149 | 17- 3186 | 3478 | 3769 | 406 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 |
| 150 | 17- 6091 | 6381 | 667 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 |
| 151 | 17- 8977 | 9264 | 9552 | 9839 | — | — | — | — | — | — |
| 151 | 18- — | — | — | — | 0126 | 0413 | 0699 | 0986 | 1272 | 1558 |
| 152 | 18- 1844 | 2129 | 2415 | 27 | 2985 | 327 | 3555 | 3839 | 4123 | 4407 |
| 153 | 18- 4691 | 4975 | 5259 | 5542 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 |
| 154 | 18- 7521 | 7803 | 8084 | 8366 | 8647 | 8928 | 9209 | 949 | 9771 | — |
| 154 | 19- — | — | — | — | — | — | — | — | — | 0051 |
| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|----|------|------|------|------|------|------|------|------|------|------|-----|
| 19 | 0332 | 0612 | 0892 | 1171 | 1451 | 173 | 201 | 2289 | 2567 | 2846 | 279 |
| 19 | 3125 | 3403 | 3681 | 3959 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 | 278 |
| 19 | 59 | 6176 | 6453 | 6729 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 | 276 |
| 19 | 8657 | 8932 | 9206 | 9481 | 9755 | — | — | — | — | — | 275 |
| 20 | — | — | — | — | — | 0029 | 0303 | 0577 | 085 | 1124 | 274 |
| 20 | 1397 | 167 | 1943 | 2216 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 | 272 |
| 20 | 412 | 4391 | 4663 | 4934 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 | 271 |
| 20 | 6826 | 7096 | 7365 | 7634 | 7904 | 8173 | 8441 | 871 | 8979 | 9247 | 269 |
| 20 | 9515 | 9783 | — | — | — | — | — | — | — | — | 268 |
| 21 | — | — | 0051 | 0319 | 0586 | 0853 | 1121 | 1388 | 1654 | 1921 | 267 |
| 21 | 2188 | 2454 | 272 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 | 266 |
| 21 | 4844 | 5109 | 5373 | 5638 | 5902 | 6166 | 643 | 6694 | 6957 | 7221 | 264 |
| 21 | 7484 | 7747 | 801 | 8273 | 8536 | 8798 | 906 | 9323 | 9585 | 9846 | 262 |
| 22 | 0108 | 037 | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 | 261 |
| 22 | 2716 | 2976 | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 | 259 |
| 22 | 5309 | 5568 | 5826 | 6084 | 6342 | 66 | 6858 | 7115 | 7372 | 763 | 258 |
| 22 | 7887 | 8144 | 84 | 8657 | 8913 | 917 | 9426 | 9682 | 9938 | — | 257 |
| 23 | — | — | — | — | — | — | — | — | — | 0193 | 256 |
| 23 | 0449 | 0704 | 096 | 1215 | 147 | 1724 | 1979 | 2234 | 2488 | 2742 | 255 |
| 23 | 2990 | 325 | 3504 | 3757 | 4011 | 4264 | 4517 | 477 | 5023 | 5276 | 253 |
| 23 | 5528 | 5781 | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 | 252 |
| 23 | 8046 | 8297 | 8548 | 8799 | 9049 | 9299 | 955 | 98 | — | — | 251 |
| 24 | — | — | — | — | — | — | — | — | 005 | 03 | 250 |
| 24 | 0549 | 0799 | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 279 | 249 |
| 24 | 3038 | 3286 | 3534 | 3782 | 403 | 4277 | 4525 | 4772 | 5019 | 5266 | 248 |
| 24 | 5513 | 5759 | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 | 246 |
| 24 | 7973 | 8219 | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | — | 245 |
| 25 | — | — | — | — | — | — | — | — | — | 0176 | 245 |
| 25 | 042 | 0664 | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 261 | 243 |
| 25 | 2853 | 3096 | 3338 | 358 | 3822 | 4064 | 4306 | 4548 | 479 | 5031 | 242 |
| 25 | 5273 | 5514 | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 | 241 |
| 25 | 7679 | 7918 | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 | 240 |
| 26 | 0071 | 031 | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 | 239 |
| 26 | 2451 | 2688 | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 | 238 |
| 26 | 4818 | 5054 | 529 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 | 237 |
| 26 | 7172 | 7406 | 7641 | 7875 | 811 | 8344 | 8578 | 8812 | 9046 | 9279 | 236 |
| 26 | 9513 | 9746 | 998 | — | — | — | — | — | — | — | 235 |
| 27 | — | — | — | 0213 | 0446 | 0679 | 0912 | 1144 | 1377 | 1609 | 234 |
| 27 | 1842 | 2074 | 2306 | 2538 | 277 | 3001 | 3233 | 3464 | 3695 | 3926 | 233 |
| 27 | 4158 | 4389 | 462 | 485 | 5081 | 5311 | 5542 | 5772 | 6003 | 6233 | 232 |
| 27 | 6462 | 6692 | 6921 | 7151 | 738 | 7609 | 7838 | 8067 | 8296 | 8525 | 231 |
| 27 | 8754 | 8982 | 9211 | 9439 | 9667 | 9895 | — | — | — | — | 230 |
| 28 | — | — | — | — | — | — | 0123 | 0351 | 0579 | 0806 | 229 |
| 28 | 1033 | 1261 | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 | 228 |
| 28 | 3301 | 3527 | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 | 227 |
| 28 | 5557 | 5782 | 6007 | 6232 | 6456 | 6681 | 6905 | 713 | 7358 | 7582 | 226 |
| 28 | 7802 | 8026 | 8249 | 8473 | 8696 | 892 | 9143 | 9366 | 9589 | 9812 | 225 |
| 29 | 0035 | 0257 | 048 | 0702 | 0925 | 1147 | 1369 | 1591 | 1812 | 2034 | 224 |
| 29 | 2256 | 2478 | 2699 | 292 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 | 223 |
| 29 | 4466 | 4687 | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6227 | 6447 | 222 |
| 29 | 6665 | 6884 | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8417 | 8636 | 221 |
| 29 | 8853 | 9071 | 9289 | 9507 | 9725 | 9943 | — | — | — | — | 220 |
| 30 | — | — | — | — | — | — | 0161 | 0378 | 0595 | 0812 | 219 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | D |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |
|-----|-----|------|------|------|------|------|------|------|------|------|------|----|
| 200 | 30- | 103 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 298 | 21 |
| 201 | 30- | 3196 | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 21 |
| 202 | 30- | 5351 | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 21 |
| 203 | 30- | 7496 | 771 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 21 |
| 204 | 30- | 963 | 9843 | — | — | — | — | — | — | — | — | 21 |
| 204 | 31- | — | — | 0056 | 0268 | 0481 | 0693 | 0906 | 1118 | 133 | 1542 | 21 |
| 205 | 31- | 1754 | 1966 | 2177 | 2389 | 26 | 2812 | 3023 | 3234 | 3445 | 3656 | 21 |
| 206 | 31- | 3867 | 4078 | 4289 | 4499 | 471 | 492 | 513 | 534 | 5551 | 576 | 21 |
| 207 | 31- | 597 | 618 | 639 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 20 |
| 208 | 31- | 8063 | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 973 | 9938 | 20 |
| 209 | 32- | 0146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 20 |
| 210 | 32- | 2219 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 20 |
| 211 | 32- | 4282 | 4488 | 4694 | 4899 | 5105 | 531 | 5516 | 5721 | 5926 | 6131 | 20 |
| 212 | 32- | 6336 | 6541 | 6745 | 695 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 20 |
| 213 | 32- | 838 | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | — | — | 20 |
| 213 | 33- | — | — | — | — | — | — | — | — | 0008 | 0211 | 20 |
| 214 | 33- | 0414 | 0617 | 0819 | 1022 | 1225 | 1427 | 163 | 1832 | 2034 | 2236 | 20 |
| 215 | 33- | 2438 | 264 | 2842 | 3044 | 3246 | 3447 | 3649 | 385 | 4051 | 4253 | 20 |
| 216 | 33- | 4454 | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 626 | 20 |
| 217 | 33- | 646 | 666 | 686 | 706 | 726 | 7459 | 7659 | 7858 | 8058 | 8257 | 20 |
| 218 | 33- | 8456 | 8656 | 8855 | 9054 | 9253 | 9451 | 965 | 9849 | — | — | 20 |
| 218 | 34- | — | — | — | — | — | — | — | — | 0047 | 0246 | 19 |
| 219 | 34- | 0444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 183 | 2028 | 2225 | 19 |
| 220 | 34- | 2423 | 262 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 | 19 |
| 221 | 34- | 4392 | 4589 | 4785 | 4981 | 5178 | 5374 | 557 | 5766 | 5962 | 6157 | 19 |
| 222 | 34- | 6353 | 6549 | 6744 | 6939 | 7135 | 733 | 7525 | 772 | 7915 | 811 | 19 |
| 223 | 34- | 8305 | 85 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 986 | — | 19 |
| 223 | 35- | — | — | — | — | — | — | — | — | — | 0054 | 19 |
| 224 | 35- | 0248 | 0442 | 0636 | 0829 | 1023 | 1216 | 141 | 1603 | 1796 | 1989 | 19 |
| 225 | 35- | 2183 | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 19 |
| 226 | 35- | 4108 | 4301 | 4493 | 4685 | 4876 | 5068 | 526 | 5452 | 5643 | 5834 | 19 |
| 227 | 35- | 6026 | 6217 | 6408 | 6599 | 679 | 6981 | 7172 | 7363 | 7554 | 7744 | 19 |
| 228 | 35- | 7935 | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 19 |
| 229 | 35- | 9835 | — | — | — | — | — | — | — | — | — | 19 |
| 229 | 36- | — | 0025 | 0215 | 0404 | 0593 | 0783 | 0972 | 1161 | 135 | 1539 | 19 |
| 230 | 36- | 1728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 19 |
| 231 | 36- | 3612 | 38 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 19 |
| 232 | 36- | 5488 | 5675 | 5862 | 6049 | 6236 | 6423 | 661 | 6796 | 6983 | 7169 | 19 |
| 233 | 36- | 7356 | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 903 | 19 |
| 234 | 36- | 9216 | 9401 | 9587 | 9772 | 9958 | — | — | — | — | — | 19 |
| 235 | 37- | — | — | — | — | — | 0143 | 0328 | 0513 | 0698 | 0883 | 19 |
| 235 | 37- | 1068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 236 | 2544 | 2728 | 19 |
| 236 | 37- | 2912 | 3096 | 328 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 | 19 |
| 237 | 37- | 4748 | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 | 19 |
| 238 | 37- | 6577 | 6759 | 6942 | 7124 | 7306 | 7488 | 767 | 7852 | 8034 | 8216 | 19 |
| 239 | 37- | 8398 | 858 | 8761 | 8943 | 9124 | 9306 | 9487 | 9668 | 9849 | — | 19 |
| 239 | 38- | — | — | — | — | — | — | — | — | — | 003 | 19 |
| 240 | 38- | 0211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 | 19 |
| 241 | 38- | 2017 | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 | 19 |
| 242 | 38- | 3815 | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 507 | 5249 | 5428 | 19 |
| 243 | 38- | 5606 | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 | 19 |
| 244 | 38- | 739 | 7568 | 7746 | 7923 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 | 19 |
| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |

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|-----|------|------|------|------|------|------|------|------|------|------|-----|
| 38- | 9166 | 9343 | 952 | 9698 | 9875 | — | — | — | — | — | 177 |
| 39- | — | — | — | — | — | 0051 | 0228 | 0405 | 0582 | 0759 | 177 |
| 39- | 0935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 | 176 |
| 39- | 2697 | 2873 | 3048 | 3224 | 34 | 3575 | 3751 | 3926 | 4101 | 4277 | 176 |
| 39- | 4452 | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 585 | 6025 | 175 |
| 39- | 6199 | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 | 174 |
| 39- | 794 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 | 173 |
| 39- | 9674 | 9847 | — | — | — | — | — | — | — | — | 173 |
| 40- | — | — | 002 | 0192 | 0365 | 0538 | 0711 | 0883 | 1056 | 1228 | 173 |
| 40- | 1401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 | 172 |
| 40- | 3121 | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 432 | 4492 | 4663 | 171 |
| 40- | 4834 | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 637 | 171 |
| 40- | 654 | 671 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 807 | 170 |
| 40- | 824 | 841 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 | 169 |
| 40- | 9933 | — | — | — | — | — | — | — | — | — | 169 |
| 41- | — | 0102 | 0271 | 044 | 0609 | 0777 | 0946 | 1114 | 1283 | 1451 | 169 |
| 41- | 162 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 | 168 |
| 41- | 33 | 3467 | 3635 | 3803 | 397 | 4137 | 4305 | 4472 | 4639 | 4806 | 167 |
| 41- | 4973 | 514 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 | 167 |
| 41- | 6641 | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 797 | 8135 | 166 |
| 41- | 8301 | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 946 | 9625 | 9791 | 165 |
| 41- | 9956 | — | — | — | — | — | — | — | — | — | 165 |
| 42- | — | 0121 | 0286 | 0451 | 0616 | 0781 | 0945 | 111 | 1275 | 1439 | 165 |
| 42- | 1604 | 1768 | 1933 | 2097 | 2261 | 2426 | 259 | 2754 | 2918 | 3082 | 164 |
| 42- | 3246 | 341 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 | 164 |
| 42- | 4882 | 5045 | 5208 | 5371 | 5534 | 5697 | 586 | 6023 | 6186 | 6349 | 163 |
| 42- | 6511 | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 | 162 |
| 42- | 8135 | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 | 162 |
| 42- | 9752 | 9914 | — | — | — | — | — | — | — | — | 162 |
| 43- | — | — | 0075 | 0236 | 0398 | 0559 | 072 | 0881 | 1042 | 1203 | 161 |
| 43- | 1364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 | 161 |
| 43- | 2969 | 313 | 329 | 345 | 361 | 377 | 393 | 409 | 4249 | 4409 | 160 |
| 43- | 4569 | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 | 159 |
| 43- | 6163 | 6322 | 6481 | 664 | 6799 | 6957 | 7116 | 7275 | 7433 | 7592 | 159 |
| 43- | 7751 | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 | 158 |
| 43- | 9333 | 9491 | 9648 | 9806 | 9964 | — | — | — | — | — | 158 |
| 44- | — | — | — | — | — | 0122 | 0279 | 0437 | 0594 | 0752 | 158 |
| 44- | 0909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 | 157 |
| 44- | 248 | 2637 | 2793 | 295 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 | 157 |
| 44- | 4045 | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 | 156 |
| 44- | 5604 | 576 | 5915 | 6071 | 6226 | 6382 | 6537 | 6692 | 6848 | 7003 | 155 |
| 44- | 7158 | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | 8242 | 8397 | 8552 | 155 |
| 44- | 8706 | 8861 | 9015 | 917 | 9324 | 9478 | 9633 | 9787 | 9941 | — | 154 |
| 45- | — | — | — | — | — | — | — | — | — | 0095 | 154 |
| 45- | 0249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 | 154 |
| 45- | 1786 | 194 | 2093 | 2247 | 24 | 2553 | 2706 | 2859 | 3012 | 3165 | 153 |
| 45- | 3318 | 3471 | 3624 | 3777 | 393 | 4082 | 4235 | 4387 | 454 | 4692 | 153 |
| 45- | 4845 | 4997 | 515 | 5302 | 5454 | 5606 | 5758 | 591 | 6062 | 6214 | 152 |
| 45- | 6366 | 6518 | 667 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 | 152 |
| 45- | 7882 | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 894 | 9091 | 9242 | 151 |
| 45- | 9392 | 9543 | 9694 | 9845 | 9995 | — | — | — | — | — | 151 |
| 46- | — | — | — | — | — | 0146 | 0296 | 0447 | 0597 | 0748 | 151 |
| 46- | 0898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 | 150 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|---------|------|------|------|------|------|------|------|------|
| 290 | 46-2398 | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 |
| 291 | 46-3893 | 4042 | 4191 | 434 | 449 | 4639 | 4788 | 4936 | 5085 |
| 292 | 46-5383 | 5532 | 568 | 5829 | 5977 | 6126 | 6274 | 6423 | 6571 |
| 293 | 46-6868 | 7016 | 7164 | 7312 | 746 | 7608 | 7756 | 7904 | 8052 |
| 294 | 46-8347 | 8495 | 8643 | 879 | 8938 | 9085 | 9233 | 938 | 9527 |
| 295 | 46-9822 | 9969 | — | — | — | — | — | — | — |
| 296 | 47- — | — | 0116 | 0263 | 041 | 0557 | 0704 | 0851 | 0998 |
| 297 | 47-1292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 |
| 298 | 47-2756 | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 |
| 299 | 47-4216 | 4362 | 4508 | 4653 | 4799 | 4944 | 509 | 5235 | 5381 |
| 300 | 47-5671 | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 |
| 301 | 47-7121 | 7266 | 7411 | 7555 | 77 | 7844 | 7989 | 8133 | 8278 |
| 302 | 47-8566 | 8711 | 8855 | 8999 | 9143 | 9287 | 9431 | 9575 | 9719 |
| 303 | 48-0007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 |
| 304 | 48-1443 | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 |
| 305 | 48-2874 | 3016 | 3159 | 3302 | 3445 | 3587 | 373 | 3872 | 4015 |
| 306 | 48-43 | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 |
| 307 | 48-5721 | 5863 | 6005 | 6147 | 6289 | 643 | 6572 | 6714 | 6855 |
| 308 | 48-7138 | 728 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 |
| 309 | 48-8551 | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9677 |
| 310 | 48-9958 | — | — | — | — | — | — | — | — |
| 311 | 49- — | 0099 | 0239 | 038 | 052 | 0661 | 0801 | 0941 | 1081 |
| 312 | 49-1362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 |
| 313 | 49-276 | 29 | 304 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 |
| 314 | 49-4155 | 4294 | 4433 | 4572 | 4711 | 485 | 4989 | 5128 | 5267 |
| 315 | 49-5544 | 5683 | 5822 | 596 | 6099 | 6238 | 6376 | 6515 | 6653 |
| 316 | 49-693 | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 |
| 317 | 49-8311 | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 |
| 318 | 49-9687 | 9824 | 9962 | — | — | — | — | — | — |
| 319 | 50- — | — | — | 0099 | 0236 | 0374 | 0511 | 0648 | 0785 |
| 320 | 50-1059 | 1196 | 1333 | 147 | 1607 | 1744 | 188 | 2017 | 2154 |
| 321 | 50-2427 | 2564 | 27 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 |
| 322 | 50-3791 | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 4878 |
| 323 | 50-515 | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 |
| 324 | 50-6505 | 664 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 |
| 325 | 50-7856 | 7991 | 8126 | 826 | 8395 | 853 | 8664 | 8799 | 8934 |
| 326 | 50-9203 | 9337 | 9471 | 9606 | 974 | 9874 | — | — | — |
| 327 | 51- — | — | — | — | — | — | 0009 | 0143 | 0277 |
| 328 | 51-0545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 |
| 329 | 51-1883 | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 |
| 330 | 51-3218 | 3351 | 3484 | 3617 | 375 | 3883 | 4016 | 4149 | 4282 |
| 331 | 51-4548 | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 |
| 332 | 51-5874 | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 68 | 6932 |
| 333 | 51-7196 | 7328 | 746 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 |
| 334 | 51-8514 | 8646 | 8777 | 8909 | 904 | 9171 | 9303 | 9434 | 9566 |
| 335 | 51-9828 | 9959 | — | — | — | — | — | — | — |
| 336 | 52- — | — | 009 | 0221 | 0353 | 0484 | 0615 | 0745 | 0876 |
| 337 | 52-1138 | 1269 | 14 | 153 | 1661 | 1792 | 1922 | 2053 | 2183 |
| 338 | 52-2444 | 2575 | 2705 | 2835 | 2966 | 3096 | 3226 | 3356 | 3486 |
| 339 | 52-3746 | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 |
| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|------|------|------|------|------|------|------|------|------|------|-----|
| 52- | 5045 | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 621 | 129 |
| 52- | 6339 | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501 | 129 |
| 52- | 763 | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 866 | 8788 | 129 |
| 52- | 8917 | 9045 | 9174 | 9302 | 943 | 9559 | 9687 | 9815 | 9943 | — | 128 |
| 53- | — | — | — | — | — | — | — | — | — | 0072 | 128 |
| 53- | 02 | 0328 | 0456 | 0584 | 0712 | 084 | 0968 | 1096 | 1223 | 1351 | 128 |
| 53- | 1479 | 1607 | 1734 | 1862 | 199 | 2117 | 2245 | 2372 | 25 | 2627 | 128 |
| 53- | 2754 | 2882 | 3009 | 3136 | 3264 | 3391 | 3518 | 3645 | 3772 | 3899 | 127 |
| 53- | 4026 | 4153 | 428 | 4407 | 4534 | 4661 | 4787 | 4914 | 5041 | 5167 | 127 |
| 53- | 5294 | 5421 | 5547 | 5674 | 58 | 5927 | 6053 | 618 | 6306 | 6432 | 126 |
| 53- | 6558 | 6685 | 6811 | 6937 | 7063 | 7189 | 7315 | 7441 | 7567 | 7693 | 126 |
| 53- | 7819 | 7945 | 8071 | 8197 | 8322 | 8448 | 8574 | 8699 | 8825 | 8951 | 126 |
| 53- | 9076 | 9202 | 9327 | 9452 | 9578 | 9703 | 9829 | 9954 | — | — | 126 |
| 54- | — | — | — | — | — | — | — | — | 0079 | 0204 | 125 |
| 54- | 0329 | 0455 | 058 | 0705 | 083 | 0955 | 108 | 1205 | 133 | 1454 | 125 |
| 54- | 1579 | 1704 | 1829 | 1953 | 2078 | 2203 | 2327 | 2452 | 2576 | 2701 | 125 |
| 54- | 2825 | 295 | 3074 | 3199 | 3323 | 3447 | 3571 | 3696 | 382 | 3944 | 124 |
| 54- | 4068 | 4192 | 4316 | 444 | 4564 | 4688 | 4812 | 4936 | 506 | 5183 | 124 |
| 54- | 5307 | 5431 | 5555 | 5678 | 5802 | 5925 | 6049 | 6172 | 6296 | 6419 | 124 |
| 54- | 6543 | 6666 | 6789 | 6913 | 7036 | 7159 | 7282 | 7405 | 7529 | 7652 | 123 |
| 54- | 7775 | 7898 | 8021 | 8144 | 8267 | 8389 | 8512 | 8635 | 8758 | 8881 | 123 |
| 54- | 9003 | 9126 | 9249 | 9371 | 9494 | 9616 | 9739 | 9861 | 9984 | — | 123 |
| 55- | — | — | — | — | — | — | — | — | — | 0106 | 123 |
| 55- | 0228 | 0351 | 0473 | 0595 | 0717 | 084 | 0962 | 1084 | 1206 | 1328 | 122 |
| 55- | 145 | 1572 | 1694 | 1816 | 1938 | 206 | 2181 | 2303 | 2425 | 2547 | 122 |
| 55- | 2668 | 279 | 2911 | 3033 | 3155 | 3276 | 3398 | 3519 | 364 | 3762 | 121 |
| 55- | 3883 | 4004 | 4126 | 4247 | 4368 | 4489 | 461 | 4731 | 4852 | 4973 | 121 |
| 55- | 5094 | 5215 | 5336 | 5457 | 5578 | 5699 | 582 | 594 | 6061 | 6182 | 121 |
| 55- | 6303 | 6423 | 6544 | 6664 | 6785 | 6905 | 7026 | 7146 | 7267 | 7387 | 120 |
| 55- | 7507 | 7627 | 7748 | 7868 | 7988 | 8108 | 8228 | 8349 | 8469 | 8589 | 120 |
| 55- | 8709 | 8829 | 8948 | 9068 | 9188 | 9308 | 9428 | 9548 | 9667 | 9787 | 120 |
| 55- | 9907 | — | — | — | — | — | — | — | — | — | 120 |
| 56- | — | 0026 | 0146 | 0265 | 0385 | 0504 | 0624 | 0743 | 0863 | 0982 | 119 |
| 56- | 1101 | 1221 | 134 | 1459 | 1578 | 1698 | 1817 | 1936 | 2055 | 2174 | 119 |
| 56- | 2293 | 2412 | 2531 | 265 | 2769 | 2887 | 3006 | 3125 | 3244 | 3362 | 119 |
| 56- | 3481 | 36 | 3718 | 3837 | 3955 | 4074 | 4192 | 4311 | 4429 | 4548 | 119 |
| 56- | 4666 | 4784 | 4903 | 5021 | 5139 | 5257 | 5376 | 5494 | 5612 | 573 | 118 |
| 56- | 5848 | 5966 | 6084 | 6202 | 632 | 6437 | 6555 | 6673 | 6791 | 6909 | 118 |
| 56- | 7026 | 7144 | 7262 | 7379 | 7497 | 7614 | 7732 | 7849 | 7967 | 8084 | 118 |
| 56- | 8202 | 8319 | 8436 | 8554 | 8671 | 8788 | 8905 | 9023 | 914 | 9257 | 117 |
| 56- | 9374 | 9491 | 9608 | 9725 | 9842 | 9959 | — | — | — | — | 117 |
| 57- | — | — | — | — | — | — | 0076 | 0193 | 0309 | 0426 | 117 |
| 57- | 0543 | 066 | 0776 | 0893 | 101 | 1126 | 1243 | 1359 | 1476 | 1592 | 117 |
| 57- | 1709 | 1825 | 1942 | 2058 | 2174 | 2291 | 2407 | 2523 | 2639 | 2755 | 116 |
| 57- | 2872 | 2988 | 3104 | 322 | 3336 | 3452 | 3568 | 3684 | 38 | 3915 | 116 |
| 7- | 4031 | 4147 | 4263 | 4379 | 4494 | 461 | 4726 | 4841 | 4957 | 5072 | 116 |
| 7- | 5188 | 5303 | 5419 | 5534 | 565 | 5765 | 588 | 5996 | 6111 | 6226 | 115 |
| 7- | 6341 | 6457 | 6572 | 6687 | 6802 | 6917 | 7032 | 7147 | 7262 | 7377 | 115 |
| 7- | 7492 | 7607 | 7722 | 7836 | 7951 | 8066 | 8181 | 8295 | 841 | 8525 | 115 |
| 7- | 8639 | 8754 | 8868 | 8983 | 9097 | 9212 | 9326 | 9441 | 9555 | 9669 | 114 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|---------|------|------|------|------|------|------|------|------|------|-----|
| 200 | 30-103 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 298 | 217 |
| 201 | 30-3196 | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 216 |
| 202 | 30-5351 | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 215 |
| 203 | 30-7496 | 771 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 213 |
| 204 | 30-963 | 9843 | — | — | — | — | — | — | — | — | 213 |
| 204 | 31- — | — | 0056 | 0268 | 0481 | 0693 | 0906 | 1118 | 133 | 1542 | 212 |
| 205 | 31-1754 | 1966 | 2177 | 2389 | 26 | 2812 | 3023 | 3234 | 3445 | 3656 | 211 |
| 206 | 31-3867 | 4078 | 4289 | 4499 | 471 | 492 | 513 | 534 | 5551 | 576 | 210 |
| 207 | 31-597 | 618 | 639 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 209 |
| 208 | 31-8063 | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 973 | 9938 | 208 |
| 209 | 32-0146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 207 |
| 210 | 32-2219 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 206 |
| 211 | 32-4282 | 4488 | 4694 | 4899 | 5105 | 531 | 5516 | 5721 | 5926 | 6131 | 205 |
| 212 | 32-6336 | 6541 | 6745 | 695 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 204 |
| 213 | 32-838 | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | — | — | 204 |
| 213 | 33- — | — | — | — | — | — | — | — | 0008 | 0211 | 203 |
| 214 | 33-0414 | 0617 | 0819 | 1022 | 1225 | 1427 | 163 | 1832 | 2034 | 2236 | 202 |
| 215 | 33-2438 | 264 | 2842 | 3044 | 3246 | 3447 | 3649 | 385 | 4051 | 4253 | 202 |
| 216 | 33-4454 | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 626 | 201 |
| 217 | 33-646 | 666 | 686 | 706 | 726 | 7459 | 7659 | 7858 | 8058 | 8257 | 200 |
| 218 | 33-8456 | 8656 | 8855 | 9054 | 9253 | 9451 | 965 | 9849 | — | — | 200 |
| 218 | 34- — | — | — | — | — | — | — | — | 0047 | 0246 | 199 |
| 219 | 34-0444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 183 | 2028 | 2225 | 198 |
| 220 | 34-2423 | 262 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 | 197 |
| 221 | 34-4392 | 4589 | 4785 | 4981 | 5178 | 5374 | 557 | 5766 | 5962 | 6157 | 196 |
| 222 | 34-6353 | 6549 | 6744 | 6939 | 7135 | 733 | 7525 | 772 | 7915 | 811 | 195 |
| 223 | 34-8305 | 85 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 986 | — | 194 |
| 223 | 35- — | — | — | — | — | — | — | — | — | 0054 | 194 |
| 224 | 35-0248 | 0442 | 0636 | 0829 | 1023 | 1216 | 141 | 1603 | 1796 | 1989 | 193 |
| 225 | 35-2183 | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 193 |
| 226 | 35-4108 | 4301 | 4493 | 4685 | 4876 | 5068 | 526 | 5452 | 5643 | 5834 | 192 |
| 227 | 35-6026 | 6217 | 6408 | 6599 | 679 | 6981 | 7172 | 7363 | 7554 | 7744 | 191 |
| 228 | 35-7935 | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 190 |
| 229 | 35-9835 | — | — | — | — | — | — | — | — | — | 189 |
| 229 | 36- — | 0025 | 0215 | 0404 | 0593 | 0783 | 0972 | 1161 | 135 | 1539 | 189 |
| 230 | 36-1728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 188 |
| 231 | 36-3612 | 38 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 188 |
| 232 | 36-5488 | 5675 | 5862 | 6049 | 6236 | 6423 | 661 | 6796 | 6983 | 7169 | 187 |
| 232 | 36-7356 | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 903 | 186 |
| 232 | 36-9216 | 9401 | 9587 | 9772 | 9958 | — | — | — | — | — | 186 |
| 233 | — | — | — | — | — | 0143 | 0328 | 0513 | 0698 | 0883 | 185 |
| 233 | 36-1068 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 236 | 2544 | 2728 | 184 |
| 234 | 36-3012 | 3096 | 328 | 3464 | 3647 | 3831 | 4015 | 4198 | 4382 | 4565 | 184 |
| 234 | 36-4748 | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 6029 | 6212 | 6394 | 183 |
| 234 | 36-6577 | 6759 | 6942 | 7124 | 7306 | 7488 | 767 | 7852 | 8034 | 8216 | 182 |
| 234 | 36-8406 | 8587 | 8768 | 8949 | 9129 | 9309 | 9487 | 9668 | 9849 | — | 182 |
| 234 | 36-1025 | — | — | — | — | — | — | — | — | 003 | 181 |
| 235 | 36-2855 | 3036 | 3217 | 3397 | 3577 | 3756 | 3936 | 4115 | 4294 | 4473 | 181 |
| 235 | 36-4684 | 4863 | 5042 | 5221 | 5399 | 5578 | 5756 | 5935 | 6113 | 6291 | 180 |
| 235 | 36-6513 | 6691 | 6869 | 7047 | 7225 | 7403 | 7581 | 7759 | 7937 | 8114 | 179 |
| 235 | 36-8342 | 8519 | 8696 | 8873 | 905 | 9227 | 9404 | 9581 | 9758 | 9935 | 178 |
| 235 | 36-1013 | — | — | — | — | — | — | — | — | — | 178 |
| 236 | 36-2842 | 302 | 320 | 3378 | 3556 | 3734 | 3912 | 409 | 4268 | 4445 | 177 |
| 236 | 36-4671 | 4849 | 5027 | 5204 | 5381 | 5558 | 5735 | 5912 | 6089 | 6266 | 176 |
| 236 | 36-6500 | 6677 | 6854 | 7031 | 7208 | 7385 | 7562 | 7739 | 7916 | 8093 | 175 |
| 236 | 36-8329 | 8506 | 8683 | 886 | 9037 | 9214 | 9391 | 9568 | 9745 | 9922 | 175 |
| 236 | 36-1004 | — | — | — | — | — | — | — | — | — | 174 |
| 237 | 36-2833 | 301 | 3188 | 3365 | 3542 | 3719 | 3896 | 4073 | 425 | 4427 | 173 |
| 237 | 36-4662 | 4839 | 5016 | 5193 | 537 | 5548 | 5725 | 5902 | 6079 | 6256 | 172 |
| 237 | 36-6491 | 6668 | 6845 | 7022 | 7199 | 7376 | 7553 | 773 | 7908 | 8085 | 171 |
| 237 | 36-8320 | 8497 | 8674 | 8851 | 9028 | 9205 | 9382 | 9559 | 9736 | 9913 | 170 |
| 237 | 36-1003 | — | — | — | — | — | — | — | — | — | 170 |
| 238 | 36-2826 | 3003 | 318 | 3357 | 3534 | 3711 | 3888 | 4065 | 4242 | 4419 | 169 |
| 238 | 36-4655 | 4832 | 5009 | 5186 | 5363 | 554 | 5719 | 5896 | 6073 | 625 | 168 |
| 238 | 36-6484 | 6661 | 6838 | 7015 | 7192 | 7369 | 7546 | 7723 | 79 | 8076 | 167 |
| 238 | 36-8313 | 849 | 8666 | 8843 | 902 | 9197 | 9374 | 9551 | 9728 | 9905 | 166 |
| 238 | 36-1002 | — | — | — | — | — | — | — | — | — | 166 |
| 239 | 36-2819 | 300 | 3177 | 3354 | 3531 | 3708 | 3885 | 4062 | 4239 | 4416 | 165 |
| 239 | 36-4648 | 4825 | 5002 | 5179 | 5356 | 5533 | 571 | 5888 | 6065 | 6242 | 164 |
| 239 | 36-6477 | 6654 | 6831 | 7008 | 7185 | 7362 | 7539 | 7716 | 7893 | 807 | 163 |
| 239 | 36-8306 | 8483 | 866 | 8837 | 9014 | 9191 | 9368 | 9545 | 9722 | 9899 | 162 |
| 239 | 36-1001 | — | — | — | — | — | — | — | — | — | 162 |
| 240 | 36-2812 | 300 | 3176 | 3353 | 353 | 3706 | 3883 | 406 | 4236 | 4413 | 161 |
| 240 | 36-4641 | 4818 | 4995 | 5172 | 5349 | 5526 | 5703 | 588 | 6059 | 6236 | 160 |
| 240 | 36-6470 | 6647 | 6824 | 7001 | 7178 | 7355 | 7532 | 7709 | 7886 | 8063 | 159 |
| 240 | 36-8299 | 8476 | 8653 | 883 | 9006 | 9183 | 936 | 9539 | 9716 | 9893 | 158 |
| 240 | 36-1000 | — | — | — | — | — | — | — | — | — | 158 |
| 241 | 36-2805 | 300 | 3175 | 3352 | 3529 | 3706 | 3883 | 406 | 4233 | 441 | 157 |
| 241 | 36-4634 | 4811 | 4988 | 5165 | 5342 | 5519 | 5696 | 5873 | 605 | 6228 | 156 |
| 241 | 36-6463 | 664 | 6821 | 700 | 7177 | 7354 | 7531 | 7708 | 7885 | 8062 | 155 |
| 241 | 36-8292 | 8469 | 8646 | 8823 | 900 | 9177 | 9354 | 9531 | 9708 | 9885 | 154 |
| 241 | 36-1000 | — | — | — | — | — | — | — | — | — | 154 |
| 242 | 36-2798 | 300 | 3174 | 3351 | 3528 | 3705 | 3882 | 4059 | 4236 | 4413 | 153 |
| 242 | 36-4627 | 4804 | 4981 | 5158 | 5335 | 5512 | 5689 | 5866 | 6043 | 622 | 152 |
| 242 | 36-6456 | 6633 | 681 | 6988 | 7165 | 7342 | 7519 | 7696 | 7873 | 805 | 151 |
| 242 | 36-8285 | 8462 | 8639 | 8816 | 8993 | 917 | 9348 | 9525 | 9702 | 9879 | 150 |
| 242 | 36-1000 | — | — | — | — | — | — | — | — | — | 150 |
| 243 | 36-2791 | 300 | 3173 | 3350 | 3527 | 3704 | 3881 | 4058 | 4235 | 4412 | 149 |
| 243 | 36-4620 | 4797 | 4974 | 5151 | 5328 | 5505 | 5682 | 5859 | 6036 | 6213 | 148 |
| 243 | 36-6449 | 6626 | 6803 | 698 | 7157 | 7334 | 7511 | 7688 | 7865 | 8042 | 147 |
| 243 | 36-8278 | 8455 | 8632 | 8809 | 8986 | 9163 | 934 | 9521 | 9698 | 9875 | 146 |
| 243 | 36-1000 | — | — | — | — | — | — | — | — | — | 146 |
| 244 | 36-2784 | 300 | 3172 | 3349 | 3526 | 3703 | 388 | 4055 | 4232 | 4409 | 145 |
| 244 | 36-4613 | 479 | 4966 | 5143 | 532 | 5499 | 5676 | 5853 | 603 | 6206 | 144 |
| 244 | 36-6442 | 6619 | 6796 | 6973 | 715 | 7328 | 7505 | 7682 | 7859 | 8036 | 143 |
| 244 | 36-8271 | 8448 | 8625 | 8802 | 8979 | 9156 | 9333 | 951 | 9688 | 9865 | 142 |
| 244 | 36-1000 | — | — | — | — | — | — | — | — | — | 142 |
| 245 | 36-2777 | 300 | 3171 | 3348 | 3525 | 3702 | 3879 | 4056 | 4233 | 441 | 141 |
| 245 | 36-4606 | 4783 | 496 | 5137 | 5314 | 5491 | 5668 | 5845 | 6022 | 62 | 140 |
| 245 | 36-6435 | 6612 | 6789 | 6966 | 7143 | 732 | 7499 | 7676 | 7853 | 803 | 139 |
| 245 | 36-8264 | 8441 | 8618 | 8795 | 8972 | 9149 | 9326 | 9503 | 968 | 9859 | 138 |
| 245 | 36-1000 | — | — | — | — | — | — | — | — | — | 138 |
| 246 | 36-2770 | 300 | 3170 | 3347 | 3524 | 3701 | 3878 | 4055 | 4232 | 4409 | 137 |
| 246 | 36-4599 | 4776 | 4953 | 513 | 5306 | 5483 | 566 | 5839 | 6016 | 6193 | 136 |
| 246 | 36-6428 | 6605 | 6782 | 6959 | 7136 | 7313 | 749 | 7666 | 7843 | 802 | 135 |
| 246 | 36-8257 | 8434 | 8611 | 8788 | 8965 | 9142 | 9319 | 9496 | 9673 | 985 | 134 |
| 246 | 36-1000 | — | — | — | — | — | — | — | — | — | 134 |
| 247 | 36-2763 | 300 | 3169 | 3346 | 3523 | 3700 | 3877 | 4054 | 4231 | 4408 | 133 |
| 247 | 36-4592 | 4769 | 4946 | 5123 | 530 | 5477 | 5654 | 5831 | 6008 | 6185 | 132 |
| 247 | 36-6421 | 6598 | 6775 | 6952 | 7129 | 7306 | 7483 | 766 | 7837 | 8014 | 131 |
| 247 | 36-8250 | 8427 | 8604 | 8781 | 8958 | 9135 | 9312 | 9489 | 9666 | 9843 | 130 |
| 247 | 36-1000 | — | — | — | — | — | — | — | — | — | 130 |
| 248 | 36-2756 | 300 | 3168 | 3345 | 3522 | 3700 | 3877 | 4054 | 4231 | 4408 | 129 |
| | | | | | | | | | | | |

| 10. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|----|------|------|------|------|------|------|------|------|------|------|
| 15 | 38 | 9166 | 9343 | 952 | 9698 | 9875 | — | — | — | — | 177 |
| 15 | 39 | — | — | — | — | — | 0051 | 0228 | 0405 | 0582 | 0759 |
| 16 | 39 | 0935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 |
| 17 | 39 | 2697 | 2873 | 3048 | 3224 | 34 | 3575 | 3751 | 3926 | 4101 | 4277 |
| 18 | 39 | 4452 | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 585 | 6025 |
| 19 | 39 | 6199 | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 |
| 20 | 39 | 794 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 |
| 21 | 39 | 9674 | 9847 | — | — | — | — | — | — | — | 173 |
| 21 | 40 | — | — | 002 | 0192 | 0365 | 0538 | 0711 | 0883 | 1056 | 1228 |
| 22 | 40 | 1401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 |
| 23 | 40 | 3121 | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 432 | 4492 | 4663 |
| 24 | 40 | 4834 | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 637 |
| 25 | 40 | 654 | 671 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 807 |
| 26 | 40 | 824 | 841 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 |
| 27 | 40 | 9933 | — | — | — | — | — | — | — | — | 169 |
| 27 | 41 | — | 0102 | 0271 | 044 | 0609 | 0777 | 0946 | 1114 | 1283 | 1451 |
| 28 | 41 | 162 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 |
| 29 | 41 | 33 | 3467 | 3635 | 3803 | 397 | 4137 | 4305 | 4472 | 4639 | 4806 |
| 30 | 41 | 4973 | 514 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 |
| 31 | 41 | 6641 | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 797 | 8135 |
| 32 | 41 | 8301 | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 946 | 9625 | 9791 |
| 33 | 41 | 9956 | — | — | — | — | — | — | — | — | 165 |
| 33 | 42 | — | 0121 | 0286 | 0451 | 0616 | 0781 | 0945 | 111 | 1275 | 1439 |
| 34 | 42 | 1604 | 1768 | 1933 | 2097 | 2261 | 2426 | 259 | 2754 | 2918 | 3082 |
| 35 | 42 | 3246 | 341 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 |
| 36 | 42 | 4882 | 5045 | 5208 | 5371 | 5534 | 5697 | 586 | 6023 | 6186 | 6349 |
| 37 | 42 | 6511 | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 |
| 38 | 42 | 8135 | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 |
| 39 | 42 | 9752 | 9914 | — | — | — | — | — | — | — | 162 |
| 39 | 43 | — | — | 0075 | 0236 | 0398 | 0559 | 072 | 0881 | 1042 | 1203 |
| 40 | 43 | 1364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 |
| 41 | 43 | 2969 | 313 | 329 | 345 | 361 | 377 | 393 | 409 | 4249 | 4409 |
| 42 | 43 | 4569 | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 |
| 43 | 43 | 6163 | 6322 | 6481 | 664 | 6799 | 6957 | 7116 | 7275 | 7433 | 7592 |
| 44 | 43 | 7751 | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 |
| 45 | 43 | 9333 | 9491 | 9648 | 9806 | 9964 | — | — | — | — | 158 |
| 45 | 44 | — | — | — | — | — | 0122 | 0279 | 0437 | 0594 | 0752 |
| 46 | 44 | 0909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 |
| 47 | 44 | 248 | 2637 | 2793 | 295 | 3106 | 3263 | 3419 | 3 | — | — |
| 48 | 44 | 4045 | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | — | — | — |
| 49 | 44 | 5604 | 576 | 5915 | 6071 | 6226 | 6382 | 6537 | — | — | — |
| 50 | 44 | 7158 | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | — | — | — |
| 51 | 44 | 8706 | 8861 | 9015 | 917 | 9324 | 9478 | 9633 | — | — | — |
| 51 | 45 | — | — | — | — | — | — | — | — | — | — |
| 52 | 45 | 0249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 13 | — | — |
| 53 | 45 | 1786 | 194 | 2093 | 2247 | 24 | 2553 | 2706 | 28 | — | — |
| 54 | 45 | 3318 | 3471 | 3624 | 3777 | 393 | 4082 | 4235 | 438 | — | — |
| 55 | 45 | 4845 | 4997 | 515 | 5302 | 5454 | 5606 | 5758 | 591 | — | — |
| 56 | 45 | 6366 | 6518 | 667 | 6821 | 6973 | 7125 | 7276 | 7428 | — | — |
| 57 | 45 | 7882 | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 894 | — | — |
| 58 | 45 | 9392 | 9543 | 9694 | 9845 | 9995 | — | — | — | — | — |
| 58 | 46 | — | — | — | — | — | 0146 | 0296 | 0447 | 05 | — |
| 59 | 46 | 0898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 209 | — |
| A | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 299 | 481 | 482 | 483 | 484 | 485 | 486 | 487 | 488 | 489 | 490 |
| 300 | 490 | 491 | 492 | 493 | 494 | 495 | 496 | 497 | 498 | 499 |
| 301 | 500 | 501 | 502 | 503 | 504 | 505 | 506 | 507 | 508 | 509 |
| 302 | 510 | 511 | 512 | 513 | 514 | 515 | 516 | 517 | 518 | 519 |
| 303 | 520 | 521 | 522 | 523 | 524 | 525 | 526 | 527 | 528 | 529 |
| 304 | 530 | 531 | 532 | 533 | 534 | 535 | 536 | 537 | 538 | 539 |
| 305 | 540 | 541 | 542 | 543 | 544 | 545 | 546 | 547 | 548 | 549 |
| 306 | 550 | 551 | 552 | 553 | 554 | 555 | 556 | 557 | 558 | 559 |
| 307 | 560 | 561 | 562 | 563 | 564 | 565 | 566 | 567 | 568 | 569 |
| 308 | 570 | 571 | 572 | 573 | 574 | 575 | 576 | 577 | 578 | 579 |
| 309 | 580 | 581 | 582 | 583 | 584 | 585 | 586 | 587 | 588 | 589 |
| 310 | 590 | 591 | 592 | 593 | 594 | 595 | 596 | 597 | 598 | 599 |
| 311 | 600 | 601 | 602 | 603 | 604 | 605 | 606 | 607 | 608 | 609 |
| 312 | 610 | 611 | 612 | 613 | 614 | 615 | 616 | 617 | 618 | 619 |
| 313 | 620 | 621 | 622 | 623 | 624 | 625 | 626 | 627 | 628 | 629 |
| 314 | 630 | 631 | 632 | 633 | 634 | 635 | 636 | 637 | 638 | 639 |
| 315 | 640 | 641 | 642 | 643 | 644 | 645 | 646 | 647 | 648 | 649 |
| 316 | 650 | 651 | 652 | 653 | 654 | 655 | 656 | 657 | 658 | 659 |
| 317 | 660 | 661 | 662 | 663 | 664 | 665 | 666 | 667 | 668 | 669 |
| 318 | 670 | 671 | 672 | 673 | 674 | 675 | 676 | 677 | 678 | 679 |
| 319 | 680 | 681 | 682 | 683 | 684 | 685 | 686 | 687 | 688 | 689 |
| 320 | 690 | 691 | 692 | 693 | 694 | 695 | 696 | 697 | 698 | 699 |
| 321 | 700 | 701 | 702 | 703 | 704 | 705 | 706 | 707 | 708 | 709 |
| 322 | 710 | 711 | 712 | 713 | 714 | 715 | 716 | 717 | 718 | 719 |
| 323 | 720 | 721 | 722 | 723 | 724 | 725 | 726 | 727 | 728 | 729 |
| 324 | 730 | 731 | 732 | 733 | 734 | 735 | 736 | 737 | 738 | 739 |
| 325 | 740 | 741 | 742 | 743 | 744 | 745 | 746 | 747 | 748 | 749 |
| 326 | 750 | 751 | 752 | 753 | 754 | 755 | 756 | 757 | 758 | 759 |
| 327 | 760 | 761 | 762 | 763 | 764 | 765 | 766 | 767 | 768 | 769 |
| 328 | 770 | 771 | 772 | 773 | 774 | 775 | 776 | 777 | 778 | 779 |
| 329 | 780 | 781 | 782 | 783 | 784 | 785 | 786 | 787 | 788 | 789 |
| 330 | 790 | 791 | 792 | 793 | 794 | 795 | 796 | 797 | 798 | 799 |
| 331 | 800 | 801 | 802 | 803 | 804 | 805 | 806 | 807 | 808 | 809 |
| 332 | 810 | 811 | 812 | 813 | 814 | 815 | 816 | 817 | 818 | 819 |
| 333 | 820 | 821 | 822 | 823 | 824 | 825 | 826 | 827 | 828 | 829 |
| 334 | 830 | 831 | 832 | 833 | 834 | 835 | 836 | 837 | 838 | 839 |
| 335 | 840 | 841 | 842 | 843 | 844 | 845 | 846 | 847 | 848 | 849 |
| 336 | 850 | 851 | 852 | 853 | 854 | 855 | 856 | 857 | 858 | 859 |
| 337 | 860 | 861 | 862 | 863 | 864 | 865 | 866 | 867 | 868 | 869 |
| 338 | 870 | 871 | 872 | 873 | 874 | 875 | 876 | 877 | 878 | 879 |
| 339 | 880 | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 889 |
| 340 | 890 | 891 | 892 | 893 | 894 | 895 | 896 | 897 | 898 | 899 |
| 341 | 900 | 901 | 902 | 903 | 904 | 905 | 906 | 907 | 908 | 909 |
| 342 | 910 | 911 | 912 | 913 | 914 | 915 | 916 | 917 | 918 | 919 |
| 343 | 920 | 921 | 922 | 923 | 924 | 925 | 926 | 927 | 928 | 929 |
| 344 | 930 | 931 | 932 | 933 | 934 | 935 | 936 | 937 | 938 | 939 |
| 345 | 940 | 941 | 942 | 943 | 944 | 945 | 946 | 947 | 948 | 949 |
| 346 | 950 | 951 | 952 | 953 | 954 | 955 | 956 | 957 | 958 | 959 |
| 347 | 960 | 961 | 962 | 963 | 964 | 965 | 966 | 967 | 968 | 969 |
| 348 | 970 | 971 | 972 | 973 | 974 | 975 | 976 | 977 | 978 | 979 |
| 349 | 980 | 981 | 982 | 983 | 984 | 985 | 986 | 987 | 988 | 989 |
| 350 | 990 | 991 | 992 | 993 | 994 | 995 | 996 | 997 | 998 | 999 |

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|----------|------|------|------|------|------|------|------|------|------|----|---|
| 11- 1807 | 1892 | 1976 | 206 | 2144 | 2229 | 2313 | 2397 | 2481 | 2566 | 84 | |
| 11- 265 | 2734 | 2818 | 2902 | 2986 | 307 | 3154 | 3238 | 3323 | 3407 | 84 | |
| 11- 3491 | 3575 | 3659 | 3742 | 3826 | 391 | 3994 | 4078 | 4162 | 4246 | 84 | |
| 11- 433 | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5 | 5084 | 84 | |
| 11- 5167 | 5251 | 5335 | 5418 | 5502 | 5586 | 5669 | 5753 | 5836 | 592 | 84 | |
| 11- 6003 | 6087 | 617 | 6254 | 6337 | 6421 | 6504 | 6588 | 6671 | 6754 | 83 | |
| 11- 6838 | 6921 | 7004 | 7088 | 7171 | 7254 | 7338 | 7421 | 7504 | 7587 | 83 | |
| 11- 7671 | 7754 | 7837 | 792 | 8003 | 8086 | 8169 | 8253 | 8336 | 8419 | 83 | |
| 11- 8502 | 8585 | 8668 | 8751 | 8834 | 8917 | 9 | 9083 | 9165 | 9248 | 83 | |
| 11- 9331 | 9414 | 9497 | 958 | 9663 | 9745 | 9828 | 9911 | 9994 | — | 83 | |
| 12- — | — | — | — | — | — | — | — | — | 0077 | 83 | |
| 12- 0159 | 0242 | 0325 | 0407 | 049 | 0573 | 0655 | 0738 | 0821 | 0903 | 83 | |
| 12- 0986 | 1068 | 1151 | 1233 | 1316 | 1398 | 1481 | 1563 | 1646 | 1728 | 82 | |
| 12- 1811 | 1893 | 1975 | 2058 | 214 | 2222 | 2305 | 2387 | 2469 | 2552 | 82 | |
| 12- 2634 | 2716 | 2798 | 2881 | 2963 | 3045 | 3127 | 3209 | 3291 | 3374 | 82 | |
| 12- 3456 | 3538 | 362 | 3702 | 3784 | 3866 | 3948 | 403 | 4112 | 4194 | 82 | |
| 12- 4276 | 4358 | 444 | 4522 | 4604 | 4685 | 4767 | 4849 | 4931 | 5013 | 82 | |
| 12- 5095 | 5176 | 5258 | 534 | 5422 | 5503 | 5585 | 5667 | 5748 | 583 | 82 | |
| 12- 5912 | 5993 | 6075 | 6156 | 6238 | 632 | 6401 | 6483 | 6564 | 6646 | 82 | |
| 12- 6727 | 6809 | 689 | 6972 | 7053 | 7134 | 7216 | 7297 | 7379 | 746 | 81 | |
| 12- 7541 | 7623 | 7704 | 7785 | 7866 | 7948 | 8029 | 811 | 8191 | 8273 | 81 | |
| 12- 8354 | 8435 | 8516 | 8597 | 8678 | 8759 | 8841 | 8922 | 9003 | 9084 | 81 | |
| 12- 9165 | 9246 | 9327 | 9408 | 9489 | 957 | 9651 | 9732 | 9813 | 9893 | 81 | |
| 12- 9974 | — | — | — | — | — | — | — | — | — | 81 | |
| 13- — | 0055 | 0136 | 0217 | 0298 | 0378 | 0459 | 054 | 0621 | 0702 | 81 | |
| 13- 0782 | 0863 | 0944 | 1024 | 1105 | 1186 | 1266 | 1347 | 1428 | 1508 | 81 | |
| 13- 1589 | 1669 | 175 | 183 | 1911 | 1991 | 2072 | 2152 | 2233 | 2313 | 81 | |
| 13- 2394 | 2474 | 2555 | 2635 | 2715 | 2796 | 2876 | 2956 | 3037 | 3117 | 80 | |
| 13- 3197 | 3278 | 3358 | 3438 | 3518 | 3598 | 3679 | 3759 | 3839 | 3919 | 80 | |
| 13- 3999 | 4079 | 416 | 424 | 432 | 44 | 448 | 456 | 464 | 472 | 80 | |
| 13- 48 | 488 | 496 | 504 | 512 | 52 | 5279 | 5359 | 5439 | 5519 | 80 | |
| 13- 5599 | 5679 | 5759 | 5838 | 5918 | 5998 | 6078 | 6157 | 6237 | 6317 | 80 | |
| 13- 6397 | 6476 | 6556 | 6635 | 6715 | 6795 | 6874 | 6954 | 7034 | 7113 | 80 | |
| 13- 7193 | 7272 | 7352 | 7431 | 7511 | 759 | 767 | 7749 | 7829 | 7908 | 79 | |
| 13- 7987 | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 | 79 | |
| 13- 8781 | 886 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 | 79 | |
| 13- 9572 | 9651 | 9731 | 981 | 9889 | 9968 | — | — | — | — | 79 | |
| 14- — | — | — | — | — | — | 0047 | 0126 | 0205 | 0284 | 79 | |
| 14- 0363 | 0442 | 0521 | 06 | 0678 | 0757 | 0836 | 0915 | 0994 | 1073 | 79 | |
| 14- 1152 | 123 | 1309 | 1388 | 1467 | 1546 | 1624 | 1703 | 1782 | 186 | 79 | |
| 14- 1939 | 2018 | 2096 | 2175 | 2254 | 2332 | 2411 | 2489 | 2568 | 2647 | 79 | |
| 14- 2725 | 2804 | 2882 | 2961 | 3039 | 3118 | 3196 | 3275 | 3353 | 3431 | 78 | |
| 14- 351 | 3588 | 3667 | 3745 | 3823 | 3902 | 398 | 4058 | 4136 | 4215 | 78 | |
| 14- 4293 | 4371 | 4449 | 4528 | 4606 | 4684 | 4762 | 484 | 4919 | 4997 | 78 | |
| 14- 5075 | 5153 | 5231 | 5309 | 5387 | 5465 | 5543 | 5621 | 5699 | 5777 | 78 | |
| 14- 5855 | 5933 | 6011 | 6089 | 6167 | 6245 | 6323 | 6401 | 6479 | 6556 | 78 | |
| 14- 6634 | 6712 | 679 | 6868 | 6945 | 7023 | 7101 | 7179 | 7256 | 7334 | 78 | |
| 14- 7412 | 7489 | 7567 | 7645 | 7722 | 78 | 7878 | 7955 | 8033 | 811 | 78 | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | | |

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|-----|----------|------|------|------|------|------|------|------|------|------|
| 560 | 74- 8188 | 8266 | 8343 | 8421 | 8498 | 8576 | 8653 | 8731 | 8808 | 8885 |
| 561 | 74- 8963 | 904 | 9118 | 9195 | 9272 | 935 | 9427 | 9504 | 9582 | 9659 |
| 562 | 74- 9736 | 9814 | 9891 | 9968 | — | — | — | — | — | — |
| 562 | 75- — | — | — | — | 0045 | 0123 | 02 | 0277 | 0354 | 0431 |
| 563 | 75- 0508 | 0586 | 0663 | 074 | 0817 | 0894 | 0971 | 1048 | 1125 | 1202 |
| 564 | 75- 1279 | 1356 | 1433 | 151 | 1587 | 1664 | 1741 | 1818 | 1895 | 1972 |
| 565 | 75- 2048 | 2125 | 2202 | 2279 | 2356 | 2433 | 2509 | 2586 | 2663 | 274 |
| 566 | 75- 2816 | 2893 | 297 | 3047 | 3123 | 32 | 3277 | 3353 | 343 | 3506 |
| 567 | 75- 3583 | 366 | 3736 | 3813 | 3889 | 3966 | 4042 | 4119 | 4195 | 4272 |
| 568 | 75- 4348 | 4425 | 4501 | 4578 | 4654 | 473 | 4807 | 4883 | 496 | 5036 |
| 569 | 75- 5112 | 5189 | 5265 | 5341 | 5417 | 5494 | 557 | 5646 | 5722 | 5799 |
| 570 | 75- 5875 | 5951 | 6027 | 6103 | 618 | 6256 | 6332 | 6408 | 6484 | 656 |
| 571 | 75- 6636 | 6712 | 6788 | 6864 | 694 | 7016 | 7092 | 7168 | 7244 | 732 |
| 572 | 75- 7396 | 7472 | 7548 | 7624 | 77 | 7775 | 7851 | 7927 | 8003 | 8079 |
| 573 | 75- 8155 | 823 | 8306 | 8382 | 8458 | 8533 | 8609 | 8685 | 8761 | 8836 |
| 574 | 75- 8912 | 8988 | 9063 | 9139 | 9214 | 929 | 9366 | 9441 | 9517 | 9592 |
| 575 | 75- 9668 | 9743 | 9819 | 9894 | 997 | — | — | — | — | — |
| 575 | 76- — | — | — | — | — | 0045 | 0121 | 0196 | 0272 | 0347 |
| 576 | 76- 0422 | 0498 | 0573 | 0649 | 0724 | 0799 | 0875 | 095 | 1025 | 1101 |
| 577 | 76- 1176 | 1251 | 1326 | 1402 | 1477 | 1552 | 1627 | 1702 | 1778 | 1853 |
| 578 | 76- 1928 | 2003 | 2078 | 2153 | 2228 | 2303 | 2378 | 2453 | 2529 | 2604 |
| 579 | 76- 2679 | 2754 | 2829 | 2904 | 2978 | 3053 | 3128 | 3203 | 3278 | 3353 |
| 580 | 76- 3428 | 3503 | 3578 | 3653 | 3727 | 3802 | 3877 | 3952 | 4027 | 4101 |
| 581 | 76- 4176 | 4251 | 4326 | 44 | 4475 | 455 | 4624 | 4699 | 4774 | 4848 |
| 582 | 76- 4923 | 4998 | 5072 | 5147 | 5221 | 5296 | 537 | 5445 | 552 | 5594 |
| 583 | 76- 5669 | 5743 | 5818 | 5892 | 5966 | 6041 | 6115 | 619 | 6264 | 6338 |
| 584 | 76- 6413 | 6487 | 6562 | 6636 | 671 | 6785 | 6859 | 6933 | 7007 | 7082 |
| 585 | 76- 7156 | 723 | 7304 | 7379 | 7453 | 7527 | 7601 | 7675 | 7749 | 7823 |
| 586 | 76- 7898 | 7972 | 8046 | 812 | 8194 | 8268 | 8342 | 8416 | 849 | 8564 |
| 587 | 76- 8638 | 8712 | 8786 | 886 | 8934 | 9008 | 9082 | 9156 | 923 | 9303 |
| 588 | 76- 9377 | 9451 | 9525 | 9599 | 9673 | 9746 | 982 | 9894 | 9968 | — |
| 588 | 77- — | — | — | — | — | — | — | — | — | 0042 |
| 589 | 77- 0115 | 0189 | 0263 | 0336 | 041 | 0484 | 0557 | 0631 | 0705 | 0778 |
| 590 | 77- 0852 | 0926 | 0999 | 1073 | 1146 | 122 | 1293 | 1367 | 144 | 1514 |
| 591 | 77- 1587 | 1661 | 1734 | 1808 | 1881 | 1955 | 2028 | 2102 | 2175 | 2248 |
| 592 | 77- 2322 | 2395 | 2468 | 2542 | 2615 | 2688 | 2762 | 2835 | 2908 | 2981 |
| 593 | 77- 3055 | 3128 | 3201 | 3274 | 3348 | 3421 | 3494 | 3567 | 364 | 3713 |
| 594 | 77- 3786 | 386 | 3933 | 4006 | 4079 | 4152 | 4225 | 4298 | 4371 | 4444 |
| 595 | 77- 4517 | 459 | 4663 | 4736 | 4809 | 4882 | 4955 | 5028 | 51 | 5173 |
| 596 | 77- 5246 | 5319 | 5392 | 5465 | 5538 | 561 | 5683 | 5756 | 5829 | 5902 |
| 597 | 77- 5974 | 6047 | 612 | 6193 | 6265 | 6338 | 6411 | 6483 | 6556 | 6629 |
| 598 | 77- 6701 | 6774 | 6846 | 6919 | 6992 | 7064 | 7137 | 7209 | 7282 | 7354 |
| 599 | 77- 7427 | 7499 | 7572 | 7644 | 7717 | 7789 | 7862 | 7934 | 8006 | 8079 |
| 600 | 77- 8151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585 | 8658 | 873 | 8802 |
| 601 | 77- 8874 | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 938 | 9452 | 9524 |
| 602 | 77- 9596 | 9669 | 9741 | 9813 | 9885 | 9957 | — | — | — | — |
| 602 | 78- — | — | — | — | — | — | 0029 | 0101 | 0173 | 0245 |
| 603 | 78- 0317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749 | 0821 | 0893 | 0965 |
| 604 | 78- 1037 | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 154 | 1612 | 1684 |

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|----|------|------|------|------|------|------|------|------|------|------|-----|
| 2- | 8389 | 8491 | 8593 | 8695 | 8797 | 89 | 9002 | 9104 | 9206 | 9308 | 102 |
| 2- | 941 | 9512 | 9613 | 9715 | 9817 | 9919 | — | — | — | — | 102 |
| 3- | — | — | — | — | — | — | 0021 | 0123 | 0224 | 0326 | 102 |
| 3- | 0428 | 053 | 0631 | 0733 | 0835 | 0936 | 1038 | 1139 | 1241 | 1342 | 102 |
| 3- | 1444 | 1545 | 1647 | 1748 | 1849 | 1951 | 2052 | 2153 | 2255 | 2356 | 101 |
| 3- | 2457 | 2559 | 266 | 2761 | 2862 | 2963 | 3064 | 3165 | 3266 | 3367 | 101 |
| 3- | 3468 | 3569 | 367 | 3771 | 3872 | 3973 | 4074 | 4175 | 4276 | 4376 | 101 |
| 3- | 4477 | 4578 | 4679 | 4779 | 488 | 4981 | 5081 | 5182 | 5283 | 5383 | 101 |
| 3- | 5484 | 5584 | 5685 | 5785 | 5886 | 5986 | 6087 | 6187 | 6287 | 6388 | 100 |
| 3- | 6488 | 6588 | 6688 | 6789 | 6889 | 6989 | 7089 | 7189 | 729 | 739 | 100 |
| 3- | 749 | 759 | 769 | 779 | 789 | 799 | 809 | 819 | 829 | 8389 | 100 |
| 3- | 8489 | 8589 | 8689 | 8789 | 8888 | 8988 | 9088 | 9188 | 9287 | 9387 | 100 |
| 3- | 9486 | 9586 | 9686 | 9785 | 9885 | 9984 | — | — | — | — | 100 |
| 4- | — | — | — | — | — | — | 0084 | 0183 | 0283 | 0382 | 99 |
| 4- | 0481 | 0581 | 068 | 0779 | 0879 | 0978 | 1077 | 1177 | 1276 | 1375 | 99 |
| 4- | 1474 | 1573 | 1672 | 1771 | 1871 | 197 | 2069 | 2168 | 2267 | 2366 | 99 |
| 4- | 2465 | 2563 | 2662 | 2761 | 286 | 2959 | 3058 | 3150 | 3255 | 3354 | 99 |
| 4- | 3453 | 3551 | 365 | 3749 | 3847 | 3946 | 4044 | 4143 | 4242 | 434 | 99 |
| 4- | 4439 | 4537 | 4636 | 4734 | 4832 | 4931 | 5029 | 5127 | 5226 | 5324 | 98 |
| 4- | 5422 | 5521 | 5619 | 5717 | 5815 | 5913 | 6011 | 611 | 6208 | 6306 | 98 |
| 4- | 6404 | 6502 | 66 | 6698 | 6796 | 6894 | 6992 | 7089 | 7187 | 7285 | 98 |
| 4- | 7383 | 7481 | 7579 | 7676 | 7774 | 7872 | 7969 | 8067 | 8165 | 8262 | 98 |
| 4- | 836 | 8458 | 8555 | 8653 | 875 | 8848 | 8945 | 9043 | 914 | 9237 | 97 |
| 4- | 9335 | 9432 | 953 | 9627 | 9724 | 9821 | 9919 | — | — | — | 97 |
| 5- | — | — | — | — | — | — | — | 0016 | 0113 | 021 | 97 |
| 5- | 0308 | 0405 | 0502 | 0599 | 0696 | 0793 | 089 | 0987 | 1083 | 1181 | 97 |
| 5- | 1278 | 1375 | 1472 | 1569 | 1666 | 1762 | 1859 | 1956 | 2053 | 215 | 97 |
| 5- | 2246 | 2343 | 244 | 2536 | 2633 | 273 | 2826 | 2923 | 3019 | 3116 | 97 |
| 5- | 3213 | 3309 | 3405 | 3502 | 3598 | 3695 | 3791 | 3888 | 3984 | 408 | 96 |
| 5- | 4177 | 4273 | 4369 | 4465 | 4562 | 4658 | 4754 | 485 | 4946 | 5042 | 96 |
| 5- | 5138 | 5235 | 5331 | 5427 | 5523 | 5619 | 5715 | 581 | 5906 | 6002 | 96 |
| 5- | 6098 | 6194 | 629 | 6386 | 6482 | 6577 | 6673 | 6769 | 6864 | 696 | 96 |
| 5- | 7056 | 7152 | 7247 | 7343 | 7438 | 7534 | 7629 | 7725 | 782 | 7916 | 96 |
| 5- | 8011 | 8107 | 8202 | 8298 | 8393 | 8488 | 8584 | 8679 | 8774 | 887 | 96 |
| 5- | 8965 | 906 | 9155 | 925 | 9346 | 9441 | 9536 | 9631 | 9726 | 9821 | 96 |
| 6- | 9916 | — | — | — | — | — | — | — | — | — | 96 |
| 6- | — | 0011 | 0106 | 0201 | 0296 | 0391 | 0486 | 058 | — | — | 96 |
| 6- | 0865 | 096 | 1055 | 115 | 1245 | 1339 | 1434 | 153 | — | — | 96 |
| 6- | 1813 | 1907 | 2002 | 2096 | 2191 | 2286 | 238 | 248 | — | — | 96 |
| 6- | 2758 | 2852 | 2947 | 3041 | 3135 | 323 | 3324 | 34 | — | — | 96 |
| 6- | 3701 | 3795 | 3889 | 3983 | 4078 | 4172 | 4266 | 43 | — | — | 96 |
| 6- | 4642 | 4736 | 483 | 4924 | 5018 | 5112 | 5206 | 529 | — | — | 96 |
| 6- | 5581 | 5675 | 5769 | 5862 | 5956 | 605 | 6143 | 623 | — | — | 96 |
| 6- | 6518 | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | — | — | 96 |
| 6- | 7453 | 7546 | 764 | 7733 | 7826 | 792 | 8013 | 8106 | — | — | 96 |
| 6- | 8386 | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | — | — | 96 |
| 6- | 9317 | 941 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | — | — | 96 |
| 7- | — | — | — | — | — | — | — | — | 00 | — | 96 |
| 7- | 0046 | 0139 | 0431 | 0524 | 0617 | 071 | 0802 | 0895 | 098 | — | 96 |
| 7- | 1173 | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | — | 96 |
| 8- | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

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|-----|-----|------|------|------|------|------|------|------|------|------|------|----|
| 655 | 81- | 6241 | 6308 | 6374 | 644 | 6506 | 6573 | 6639 | 6705 | 6771 | 6838 | 66 |
| 656 | 81- | 6904 | 697 | 7036 | 7102 | 7169 | 7235 | 7301 | 7367 | 7433 | 7499 | 66 |
| 657 | 81- | 7565 | 7631 | 7698 | 7764 | 783 | 7896 | 7962 | 8028 | 8094 | 816 | 66 |
| 658 | 81- | 8226 | 8292 | 8358 | 8424 | 849 | 8556 | 8622 | 8688 | 8754 | 882 | 66 |
| 659 | 81- | 8885 | 8951 | 9017 | 9083 | 9149 | 9215 | 9281 | 9346 | 9412 | 9478 | 66 |
| 660 | 81- | 9544 | 961 | 9676 | 9741 | 9807 | 9873 | 9939 | — | — | — | 66 |
| 660 | 82- | — | — | — | — | — | — | 0004 | 007 | 0136 | — | 66 |
| 661 | 82- | 0201 | 0267 | 0333 | 0399 | 0464 | 053 | 0595 | 0661 | 0727 | 0792 | 66 |
| 662 | 82- | 0858 | 0924 | 0989 | 1055 | 112 | 1186 | 1251 | 1317 | 1382 | 1448 | 66 |
| 663 | 82- | 1514 | 1579 | 1645 | 171 | 1775 | 1841 | 1906 | 1972 | 2037 | 2103 | 66 |
| 664 | 82- | 2168 | 2233 | 2299 | 2364 | 243 | 2495 | 256 | 2626 | 2691 | 2756 | 66 |
| 665 | 82- | 2822 | 2887 | 2952 | 3018 | 3083 | 3148 | 3213 | 3279 | 3344 | 3409 | 66 |
| 666 | 82- | 3474 | 3539 | 3605 | 367 | 3735 | 38 | 3865 | 393 | 3996 | 4061 | 66 |
| 667 | 82- | 4126 | 4191 | 4256 | 4321 | 4386 | 4451 | 4516 | 4581 | 4646 | 4711 | 66 |
| 668 | 82- | 4776 | 4841 | 4906 | 4971 | 5036 | 5101 | 5166 | 5231 | 5296 | 5361 | 66 |
| 669 | 82- | 5426 | 5491 | 5556 | 5621 | 5686 | 5751 | 5815 | 588 | 5945 | 601 | 66 |
| 670 | 82- | 6075 | 614 | 6204 | 6269 | 6334 | 6399 | 6464 | 6528 | 6593 | 6658 | 66 |
| 671 | 82- | 6723 | 6787 | 6852 | 6917 | 6981 | 7046 | 7111 | 7175 | 724 | 7305 | 66 |
| 672 | 82- | 7369 | 7434 | 7499 | 7563 | 7628 | 7692 | 7757 | 7821 | 7886 | 7951 | 66 |
| 673 | 82- | 8015 | 808 | 8144 | 8209 | 8273 | 8338 | 8402 | 8467 | 8531 | 8595 | 66 |
| 674 | 82- | 866 | 8724 | 8789 | 8853 | 8918 | 8982 | 9046 | 9111 | 9175 | 9239 | 66 |
| 675 | 82- | 9304 | 9368 | 9432 | 9497 | 9561 | 9625 | 969 | 9754 | 9818 | 9882 | 66 |
| 676 | 82- | 9947 | — | — | — | — | — | — | — | — | — | 66 |
| 676 | 83- | — | 0011 | 0075 | 0139 | 0204 | 0268 | 0332 | 0396 | 046 | 0525 | 66 |
| 677 | 83- | 0589 | 0653 | 0717 | 0781 | 0845 | 0909 | 0973 | 1037 | 1102 | 1166 | 66 |
| 678 | 83- | 123 | 1294 | 1358 | 1422 | 1486 | 155 | 1614 | 1678 | 1742 | 1806 | 66 |
| 679 | 83- | 187 | 1934 | 1998 | 2062 | 2126 | 2189 | 2253 | 2317 | 2381 | 2445 | 66 |
| 680 | 83- | 2509 | 2573 | 2637 | 27 | 2764 | 2828 | 2892 | 2956 | 302 | 3083 | 66 |
| 681 | 83- | 3147 | 3211 | 3275 | 3338 | 3402 | 3466 | 353 | 3593 | 3657 | 3721 | 66 |
| 682 | 83- | 3784 | 3848 | 3912 | 3975 | 4039 | 4103 | 4166 | 423 | 4294 | 4357 | 66 |
| 683 | 83- | 4421 | 4484 | 4548 | 4611 | 4675 | 4739 | 4802 | 4866 | 4929 | 4993 | 66 |
| 684 | 83- | 5056 | 512 | 5183 | 5247 | 531 | 5373 | 5437 | 55 | 5594 | 5657 | 66 |
| 685 | 83- | 5691 | 5754 | 5817 | 5881 | 5944 | 6007 | 6071 | 6134 | 6197 | 6261 | 66 |
| 686 | 83- | 6324 | 6387 | 6451 | 6514 | 6577 | 6641 | 6704 | 6767 | 683 | 6894 | 66 |
| 687 | 83- | 6957 | 702 | 7083 | 7146 | 721 | 7273 | 7336 | 7399 | 7462 | 7525 | 66 |
| 688 | 83- | 7588 | 7652 | 7715 | 7778 | 7841 | 7904 | 7967 | 803 | 8093 | 8156 | 66 |
| 689 | 83- | 8219 | 8282 | 8345 | 8408 | 8471 | 8534 | 8597 | 866 | 8723 | 8786 | 66 |
| 690 | 83- | 8849 | 8912 | 8975 | 9038 | 9101 | 9164 | 9227 | 9289 | 9352 | 9415 | 66 |
| 691 | 83- | 9478 | 9541 | 9604 | 9667 | 9729 | 9792 | 9855 | 9918 | 9981 | — | 66 |
| 691 | 84- | — | — | — | — | — | — | — | — | — | 0043 | 66 |
| 692 | 84 | 0106 | 0169 | 0232 | 0294 | 0357 | 042 | 0482 | 0545 | 0608 | 0671 | 66 |
| 693 | 84 | 0733 | 0796 | 0859 | 0921 | 0984 | 1046 | 1109 | 1172 | 1234 | 1297 | 66 |
| 694 | 84 | 1359 | 1422 | 1485 | 1547 | 161 | 1672 | 1735 | 1797 | 186 | 1922 | 66 |
| 695 | 84 | 1985 | 2047 | 211 | 2172 | 2235 | 2297 | 236 | 2422 | 2484 | 2547 | 66 |
| 696 | 84 | 2609 | 2672 | 2734 | 2796 | 2859 | 2921 | 2983 | 3046 | 3108 | 317 | 66 |
| 697 | 84 | 3233 | 3295 | 3357 | 342 | 3482 | 3544 | 3606 | 3669 | 3731 | 3793 | 66 |
| 698 | 84 | 3855 | 3918 | 398 | 4042 | 4104 | 4166 | 4229 | 4291 | 4353 | 4415 | 66 |
| 699 | 84 | 4477 | 4539 | 4601 | 4664 | 4726 | 4788 | 485 | 4912 | 4974 | 5036 | 66 |
| 700 | 84 | 5098 | 516 | 5222 | 5284 | 5346 | 5408 | 547 | 5532 | 5594 | 5656 | 66 |
| 701 | 84 | 5718 | 578 | 5842 | 5904 | 5966 | 6028 | 609 | 6151 | 6213 | 6275 | 66 |
| 702 | 84 | 6337 | 6399 | 6461 | 6523 | 6585 | 6646 | 6708 | 677 | 6832 | 6894 | 66 |
| 703 | 84 | 6955 | 7017 | 7079 | 7141 | 7202 | 7264 | 7326 | 7388 | 7449 | 7511 | 66 |
| 704 | 84 | 7573 | 7634 | 7696 | 7758 | 7819 | 7881 | 7943 | 8004 | 8066 | 8127 | 66 |
| No. | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|---------|------|------|------|------|------|------|------|------|------|----|
| 805 | 78-1755 | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 | 72 |
| 606 | 78-2473 | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 | 72 |
| 607 | 78-3189 | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 | 71 |
| 608 | 78-3904 | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 | 71 |
| 609 | 78-4617 | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 | 71 |
| 110 | 78-533 | 5401 | 5472 | 5543 | 5615 | 5686 | 5757 | 5828 | 5899 | 5970 | 71 |
| 611 | 78-6041 | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 | 71 |
| 612 | 78-6751 | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 | 71 |
| 613 | 78-746 | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 | 71 |
| 614 | 78-8168 | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 | 71 |
| 115 | 78-8875 | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9511 | 71 |
| 616 | 78-9581 | 9651 | 9722 | 9792 | 9863 | 9933 | — | — | — | — | 70 |
| 616 | 79-— | — | — | — | — | — | 0004 | 0074 | 0144 | 0215 | 70 |
| 617 | 79-0285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707 | 0778 | 0848 | 0918 | 70 |
| 618 | 79-0988 | 1059 | 1129 | 1199 | 1269 | 1340 | 1411 | 1481 | 1551 | 1622 | 70 |
| 619 | 79-1691 | 1761 | 1831 | 1901 | 1971 | 2041 | 2111 | 2181 | 2252 | 2322 | 70 |
| 120 | 79-2392 | 2462 | 2532 | 2602 | 2672 | 2742 | 2812 | 2882 | 2952 | 3022 | 70 |
| 621 | 79-3092 | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 | 70 |
| 622 | 79-379 | 386 | 393 | 4 | 407 | 4139 | 4209 | 4279 | 4349 | 4418 | 70 |
| 623 | 79-4488 | 4558 | 4627 | 4697 | 4767 | 4836 | 4906 | 4976 | 5045 | 5115 | 70 |
| 624 | 79-5185 | 5254 | 5324 | 5393 | 5463 | 5532 | 5602 | 5672 | 5741 | 5811 | 70 |
| 125 | 79-588 | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 | 69 |
| 626 | 79-6574 | 6644 | 6713 | 6782 | 6852 | 6921 | 6991 | 7060 | 7129 | 7198 | 69 |
| 527 | 79-7268 | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 | 69 |
| 528 | 79-796 | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 | 69 |
| 529 | 79-8651 | 8720 | 8789 | 8858 | 8927 | 8996 | 9065 | 9134 | 9203 | 9272 | 69 |
| 130 | 79-9341 | 9409 | 9478 | 9547 | 9616 | 9685 | 9754 | 9823 | 9892 | 9961 | 69 |
| 531 | 80-0029 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442 | 0511 | 0580 | 0648 | 69 |
| 532 | 80-0717 | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 | 69 |
| 533 | 80-1404 | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 | 69 |
| 534 | 80-2089 | 2158 | 2226 | 2295 | 2363 | 2432 | 25 | 2568 | 2637 | 2705 | 69 |
| 135 | 80-2774 | 2842 | 2911 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 | 68 |
| 536 | 80-3457 | 3525 | 3594 | 3662 | 3731 | 3798 | 3867 | 3935 | 4003 | 4071 | 68 |
| 537 | 80-4139 | 4208 | 4276 | 4344 | 4412 | 4480 | 4548 | 4616 | 4685 | 4753 | 68 |
| 538 | 80-4821 | 4889 | 4957 | 5025 | 5093 | 5161 | 5229 | 5297 | 5365 | 5433 | 68 |
| 539 | 80-5501 | 5569 | 5637 | 5705 | 5773 | 5841 | 5908 | 5976 | 6044 | 6112 | 68 |
| 140 | 80-618 | 6248 | 6316 | 6384 | 6451 | 6519 | 6587 | 6655 | 6723 | 6791 | 68 |
| 541 | 80-6858 | 6926 | 6994 | 7061 | 7129 | 7197 | 7264 | 7332 | 74 | 7467 | 68 |
| 542 | 80-7535 | 7603 | 7671 | 7738 | 7806 | 7873 | 7941 | 8008 | 8076 | 8143 | 68 |
| 543 | 80-8211 | 8279 | 8346 | 8414 | 8481 | 8549 | 8616 | 8684 | 8751 | 8818 | 67 |
| 544 | 80-8886 | 8953 | 9021 | 9088 | 9156 | 9223 | 9291 | 9358 | 9425 | 9492 | 67 |
| 145 | 80-956 | 9627 | 9694 | 9762 | 9829 | 9896 | 9964 | — | — | — | 67 |
| 545 | 81-— | — | — | — | — | — | — | 0031 | 0098 | 0165 | 67 |
| 546 | 81-0233 | 0301 | 0367 | 0434 | 0501 | 0569 | 0636 | 0703 | 0770 | 0837 | 67 |
| 547 | 81-0904 | 0971 | 1039 | 1106 | 1173 | 1240 | 1307 | 1374 | 1441 | 1508 | 67 |
| 548 | 81-1575 | 1642 | 1709 | 1776 | 1843 | 1910 | 1977 | 2044 | 2111 | 2178 | 67 |
| 549 | 81-2245 | 2312 | 2379 | 2445 | 2512 | 2579 | 2646 | 2713 | 2780 | 2847 | 67 |
| 550 | 81-2913 | 2980 | 3047 | 3114 | 3181 | 3248 | 3315 | 3382 | 3449 | 3516 | 67 |
| 551 | 81-3581 | 3648 | 3714 | 3781 | 3848 | 3915 | 3982 | 4049 | 4116 | 4183 | 67 |
| 552 | 81-4248 | 4314 | 4381 | 4447 | 4514 | 4581 | 4648 | 4715 | 4782 | 4849 | 67 |
| 553 | 81-4913 | 4980 | 5046 | 5113 | 5179 | 5246 | 5313 | 5380 | 5447 | 5514 | 67 |
| 554 | 81-5578 | 5644 | 5711 | 5777 | 5843 | 5910 | 5977 | 6044 | 6111 | 6178 | 67 |
| Na | 0 | 1 | 2 | 3 | 4 | 5 | | | | | |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----|
| 655 | 81- 6241 | 6308 | 6374 | 644 | 6506 | 6573 | 6639 | 6705 | 6771 | 6838 | 66 |
| 656 | 81- 6904 | 697 | 7036 | 7102 | 7169 | 7235 | 7301 | 7367 | 7433 | 7499 | 66 |
| 657 | 81- 7595 | 7631 | 7698 | 7764 | 783 | 7896 | 7962 | 8028 | 8094 | 816 | 66 |
| 658 | 81- 8226 | 8292 | 8358 | 8424 | 849 | 8556 | 8622 | 8688 | 8754 | 882 | 66 |
| 659 | 81- 8885 | 8951 | 9017 | 9083 | 9149 | 9215 | 9281 | 9346 | 9412 | 9478 | 66 |
| 660 | 81- 9544 | 961 | 9676 | 9741 | 9807 | 9873 | 9939 | — | — | — | 66 |
| 660 | 82- — | — | — | — | — | — | — | 0004 | 007 | 0136 | 66 |
| 661 | 82- 0201 | 0267 | 0333 | 0399 | 0464 | 053 | 0595 | 0661 | 0727 | 0792 | 66 |
| 662 | 82- 0858 | 0924 | 0989 | 1055 | 112 | 1186 | 1251 | 1317 | 1382 | 1448 | 66 |
| 663 | 82- 1514 | 1579 | 1645 | 171 | 1775 | 1841 | 1906 | 1972 | 2037 | 2103 | 65 |
| 664 | 82- 2168 | 2233 | 2299 | 2364 | 243 | 2495 | 256 | 2626 | 2691 | 2756 | 65 |
| 665 | 82- 2822 | 2887 | 2952 | 3018 | 3083 | 3148 | 3213 | 3279 | 3344 | 3409 | 65 |
| 666 | 82- 3474 | 3539 | 3605 | 367 | 3735 | 38 | 3865 | 393 | 3996 | 4061 | 65 |
| 667 | 82- 4126 | 4191 | 4256 | 4321 | 4386 | 4451 | 4516 | 4581 | 4646 | 4711 | 65 |
| 668 | 82- 4776 | 4841 | 4906 | 4971 | 5036 | 5101 | 5166 | 5231 | 5296 | 5361 | 65 |
| 669 | 82- 5426 | 5491 | 5556 | 5621 | 5686 | 5751 | 5815 | 588 | 5945 | 601 | 65 |
| 670 | 82- 6075 | 614 | 6204 | 6269 | 6334 | 6399 | 6464 | 6528 | 6593 | 6658 | 65 |
| 671 | 82- 6723 | 6787 | 6852 | 6917 | 6981 | 7046 | 7111 | 7175 | 724 | 7305 | 65 |
| 672 | 82- 7399 | 7434 | 7499 | 7563 | 7628 | 7692 | 7757 | 7821 | 7886 | 7951 | 65 |
| 673 | 82- 8015 | 808 | 8144 | 8209 | 8273 | 8338 | 8402 | 8467 | 8531 | 8595 | 64 |
| 674 | 82- 866 | 8724 | 8789 | 8853 | 8918 | 8982 | 9046 | 9111 | 9175 | 9239 | 64 |
| 675 | 82- 9304 | 9368 | 9432 | 9497 | 9561 | 9625 | 969 | 9754 | 9818 | 9882 | 64 |
| 676 | 82- 9947 | — | — | — | — | — | — | — | — | — | 64 |
| 676 | 83- — | 0011 | 0075 | 0139 | 0204 | 0268 | 0332 | 0396 | 046 | 0525 | 64 |
| 677 | 83- 0589 | 0653 | 0717 | 0781 | 0845 | 0909 | 0973 | 1037 | 1102 | 1166 | 64 |
| 678 | 83- 123 | 1294 | 1358 | 1422 | 1486 | 155 | 1614 | 1678 | 1742 | 1806 | 64 |
| 679 | 83- 187 | 1934 | 1998 | 2062 | 2126 | 2189 | 2253 | 2317 | 2381 | 2445 | 64 |
| 680 | 83- 2509 | 2573 | 2637 | 27 | 2764 | 2828 | 2892 | 2956 | 302 | 3083 | 64 |
| 681 | 83- 3147 | 3211 | 3275 | 3338 | 3402 | 3466 | 353 | 3593 | 3657 | 3721 | 64 |
| 682 | 83- 3784 | 3848 | 3912 | 3975 | 4039 | 4103 | 4166 | 423 | 4294 | 4357 | 64 |
| 683 | 83- 4421 | 4484 | 4548 | 4611 | 4675 | 4739 | 4802 | 4866 | 4929 | 4993 | 63 |
| 684 | 83- 5056 | 512 | 5183 | 5247 | 531 | 5373 | 5437 | 55 | 5564 | 5627 | 63 |
| 685 | 83- 5691 | 5754 | 5817 | 5881 | 5944 | 6007 | 6071 | 6134 | 6197 | 6261 | 63 |
| 686 | 83- 6324 | 6387 | 6451 | 6514 | 6577 | 6641 | 6704 | 6767 | 683 | 6894 | 63 |
| 687 | 83- 6957 | 702 | 7083 | 7146 | 721 | 7273 | 7336 | 7399 | 7462 | 7525 | 63 |
| 688 | 83- 7583 | 7652 | 7715 | 7778 | 7841 | 7904 | 7967 | 803 | 8093 | 8156 | 63 |
| 689 | 83- 8219 | 8282 | 8345 | 8408 | 8471 | 8534 | 8597 | 866 | 8723 | 8786 | 63 |
| 690 | 83- 8849 | 8912 | 8975 | 9038 | 9101 | 9164 | 9227 | 9289 | 9352 | 9415 | 63 |
| 691 | 83- 9478 | 9541 | 9604 | 9667 | 9729 | 9792 | 9855 | 9918 | 9981 | — | 63 |
| 691 | 84- — | — | — | — | — | — | — | — | — | 0043 | 63 |
| 692 | 84- 0106 | 0169 | 0232 | 0294 | 0357 | 042 | 0482 | 0545 | 0608 | 0671 | 63 |
| 693 | 84- 0733 | 0796 | 0859 | 0921 | 0984 | 1046 | 1109 | 1172 | 1234 | 1297 | 63 |
| 694 | 84- 1359 | 1422 | 1485 | 1547 | 161 | 1672 | 1735 | 1797 | 186 | 1922 | 63 |
| 695 | 84- 1985 | 2047 | 211 | 2172 | 2235 | 2297 | 236 | 2422 | 2484 | 2547 | 63 |
| 696 | 84- 2609 | 2672 | 2734 | 2796 | 2859 | 2921 | 2983 | 3046 | 3108 | 317 | 63 |
| 697 | 84- 3233 | 3295 | 3357 | 342 | 3482 | 3544 | 3606 | 3669 | 3731 | 3793 | 63 |
| 698 | 84- 3855 | 3918 | 398 | 4042 | 4104 | 4166 | 4229 | 4291 | 4353 | 4415 | 63 |
| 699 | 84- 4477 | 4539 | 4601 | 4664 | 4726 | 4788 | 485 | 4912 | 4974 | 5036 | 63 |
| 700 | 84- 5098 | 516 | 5222 | 5284 | 5346 | 5408 | 547 | 5532 | 5594 | 5656 | 63 |
| 701 | 84- 5718 | 578 | 5842 | 5904 | 5966 | 6028 | 609 | 6151 | 6213 | 6275 | 63 |
| 702 | 84- 6337 | 6399 | 6461 | 6523 | 6585 | 6646 | 6708 | 677 | 6832 | 6894 | 63 |
| 703 | 84- 7017 | 7079 | 7141 | 7202 | 7264 | 7326 | 7388 | 7449 | 7511 | 7573 | 63 |
| 704 | 84- 7758 | 7819 | 7881 | 7943 | 8004 | 8066 | 8127 | 8189 | 8251 | 8313 | 63 |
| 705 | 84- 8449 | 8511 | 8573 | 8635 | 8697 | 8759 | 8821 | 8883 | 8945 | 9007 | 63 |
| 706 | 84- 9160 | 9222 | 9284 | 9346 | 9408 | 9470 | 9532 | 9594 | 9656 | 9718 | 63 |
| 707 | 84- 9829 | 9891 | 9953 | 10015 | 10077 | 10139 | 10201 | 10263 | 10325 | 10387 | 63 |
| 708 | 84- 10649 | 10711 | 10773 | 10835 | 10897 | 10959 | 11021 | 11083 | 11145 | 11207 | 63 |
| 709 | 84- 11569 | 11631 | 11693 | 11755 | 11817 | 11879 | 11941 | 12003 | 12065 | 12127 | 63 |
| 710 | 84- 12489 | 12551 | 12613 | 12675 | 12737 | 12799 | 12861 | 12923 | 12985 | 13047 | 63 |
| 711 | 84- 13409 | 13471 | 13533 | 13595 | 13657 | 13719 | 13781 | 13843 | 13905 | 13967 | 63 |
| 712 | 84- 14329 | 14391 | 14453 | 14515 | 14577 | 14639 | 14701 | 14763 | 14825 | 14887 | 63 |
| 713 | 84- 15249 | 15311 | 15373 | 15435 | 15497 | 15559 | 15621 | 15683 | 15745 | 15807 | 63 |
| 714 | 84- 16169 | 16231 | 16293 | 16355 | 16417 | 16479 | 16541 | 16603 | 16665 | 16727 | 63 |
| 715 | 84- 17089 | 17151 | 17213 | 17275 | 17337 | 17399 | 17461 | 17523 | 17585 | 17647 | 63 |
| 716 | 84- 18009 | 18071 | 18133 | 18195 | 18257 | 18319 | 18381 | 18443 | 18505 | 18567 | 63 |
| 717 | 84- 18929 | 18991 | 19053 | 19115 | 19177 | 19239 | 19301 | 19363 | 19425 | 19487 | 63 |
| 718 | 84- 19849 | 19911 | 19973 | 20035 | 20097 | 20159 | 20221 | 20283 | 20345 | 20407 | 63 |
| 719 | 84- 20769 | 20831 | 20893 | 20955 | 21017 | 21079 | 21141 | 21203 | 21265 | 21327 | 63 |
| 720 | 84- 21689 | 21751 | 21813 | 21875 | 21937 | 21999 | 22061 | 22123 | 22185 | 22247 | 63 |
| 721 | 84- 22609 | 22671 | 22733 | 22795 | 22857 | 22919 | 22981 | 23043 | 23105 | 23167 | 63 |
| 722 | 84- 23529 | 23591 | 23653 | 23715 | 23777 | 23839 | 23901 | 23963 | 24025 | 24087 | 63 |
| 723 | 84- 24449 | 24511 | 24573 | 24635 | 24697 | 24759 | 24821 | 24883 | 24945 | 25007 | 63 |
| 724 | 84- 25369 | 25431 | 25493 | 25555 | 25617 | 25679 | 25741 | 25803 | 25865 | 25927 | 63 |
| 725 | 84- 26289 | 26351 | 26413 | 26475 | 26537 | 26599 | 26661 | 26723 | 26785 | 26847 | 63 |
| 726 | 84- 27209 | 27271 | 27333 | 27395 | 27457 | 27519 | 27581 | 27643 | 27705 | 27767 | 63 |
| 727 | 84- 28129 | 28191 | 28253 | 28315 | 28377 | 28439 | 28501 | 28563 | 28625 | 28687 | 63 |
| 728 | 84- 29049 | 29111 | 29173 | 29235 | 29297 | 29359 | 29421 | 29483 | 29545 | 29607 | 63 |
| 729 | 84- 29969 | 30031 | 30093 | 30155 | 30217 | 30279 | 30341 | 30403 | 30465 | 30527 | 63 |
| 730 | 84- 30889 | 30951 | 31013 | 31075 | 31137 | 31199 | 31261 | 31323 | 31385 | 31447 | 63 |
| 731 | 84- 31809 | 31871 | 31933 | 31995 | 32057 | 32119 | 32181 | 32243 | 32305 | 32367 | 63 |
| 732 | 84- 32729 | 32791 | 32853 | 32915 | 32977 | 33039 | 33101 | 33163 | 33225 | 33287 | 63 |
| 733 | 84- 33649 | 33711 | 33773 | 33835 | 33897 | 33959 | 34021 | 34083 | 34145 | 34207 | 63 |
| 734 | 84- 34569 | 34631 | 34693 | 34755 | 34817 | 34879 | 34941 | 35003 | 35065 | 35127 | 63 |
| 735 | 84- 35489 | 35551 | 35613 | 35675 | 35737 | 35799 | 35861 | 35923 | 35985 | 36047 | 63 |
| 736 | 84- 36409 | 36471 | 36533 | 36595 | 36657 | 36719 | 36781 | 36843 | 36905 | 36967 | 63 |
| 737 | 84- 37329 | 37391 | 37453 | 37515 | 37577 | 37639 | 37701 | 37763 | 37825 | 37887 | 63 |
| 738 | 84- 38249 | 38311 | 38373 | 38435 | 38497 | 38559 | 38621 | 38683 | 38745 | 38807 | 63 |
| 739 | 84- 39169 | 39231 | 39293 | 39355 | 39417 | 39479 | 39541 | 39603 | 39665 | 39727 | 63 |
| 740 | 84- 40089 | 40151 | 40213 | 40275 | 40337 | 40399 | 40461 | 40523 | 40585 | 40647 | 63 |
| 741 | 84- 41009 | 41071 | 41133 | 41195 | 41257 | 41319 | 41381 | 41443 | 41505 | 41567 | 63 |
| 742 | 84- 41929 | 41991 | 42053 | 42115 | 42177 | 42239 | 42301 | 42363 | 42425 | 42487 | 63 |
| 743 | 84- 42849 | 42911 | 42973 | 43035 | 43097 | 43159 | 43221 | 43283 | 43345 | 43407 | 63 |
| 744 | 84- 43769 | 43831 | 43893 | 43955 | 44017 | 44079 | 44141 | 44203 | 44265 | 44327 | 63 |
| 745 | 84- 44689 | 44751 | 44813 | 44875 | 44937 | 44999 | 45061 | 45123 | 45185 | 45247 | 63 |
| 746 | 84- 45609 | 45671 | 45733 | 45795 | 45857 | 45919 | 45981 | 46043 | 46105 | 46167 | 63 |
| 747 | 84- 46529 | 46591 | 46653 | 46715 | 46777 | 46839 | 46901 | 46963 | 47025 | 47087 | 63 |
| 748 | 84- 47449 | 47511 | 47573 | 47635 | 47697 | 47759 | 47821 | 47883 | 47945 | 48007 | 63 |
| 749 | 84- 48369 | 48431 | 48493 | 48555 | 48617 | 48679 | 48741 | 48803 | 48865 | 48927 | 63 |
| 750 | 84- 49289 | 49351 | 49413 | 49475 | 49537 | 49599 | 49661 | 49723 | 49785 | 49847 | 63 |
| 751 | 84- 50209 | 50271 | 50333 | 50395 | 50457 | 50519 | 50581 | 50643 | 50705 | 50767 | 63 |
| 752 | 84- 51129 | 51191 | 51253 | 51315 | 51377 | 51439 | 51501 | 51563 | 51625 | 51687 | 63 |
| 753 | 84- 52049 | 52111 | 52173 | 52235 | 52297 | 52359 | 52421 | 52483 | 52545 | 52607 | 63 |
| 754 | 84- 52969 | 53031 | 53093 | 53155 | 53217 | 53279 | 53341 | 53403 | 53465 | 53527 | 63 |
| 755 | 84- 53889 | 53951 | 54013 | 54075 | 54137 | 54199 | 54261 | 54323 | 54385 | 54447 | 63 |
| 756 | 84- | | | | | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|---|------|------|------|------|------|------|------|------|------|----|
| 9 | 8251 | 8312 | 8374 | 8435 | 8497 | 8559 | 8621 | 8682 | 8743 | 62 |
| 0 | 8866 | 8928 | 8989 | 9051 | 9112 | 9174 | 9235 | 9297 | 9358 | 61 |
| 1 | 9481 | 9542 | 9604 | 9665 | 9726 | 9788 | 9849 | 9911 | 9972 | 61 |
| 2 | 0095 | 0156 | 0217 | 0279 | 034 | 0401 | 0462 | 0524 | 0585 | 61 |
| 3 | 0707 | 0769 | 083 | 0891 | 0952 | 1014 | 1075 | 1136 | 1197 | 61 |
| 4 | 1321 | 1381 | 1442 | 1503 | 1564 | 1625 | 1686 | 1747 | 1809 | 61 |
| 5 | 1931 | 1992 | 2053 | 2114 | 2175 | 2236 | 2297 | 2358 | 2419 | 61 |
| 6 | 2541 | 2602 | 2663 | 2724 | 2785 | 2846 | 2907 | 2968 | 3029 | 61 |
| 7 | 315 | 3211 | 3272 | 3333 | 3394 | 3455 | 3516 | 3577 | 3637 | 61 |
| 8 | 3759 | 382 | 3881 | 3941 | 4002 | 4063 | 4124 | 4185 | 4245 | 61 |
| 9 | 4367 | 4428 | 4488 | 4549 | 461 | 467 | 4731 | 4792 | 4852 | 61 |
| 0 | 4974 | 5034 | 5095 | 5156 | 5216 | 5277 | 5337 | 5398 | 5459 | 61 |
| 1 | 558 | 564 | 5701 | 5761 | 5822 | 5882 | 5943 | 6003 | 6064 | 61 |
| 2 | 6185 | 6245 | 6306 | 6366 | 6427 | 6487 | 6548 | 6608 | 6668 | 60 |
| 3 | 6789 | 685 | 691 | 697 | 7031 | 7091 | 7152 | 7212 | 7272 | 60 |
| 4 | 7393 | 7453 | 7513 | 7574 | 7634 | 7694 | 7755 | 7815 | 7875 | 60 |
| 5 | 7995 | 8056 | 8116 | 8176 | 8236 | 8297 | 8357 | 8417 | 8477 | 60 |
| 6 | 8597 | 8657 | 8718 | 8778 | 8838 | 8898 | 8958 | 9018 | 9078 | 60 |
| 7 | 9198 | 9258 | 9318 | 9379 | 9439 | 9499 | 9559 | 9619 | 9679 | 60 |
| 8 | 9799 | 9859 | 9918 | 9978 | — | — | — | — | — | 60 |
| 9 | — | — | — | — | 0038 | 0098 | 0158 | 0218 | 0278 | 60 |
| 0 | 0398 | 0458 | 0518 | 0578 | 0637 | 0697 | 0757 | 0817 | 0877 | 60 |
| 1 | 0996 | 1056 | 1116 | 1176 | 1236 | 1295 | 1355 | 1415 | 1475 | 60 |
| 2 | 1594 | 1654 | 1714 | 1773 | 1833 | 1893 | 1952 | 2012 | 2072 | 60 |
| 3 | 2191 | 2251 | 231 | 237 | 243 | 2489 | 2549 | 2608 | 2668 | 60 |
| 4 | 2787 | 2847 | 2906 | 2966 | 3025 | 3085 | 3144 | 3204 | 3263 | 60 |
| 5 | 3382 | 3442 | 3501 | 3561 | 362 | 368 | 3739 | 3799 | 3858 | 59 |
| 6 | 3977 | 4036 | 4096 | 4155 | 4214 | 4274 | 4333 | 4392 | 4452 | 59 |
| 7 | 457 | 463 | 4689 | 4748 | 4808 | 4867 | 4926 | 4985 | 5045 | 59 |
| 8 | 5163 | 5222 | 5282 | 5341 | 54 | 5459 | 5519 | 5578 | 5637 | 59 |
| 9 | 5755 | 5814 | 5874 | 5933 | 5992 | 6051 | 6111 | 6169 | 6228 | 59 |
| 0 | 6346 | 6405 | 6465 | 6524 | 6583 | 6642 | 6701 | 676 | 6819 | 59 |
| 1 | 6937 | 6996 | 7055 | 7114 | 7173 | 7232 | 7291 | 735 | 7409 | 59 |
| 2 | 7526 | 7585 | 7644 | 7703 | 7762 | 7821 | 788 | 7939 | 7998 | 59 |
| 3 | 8115 | 8174 | 8233 | 8292 | 835 | 8409 | 8468 | 8527 | 8586 | 59 |
| 4 | 8703 | 8762 | 8821 | 8879 | 8938 | 8997 | 9056 | 9114 | 9173 | 59 |
| 5 | 929 | 9349 | 9408 | 9466 | 9525 | 9584 | 9642 | 9701 | 976 | 59 |
| 6 | 9877 | 9935 | 9994 | — | — | — | — | — | — | 59 |
| 7 | — | — | — | 0053 | 0111 | 017 | 0228 | 0287 | 0345 | 59 |
| 8 | 0462 | 0521 | 0579 | 0638 | 0696 | 0755 | 0813 | 0872 | 093 | 58 |
| 9 | 1047 | 1106 | 1164 | 1223 | 1281 | 1339 | 1398 | 1456 | 1515 | 58 |
| 0 | 1631 | 169 | 1748 | 1806 | 1865 | 1923 | 1981 | 204 | 2096 | 58 |
| 1 | 2215 | 2273 | 2331 | 2389 | 2448 | 2506 | 2564 | 2622 | 2681 | 58 |
| 2 | 2797 | 2855 | 2913 | 2972 | 303 | 3088 | 3146 | — | — | 58 |
| 3 | 3379 | 3437 | 3495 | 3553 | 3611 | 3669 | 3727 | — | — | 58 |
| 4 | 396 | 4018 | 4076 | 4134 | 4192 | 425 | 430 | — | — | 58 |
| 5 | 454 | 4598 | 4656 | 4714 | 4772 | 483 | 488 | — | — | 58 |
| 6 | 5119 | 5177 | 5235 | 5293 | 5351 | 5409 | 5466 | — | — | 58 |
| 7 | 5698 | 5756 | 5813 | 5871 | 5929 | 5987 | 6045 | — | — | 58 |
| 8 | 6276 | 6333 | 6391 | 6449 | 6507 | 6564 | 6622 | — | — | 58 |
| 9 | 6853 | 691 | 6968 | 7026 | 7083 | 7141 | 7198 | — | — | 58 |
| 0 | 7429 | 7487 | 7544 | 7602 | 7659 | — | — | — | — | 58 |

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| 855 | 93-1966 | 2017 | 2068 | 2118 | 2169 | 222 | 2271 | 2322 | 2372 | 2423 | 51 |
| 856 | 93-2474 | 2524 | 2575 | 2626 | 2677 | 2727 | 2778 | 2829 | 2879 | 293 | 51 |
| 857 | 93-2981 | 3031 | 3082 | 3133 | 3183 | 3234 | 3285 | 3335 | 3386 | 3437 | 51 |
| 858 | 93-3487 | 3538 | 3589 | 3639 | 369 | 374 | 3791 | 3841 | 3892 | 3943 | 51 |
| 859 | 93-3993 | 4044 | 4094 | 4145 | 4195 | 4246 | 4296 | 4347 | 4397 | 4448 | 51 |
| 860 | 93-4498 | 4549 | 4599 | 465 | 47 | 4751 | 4801 | 4852 | 4902 | 4953 | 50 |
| 861 | 93-5003 | 5054 | 5104 | 5154 | 5205 | 5255 | 5306 | 5356 | 5406 | 5457 | 50 |
| 862 | 93-5507 | 5558 | 5608 | 5658 | 5709 | 5759 | 5809 | 586 | 591 | 596 | 50 |
| 863 | 93-6011 | 6061 | 6111 | 6162 | 6212 | 6262 | 6313 | 6363 | 6413 | 6463 | 50 |
| 864 | 93-6514 | 6564 | 6614 | 6665 | 6715 | 6765 | 6815 | 6865 | 6916 | 6966 | 50 |
| 865 | 93-7016 | 7066 | 7117 | 7167 | 7217 | 7267 | 7317 | 7367 | 7418 | 7468 | 50 |
| 866 | 93-7518 | 7568 | 7618 | 7668 | 7718 | 7769 | 7819 | 7869 | 7919 | 7969 | 50 |
| 867 | 93-8019 | 8069 | 8119 | 8169 | 8219 | 8269 | 8319 | 837 | 842 | 847 | 50 |
| 868 | 93-852 | 857 | 862 | 867 | 872 | 877 | 882 | 887 | 892 | 897 | 50 |
| 869 | 93-902 | 907 | 912 | 917 | 922 | 927 | 932 | 9369 | 9419 | 9469 | 50 |
| 870 | 93-9519 | 9569 | 9619 | 9669 | 9719 | 9769 | 9819 | 9869 | 9918 | 9968 | 50 |
| 871 | 94-0018 | 0068 | 0118 | 0168 | 0218 | 0267 | 0317 | 0367 | 0417 | 0467 | 50 |
| 872 | 94-0516 | 0566 | 0616 | 0666 | 0716 | 0765 | 0815 | 0865 | 0915 | 0964 | 50 |
| 873 | 94-1014 | 1064 | 1114 | 1163 | 1213 | 1263 | 1313 | 1362 | 1412 | 1462 | 50 |
| 874 | 94-1511 | 1561 | 1611 | 166 | 171 | 176 | 1809 | 1859 | 1909 | 1958 | 50 |
| 875 | 94-2008 | 2058 | 2107 | 2157 | 2207 | 2256 | 2306 | 2355 | 2405 | 2455 | 50 |
| 876 | 94-2504 | 2554 | 2603 | 2653 | 2702 | 2752 | 2801 | 2851 | 2901 | 295 | 50 |
| 877 | 94-3 | 3049 | 3099 | 3148 | 3198 | 3247 | 3297 | 3346 | 3396 | 3445 | 50 |
| 878 | 94-3495 | 3544 | 3593 | 3643 | 3692 | 3742 | 3791 | 3841 | 389 | 3939 | 50 |
| 879 | 94-3989 | 4038 | 4088 | 4137 | 4186 | 4236 | 4285 | 4335 | 4384 | 4433 | 50 |
| 880 | 94-4483 | 4532 | 4581 | 4631 | 468 | 4729 | 4779 | 4828 | 4877 | 4927 | 50 |
| 881 | 94-4976 | 5025 | 5074 | 5124 | 5173 | 5222 | 5272 | 5321 | 537 | 5419 | 50 |
| 882 | 94-5469 | 5518 | 5567 | 5616 | 5665 | 5715 | 5764 | 5813 | 5862 | 5912 | 50 |
| 883 | 94-5961 | 601 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | 50 |
| 884 | 94-6452 | 6501 | 6551 | 66 | 6649 | 6698 | 6747 | 6796 | 6845 | 6894 | 50 |
| 885 | 94-6943 | 6992 | 7041 | 709 | 714 | 7189 | 7238 | 7287 | 7336 | 7385 | 50 |
| 886 | 94-7434 | 7483 | 7532 | 7581 | 763 | 7679 | 7728 | 7777 | 7826 | 7875 | 50 |
| 887 | 94-7924 | 7973 | 8022 | 807 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | 50 |
| 888 | 94-8413 | 8462 | 8511 | 856 | 8609 | 8657 | 8706 | 8755 | 8804 | 8853 | 50 |
| 889 | 94-8902 | 8951 | 8999 | 9048 | 9097 | 9146 | 9195 | 9244 | 9292 | 9341 | 50 |
| 890 | 94-939 | 9439 | 9488 | 9536 | 9585 | 9634 | 9683 | 9731 | 978 | 9829 | 50 |
| 891 | 94-9878 | 9926 | 9975 | — | — | — | — | — | — | — | 50 |
| 891 | 95-— | — | — | 0024 | 0073 | 0121 | 017 | 0219 | 0267 | 0316 | 50 |
| 892 | 95-0365 | 0414 | 0462 | 0511 | 056 | 0608 | 0657 | 0706 | 0754 | 0803 | 50 |
| 893 | 95-0851 | 09 | 0949 | 0997 | 1046 | 1095 | 1143 | 1192 | 124 | 1289 | 50 |
| 894 | 95-1338 | 1386 | 1435 | 1483 | 1532 | 158 | 1629 | 1677 | 1726 | 1775 | 50 |
| 895 | 95-1823 | 1872 | 192 | 1969 | 2017 | 2066 | 2114 | 2163 | 2211 | 226 | 50 |
| 896 | 95-2308 | 2356 | 2405 | 2453 | 2502 | 255 | 2599 | 2647 | 2696 | 2744 | 50 |
| 897 | 95-2792 | 2841 | 2889 | 2938 | 2986 | 3034 | 3083 | 3131 | 318 | 3228 | 50 |
| 898 | 95-3276 | 3325 | 3373 | 3421 | 347 | 3518 | 3566 | 3615 | 3663 | 3711 | 50 |
| 899 | 95-376 | 3808 | 3856 | 3905 | 3953 | 4001 | 4049 | 4098 | 4146 | 4194 | 50 |
| 900 | 95-4243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 458 | 4628 | 4677 | 50 |
| 901 | 95-4725 | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 511 | 5158 | 50 |
| 902 | 95-5207 | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 564 | 50 |
| 903 | 95-5688 | 5736 | 5784 | 5832 | 588 | 5928 | 5976 | 6024 | 6072 | 612 | 50 |
| 904 | 95-6168 | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 | 50 |
| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D | |
|---|-----|------|------|------|------|------|------|------|------|------|------|----|
| 5 | 90- | 5796 | 585 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 | 54 |
| 6 | 90- | 6335 | 6389 | 6443 | 6497 | 6551 | 6604 | 6658 | 6712 | 6766 | 6820 | 54 |
| 7 | 90- | 6874 | 6927 | 6981 | 7035 | 7089 | 7143 | 7196 | 725 | 7304 | 7358 | 54 |
| 8 | 90- | 7411 | 7465 | 7519 | 7573 | 7626 | 768 | 7734 | 7787 | 7841 | 7895 | 54 |
| 9 | 90- | 7949 | 8002 | 8056 | 811 | 8163 | 8217 | 827 | 8324 | 8378 | 8431 | 54 |
| 0 | 90- | 8485 | 8539 | 8592 | 8646 | 8699 | 8753 | 8807 | 886 | 8914 | 8967 | 54 |
| 1 | 90- | 9021 | 9074 | 9128 | 9181 | 9235 | 9289 | 9342 | 9396 | 9449 | 9503 | 54 |
| 2 | 90- | 9556 | 9609 | 9663 | 9716 | 977 | 9823 | 9877 | 993 | 9984 | — | 54 |
| 3 | 91- | — | — | — | — | — | — | — | — | — | 0037 | 53 |
| 4 | 91- | 0091 | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 | 53 |
| 5 | 91- | 0624 | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 | 53 |
| 6 | 91- | 1158 | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 153 | 1584 | 1637 | 53 |
| 7 | 91- | 169 | 1743 | 1797 | 185 | 1903 | 1956 | 2009 | 2063 | 2116 | 2169 | 53 |
| 8 | 91- | 2222 | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2647 | 27 | 53 |
| 9 | 91- | 2753 | 2806 | 2859 | 2913 | 2966 | 3019 | 3072 | 3125 | 3178 | 3231 | 53 |
| 0 | 91- | 3284 | 3337 | 339 | 3443 | 3496 | 3549 | 3602 | 3655 | 3708 | 3761 | 53 |
| 1 | 91- | 3814 | 3867 | 392 | 3973 | 4026 | 4079 | 4132 | 4184 | 4237 | 429 | 53 |
| 2 | 91- | 4343 | 4396 | 4449 | 4502 | 4555 | 4608 | 466 | 4713 | 4766 | 4819 | 53 |
| 3 | 91- | 4872 | 4925 | 4977 | 503 | 5083 | 5136 | 5189 | 5241 | 5294 | 5347 | 53 |
| 4 | 91- | 54 | 5453 | 5505 | 5558 | 5611 | 5664 | 5716 | 5769 | 5822 | 5875 | 53 |
| 5 | 91- | 5927 | 598 | 6033 | 6085 | 6138 | 6191 | 6243 | 6296 | 6349 | 6401 | 53 |
| 6 | 91- | 6454 | 6507 | 6559 | 6612 | 6664 | 6717 | 677 | 6822 | 6875 | 6927 | 53 |
| 7 | 91- | 698 | 7033 | 7085 | 7138 | 719 | 7243 | 7295 | 7348 | 74 | 7453 | 53 |
| 8 | 91- | 7506 | 7558 | 7611 | 7663 | 7716 | 7768 | 782 | 7873 | 7925 | 7978 | 52 |
| 9 | 91- | 803 | 8083 | 8135 | 8188 | 824 | 8293 | 8345 | 8397 | 845 | 8502 | 52 |
| 0 | 91- | 8555 | 8607 | 8659 | 8712 | 8764 | 8816 | 8869 | 8921 | 8973 | 9026 | 52 |
| 1 | 91- | 9078 | 913 | 9183 | 9235 | 9287 | 934 | 9392 | 9444 | 9496 | 9549 | 52 |
| 2 | 91- | 9601 | 9653 | 9706 | 9758 | 981 | 9862 | 9914 | 9967 | — | — | 52 |
| 3 | 92- | — | — | — | — | — | — | — | — | 0019 | 0071 | 52 |
| 4 | 92- | 0123 | 0176 | 0228 | 028 | 0332 | 0384 | 0436 | 0489 | 0541 | 0593 | 52 |
| 5 | 92- | 0645 | 0697 | 0749 | 0801 | 0853 | 0906 | 0958 | 101 | 1062 | 1114 | 52 |
| 6 | 92- | 1166 | 1218 | 127 | 1322 | 1374 | 1426 | 1478 | 153 | 1582 | 1634 | 52 |
| 7 | 92- | 1686 | 1738 | 179 | 1842 | 1894 | 1946 | 1998 | 205 | 2102 | 2154 | 52 |
| 8 | 92- | 2206 | 2258 | 231 | 2362 | 2414 | 2466 | 2518 | 257 | 2622 | 2674 | 52 |
| 9 | 92- | 2725 | 2777 | 2829 | 2881 | 2933 | 2985 | 3037 | 3089 | 314 | 3192 | 52 |
| 0 | 92- | 3244 | 3296 | 3348 | 3399 | 3451 | 3503 | 3555 | 3607 | 3658 | 371 | 52 |
| 1 | 92- | 3762 | 3814 | 3865 | 3917 | 3969 | 4021 | 4072 | 4124 | 4176 | 4228 | 52 |
| 2 | 92- | 4279 | 4331 | 4383 | 4434 | 4486 | 4538 | 4589 | 4641 | 4693 | 4744 | 52 |
| 3 | 92- | 4796 | 4848 | 4899 | 4951 | 5003 | 5054 | 5106 | 5157 | 5209 | 5261 | 52 |
| 4 | 92- | 5312 | 5364 | 5415 | 5467 | 5518 | 557 | 5621 | 5673 | 5725 | 5776 | 52 |
| 5 | 92- | 5828 | 5879 | 5931 | 5982 | 6034 | 6085 | 6137 | 6188 | 624 | 6291 | 51 |
| 6 | 92- | 6342 | 6394 | 6445 | 6497 | 6548 | 66 | 6651 | 6702 | 6754 | 6805 | 51 |
| 7 | 92- | 6857 | 6908 | 6959 | 7011 | 7062 | 7114 | 7165 | 7216 | 7268 | 7319 | 51 |
| 8 | 92- | 737 | 7422 | 7473 | 7524 | 7576 | 7627 | 7678 | 773 | 7781 | 7832 | 51 |
| 9 | 92- | 7883 | 7935 | 7986 | 8037 | 8088 | 814 | 8191 | 8242 | 8293 | 8345 | 51 |
| 0 | 92- | 8396 | 8447 | 8498 | 8549 | 8601 | 8652 | 8703 | 8754 | 8805 | 8857 | 51 |
| 1 | 92- | 8908 | 8959 | 901 | 9061 | 9112 | 9163 | 9215 | 9266 | 9317 | 9368 | 51 |
| 2 | 92- | 9419 | 947 | 9521 | 9572 | 9623 | 9674 | 9725 | 9776 | 9827 | 9879 | 51 |
| 3 | 92- | 993 | 9981 | — | — | — | — | — | — | — | — | 51 |
| 4 | 93- | — | — | 0032 | 0083 | 0134 | 0185 | 0236 | 0287 | 0338 | 0389 | 51 |
| 5 | 93- | 044 | 0491 | 0542 | 0592 | 0643 | 0694 | 0745 | 0796 | 0847 | 0898 | 51 |
| 6 | 93- | 0949 | 1 | 1051 | 1102 | 1153 | 1203 | 1254 | 1305 | 1356 | 1407 | 51 |
| 7 | 93- | 1458 | 1509 | 156 | 161 | 1661 | 1712 | 1763 | 1814 | 1865 | 1915 | 51 |
| 8 | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | | |

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D |
|-----|---------|------|------|------|------|------|------|------|------|------|----|
| 955 | 98-0003 | 0049 | 0094 | 014 | 0185 | 0231 | 0276 | 0322 | 0367 | 0412 | 45 |
| 956 | 98-0458 | 0503 | 0549 | 0594 | 064 | 0685 | 073 | 0776 | 0821 | 0867 | 45 |
| 957 | 98-0912 | 0957 | 1003 | 1048 | 1093 | 1139 | 1184 | 1229 | 1275 | 132 | 45 |
| 958 | 98-1366 | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 | 45 |
| 959 | 98-1819 | 1864 | 1909 | 1954 | 2 | 2045 | 209 | 2135 | 2181 | 2226 | 45 |
| 960 | 98-2271 | 2316 | 2362 | 2407 | 2452 | 2497 | 2543 | 2588 | 2633 | 2678 | 45 |
| 961 | 98-2723 | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 304 | 3085 | 313 | 45 |
| 962 | 98-3175 | 322 | 3265 | 331 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 | 45 |
| 963 | 98-3626 | 3671 | 3716 | 3762 | 3807 | 3852 | 3897 | 3942 | 3987 | 4032 | 45 |
| 964 | 98-4077 | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 | 45 |
| 965 | 98-4527 | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 | 45 |
| 966 | 98-4977 | 5022 | 5067 | 5112 | 5157 | 5202 | 5247 | 5292 | 5337 | 5382 | 45 |
| 967 | 98-5426 | 5471 | 5516 | 5561 | 5606 | 5651 | 5696 | 5741 | 5786 | 583 | 45 |
| 968 | 98-5875 | 592 | 5965 | 601 | 6055 | 61 | 6144 | 6189 | 6234 | 6279 | 45 |
| 969 | 98-6324 | 6369 | 6413 | 6458 | 6503 | 6548 | 6593 | 6637 | 6682 | 6727 | 45 |
| 970 | 98-6772 | 6817 | 6861 | 6906 | 6951 | 6996 | 704 | 7085 | 713 | 7175 | 45 |
| 971 | 98-7219 | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 | 45 |
| 972 | 98-7666 | 7711 | 7756 | 78 | 7845 | 789 | 7934 | 7979 | 8024 | 8068 | 45 |
| 973 | 98-8113 | 8157 | 8202 | 8247 | 8291 | 8336 | 8381 | 8425 | 847 | 8514 | 45 |
| 974 | 98-8559 | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 896 | 45 |
| 975 | 98-9005 | 9049 | 9094 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 | 45 |
| 976 | 98-945 | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 985 | 44 |
| 977 | 98-9895 | 9939 | 9983 | — | — | — | — | — | — | — | 44 |
| 977 | 99-— | — | — | 0028 | 0072 | 0117 | 0161 | 0206 | 025 | 0294 | 44 |
| 978 | 99-0339 | 0383 | 0428 | 0472 | 0516 | 0561 | 0605 | 065 | 0694 | 0738 | 44 |
| 979 | 99-0783 | 0827 | 0871 | 0916 | 096 | 1004 | 1049 | 1093 | 1137 | 1182 | 44 |
| 980 | 99-1226 | 127 | 1315 | 1359 | 1403 | 1448 | 1492 | 1536 | 158 | 1625 | 44 |
| 981 | 99-1669 | 1713 | 1758 | 1802 | 1846 | 189 | 1935 | 1979 | 2023 | 2067 | 44 |
| 982 | 99-2111 | 2156 | 22 | 2244 | 2288 | 2333 | 2377 | 2421 | 2465 | 2509 | 44 |
| 983 | 99-2554 | 2598 | 2642 | 2686 | 273 | 2774 | 2819 | 2863 | 2907 | 2951 | 44 |
| 984 | 99-2995 | 3039 | 3083 | 3127 | 3172 | 3216 | 326 | 3304 | 3348 | 3392 | 44 |
| 985 | 99-3436 | 348 | 3524 | 3568 | 3613 | 3657 | 3701 | 3745 | 3789 | 3833 | 44 |
| 986 | 99-3877 | 3921 | 3965 | 4009 | 4053 | 4097 | 4141 | 4185 | 4229 | 4273 | 44 |
| 987 | 99-4317 | 4361 | 4405 | 4449 | 4493 | 4537 | 4581 | 4625 | 4669 | 4713 | 44 |
| 988 | 99-4757 | 4801 | 4845 | 4889 | 4933 | 4977 | 5021 | 5065 | 5108 | 5152 | 44 |
| 989 | 99-5196 | 524 | 5284 | 5328 | 5372 | 5416 | 546 | 5504 | 5547 | 5591 | 44 |
| 990 | 99-5635 | 5679 | 5723 | 5767 | 5811 | 5854 | 5898 | 5942 | 5986 | 603 | 44 |
| 991 | 99-6074 | 6117 | 6161 | 6205 | 6249 | 6293 | 6337 | 638 | 6424 | 6468 | 44 |
| 992 | 99-6512 | 6555 | 6599 | 6643 | 6687 | 6731 | 6774 | 6818 | 6862 | 6906 | 44 |
| 993 | 99-6949 | 6993 | 7037 | 708 | 7124 | 7168 | 7212 | 7255 | 7299 | 7343 | 44 |
| 994 | 99-7386 | 743 | 7474 | 7517 | 7561 | 7605 | 7648 | 7692 | 7736 | 7779 | 44 |
| 995 | 99-7823 | 7867 | 791 | 7954 | 7998 | 8041 | 8085 | 8129 | 8172 | 8216 | 44 |
| 996 | 99-8259 | 8303 | 8347 | 839 | 8434 | 8477 | 8521 | 8564 | 8608 | 8652 | 44 |
| 997 | 99-8695 | 8739 | 8782 | 8826 | 8869 | 8913 | 8956 | 9 | 9043 | 9087 | 44 |
| 998 | 99-9131 | 9174 | 9218 | 9261 | 9305 | 9348 | 9392 | 9435 | 9479 | 9522 | 44 |
| 999 | 99-9565 | 9609 | 9652 | 9696 | 9739 | 9783 | 9826 | 987 | 9913 | 9957 | 44 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |

Hyperbolic Logarithms of Numbers.

From 1.01 to 30.

following table, the numbers range from 1.01 to 30, advancing by .01, the whole number 10; and thence by larger intervals up to 30. The hyperbolic logarithms of numbers, or Neperian logarithms, as they are sometimes termed, are computed by multiplying the common logarithms of numbers by the constant multiplier, 2.302 585.

The hyperbolic logarithms of numbers intermediate between those which are given in the table may be readily obtained by interpolating proportional parts.

| | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|----|-------|------|-------|------|-------|------|-------|------|--------|
| | .0099 | 1.41 | .3436 | 1.81 | .5933 | 2.21 | .793 | 2.61 | .9594 |
| 1 | .0198 | 1.42 | .3507 | 1.82 | .5988 | 2.22 | .7975 | 2.62 | .9632 |
| 2 | .0296 | 1.43 | .3577 | 1.83 | .6043 | 2.23 | .802 | 2.63 | .967 |
| 3 | .0392 | 1.44 | .3646 | 1.84 | .6098 | 2.24 | .8065 | 2.64 | .9708 |
| 4 | .0488 | 1.45 | .3716 | 1.85 | .6152 | 2.25 | .8109 | 2.65 | .9746 |
| 5 | .0583 | 1.46 | .3784 | 1.86 | .6206 | 2.26 | .8154 | 2.66 | .9783 |
| 6 | .0677 | 1.47 | .3853 | 1.87 | .6259 | 2.27 | .8198 | 2.67 | .9821 |
| 7 | .077 | 1.48 | .392 | 1.88 | .6313 | 2.28 | .8242 | 2.68 | .9858 |
| 8 | .0862 | 1.49 | .3988 | 1.89 | .6366 | 2.29 | .8286 | 2.69 | .9895 |
| 9 | .0953 | 1.5 | .4055 | 1.9 | .6419 | 2.3 | .8329 | 2.7 | .9933 |
| 10 | .1044 | 1.51 | .4121 | 1.91 | .6471 | 2.31 | .8372 | 2.71 | .9969 |
| 11 | .1133 | 1.52 | .4187 | 1.92 | .6523 | 2.32 | .8416 | 2.72 | 1.0006 |
| 12 | .1222 | 1.53 | .4253 | 1.93 | .6575 | 2.33 | .8458 | 2.73 | 1.0043 |
| 13 | .131 | 1.54 | .4318 | 1.94 | .6627 | 2.34 | .8502 | 2.74 | 1.008 |
| 14 | .1398 | 1.55 | .4383 | 1.95 | .6678 | 2.35 | .8544 | 2.75 | 1.0116 |
| 15 | .1484 | 1.56 | .4447 | 1.96 | .6729 | 2.36 | .8587 | 2.76 | 1.0152 |
| 16 | .157 | 1.57 | .4511 | 1.97 | .678 | 2.37 | .8629 | 2.77 | 1.0188 |
| 17 | .1655 | 1.58 | .4574 | 1.98 | .6831 | 2.38 | .8671 | 2.78 | 1.0225 |
| 18 | .174 | 1.59 | .4637 | 1.99 | .6881 | 2.39 | .8713 | 2.79 | 1.026 |
| 19 | .1823 | 1.6 | .47 | 2 | .6931 | 2.4 | .8755 | 2.8 | 1.0296 |
| 20 | .1906 | 1.61 | .4762 | 2.01 | .6981 | 2.41 | .8796 | 2.81 | 1.0332 |
| 21 | .1988 | 1.62 | .4824 | 2.02 | .7031 | 2.42 | .8838 | 2.82 | 1.0367 |
| 22 | .207 | 1.63 | .4886 | 2.03 | .708 | 2.43 | .8879 | 2.83 | 1.0403 |
| 23 | .2151 | 1.64 | .4947 | 2.04 | .7129 | 2.44 | .892 | 2.84 | 1.0438 |
| 24 | .2231 | 1.65 | .5008 | 2.05 | .7178 | 2.45 | .8961 | 2.85 | 1.0473 |
| 25 | .2311 | 1.66 | .5068 | 2.06 | .7227 | 2.46 | .9002 | 2.86 | 1.0508 |
| 26 | .239 | 1.67 | .5128 | 2.07 | .7275 | 2.47 | .9042 | 2.87 | 1.0543 |
| 27 | .2469 | 1.68 | .5188 | 2.08 | .7324 | 2.48 | .9083 | 2.88 | 1.0578 |
| 28 | .2546 | 1.69 | .5247 | 2.09 | .7372 | 2.49 | .9123 | 2.89 | 1.0613 |
| 29 | .2624 | 1.7 | .5306 | 2.1 | .7419 | 2.5 | .9163 | 2.9 | 1.0647 |
| 30 | .27 | 1.71 | .5365 | 2.11 | .7467 | 2.51 | .9203 | 2.91 | 1.068 |
| 31 | .2776 | 1.72 | .5423 | 2.12 | .7514 | 2.52 | .9243 | 2.92 | 1.07 |
| 32 | .2852 | 1.73 | .5481 | 2.13 | .7561 | 2.53 | .9282 | 2.93 | 1.07 |
| 33 | .2927 | 1.74 | .5539 | 2.14 | .7608 | 2.54 | .9322 | 2.94 | 1.07 |
| 34 | .3001 | 1.75 | .5596 | 2.15 | .7655 | 2.55 | .9361 | 2.95 | 1.07 |
| 35 | .3075 | 1.76 | .5653 | 2.16 | .7701 | 2.56 | | 2.96 | 1.07 |
| 36 | .3148 | 1.77 | .571 | 2.17 | .7747 | 2.57 | | 2.97 | 1.07 |
| 37 | .3221 | 1.78 | .5766 | 2.18 | .7793 | 2.58 | | | |
| 38 | .3293 | 1.79 | .5822 | 2.19 | .7839 | 2.59 | | | |
| 39 | .3365 | 1.8 | .5878 | 2.2 | .7885 | 2.6 | | | |

| | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|---|--------|------|--------|------|--------|------|--------|------|--------|
| 1 | 1.7066 | 6.01 | 1.7934 | 6.51 | 1.8733 | 7.01 | 1.9473 | 7.51 | 2.0162 |
| 2 | 1.7084 | 6.02 | 1.7951 | 6.52 | 1.8749 | 7.02 | 1.9488 | 7.52 | 2.0176 |
| 3 | 1.7102 | 6.03 | 1.7967 | 6.53 | 1.8764 | 7.03 | 1.9502 | 7.53 | 2.0189 |
| 4 | 1.712 | 6.04 | 1.7984 | 6.54 | 1.8779 | 7.04 | 1.9516 | 7.54 | 2.0202 |
| 5 | 1.7138 | 6.05 | 1.8001 | 6.55 | 1.8795 | 7.05 | 1.953 | 7.55 | 2.0215 |
| 6 | 1.7156 | 6.06 | 1.8017 | 6.56 | 1.881 | 7.06 | 1.9544 | 7.56 | 2.0229 |
| 7 | 1.7174 | 6.07 | 1.8034 | 6.57 | 1.8825 | 7.07 | 1.9559 | 7.57 | 2.0242 |
| 8 | 1.7192 | 6.08 | 1.805 | 6.58 | 1.884 | 7.08 | 1.9573 | 7.58 | 2.0255 |
| 9 | 1.721 | 6.09 | 1.8066 | 6.59 | 1.8856 | 7.09 | 1.9587 | 7.59 | 2.0268 |
| | 1.7228 | 6.1 | 1.8083 | 6.6 | 1.8871 | 7.1 | 1.9601 | 7.6 | 2.0281 |
| 1 | 1.7246 | 6.11 | 1.8099 | 6.61 | 1.8886 | 7.11 | 1.9615 | 7.61 | 2.0295 |
| 2 | 1.7263 | 6.12 | 1.8116 | 6.62 | 1.8901 | 7.12 | 1.9629 | 7.62 | 2.0308 |
| 3 | 1.7281 | 6.13 | 1.8132 | 6.63 | 1.8916 | 7.13 | 1.9643 | 7.63 | 2.0321 |
| 4 | 1.7299 | 6.14 | 1.8148 | 6.64 | 1.8931 | 7.14 | 1.9657 | 7.64 | 2.0334 |
| 5 | 1.7317 | 6.15 | 1.8165 | 6.65 | 1.8946 | 7.15 | 1.9671 | 7.65 | 2.0347 |
| 6 | 1.7334 | 6.16 | 1.8181 | 6.66 | 1.8961 | 7.16 | 1.9685 | 7.66 | 2.036 |
| 7 | 1.7352 | 6.17 | 1.8197 | 6.67 | 1.8976 | 7.17 | 1.9699 | 7.67 | 2.0373 |
| 8 | 1.737 | 6.18 | 1.8213 | 6.68 | 1.8991 | 7.18 | 1.9713 | 7.68 | 2.0386 |
| 9 | 1.7387 | 6.19 | 1.8229 | 6.69 | 1.9006 | 7.19 | 1.9727 | 7.69 | 2.0399 |
| | 1.7405 | 6.2 | 1.8245 | 6.7 | 1.9021 | 7.2 | 1.9741 | 7.7 | 2.0412 |
| 1 | 1.7422 | 6.21 | 1.8262 | 6.71 | 1.9036 | 7.21 | 1.9755 | 7.71 | 2.0425 |
| 2 | 1.744 | 6.22 | 1.8278 | 6.72 | 1.9051 | 7.22 | 1.9769 | 7.72 | 2.0438 |
| 3 | 1.7457 | 6.23 | 1.8294 | 6.73 | 1.9066 | 7.23 | 1.9782 | 7.73 | 2.0451 |
| 4 | 1.7475 | 6.24 | 1.831 | 6.74 | 1.9081 | 7.24 | 1.9796 | 7.74 | 2.0464 |
| 5 | 1.7492 | 6.25 | 1.8326 | 6.75 | 1.9095 | 7.25 | 1.981 | 7.75 | 2.0477 |
| 6 | 1.7509 | 6.26 | 1.8342 | 6.76 | 1.911 | 7.26 | 1.9824 | 7.76 | 2.049 |
| 7 | 1.7527 | 6.27 | 1.8358 | 6.77 | 1.9125 | 7.27 | 1.9838 | 7.77 | 2.0503 |
| 8 | 1.7544 | 6.28 | 1.8374 | 6.78 | 1.914 | 7.28 | 1.9851 | 7.78 | 2.0516 |
| 9 | 1.7561 | 6.29 | 1.839 | 6.79 | 1.9155 | 7.29 | 1.9865 | 7.79 | 2.0528 |
| | 1.7579 | 6.3 | 1.8405 | 6.8 | 1.9169 | 7.3 | 1.9879 | 7.8 | 2.0541 |
| 1 | 1.7596 | 6.31 | 1.8421 | 6.81 | 1.9184 | 7.31 | 1.9892 | 7.81 | 2.0554 |
| 2 | 1.7613 | 6.32 | 1.8437 | 6.82 | 1.9199 | 7.32 | 1.9906 | 7.82 | 2.0567 |
| 3 | 1.763 | 6.33 | 1.8453 | 6.83 | 1.9213 | 7.33 | 1.992 | 7.83 | 2.058 |
| 4 | 1.7647 | 6.34 | 1.8469 | 6.84 | 1.9228 | 7.34 | 1.9933 | 7.84 | 2.0592 |
| 5 | 1.7664 | 6.35 | 1.8485 | 6.85 | 1.9242 | 7.35 | 1.9947 | 7.85 | 2.0605 |
| 6 | 1.7681 | 6.36 | 1.85 | 6.86 | 1.9257 | 7.36 | 1.9961 | 7.86 | 2.0618 |
| 7 | 1.7699 | 6.37 | 1.8516 | 6.87 | 1.9272 | 7.37 | 1.9974 | 7.87 | 2.0631 |
| 8 | 1.7716 | 6.38 | 1.8532 | 6.88 | 1.9286 | 7.38 | 1.9988 | 7.88 | 2.0643 |
| 9 | 1.7733 | 6.39 | 1.8547 | 6.89 | 1.9301 | 7.39 | 2.0001 | 7.89 | 2.0656 |
| | 1.775 | 6.4 | 1.8563 | 6.9 | 1.9315 | 7.4 | 2.0015 | 7.9 | 2.0669 |
| 1 | 1.7766 | 6.41 | 1.8579 | 6.91 | 1.933 | 7.41 | 2.0028 | 7.91 | 2.0682 |
| 2 | 1.7783 | 6.42 | 1.8594 | 6.92 | 1.9344 | 7.42 | 2.0042 | 7.92 | 2.0695 |
| 3 | 1.78 | 6.43 | 1.861 | 6.93 | 1.9359 | 7.43 | 2.0055 | 7.93 | 2.0708 |
| 4 | 1.7817 | 6.44 | 1.8625 | 6.94 | 1.9373 | 7.44 | | 7.94 | 2.0721 |
| 5 | 1.7834 | 6.45 | 1.8641 | 6.95 | 1.9387 | 7.45 | | 7.95 | 2.0734 |
| 6 | 1.7851 | 6.46 | 1.8656 | 6.96 | 1.9402 | | | | 2.0747 |
| 7 | 1.7867 | 6.47 | 1.8672 | 6.97 | 1.9416 | | | | 2.0760 |
| 8 | 1.7884 | 6.48 | 1.8687 | 6.98 | 1.943 | | | | 2.0773 |
| 9 | 1.7901 | 6.49 | 1.8703 | 6.99 | 1.9445 | | | | 2.0786 |
| | 1.7918 | 6.5 | 1.8718 | 7 | 1.9459 | | | | 2.0799 |

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | L |
|------|--------|-------|--------|-------|--------|------|--------|------|-----|
| 8.01 | 2.0807 | 8.41 | 2.1294 | 8.81 | 2.1759 | 9.21 | 2.2203 | 9.61 | 2.2 |
| 8.02 | 2.0819 | 8.42 | 2.1306 | 8.82 | 2.177 | 9.22 | 2.2214 | 9.62 | 2.2 |
| 8.03 | 2.0832 | 8.43 | 2.1318 | 8.83 | 2.1782 | 9.23 | 2.2225 | 9.63 | 2.2 |
| 8.04 | 2.0844 | 8.44 | 2.133 | 8.84 | 2.1793 | 9.24 | 2.2235 | 9.64 | 2.2 |
| 8.05 | 2.0857 | 8.45 | 2.1342 | 8.85 | 2.1804 | 9.25 | 2.2246 | 9.65 | 2.2 |
| 8.06 | 2.0869 | 8.46 | 2.1353 | 8.86 | 2.1815 | 9.26 | 2.2257 | 9.66 | 2.2 |
| 8.07 | 2.0882 | 8.47 | 2.1365 | 8.87 | 2.1827 | 9.27 | 2.2268 | 9.67 | 2.2 |
| 8.08 | 2.0894 | 8.48 | 2.1377 | 8.88 | 2.1838 | 9.28 | 2.2279 | 9.68 | 2.2 |
| 8.09 | 2.0906 | 8.49 | 2.1389 | 8.89 | 2.1849 | 9.29 | 2.2289 | 9.69 | 2.2 |
| 8.1 | 2.0919 | 8.5 | 2.1401 | 8.9 | 2.1861 | 9.3 | 2.23 | 9.7 | 2.2 |
| 8.11 | 2.0931 | 8.51 | 2.1412 | 8.91 | 2.1872 | 9.31 | 2.2311 | 9.71 | 2.2 |
| 8.12 | 2.0943 | 8.52 | 2.1424 | 8.92 | 2.1883 | 9.32 | 2.2322 | 9.72 | 2.2 |
| 8.13 | 2.0956 | 8.53 | 2.1436 | 8.93 | 2.1894 | 9.33 | 2.2332 | 9.73 | 2.2 |
| 8.14 | 2.0968 | 8.54 | 2.1448 | 8.94 | 2.1905 | 9.34 | 2.2343 | 9.74 | 2.2 |
| 8.15 | 2.098 | 8.55 | 2.1459 | 8.95 | 2.1917 | 9.35 | 2.2354 | 9.75 | 2.2 |
| 8.16 | 2.0992 | 8.56 | 2.1471 | 8.96 | 2.1928 | 9.36 | 2.2364 | 9.76 | 2.2 |
| 8.17 | 2.1005 | 8.57 | 2.1483 | 8.97 | 2.1939 | 9.37 | 2.2375 | 9.77 | 2.2 |
| 8.18 | 2.1017 | 8.58 | 2.1494 | 8.98 | 2.195 | 9.38 | 2.2386 | 9.78 | 2.2 |
| 8.19 | 2.1029 | 8.59 | 2.1506 | 8.99 | 2.1961 | 9.39 | 2.2396 | 9.79 | 2.2 |
| 8.2 | 2.1041 | 8.6 | 2.1518 | 9 | 2.1972 | 9.4 | 2.2407 | 9.8 | 2.2 |
| 8.21 | 2.1054 | 8.61 | 2.1529 | 9.01 | 2.1983 | 9.41 | 2.2418 | 9.81 | 2.2 |
| 8.22 | 2.1066 | 8.62 | 2.1541 | 9.02 | 2.1994 | 9.42 | 2.2428 | 9.82 | 2.2 |
| 8.23 | 2.1078 | 8.63 | 2.1552 | 9.03 | 2.2006 | 9.43 | 2.2439 | 9.83 | 2.2 |
| 8.24 | 2.109 | 8.64 | 2.1564 | 9.04 | 2.2017 | 9.44 | 2.245 | 9.84 | 2.2 |
| 8.25 | 2.1102 | 8.65 | 2.1576 | 9.05 | 2.2028 | 9.45 | 2.246 | 9.85 | 2.2 |
| 8.26 | 2.1114 | 8.66 | 2.1587 | 9.06 | 2.2039 | 9.46 | 2.2471 | 9.86 | 2.2 |
| 8.27 | 2.1126 | 8.67 | 2.1599 | 9.07 | 2.205 | 9.47 | 2.2481 | 9.87 | 2.2 |
| 8.28 | 2.1138 | 8.68 | 2.161 | 9.08 | 2.2061 | 9.48 | 2.2492 | 9.88 | 2.2 |
| 8.29 | 2.115 | 8.69 | 2.1622 | 9.09 | 2.2072 | 9.49 | 2.2502 | 9.89 | 2.2 |
| 8.3 | 2.1163 | 8.7 | 2.1633 | 9.1 | 2.2083 | 9.5 | 2.2513 | 9.9 | 2.2 |
| 8.31 | 2.1175 | 8.71 | 2.1645 | 9.11 | 2.2094 | 9.51 | 2.2523 | 9.91 | 2.2 |
| 8.32 | 2.1187 | 8.72 | 2.1656 | 9.12 | 2.2105 | 9.52 | 2.2534 | 9.92 | 2.2 |
| 8.33 | 2.1199 | 8.73 | 2.1668 | 9.13 | 2.2116 | 9.53 | 2.2544 | 9.93 | 2.2 |
| 8.34 | 2.1211 | 8.74 | 2.1679 | 9.14 | 2.2127 | 9.54 | 2.2555 | 9.94 | 2.2 |
| 8.35 | 2.1223 | 8.75 | 2.1691 | 9.15 | 2.2138 | 9.55 | 2.2565 | 9.95 | 2.2 |
| 8.36 | 2.1235 | 8.76 | 2.1702 | 9.16 | 2.2148 | 9.56 | 2.2576 | 9.96 | 2.2 |
| 8.37 | 2.1247 | 8.77 | 2.1713 | 9.17 | 2.2159 | 9.57 | 2.2586 | 9.97 | 2.2 |
| 8.38 | 2.1259 | 8.78 | 2.1725 | 9.18 | 2.217 | 9.58 | 2.2597 | 9.98 | 2.2 |
| 8.39 | 2.1271 | 8.79 | 2.1736 | 9.19 | 2.2181 | 9.59 | 2.2607 | 9.99 | 2.2 |
| 8.4 | 2.1283 | 8.8 | 2.1748 | 9.2 | 2.2192 | 9.6 | 2.2618 | 10 | 2.2 |
| 8.41 | 2.1295 | 12.25 | 2.5052 | 14.25 | 2.6567 | 17.5 | 2.8621 | 23 | 3.2 |
| 8.42 | 2.1307 | 12.5 | 2.5262 | 14.5 | 2.674 | 18 | 2.8904 | 24 | 3.2 |
| 8.43 | 2.1319 | 12.75 | 2.5473 | 14.75 | 2.6913 | 18.5 | 2.9173 | 25 | 3.2 |
| 8.44 | 2.1331 | 13 | 2.5684 | 15 | 2.7081 | 19 | 2.9444 | 26 | 3.2 |
| 8.45 | 2.1343 | 13.25 | 2.5895 | 15.25 | 2.7248 | 19.5 | 2.9703 | 27 | 3.2 |
| 8.46 | 2.1355 | 13.5 | 2.6106 | 15.5 | 2.7416 | 20 | 2.9957 | 28 | 3.2 |
| 8.47 | 2.1367 | 13.75 | 2.6317 | 15.75 | 2.7584 | 20.5 | 3.0211 | 29 | 3.2 |
| 8.48 | 2.1379 | 14 | 2.6528 | 16 | 2.7752 | 21 | 3.0465 | 30 | 3.2 |
| 8.49 | 2.1391 | 14.25 | 2.6739 | 16.25 | 2.792 | 21.5 | 3.0719 | 31 | 3.2 |
| 8.5 | 2.1403 | 14.5 | 2.695 | 16.5 | 2.8088 | 22 | 3.0973 | 32 | 3.2 |
| 8.51 | 2.1415 | 14.75 | 2.7161 | 16.75 | 2.8256 | 22.5 | 3.1227 | 33 | 3.2 |
| 8.52 | 2.1427 | 15 | 2.7372 | 17 | 2.8424 | 23 | 3.1481 | 34 | 3.2 |
| 8.53 | 2.1439 | 15.25 | 2.7583 | 17.25 | 2.8592 | 23.5 | 3.1735 | 35 | 3.2 |
| 8.54 | 2.1451 | 15.5 | 2.7794 | 17.5 | 2.876 | 24 | 3.1989 | 36 | 3.2 |
| 8.55 | 2.1463 | 15.75 | 2.8005 | 17.75 | 2.8928 | 24.5 | 3.2243 | 37 | 3.2 |
| 8.56 | 2.1475 | 16 | 2.8216 | 18 | 2.9096 | 25 | 3.2497 | 38 | 3.2 |
| 8.57 | 2.1487 | 16.25 | 2.8427 | 18.25 | 2.9264 | 25.5 | 3.2751 | 39 | 3.2 |
| 8.58 | 2.1499 | 16.5 | 2.8638 | 18.5 | 2.9432 | 26 | 3.3005 | 40 | 3.2 |
| 8.59 | 2.1511 | 16.75 | 2.8849 | 18.75 | 2.96 | 26.5 | 3.3259 | 41 | 3.2 |
| 8.6 | 2.1523 | 17 | 2.906 | 19 | 2.9768 | 27 | 3.3513 | 42 | 3.2 |
| 8.61 | 2.1535 | 17.25 | 2.9271 | 19.25 | 2.9936 | 27.5 | 3.3767 | 43 | 3.2 |
| 8.62 | 2.1547 | 17.5 | 2.9482 | 19.5 | 3.0104 | 28 | 3.4021 | 44 | 3.2 |
| 8.63 | 2.1559 | 17.75 | 2.9693 | 19.75 | 3.0272 | 28.5 | 3.4275 | 45 | 3.2 |
| 8.64 | 2.1571 | 18 | 2.9904 | 20 | 3.044 | 29 | 3.4529 | 46 | 3.2 |
| 8.65 | 2.1583 | 18.25 | 3.0115 | 20.25 | 3.0608 | 29.5 | 3.4783 | 47 | 3.2 |
| 8.66 | 2.1595 | 18.5 | 3.0326 | 20.5 | 3.0776 | 30 | 3.5037 | 48 | 3.2 |
| 8.67 | 2.1607 | 18.75 | 3.0537 | 20.75 | 3.0944 | 30.5 | 3.5291 | 49 | 3.2 |
| 8.68 | 2.1619 | 19 | 3.0748 | 21 | 3.1112 | 31 | 3.5545 | 50 | 3.2 |
| 8.69 | 2.1631 | 19.25 | 3.0959 | 21.25 | 3.128 | 31.5 | 3.5799 | 51 | 3.2 |
| 8.7 | 2.1643 | 19.5 | 3.117 | 21.5 | 3.1448 | 32 | 3.6053 | 52 | 3.2 |
| 8.71 | 2.1655 | 19.75 | 3.1381 | 21.75 | 3.1616 | 32.5 | 3.6307 | 53 | 3.2 |
| 8.72 | 2.1667 | 20 | 3.1592 | 22 | 3.1784 | 33 | 3.6561 | 54 | 3.2 |
| 8.73 | 2.1679 | 20.25 | 3.1803 | 22.25 | 3.1952 | 33.5 | 3.6815 | 55 | 3.2 |
| 8.74 | 2.1691 | 20.5 | 3.2014 | 22.5 | 3.212 | 34 | 3.7069 | 56 | 3.2 |
| 8.75 | 2.1703 | 20.75 | 3.2225 | 22.75 | 3.2288 | 34.5 | 3.7323 | 57 | 3.2 |
| 8.76 | 2.1715 | 21 | 3.2436 | 23 | 3.2456 | 35 | 3.7577 | 58 | 3.2 |
| 8.77 | 2.1727 | 21.25 | 3.2647 | 23.25 | 3.2624 | 35.5 | 3.7831 | 59 | 3.2 |
| 8.78 | 2.1739 | 21.5 | 3.2858 | 23.5 | 3.2792 | 36 | 3.8085 | 60 | 3.2 |
| 8.79 | 2.1751 | 21.75 | 3.3069 | 23.75 | 3.296 | 36.5 | 3.8339 | 61 | 3.2 |
| 8.8 | 2.1763 | 22 | 3.328 | 24 | 3.3128 | 37 | 3.8593 | 62 | 3.2 |
| 8.81 | 2.1775 | 22.25 | 3.3491 | 24.25 | 3.3296 | 37.5 | 3.8847 | 63 | 3.2 |
| 8.82 | 2.1787 | 22.5 | 3.3702 | 24.5 | 3.3464 | 38 | 3.9101 | 64 | 3.2 |
| 8.83 | 2.1799 | 22.75 | 3.3913 | 24.75 | 3.3632 | 38.5 | 3.9355 | 65 | 3.2 |
| 8.84 | 2.1811 | 23 | 3.4124 | 25 | 3.38 | 39 | 3.9609 | 66 | 3.2 |
| 8.85 | 2.1823 | 23.25 | 3.4335 | 25.25 | 3.3968 | 39.5 | 3.9863 | 67 | 3.2 |
| 8.86 | 2.1835 | 23.5 | 3.4546 | 25.5 | 3.4136 | 40 | 4.0117 | 68 | 3.2 |
| 8.87 | 2.1847 | 23.75 | 3.4757 | 25.75 | 3.4304 | 40.5 | 4.0371 | 69 | 3.2 |
| 8.88 | 2.1859 | 24 | 3.4968 | 26 | 3.4472 | 41 | 4.0625 | 70 | 3.2 |
| 8.89 | 2.1871 | 24.25 | 3.5179 | 26.25 | 3.464 | 41.5 | 4.0879 | 71 | 3.2 |
| 8.9 | 2.1883 | 24.5 | 3.539 | 26.5 | 3.4808 | 42 | 4.1133 | 72 | 3.2 |
| 8.91 | 2.1895 | 24.75 | 3.5601 | 26.75 | 3.4976 | 42.5 | 4.1387 | 73 | 3.2 |
| 8.92 | 2.1907 | 25 | 3.5812 | 27 | 3.5144 | 43 | 4.1641 | 74 | 3.2 |
| 8.93 | 2.1919 | 25.25 | 3.6023 | 27.25 | 3.5312 | 43.5 | 4.1895 | 75 | 3.2 |
| 8.94 | 2.1931 | 25.5 | 3.6234 | 27.5 | 3.548 | 44 | 4.2149 | 76 | 3.2 |
| 8.95 | 2.1943 | 25.75 | 3.6445 | 27.75 | 3.5648 | 44.5 | 4.2403 | 77 | 3.2 |
| 8.96 | 2.1955 | 26 | 3.6656 | 28 | 3.5816 | 45 | 4.2657 | 78 | 3.2 |
| 8.97 | 2.1967 | 26.25 | 3.6867 | 28.25 | 3.5984 | 45.5 | 4.2911 | 79 | 3.2 |
| 8.98 | 2.1979 | 26.5 | 3.7078 | 28.5 | 3.6152 | 46 | 4.3165 | 80 | 3.2 |
| 8.99 | 2.1991 | 26.75 | 3.7289 | 28.75 | 3.632 | 46.5 | 4.3419 | 81 | 3.2 |
| 9 | 2.2003 | 27 | 3.75 | 29 | 3.6488 | 47 | 4.3673 | 82 | 3.2 |
| 9.01 | 2.2015 | 27.25 | 3.7711 | 29.25 | 3.6656 | 47.5 | 4.3927 | 83 | 3.2 |
| 9.02 | 2.2027 | 27.5 | 3.7922 | 29.5 | 3.6824 | 48 | 4.4181 | 84 | 3.2 |
| 9.03 | 2.2039 | 27.75 | 3.8133 | 29.75 | 3.6992 | 48.5 | 4.4435 | 85 | 3.2 |
| 9.04 | 2.2051 | 28 | 3.8344 | 30 | 3.716 | 49 | 4.4689 | 86 | 3.2 |
| 9.05 | 2.2063 | 28.25 | 3.8555 | 30.25 | 3.7328 | 49.5 | 4.4943 | 87 | 3.2 |
| 9.06 | 2.2075 | 28.5 | 3.8766 | 30.5 | 3.7496 | 50 | 4.5197 | 88 | 3.2 |
| 9.07 | 2.2087 | 28.75 | 3.8977 | 30.75 | 3.7664 | 50.5 | 4.5451 | 89 | 3.2 |
| 9.08 | 2.2099 | 29 | 3.9188 | 31 | 3.7832 | 51 | 4.5705 | 90 | 3.2 |
| 9.09 | 2.2111 | 29.25 | 3.9399 | 31.25 | 3.7999 | 51.5 | 4.5959 | 91 | 3.2 |
| 9.1 | 2.2123 | 29.5 | 3.961 | 31.5 | 3.8167 | 52 | 4.6213 | 92 | 3.2 |
| 9.11 | 2.2135 | 29.75 | 3.9821 | 31.75 | 3.8335 | 52.5 | 4.6467 | 93 | 3.2 |
| 9.12 | 2.2147 | 30 | 4.0032 | 32 | 3.8503 | 53 | 4.6721 | 94 | 3.2 |
| 9.13 | 2.2159 | 30.25 | 4.0243 | 32.25 | 3.8671 | 53.5 | 4.6975 | 95 | 3.2 |
| 9.14 | 2.2171 | 30.5 | 4.0454 | 32.5 | 3.8839 | 54 | 4.7229 | 96 | 3.2 |
| 9.15 | 2.2183 | 30.75 | 4.0665 | 32.75 | 3.9007 | 54.5 | 4.7483 | 97 | 3.2 |
| 9.16 | 2.2195 | 31 | 4.0876 | 33 | 3.9175 | 55 | 4.7737 | 98 | 3.2 |
| 9.17 | 2.2207 | 31.25 | 4.1087 | 33.25 | 3.9343 | 55.5 | 4.7991 | 99 | 3.2 |
| 9.18 | 2.2219 | 31.5 | 4.1298 | 33.5 | 3.9511 | 56 | 4.8245 | 100 | 3.2 |
| 9.19 | 2.2231 | 31.75 | 4.1509 | 33.75 | 3.9679 | 56.5 | 4.8499 | | |
| 9.2 | 2.2243 | 32 | 4.172 | 34 | 3.9847 | 57 | 4.8753 | | |
| 9.21 | 2.2255 | 32.25 | 4.1931 | 34.25 | 4.0015 | 57.5 | 4.9007 | | |
| 9.22 | 2.2267 | 32.5 | 4.2142 | 34.5 | 4.0183 | 58 | 4.9261 | | |
| 9.23 | 2.2279 | 32.75 | 4.2353 | 34.75 | 4.0351 | 58.5 | 4.9515 | | |
| 9.24 | 2.2291 | | | | | | | | |

MENSURATION OF AREAS, LINES, AND SURFACES.

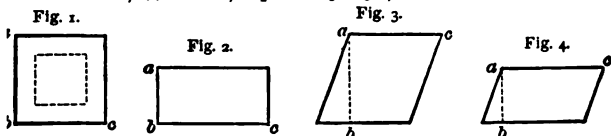
Parallelograms.

DEFINITION.—Quadrilaterals, having their opposite sides parallel.

o Compute Area of a Square, Rectangle, Rhombus, or Rhomboid.—Figs. 1, 2, 3, and 4.

RULE.—Multiply length by breadth or height.

Or, $l \times b = \text{area}$, l representing length, and b breadth.



EXAMPLE.—Sides a , b , c , Fig. 1, are 5 feet 6 ins.; what is area?

$$5.5 \times 5.5 = 30.25 \text{ square feet.}$$

NOTE 1.—Opposite angles of a Rhombus and a Rhomboid are equal.

2.—In any parallelogram the four angles equal 360° .

3.—Side of a square multiplied by 1.52 is equal to side of an equilateral triangle equal area.

Gnomon.

DEFINITION.—Space included between the lines forming two similar parallelograms, of which smaller is inscribed within larger, so that one angle in each is common to both, as shown by dotted lines, Fig. 1.

To Compute Area of a Gnomon.—Fig. 1.

RULE.—Ascertain areas of the two parallelograms, and subtract less from greater.

Or, $a - a' = \text{area}$, a and a' representing areas.

EXAMPLE.—Sides of a gnomon are 10 by 10 and 6 by 6 ins.; what is its area?

$$10 \times 10 = 100, \text{ and } 6 \times 6 = 36. \text{ Then } 100 - 36 = 64 \text{ square ins.}$$

Triangles.

DEFINITION.—Plain superficies having three sides and angles.

To Compute Area of a Triangle.—Figs. 5, 6, and 7.

RULE.—Multiply base by height, and divide product by 2.

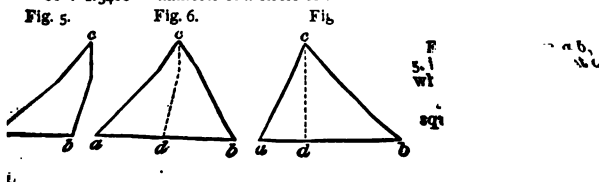
$$\text{Or, } \frac{ab \times cd}{2} \quad \text{Or, } \frac{b \times h}{2} = \text{area, } b \text{ representing base, and } h \text{ height.}$$

NOTE 1.—*Hypotenuse* of a right angle is side opposite to right angle.

2.—Perpendicular height of a triangle = twice its area divided by its base.

3.—Perpendicular height of an equilateral triangle = a side $\times .866$.

4.—Side of an equilateral triangle $\times .65825$ = side of a square of equal area,
 Or $\div 1.3468$ = diameter of a circle of equal area.



| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. |
|-------|--------|-------|--------|-------|--------|-------|--------|-------|
| 8.01 | 2.0807 | 8.41 | 2.1294 | 8.81 | 2.1759 | 9.21 | 2.2203 | 9.6 |
| 8.02 | 2.0819 | 8.42 | 2.1306 | 8.82 | 2.1777 | 9.22 | 2.2214 | 9.6 |
| 8.03 | 2.0832 | 8.43 | 2.1318 | 8.83 | 2.1782 | 9.23 | 2.2225 | 9.6 |
| 8.04 | 2.0844 | 8.44 | 2.133 | 8.84 | 2.1793 | 9.24 | 2.2235 | 9.6 |
| 8.05 | 2.0857 | 8.45 | 2.1342 | 8.85 | 2.1804 | 9.25 | 2.2246 | 9.6 |
| 8.06 | 2.0869 | 8.46 | 2.1353 | 8.86 | 2.1815 | 9.26 | 2.2257 | 9.6 |
| 8.07 | 2.0882 | 8.47 | 2.1365 | 8.87 | 2.1827 | 9.27 | 2.2268 | 9.6 |
| 8.08 | 2.0894 | 8.48 | 2.1377 | 8.88 | 2.1838 | 9.28 | 2.2279 | 9.6 |
| 8.09 | 2.0906 | 8.49 | 2.1389 | 8.89 | 2.1849 | 9.29 | 2.2289 | 9.6 |
| 8.1 | 2.0919 | 8.5 | 2.1401 | 8.9 | 2.1861 | 9.3 | 2.23 | 9.6 |
| 8.11 | 2.0931 | 8.51 | 2.1412 | 8.91 | 2.1872 | 9.31 | 2.2311 | 9.6 |
| 8.12 | 2.0943 | 8.52 | 2.1424 | 8.92 | 2.1883 | 9.32 | 2.2322 | 9.6 |
| 8.13 | 2.0956 | 8.53 | 2.1436 | 8.93 | 2.1894 | 9.33 | 2.2332 | 9.6 |
| 8.14 | 2.0968 | 8.54 | 2.1448 | 8.94 | 2.1905 | 9.34 | 2.2343 | 9.6 |
| 8.15 | 2.098 | 8.55 | 2.1459 | 8.95 | 2.1917 | 9.35 | 2.2354 | 9.6 |
| 8.16 | 2.0992 | 8.56 | 2.1471 | 8.96 | 2.1928 | 9.36 | 2.2364 | 9.6 |
| 8.17 | 2.1005 | 8.57 | 2.1483 | 8.97 | 2.1939 | 9.37 | 2.2375 | 9.6 |
| 8.18 | 2.1017 | 8.58 | 2.1494 | 8.98 | 2.195 | 9.38 | 2.2386 | 9.6 |
| 8.19 | 2.1029 | 8.59 | 2.1506 | 8.99 | 2.1961 | 9.39 | 2.2396 | 9.6 |
| 8.2 | 2.1041 | 8.6 | 2.1518 | 9 | 2.1972 | 9.4 | 2.2407 | 9.6 |
| 8.21 | 2.1054 | 8.61 | 2.1529 | 9.01 | 2.1983 | 9.41 | 2.2418 | 9.6 |
| 8.22 | 2.1066 | 8.62 | 2.1541 | 9.02 | 2.1994 | 9.42 | 2.2428 | 9.6 |
| 8.23 | 2.1078 | 8.63 | 2.1552 | 9.03 | 2.2006 | 9.43 | 2.2439 | 9.6 |
| 8.24 | 2.109 | 8.64 | 2.1564 | 9.04 | 2.2017 | 9.44 | 2.245 | 9.6 |
| 8.25 | 2.1102 | 8.65 | 2.1576 | 9.05 | 2.2028 | 9.45 | 2.246 | 9.6 |
| 8.26 | 2.1114 | 8.66 | 2.1587 | 9.06 | 2.2039 | 9.46 | 2.2471 | 9.6 |
| 8.27 | 2.1126 | 8.67 | 2.1599 | 9.07 | 2.205 | 9.47 | 2.2481 | 9.6 |
| 8.28 | 2.1138 | 8.68 | 2.161 | 9.08 | 2.2061 | 9.48 | 2.2492 | 9.6 |
| 8.29 | 2.115 | 8.69 | 2.1622 | 9.09 | 2.2072 | 9.49 | 2.2502 | 9.6 |
| 8.3 | 2.1163 | 8.7 | 2.1633 | 9.1 | 2.2083 | 9.5 | 2.2513 | 9.6 |
| 8.31 | 2.1175 | 8.71 | 2.1645 | 9.11 | 2.2094 | 9.51 | 2.2523 | 9.6 |
| 8.32 | 2.1187 | 8.72 | 2.1656 | 9.12 | 2.2105 | 9.52 | 2.2534 | 9.6 |
| 8.33 | 2.1199 | 8.73 | 2.1668 | 9.13 | 2.2116 | 9.53 | 2.2544 | 9.6 |
| 8.34 | 2.1211 | 8.74 | 2.1679 | 9.14 | 2.2127 | 9.54 | 2.2555 | 9.6 |
| 8.35 | 2.1223 | 8.75 | 2.1691 | 9.15 | 2.2138 | 9.55 | 2.2565 | 9.6 |
| 8.36 | 2.1235 | 8.76 | 2.1702 | 9.16 | 2.2148 | 9.56 | 2.2576 | 9.6 |
| 8.37 | 2.1247 | 8.77 | 2.1713 | 9.17 | 2.2159 | 9.57 | 2.2586 | 9.6 |
| 8.38 | 2.1258 | 8.78 | 2.1724 | 9.18 | 2.217 | 9.58 | 2.2597 | 9.6 |
| 8.39 | 2.127 | 8.79 | 2.1735 | 9.19 | 2.2181 | 9.59 | 2.2607 | 9.6 |
| 8.4 | 2.1282 | 8.8 | 2.1746 | 9.2 | 2.2192 | 9.6 | 2.2618 | 9.6 |
| 10.35 | 2.3577 | 10.35 | 2.3577 | 10.35 | 2.3577 | 10.35 | 2.3577 | 10.35 |
| 10.5 | 2.3577 | 10.5 | 2.3577 | 10.5 | 2.3577 | 10.5 | 2.3577 | 10.5 |
| 10.75 | 2.3577 | 10.75 | 2.3577 | 10.75 | 2.3577 | 10.75 | 2.3577 | 10.75 |

OF AREAS, LINES, AND SURFACES

Parallelograms.

ms, having their opposite sides parallel

s of a Square, Rectangle, Rhombus, or
bold.—Figs. 1, 2, 3, and 4.

ph by breadth or height.

area, l representing length, and b breadth

Fig. 3

Fig. 2

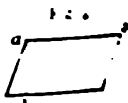
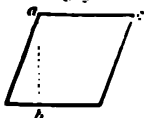


Fig. 1, are 5 feet 6 ins., what is area?

$55 \times 5.5 = 302.5$ square feet

of a Rhombus and a Rhombus, are equal

the four angles equal 90° .

plied by 1.52 is equal to side of an equilateral triangle

Gnomon.

nded between the lines forming two similar paralle-
s inscribed within larger, so that the angles are equal
by dotted lines, Fig. 1.

Area of a Gnomon.—Fig. 1.

s of the two parallelograms, and I subtract less from

= area, a and a' representing area of

non are 10 by 11 and 6 by 6 ins., what is area?

$6 \times 6 = 36$. Then $11 \times 10 = 110$ square ins.

Triangles.

ms having three sides and angles

of a Triangle.—Figs. 5, 6, and 7.

ight, and divide product by 2

b representing base, and h height

the side opposite the right angle

twice its area divided by base

angle = area of square

side of a square of equal area.

Find area—Base a b, Fig.
c is height and height c b, 6;
what is area?

$4 \times 6 = 24$ square feet
 $24 \div 2 = 12$

ES. 347

is are given.
e third of it
multiplied by

half arc, a b, is

ard of it more.

$104.26 = 116.427$

RULE.—Ascend-

of a Circle.

imeter of circle,

a Circle, p. 267,

it.

ght, take square

$\times 2 = c$.

13.)

ce from centre.

re.—Fig. 23.

ference, and prod-

= surface.

sphere, Fig. 23, hav-

re ins.

phere.—Fig. 24.

here, and add product

a b c, Fig. 24, is 36 ins.,
is convex surface, and

of segment multiplied by

er and versed sine being
ng equal to chord of arc,

$5 - 28^2 = 96$.

$1 + 11309.76 = 18548.0064$

ired, area or areas of base

circumference of a circle to

336 MENSURATION OF AREAS, LINES, AND SURFACES.

To Compute Area of a Triangle by Length of its Sides.— Figs. 6 and 7.

RULE.—From half sum of the three sides subtract each side separately; then multiply half sum and the three remainders continually together, and take square root of product.

Or, $\sqrt{(s-a) \times (s-b) \times (s-c)} = S$ = area, a, b, c representing sides, and S half sum of the three sides.

EXAMPLE.—Sides of a triangle, Figs. 6 and 7, are 30, 40, and 50 feet; what is area?

$$\frac{30+40+50}{2} = \frac{120}{2} = 60, \text{ or half sum of sides.}$$

$$\begin{array}{r} 60-30=30 \\ 60-40=20 \\ 60-50=10 \end{array} \left. \vphantom{\begin{array}{r} 60-30=30 \\ 60-40=20 \\ 60-50=10 \end{array}} \right\} \text{remainders.}$$

Whence, $30 \times 20 \times 10 \times 60 = 360000$, and $\sqrt{360000} = 600$ square feet.

When all Sides are Equal. **RULE.**—Square length of a side, and multiply product by .433.

Or, $S^2 \times .433 = \text{area}$, S representing length of a side.

To Compute Length of One Side of a Right-Angled Triangle.

When Length of the other Two Sides are given.

To Ascertain Hypotenuse.—Fig. 5.

RULE.—Add together squares of the two legs, and take square root of sum.

$$\text{Or, } \sqrt{a^2 + b^2} = \text{hypotenuse.} \quad \text{Or, } \sqrt{b^2 + h^2}.$$

EXAMPLE.—Base, a, b , Fig. 5, is 30 ins., and height, b, c , 40; what is length of hypotenuse?

$$30^2 + 40^2 = 2500, \text{ and } \sqrt{2500} = 50 \text{ ins.}$$

To Ascertain other Leg.

When Hypotenuse and One of the Legs are given.—Fig. 5. **RULE.**—Subtract square of given leg from square of hypotenuse, and take square root of remainder.

$$\text{Or, } \sqrt{\text{hyp.}^2 - \left\{ \begin{array}{l} b^2 = h. \\ h^2 = b. \end{array} \right.} \quad \text{Or, } \sqrt{a^2 - \left\{ \begin{array}{l} a^2 = b. c. \\ b^2 = a. b. \end{array} \right.}$$

EXAMPLE.—Base of a triangle, a, b , Fig. 5, is 30 feet, and hypotenuse, a, c , 50; what is height of it?

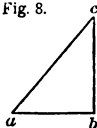
$$50^2 - 30^2 = 1600, \text{ and } \sqrt{1600} = 40 \text{ feet.}$$

To Compute Length of a Side.

When Hypotenuse of a Right-angled Triangle of Equal Sides alone is given.—Fig. 8. **RULE.**—Divide hypotenuse by 1.414 213.

$$\text{Or, } \frac{\text{hyp.}}{1.414\,213} = \text{length of a side.}$$

Fig. 8.



EXAMPLE.—Hypotenuse a, c of a right-angled triangle, Fig. 8, is 300 feet; what is length of its sides?

$$300 \div 1.414\,213 = 212.1321 \text{ feet.}$$

To Compute Perpendicular or Height of a Triangle.

When Base and Area alone are given.—Fig. 9. **RULE.**—Divide twice by its base. Or, $2a \div b = h$.

EXAMPLE.—Area of a triangle, Fig. 9, is 10 feet, and length of its base, a, b , 5; what is height, c, d ?

$$10 \times 2 = 20, \text{ and } 20 \div 5 = 4 \text{ feet.}$$

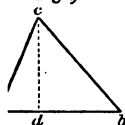
o Compute Perpendicular or Height of a Triangle.

When Base and Two Sides are given. RULE.—As base is to sum of the sides, so is difference of sides to difference of divisions of base. Half this difference being added to or subtracted from half base will give the two divisions thereof. Hence, as the sides and their opposite division of base constitute a right-angled triangle, the perpendicular thereof is readily ascertained preceding rules.

$$\text{Or, } \frac{bc + ca \times bc \sim ca}{ba} = bd \sim da.$$

$$\text{Or, } \frac{ac^2 + ab^2 - bc^2}{2ab} = ad; \text{ whence } \sqrt{ac^2 - ad^2} = dc.$$

Fig. 9.



EXAMPLE.—Three sides of a triangle, abc , Fig. 9, are 9.928, 8, and 5 feet; what is length of perpendicular on longest side? As $9.928 : 8 + 5 :: 8 \sim 5 : 3.928 = \text{difference of divisions of the base.}$

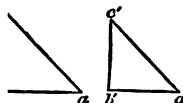
Then $3.928 \div 2 = 1.964$, which, added to $\frac{9.928}{2} = 4.964 + 1.964 = 6.928 = \text{length of longest division of base.}$

Hence, there is a right-angled triangle with its base 6.928, and its hypotenuse 8; consequently, its remaining side or perpendicular is $\sqrt{8^2 - 6.928^2} = 4 \text{ feet.}$

When any Two of the Dimensions of a Triangle and One of the corresponding Dimensions of a similar Figure are given, and it is required to ascertain the other corresponding Dimensions of the last Figure.

Fig. 10.

Fig. 11.



Let $abc, a'b'c'$, be two similar triangles, Figs. 10 and 11.

Then $ab : bc :: a'b' : b'c'$, or $a'b' : b'c' :: ab : bc$.

NOTE.—Same proportion holds with respect to the similar lineal parts of any other similar figures, whether plane or solid.

EXAMPLE.—Shadow of a vertical stake 4 feet in length was 5 feet; at same time, shadow of a tree, both on level ground, was 83 feet; what was height of tree?

$$5 a'b' : 4 b'c' :: 83 ab : 66.4 \text{ feet.}$$

To Compute Acreage.

Divide area into convenient triangles, and multiply base of each triangle links by half perpendicular in links; cut off 5 figures at the right, remaining figures will give acres; multiply the 5 figures so cut off by 4, and again cut off 5, and remainder will give roods; multiply the 5 by 40, and again cut off 5 for perches.

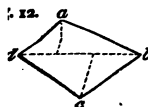
Trapezium.

DEFINITION.—A Quadrilateral having unequal sides of which no two are parallel:

To Compute Area of a Trapezium.—Fig. 12.

RULE.—Multiply diagonal by sum of the two perpendiculars falling upon from the opposite angles, and divide product by 2.

$$\text{Or, } \frac{db \times a + c}{2} = \text{area.}$$



EXAMPLE.—Diagonal db , Fig. 12, is 125 feet, and perpendiculars a and c 50 and 37; what is area?

$$125 \times 50 + 37 = 10875, \text{ and } 10875 \div 2 = 5437.5$$

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When the Two opposite Angles are Supplements to each other, that is, in a Trapezium can be inscribed in a Circle, the Sum of its opposite Angles being equal to Two Right Angles, or 180° . **RULE.**—From half sum of four sides, subtract each side severally; then multiply the four remain continually together, and take square root of product.

EXAMPLE.—In a trapezium the sides are 15, 13, 14, and 12 feet; its opposite angles being supplements to each other, required its area.

$$15 + 13 + 14 + 12 = 54, \text{ and } \frac{54}{2} = 27.$$

$$\frac{27}{15} \quad \frac{27}{13} \quad \frac{27}{14} \quad \frac{27}{12}$$

$$12 \times 14 \times 13 \times 15 = 32760, \text{ and } \sqrt{32760} = 180.997 \text{ square feet.}$$

Trapezoid.

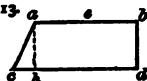
DEFINITION.—A Quadrilateral with only one pair of opposite sides parallel.

To Compute Area of a Trapezoid.—Fig. 13.

RULE.—Multiply sum of the parallel sides by perpendicular distance between them, and divide product by 2.

$$\text{Or, } \frac{a b + d c \times a h}{2} \quad \text{Or, } \frac{s + s' \times h}{2} = \text{area, } s \text{ and } s' \text{ representing sides.}$$

Fig. 13.



EXAMPLE.—Parallel sides $a b, c d$, Fig. 13, are 100 and 132 feet, and distance between them 62.5 feet; what is area?
 $100 + 132 \times 62.5 = 14500$, and $14500 \div 2 = 7250$ sq feet.

Polygons.

DEFINITION.—Plane figures having three or more sides, and are either regular or irregular, according as their sides or angles are equal or unequal, and they are named from the number of their sides and angles.

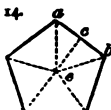
Regular Polygons.

To Compute Area of a Regular Polygon.—Fig. 14.

RULE.—Multiply length of a side by perpendicular distance to centre multiply product by number of sides, and divide it by 2.

$$\text{Or, } \frac{a b \times c c \times n}{2} = \text{area, } n \text{ representing number of sides.}$$

Fig. 14.



EXAMPLE.—What is area of a pentagon, side $a b$, Fig. 14, 5 feet, and distance $c c$ 4.25 feet?

$$5 \times 4.25 \times 5 (n) = 106.25 = \text{product of length of a side, distance to centre, and number of sides.}$$

$$\text{Then, } 106.25 \div 2 = 53.125 \text{ square feet.}$$

To Compute Radius of a Circle that contains a Given Polygon.

When Length of a Perpendicular from Centre alone is given. **RULE.**—Multiply distance from centre to a side of the polygon, by unit in column of following Table.

EXAMPLE.—What is radius of a circle that contains a hexagon, distance to side being 4.33 inches?

$$4.33 \times 1.156 = 5 \text{ ins.}$$

To Compute Length of a Side of a Polygon that is contained in a Given Circle.

Radius of Circle is given. **RULE.**—Multiply radius of circle by unit in column B of following Table.

—What is length of side of a pentagon contained in a circle of 5 feet?

$$5 \div 2 = 2.5 \text{ radius, and } 2.5 \times 1.1756 = 2.94 \text{ feet.}$$

To Compute Radius of a Circumscribing Circle.

When Length of a Side is given. **RULE.**—Multiply length of a side of the polygon, by unit in column C of following Table.

EXAMPLE.—What is radius of a circle that will contain a hexagon, a side being 5 ins?

$$5 \times 1 = 5 \text{ ins.}$$

o Compute Radius of a Circle that can be Inscribed in a Given Polygon.

When Length of a Side is given. **RULE.**—Multiply length of a side of polygon, by unit in column D of following Table.

EXAMPLE.—What is radius of the circle that is bounded by a hexagon, its sides ing 5 inches?

$$5 \times .866 = 4.33 \text{ ins.}$$

To Compute Area of a Regular Polygon.

When Length of a Side only is given. **RULE.**—Multiply square of side, by multiplier opposite to term of polygon in following Table:

| o. of sides. | POLYGON. | AREA. | A. Radius of Circumscribed Circle. | B. Length of a Side. | C. Radius of Circumscrib- ing Circle. | D. Radius of Inscribed Circle. |
|-----------------|-----------|-----------|---|----------------------------|--|---|
| 3 | Trigon | .433 01 | 2 | 1 732 | .5773 | .2887 |
| 4 | Tetragon | 1 | 1.414 | 1.4142 | .7071 | .5 |
| 5 | Pentagon | 1.720 48 | 1.238 | 1.1756 | .8506 | .6882 |
| 6 | Hexagon | 2.598 08 | 1.156 | 1 | 1 | .866 |
| 7 | Heptagon | 3.633 91 | 1.11 | .8677 | 1.1524 | 1.0383 |
| 8 | Octagon | 4.828 43 | 1.083 | .7653 | 1.3066 | 1.2071 |
| 9 | Nonagon | 6.181 82 | 1.064 | .684 | 1.4619 | 1.3737 |
| 10 | Decagon | 7.694 21 | 1.051 | .618 | 1.618 | 1.5388 |
| 11 | Undecagon | 9.365 64 | 1.042 | .5634 | 1.7747 | 1.7028 |
| 12 | Dodecagon | 11.196 15 | 1.037 | .5176 | 1.9319 | 1.866 |

EXAMPLE.—What is area of a square (tetragon) when length of its sides is 71.0678 inches?

$$71.0678^2 = 50, \text{ and } 50 \times 1 = 50 \text{ square ins.}$$

o Compute Length of a Side and Radii of a Regular Polygon.

When Area alone is given. **RULE.**—Multiply square root of area of polygon by multiplier in column E of the following table for length of side; by multiplier in column G for radius of circumscribing circle, and by multiplier column H for radius of inscribed circle or perpendicular.

| o. of sides. | POLYGON. | E. Length of Side. | G. Radius of Circumscrib- ing Circle. | H. Radius of Inscribed Circle. | Angle. | Angle of Polygon. | Tangent. |
|-----------------|-----------|--------------------------|--|---|-----------|----------------------|----------|
| 3 | Trigon | 1.5197 | .8774 | .4387 | 120° | 60° | .5774 |
| 4 | Tetragon | 1 | .7071 | .5 | 90 | 90 | 1 |
| 5 | Pentagon | .7624 | .6485 | .5247 | 72 | 108 | 1.3764 |
| 6 | Hexagon | .6204 | .6204 | .5373 | 60 | 120 | 1.7321 |
| 7 | Heptagon | .5246 | .6045 | .5446 | 51 25.71' | 128 34.29' | 2.0765 |
| 8 | Octagon | .4551 | .5946 | .5493 | 45 | 135 | 2.4142 |
| 9 | Nonagon | .4022 | .588 | .5525 | 40 | 140 | 2.7475 |
| 10 | Decagon | .3605 | .5833 | .5548 | 36 | 144 | 3.0777 |
| 11 | Undecagon | .3268 | .5799 | .5564 | 32 43.64' | 147 16.36' | 3.4057 |
| 12 | Dodecagon | .2989 | .5774 | .5577 | 30 | 150 | 3.7321 |

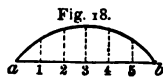
EXAMPLE 1.—Area of a square (tetragon) is 16 inches; what is

$$\sqrt{16} = 4, \text{ and } 4 \times 1 = 4 \text{ ins.}$$

EXAMPLE 2.—Area of an octagon is 70.698 yards; what is diameter?

$$\sqrt{70.698} \times .5946 = 5, \text{ and } 5 \times 2 = 10 \text{ yards.}$$

To Compute an Area bounded by a Curve.—Fig. 18.
(Simpson's Rule.)



OPERATION.—Divide line *ab* into any number of equal parts, by perpendiculars from base, as 1, 2, 3, etc., which will give an odd number of points of division. Measure lengths of these perpendiculars or ordinates, and proceed as follows:
To sum of lengths of first and last ordinates, add four times sum of lengths of all even numbered ordinates and twice sum of odd; multiply their sum by one third of distance between ordinates, and product will give area required.

ILLUSTRATION.—Water-line of a vessel has a length of 80 feet, and ordinates *a*, 1, 1.2, 1.5, 2, 1.9, 1.5, 1.1, and *b*, each 10 feet apart; what is its area?

| Even. | Odd. | Sum. |
|---------------|---------------|--|
| 1 | 1.2 | first 0 |
| 1.5 | 2 | last 0 |
| 1.9 | 1.5 | even 22 |
| 1.1 | | odd 9.4 |
| 5.5 × 4 = 22. | 4.7 × 2 = 9.4 | 31.4 × 10 = 314, which ÷ 3 = 104.66 square feet. |

Circle.

Diameter is a right line drawn through its centre, bounded by its periphery.

Radius is a right line drawn from its centre to its circumference.

Circumference is assumed to be divided into 360 equal parts, termed *degrees*; each degree is divided into 60 parts, termed *minutes*; each minute into 60 parts, termed *seconds*; and each second into 60 parts, termed *thirds*, and so on.

To Compute Circumference of a Circle.

RULE.—Multiply diameter by 3.1416.

Or, as 7 is to 22, so is diameter to circumference.

Or, as 113 is to 355, so is diameter to circumference.

EXAMPLE.—Diameter of a circle is 1.25 inches; what is its circumference?

$$1.25 \times 3.1416 = 3.927 \text{ ins.}$$

To Compute Diameter of a Circle.

RULE.—Divide circumference by 3.1416.

Or, as 22 is to 7, so is circumference to diameter.

NOTE.—Divide area by .7854, and square root of quotient will give diameter of circle.

To Compute Area of a Circle.

RULE.—Multiply square of diameter by .7854.

Or, multiply square of circumference by .09958.

Or, multiply half circumference by half diameter.

Or, multiply square of radius by 3.1416.

Or, $p \cdot r^2 = \text{area}$, r representing radius.

EXAMPLE.—The diameter of a circle is 8 inches; what is the area of it?

$$8^2 = 64, \text{ and } 64 \times .7854 = 50.2656 \text{ ins.}$$

Proportions of a Circle, its Equal, Inscribed, and Circumscribed Squares.

CIRCLE.

1. Diameter × .8862 } = Side of an Equal Square.

2. Circumference × .2821 }

3. Diameter × .7071 }

4. Circumference × .2251 } = Side of Inscribed Square.

5. Area × .9003 ÷ diam. }

6. Diameter × 1.3468 } = Side of an Equilateral Triangle.

SQUARE.

1. Side × 1.1442 = Diameter of its Circumscribing Circle.

2. Side × 1.4142 = Circumference of its Circumscribing Circle.

3. Side = Diameter

4. Side = Circumference } of an Equal Circle.

5. Side = Circle inches }

thin a circle is one half area of one described without

To Compute Side of Greatest Square that can be Inscribed in a Circle.

RULE.—Multiply diameter by .7071, or take twice square of radius.

Useful Factors.

In which p or π represents Circumference of a Circle.

Diameter = 1.

| | | |
|-----------------------------------|----------------------------|-----------------------------------|
| $p = 3.141592653589+$ | $\frac{4}{3} p = 4.18879+$ | $\sqrt{p} = 1.772453$ |
| $2 p = 6.283185307179+$ | $\frac{3}{4} p = .523598+$ | $\sqrt{\frac{2}{p}} = .797884$ |
| $4 p = 12.566370614359+$ | $\frac{3}{2} p = .392699+$ | $\text{Log. } p = .49714987$ |
| $\frac{1}{2} p = 1.570796326794+$ | $\frac{1}{4} p = .261799+$ | $\frac{1}{2} \sqrt{p} = .886226+$ |
| $\frac{1}{4} p = .785398163397+$ | $\frac{1}{8} p = .008726+$ | $\frac{1}{36} p = .113097335+$ |

Diameter = 10.

| | |
|--|-------------|
| 1. Chord of arc of semicircle | = 10 |
| 2. Chord of half arc of semicircle | = 7.071067 |
| 3. Versed sine of arc of semicircle. | = 5 |
| 4. Versed sine of half arc of semicircle | = 1.464466 |
| 5. Chord of half arc, of half of arc of semicircle | = 3.82683 |
| 6. Half chord, of chord of half arc | = 3.535533 |
| 7. Length of arc of semicircle | = 15.707963 |
| 8. Length of half arc of semicircle | = 7.853981 |
| 9. Square of chord, of half arc of semicircle (2) | = 50 |
| 10. Square root of versed sine of half arc (4) | = 1.210151 |
| 11. Square of versed sine of half arc (4) | = 2.144664 |
| 12. Square of chord of half arc, of half arc of semicircle (5) | = 14.64467 |
| 13. Square of half chord, of chord of half arc (6) | = 12.5 |

NOTE.—In all computations p is taken at 3.1416, $\frac{1}{2} p$ at .7854, $\frac{1}{4} p$ at .5236; and whenever the decimal figure next to the one last taken exceeds 5, one is added. Thus, 3.14159 for four places of decimals is taken as 3.1416.

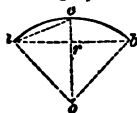
To Compute Length of an Arc of a Circle.—Fig. 19.

When Number of Degrees and Radius are given. RULE 1.—Multiply number of degrees in the arc by 3.1416 times the radius, and divide by 180.

2.—Multiply radius of circle by .01745329, and product by degrees in arc.

If length is required for minutes, multiply radius by .000290889; if for seconds, by .000004848.

Fig. 19.



EXAMPLE 1.—Number of degrees in an arc, $o a b$, Fig. 19, are 90, and radius, $o b$, 5 inches; what is length of arc?

$90 \times (3.1416 \times 5) = 1413.72$, which $\div 180 = 7.854$ ins.

2.—Radius of an arc is 10, and measure of its angle $44^\circ 30'$; what is length of arc?

$10 \times .01745329 = .1745329$, which $\times 44 = 7.6794476$, length for 44° .

$10 \times .000290889 = .00290889$, which $\times 30 = .0872667$, length for $30'$.

$10 \times .000004848 = .00004848$, which $\times 30 = .0014544$, length for $30''$.

Then $\left. \begin{array}{r} 7.6794476 \\ .0872667 \\ .0014544 \end{array} \right\} = 7.7681687$ ins.

Or, reduce minutes and seconds to decimal of a degree, and multiply

See Rule, page 93. $30' 30'' = .5083$, and $.1745329$ from above

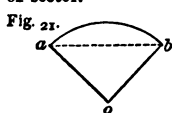
is 163 ins.

Sector of a Circle.

DEFINITION.—A part of a circle bounded by an arc and two radii.

To Compute Area of a Sector of a Circle.

When Degrees in the Arc are given.—Fig. 21. RULE.—As 360 is to number of degrees in a sector, so is area of circle of which sector is a part to area of sector.



Or, $\frac{da}{360} = \text{area}$, d representing degrees in arc, and a area of circle.

EXAMPLE.—Radius of a circle, $o a$, Fig. 21, is 5 ins., and number of degrees of sector, $a b o$, is $22^\circ 30'$; what is area?

Area of a circle of 5 ins. radius = 78.54 ins.

Then, as $360^\circ : 22^\circ 30' :: 78.54 : 4.90875$ ins.

When Length of the Arc, etc., are given. RULE.—Multiply length of arc by half length of radius, and product is area.

Or, $b \times r \div 2 = \text{area}$, b representing arc, and r radius.

Segment of a Circle.

DEFINITION.—A part of a circle bounded by an arc and a chord.

To Compute Area of a Segment of a Circle.

When Chord and Versed Sine of Arc, and Radius or Diameter of Circle are given.

When Segment is less than a Semicircle, as $a b c$, Fig. 21. RULE.—Ascertain area of sector having same arc as segment; then ascertain area of triangle formed by chord of segment and radii of sector, and take difference of these areas.

NOTE.—Subtract versed sine from radius; multiply remainder by one half of chord of arc, and product will give area of triangle.

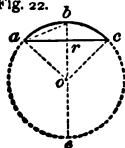
Or, $a - a' = \text{area}$, a and a' representing areas of sector and triangle.

When Segment is greater than a Semicircle. RULE.—Ascertain, by preceding rule, area of lesser portion of circle; subtract it from area of whole circle, and remainder will give area.

Or, $a - a' = \text{area}$, a and a' representing areas of circle and lesser portion.

See Table of Areas of Segments, page 267.

Fig. 22. EXAMPLE.—Chord, $a c$, Fig. 22, is 14.142; diameter, $b e$, is 20 ins.; and versed sine, $b r$, is 2.929; what is area of segment?



$14.142 \div 2 = 7.071 = \text{half chord of arc}$.

$\sqrt{7.071^2 + 2.929^2} = 7.654 = \text{square root of sum of squares of half chord of arc and versed sine, which is chord } a b \text{ of half arc } a b c$.

By Rule, page 344,

$7.654 \times 2 \times 2.929 \times 10 = 448.371 = \text{twice chord of half arc by 10 times versed sine}$.

$60 - 2.929 \times 27 = 1120.917 = 60 \text{ times diameter subtracted from } 27 \text{ times sine}$.

$448.371 + 1120.917 = .4$, and $.4$ added to 7.654×2 (twice chord of half arc) $\frac{10}{2} = 78.54 = \text{the arc multiplied by half length of radius}$.

$\frac{10}{2} = 78.54 = \text{the arc multiplied by half length of radius, and then subtracted from a radius, which is height of triangle}$.

When the Chords of Arc, and of half of Arc, and Versed Sine are given.
RULE.—To chord of whole arc add chord of half arc and one third of it more; multiply this sum by versed sine, and this product, multiplied by .404 26, will give area nearly.

$$\text{Or, } c + c' + \frac{c'}{3} \times v. \sin. \times .404\ 26 = \text{area nearly.}$$

EXAMPLE.—Chord of a segment, $a c$, Fig. 22, is 28 feet; chord of half arc, $a b$, is 15; and versed sine, $b r$, 5; what is area of segment?

$28 + 15 + \frac{15}{3} = \text{chord of arc added to chord of half arc and one third of it more}$
 $48 \times 6 = 288 = \text{product of above sum and versed sine. Hence } 288 \times .404\ 26 = 116.42;$
 square feet.

When the Chord of Arc and Versed Sine only are given. **RULE.**—Ascertain chord of half arc, and proceed as before.

To Compute Chord and Height of a Segment of a Circle.

When Area is given. **RULE.**—Divide area by square of diameter of circle take tab. height for area from table of Areas of Segments of a Circle, p. 267, multiply it by diameter, and product will give required height.

From diameter subtract height, multiply remainder by height, take square root of product and multiply it by 2 for required chord.

$$\text{Or, } \frac{a}{d^2} = (\text{tab. area for height}) \times d = h, \text{ and } \sqrt{d - h} \times h \times 2 = c.$$

Circular Measure. (See Rule, page 113.)

Sphere.

DEFINITION.—A figure, surface of which is at a uniform distance from centre.

To Compute Convex Surface of a Sphere.—Fig. 23.

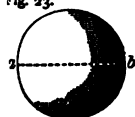
Fig. 23.

RULE.—Multiply diameter by circumference, and product will give surface.

$$\text{Or, } 4 p r^2 = \text{surface.}^* \quad \text{Or, } p d^2 = \text{surface.}$$

EXAMPLE.—What is convex surface of a sphere, Fig. 23, having a diameter, $a b$, of 10 ins?

$$10 \times 31.416 = 314.16 \text{ square ins.}$$



Segment of a Sphere.

DEFINITION.—A section of a sphere.

To Compute Surface of a Segment of a Sphere.—Fig. 24.

RULE.—Multiply height by the circumference of sphere, and add product to the area of base.

$$\text{Or, } 2 p r h = \text{convex surface alone.}$$

EXAMPLE.—Height, $b o$, of a segment, $a b c$, Fig. 24, is 36 ins., and diameter, $b c$, of sphere 100; what is convex surface, and what whole surface?

$$36 \times 100 \times 3.1416 = 11\ 309.76 = \text{height of segment multiplied by circumference of sphere.}$$

To ascertain area of base; diameter and versed sine being given, diameter of base of segment, being equal to chord of arc, is, by Rule, page 344,

$$100 - 36 \times 2 = 28 \quad \sqrt{100^2 - 28^2} = 96.$$

$$96^2 \times .7854 = 7238.2464 = \text{convex surface, and } 7238.2464 + 11\ 309.76 = 18\ 548.0064 = \text{convex surface added to area of base} = \text{square ins.}$$

NOTE.—When convex surface of a figure alone is required, area of base must be omitted.

^{*} p or π represents in this, and in all cases where it is used, ratio of circumference to diameter, or .31416.

When the Diameter of Base of Segment and Height of it are alone given. RULE.—Add square of half diameter of base to the square of height; divide this sum by height, and result will give diameter of sphere.

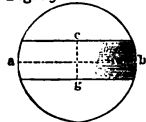
$$\text{Or, } \frac{d^2}{4} \div h + h = \text{diameter.}$$

Spherical Zone (or Frustum of a Sphere).

DEFINITION.—The part of a sphere included between two parallel chords.

To Compute Surface of a Spherical Zone.—Fig. 25.

Fig. 25.



RULE.—Multiply height by the circumference of sphere, and add product to area of the two ends.

$$\text{Or, } h c + a + a' = \text{surface.}$$

$$\text{Or, } 2 p r h = \text{convex surface alone.}$$

EXAMPLE.—Diameter of a sphere, $a b$, Fig. 25, from which a zone, $c g$, is cut, is 25 inches, and height, $c g$, is 8; what is convex surface?

$$25 \times 3.1416 \times 8 = 628.32 = \text{height} \times \text{circumference of sphere} = \text{square ins.}$$

When the Diameter of Sphere is not given. RULE.—Multiply mean length of the two chords by half their difference; divide this product by breadth of zone, and to quotient add breadth. To square of this sum add square of lesser chord, and square root of their sum will give diameter of sphere.

$$\text{Or, } \sqrt{\left(\frac{l+l'}{2} \times \frac{l-l'}{2} \div b + b + l'^2\right)} = d.$$

Spheroids or Ellipsoids.

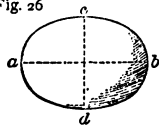
DEFINITION.—Figures generated by the revolution of a semi-ellipse about one of its diameters.

When revolution is about Transverse diameter they are Prolate, and when it is about Conjugate they are Oblate.

To Compute Surface of a Spheroid.—Fig. 26.

When Spheroid is Prolate. RULE.—Square diameters, and multiply square root of half their sum by 3.1416, and this product by conjugate diameter.

Fig. 26



$$\text{Or, } \sqrt{\frac{d^2 + d'^2}{2}} \times 3.1416 \times d = \text{surface, } d \text{ and } d' \text{ representing conjugate and transverse diameters.}$$

EXAMPLE.—A prolate spheroid, Fig. 26, has diameters, $c d$ and $a b$, of 10 and 14 inches; what is its surface?

$$10^2 + 14^2 = 296 = \text{sum of squares of diameters.}$$

$$296 \div 2 = 148, \text{ and } \sqrt{148} = 12.1655 = \text{square root of half sum of squares of diameters.}$$

$$12.1655 \times 3.1416 \times 10 = 382.191 \text{ ins.} = \text{product of root above obtained} \times 3.1416, \text{ conjugate diameter.}$$

Spheroid is Oblate. RULE.—Square diameters, and multiply square root of half their sum by 3.1416, and this product by transverse diameter.

$$\text{Or, } \sqrt{\frac{d^2 + d'^2}{2}} \times 3.1416 \times d' = \text{surface.}$$

An oblate spheroid has diameters of 14 and 10 inches; what is its

$$296 = \text{sum of squares of diameters.}$$

$$12.1655 = \text{square root of half sum of squares of di-}$$

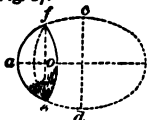
$$* \text{ above obtained} \times 3.1416$$

To Compute Convex Surface of a Segment of a Spheroid.—Figs. 27 and 28.

RULE.—Square diameters, and take square root of half their sum; then, as diameter from which the segment is cut is to this root, so is the height of segment to proportionate height required. Multiply product of other diameter and 3.1416 by proportionate height of segment, and this last product will give surface.

$$\text{Or, } \frac{\sqrt{d^2 + d'^2 + 2}}{d \text{ or } d'} \times h \times d' \text{ or } d \times 3.1416 = \text{surface.}$$

Fig. 27.



EXAMPLE.—Height, *c o*, of a segment, *e f*, of a prolate spheroid, Fig. 27, is 4 inches, diameters being 10 and 14; what is convex surface of it?

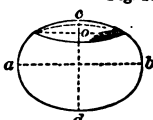
Square root of half sum of squares of diameters, 12.1655.

Then 14 : 12.1655 :: 4 : 3.4758 = height of segment, proportionate to mean of

diameters, and $10 \times 3.1416 \times 3.4758 = 109.1957$ ins.

2.—Height, *c o*, of a segment of an oblate spheroid, Fig. 28, is 4 inches, the diameters being 14 and 10; what is convex surface of it? 214.0272 square ins.

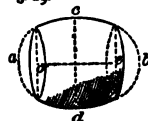
Fig. 28.



To Compute Convex Surface of a Frustum or Zone of a Spheroid.—Figs. 29 and 30.

RULE.—Proceed as by previous rule for surface of a segment, and obtain proportionate height of frustum; then multiply product of diameter parallel to base of frustum and 3.1416 by proportionate height of frustum, and it will give surface.

Fig. 29.



EXAMPLE.—Middle frustum, *o e*, of a prolate spheroid, Fig. 29, is 6 inches, diameters of spheroid being 10 and 14; what is its convex surface?

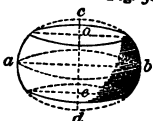
Mean diameter, as per preceding example, is 12.1655.

Diameter parallel to base of frustum is 10.

Then 14 : 12.1655 :: 6 : 5.2138, and $10 \times 3.1416 \times 5.2138 = 163.7967$ square ins.

2.—Middle frustum of an oblate spheroid, as *o e*, Fig. 30, is 2 inches in height, diameters of spheroid, as in preceding examples, being 10 and 14; what is its convex surface? 107.0136 square ins.

Fig. 30.



Circular Zone.

DEFINITION.—A part of a circle included between two parallel chords.

To Compute Area of a Circular Zone.

RULE.—From area of circle subtract areas of segments.

Or, see Table of Areas of Zones, page 269.

When Diameter of Circle is not given.—Multiply mean length of the two chords by half their difference; divide this product by breadth of zone, and quotient add the breadth.

To square of this sum add square of lesser chord, and square root of their sum will give diameter of circle.

EXAMPLE.—Greater chord, *h g*, is 90 inches; lesser, *a c*, is 80; and breadth, *e*, is 72.526; what is its diameter?

$$\frac{80 + 90}{2} \times \frac{90 - 80}{2} = 85 \times 5 = 425, \text{ and } \frac{425}{72.526} + 72.526 = 78.3$$

Then $\sqrt{78.3^2 + 80^2} = \sqrt{12544.2} = 112 = \text{diameter.}$

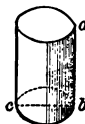
Cylinder.

DEFINITION.—A figure formed by revolution of a right-angled parallelogram around one of its sides.

To Compute Surface of a Cylinder.—Fig. 31.

RULE.—Multiply length by circumference, and add product to area of the two ends.

Fig. 31.



Or, $l c + 2 a = s$, a representing area of end.

NOTE.—When internal or convex surface alone is wanted, areas of ends are omitted.

EXAMPLE.—Diameter of a cylinder, $b c$, Fig. 31, is 30 inches, and its length, a , 50; what is its surface?

$$30 \times 3.1416 = 94.248, \text{ and } 94.248 \times 50 = 4712.4.$$

Then $30^2 \times .7854 = 706.86 = \text{area of one end}$; $706.86 \times 2 = 1413.72 = \text{area of both ends}$, and $4712.4 + 1413.72 = 6126.12 \text{ square ins.}$

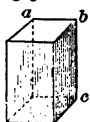
Prisms.

DEFINITION.—Figures, sides of which are parallelograms, and ends equal and parallel.

NOTE.—When ends are triangles, they are termed *triangular prisms*; when they are square, *square* or *right prisms*; and when they are a pentagon, *pentagonal prisms*, etc.

To Compute Surface of a Right Prism.—Figs. 32 and 33.

Fig. 32.



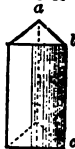
RULE.—Ascertain areas of ends and sides, and add them together.

Or, $2 a + n a' = s$, a representing area of ends, a' area of sides, and n their number.

EXAMPLE.—Side, $a b$, Fig. 32, of a square prism is 12 inches, and length, $b c$, 30; what is surface?

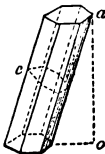
$12 \times 12 = 144 = \text{area of one end}$; $144 \times 2 = 288 = \text{area of both ends}$; $12 \times 30 = 360 = \text{area of one side}$; $360 \times 4 = 1440 = \text{area of four sides}$, and $288 + 1440 = 1728 \text{ sq. ins.}$

Fig. 33.



To Compute Surface of an Oblique or Irregular Prism.—Fig. 34.

Fig. 34.



RULE.—Multiply perimeter of one end, by perpendicular height, $a o$. Or, multiply perimeter as at c , at a right angle to sides by actual length of figure, and add area of ends.

EXAMPLE.—Sides, $a c$, of an oblique hexagonal prism, Fig. 34, are 10 inches, and perpendicular height, $a o$, is 5 feet; what is its surface?

$$10 \times 6 = 60 \text{ ins.} = \text{length of sides.}$$

$60 \times 5 \times 12 = 3600 \text{ square ins.} = \text{area of sides}$, and by table, page 339, $100 \times 2.59808 \times 2 = 519.616 \text{ square ins.}$, which added to 3600 = 4119.616 square ins.

Wedge.

DEFINITION.—A wedge is a prolate triangular prism, and its surface is computed as for that of a right prism.

To Compute Surface of a Wedge.—Fig. 35.

EXAMPLE.—Back of a wedge, $a b c d$, Fig. 35, is 20 by 2 inches, and its end, $e f$, 20 by 2; what is its surface?



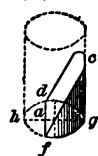
$20^2 + 2^2 = 401 = \text{sum of squares of half base, } a f, \text{ and height, } e f, \text{ of triangle, } e f a.$

$= 20.025 = \text{square root of above sum} = \text{length of } e a.$

Then $20.025 \times 20 \times 2 = 801 = \text{area of sides.}$

$\times 2 = 40 = \text{area of back}$; and $20 \times 2 + 2 \times 2 = 40 = 801 + 40 = 841 \text{ square ins.}$

When Section passes through Base of Cylinder, and Versed Sine, *a*, exceeds Sine, or when Base exceeds a Semicircle, Fig. 40. RULE 4.—Multiply sine of half the arc of base by diameter of cylinder, and the product add product of arc and the excess of versed sine the sine of base. Multiply sum thus found by quotient height divided by versed sine.

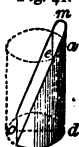


EXAMPLE.—Sine, *a*, of half arc of an ungula, Fig. 40, is 12 in versed sine, *a*, is 16; height, *c*, 16; and diameter of cylinder 25 inches; what is curved surface?

$12 \times 25 = 300 = \text{sine of half arc by diameter of cylinder, and } h$
of arc of base, Rule, page 344 = arc of *d h f*—circumference of *b*
46.392.

Then $46.392 \times 16 = 742.272$, and $300 + 162.272 = 462.372$; $16 \div 16 = 1$
 $462.372 \times 1 = 462.372$ square ins.

Fig. 41.



NOTE.—When sine of an arc is 0, the versed sine is equal to diam

When Section passes obliquely through both Ends of Cylinder, Fig. 41. RULE 5.—Conceive section to be continued to *m*, meets side of cylinder produced; then, as difference of versines, *a e* and *d o*, of arcs of two ends of ungula is to versed *a e*, of arc of the less end, so is height of cylinder, *a d*, to part of side produced.

Ascertain surface of each of ungulas thus found by Rule 4, and their difference will give curved surface.

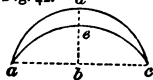
Lune.

DEFINITION.—Space between intersecting arcs of two eccentric circles.

To Compute Area of a Lune.—Fig. 42.

RULE.—Ascertain areas of the two segments from which lune is formed and their difference will give area.

Fig. 42.



EXAMPLE.—Length of chord *a c*, Fig. 42, is 20 inches, *h* *e d* is 3, and *e b* 2; what is area of lune?

By Rule 2, page 345, diameters of circles of which lune is formed are thus ascertained:

$$\text{For } a d c, \frac{10^2 + (3+2)^2}{5} = 25. \quad \text{For } a e c, \frac{10^2 + 2^2}{2} = 50.$$

Then, by Rule for Areas of Segments of a Circle, page 267,

Area of *a d c* is 70.5577 sq. ins.

" *a e c* " 27.1638 "

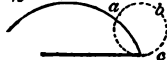
Their difference 43.3939 sq. ins.

Cycloid.

DEFINITION.—A curve generated by revolution of a circle on a plane.

To Compute Area of a Cycloid.—Fig. 43.

Fig. 43.



RULE.—Multiply area of generating circle by

EXAMPLE.—Generating circle of a cycloid, *a b c*, Fig. 43, has an area of 115.45 sq. inches; what is area of cycloid?

$$115.45 \times 3 = 346.35 \text{ square ins.}$$

To Compute Length of a Cycloidal Curve.

—Multiply diameter of generating circle by 4.

EXAMPLE.—Diameter of generating circle of a cycloid, Fig. 43, is 8 inches; what is length of curve?

$$8 \times 4 = 32 = \text{product of diameter and 4} = \text{ins.}$$

This is line of swiftest descent; that is, a body will fall from one point to another, in less time than through

Circular Rings.

DEFINITION.—Space between two concentric circles.

To Compute Sectional Area of a Circular Ring.—Fig. 44.

RULE.—From area of greater circle subtract that of less.

Cylindrical Rings.

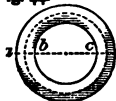
DEFINITION.—A ring formed by curvature of a cylinder.

To Compute Surface of a Cylindrical Ring.—Fig. 44.

RULE.—To diameter of body of the ring add inner diameter of the ring; multiply this sum by diameter of the body, and product by 9.8696.

Fig. 44.

Or, $c \times l = \text{surface}$.



EXAMPLE.—Diameter of body of a cylindrical ring, a b , Fig. 44, is 2 inches, and inner diameter, b c , is 18; what is surface of it?

$2 + 18 = 20 = \text{thickness of ring added to inner diameter.}$

$20 \times 2 \times 9.8696 = \text{sum above obtained} \times \text{thickness of ring, and that product by } 9.8696 = 394.784 \text{ ins.}$

Link.

DEFINITION.—An elongated ring.

To Compute Surface of a Link.—Figs. 45 and 46.

RULE.—Multiply length of axis of link by circumference of a section of ody, a b .

Or, $l \times c = \text{surface}$.

To Compute Length of Axis and Circumference.

When Ring is Elongated. **RULE.**—To less diameter add the diameter of the body of the link, and multiply sum by 3.1416; subtract less diameter from greater, multiply remainder by 2, and sum of these products is length of axis.

Fig. 45.



EXAMPLE.—Link of a chain, Fig. 45, is 1 inch in diameter of body, a b , and its inner diameters, b c and e f , are 12.5 and 2.5 inches; what is its circumference?

$2.5 + 1 \times 3.1416 = 10.9956 = \text{length of axis of ends.}$

$12.5 - 2.5 \times 2 = 20 = \text{length of sides of body.}$

Then $10.9956 + 20 = 30.9956 = \text{length of axis of link, and } 30.9956 \times 3.1416 \text{ (cir. of 1 inch)} = 97.3758 \text{ square ins.}$

Fig. 46.



When Ring is Elliptical, Fig. 46. **RULE.**—Square diameters of axes of link, multiply square root of half their sum by 3.1416, and product is length of axis.

Cones.

DEFINITION.—A figure described by revolution of a right-angled triangle about one of its legs.

For Sections of a Cone, see Conic Sections, page 379.

To Compute Surface of a Cone.—Fig. 47.

RULE.—Multiply perimeter or circumference of base by slant height, one of cone; divide product by 2, and add the quotient to area of the base.

Fig. 47.



Or, $c \times h \div 2 + a' = \text{surface, } c \text{ representing radius of base.}$

EXAMPLE.—Diameter, a b , Fig. 47, of base and slant height, a c , 15; what is surface of it?

Circum. of 3 feet = 9.4248, and $\frac{9.4248}{2}$

$\frac{1}{2}$ face of side; area of base $3 = 7.068$, and $7.068 + 4.7124 = 11.7804 \text{ square feet.}$

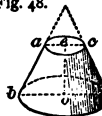
354 MENSURATION OF AREAS, LINES, AND SURFACES.

To Compute Surface of the Frustum of a Cone.—
Fig. 48.

RULE.—Multiply sum of perimeters of two ends by slant height of frustum; divide product by 2, and add it to areas of two ends.

$$\text{Or, } \frac{c + c' \times h}{2} + a + a' = \text{surface}$$

EXAMPLE.—Frustum, $a b c d$, Fig. 48, has a slant height, $c d$, of 26 inches, and circumferences of its ends are 15.71 and 22 inches respectively; what is its surface?



$$\frac{15.71 + 22 \times 26}{2} = 490.23 = \text{surface of sides; } \left(\frac{15.71}{3.1416} \right)^2 \times .7854 + \left(\frac{22}{3.1416} \right)^2 \times .7854 = 58.119 = \text{areas of ends. Then } 490.23 + 58.119 = 548.349 \text{ square ins.}$$

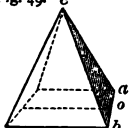
Pyramids.

DEFINITION.—A figure, base of which has three or more sides, and sides of which are plane triangles.

To Compute Surface of a Pyramid.—Figs. 49 and 50.

RULE.—Multiply perimeter of base by slant height; divide product by 2, and add it to area of base.

Fig. 49.



$$\text{Or, } \frac{c h}{2} + a = \text{surface.}$$

EXAMPLE.—Side of a quadrangular pyramid, $a b$, Fig. 49, is 12 inches, and its slant height, $o c$, 40; what is its surface?

$$12 \times 4 = 48 = \text{perimeter of base. } \frac{48 \times 40}{2} = 960 = \text{area of sides, and } 12 \times 12 + 960 = 1104 \text{ square ins.}$$

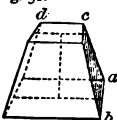
Fig. 50.



To Compute Surface of Frustum of a Pyramid.—
Fig. 51.

RULE.—Multiply sum of perimeters of two ends by slant height; divide product by 2, and add it to areas of ends.

Fig. 51.



EXAMPLE.—Sides $a b, c d$, Fig. 51, of frustum of a quadrangular pyramid are 10 and 9 inches, and its slant height is 20; what is its surface?

$$10 \times 4 = 40, \text{ and } 9 \times 4 = 36; 40 + 36 = 76 = \text{sum of perimeters. } 76 \times 20 = 1520, \text{ and } \frac{1520}{2} = 760 = \text{area of sides; } 10 \times 10 = 100, \text{ and } 9 \times 9 = 81. \text{ Then } 100 + 81 + 760 = 941 = \text{square ins.}$$

When Pyramid is Irregular sided or Oblique. **RULE.**—The surfaces of each of the sides and ends must be computed and added together.

Helix (Screw).

DEFINITION.—A line generated by progressive rotation of a point around an axis distant from its centre.

To Compute Length of a Helix.—Fig. 52.

RULE.—Multiply circumference described by generating point, add circumference of base, divide by 2, and extract square root of the result.

Fig. 52.



Or, $\sqrt{(p^2 + l^2)} n = \text{length, } n \text{ representing number of revolutions.}$

EXAMPLE.—What is length of a helical line, Fig. 52, running 3.5 times around a cylinder of 22 inches in circumference, and advancing 16 inches in each revolution?

$22^2 + 16^2 = 740 = \text{sum of squares of circumference and of distance advanced.}^*$ Then $\sqrt{740} \times 3.5 = 95.21 \text{ ins.}$

To Compute Length of a Revolution of Thread of a Screw.

RULE.—Proceed as above for length and omit number of revolutions.

Spirals.

DEFINITION.—Lines generated by the progressive rotation of a point around a fixed axis.

A *Plane Spiral* is when the point rotates around a central point.

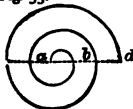
A *Conical Spiral* is when the point rotates around an axis at a progressing distance from its centre, as around a cone.

To Compute Length of a Plane Spiral Line.—Fig. 53.

RULE.—Add together greater and less diameters; divide their sum by 2; multiply quotient by 3.1416, and again by number of revolutions.

Or, when circumferences are given, take their mean length, and multiply it by number of revolutions.

Fig. 53.



Or, $d + d' \div 2 \times 3.1416 n = \text{length of line; } P \times n = \text{radius, and } p r^2 \div l = \text{pitch. } P \text{ representing the pitch.}$

EXAMPLE.—Less and greater diameters of a plane spiral spring, as *a b, c d*, Fig. 53, are 2 and 20 inches, and number of revolutions 10; what is length of it?

$\frac{2 + 20}{2} \div 2 = 11 = \text{sum of diameters} \div 2; 11 \times 3.1416 = 34.5576.$

Then $34.5576 \times 10 = 345.576 \text{ inches.}$

NOTE.—Above rule is applicable to winding engines, see page 862, where it is required to ascertain length of a rope, its thickness, number of revolutions, diameter of drum, etc.

To Compute Length of a Conical Spiral Line.—Fig. 54.

RULE.—Add together greater and less diameters; divide their sum by 2; multiply quotient by 3.1416.

To square of product of this circumference and number of revolutions of spiral, add square of height of its axis, and take square root of the sum.

Fig. 54.



Or, $\sqrt{(d + d' \div 2 \times 3.1416 n + h^2)} = \text{length of line.}$

EXAMPLE.—Greater and less diameters of a conical spiral, Fig. 54, are 20 and 2 inches; its height, *c d*, 10; and number of revolutions 10; what is length of it?

$\frac{20 + 2}{2} \div 2 = 11 \times 3.1416 = 34.5576 = \text{sum of diameters} \div 2, \text{ and } 3.1416; 34.5576 \times 10 = 345.576.$

Then $\sqrt{345.576^2 + 10^2} = 345.72 \text{ inches.}$

Spindles.

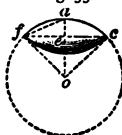
DEFINITION.—Figures generated by revolution of a plane area, revolved about a chord perpendicular to its axis, or about its diameter are designated by the name of the arc or curve from which they are generated, Circular, Elliptic, Parabolic, etc.

Circular Spindle.

To Compute Convex Surface of a Circular Spindle, Zone or Segment of it.—Figs. 55, 56, and 57.

- RULE.**—Multiply length by radius of revolving arc; multiply this arc by central distance, or distance between centre of spindle and centre of revolving arc; subtract this product from former, double remainder, and multiply it by 3.1416.

Fig. 55.



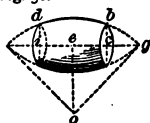
Or, $l r - \left(a \sqrt{r^2 - \left(\frac{c}{2} \right)^2} \right) 2 p = \text{surface}$, a representing length of arc, and c the spindle chord.

EXAMPLE.—What is surface of a circular spindle, Fig. 55, length of it, $f c$, being 14.142 inches, radius of its arc, $o c$, 10, and central distance, $o e$, 7.071?

$14.142 \times 10 = 141.42 = \text{length} \times \text{radius}$. Length of arc, $f a c$, by Rules, page 344 = 15.708.

$15.708 \times 7.071 = 111.0713 = \text{length of arc} \times \text{central distance}$; $141.42 - 111.0713 = 30.3487 = \text{difference of products}$. Then $30.3487 \times 2 \times 3.1416 = 190.687 \text{ square ins.}$

Fig. 56.



EXAMPLE.—What is convex surface of zone of a circular spindle, Fig. 56, length of it, $a b$, being 7.653 inches, radius of its arc, $o g$, 10, central distance, $o e$, 7.071, and length of its side, $a d$, 7.854 inches?

$7.653 \times 10 = 76.53 = \text{length} \times \text{radius}$; $7.854 \times 7.071 = 55.5356 = \text{length of arc} \times \text{central distance}$; $76.53 - 55.5356 = 20.9944 = \text{difference of products}$.

Then $20.9944 \times 2 \times 3.1416 = 131.912 \text{ square ins.}$

Zone.

Segment.

EXAMPLE.—What is convex surface of a segment of a circular spindle, Fig. 57, length of it, $a c$, being 3.2495 inches, radius of its arc, $o g$, 10, central distance, $o e$, 7.071, and length of its side, $a d$, 3.927 inches?

$3.2495 \times 10 = 32.495 = \text{length} \times \text{radius}$; $3.927 \times 7.071 = 27.7678 = \text{length of arc} \times \text{central distance}$; $32.495 - 27.7678 = 4.7272 = \text{difference of products}$.

Then $4.7272 \times 2 \times 3.1416 = 29.702 \text{ square ins.}$

GENERAL FORMULA.— $S = 2 (l r - a c) p = \text{surface}$, l representing length of spindle, segment, or zone, a length of its revolving arc, r radius of generating circle, and c central distance.

ILLUSTRATION.—Length of a circular spindle is 14.142 inches, length of its revolving arc is 15.708, radius of its generating circle is 10, and distance of its centre from centre of the circle from which it is generated is 7.071; what is its surface?

$2 \times (14.142 \times 10 - 15.708 \times 7.071) \times 3.1416 = 190.687 \text{ square inches.}$

NOTE.—Surface of a frustum of a spindle may be obtained by division of the surface of a zone.

Cycloidal Spindle.

To Compute Convex Surface of a Cycloidal Spindle.—Fig. 58.

—Multiply area of generating circle by 64, and divide it by 3.



Or, $\frac{a \times 64}{3} = \text{surface}$.

EXAMPLE.—Area of generating circle, $a b c$, of a cycloid $d e$, is 32 inches; what is surface of spindle?

$32 \times 64 = 2048 = \text{area of circle} \times 64$, and $2048 \div 3 =$

682.666 = surface of cycloidal spindle is twice area of

Ellipsoid, Paraboloid, or Hyperboloid of Revolution.

DEFINITION.—Figures alike to a cone, generated by revolution of a curve section around its axis.

NOTE.—These figures are usually known as *Conoids*.

When they are generated by revolution of an ellipse, they are termed *Ellipsoids*; and when by a parabola, *Paraboloids*, etc.

Revolution of an arc of a conic section around the axis of the curve will give a segment of a conoid.

Ellipsoid.

To Compute Convex Surface of an Ellipsoid.—Fig. 59.

RULE.—Add together square of base and four times square of height; multiply square root of half their sum by 3.1416, and this product by radius of the base.

Fig. 59.
$$\text{Or, } \sqrt{\frac{b^2 + 4h^2}{2}} \times 3.1416 \times r = \text{surface.}$$

EXAMPLE.—Base, a, b , of an ellipsoid, Fig. 59, is 20 inches, and vertical height, c, d , 7; what is its surface?

$10^2 + 7^2 \times 4 = 274 = \text{sum of square of base and 4 times square of height; } 274 \div 2 = 137. \text{ And } \sqrt{137} = 11.7047 \text{ square root of above sum. Then } 11.7047 \times 3.1416 \times 10 = 369.007 \text{ square ins.}$

To Compute Convex Surface of a Segment, Frustum, or Zone of an Ellipsoid.—Fig. 59.

See Rules for Convex Surface of a Segment, Frustum, or Zone of a spheroid or Ellipsoid, pages 343-9.

$$d \text{ or } d' \times 3.1416 \times h = \text{surface,}$$

$$\text{and } \frac{\text{mean diam.} \times h}{d \text{ or } d'} = h; \text{ then } d \times 3.1416 \times h = \text{surface.}$$

Paraboloid.

To Compute Convex Surface of a Paraboloid.—Fig. 60.

RULE.—From cube of square root of sum of four times square of height, and square of radius of base, subtract cube of radius of base; multiply remainder by quotient of 3.1416 times radius of base divided by six times square of height.

Fig. 60.
$$\text{Or, } (\sqrt{4h^2 + r^2})^3 - r^3 \times \frac{\pi \times r}{6 \times h^2} = \text{surface.}$$

EXAMPLE.—Axis, b, d , of a paraboloid, Fig. 60, is 20 inches; radius, a, c , of its base is 15 inches; what is its convex surface?

$40^2 \times 4 = 6400 = 4 \text{ times square of height; } 15^2 = 225 = \text{square of radius of base; } 6400 + 225 = 6625 = \text{sum of above product and square of radius of base; } \sqrt{6625} = 81.3953 = \text{square root of sum of 4 times square of height and square of radius of base; } 81.3953^3 = 545536 = \text{remainder of cube of root of sum of 4 times square of height and square of radius of base, less cube of radius of base; } 15^3 = 3375 = \text{cube of radius of base; } 545536 - 3375 = 542161 = \text{remainder of cube of root of sum of 4 times square of height and square of radius of base, less cube of radius of base; } 15 \div 6 \times 20 = 1.25 = \text{quotient of 3.1416 times radius of base divided by 6 times square of height; then } 542161 \times 1.25 = 3213.46 \text{ square ins.}$

Cylinder Sections.

To Compute Surface of a Cylinder Section.—Fig. 61.

RULE.—From entire surface of cylinder surface of the two ungulas, r, o, o, n , as per Fig. 61, and multiply result by 4.

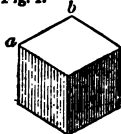
MENSURATION OF VOLUMES.

Cubes and Parallelopipeds.

Cube.

DEFINITION.—A volume contained by six equal square sides.

Fig. 1.



To Compute Volume of a Cube.—**Fig. 1.**
RULE.—Multiply a side of cube by itself, and that product again by a side.

Or, $s^3 = V$, s representing length of a side, and V volume.

EXAMPLE.—Side, a or b , Fig. 1, is 12 inches; what is volume?
 $12 \times 12 \times 12 = 1728$ cube ins.

Parallelopipedon.

DEFINITION.—A volume contained by six quadrilateral sides, every opposite of which are equal and parallel.

To Compute Volume of a Parallelopipedon.

—**Fig. 2.**

RULE.—Multiply length by breadth, and that product again by depth.

Or, $l b d = V$.



Prisms, Prismoids, and Wedges.

Prisms.

DEFINITION.—Volumes, ends of which are equal, similar, and parallel plan sides of which are parallelograms.

NOTE.—When ends of a prism or prismoid are triangles, it is termed a *triangular prism* or *prismoid*; when rhomboids, a *rhomboidal prism*, and when square, a *square prism*, etc.

Fig. 3.



To Compute Volume of a Prism.—**Figs. 3 and 4.**

RULE.—Multiply area of base by height.

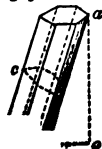
Or, $a h = V$.

EXAMPLE.—A triangular prism, $a b c$, Fig. 4, has sides of 2.5 feet, and a length, $c b$, of 10; what is its volume?

By Rule, page 339, $2.5^2 \times .433 = 2.70625 =$ area of end $a b$, and $2.70625 \times 10 = 27.0625$ cube feet.



Fig. 5.



When a Prism is Oblique or Irregular.

RULE.—Multiply area of an end by height, as a multiply area taken at a right angle to sides, as at actual length.

To Compute Volume of any Frustum of a Prism, whether Right or Oblique.—**Figs. 6 and 7.**



RULE.—Multiply area of base by perpendicular distances between it and centre of gravity of other end.

EXAMPLE.—Area of base, $a b c$, of frustum of a cylindrical prism, Fig. 6, is 15 inches; centre of gravity, c , is 12; what is its volume?

$2 = 120$ cube ins.

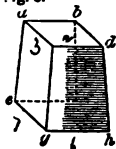
Prismoids.*

To Compute Volume of a Prismoid.—Fig. 8.

RULE.—To sum of areas of the two ends add four times area of middle section, parallel to them, and multiply this sum by one sixth of perpendicular height.

NOTE.—This is the general rule, and known as the *Prismoidal Formula*, and it applies equally to all figures of proportionate or dissimilar ends.

Fig. 8.



Or, $a + a' + 4m \times h \div 6 = V$, a and a' representing areas of ends, and m area of middle section.

EXAMPLE.—What is volume of a rectangular prismoid, Fig. 8, lengths and breadths, eg and gh , ab and bd , of two ends being 7×6 and 3×2 inches, and height 15 feet?

$7 \times 6 + 3 \times 2 = 42 + 6 = 48 = \text{sum of areas of two ends}$; $7 + 3 \div 2 = 5 = \text{length of middle section}$; $6 + 2 \div 2 = 4 = \text{breadth of middle section}$; $5 \times 4 \times 4 = 80 = \text{four times area of middle section}$.

$$\text{Then } 48 + 80 \times \frac{15 \times 12}{6} = 128 \times 30 = 3840 \text{ cube ins.}$$

NOTE 1.—Length and breadth of middle section are respectively equal to half sum of lengths and breadths of the two ends.

2.—Prismoids, alike to prisms, derive their designation from figure of their ends, as triangular, square, rectangular, pentagonal, etc.

When it is *Irregular or Oblique* and their ends are united by plane or curved surfaces, through which and every point of them, a right line may be drawn from one of the ends or parallel faces to the other.—Figs. 9, 10, and 11.

Fig. 9.

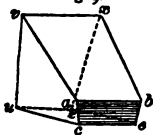


Fig. 10.

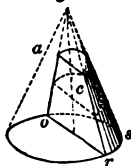
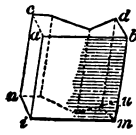


Fig. 11.



EXAMPLE.—Areas of ends, ac and ors , Fig. 10, $abcd$, and $imnu$, Fig. 11, and $bc e$ and $vxwz$, Fig. 9, are each 10 and 30 inches, that of their middle section q , and their perpendicular heights 18; what is their volume?

$$10 + 30 + 20 \times 4 = 120 = \text{sum of areas of ends} + 4 \text{ times middle section. And } 120 \times \frac{18}{6} = 360 \text{ cube ins.}$$

Wedge.

To Compute Volume of a Wedge.—Fig. 12.

RULE.—To length of edge add twice length of back; multiply this sum by perpendicular height, and then by breadth of back, and take one sixth product.

Fig. 12.



$$\text{Or, } (l + l' \times 2 \times h b) \div 6 = V.$$

EXAMPLE.—Length of edge of a wedge, eg , is 20 inches, back $abcd$, is 20 by 2, and its height, ef , 20; what is its volume?

$20 + 20 \times 2 = 60 = \text{length of edge added to twice length of back}$; $60 \times 20 \times 2 = 2400 = \text{above sum multiplied by height}$; that product by breadth of back.

$$\text{Then } 2400 \div 6 = 400 \text{ cube ins.}$$

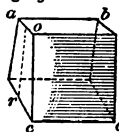
NOTE.—When a wedge is a true prism,

Fig. 12, volume of it is equal to area of an end multiplied by its

base or embankment of a road, when terminated by parallel cur.

To Compute Frustum of a Wedge.—Fig. 13.

Fig. 13.



RULE.—To sum of areas of both ends, add 4 times area of section parallel to and equally distant from both ends, and multiply sum by one sixth of length.

$$\text{Or, } A + a + 4a' \times \frac{l}{6} = V.$$

EXAMPLE.—Lengths of edge and back of a frustum of a wedge ab and cd are 20×1 and 20×2 ins., and height or is 20 ins.; what is its volume?

$$\frac{20 \times \frac{2+1}{2}}{2} \times 2 + 4 \times \left(20 \times \frac{2+1}{2}\right) \times \frac{20}{6} = 60 + 120 \times \frac{20}{6} = 600 \text{ cube ins.}$$

NOTE.—When frustum is a true prism, as represented Fig. 13, volume of it is equal to mean area of ends multiplied by its length.

Regular Bodies (Polyhedrons).

DEFINITION.—A regular body is a solid contained under a certain number of similar and equal plane faces,* all of which are equal regular polygons.

NOTE 1.—Whole number of regular bodies which can possibly be formed is five.

2.—A sphere may always be inscribed within, and may always be circumscribed about a regular body or polyhedron, which will have a common centre.

Fig. 14.

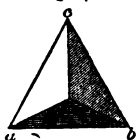


Fig. 15.

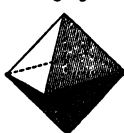


Fig. 16.

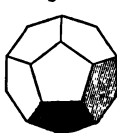


Fig. 17.



1. Tetrahedron, or Pyramid, Fig. 14, which has four triangular faces.
2. Hexahedron, or Cube, Fig. 15, which has six square faces.
3. Octahedron, Fig. 16, which has eight triangular faces.
4. Dodecahedron, Fig. 17, which has twelve pentagonal faces.
5. Icosahedron, Fig. 18, which has twenty triangular faces.

To Compute Elements of any Regular Body.—Figs. 14, 15, 16, and 17.

To Compute Radius of a Sphere that will Circumscribe a given Regular Body, or that may be Inscribed within it.

When Linear Edge is given. **RULE.**—Multiply it by multiplier opposite to body in columns A and B in following Table, under head of element required.

EXAMPLE.—Linear edge of a hexahedron or cube, Fig. 1, is 2 inches; what are of circumscribing and inscribed spheres?

$$2.86602 = 1.73204 \text{ inches} = \text{radius of circumscribing sphere}; 2 \times .5 = 1 \text{ inch} = \text{radius of inscribed sphere.}$$

When Surface is given. **RULE.**—Multiply square root of it by multiplier in columns C and D in following Table, under head of element required.

RULE.—Multiply cube root of it by multiplier in columns E and F in following Table, under head of element required.

* is termed dihedral angle.

*en one of the Radii of Circumscribing or Inscribed Sphere alone is re-
quired, the other being given.* RULE.—Multiply given radius by multiplier
opposite to body in columns G and H in Table, page 364, under head of
radius.

To Compute Linear Edge.

en Radius of Circumscribing or Inscribed Sphere is given. RULE.—
Multiply radius given by multiplier opposite to body in columns I and K in
page 364.

en Surface is given. RULE.—Multiply square root of it by multiplier
opposite to body in column L in Table, page 364.

en Volume is given. RULE.—Multiply cube root of it by multiplier
opposite to body in column M in Table, page 364.

To Compute Surface.

en Radius of Circumscribing Sphere is given. RULE.—Multiply square
root of it by multiplier opposite to body in column N in Table, page 364.

en Radius of Inscribed Sphere is given. RULE.—Multiply square of
radius given by multiplier opposite to body in column O in Table, page 364.

en Linear Edge is given. RULE.—Multiply square of edge by multi-
plier opposite to body in column P in Table, page 364.

en Volume is given. RULE.—Extract cube root of volume, and multi-
ply it by multiplier opposite to body in column Q in Table,
page 364.

To Compute Volume.

en Linear Edge is given. RULE.—Cube linear edge, and multiply it
by multiplier opposite to body in column R in Table, page 364.

en Radius of Circumscribing Sphere is given. RULE.—Multiply cube
root of radius given by multiplier opposite to body in column S in Table,
page 364.

en Radius of Inscribed Sphere is given. RULE.—Multiply cube of
radius given by multiplier opposite to body in column T in Table, page 364.

en Surface is given. RULE.—Cube surface given, extract square root,
multiply the root by multiplier opposite to body in column U in Table,
page 364.

Cylinder.



To Compute Volume of a Solid Cylinder.—
Fig. 18.

RULE.—Multiply area of base by height.

EXAMPLE.—Diameter of a cylinder, a , is 3 feet, and its length, b ,
7 feet; what is its volume?

Area of 3 feet = 7.068. Then $7.068 \times 7 = 49.476$ cube feet.

To Compute Volume of a Hollow Cylinder.

R.—Subtract volume of internal cylinder from that of cylinder.

Cone.



To Compute Volume of

Fig. 16
height,

RULE.—Multiply area of base
and take one third of product

EXAMPLE.—Diameter, a , of b
height, c , 32.5 inches; what is

of 15 inches = 176.7146. Then $\frac{176.715 \times 32.5}{3}$

Units for Elements of the Regular Bodies.

| | A. | B. | C. | D. | E. | F. | G. | H. | I. | K. |
|------------------|---|--|---|--|--|---|--|--|---|--|
| | Radius of Circumscribing Sphere. By Linear Edge. | Radius of Inscribed Sphere. By Linear Edge. | Radius of Circumscribing Sphere. By Surface. | Radius of Inscribed Sphere. By Surface. | Radius of Circumscribing Sphere. By Volume. | Radius of Inscribed Sphere. By Volume. | Radius of Inscribed Sphere. By Circumscribing Sphere. | Radius of Circumscribing Sphere. By Inscribed Sphere. | Linear Edge. By Radius of Circumscribing Sphere. | Linear Edge. By Radius of Inscribed Sphere. |
| Tetrahedron | .612 37 | .204 12 | .465 3 | .155 1 | 1.248 96 | .416 34 | .333 33 | 3 | 1.632 99 | 4.868 98 |
| Hexahedron | .866 02 | .5 | .353 55 | .204 12 | .866 02 | .5 | .577 35 | 1.732 05 | 1.154 7 | 2 |
| Octahedron | .707 11 | .408 25 | .379 92 | .219 35 | .908 06 | .524 56 | .577 35 | 1.732 05 | 1.414 21 | 2.449 49 |
| Dodecahedron .. | 1.401 26 | 1.113 52 | .308 39 | .245 07 | .710 75 | .564 8 | .704 65 | 1.258 41 | .713 64 | .898 06 |
| Icosahedron | .951 06 | .755 76 | .323 18 | .256 81 | .733 29 | .582 71 | .794 65 | 1.258 41 | 1.051 46 | 1.323 17 |
| | L. | M. | N. | O. | P. | Q. | R. | S. | T. | U. |
| FIGURE. | Linear Edge. By Surface. | Linear Edge. By Volume. | Surface. By Radius of Circumscribing Sphere. | Surface. By Radius of Inscribed Sphere. | Surface. By Linear Edge. | Surface. By Volume. | Volume. By Linear Edge. | Volume. By Radius of Circumscribing Sphere. | Volume. By Radius of Inscribed Sphere. | Volume. By Surface. |
| Tetrahedron ... | .759 84 | 2.039 55 | 4.618 8 | 41.569 22 | 1.732 05 | 7.205 62 | .117 85 | .513 2 | 13.856 41 | .051 7 |
| Hexahedron ... | .408 25 | 1 | 8 | 23 | 6 | 6 | 1 | 1.003 5 | 8 | .068 04 |

To Compute Volume of Frustum of a Cone.—Fig. 20.

RULE.—Add together squares of the diameters or circumferences of greater and lesser ends and product of the two diameters or circumferences; multiply their sum respectively by .7854 or .07958, and this product by height; then divide this last product by 3.

$$\text{Or, } d^2 + d'^2 + \overline{d \times d'} \times .7854 h \div 3 = V.$$

$$\text{Or, } c^2 + c'^2 + \overline{c \times c'} \times .07958 h \div 3 = V.$$

Fig. 20.



EXAMPLE.—What is volume of frustum of a cone, diameters of greater and lesser ends, $b d, a c$, being 5 and 3 feet, and height, $e o, g$?

$$5^2 + 3^2 + 5 \times 3 = 49; \text{ and } 49 \times .7854 = 38.4846 = \text{above sum by } .7854; \text{ and } \frac{38.4846 \times 9}{3} = 115.4538 \text{ cube feet.}$$

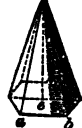
Pyramid.

NOTE.—Volume of a pyramid is equal to one third of that of a prism having equal base and altitude.

To Compute Volume of a Pyramid.—Fig. 21.

RULE.—Multiply area of base by perpendicular height, and take one third of product.

Fig. 21.



EXAMPLE.—What is the volume of a hexagonal pyramid, Fig. 21, a side, $a b$, being 40 feet, and its height, $e c, 60$?

$$40^2 \times 2.5981 \text{ (tabular multiplier, page 339)} = 4156.96 = \text{area of base.}$$

$$\frac{4156.96 \times 60}{3} = 83139.2 \text{ cube feet.}$$

To Compute Volume of Frustum of a Pyramid.—Fig. 22.

RULE.—Add together squares of sides of greater and lesser ends, and product of these two sides; multiply sum by tabular multiplier for areas in table, page 339, and this product by height; then divide last product by 3.

$$\text{Or, } s^2 + s'^2 + s \times s' \times \text{tab. mult.} \times h \div 3 = V.$$

When Areas of Ends are known, or can be obtained without reference to tabular multiplier, use following.

Fig. 22.



$$\text{Or, } a + a' + \sqrt{a \times a'} \times h \div 3 = V.$$

EXAMPLE.—What is the volume of the frustum of a hexagonal pyramid, Fig. 22, the lengths of the sides of the greater and lesser ends, $a b, c d$, being respectively 3.75 and 2.5 feet, and its perpendicular height, $e o, 7.5$?

$$3.75^2 + 2.5^2 = 20.3125 = \text{sum of squares of sides of greater and lesser ends; } 20.3125 + 3.75 \times 2.5 = 29.6875 = \text{above sum added product of the two sides; } 29.6875 \times 2.5981 \times 7.5 = 578.48 \times \text{tab. mult., and again by the height, which, } \div 3 = 192.83 \text{ cube feet.}$$

When Ends of a Pyramid are not those of a Regular Polygon, or when Areas of Ends are given

RULE.—Add together areas of the two ends and square root of their product; multiply sum by height, and take one third of product.

$$\text{Or, } a + a' + \sqrt{a \times a'} \times h \div 3 = V.$$

EXAMPLE.—What is the volume of an irregular-sided frustum of the two ends being 22 and 88 inches, and the length 20?

$$22 + 88 = 110 = \text{sum of areas of ends; } 22 \times 88 = 1936, \sqrt{1936} = 44 = \text{square root of areas. Then } \frac{110 + 44 \times 20}{3} = 1026.66 \text{ cu. in.}$$

Spherical Pyramid.

A Spherical Pyramid is that part of a sphere included within three or more adjoining plane surfaces meeting at centre of sphere. The spherical polygon defined by these plane surfaces of pyramid is termed the base, and the lateral faces are sectors of circles.

NOTE.—To compute the Elements of Spherical Pyramids, see Docharty and Hackley's Geometry.

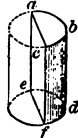
Cylindrical Ungulas.

DEFINITION.—Cylindrical Ungulas are frusta of cylinders. Conical Ungulas are frusta of cones.

To Compute Volume of a Cylindrical Ungula.—Fig. 23.

1. When Section is parallel to Axis of Cylinder. RULE.—Multiply area of base by height of the cylinder.

Fig. 23.



$$\text{Or, } a h = V.$$

EXAMPLE.—Area of base, $d e f$, Fig. 23, of a cylindrical ungula is 15.5 inches, and its height, $a e$, 20; what is its volume?

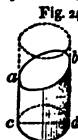
$$15.5 \times 20 = 310 \text{ cube ins.}$$

2. When Section passes Obliquely through opposite sides of Cylinder, Fig. 24. RULE.—Multiply area of base of cylinder by half sum of greatest and least lengths of ungula.

$$\text{Or, } a \times \frac{l + l'}{2} = V.$$

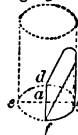
EXAMPLE.—Area of base, $c d$, of a cylindrical ungula, Fig. 24, is 25 inches, and the greater and less heights of it, $a c$, $b d$, are 15 and 17; what is its volume?

$$25 \times \frac{15 + 17}{2} = 400 \text{ cube ins.}$$



3. When Section passes through Base of Cylinder and one of its Sides, and Versed Sine does not exceed Sine, or the Base is equal to or less than a Semicircle, Fig. 25. RULE.—From two thirds of cube of sine of half arc of base subtract product of area of base and cosine * of half arc. Multiply difference thus found by quotient arising from height divided by versed sine.

Fig. 25.



$$\text{Or, } \frac{2 \sin^3}{3} - a c \times \frac{h}{v \sin} = V, v \sin. \text{ representing versed sine.}$$

EXAMPLE.—Sine, $a d$, of half arc, $d e f$, of base of an ungula, Fig. 25, is 5 inches, diameter of cylinder is 10, and height, $e g$, of ungula 10; what is its volume?

Two thirds of $5^3 = 83.333 = \text{two thirds of cube of sine}$. As versed sine and radius of base are equal, cosine is 0. Hence, area of base \times cosine = 0, and $83.333 - 0 \times 10 \div 5 = 166.666 \text{ cube ins.}$

4. When Section passes through Base of Cylinder, and Versed Sine exceeds Radius, or when the Base exceeds a Semicircle, Fig. 26. RULE.—To two thirds of cube of sine of half arc of base add product of area of base and cosine. Multiply sum thus found by quotient arising from height, divided by versed sine.

$$\text{Or, } \frac{2 \sin^3}{3} + a c \times \frac{h}{v \sin} = V.$$

EXAMPLE.—Sine, $a d$, of half arc of an ungula, Fig. 26, is 12 inches, sine, $a g$, is 16, height, $g c$, 10, and diameter of cylinder 15

what is its volume?

$$\begin{aligned} \frac{2}{3} \text{ of } 12^3 &= 1152 = \text{two thirds of cube of sine of half arc of} \\ &= 131.78; 1152 + 131.78 \times 16 = 2312.23 = \\ &= \text{of sine of half the arc of base, and product of} \\ &= 2312.23 \times 10 \div 16 = 1445.14 \text{ cube ins.} \end{aligned}$$

5. When Section passes Obliquely through both Ends of Cylinder, Fig. 27.
RULE.—Conceive section to be continued till it meets side of cylinder produced; then, as the difference of versed sines of the arcs of the two ends of ungula is to the versed sine of arc of less end, so is the height of cylinder to the part of side produced.

Ascertain volume of each of the ungulas by Rules 3 and 4, and take their difference.

Fig. 27.



Or, $\frac{v. \sin. h}{v. \sin. - v. \sin.} = h', v. \sin. \text{ and } v. \sin. \text{ representing versed sines of arcs of the two ends, } h \text{ height of cylinder, and } h' \text{ height of part produced.}$

EXAMPLE.—Versed sines, ae, do , and sines, e and o , of arcs of two ends of an ungula, Fig. 27, are assumed to be respectively 8.5 and 25, and 11.5 and 0 inches, length of ungula, bo , within cylinder, cut from one having 25 inches diameter, do , is 20 inches; what is height of ungula produced beyond cylinder, and what is volume of it?

$25 \sim 8.5 : 8.5 :: 20 : 10.303 = \text{height of ungula produced beyond cylinder.}$

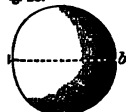
Greater ungula, sine o being 0, versed sine = the diameter. Base of ungula being a circle of 25 inches diameter, area = 490.875. Versed sine and diameter of base being equal (25), sine = 0. $490.875 \times 25 \sim \frac{25}{2} = 6135.9375 = \text{product of area of base and cosine, or excess of versed sine over sine of base.}$ $30.303 \div 25 = 1.21212 = \text{quotient of height} \div \text{versed sine.}$

Then $6135.9375 \times 1.21212 = 7437.4926 \text{ cube inches}$; and by Rules 3 and 4, volumes of less and greater ungulas = 515.444, and $6922.0486 = 7437.4926 \text{ cube inches.}$

Sphere.

DEFINITION.—A solid, surface of which is at a uniform distance from the centre.

Fig. 28.



To Compute Volume of a Sphere.—Fig. 28.

RULE.—Multiply cube of diameter by .5236.

Or, $d^3 \times .5236 = V, d \text{ representing diameter.}$

EXAMPLE.—What is volume of a sphere, Fig. 28, its diameter, ab , being 10 inches?

$10^3 = 1000$, and $1000 \times .5236 = 523.6 \text{ cube ins.}$

To Compute Volume of a Hollow Sphere.

RULE.—Subtract volume of internal space from that of sphere.

Or, $V - v = \text{volume.}$

Segment of a Sphere.

DEFINITION.—A section of a sphere.

To Compute Volume of a Segment of a Sphere.—Fig. 29.

RULE 1.—To three times square of radius of its base add square of height; multiply this sum by height, and product by .5236.

Fig. 29.



Or, $3r^2 + h^2 \times h \times .5236 = V.$

2.—From three times diameter of sphere subtract twice height of segment; multiply this remainder by height, and product by .5236.

Or, $3d - 2h \times h^2 \times .5236 =$

EXAMPLE.—Segment of a sphere, Fig. 29, inches for its base, and a height, bo , of 4;

$7^2 \times 3 + 4^2 = 163 = \text{the sum of three times height; } 163 \times 4 \times .5236 = 331.3872 \text{ cube ins.}$

Spherical Zone (or Frustum of a Sphere).

DEFINITION.—Part of a sphere included between two parallel chords.

To Compute Volume of a Spherical Zone.—Fig. 30.

DEFINITION.—Part of a sphere included between two parallel planes.

RULE.—To sum of squares of the radii of the two ends add one third of square of height of zone; multiply this sum by height, and again by 1.5708.

Fig. 30.

$$\text{Or, } r^2 + r'^2 + \frac{h^2}{3} \times h \times 1.5708 = V.$$



EXAMPLE.—What is the volume of a spherical zone, Fig. 30, greater and less diameters, fh and de , being 20 and 15 inches, and distance between them, or height of zone, cg , being 10 ins?

$10^2 + 7.5^2 = 156.25 = \text{sum of squares of radii of the two ends.}$
 $156.25 + 10^2 \div 3 = 189.58 = \text{above sum added to one third of square of the height.}$

Then $189.58 \times 10 \times 1.5708 = 2977.9226 \text{ cube ins.}$

Cylindrical Ring.

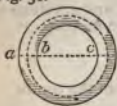
DEFINITION.—A ring formed by the curvature of a cylinder.

To Compute Volume of a Cylindrical Ring.—Fig. 31.

RULE.—To diameter of body of ring add inner diameter of ring; multiply sum by square of diameter of body, and product by 2.4674.

Fig. 31.

$$\text{Or, } d + d' \times d^2 \times 2.4674 = V.$$



Or, $a \times l = V$, a representing area of section of body, and l length of axis of body..

EXAMPLE.—What is volume of an anchor ring, Fig. 31, diameter of metal, ab , being 3 inches, and inner diameter of ring, bc , 8?

$3 + 8 \times 3^2 = 99 = \text{product of sum of diameters and square of diameter of body of ring.}$

Then $99 \times 2.4674 = 244.2726 \text{ cube ins.}$

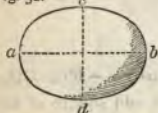
Spheroids (Ellipsoids).

DEFINITION.—Solids generated by the revolution of a semi-ellipse about one of its diameters. When the revolution is about the transverse diameter they are termed Prolate, and when about the conjugate they are Oblate.

To Compute Volume of a Spheroid.—Fig. 32.

RULE.—Multiply square of revolving axis by fixed axis, and this product by .5236.

Fig. 32.



Or, $a^2 \times a' \times .5236 = V$, a and a' representing revolving and fixed axes.

Or, $4 \div 3 \times 3.1416 \times r^2 \times r' = V$, r and r' representing semi-axis

EXAMPLE.—In a prolate spheroid, Fig. 32, fixed axis, ab , 14 inches, and revolving axis, cd , 10; what is its volume?

$10^2 \times 14 = 1400 = \text{product of square of revolving axis and fixed axis.}$ Then $1400 \times .5236 = 733.04 \text{ cube ins.}$


NOTE.—Volume of a spheroid is equal to $\frac{2}{3}$ of a cylinder that will circumscribe it.

Segments of Spheroids.

To Compute Volume of Segment of a Spheroid.—Fig. 33.

When Base, ef , is Circular, or parallel to revolving Axis, as cd , Fig. 33, or as ef to Axis ab , Fig. 34. RULE.—Multiply fixed axis by 3, height of segment by 2, and subtract one product from the other; multiply remainder by square of height of segment, and product by .5236. Then, as square of fixed axis is to square of revolving axis, so is last product to volume of segment.


$$\text{Or, } \frac{3a - 2h^2 \times .5236 \times a^2}{a^3} = V.$$

 **EXAMPLE**—In a prolate spheroid, Fig. 33, fixed or transverse axis, $a b$, is 100 inches, revolving or conjugate, $c d$, 60, and height of segment, $a o$, 10; what is its volume?

$100 \times 3 - 10 \times 2 = 280 = \text{twice the height of segment subtracted from three times fixed axis}; 280 \times 10^2 \times .5236 = \text{product of above remainder, square of height, and .5236. Then } 59.8 : 5277.888 \text{ cube ins.}$

$e f$, is Elliptical, or perpendicular to revolving Axis, $a b$, Fig. to Axis $c d$, Fig. 34. **RULE**.—Multiply fixed axis by 3, segment by 2, and subtract one from the other; multiply reverse of height of segment, and product by .5236. Then, as revolving axis, so is last product to volume of segment.

$$\text{Or, } \frac{3a' - 2h^2 \times .5236 \times a}{a^2} = V.$$

 **EXAMPLE**—Diameters of an oblate spheroid, Fig. 34, are 100 and 60 inches, and height of a segment thereof is 12; what is its volume?

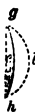
$100 \times 3 - 12 \times 2 = 296 = \text{twice the height of the segment subtracted from three times the revolving axis}; 296 \times 12^2 \times .5236 = \text{product of above remainder, the square of height, and .5236. } \therefore 20809.9584 : 12485.975 \text{ cube ins.}$

Frusta of Spheroids.

to Volume of Middle Frustum of a Spheroid.—
Fig. 35.

$e f$ and $g h$, are Circular, or parallel to revolving Axis, as $c d$, Fig. 36. **RULE**.—To twice square of revolving axis add neter of either end; multiply this sum by length of frustum, y .2618.


$$\text{Or, } 2a'^2 + d^2 \times l .2618 = V.$$

 **EXAMPLE**—Middle frustum of a prolate spheroid, Fig. 35, is 36 inches in length, diameter of it being, in middle, $c d$, 50, and at its ends, $e f$ and $g h$, 40; what is its volume?

$50^2 \times 2 + 40^2 = 6600 = \text{sum of twice square of middle diameter added to square of diameter of ends. Then } 6600 \times 36 \times .2618 = 62203.68 \text{ cube ins.}$

$e f$ and $g h$, are Elliptical, or perpendicular to revolving Axis, $e f$ and $g h$ to Axis, $c d$, Fig. 36. **RULE**.—To twice product and conjugate diameters of middle section, add product of 1 conjugate of either end; multiply this sum by length of product by .2618.

$$\text{Or, } d' \times 2 + \overline{d d'} l \times .2618 = V.$$

 **EXAMPLE**—In middle frustum of a prolate spheroid, Fig. 36, diameters of its middle section are 50 and 30 inches, its ends 40 and 24, and its length, $o t$, 18; what is its volume?

$50 \times 30 \times 2 = 3000 = \text{twice product of transverse and conjugate diameters}; 3000 + 40 \times 24 = 3960 = \text{sum of above product and product of transverse and conjugate diameters of ends.}$

$\therefore 3960 \times .2618 = 10367.104 \text{ cube ins.}$

Links.

DEFINITION.—Elongated or Elliptical rings.

Elongated or Elliptical Links.

To Compute Volume of an Elongated or Elliptical Link.
—Figs. 37 and 38.

RULE.—Multiply area of a section of the body of link by its length or circumference of its axis.

Or, $a l$ or $c = V$.

NOTE.—By Rule, page 353, Circumference or length of axis of an Elongated Link = the sum of 3.1416 times sum of less diameter added to thickness of ring multiplied by twice remainder of less diameter subtracted from greater.

Also, Circumference or length of axis of an Elliptical ring = square root of sum of diameters added to thickness of ring or axes squared $\times 3.1416$.

Fig. 37.

EXAMPLE.—Elongated link of a chain, Fig. 37, is 1 inch in diameter of body, a , and its inner diameters, b , c and e , are 10 and 2.5 inches. What is its volume?Area of 1 inch = .7854; $2.5 + 1 \times 3.1416 = 10.9956 = 3.1416$ times of less diameter and thickness of ring = length of axis of ends; $10 \times 2 = 15 =$ twice remainder of the less diameter subtracted from greater = length of sides of body.Then $10.9956 + 15 = 25.9956 =$ length of axis of length.Hence $.7854 \times 25.9956 = 20.417$ cube ins.

2.—Elliptical link of a chain, Fig. 38, is of the same dimensions as preceding; what is its volume?

 $2.5 + 1 + 10 + 1 = 133.25 =$ diameter of axes squared; $\sqrt{\frac{133.25}{2}} = 3.1416$
 $= 25.643 =$ square root of half sum of diameters squared $\times 3.1416 =$ circumference of axis of ring. Area of 1 inch = .7854.Then $25.643 \times .7854 = 20.14$ cube ins.

Spherical Sector.

DEFINITION.—A figure generated by the revolution of a sector of a circle about a straight line through the vertex of the sector as an axis.

NOTE.—Arc of sector generates surface of a zone, termed base of sector of sphere, and the radii generate surfaces of two cones, having a vertex in common with the sector at the centre of the sphere.

To Compute Volume of a Spherical Sector.—Fig. 39.

RULE.—Multiply external surface of zone, which is base of sector, by one third of the radius of sphere.

Or, $a r \div 3 = V$, a representing area of base.

NOTE.—Surface of a spherical sector = sum of surface of zone and surfaces of two cones.

Fig. 39.

EXAMPLE.—What is volume of a spherical sector, 39, generated by sector, $c a h$, height of zone, $a b e$, and radius, $g h$, of sphere 15? $12 \times 94.248 = 1130.976 =$ height of zone \times circumference of sphere = external surface of zone (see page 350). $1130.976 \times 15 \div 3 =$ surface \times one third of radius = 5654.88 cube ins.

Spindles.

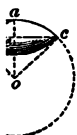
DEFINITION.—Figures generated by revolution of a plane area bounded by a curve when the curve is revolved about a chord perpendicular to its axis or about a double ordinate, and they are designated by the name of arc from which they are generated, as Circular, Elliptic, Parabolic, etc.

Circular Spindle.

Compute Volume of a Circular Spindle.—Fig. 40.

E.—Multiply central distance by half area of revolving segment;† product from one third of cube of half length, and multiply result by 12.5664.

$$\frac{\div 2)^3}{3} - \left(c \times \frac{a}{2}\right) \times 12.5664 = V, a \text{ representing area of revolving segment.}$$



EXAMPLE.—What is volume of a circular spindle, Fig. 40, when central distance, *oe*, is 7.071 067 inches, length, *fc*, 14.142 13, and radius, *oc*, 10?

NOTE.—Area of revolving segment; *fc*, being = side of square that can be inscribed in a circle of 20, is $20^2 \times .7854 = 14.142 13^2 \div 4 = 28.54$ area.

$7.071 067 \times 28.54 \div 2 = 100.9041 =$ central distance \times half area of revolving segment; $\frac{7.071 67^3}{3} - 100.9041 = 16.947 =$ remainder of product and one third of cube of half length.

Then $16.947 \times 12.5664 = 212.9628$ cube ins.

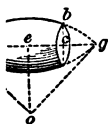
Frustum or Zone of a Circular Spindle.*

Compute Volume of a Frustum or Zone of a Circular Spindle.—Fig. 41.

E.—From square of half length of whole spindle take one third of half length of frustum, and multiply remainder by said half length of whole spindle; multiply central distance by revolving area which generates frustum; subtract this product from former, and multiply remainder by

$$\frac{\div 2 - \frac{l' \div 2}{3}}{2} \times \frac{l'}{2} - (c \times a) \times 6.2832 = V, l \text{ and } l' \text{ representing lengths of whole spindle, and } a \text{ area of revolving section of frustum.}$$

—Revolving area of frustum can be obtained by dividing its plane into a circle and a parallelogram.



EXAMPLE.—Length of middle frustum of a circular spindle, *ic*, Fig. 41, is 6 inches; length of spindle, *fg*, is 8; central distance, *oe*, is 3; and area of revolving or generating segment is 10; what is volume of frustum?

$(8 \div 2)^2 - \frac{(6 \div 2)^2}{3} = 13$, and $13 \times 3 = 39 =$ product of $\frac{l}{2}$ length of frustum, and remainder of one third square of half length of frustum subtracted from square of half length of whole spindle.

$39 - 3 \times 10 = 9 =$ product of central distance and area of segment subtracted from preceding product.

Then $9 \times 6.2832 = 56.5488$ cube ins.

Segment of a Circular Spindle.

Compute Volume of a Segment of a Circular Spindle.—Fig. 42.

E.—Subtract length of segment from half length of spindle; double result, and ascertain volume of a middle frustum of this length. Subtract from volume of whole spindle, and halve remainder.†

$$l - c \div 2 = V, C \text{ and } c \text{ representing volume of spindle and middle frustum.}$$

†The frustum of a Circular Spindle is one of the various forms of casks, and is applicable to segment of any Spindle or any Conoid, volume of the figure and frustum obtained.

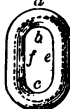
Links.

DEFINITION.—Elongated or Elliptical rings.

Elongated or Elliptical Links.

To Compute Volume of an Elongated or Elliptical Link.
—Figs. 37 and 38.

RULE.—Multiply area of a section of the body of link by its length, or circumference of its axis.

Or, $a l$ or $c = V$.NOTE.—By Rule, page 353, Circumference or length of axis of an Elongated link = the sum of 3.1416 times sum of less diameter added to thickness of ring, and product of twice remainder of less diameter subtracted from greater.Also, Circumference or length of axis of an Elliptical ring = square root of half sum of diameters added to thickness of ring or axes squared $\times 3.1416$.Fig. 37. EXAMPLE.—Elongated link of a chain, Fig. 37, is 1 inch in diameter of body, $a b$, and its inner diameters, $b c$ and $e f$, are 10 and 2.5 inches; what is its volume?Area of 1 inch = $.7854$; $2.5 + 1 \times 3.1416 = 10.9956 = 3.1416$ times sum of less diameter and thickness of ring = length of axis of ends; $10 - 2.5 \times 2 = 15 =$ twice remainder of the less diameter subtracted from greater = length of sides of body.Then $10.9956 + 15 = 25.9956 =$ length of axis of length.Hence $.7854 \times 25.9956 = 20.417$ cube ins.

2.—Elliptical link of a chain, Fig. 38, is of the same dimensions as preceding; what is its volume?

$$\frac{2.5 + 1}{2} + \frac{10 + 1}{2} = 133.25 = \text{diameter of axes squared}; \sqrt{\frac{133.25}{2}} \times 3.1416$$

$$= 25.643 = \text{square root of half sum of diameters squared} \times 3.1416 = \text{circumference of axis of ring. Area of } 1 \text{ inch} = .7854.$$
Then $25.643 \times .7854 = 20.14$ cube ins.

Fig. 38.



Spherical Sector.

DEFINITION.—A figure generated by the revolution of a sector of a circle about a straight line through the vertex of the sector as an axis.

NOTE.—Arc of sector generates surface of a zone, termed base of sector of a sphere, and the radii generate surfaces of two cones, having a vertex in common with the sector at the centre of the sphere.

To Compute Volume of a Spherical Sector.—Fig. 39.

RULE.—Multiply external surface of zone, which is base of sector, by one third of the radius of sphere.

Or, $a r \div 3 = V$, a representing area of base.

—Surface of a spherical sector = sum of surface of zone and surfaces of the

EXAMPLE.—What is volume of a spherical sector, Fig. 39, generated by sector, $c a h$, height of zone, $a b c d$, being $a o$, 12 inches, and radius, $g h$, of sphere 15 ? $12 \times 94.248 = 1130.976 =$ height of zone \times circumference of sphere = external surface of zone (see page 350). $1130.976 \times 15 \div 3 =$ surface \times one third of radius = 5654.88 cube ins.

Spindles.

—Plane area bounded by a curve near to its axis or about it & arc from which they

Circular Spindle.

Compute Volume of a Circular Spindle.—Fig. 40.

Multiply central distance by half area of revolving segment; product from one third of cube of half length, and multiply by 12.5664.

$$\frac{l^3}{3} - \left(c \times \frac{a}{2} \right) \times 12.5664 = V, a \text{ representing area of revolving segment.}$$

EXAMPLE.—What is volume of a circular spindle, Fig. 40, when central distance, $o e$, is 7.071 067 inches, length, $f c$, 14.142 13, and radius, $o c$, 10?



NOTE.—Area of revolving segment; $f c$, being side of square that can be inscribed in a circle of 20, is $20^2 \times .7854 = 14.142 13^2 \div 4 = 28.54$ area.

$$7.071 067 \times 28.54 \div 2 = 100.9041 = \text{central distance} \times \text{half area of revolving segment}; \frac{7.071 067^3}{3} - 100.9041 = 16.947 = \text{remainder of}$$

act and one third of cube of half length.

$$\text{Then } 16.947 \times 12.5664 = 212.9628 \text{ cube ins.}$$

Frustum or Zone of a Circular Spindle.*

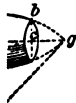
Compute Volume of a Frustum or Zone of a Circular Spindle.—Fig. 41.

From square of half length of whole spindle take one third of half length of frustum, and multiply remainder by said half length; multiply central distance by revolving area which generates n ; subtract this product from former, and multiply remainder by

$$\frac{l'^2 \div 2}{3} \times \frac{l'}{2} - (c \times a) \times 6.2832 = V, l \text{ and } l' \text{ representing lengths of}$$

of frustum, and a area of revolving section of frustum.

Revolving area of frustum can be obtained by dividing its plane into a circle and a parallelogram.



EXAMPLE.—Length of middle frustum of a circular spindle, $i c$, Fig. 41, is 6 inches; length of spindle, $f g$, is 8; central distance, $o e$, is 3; and area of revolving or generating segment is 10; what is volume of frustum?

$$(8 \div 2)^2 - \frac{(6 \div 2)^2}{3} = 13, \text{ and } 13 \times 3 = 39 = \text{product of } \frac{1}{2} \text{ length of frustum, and remainder of one third square of half length of frustum subtracted from square of half length of}$$

$- 3 \times 10 = 9 = \text{product of central distance and area of segment subtracting product.}$

$$\text{Then } 9 \times 6.2832 = 56.5488 \text{ cube ins.}$$

Segment of a Circular Spindle.

Compute Volume of a Segment of a Circular Spindle.—Fig. 42.

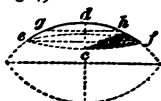
Subtract length of segment from half length and ascertain volume of a middle frustum; subtract from volume of whole spindle, and halve $l \div 2 = V, C$ and c representing volume of spindle

NOTE.—A Segment of a Circular Spindle is one of the various forms of a Spindle, and is applicable to segment of any Spindle or any Conoid, etc.

To Compute Volume of a Segment of a Parabolic Spindle.—Fig. 49.

RULE.—Add together square of diameter of base of segment and square of double diameter in middle between base and vertex; multiply sum by height of segment, and product by .1309.

Fig. 49.



$$\text{Or, } d^2 + d'^2 l \times .1309 = V.$$

EXAMPLE.—Segment of a parabolic spindle, Fig. 49, has diameters, ef and gh , of 15 and 8.75 inches, and height cd , is 2.5; what is its volume?

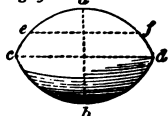
$15^2 + 8.75 \times 2 = 531.25 = \text{sum of squares of base end of double diameter in middle of segment. Then } 531.25 \times 2.5 \times .1309 = 173.852 \text{ cube ins.}$

Hyperbolic Spindle.

To Compute Volume of a Hyperbolic Spindle.—Fig. 50.

RULE.—To square of diameter add square of double diameter at one fourth of its length; multiply sum by length, and product by .1309.*

Fig. 50.



$$\text{Or, } d^2 + 2 d'^2 l \times .1309 = V.$$

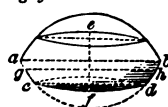
EXAMPLE.—Length, ab , Fig. 50, of a hyperbolic spindle is 100 inches, and its diameters, cd and ef , are 150 and 110; what is its volume?

$150^2 + 110 \times 2 \times 100 = 709000 = \text{product of sum of squares of greatest diameter and of twice diameter at one fourth of length of spindle and length. Then } 709000 \times .1309 = 928081 \text{ cube inches.}$

To Compute Volume of Middle Frustum of a Hyperbolic Spindle.—Fig. 51.

RULE.—Add together squares of greatest and least diameters and square of double diameter in middle between the two; multiply this sum by length, and product by .1309.†

Fig. 51.



$$\text{Or, } d^2 + d'^2 + (2 d'')^2 l \times .1309 = V.$$

EXAMPLE.—Diameters, ab and cd , of middle frustum of hyperbolic spindle, Fig. 51, are 150 and 110 inches; diameter, gh , 140; and length, ef , 50; what is its volume?

$150^2 + 110^2 + 140 \times 2 = 113000 = \text{sum of squares of greatest and least diameters and of double middle diameter. Then } 113000 \times 50 \times .1309 = 739585 \text{ cube ins.}$

To Compute Volume of a Segment of a Hyperbolic Spindle.—Fig. 52.

RULE.—Add together square of diameter of base of segment and square of double diameter in middle between base and vertex; multiply sum by height of segment, and product by .1309.

$$\text{Or, } d^2 + d'^2 l \times .1309 = V.$$

EXAMPLE.—Segment of a hyperbolic spindle, Fig. 52, has diameters, ef and gh , of 110 and 65 inches, and its length, ab , 25; what is its volume?

$110^2 + 65 \times 2 = 29000 = \text{sum of squares of diameter of base and of double middle diameter. Then } 29000 \times 25 \times .1309 = 94902.5 \text{ cube ins.}$

Ellipsoid, Paraboloid, and Hyperboloid of Revolution* (Conoids).

DEFINITION.—Figures like to a cone, described by revolution of a conic section round and at a right angle to plane of their fixed axes.

Ellipsoid of Revolution (Spheroid).

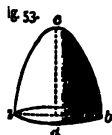
DEFINITION.—An ellipsoid of revolution is a semi-spheroid. (See page 368.)

Paraboloid of Revolution.†

To Compute Volume of a Paraboloid of Revolution.—
Fig. 53.

RULE.—Multiply area of base by half height.

$$\text{Or, } a h \div 2 = V.$$



NOTE.—This rule will hold for any segment of paraboloid, whether base be perpendicular or oblique to axis of solid.

EXAMPLE.—Diameter, a b , of base of a paraboloid of revolution, Fig. 53, is 20 inches, and its height, d c , 20; what is its volume?

Area of 20 inches diameter of base = 314.16. Then $314.16 \times 20 \div 2 = 3141.6$ cube ins.

Frustum of a Paraboloid of Revolution.

To Compute Volume of a Frustum of a Paraboloid of Revolution.—Fig. 54.

Fig. 54

RULE.—Multiply sum of squares of diameters by height of frustum, and this product by .3927.

$$\text{Or, } (d^2 + d'^2) h \times .3927 = V.$$



EXAMPLE.—Diameters, a b and d c , of the base and vertex of frustum of a paraboloid of revolution, Fig. 54, are 20 and 11.5 inches, and its height, e f , 12.6; what is its volume?

$20^2 + 11.5^2 = 532.25 = \text{sum of squares of diameters.}$ Then $532.25 \times 12.6 \times .3927 = 2633.5837$ cube ins.

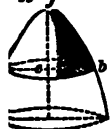
Segment of a Paraboloid of Revolution.

To Compute Volume of Segment of a Paraboloid of Revolution.—Fig. 55.

Fig. 55

RULE.—Multiply area of base by half height.

$$\text{Or, } a \times h \div 2 = V.$$



NOTE.—This rule will hold for any segment of paraboloid, whether base be perpendicular or oblique to axis of solid.

EXAMPLE.—Diameter, a b , of the base of a segment of a paraboloid of revolution; Fig. 55, is 11.5 inches, and its height, e f , 17.4; what is its volume?

Area of 11.5 inches diameter of base = 103.869. Then $103.869 \times 17.4 \div 2 = 904.315$ cube ins.

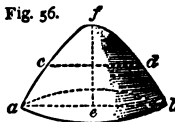
Hyperboloid of Revolution

To Compute Volume of a Hyperboloid of Revolution.—Fig. 56.

RULE.—To square of radius of base add square a , by this sum by height, and product by .5236.

* They have been known as Conoids. For the definition of Paraboloid of Revolution is .5 of its circumference.

Fig. 56.



Or, $r^2 + d^2 \div 2 \times h \times .5236 = V$, d representing middle diameter
EXAMPLE.—Base, $a b$, of a hyperboloid of revolution, Fig. 56, is 80 inches; middle diameter, $c d$, 66; and height, $e f$, 60; what is its volume?

$80 \div 2 + 66^2 = 5956 = \text{sum of square of radius of base and middle diam.}$ Then $5956 \times 60 \times .5236 = 87\ 113.7$ cube ins.

Segment of a Hyperboloid of Revolution.

To Compute Volume of Segment of a Hyperboloid of Revolution, as Fig. 56.

RULE.—To square of radius of base add square of middle diameter; multiply this sum by height, and product by .5236.

Or, $r^2 + d'^2 \div 2 \times h \times .5236 = V$, r representing radius of base.

EXAMPLE.—Radius, $a e$, of base of a segment of a hyperboloid of revolution, as Fig. 56, is 21 inches; its middle diameter, $c d$, is 30; and its height, $e f$, 15; what is its volume?

$21^2 + 30^2 \times 15 = 20115 = \text{product of sum of squares of radius of base and middle diameter multiplied by height.}$ Then $20115 \times .5236 = 10\ 532.214$ cube ins.

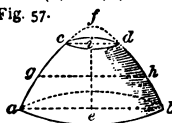
Frustum of a Hyperboloid of Revolution.

To Compute Volume of Frustum of a Hyperboloid of Revolution.—Fig. 57.

RULE.—Add together squares of greatest and least semi-diameters and square of diameter in middle of the two; multiply this sum by height, and product by .5236.

Or, $\left(\frac{d}{2}\right)^2 + \left(\frac{d'}{2}\right)^2 + d''^2 \div 2 \times h \times .5236 = V$, d , d' , and d'' representing several diameters.

Fig. 57.



EXAMPLE.—Frustum of a hyperboloid of revolution, Fig. 57, is in height, $e f$, 50 inches; diameters of greater and lesser ends, $a b$ and $c d$, are 110 and 42; and that of middle diameter, $g h$, is 80; what is volume?

$110 \div 2 = 55$, and $42 \div 2 = 21$. Hence $55^2 + 21^2 + 80^2 = 9866 = \text{sum of squares of semi-diameters of ends and of middle diam.}$ Then $9866 \times 50 \times .5236 = 258\ 291.88$ cube ins.

Any Figure of Revolution.

To Compute Volume of any Figure of Revolution.—Fig. 58.

RULE.—Multiply area of generating surface by circumference described by its centre of gravity.

Or, $a \ 2 \pi \ p = V$, r representing radius of centre of gravity.

Fig. 58.

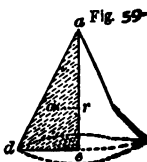


ILLUSTRATION 1.—If generating surface, $a b c d$, of cylinder, Fig. 58, is 5 inches in width and 10 in height, then will $a b = 5$ and $b d = 10$, and centre of gravity will be in o , the radius of which is $r o = 5 \div 2 = 2.5$. Hence $10 \times 5 = 50 = \text{area of generating surface.}$

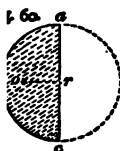
Then $50 \times 2.5 \times 2 \times 3.1416 = 785.4 = \text{area of generating surface} \times \text{circumference of its centre of gravity} = \text{volume of cylinder.}$

\therefore cylinder 10 inches in diameter and 10 .7854 = 78.54, and $78.54 \times 10 = 785.4$.

\therefore of a cone, Fig. 59, is $a e = 10$, $d e = \text{area of triangle} = 10 \times 5 \div 2 = 25$, o , and $o r$, by Rule, page 607,



$\therefore \text{taking surface} \times \text{circumference}$



3.—If generating surface of a sphere, Fig. 60, is abc , and $ae = 10$, abc will be $\left(\frac{10^2 \times .7854}{2}\right) = 39.27$, centre of gravity of which is in o , and by Rule, page 607, $or = 2.122$.

Hence, $39.27 \times 2.122 \times 2 \times 3.1416 = 523.6 = \text{area of generating surface} \times \text{circumference of its centre of gravity} = \text{volume of sphere}$.

Irregular Bodies.

To Compute Volume of an Irregular Body.

RULE.—Weigh it both in and out of fresh water, and note difference in s .; then, as 62.5^* is to this difference, so is 1728^\dagger to number of cube inches in body.

Or, divide difference in lbs. by 62.5 , and quotient will give volume in ab feet.

NOTE.—If salt water is to be used, ascertained weight of a cube foot of it, or 64 , is to be used for 62.5 .

EXAMPLE.—An irregular-shaped body weighs 15 lbs. in water, and 30 out; what is its volume in cube inches?

$30 - 15 = 15 = \text{difference of weights in and out of water}$.

$62.5 : 15 :: 1728 : 414.72 = \text{volume in cube ins.}$

Or, $15 \div 62.5 = .24$, and $.24 \times 1728 = 414.72 = \text{volume in cube ins.}$

CASK GAUGING.

Varieties of Casks.

To Compute Volume of a Cask.

1st Variety. Ordinary form of middle frustum of a Prolate Spheroid.

This class comprises all casks having a spherical outline of staves, as Rum puncheons, Whiskey barrels, etc.

RULE.—To twice square of bung diameter add square of head diameter; multiply this sum by length of the cask, and product by $.2618$, and it will give volume in cube inches, which, being divided by 231 , will give result in ab gallons.

2d Variety. Middle frustum of a Parabolic Spindle.

This class comprises all casks in which curve of staves quickens at the chime, as Brandy casks and Provision barrels.

RULE.—To square of a head diameter add double square of bung diameter, and from sum subtract $.4$ of square of difference of diameters; multiply remainder by length, and product by $.2618$, which, being divided by 231 , will give volume in gallons.

3d Variety. Middle frustum of a Paraboloid.

This class comprises all casks in which curve of staves quickens slightly s ab , as Wine casks.

RULE.—To square of bung diameter add square of head diameter; multiply sum by length, and product by $.3927$, which, being divided by 2 , will give volume in gallons.

4th Variety. Two equal frustums of Cones.

This class comprises all casks in which curve of staves quickens sharply s ab , as Gin pipes.

RULE.—To square of difference of diameters to three times sum by length, and product by $.06566$, inches, which, being divided by 231 , will

ab of fresh water.

1728

\dagger Number of inch.

EXAMPLE.—Bung and head diameters of a cask are 24 and 16 inches, and length 36; what is its volume in gallons?

$\frac{24 - 16 + (24 + 16)^2 \times 3}{11 \ 497 \ 329}$, which $\times 36 = 175 \ 104$, and $175 \ 104 \times .06566 = 11 \ 497 \ 329$, which $\div 231 = 49 \ 77$ gallons.

Generally.

$Dd + M^2 .001 \ 692 \ L = U. \ S. \ gallons$, and $.001 \ 416 \ 2 = Imperial \ gallons$.

D, d, and M representing interior, head and bung diameters, and L length of c in inches.

To Ascertain Mean Diameter of a Cask.

RULE.—Subtract head diameter from bung diameter in inches, and multiply difference by following units for the four varieties; add product head diameter, and sum will give mean diameter of varieties required.

| | | | |
|------------------|-----|------------------|-----|
| 1st Variety..... | .7 | 3d Variety..... | .56 |
| 2d Variety..... | .68 | 4th Variety..... | .52 |

EXAMPLE.—Bung and head diameters of a cask of 1st variety are 24 and 20 in es; what is its mean diameter?

$24 - 20 = 4$, and $4 \times .7 = 2.8$, which, added to 20, = 22.8 ins.

ULLAGE CASKS.

To Compute Volume of Ullage Casks.

When a cask is only partly filled, it is termed an *ullage cask*, and is considered in two positions, viz., as lying on its side, when it is termed a *Segment Lying*, or as standing on its end, when it is termed a *Segment Standing*.

To Ullage a Lying Cask.

RULE.—Divide wet inches (depth of liquid) by bung diameter; find quotient in column of versed sines in table of circular segments, page 267, take its corresponding segment; multiply this segment by capacity of c in gallons, and product by 1.25 for ullage required.

EXAMPLE.—Capacity of a cask is 90 gallons, bung diameter being 32 inches; w is its volume at 8 inches depth?

$8 \div 32 = .25$, tab. seg. of which is .153 55, which $\times 90 = 13 \ 8195$, and again $\times 1.2 = 17 \ 2744$ gallons.

To Ullage a Standing Cask.

RULE.—Add together square of diameter at surface of liquor, square head diameter, and square of double diameter taken in middle between two; multiply sum by wet inches, and product by .1309, and divide by: for result in gallons.

To Compute Volume of a Cask by Four Dimension

RULE.—Add together squares of bung and head diameters, and square double diameter taken in middle between bung and head; multiply the s by length of cask, and product by .1309, and divide this product by 231 result in gallons.

To Compute Volume of any Cask from Three Dimensions only.

Divide into one sum 39 times square of bung diameter, 25 times square of head diameter, and 26 times product of the two diameters; multiply sum by length of cask, and product by .008 726; and divide quotient by 231.

and for description and use
307-23.

CONIC SECTIONS.

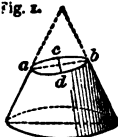
A *Cone* is a figure described by revolution of a right-angled triangle about one of its legs, or it is a solid having a circle for its base, and terminated in a vertex.

Conic Sections are figures made by a plane cutting a cone.

If a cone is cut by a plane through vertex and base, section will be a triangle, and if cut by a plane parallel to its base, section will be a circle.

Axis is line about which section revolves. *Base* is circle which is described by revolving base of triangle.

Fig. 2.



An *Ellipse* is a figure generated by an oblique plane cutting a cone above its base.

Transverse axis or diameter is longest right line that can be drawn in it, as *ab*, Fig. 1.

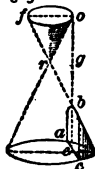
Conjugate axis or diameter is a line drawn through centre of ellipse perpendicular to transverse axis, as *cd*.

A *Parabola* is a figure generated by a plane cutting a cone parallel to its side, as *abc*, Fig. 2.

Axis is a right line drawn from vertex to middle of base, as *bo*.

NOTE.—A parabola has not a conjugate diameter.

Fig. 3.



A *Hyperbola* is a figure generated by a plane cutting a cone at any angle with base greater than that of side of cone, as *abc*, Fig. 3.

Transverse axis or diameter, *ob*, is that part of axis, *eb*, which, if continued, as at *o*, would join an opposite cone, *ofr*.

Conjugate axis or diameter is a right line drawn through centre, *g*, of transverse axis, and perpendicular to it.

Straight line through foci is indefinite transverse axis; that part of it between vertices of curves, as *ob*, is definite transverse axis. Its middle point, *g*, is centre of curve.

Eccentricity of a hyperbola is ratio obtained by dividing distance from centre to either focus by semi-transverse axis.

Parameter is cord of curve drawn through focus at right angles to axis.

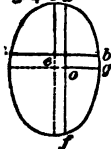
Asymptotes of a hyperbola are two right lines to which the curve continually approaches, touches at an infinite distance but does not pass; they are prolongations of diagonals of rectangle constructed on extremes of the axes.

Two hyperbolas are *conjugate* when transverse axis of one is conjugate of the other, and contrariwise.

General Definitions.

An *Ordinate* is a right line from any point of a curve to either of diameters *cd*, and *a b* and *d f*, are double ordinates; *cb*, Fig. 5, is an *ordinate* and *a b* an *abscissa*.

Fig. 4. c d



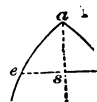
An *Abscissa* is that part of diameter which is contained between vertex and an ordinate, as *ce*, *go*, Fig. 4, and *a b*, Fig. 5.

Parameter of any diameter is equal to four times distance from focus to vertex of curve; parameter of axis is least possible, and is termed parameter of curve.

Parameter of curve of a conic section is equal to chord of curve drawn through focus perpendicular to axis.

Parameter of transverse axis is least, and is termed parameter

Parameter of a conic section and foci are sufficient elements to determine the curve.



A *Focus* is a point on principal axis where double ordinate to axis, through p is equal to parameter, as ef , Fig. 5.

It may be determined arithmetically thus: Divide square of ordinate by times abscissa, and quotient will give focal distances, as and s , in preceding fig. Fig. 6.

Directrix of a conic section is a right line at right angle to major axis, and it is in such a position that

$$f : g :: u : o.$$

Here ad , Fig. 6, is directrix, and o is offset to directrix.

Latus Rectum, or principal parameter, passes through a focus it is a double ordinate, which is a third proportion to the a

$$\text{Or, } A : a :: a : L.$$

A and a representing major and minor axes. (See *Har Mensuration*, page 232.)

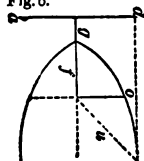
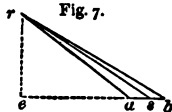


Fig. 7.



A *Conoid* is a warped surface generated by a r line being moved in such a manner that it will touch a straight line and curve, and continue parallel given plane. Straight line and curve are called *rectrices*, plane a *directrix*, and moving line *generatrix*.

Thus, let $ab a'$, Fig. 7, be a circle in a horizontal plane and $d a'$ projection of right lines perpendicular to a vertical plane, $r' b e$; if right lines, $d a$, $r s$, $r' b$, $r'' s$, and be moved so as to touch circle and right line $d a'$ at constantly parallel to plane $r' b e$, it will generate a conoid $d a b a' d'$.

Radii vectores are lines drawn from the foci to any point in the curve; her *radius vector* is one of these lines.

Traced angle is angle formed by the radii vectores and the transverse diameter

Ellipsoid, Paraboloid, and Hyperboloid of Revolution—Figures generated by the revolution of an ellipse, parabola, etc., around their axes. (See *Illustration of Surfaces and Solids*, pages 357-75.)

NOTE 1.—All figures which can possibly be formed by cutting of a cone are mentioned in these definitions, and are five following—viz., a *Triangle*, a *Circle*, an *Ellipse*, a *Parabola*, and a *Hyperbola*; but last three only are termed *Conic Sections*.

2.—In *Parabola* parameter of any diameter is a third proportional to abscissa and ordinate of any point of curve, abscissa and ordinate being referred to diameter and tangent at its vertex.

3.—In *Ellipse* and *Hyperbola* parameter of any diameter is a third proportional to diameter and its conjugate.

To Determine Parameter of an Ellipse or Hyperbola

Fig. 8.



RULE.—Divide product of conjugate diameter, multiplied by itself, by transverse, and quotient is equal to parameter.

In annexed Figs. 8 and 9, of an *Ellipse* and *Hyperbola*, transverse and conjugate diameters, ab , cd , are each 30 and 20.

Then $30 : 20 :: 20 : 13.333 = \text{parameter}$.

Parameter of curve $= ef$, a double ordinate passing through focus, e .



Ellipse.

Describe Ellipses. (See *Geometry*, page 226.)

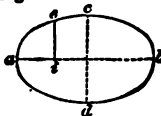
Compute Terms of an Ellipse.

If f and e of an Ellipse are given, viz., Transverse axis, and its Abscissa, to ascertain

To Compute Ordinate.

Transverse and Conjugate Diameters and Abscissa being given. RULE.—As transverse diameter is to conjugate, so is square root of product of abscissæ to ordinate which divides them.

Fig. 10.



EXAMPLE.—Transverse diameter, $a b$, of an ellipse, is 25; conjugate, $c d$, 16; and abscissa, $a i$, 7; what length of ordinate, $i e$?

$$25 - 7 = 18 \text{ less abscissa; } \sqrt{7 \times 18} = 11.225.$$

Hence $25 : 16 :: 11.225 : 7.184$ ordinate.

Or, $\sqrt{c^2 - \left(\frac{c x}{t}\right)^2} = \text{any ordinate, } c \text{ and } t \text{ represent semi-conjugate and transverse diameters, and } x \text{ distance of ordinate from centre figure.}$

To Compute Abscissæ.

Transverse and Conjugate Diameters and Ordinate being given. RULE.—As conjugate diameter is to transverse, so is square root of difference of squares of ordinate and semi-conjugate to distance between ordinate and centre; and this distance added to, or subtracted from, semi-transverse, will give abscissæ required.

EXAMPLE.—Transverse diameter, $a b$, of an ellipse, Fig. 10, is 25; conjugate, 16; and ordinate, $i e$, 7.184; what is abscissa, $i b$?

$$\sqrt{8^2 - 7.184^2} = 3.519943. \text{ Hence, as } 16 : 25 :: 3.52 : 5.5.$$

Then $25 \div 2 = 12.5$, and $12.5 + 5.5 = 18 = b i$, } abscissa.

$25 \div 2 = 12.5$, and $12.5 - 5.5 = 7 = a i$, }

To Compute Transverse Diameter.

Conjugate, Ordinate, and Abscissa being given. RULE.—To or from semi-conjugate, according as great or less abscissa is used, add or subtract square root of difference of squares of ordinate and semi-conjugate. Then, as this sum or difference is to abscissa, so is conjugate to transverse.

EXAMPLE.—Conjugate diameter, $c d$, of an ellipse, Fig. 10, is 16; ordinate 7.184; and abscissæ, $b i$ and $a i$, 18 and 7; what is length of transverse diameter?

$$\sqrt{(16 \div 2)^2 - 7.184^2} = 3.52.$$

$16 \div 2 + 3.52 : 18 :: 16 : 25$; $16 \div 2 - 3.52 : 7 :: 16 : 25$ transverse diameter

To Compute Conjugate Diameter.

Transverse, Ordinate, and Abscissa being given. RULE.—As square root of product of abscissæ is to ordinate, so is transverse diameter to conjugate.

EXAMPLE.—Transverse diameter, $a b$, of an ellipse, Fig. 10, is 25; ordinate 7.184; and abscissæ, $b i$ and $a i$, 18 and 7; what is length of conjugate diameter?

$$\sqrt{18 \times 7} = 11.225. \text{ Hence } 11.225 : 7.184 :: 25 : 16 \text{ conjugate diameter.}$$

To Compute Circumference of an Ellipse.

RULE.—Multiply square root of half sum of the squares of two diameters by 3.1416 .

EXAMPLE.—Transverse and conjugate diameters, $a b$ and $c d$, of an ellipse, Fig. 10, are 24 and 20; what is its circumference?

$$\frac{24^2 + 20^2}{2} = 488, \text{ and } \sqrt{488} = 22.09. \text{ Hence } 22.09 \times 3.1416 = 69.398 \text{ circumference.}$$

To Compute Area of an Ellipse.

RULE.—Multiply the diameters together, and the product by $.7854$. Or one diameter by $.7854$, and the product by the other.

EXAMPLE.—The transverse diameter of an ellipse, $a b$, Fig. 10, is 24; and conjugate, $c d$, 20; what is its area?

$$24 \times 20 \times .7854 = 84.8232 \text{ area.}$$

NOTE.—Area of an ellipse is a mean proportional between diameter of one being major axis and of the other minor axis.

ILLUSTRATION.—Area of circle of 40 = 1256.64; area of circle of 20 = 314.16, mean of the two circles 12. Therefore the conjugate diameter of an ellipse of an area of 100, being 40, is 25 feet, as $40 \times 25 \times .7854 = 785.4$ sq. ins.

To Compute Transverse Diameter,

Conjugate, Ordinate, and an Abscissa being given. RULE.—Add square of o to square of semi-conjugate, and extract square root of their sum.

Take sum or difference of semi-conjugate and this root, according as gre lesser abscissa is used. Then, as square of ordinate is to product of absc conjugate, so is sum or difference above ascertained to transverse diameter re

NOTE.—When the greater abscissa is used, the difference is taken, as trariwise.

EXAMPLE.—Conjugate diameter, $d f$, of a hyperbola, Fig. 15, is 72; ordin 48; and lesser abscissa, $a e$, 40; what is length of transverse diameter, $a t$?

$$\sqrt{48^2 + (72 \div 2)^2} = 60, \text{ and } 60 + 72 \div 2 = 96 \text{ lesser abscissa, and } 40 \times 72 = \\ \text{Hence, } 48^2 : 2880 :: 96 : 120 \text{ transverse diameter.}$$

To Compute Length of any Arc of a Hyperbola, mencing at Vertex.

RULE.—To 19 times transverse diameter add 21 times parameter of axis.

To 9 times transverse diameter add 21 times parameter, and multiply these sums respectively by quotient of lesser abscissa divided by transverse diameter.

To each of products thus ascertained add 15 times parameter, and divide by latter; then this quotient, multiplied by ordinate, will give length of arc,

NOTE.—To Compute Parameter, divide square of conjugate by transverse diameter.

Fig. 16. **EXAMPLE.**—In hyperbola, $a b c$, Fig. 16, transverse diameter conjugate, 72, ordinate, $e c$, 48, and lesser abscissa, $a e$, 40; what is length of arc, $a b$?

$$\frac{72^2}{120} = 43.2 \text{ parameter. } 120 \times 19 + 43.2 \times 21 \times \frac{40}{120} = 1062. \\ \frac{120 \times 9 + 43.2 \times 21 \times \frac{40}{120}}{43.2 \times 15} = 662.4. \text{ Then } 1062.4 + 43.2 \times 15 = \\ + 43.2 \times 15 = 1,305, \text{ which } \times 48 = 62.64 \text{ length.}$$

NOTE.—As transverse diameter is to conjugate, so is conjugate to parameter. (See Rule, page 380.)

To Compute Area of a Hyperbola,

Transverse, Conjugate, and Lesser Abscissa being given. RULE.—To product of transverse diameter and lesser abscissa add five sevenths of square of this and multiply square root of sum by 21.

Add 4 times square root of product of transverse diameter and lesser abscissa product last ascertained, and divide sum by 75.

Divide 4 times product of conjugate diameter and lesser abscissa by transverse diameter, and this last quotient, multiplied by former, will give area, nearly.

EXAMPLE.—Transverse diameter of a hyperbola, Fig. 16, is 60, conjugate lesser abscissa or height, $a e$, 20; what is area of figure?

$$60 \times 20 + \frac{5}{7} \text{ of } 20^2 = 1485.7143, \text{ and } \sqrt{1485.7143 \times 21} = 809.43, \text{ and } \sqrt{60} \\ 4 + 809.43 = 901.02, \text{ which } \div 75 = 12.0136 \text{ and } \frac{36 \times 20 \times 4}{60} \times 12.0136 = 576.65$$

For ordinates of a parabola in divisions of eighths and tenths, see page 229.

Delta Metal.

Delta Metal is an improved composition of Aluminium and its alloys; malleable, capable of being cast, forged, and hot rolled.

Tensile Strength per Sq. Inch.

| | | | |
|---------------------|------------|---------------------------|------|
| Cast, annealed..... | 48380 lbs. | Hot rolled, annealed..... | 605 |
| Hot rolled..... | 75200 " | Wrought, No. 22 W.G..... | 1400 |

PLANE TRIGONOMETRY.

By *Plane Trigonometry* is ascertained how to compute or determine any of the seven elements of a plane or rectilinear triangle from the other three, for when any three of them are given, one of which being side or the area, the remaining elements may be determined; and this operation is termed *Solving the Triangle*.

The determination of the mutual relation of the Sines, Tangents, Secants, &c., of the sums, differences, multiples, etc., of arcs or angles is also classed under this head.

For Diagram and Explanation of Terms, see Geometry, pp. 219-21.

Right-angled Triangles.

For Solution by Lines and Areas, see Mensuration of Areas, Lines, and Surfaces, pp. 335-39.

To Compute a Side.

When a Side and its Opposite Angle is given. RULE.—As sine of angle opposite given side is to sine of angle opposite required side, so is given side to required side.

To Compute an Angle.

RULE.—As side opposite to given angle is to side opposite to required angle, so is sine of given angle to sine of required angle.

To Compute Base or Perpendicular in a Right-angled Triangle.

When Angles and One Side next Right Angle are given. RULE.—As radius is to tangent of angle adjacent to given side, so is this side to other side.

To Compute the other Side.

When Two Sides and Included Angle are given. RULE.—As sum of two given sides is to their difference, so is tangent of half sum of their opposite angles to tangent of half their difference; add this half difference to half sum, to ascertain greater angle; and subtract half difference from half sum, to ascertain less angle. The other side may then be ascertained by Rule above.

To Compute Angles.

When Sides are given. RULE.—As one side is to other side, so is radius to tangent of angle adjacent to first side.

To Compute an Angle.

When Three Sides are given. RULE 1.—Subtract sum of logarithms of sides which contain required angle, from 20; to remainder add logarithm of half sum of three sides, and that of difference between this half sum and side opposite to required angle. Half the sum of these three logarithmic sines is logarithmic cosine of half required angle. The other angles may be ascertained by Rule above.

2.—Subtract sum of logarithms of two sides and half sum of the three sides, from 20, and to remainder add logarithm of half required angle.

NOTE.—In all ordinary cases either of these rules will serve. Rule 1 should be used when required angle is less than 90° .

in require
—

EXAMPLE.—The sides of a triangle are 3, 4, and 5; what are the angles of the hypotenuse?

20 — (Log. 4 = .60206 + Log. 5 = .69897) = 18.69897; Log. 3 + 4 + 5 + 2 — 4 = .30103; and Log. 3 + 4 + 5 + 2 — 5 = 0.

Then 18.69897 + .30103 = 19, which + 2 = 9.5 = log. sin. of half angle = 18° 26', which $\times 2 = 36^\circ 52'$ angle.

Hence $90^\circ - 36^\circ 52' = 53^\circ 8'$ remaining angle.

In following figures, 1 and 2:

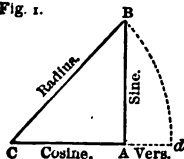
A = 90° , B = 45° , C = 45° , Radius = 1, Secant = 1.4142, Cosine = .7071, Sin. $45^\circ = .7071$, Tangent = 1, Area = .25.

By Sin., Tan., Sec., etc., A B, etc., is expressed Sine, Tangent, Secant, etc., of angles, A, B, etc.

To Compute Sides A C and B C.—Figs. 1 and 2.

When Hyp., Side B A, and Angles B and C are given.

Fig. 1.



$$\frac{\text{Sin. } B \times \text{B A}}{\text{Sin. } C} = \text{A C.}$$

$$\text{B A} \times \text{Cot. } C = \text{A C.}$$

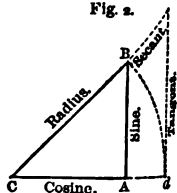
$$\text{Hyp.} \times \text{Cos. } C = \text{A C.}$$

$$\text{Hyp.} \times \text{Sin. } B = \text{A C.}$$

$$\frac{\text{B A}}{\text{Sin. } C} = \text{B C.}$$

$$\frac{\text{A C}}{\text{Sin. } B} = \text{B C.}$$

Fig. 2.



To Compute Side A C and Angles.

When Hyp. and Side B A are given.—Fig. 1 and 2.

$$\frac{\text{A C}}{\text{Hyp.}} = \text{Sin. } B.$$

$$\frac{\text{B A}}{\text{Hyp.}} = \text{Sin. } C.$$

$$\frac{\text{B A} \times \text{Sin. } B}{\text{Sin. } C} = \text{A C.}$$

$$\text{B C} \times \text{Sin. } B = \text{A C}$$

To Compute Side B C and Hyp. or Angles.

When both Sides are given.—Fig. 2.

$$\frac{\text{A C}}{\text{B A}} = \text{Tan. } B.$$

$$\frac{\text{B A}}{\text{Sin. } C} = \text{B C.}$$

$$\sqrt{\text{A C}^2 + \text{B A}^2} = \text{B C.}$$

$$\frac{\text{B A}}{\text{A C}} = \text{Tan. } C.$$

$$\frac{\text{B A}}{\text{B C}} = \text{Sin. } C.$$

$$\frac{\text{A C}}{\text{B C}} = \text{Sin. } B.$$

To Compute Sides.—Figs. 3 and 4.

Fig. 3.



When a Side and an Angle are given.

$$\text{B C} \times \text{Cos. } B = \text{B A.}$$

$$\text{B C} \times \text{Sin. } B = \text{A C.}$$

$$\text{A B} \times \text{Sec. } B = \text{B C.}$$

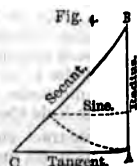
$$\frac{\text{A C} \times \text{Tan. } C}{\text{Rad.}} = \text{B A.}$$

$$\frac{\text{A C} \times \text{Sin. } C}{\text{Sin. } B} = \text{B A.}$$

$$\frac{\text{A C} \times \text{Sec. } C}{\text{Rad.}} = \text{B C.}$$

$$\frac{\text{A C} \times \text{Rad.}}{\text{Sin. } B} = \text{B C.}$$

Fig. 4.



a right-angled triangle, C A, is assumed to be radius to that radius; Or, dividing each of the side and secant of C respectively to radii

| | 40° | | 50° | | 60° | | 70° | | Prop. parts | 4 |
|----|----------|----------|----------|----------|----------|----------|----------|----------|-------------|---|
| | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | | |
| 0 | .06976 | .99756 | .08716 | .93619 | .10453 | .99452 | .12187 | .99255 | 60 | 4 |
| 1 | .07005 | .99754 | .08745 | .93617 | .10482 | .99449 | .12216 | .99251 | 59 | 4 |
| 2 | .07034 | .99752 | .08774 | .93614 | .10511 | .99446 | .12245 | .99248 | 58 | 4 |
| 3 | .07063 | .9975 | .08803 | .93612 | .1054 | .99443 | .12274 | .99244 | 57 | 4 |
| 4 | .07092 | .99748 | .08831 | .93609 | .10569 | .9944 | .12302 | .9924 | 56 | 4 |
| 5 | .07121 | .99746 | .0886 | .93607 | .10597 | .99437 | .12331 | .99237 | 55 | 4 |
| 6 | .0715 | .99744 | .08889 | .93604 | .10626 | .99434 | .1236 | .99233 | 54 | 4 |
| 7 | .07179 | .99742 | .08918 | .93602 | .10655 | .99431 | .12389 | .9923 | 53 | 4 |
| 8 | .07208 | .9974 | .08947 | .93599 | .10684 | .99428 | .12418 | .99226 | 52 | 3 |
| 9 | .07237 | .99738 | .08976 | .93596 | .10713 | .99424 | .12447 | .99222 | 51 | 3 |
| 10 | .07266 | .99736 | .09005 | .93594 | .10742 | .99421 | .12476 | .99219 | 50 | 3 |
| 11 | .07295 | .99734 | .09034 | .93591 | .10771 | .99418 | .12504 | .99215 | 49 | 3 |
| 12 | .07324 | .99731 | .09063 | .93588 | .108 | .99415 | .12533 | .99211 | 48 | 3 |
| 13 | .07353 | .99729 | .09092 | .93586 | .10829 | .99412 | .12562 | .99208 | 47 | 3 |
| 14 | .07382 | .99727 | .09121 | .93583 | .10858 | .99409 | .12591 | .99204 | 46 | 3 |
| 15 | .07411 | .99725 | .0915 | .9358 | .10887 | .99406 | .1262 | .992 | 45 | 3 |
| 16 | .0744 | .99723 | .09179 | .93578 | .10916 | .99402 | .12649 | .99197 | 44 | 3 |
| 17 | .07469 | .99721 | .09208 | .93575 | .10945 | .99399 | .12678 | .99193 | 43 | 3 |
| 18 | .07498 | .99719 | .09237 | .93572 | .10973 | .99396 | .12706 | .99189 | 42 | 3 |
| 19 | .07527 | .99716 | .09266 | .9357 | .11002 | .99393 | .12735 | .99186 | 41 | 3 |
| 20 | .07556 | .99714 | .09295 | .93567 | .11031 | .9939 | .12764 | .99182 | 40 | 3 |
| 21 | .07585 | .99712 | .09324 | .93564 | .1106 | .99386 | .12793 | .99178 | 39 | 3 |
| 22 | .07614 | .9971 | .09353 | .93562 | .11089 | .99383 | .12822 | .99175 | 38 | 3 |
| 23 | .07643 | .99708 | .09382 | .93559 | .11118 | .9938 | .12851 | .99171 | 37 | 2 |
| 24 | .07672 | .99705 | .09411 | .93556 | .11147 | .99377 | .1288 | .99167 | 36 | 2 |
| 25 | .07701 | .99703 | .0944 | .93553 | .11176 | .99374 | .12908 | .99163 | 35 | 2 |
| 26 | .0773 | .99701 | .09469 | .93551 | .11205 | .9937 | .12937 | .9916 | 34 | 2 |
| 27 | .07759 | .99699 | .09498 | .93548 | .11234 | .99367 | .12966 | .99156 | 33 | 2 |
| 28 | .07788 | .99696 | .09527 | .93545 | .11263 | .99364 | .12995 | .99152 | 32 | 2 |
| 29 | .07817 | .99694 | .09556 | .93542 | .11292 | .9936 | .13024 | .99148 | 31 | 2 |
| 30 | .07846 | .99692 | .09585 | .9354 | .11321 | .99357 | .13053 | .99144 | 30 | 2 |
| 31 | .07875 | .99689 | .09614 | .93537 | .11350 | .99354 | .13081 | .99141 | 29 | 2 |
| 32 | .07904 | .99687 | .09642 | .93534 | .11378 | .99351 | .1311 | .99137 | 28 | 2 |
| 33 | .07933 | .99685 | .09671 | .93531 | .11407 | .99347 | .13139 | .99133 | 27 | 2 |
| 34 | .07962 | .99683 | .097 | .93528 | .11436 | .99344 | .13168 | .99129 | 26 | 2 |
| 35 | .07991 | .9968 | .09729 | .93526 | .11465 | .99341 | .13197 | .99125 | 25 | 2 |
| 36 | .0802 | .99678 | .09758 | .93523 | .11494 | .99337 | .13226 | .99122 | 24 | 2 |
| 37 | .08049 | .99676 | .09787 | .9352 | .11523 | .99334 | .13254 | .99118 | 23 | 2 |
| 38 | .08078 | .99673 | .09816 | .93517 | .11552 | .99331 | .13283 | .99114 | 22 | 1 |
| 39 | .08107 | .99671 | .09845 | .93514 | .1158 | .99327 | .13312 | .9911 | 21 | 1 |
| 40 | .08136 | .99668 | .09874 | .93511 | .11609 | .99324 | .13341 | .99106 | 20 | 1 |
| 41 | .08165 | .99666 | .09903 | .93508 | .11638 | .9932 | .1337 | .99102 | 19 | 1 |
| 42 | .08194 | .99664 | .09932 | .93506 | .11667 | .99317 | .13399 | .99098 | 18 | 1 |
| 43 | .08223 | .99661 | .09961 | .93503 | .11696 | .99314 | .13427 | .99094 | 17 | 1 |
| 44 | .08252 | .99659 | .0999 | .935 | .11725 | .9931 | .13456 | .99091 | 16 | 1 |
| 45 | .08281 | .99657 | .10019 | .99497 | .11754 | .99307 | .13485 | .99087 | 15 | 1 |
| 46 | .0831 | .99654 | .10048 | .99494 | .11783 | .99303 | .13514 | .99083 | 14 | 1 |
| 47 | .08339 | .99652 | .10077 | .99491 | .11812 | .993 | .13543 | .99079 | 13 | 1 |
| 48 | .08368 | .99649 | .10106 | .99488 | .1184 | .99297 | .13572 | .99075 | 12 | 1 |
| 49 | .08397 | .99647 | .10135 | .99485 | .11869 | .99293 | .136 | .99071 | 11 | 1 |
| 50 | .08426 | .99644 | .10164 | .99482 | .11898 | .9929 | .13629 | .99067 | 10 | 1 |
| 51 | .08455 | .99642 | .10192 | .99479 | .11927 | .99286 | .13658 | .99063 | 9 | 1 |
| 52 | .08484 | .99639 | .10221 | .99476 | .11956 | .99283 | .13687 | .99059 | 8 | 1 |
| 53 | .08513 | .99637 | .1025 | .99473 | .11985 | .99279 | .13716 | .99055 | 7 | 0 |
| 54 | .08542 | .99635 | .10279 | .9947 | .12014 | .99276 | .13744 | .99051 | 6 | 0 |
| 55 | .08571 | .99632 | .10308 | .99467 | .12043 | .99272 | .13773 | .99047 | 5 | 0 |
| 56 | .086 | .9963 | .10337 | .99464 | .12071 | .99269 | .13802 | .99043 | 4 | 0 |
| 57 | .08629 | .99627 | .10366 | .99461 | .121 | .99265 | .13831 | .99039 | 3 | 0 |
| 58 | .08658 | .99625 | .10395 | .99458 | .12129 | .99262 | .1386 | .99035 | 2 | 0 |
| 59 | .08687 | .99622 | .10424 | .99455 | .12158 | .99258 | .13889 | .99031 | 1 | 0 |
| 60 | .08716 | .99619 | .10453 | .99452 | .12187 | .99255 | .13917 | .99027 | 0 | 0 |
| | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | | |
| | 86° | | 84° | | 83° | | 82° | | | |

| Prop. as parts. | ° | 80° | | 90° | | 100° | | 110° | | |
|--------------------|----|----------|----------|----------|----------|----------|----------|----------|----------|----|
| | | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | |
| 0 | 0 | .13917 | .99027 | .15643 | .98769 | .17365 | .98481 | .19081 | .98163 | 60 |
| 0 | 1 | .13946 | .99023 | .15672 | .98764 | .17393 | .98476 | .19109 | .98157 | 59 |
| 1 | 2 | .13975 | .99019 | .15701 | .98760 | .17422 | .98471 | .19138 | .98152 | 58 |
| 1 | 3 | .14004 | .99015 | .15731 | .98755 | .17451 | .98466 | .19167 | .98146 | 57 |
| 2 | 4 | .14033 | .99011 | .15758 | .98751 | .17479 | .98461 | .19195 | .98141 | 56 |
| 2 | 5 | .14061 | .99006 | .15787 | .98746 | .17508 | .98455 | .19224 | .98135 | 55 |
| 3 | 6 | .1409 | .99002 | .15816 | .98741 | .17537 | .9845 | .19252 | .98129 | 54 |
| 3 | 7 | .14119 | .98998 | .15845 | .98737 | .17565 | .98445 | .19281 | .98124 | 53 |
| 4 | 8 | .14148 | .98994 | .15873 | .98732 | .17594 | .9844 | .19309 | .98118 | 52 |
| 4 | 9 | .14177 | .9899 | .15902 | .98728 | .17623 | .98435 | .19338 | .98112 | 51 |
| 5 | 10 | .14205 | .98986 | .15931 | .98723 | .17651 | .9843 | .19366 | .98107 | 50 |
| 5 | 11 | .14234 | .98982 | .15959 | .98718 | .1768 | .98425 | .19395 | .98101 | 49 |
| 6 | 12 | .14263 | .98978 | .15988 | .98714 | .17708 | .9842 | .19423 | .98096 | 48 |
| 6 | 13 | .14292 | .98973 | .16017 | .98709 | .17737 | .98414 | .19452 | .9809 | 47 |
| 7 | 14 | .1432 | .98969 | .16046 | .98704 | .17766 | .98409 | .19481 | .98084 | 46 |
| 7 | 15 | .14349 | .98965 | .16074 | .987 | .17794 | .98404 | .19509 | .98079 | 45 |
| 7 | 16 | .14378 | .98961 | .16103 | .98695 | .17823 | .98399 | .19538 | .98073 | 44 |
| 8 | 17 | .14407 | .98957 | .16132 | .9869 | .17852 | .98394 | .19566 | .98067 | 43 |
| 8 | 18 | .14436 | .98953 | .1616 | .98686 | .1788 | .98389 | .19595 | .98061 | 42 |
| 9 | 19 | .14464 | .98948 | .16189 | .98681 | .17909 | .98383 | .19623 | .98056 | 41 |
| 9 | 20 | .14493 | .98944 | .16218 | .98676 | .17937 | .98378 | .19652 | .9805 | 40 |
| 10 | 21 | .14522 | .9894 | .16246 | .98671 | .17966 | .98373 | .1968 | .98044 | 39 |
| 10 | 22 | .14551 | .98936 | .16275 | .98667 | .17995 | .98368 | .19709 | .98039 | 38 |
| 11 | 23 | .1458 | .98931 | .16304 | .98662 | .18023 | .98362 | .19737 | .98033 | 37 |
| 11 | 24 | .14608 | .98927 | .16333 | .98657 | .18052 | .98357 | .19766 | .98027 | 36 |
| 12 | 25 | .14637 | .98923 | .16361 | .98652 | .18081 | .98352 | .19794 | .98021 | 35 |
| 12 | 26 | .14666 | .98919 | .1639 | .98648 | .18109 | .98347 | .19823 | .98016 | 34 |
| 13 | 27 | .14695 | .98914 | .16419 | .98643 | .18138 | .98341 | .19851 | .9801 | 33 |
| 13 | 28 | .14723 | .9891 | .16447 | .98638 | .18166 | .98336 | .1988 | .98004 | 32 |
| 14 | 29 | .14752 | .98906 | .16476 | .98633 | .18195 | .98331 | .19908 | .97998 | 31 |
| 14 | 30 | .14781 | .98902 | .16505 | .98629 | .18224 | .98325 | .19937 | .97992 | 30 |
| 15 | 31 | .1481 | .98897 | .16533 | .98624 | .18252 | .9832 | .19965 | .97987 | 29 |
| 15 | 32 | .14838 | .98893 | .16562 | .98619 | .18281 | .98315 | .19994 | .97981 | 28 |
| 15 | 33 | .14867 | .98889 | .16591 | .98614 | .18309 | .9831 | .20022 | .97975 | 27 |
| 16 | 34 | .14896 | .98884 | .1662 | .98609 | .18338 | .98304 | .20051 | .97969 | 26 |
| 16 | 35 | .14925 | .9888 | .16648 | .98604 | .18367 | .98299 | .20079 | .97963 | 25 |
| 17 | 36 | .14954 | .98876 | .16677 | .986 | .18395 | .98294 | .20108 | .97958 | 24 |
| 17 | 37 | .14982 | .98871 | .16706 | .98595 | .18424 | .98288 | .20136 | .97952 | 23 |
| 18 | 38 | .15011 | .98867 | .16734 | .9859 | .18452 | .98283 | .20165 | .97946 | 22 |
| 18 | 39 | .1504 | .98863 | .16763 | .98585 | .18481 | .98277 | .20193 | .9794 | 21 |
| 19 | 40 | .15069 | .98858 | .16792 | .9858 | .18509 | .98272 | .20222 | .97934 | 20 |
| 19 | 41 | .15097 | .98854 | .1682 | .98575 | .18538 | .98267 | .2025 | .97928 | 19 |
| 20 | 42 | .15126 | .98849 | .16849 | .9857 | .18567 | .98261 | .20279 | .97922 | 18 |
| 20 | 43 | .15155 | .98845 | .16878 | .98565 | .18595 | .98256 | .20307 | .97916 | 17 |
| 21 | 44 | .15184 | .98841 | .16906 | .98561 | .18624 | .9825 | .20336 | .9791 | 16 |
| 21 | 45 | .15212 | .98836 | .16935 | .98556 | .18652 | .98245 | .20364 | .97905 | 15 |
| 21 | 46 | .15241 | .98832 | .16964 | .98551 | .18681 | .9824 | .20393 | .97899 | 14 |
| 22 | 47 | .1527 | .98827 | .16992 | .98546 | .1871 | .98234 | .20421 | .97893 | 13 |
| 22 | 48 | .15299 | .98823 | .17021 | .98541 | .18738 | .98229 | .2045 | .97887 | 12 |
| 23 | 49 | .15327 | .98818 | .1705 | .98536 | .18767 | .98223 | .20478 | .97881 | 11 |
| 23 | 50 | .15356 | .98814 | .17078 | .98531 | .18795 | .98218 | .20507 | .97875 | 10 |
| 24 | 51 | .15385 | .98809 | .17107 | .98526 | .18824 | .98212 | .20535 | .97869 | 9 |
| 24 | 52 | .15414 | .98805 | .17136 | .98521 | .18852 | .98207 | .20563 | .97863 | 8 |
| 25 | 53 | .15442 | .988 | .17164 | .98516 | .18881 | .98201 | .20592 | .97857 | 7 |
| 25 | 54 | .15471 | .98796 | .17193 | .98511 | .1891 | .98196 | .2062 | .97851 | 6 |
| 26 | 55 | .155 | .98791 | .17222 | .98506 | .18938 | .9819 | .20649 | .97845 | 5 |
| 26 | 56 | .15529 | .98787 | .1725 | .98501 | .18967 | .98185 | .20677 | .97839 | 4 |
| 27 | 57 | .15557 | .98782 | .17279 | .98496 | .18995 | .98179 | .20706 | .97833 | 3 |
| 27 | 58 | .15586 | .98778 | .17308 | .98491 | .19024 | .98174 | .20734 | .97827 | 2 |
| 28 | 59 | .15615 | .98773 | .17336 | .98486 | .19052 | .98168 | .20763 | .97821 | 1 |
| 28 | 60 | .15643 | .98769 | .17365 | .98481 | .19081 | .98163 | .20791 | .97815 | 0 |
| | | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | |

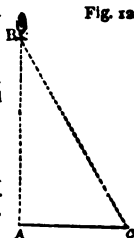
Fig. 11.

When the Objects can be aligned.—

Fig. 11.

OPERATION.—Align c B, Fig. 11, at A, measure a base line at any angle thereto, as $A c$, and angles $c A c$, $c o A$, and $B o A$. Then proceed as per formula, page 386, to deduce c B.

To Compute Distance from a Given Point to an Inaccessible Object.—Fig. 12.

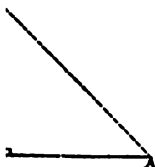


c .—Measure a level line, $A c$, Fig. 12, and ascertain angles, $B A c$, $B c A$ ng side, $A c$, and two angles, proceed as per formula, page 386, to de-
h.

Compute Height of an Elevated Point.—Fig. 13.

Fig. 13.

OPERATION.—Measure distance on a horizontal line, $A c$, Fig. 13; ascertain Angle $B A c$. Then proceed as per formulas, pp. 386-8, to ascertain $B c$.



When a Horizontal Base is not Attainable.—Fig. 14.

OPERATION.—Measure or compute distance $A c$, Fig.

in angle of depression $c A o$ and of elevation n proceed as per formula, page 386, to ascertain $B c$.

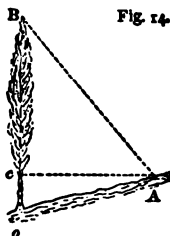


Fig. 14.

When a Full Base Line is not Attainable.—Fig. 15.

OPERATION.—Measure a base line, $A c$, Fig. 15, and ascertain angles $A c B$, $c A B$.

Then proceed as per formula, page 386, to ascertain d B.

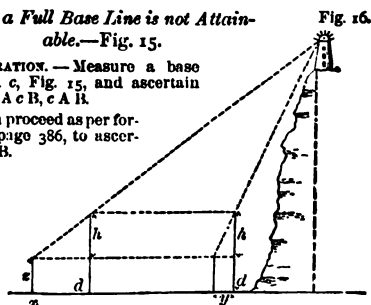


Fig. 16.

in of an Instrument.—Fig. 16.

h .—Lay off any suitable and level distance, d , set up e elevation from base line $d d$, and note distance of object range with tops of the staffs; deduct d ascertain heights h .

$+s = \text{height}$ s representing height of line of
 $s d$.

K K*

Natural Sines and Cosines.

| Prop. | | 0° | | 1° | | 2° | | 3° | | |
|--------|----|----------|---------|----------|---------|----------|---------|----------|---------|----|
| Parts. | * | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | |
| 29 | 0 | .00000 | 1 | .01745 | .99985 | .0349 | .99939 | .05234 | .99863 | 60 |
| 0 | 1 | .00029 | 1 | .01774 | .99984 | .03519 | .99938 | .05263 | .99861 | 59 |
| 1 | 2 | .00058 | 1 | .01803 | .99984 | .03548 | .99937 | .05292 | .99859 | 58 |
| 2 | 3 | .00087 | 1 | .01832 | .99983 | .03577 | .99936 | .05321 | .99858 | 57 |
| 3 | 4 | .00116 | 1 | .01862 | .99983 | .03606 | .99935 | .05350 | .99857 | 56 |
| 4 | 5 | .00145 | 1 | .01891 | .99982 | .03635 | .99934 | .05379 | .99855 | 55 |
| 5 | 6 | .00175 | 1 | .0192 | .99982 | .03664 | .99933 | .05408 | .99854 | 54 |
| 6 | 7 | .00204 | 1 | .01949 | .99981 | .03693 | .99932 | .05437 | .99852 | 53 |
| 7 | 8 | .00233 | 1 | .01978 | .9998 | .03723 | .99931 | .05466 | .99851 | 52 |
| 8 | 9 | .00262 | 1 | .02007 | .99979 | .03752 | .9993 | .05495 | .99849 | 51 |
| 9 | 10 | .00291 | 1 | .02036 | .99979 | .03781 | .99929 | .05524 | .99847 | 50 |
| 10 | 11 | .0032 | .99999 | .02065 | .99979 | .0381 | .99927 | .05553 | .99846 | 49 |
| 11 | 12 | .00349 | .99999 | .02094 | .99978 | .03839 | .99926 | .05582 | .99844 | 48 |
| 12 | 13 | .00378 | .99999 | .02123 | .99977 | .03868 | .99925 | .05611 | .99842 | 47 |
| 13 | 14 | .00407 | .99999 | .02152 | .99977 | .03897 | .99924 | .0564 | .99841 | 46 |
| 14 | 15 | .00436 | .99999 | .02181 | .99976 | .03926 | .99923 | .05669 | .99839 | 45 |
| 15 | 16 | .00465 | .99999 | .02211 | .99976 | .03955 | .99922 | .05698 | .99838 | 44 |
| 16 | 17 | .00495 | .99999 | .0224 | .99975 | .03984 | .99921 | .05727 | .99836 | 43 |
| 17 | 18 | .00524 | .99999 | .02269 | .99974 | .04013 | .99919 | .05756 | .99834 | 42 |
| 18 | 19 | .00553 | .99998 | .02298 | .99974 | .04042 | .99918 | .05785 | .99833 | 41 |
| 19 | 20 | .00582 | .99998 | .02327 | .99973 | .04071 | .99917 | .05814 | .99831 | 40 |
| 20 | 21 | .00611 | .99998 | .02356 | .99972 | .041 | .99916 | .05843 | .99829 | 39 |
| 21 | 22 | .0064 | .99998 | .02385 | .99972 | .04129 | .99915 | .05873 | .99827 | 38 |
| 22 | 23 | .00669 | .99998 | .02414 | .99971 | .04159 | .99913 | .05902 | .99826 | 37 |
| 23 | 24 | .00698 | .99998 | .02443 | .9997 | .04188 | .99912 | .05931 | .99824 | 36 |
| 24 | 25 | .00727 | .99997 | .02472 | .99969 | .04217 | .99911 | .0596 | .99822 | 35 |
| 25 | 26 | .00756 | .99997 | .02501 | .99969 | .04246 | .9991 | .05989 | .99821 | 34 |
| 26 | 27 | .00785 | .99997 | .0253 | .99968 | .04275 | .99909 | .06018 | .99819 | 33 |
| 27 | 28 | .00814 | .99997 | .0256 | .99967 | .04304 | .99907 | .06047 | .99817 | 32 |
| 28 | 29 | .00844 | .99996 | .02589 | .99966 | .04333 | .99906 | .06076 | .99815 | 31 |
| 29 | 30 | .00873 | .99996 | .02618 | .99966 | .04362 | .99905 | .06105 | .99813 | 30 |
| 30 | 31 | .00902 | .99996 | .02647 | .99965 | .04391 | .99904 | .06134 | .99812 | 29 |
| 31 | 32 | .00931 | .99996 | .02676 | .99964 | .0442 | .99902 | .06163 | .9981 | 28 |
| 32 | 33 | .0096 | .99995 | .02705 | .99963 | .04449 | .99901 | .06192 | .99808 | 27 |
| 33 | 34 | .00989 | .99995 | .02734 | .99963 | .04478 | .999 | .06221 | .99806 | 26 |
| 34 | 35 | .01018 | .99995 | .02763 | .99962 | .04507 | .99898 | .0625 | .99804 | 25 |
| 35 | 36 | .01047 | .99995 | .02792 | .99961 | .04536 | .99897 | .06279 | .99803 | 24 |
| 36 | 37 | .01076 | .99994 | .02821 | .9996 | .04565 | .99896 | .06308 | .99801 | 23 |
| 37 | 38 | .01105 | .99994 | .0285 | .99959 | .04594 | .99894 | .06337 | .99799 | 22 |
| 38 | 39 | .01134 | .99994 | .02879 | .99959 | .04623 | .99893 | .06366 | .99797 | 21 |
| 39 | 40 | .01164 | .99993 | .02908 | .99958 | .04653 | .99892 | .06395 | .99795 | 20 |
| 40 | 41 | .01193 | .99993 | .02938 | .99957 | .04682 | .9989 | .06424 | .99793 | 19 |
| 41 | 42 | .01222 | .99993 | .02967 | .99956 | .04711 | .99889 | .06453 | .99792 | 18 |
| 42 | 43 | .01251 | .99992 | .02996 | .99955 | .0474 | .99888 | .06482 | .9979 | 17 |
| 43 | 44 | .0128 | .99992 | .03025 | .99954 | .04769 | .99886 | .06511 | .99788 | 16 |
| 44 | 45 | .01309 | .99991 | .03054 | .99953 | .04798 | .99885 | .0654 | .99786 | 15 |
| 45 | 46 | .01338 | .99991 | .03083 | .99952 | .04827 | .99883 | .06569 | .99784 | 14 |
| 46 | 47 | .01367 | .99991 | .03112 | .99952 | .04856 | .99882 | .06598 | .99782 | 13 |
| 47 | 48 | .01396 | .9999 | .03141 | .99951 | .04885 | .99881 | .06627 | .9978 | 12 |
| 48 | 49 | .01425 | .9999 | .0317 | .9995 | .04914 | .99879 | .06656 | .99778 | 11 |
| 49 | 50 | .01454 | .99989 | .03199 | .99949 | .04943 | .99878 | .06685 | .99776 | 10 |
| 50 | 51 | .01483 | .99989 | .03228 | .99948 | .04972 | .99876 | .06714 | .99774 | 9 |
| 51 | 52 | .01512 | .99988 | .03257 | .99947 | .05001 | .99875 | .06743 | .99772 | 8 |
| 52 | 53 | .01541 | .99988 | .03286 | .99946 | .0503 | .99873 | .06772 | .9977 | 7 |
| 53 | 54 | .0157 | .99988 | .03315 | .99945 | .05059 | .99872 | .06802 | .99768 | 6 |
| 54 | 55 | .016 | .99987 | .03344 | .99944 | .05088 | .9987 | .06831 | .99766 | 5 |
| 55 | 56 | .01629 | .99987 | .03373 | .99943 | .05117 | .99869 | .0686 | .99764 | 4 |
| 56 | 57 | .01658 | .99986 | .03402 | .99942 | .05146 | .99867 | .06889 | .99762 | 3 |
| 57 | 58 | .01687 | .99986 | .03431 | .99941 | .05175 | .99866 | .06918 | .9976 | 2 |
| 58 | 59 | .01716 | .99985 | .0346 | .9994 | .05205 | .99864 | .06947 | .99758 | 1 |
| 59 | 60 | .01745 | .99985 | .03489 | .99939 | .05234 | .99863 | .06976 | .99756 | |

| | 40° | | 50° | | 60° | | 70° | | Prod. | Part. |
|----|----------|----------|----------|----------|----------|----------|----------|----------|-------|-------|
| | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | | |
| 0 | .06976 | .99756 | .08716 | .93619 | .10453 | .99452 | .12187 | .99255 | 60 | 4 |
| 1 | .07005 | .99754 | .08745 | .93617 | .10482 | .99449 | .12217 | .99251 | 59 | 4 |
| 2 | .07034 | .99752 | .08774 | .93614 | .10511 | .99446 | .12245 | .99248 | 58 | 4 |
| 3 | .07063 | .9975 | .08803 | .93612 | .1054 | .99443 | .12274 | .99244 | 57 | 4 |
| 4 | .07092 | .99748 | .08831 | .93609 | .10569 | .9944 | .12302 | .9924 | 56 | 4 |
| 5 | .07121 | .99746 | .0886 | .93607 | .10597 | .99437 | .12331 | .99237 | 55 | 4 |
| 6 | .0715 | .99744 | .08889 | .93604 | .10626 | .99434 | .1236 | .99233 | 54 | 4 |
| 7 | .07179 | .99742 | .08918 | .93602 | .10655 | .99431 | .12389 | .9923 | 53 | 4 |
| 8 | .07208 | .9974 | .08947 | .93599 | .10684 | .99428 | .12418 | .99226 | 52 | 3 |
| 9 | .07237 | .99738 | .08976 | .93596 | .10713 | .99424 | .12447 | .99222 | 51 | 3 |
| 10 | .07266 | .99736 | .09005 | .93594 | .10742 | .99421 | .12476 | .99219 | 50 | 3 |
| 11 | .07295 | .99734 | .09034 | .93591 | .10771 | .99418 | .12504 | .99215 | 49 | 3 |
| 12 | .07324 | .99731 | .09063 | .93588 | .108 | .99415 | .12533 | .99211 | 48 | 3 |
| 13 | .07353 | .99729 | .09092 | .93586 | .10829 | .99412 | .12562 | .99208 | 47 | 3 |
| 14 | .07382 | .99727 | .09121 | .93583 | .10858 | .99409 | .12591 | .99204 | 46 | 3 |
| 15 | .07411 | .99725 | .0915 | .9358 | .10887 | .99406 | .1262 | .992 | 45 | 3 |
| 16 | .0744 | .99723 | .09179 | .93578 | .10916 | .99402 | .12649 | .99197 | 44 | 3 |
| 17 | .07469 | .99721 | .09208 | .93575 | .10945 | .99399 | .12678 | .99193 | 43 | 3 |
| 18 | .07498 | .99719 | .09237 | .93572 | .10973 | .99396 | .12706 | .99189 | 42 | 3 |
| 19 | .07527 | .99716 | .09266 | .9357 | .11002 | .99393 | .12735 | .99186 | 41 | 3 |
| 20 | .07556 | .99714 | .09295 | .93567 | .11031 | .9939 | .12764 | .99182 | 40 | 3 |
| 21 | .07585 | .99712 | .09324 | .93564 | .1106 | .99386 | .12793 | .99178 | 39 | 3 |
| 22 | .07614 | .9971 | .09353 | .93562 | .11089 | .99383 | .12822 | .99175 | 38 | 3 |
| 23 | .07643 | .99708 | .09382 | .93559 | .11118 | .9938 | .12851 | .99171 | 37 | 2 |
| 24 | .07672 | .99705 | .09411 | .93556 | .11147 | .99377 | .1288 | .99167 | 36 | 2 |
| 25 | .07701 | .99703 | .0944 | .93553 | .11176 | .99374 | .12908 | .99163 | 35 | 2 |
| 26 | .0773 | .99701 | .09469 | .93551 | .11205 | .9937 | .12937 | .9916 | 34 | 2 |
| 27 | .07759 | .99699 | .09498 | .93548 | .11234 | .99367 | .12966 | .99156 | 33 | 2 |
| 28 | .07788 | .99696 | .09527 | .93545 | .11263 | .99364 | .12995 | .99152 | 32 | 2 |
| 29 | .07817 | .99694 | .09556 | .93542 | .11291 | .9936 | .13024 | .99148 | 31 | 2 |
| 30 | .07846 | .99692 | .09585 | .9354 | .1132 | .99357 | .13053 | .99144 | 30 | 2 |
| 31 | .07875 | .9969 | .09614 | .93537 | .11349 | .99354 | .13081 | .99141 | 29 | 2 |
| 32 | .07904 | .99687 | .09642 | .93534 | .11378 | .99351 | .1311 | .99137 | 28 | 2 |
| 33 | .07933 | .99685 | .09671 | .93531 | .11407 | .99347 | .13139 | .99133 | 27 | 2 |
| 34 | .07962 | .99683 | .097 | .93528 | .11436 | .99344 | .13168 | .99129 | 26 | 2 |
| 35 | .07991 | .9968 | .09729 | .93526 | .11465 | .99341 | .13197 | .99125 | 25 | 2 |
| 36 | .0802 | .99678 | .09758 | .93523 | .11494 | .99337 | .13226 | .99122 | 24 | 2 |
| 37 | .08049 | .99676 | .09787 | .9352 | .11523 | .99334 | .13254 | .99118 | 23 | 2 |
| 38 | .08078 | .99673 | .09816 | .93517 | .11552 | .99331 | .13283 | .99114 | 22 | 1 |
| 39 | .08107 | .99671 | .09845 | .93514 | .1158 | .99327 | .13312 | .9911 | 21 | 1 |
| 40 | .08136 | .99668 | .09874 | .93511 | .11609 | .99324 | .13341 | .99106 | 20 | 1 |
| 41 | .08165 | .99666 | .09903 | .93508 | .11638 | .9932 | .1337 | .99102 | 19 | 1 |
| 42 | .08194 | .99664 | .09932 | .93506 | .11667 | .99317 | .13399 | .99098 | 18 | 1 |
| 43 | .08223 | .99661 | .09961 | .93503 | .11696 | .99314 | .13427 | .99094 | 17 | 1 |
| 44 | .08252 | .99659 | .0999 | .935 | .11725 | .9931 | .13456 | .99091 | 16 | 1 |
| 45 | .08281 | .99657 | .10019 | .93497 | .11754 | .99307 | .13485 | .99087 | 15 | 1 |
| 46 | .0831 | .99654 | .10048 | .93494 | .11783 | .99303 | .13514 | .99083 | 14 | 1 |
| 47 | .08339 | .99652 | .10077 | .93491 | .11812 | .993 | .13543 | .99079 | 13 | 1 |
| 48 | .08368 | .99649 | .10106 | .93488 | .1184 | .99297 | .13572 | .99075 | 12 | 1 |
| 49 | .08397 | .99647 | .10135 | .93485 | .11869 | .99293 | .136 | .99071 | 11 | 1 |
| 50 | .08426 | .99644 | .10164 | .93482 | .11898 | .9929 | .13629 | .99067 | 10 | 0 |
| 51 | .08455 | .99642 | .10192 | .93479 | .11927 | .99286 | .13658 | .99063 | 9 | 8 |
| 52 | .08484 | .99639 | .10221 | .93476 | .11956 | .99283 | .13687 | .99059 | 8 | |
| 53 | .08513 | .99637 | .1025 | .93473 | .11985 | .99279 | .13716 | .99055 | 7 | |
| 54 | .08542 | .99635 | .10279 | .9347 | .12014 | .99276 | .13744 | .99051 | 6 | |
| 55 | .08571 | .99632 | .10308 | .93467 | .12043 | .99272 | .13773 | .99047 | 5 | |
| 56 | .086 | .9963 | .10337 | .93464 | .12071 | .99269 | .13802 | .99043 | 4 | |
| 57 | .08629 | .99627 | .10366 | .93461 | .121 | .99265 | .13831 | .99039 | 3 | |
| 58 | .08658 | .99625 | .10395 | .93458 | .12129 | .99262 | .1386 | .99035 | 2 | |
| 59 | .08687 | .99622 | .10424 | .93455 | .12158 | .99258 | .13889 | .99031 | 1 | |
| 60 | .08716 | .9962 | .10453 | .93452 | .12187 | .99255 | .13917 | .99027 | 0 | |
| | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | | |

| Prop. partia. 26 | | 24° | | 25° | | 26° | | 27° | | Prop. partia. 26 |
|------------------------|----|----------|---------|----------|---------|----------|---------|----------|---------|------------------------|
| | | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | |
| 0 | 0 | .40674 | .91355 | .42262 | .90631 | .43837 | .89879 | .45399 | .89101 | 60 |
| 0 | 1 | .407 | .91343 | .42288 | .90618 | .43863 | .89867 | .45425 | .89087 | 59 |
| 1 | 2 | .40727 | .91331 | .42315 | .90606 | .43889 | .89854 | .45451 | .89074 | 58 |
| 1 | 3 | .40753 | .91319 | .42341 | .90594 | .43916 | .89841 | .45477 | .89061 | 57 |
| 2 | 4 | .4078 | .91307 | .42367 | .90582 | .43942 | .89828 | .45503 | .89048 | 56 |
| 2 | 5 | .40806 | .91295 | .42394 | .90569 | .43968 | .89816 | .45529 | .89035 | 55 |
| 3 | 6 | .40833 | .91283 | .4242 | .90557 | .43994 | .89803 | .45554 | .89021 | 54 |
| 3 | 7 | .4086 | .91272 | .42446 | .90545 | .4402 | .8979 | .4558 | .89008 | 53 |
| 3 | 8 | .40886 | .9126 | .42473 | .90532 | .44046 | .89777 | .45606 | .88995 | 52 |
| 4 | 9 | .40913 | .91248 | .42499 | .9052 | .44072 | .89764 | .45632 | .88981 | 51 |
| 4 | 10 | .40939 | .91236 | .42525 | .90507 | .44098 | .89752 | .45658 | .88968 | 50 |
| 5 | 11 | .40966 | .91224 | .42552 | .90495 | .44124 | .89739 | .45684 | .88955 | 49 |
| 5 | 12 | .40992 | .91212 | .42578 | .90483 | .44151 | .89726 | .4571 | .88942 | 48 |
| 6 | 13 | .41019 | .912 | .42604 | .9047 | .44177 | .89713 | .45736 | .88928 | 47 |
| 6 | 14 | .41045 | .91188 | .42631 | .90458 | .44203 | .897 | .45762 | .88915 | 46 |
| 7 | 15 | .41072 | .91176 | .42657 | .90446 | .44229 | .89687 | .45787 | .88902 | 45 |
| 7 | 16 | .41098 | .91164 | .42683 | .90433 | .44255 | .89674 | .45813 | .88888 | 44 |
| 7 | 17 | .41125 | .91152 | .42709 | .90421 | .44281 | .89662 | .45839 | .88875 | 43 |
| 8 | 18 | .41151 | .9114 | .42736 | .90408 | .44307 | .89649 | .45865 | .88862 | 42 |
| 8 | 19 | .41178 | .91128 | .42762 | .90396 | .44333 | .89636 | .45891 | .88848 | 41 |
| 9 | 20 | .41204 | .91116 | .42788 | .90383 | .44359 | .89623 | .45917 | .88835 | 40 |
| 9 | 21 | .41231 | .91104 | .42815 | .90371 | .44385 | .8961 | .45942 | .88822 | 39 |
| 10 | 22 | .41257 | .91092 | .42841 | .90358 | .44411 | .89597 | .45968 | .88808 | 38 |
| 10 | 23 | .41284 | .9108 | .42867 | .90346 | .44437 | .89584 | .45994 | .88795 | 37 |
| 10 | 24 | .4131 | .91068 | .42894 | .90334 | .44464 | .89571 | .4602 | .88782 | 36 |
| 11 | 25 | .41337 | .91056 | .4292 | .90321 | .4449 | .89558 | .46046 | .88768 | 35 |
| 11 | 26 | .41363 | .91044 | .42946 | .90309 | .44516 | .89545 | .46072 | .88755 | 34 |
| 12 | 27 | .4139 | .91032 | .42972 | .90296 | .44542 | .89532 | .46097 | .88741 | 33 |
| 12 | 28 | .41416 | .9102 | .42999 | .90284 | .44568 | .89519 | .46123 | .88728 | 32 |
| 13 | 29 | .41443 | .91008 | .43025 | .90271 | .44594 | .89506 | .46149 | .88715 | 31 |
| 13 | 30 | .41469 | .90996 | .43051 | .90259 | .4462 | .89493 | .46175 | .88701 | 30 |
| 13 | 31 | .41496 | .90984 | .43077 | .90246 | .44646 | .8948 | .46201 | .88688 | 29 |
| 14 | 32 | .41522 | .90972 | .43104 | .90233 | .44672 | .89467 | .46226 | .88674 | 28 |
| 14 | 33 | .41549 | .9096 | .4313 | .90221 | .44698 | .89454 | .46252 | .88661 | 27 |
| 15 | 34 | .41575 | .90948 | .43156 | .90208 | .44724 | .89441 | .46278 | .88647 | 26 |
| 15 | 35 | .41602 | .90936 | .43182 | .90196 | .4475 | .89428 | .46304 | .88634 | 25 |
| 16 | 36 | .41628 | .90924 | .43209 | .90183 | .44776 | .89415 | .4633 | .8862 | 24 |
| 16 | 37 | .41655 | .90911 | .43235 | .90171 | .44802 | .89402 | .46355 | .88607 | 23 |
| 16 | 38 | .41681 | .90899 | .43261 | .90158 | .44828 | .89389 | .46381 | .88593 | 22 |
| 17 | 39 | .41707 | .90887 | .43287 | .90146 | .44854 | .89376 | .46407 | .8858 | 21 |
| 17 | 40 | .41734 | .90875 | .43313 | .90133 | .4488 | .89363 | .46433 | .88566 | 20 |
| 18 | 41 | .4176 | .90863 | .4334 | .9012 | .44906 | .8935 | .46458 | .88553 | 19 |
| 18 | 42 | .41787 | .90851 | .43366 | .90108 | .44932 | .89337 | .46484 | .88539 | 18 |
| 19 | 43 | .41813 | .90839 | .43392 | .90095 | .44958 | .89324 | .4651 | .88526 | 17 |
| 19 | 44 | .4184 | .90826 | .43418 | .90082 | .44984 | .89311 | .46536 | .88512 | 16 |
| 20 | 45 | .41866 | .90814 | .43445 | .9007 | .4501 | .89298 | .46561 | .88499 | 15 |
| 20 | 46 | .41892 | .90802 | .43471 | .90057 | .45036 | .89285 | .46587 | .88485 | 14 |
| 20 | 47 | .41919 | .9079 | .43497 | .90045 | .45062 | .89272 | .46613 | .88472 | 13 |
| 21 | 48 | .41945 | .90778 | .43523 | .90032 | .45088 | .89259 | .46639 | .88458 | 12 |
| 21 | 49 | .41972 | .90766 | .43549 | .90019 | .45114 | .89245 | .46664 | .88445 | 11 |
| 22 | 50 | .41998 | .90753 | .43575 | .90007 | .4514 | .89232 | .4669 | .88431 | 10 |
| 22 | 51 | .42024 | .90741 | .43602 | .89994 | .45166 | .89219 | .46716 | .88417 | 9 |
| 23 | 52 | .42051 | .90729 | .43628 | .89981 | .45192 | .89206 | .46742 | .88404 | 8 |
| 23 | 53 | .42077 | .90717 | .43654 | .89968 | .45218 | .89193 | .46767 | .8839 | 7 |
| 23 | 54 | .42104 | .90704 | .4368 | .89956 | .45243 | .8918 | .46793 | .88377 | 6 |
| 24 | 55 | .4213 | .90692 | .43706 | .89943 | .45269 | .89167 | .46819 | .88363 | 5 |
| 24 | 56 | .42156 | .9068 | .43733 | .8993 | .45295 | .89153 | .46844 | .88349 | 4 |
| 25 | 57 | .42183 | .90668 | .43759 | .89918 | .45321 | .8914 | .4687 | .88336 | 3 |
| 25 | 58 | .42209 | .90655 | .43785 | .89905 | .45347 | .89127 | .46896 | .88322 | 2 |
| 26 | 59 | .42235 | .90643 | .43811 | .89892 | .45373 | .89114 | .46921 | .88308 | 1 |
| 26 | 60 | .42262 | .90631 | .43837 | .89879 | .45399 | .89101 | .46947 | .88295 | 0 |
| | | N. cos. | | N. sine. | | N. cos. | | N. sine. | | |
| | | 65° | | 64° | | 63° | | 62° | | |

| | 28° | | 29° | | 30° | | 31° | | Prop. partie |
|----|----------|---------|----------|---------|----------|---------|----------|---------|-----------------|
| | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | 14 |
| 0 | .46947 | .88295 | .48481 | .87462 | .5 | .86603 | .51504 | .85717 | 60 |
| 1 | .46973 | .88281 | .48506 | .87448 | .50023 | .86588 | .51529 | .85702 | 59 |
| 2 | .46999 | .88267 | .48532 | .87434 | .5005 | .86573 | .51554 | .85687 | 58 |
| 3 | .47024 | .88254 | .48557 | .8742 | .50076 | .86559 | .51579 | .85672 | 57 |
| 4 | .4705 | .8824 | .48583 | .87406 | .50101 | .86544 | .51604 | .85657 | 56 |
| 5 | .47076 | .88226 | .48608 | .87391 | .50126 | .8653 | .51628 | .85642 | 55 |
| 6 | .47101 | .88213 | .48634 | .87377 | .50151 | .86515 | .51653 | .85627 | 54 |
| 7 | .47127 | .88199 | .48659 | .87363 | .50176 | .86501 | .51678 | .85612 | 53 |
| 8 | .47153 | .88185 | .48684 | .87349 | .50201 | .86486 | .51703 | .85597 | 52 |
| 9 | .47178 | .88172 | .4871 | .87335 | .50227 | .86471 | .51728 | .85582 | 51 |
| 10 | .47204 | .88158 | .48735 | .87321 | .50252 | .86457 | .51753 | .85567 | 50 |
| 11 | .47229 | .88144 | .48761 | .87306 | .50277 | .86442 | .51778 | .85551 | 49 |
| 12 | .47255 | .8813 | .48786 | .87292 | .50302 | .86427 | .51803 | .85536 | 48 |
| 13 | .47281 | .88117 | .48811 | .87278 | .50327 | .86413 | .51828 | .85521 | 47 |
| 14 | .47306 | .88103 | .48837 | .87264 | .50352 | .86398 | .51852 | .85506 | 46 |
| 15 | .47332 | .88089 | .48862 | .8725 | .50377 | .86384 | .51877 | .85491 | 45 |
| 16 | .47358 | .88075 | .48888 | .87235 | .50403 | .86369 | .51902 | .85476 | 44 |
| 17 | .47383 | .88062 | .48913 | .87221 | .50428 | .86354 | .51927 | .85461 | 43 |
| 18 | .47409 | .88048 | .48938 | .87207 | .50453 | .8634 | .51952 | .85446 | 42 |
| 19 | .47434 | .88034 | .48964 | .87193 | .50478 | .86325 | .51977 | .85431 | 41 |
| 20 | .4746 | .8802 | .48989 | .87178 | .50503 | .8631 | .52002 | .85416 | 40 |
| 21 | .47486 | .88006 | .49014 | .87164 | .50528 | .86295 | .52026 | .85401 | 39 |
| 22 | .47511 | .87993 | .4904 | .8715 | .50553 | .8628 | .52051 | .85385 | 38 |
| 23 | .47537 | .87979 | .49065 | .87136 | .50578 | .86266 | .52076 | .8537 | 37 |
| 24 | .47562 | .87965 | .4909 | .87121 | .50603 | .86251 | .52101 | .85355 | 36 |
| 25 | .47588 | .87951 | .49116 | .87107 | .50628 | .86237 | .52126 | .8534 | 35 |
| 26 | .47614 | .87937 | .49141 | .87093 | .50654 | .86222 | .52151 | .85325 | 34 |
| 27 | .47639 | .87923 | .49166 | .87079 | .50679 | .86207 | .52175 | .8531 | 33 |
| 28 | .47665 | .87909 | .49192 | .87064 | .50704 | .86192 | .522 | .85294 | 32 |
| 29 | .4769 | .87896 | .49217 | .8705 | .50729 | .86178 | .52225 | .85279 | 31 |
| 30 | .47716 | .87883 | .49242 | .87036 | .50754 | .86163 | .5225 | .85264 | 30 |
| 31 | .47741 | .87868 | .49268 | .87021 | .50779 | .86148 | .52275 | .85249 | 29 |
| 32 | .47767 | .87854 | .49293 | .87007 | .50804 | .86133 | .52299 | .85234 | 28 |
| 33 | .47793 | .8784 | .49318 | .86993 | .50829 | .86119 | .52324 | .85218 | 27 |
| 34 | .47818 | .87826 | .49344 | .86978 | .50854 | .86104 | .52349 | .85203 | 26 |
| 35 | .47844 | .87812 | .49369 | .86964 | .50879 | .86089 | .52374 | .85188 | 25 |
| 36 | .47869 | .87798 | .49394 | .86949 | .50904 | .86074 | .52399 | .85173 | 24 |
| 37 | .47895 | .87784 | .49419 | .86935 | .50929 | .86059 | .52423 | .85157 | 23 |
| 38 | .4792 | .8777 | .49445 | .86921 | .50954 | .86045 | .52448 | .85142 | 22 |
| 39 | .47946 | .87756 | .4947 | .86906 | .50979 | .8603 | .52473 | .85127 | 21 |
| 40 | .47971 | .87743 | .49495 | .86892 | .51004 | .86015 | .52498 | .85112 | 20 |
| 41 | .47997 | .87729 | .49521 | .86878 | .51029 | .86 | .52522 | .85096 | 19 |
| 42 | .48022 | .87715 | .49546 | .86863 | .51054 | .85985 | .52547 | .85081 | 18 |
| 43 | .48048 | .87701 | .49571 | .86849 | .51079 | .8597 | .52572 | .85066 | 17 |
| 44 | .48073 | .87687 | .49596 | .86834 | .51104 | .85956 | .52597 | .85051 | 16 |
| 45 | .48099 | .87673 | .49622 | .8682 | .51129 | .85941 | .52621 | .85035 | 15 |
| 46 | .48124 | .87659 | .49647 | .86805 | .51154 | .85926 | .52646 | .8502 | 14 |
| 47 | .4815 | .87645 | .49672 | .86791 | .51179 | .85911 | .52671 | .85005 | 13 |
| 48 | .48175 | .87631 | .49697 | .86777 | .51204 | .85896 | .52696 | .84989 | 12 |
| 49 | .48201 | .87617 | .49723 | .86762 | .51229 | .85881 | .5272 | .84974 | 11 |
| 50 | .48226 | .87603 | .49748 | .86748 | .51254 | .85866 | .52745 | .84959 | 10 |
| 51 | .48252 | .87589 | .49773 | .86733 | .51279 | .85851 | .5277 | .84943 | 9 |
| 52 | .48277 | .87575 | .49798 | .86719 | .51304 | .85836 | .52794 | .84928 | 8 |
| 53 | .48303 | .87561 | .49824 | .86704 | .51329 | .85821 | .52819 | | |
| 54 | .48328 | .87546 | .49849 | .86689 | .51354 | .85806 | .5284 | | |
| 55 | .48354 | .87532 | .49874 | .86675 | .51379 | .85792 | .5286 | | |
| 56 | .48379 | .87518 | .49899 | .86661 | .51404 | .85777 | .5288 | | |
| 57 | .48405 | .87504 | .49924 | .86646 | .51429 | .85762 | .529 | | |
| 58 | .4843 | .8749 | .4995 | .86632 | .51454 | .85747 | .5292 | | |
| 59 | .48456 | .87476 | .49975 | .86617 | .51479 | .85732 | .5294 | | |
| 60 | .48482 | .87462 | .5 | .86603 | .51504 | .85717 | .5296 | | |
| | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | |

| Deg. Parts | 23 | 32° | | 33° | | 34° | | 35° | | 60 |
|---------------|----|----------|----------|----------|----------|----------|----------|----------|----------|----|
| | | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | |
| 0 | 0 | .52992 | .84805 | .54464 | .83867 | .55919 | .82904 | .57358 | .81915 | 60 |
| 0 | 1 | .53017 | .84789 | .54488 | .83851 | .55943 | .82887 | .57381 | .81899 | 59 |
| 1 | 2 | .53041 | .84774 | .54513 | .83835 | .55968 | .82871 | .57405 | .81882 | 58 |
| 1 | 3 | .53066 | .84759 | .54537 | .83819 | .55992 | .82855 | .57429 | .81865 | 57 |
| 2 | 4 | .53091 | .84743 | .54561 | .83804 | .56016 | .82839 | .57453 | .81848 | 56 |
| 2 | 5 | .53115 | .84728 | .54586 | .83788 | .5604 | .82822 | .57477 | .81832 | 55 |
| 2 | 6 | .5314 | .84712 | .5461 | .83772 | .56064 | .82806 | .57501 | .81815 | 54 |
| 3 | 7 | .53164 | .84697 | .54635 | .83756 | .56088 | .8279 | .57524 | .81798 | 53 |
| 3 | 8 | .53189 | .84681 | .54659 | .8374 | .56112 | .82773 | .57548 | .81782 | 52 |
| 3 | 9 | .53214 | .84666 | .54683 | .83724 | .56136 | .82757 | .57572 | .81765 | 51 |
| 4 | 10 | .53238 | .8465 | .54708 | .83708 | .5616 | .82741 | .57596 | .81748 | 50 |
| 4 | 11 | .53263 | .84635 | .54732 | .83692 | .56184 | .82724 | .57619 | .81731 | 49 |
| 5 | 12 | .53288 | .84619 | .54756 | .83676 | .56208 | .82708 | .57643 | .81714 | 48 |
| 5 | 13 | .53312 | .84604 | .54781 | .8366 | .56232 | .82692 | .57667 | .81698 | 47 |
| 5 | 14 | .53337 | .84588 | .54805 | .83645 | .56256 | .82675 | .57691 | .81681 | 46 |
| 5 | 15 | .53361 | .84573 | .54829 | .83629 | .5628 | .82659 | .57715 | .81664 | 45 |
| 6 | 16 | .53386 | .84557 | .54854 | .83613 | .56305 | .82643 | .57738 | .81647 | 44 |
| 7 | 17 | .53411 | .84542 | .54878 | .83597 | .56329 | .82626 | .57762 | .81631 | 43 |
| 7 | 18 | .53435 | .84526 | .54902 | .83581 | .56353 | .8261 | .57786 | .81614 | 42 |
| 7 | 19 | .5346 | .84511 | .54927 | .83565 | .56377 | .82593 | .5781 | .81597 | 41 |
| 8 | 20 | .53484 | .84495 | .54951 | .83549 | .56401 | .82577 | .57833 | .8158 | 40 |
| 8 | 21 | .53509 | .8448 | .54975 | .83533 | .56425 | .82561 | .57857 | .81563 | 39 |
| 8 | 22 | .53534 | .84464 | .54999 | .83517 | .56449 | .82544 | .57881 | .81546 | 38 |
| 9 | 23 | .53558 | .84448 | .55024 | .83501 | .56473 | .82528 | .57904 | .8153 | 37 |
| 9 | 24 | .53583 | .84433 | .55048 | .83485 | .56497 | .82511 | .57928 | .81513 | 36 |
| 10 | 25 | .53607 | .84417 | .55072 | .83469 | .56521 | .82495 | .57952 | .81496 | 35 |
| 10 | 26 | .53632 | .84402 | .55097 | .83453 | .56545 | .82478 | .57976 | .81479 | 34 |
| 10 | 27 | .53656 | .84386 | .55121 | .83437 | .56569 | .82462 | .57999 | .81462 | 33 |
| 11 | 28 | .53681 | .8437 | .55145 | .83421 | .56593 | .82446 | .58023 | .81445 | 32 |
| 11 | 29 | .53705 | .84355 | .55169 | .83405 | .56617 | .82429 | .58047 | .81428 | 31 |
| 12 | 30 | .5373 | .84339 | .55194 | .83389 | .56641 | .82413 | .5807 | .81412 | 30 |
| 12 | 31 | .53754 | .84324 | .55218 | .83373 | .56665 | .82396 | .58094 | .81395 | 29 |
| 12 | 32 | .53779 | .84308 | .55242 | .83356 | .56689 | .8238 | .58118 | .81378 | 28 |
| 13 | 33 | .53804 | .84292 | .55266 | .8334 | .56713 | .82363 | .58141 | .81361 | 27 |
| 13 | 34 | .53828 | .84277 | .55291 | .83324 | .56736 | .82347 | .58165 | .81344 | 26 |
| 13 | 35 | .53853 | .84261 | .55315 | .83308 | .5676 | .8233 | .58189 | .81327 | 25 |
| 14 | 36 | .53877 | .84245 | .55339 | .83292 | .56784 | .82314 | .58212 | .8131 | 24 |
| 14 | 37 | .53902 | .8423 | .55363 | .83276 | .56808 | .82297 | .58236 | .81293 | 23 |
| 15 | 38 | .53926 | .84214 | .55388 | .8326 | .56832 | .82281 | .5826 | .81276 | 22 |
| 15 | 39 | .53951 | .84198 | .55412 | .83244 | .56856 | .82264 | .58283 | .81259 | 21 |
| 15 | 40 | .53975 | .84182 | .55436 | .83228 | .5688 | .82248 | .58307 | .81242 | 20 |
| 16 | 41 | .54 | .84167 | .5546 | .83212 | .56904 | .82231 | .5833 | .81225 | 19 |
| 16 | 42 | .54024 | .84151 | .55484 | .83195 | .56928 | .82214 | .58354 | .81208 | 18 |
| 16 | 43 | .54049 | .84135 | .55509 | .83179 | .56952 | .82198 | .58378 | .81191 | 17 |
| 17 | 44 | .54073 | .84119 | .55533 | .83163 | .56976 | .82181 | .58401 | .81174 | 16 |
| 17 | 45 | .54097 | .84104 | .55557 | .83147 | .57 | .82165 | .58425 | .81157 | 15 |
| 18 | 46 | .54122 | .84088 | .55581 | .83131 | .57024 | .82148 | .58449 | .8114 | 14 |
| 18 | 47 | .54146 | .84072 | .55605 | .83115 | .57047 | .82132 | .58472 | .81123 | 13 |
| 18 | 48 | .54171 | .84057 | .5563 | .83098 | .57071 | .82115 | .58496 | .81106 | 12 |
| 19 | 49 | .54195 | .84041 | .55654 | .83082 | .57095 | .82098 | .58519 | .81089 | 11 |
| 19 | 50 | .5422 | .84025 | .55678 | .83066 | .57119 | .82082 | .58543 | .81072 | 10 |
| 19 | 51 | .54244 | .84009 | .55702 | .8305 | .57143 | .82065 | .58567 | .81055 | 9 |
| 19 | 52 | .54269 | .83994 | .55726 | .83034 | .57167 | .82048 | .5859 | .81038 | 8 |
| 19 | 53 | .54293 | .83978 | .5575 | .83017 | .57191 | .82032 | .58614 | .81021 | 7 |
| 19 | 54 | .54317 | .83962 | .55775 | .83001 | .57215 | .82015 | .58637 | .81004 | 6 |
| 19 | 55 | .54341 | .83946 | .55799 | .82985 | .57238 | .81999 | .58661 | .80987 | 5 |
| 19 | 56 | .54365 | .8393 | .55823 | .82969 | .57262 | .81982 | .58684 | .8097 | 4 |
| 19 | 57 | .54389 | .83914 | .55847 | .82953 | .57286 | .81965 | .58708 | .80953 | 3 |
| 19 | 58 | .54413 | .83898 | .55871 | .82937 | .5731 | .81949 | .58731 | .80936 | 2 |
| 19 | 59 | .54437 | .83882 | .55895 | .82921 | .57334 | .81932 | .58755 | .80919 | 1 |
| 19 | 60 | .54461 | .83866 | .55919 | .82905 | .57358 | .81915 | .58779 | .80902 | 0 |
| | | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | |

| 36° | | 37° | | 38° | | 39° | | Prop. Parts | 18 |
|----------|----------|----------|----------|----------|----------|----------|----------|----------------|----|
| N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | | |
| 58779 | .80902 | .60182 | .79864 | .61566 | .78801 | .62932 | .77715 | 60 | 18 |
| 58802 | .80885 | .60205 | .79846 | .61589 | .78783 | .62955 | .77696 | 59 | 18 |
| 58826 | .80867 | .60228 | .79829 | .61612 | .78765 | .62977 | .77678 | 58 | 17 |
| 58849 | .8085 | .60251 | .79811 | .61635 | .78747 | .63 | .7766 | 57 | 17 |
| 58873 | .80833 | .60274 | .79793 | .61658 | .78729 | .63022 | .77641 | 56 | 17 |
| 58896 | .80816 | .60298 | .79776 | .61681 | .78711 | .63045 | .77623 | 55 | 17 |
| 5892 | .80799 | .60321 | .79758 | .61704 | .78694 | .63068 | .77605 | 54 | 16 |
| 58943 | .80782 | .60344 | .79741 | .61726 | .78676 | .6309 | .77586 | 53 | 16 |
| 58967 | .80765 | .60367 | .79723 | .61749 | .78658 | .63113 | .77568 | 52 | 16 |
| 5899 | .80748 | .6039 | .79706 | .61772 | .7864 | .63135 | .7755 | 51 | 15 |
| 59014 | .8073 | .60414 | .79688 | .61795 | .78622 | .63158 | .77531 | 50 | 15 |
| 59037 | .80713 | .60437 | .79671 | .61818 | .78604 | .6318 | .77513 | 49 | 15 |
| 59061 | .80696 | .6046 | .79653 | .61841 | .78586 | .63203 | .77494 | 48 | 14 |
| 59084 | .80679 | .60483 | .79635 | .61864 | .78568 | .63225 | .77476 | 47 | 14 |
| 59108 | .80662 | .60506 | .79618 | .61887 | .7855 | .63248 | .77458 | 46 | 14 |
| 59131 | .80644 | .60529 | .796 | .61909 | .78532 | .63271 | .77439 | 45 | 14 |
| 59154 | .80627 | .60553 | .79583 | .61932 | .78514 | .63293 | .77421 | 44 | 13 |
| 59178 | .8061 | .60576 | .79565 | .61955 | .78496 | .63316 | .77402 | 43 | 13 |
| 59201 | .80593 | .60599 | .79547 | .61978 | .78478 | .63338 | .77384 | 42 | 13 |
| 59225 | .80576 | .60622 | .7953 | .62001 | .7846 | .63361 | .77366 | 41 | 12 |
| 59248 | .80558 | .60645 | .79512 | .62024 | .78442 | .63383 | .77347 | 40 | 12 |
| 59272 | .80541 | .60668 | .79494 | .62046 | .78424 | .63406 | .77329 | 39 | 12 |
| 59295 | .80524 | .60691 | .79477 | .62069 | .78405 | .63428 | .7731 | 38 | 11 |
| 59318 | .80507 | .60714 | .79459 | .62092 | .78387 | .63451 | .77292 | 37 | 11 |
| 59342 | .80489 | .60738 | .79441 | .62115 | .78369 | .63473 | .77273 | 36 | 11 |
| 59365 | .80472 | .60761 | .79424 | .62138 | .78351 | .63496 | .77255 | 35 | 11 |
| 59389 | .80455 | .60784 | .79406 | .6216 | .78333 | .63518 | .77236 | 34 | 10 |
| 59412 | .80438 | .60807 | .79388 | .62183 | .78315 | .6354 | .77218 | 33 | 10 |
| 59436 | .8042 | .6083 | .79371 | .62206 | .78297 | .63563 | .77199 | 32 | 10 |
| 59459 | .80403 | .60853 | .79353 | .62229 | .78279 | .63585 | .77181 | 31 | 9 |
| 59482 | .80386 | .60876 | .79335 | .62251 | .78261 | .63608 | .77162 | 30 | 9 |
| 59506 | .80368 | .60899 | .79318 | .62274 | .78243 | .6363 | .77144 | 29 | 9 |
| 59529 | .80351 | .60922 | .793 | .62297 | .78225 | .63653 | .77125 | 28 | 8 |
| 59552 | .80334 | .60945 | .79282 | .6232 | .78206 | .63675 | .77107 | 27 | 8 |
| 59576 | .80316 | .60968 | .79264 | .62342 | .78188 | .63698 | .77088 | 26 | 8 |
| 59599 | .80299 | .60991 | .79247 | .62365 | .7817 | .6372 | .7707 | 25 | 8 |
| 59622 | .80282 | .61015 | .79229 | .62388 | .78152 | .63742 | .77051 | 24 | 7 |
| 59646 | .80264 | .61038 | .79211 | .62411 | .78134 | .63765 | .77033 | 23 | 7 |
| 59669 | .80247 | .61061 | .79193 | .62433 | .78116 | .63787 | .77014 | 22 | 7 |
| 59693 | .8023 | .61084 | .79176 | .62456 | .78098 | .6381 | .76996 | 21 | 6 |
| 59716 | .80212 | .61107 | .79158 | .62479 | .78079 | .63832 | .76977 | 20 | 6 |
| 59739 | .80195 | .6113 | .7914 | .62502 | .78061 | .63854 | .76959 | 19 | 6 |
| 59763 | .80178 | .61153 | .79122 | .62524 | .78043 | .63877 | .7694 | 18 | 5 |
| 59786 | .8016 | .61176 | .79105 | .62547 | .78025 | .63899 | .76921 | 17 | 5 |
| 59809 | .80143 | .61199 | .79087 | .6257 | .78007 | .63922 | .76903 | 16 | 5 |
| 59832 | .80125 | .61222 | .79069 | .62592 | .77988 | .63944 | .76884 | 15 | 5 |
| 59856 | .80108 | .61245 | .79051 | .62615 | .7797 | .63966 | .76866 | 14 | 4 |
| 59879 | .80091 | .61268 | .79033 | .62638 | .77952 | .63989 | .76847 | 13 | 4 |
| 59902 | .80073 | .61291 | .79016 | .6266 | .77934 | .64011 | .76828 | 12 | 4 |
| 59926 | .80056 | .61314 | .78998 | .62683 | .77916 | .64033 | .7681 | 11 | 3 |
| 59949 | .80038 | .61337 | .7898 | .62706 | .77897 | .64056 | .76791 | 10 | 3 |
| 59972 | .80021 | .6136 | .78962 | .62728 | .77879 | .64078 | .76772 | 9 | 3 |
| 59995 | .80003 | .61383 | .78944 | .62751 | .77861 | .641 | .76754 | 8 | 2 |
| 60019 | .79986 | .61406 | .78926 | .62774 | .77843 | .64123 | .76735 | 7 | 2 |
| 60042 | .79968 | .61429 | .78908 | .62796 | .77824 | .64145 | .76717 | 6 | 2 |
| 60065 | .79951 | .61451 | .78891 | .62819 | .77806 | .64167 | .76698 | 5 | 2 |
| 60089 | .79934 | .61474 | .78873 | .62842 | .77788 | .6419 | .76679 | 4 | 1 |
| 60112 | .79916 | .61497 | .78855 | .62864 | .77769 | .64212 | .76661 | 3 | 1 |
| 60135 | .79899 | .6152 | .78837 | .62887 | .77751 | .64234 | .76642 | 2 | 1 |
| 60158 | .79881 | .61543 | .78819 | .62909 | .77733 | .64256 | .76623 | 1 | 0 |
| 60182 | .79864 | .61566 | .78801 | .62932 | .77715 | .64279 | .76604 | 0 | 0 |
| N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | N. cos. | N. sine. | | |
| 53° | | 52° | | 51° | | 50° | | | |

| S. sine. | S. cos. | 40° | | 41° | | 42° | | 43° | | S. sine. | S. cos. |
|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|----------|---------|
| | | S. sine. | S. cos. | S. sine. | S. cos. | S. sine. | S. cos. | S. sine. | S. cos. | | |
| 1 | 0 | 65279 | 66044 | 65285 | 66041 | 65291 | 66037 | 65297 | 66033 | 65303 | 66029 |
| 2 | 1 | 65281 | 66042 | 65287 | 66039 | 65293 | 66035 | 65299 | 66031 | 65305 | 66027 |
| 3 | 2 | 65283 | 66040 | 65289 | 66037 | 65295 | 66033 | 65301 | 66029 | 65307 | 66025 |
| 4 | 3 | 65285 | 66038 | 65291 | 66035 | 65297 | 66031 | 65303 | 66027 | 65309 | 66023 |
| 5 | 4 | 65287 | 66036 | 65293 | 66033 | 65299 | 66029 | 65305 | 66025 | 65311 | 66021 |
| 6 | 5 | 65289 | 66034 | 65295 | 66031 | 65301 | 66027 | 65307 | 66023 | 65313 | 66019 |
| 7 | 6 | 65291 | 66032 | 65297 | 66029 | 65303 | 66025 | 65309 | 66021 | 65315 | 66017 |
| 8 | 7 | 65293 | 66030 | 65299 | 66027 | 65305 | 66023 | 65311 | 66019 | 65317 | 66015 |
| 9 | 8 | 65295 | 66028 | 65301 | 66025 | 65307 | 66021 | 65313 | 66017 | 65319 | 66013 |
| 10 | 9 | 65297 | 66026 | 65303 | 66023 | 65309 | 66019 | 65315 | 66015 | 65321 | 66011 |
| 11 | 10 | 65299 | 66024 | 65305 | 66021 | 65311 | 66017 | 65317 | 66013 | 65323 | 66009 |
| 12 | 11 | 65301 | 66022 | 65307 | 66019 | 65313 | 66015 | 65319 | 66011 | 65325 | 66007 |
| 13 | 12 | 65303 | 66020 | 65309 | 66017 | 65315 | 66013 | 65321 | 66009 | 65327 | 66005 |
| 14 | 13 | 65305 | 66018 | 65311 | 66015 | 65317 | 66011 | 65323 | 66007 | 65329 | 66003 |
| 15 | 14 | 65307 | 66016 | 65313 | 66013 | 65319 | 66009 | 65325 | 66005 | 65331 | 66001 |
| 16 | 15 | 65309 | 66014 | 65315 | 66011 | 65321 | 66007 | 65327 | 66003 | 65333 | 65999 |
| 17 | 16 | 65311 | 66012 | 65317 | 66009 | 65323 | 66005 | 65329 | 66001 | 65335 | 65997 |
| 18 | 17 | 65313 | 66010 | 65319 | 66007 | 65325 | 66003 | 65331 | 65999 | 65337 | 65995 |
| 19 | 18 | 65315 | 66008 | 65321 | 66005 | 65327 | 66001 | 65333 | 65997 | 65339 | 65993 |
| 20 | 19 | 65317 | 66006 | 65323 | 66003 | 65329 | 65999 | 65335 | 65995 | 65341 | 65991 |
| 21 | 20 | 65319 | 66004 | 65325 | 66001 | 65331 | 65997 | 65337 | 65993 | 65343 | 65989 |
| 22 | 21 | 65321 | 66002 | 65327 | 65999 | 65333 | 65995 | 65339 | 65991 | 65345 | 65987 |
| 23 | 22 | 65323 | 66000 | 65329 | 65997 | 65335 | 65993 | 65341 | 65989 | 65347 | 65985 |
| 24 | 23 | 65325 | 65998 | 65331 | 65995 | 65337 | 65991 | 65343 | 65987 | 65349 | 65983 |
| 25 | 24 | 65327 | 65996 | 65333 | 65993 | 65339 | 65989 | 65345 | 65985 | 65351 | 65981 |
| 26 | 25 | 65329 | 65994 | 65335 | 65991 | 65341 | 65987 | 65347 | 65983 | 65353 | 65979 |
| 27 | 26 | 65331 | 65992 | 65337 | 65989 | 65343 | 65985 | 65349 | 65981 | 65355 | 65977 |
| 28 | 27 | 65333 | 65990 | 65339 | 65987 | 65345 | 65983 | 65351 | 65979 | 65357 | 65975 |
| 29 | 28 | 65335 | 65988 | 65341 | 65985 | 65347 | 65981 | 65353 | 65977 | 65359 | 65973 |
| 30 | 29 | 65337 | 65986 | 65343 | 65983 | 65349 | 65979 | 65355 | 65975 | 65361 | 65971 |
| 31 | 30 | 65339 | 65984 | 65345 | 65981 | 65351 | 65977 | 65357 | 65973 | 65363 | 65969 |
| 32 | 31 | 65341 | 65982 | 65347 | 65979 | 65353 | 65975 | 65359 | 65971 | 65365 | 65967 |
| 33 | 32 | 65343 | 65980 | 65349 | 65977 | 65355 | 65973 | 65361 | 65969 | 65367 | 65965 |
| 34 | 33 | 65345 | 65978 | 65351 | 65975 | 65357 | 65971 | 65363 | 65967 | 65369 | 65963 |
| 35 | 34 | 65347 | 65976 | 65353 | 65973 | 65359 | 65969 | 65365 | 65965 | 65371 | 65961 |
| 36 | 35 | 65349 | 65974 | 65355 | 65971 | 65361 | 65967 | 65367 | 65963 | 65373 | 65959 |
| 37 | 36 | 65351 | 65972 | 65357 | 65969 | 65363 | 65965 | 65369 | 65961 | 65375 | 65957 |
| 38 | 37 | 65353 | 65970 | 65359 | 65967 | 65365 | 65963 | 65371 | 65959 | 65377 | 65955 |
| 39 | 38 | 65355 | 65968 | 65361 | 65965 | 65367 | 65961 | 65373 | 65957 | 65379 | 65953 |
| 40 | 39 | 65357 | 65966 | 65363 | 65963 | 65369 | 65959 | 65375 | 65955 | 65381 | 65951 |
| 41 | 40 | 65359 | 65964 | 65365 | 65961 | 65371 | 65957 | 65377 | 65953 | 65383 | 65949 |
| 42 | 41 | 65361 | 65962 | 65367 | 65959 | 65373 | 65955 | 65379 | 65951 | 65385 | 65947 |
| 43 | 42 | 65363 | 65960 | 65369 | 65957 | 65375 | 65953 | 65381 | 65949 | 65387 | 65945 |
| 44 | 43 | 65365 | 65958 | 65371 | 65955 | 65377 | 65951 | 65383 | 65947 | 65389 | 65943 |
| 45 | 44 | 65367 | 65956 | 65373 | 65953 | 65379 | 65949 | 65385 | 65945 | 65391 | 65941 |
| 46 | 45 | 65369 | 65954 | 65375 | 65951 | 65381 | 65947 | 65387 | 65943 | 65393 | 65939 |
| 47 | 46 | 65371 | 65952 | 65377 | 65949 | 65383 | 65945 | 65389 | 65941 | 65395 | 65937 |
| 48 | 47 | 65373 | 65950 | 65379 | 65947 | 65385 | 65943 | 65391 | 65939 | 65397 | 65935 |
| 49 | 48 | 65375 | 65948 | 65381 | 65945 | 65387 | 65941 | 65393 | 65937 | 65399 | 65933 |
| 50 | 49 | 65377 | 65946 | 65383 | 65943 | 65389 | 65939 | 65395 | 65935 | 65401 | 65931 |
| 51 | 50 | 65379 | 65944 | 65385 | 65941 | 65391 | 65937 | 65397 | 65933 | 65403 | 65929 |
| 52 | 51 | 65381 | 65942 | 65387 | 65939 | 65393 | 65935 | 65399 | 65931 | 65405 | 65927 |
| 53 | 52 | 65383 | 65940 | 65389 | 65937 | 65395 | 65933 | 65401 | 65929 | 65407 | 65925 |
| 54 | 53 | 65385 | 65938 | 65391 | 65935 | 65397 | 65931 | 65403 | 65927 | 65409 | 65923 |
| 55 | 54 | 65387 | 65936 | 65393 | 65933 | 65399 | 65929 | 65405 | 65925 | 65411 | 65921 |
| 56 | 55 | 65389 | 65934 | 65395 | 65931 | 65401 | 65927 | 65407 | 65923 | 65413 | 65919 |
| 57 | 56 | 65391 | 65932 | 65397 | 65929 | 65403 | 65925 | 65409 | 65921 | 65415 | 65917 |
| 58 | 57 | 65393 | 65930 | 65399 | 65927 | 65405 | 65923 | 65411 | 65919 | 65417 | 65915 |
| 59 | 58 | 65395 | 65928 | 65401 | 65925 | 65407 | 65921 | 65413 | 65917 | 65419 | 65913 |
| 60 | 59 | 65397 | 65926 | 65403 | 65923 | 65409 | 65919 | 65415 | 65915 | 65421 | 65911 |

| 44° | | | | 44° | | | |
|-----|----------|----------|--------------|-----|----------|----------|--------------|
| | N. sine. | N. cos. | Prop. parts. | | N. sine. | N. cos. | Prop. parts. |
| | | | 19 | | | | 9 |
| 0 | .69466 | .71934 | 60 | 19 | .70112 | .71305 | 29 |
| 1 | .69487 | .71914 | 59 | 10 | .70132 | .71284 | 28 |
| 2 | .69508 | .71894 | 58 | 18 | .70153 | .71264 | 27 |
| 3 | .69529 | .71873 | 57 | 18 | .70174 | .71243 | 26 |
| 4 | .69549 | .71853 | 56 | 13 | .70195 | .71223 | 25 |
| 5 | .6957 | .71833 | 55 | 17 | .70215 | .71203 | 24 |
| 6 | .69591 | .71813 | 54 | 17 | .70236 | .71182 | 23 |
| 7 | .69612 | .71792 | 53 | 14 | .70257 | .71162 | 22 |
| 8 | .69633 | .71772 | 52 | 16 | .70277 | .71141 | 21 |
| 9 | .69654 | .71752 | 51 | 16 | .70298 | .71121 | 20 |
| 10 | .69675 | .71732 | 50 | 16 | .70319 | .711 | 19 |
| 11 | .69696 | .71711 | 49 | 16 | .70339 | .7108 | 18 |
| 12 | .69717 | .71691 | 48 | 15 | .7036 | .71059 | 17 |
| 13 | .69737 | .71671 | 47 | 15 | .70381 | .71039 | 16 |
| 14 | .69758 | .7165 | 46 | 15 | .70401 | .71019 | 15 |
| 15 | .69779 | .7163 | 45 | 14 | .70422 | .70998 | 14 |
| 16 | .698 | .7161 | 44 | 14 | .70443 | .70978 | 13 |
| 17 | .69821 | .7159 | 43 | 14 | .70463 | .70957 | 12 |
| 18 | .69842 | .71569 | 42 | 13 | .70484 | .70937 | 11 |
| 19 | .69862 | .71549 | 41 | 13 | .70505 | .70916 | 10 |
| 20 | .69883 | .71529 | 40 | 13 | .70525 | .70896 | 9 |
| 21 | .69904 | .71508 | 39 | 12 | .70546 | .70875 | 8 |
| 22 | .69925 | .71488 | 38 | 12 | .70567 | .70855 | 7 |
| 23 | .69946 | .71468 | 37 | 12 | .70587 | .70834 | 6 |
| 24 | .69966 | .71447 | 36 | 11 | .70608 | .70813 | 5 |
| 25 | .69987 | .71427 | 35 | 11 | .70628 | .70793 | 4 |
| 26 | .70008 | .71407 | 34 | 11 | .70649 | .70772 | 3 |
| 27 | .70029 | .71386 | 33 | 10 | .7067 | .70752 | 2 |
| 28 | .70049 | .71366 | 32 | 10 | .7069 | .70731 | 1 |
| 29 | .7007 | .71345 | 31 | 10 | .70711 | .70711 | 0 |
| 30 | .70091 | .71325 | 30 | 10 | | | |
| | N. cos. | N. sine. | | | N. cos. | N. sine. | |
| 45° | | | | 45° | | | |

Preceding Table contains Natural Sine and Cosine for every minute the Quadrant to Radius 1.

If Degrees are taken at head of columns, Minutes, Sine, and Cosine must be taken from head also; and if they are taken at foot of column, Minutes, Sine, must be taken from foot also.

ILLUSTRATION.—.3173 is sine of $18^{\circ} 30'$, and cosine of $71^{\circ} 30'$.

To Compute Sine or Cosine for Seconds.

When Angle is less than 45° . RULE.—Ascertain sine or cosine of angle in degrees and minutes from Table; take difference between it and sine or cosine of angle next below it. Look for this difference or remainder in the column of *Proportional Parts*, on left side if Sine is required, at head of column on right side; and in the respective columns, opposite to number of seconds of angle in column, is number or correction in seconds to be added to Sine, or subtracted from cosine of angle.

ILLUSTRATION I.—What is sine of $8^{\circ} 9' 10''$?

Sine of $8^{\circ} 9'$, per Table = .14177; } .00028 difference
Sine of $8^{\circ} 10'$, " = .14205; }

In the side column of *proportional parts*, under 28, and opposite to 10, which, being added to .14177 = .14182 Sine.

These instances will give a unit too much, but this, in general, is so.

2.—What is cosine of $80^{\circ} 9' 10''$?

Cosine of $80^{\circ} 9'$, per Table = .98990; }
Cosine of $80^{\circ} 10'$, " = .98986; } .00004 difference.

In right-side column of *proportional parts*, under 4, and opposite to $10'$, is 1, the correction for $10'$, which, being subtracted from .98990 = .98989 cosine.

When Angle exceeds 45° . RULE.—Ascertain sine or cosine for angle in degrees and minutes from Table, taking degrees at the foot of it; then take difference between it and sine or cosine of angle next above it. Look for remainder, if *Sine* is required, at head of column of *Proportional Parts*, on right side; and if *Cosine* is required, at head of column on left side; and in these respective columns, opposite to seconds of angle, is number or correction in seconds to be added to Sine, or subtracted from Cosine of angle.

ILLUSTRATION.—What is the Sine and Cosine of $81^{\circ} 50' 50''$?

Sine of $81^{\circ} 50'$, per Table = .98986; }
Sine of $81^{\circ} 51'$, " = .98999; } .00004 difference.

In right-side column of *proportional parts*, and opposite to $50'$, is 3, which, added to .98986 = .98989 Sine.

Cosine of $81^{\circ} 50'$, per Table = .14205; }
Cosine of $81^{\circ} 51'$, " = .14177; } .00025 difference.

In left-side column of *proportional parts*, and opposite to $50'$, is 24, which, subtracted from .14205 = .14181 Cosine.

To Ascertain or Compute Number of Degrees, Minutes, and Seconds of a given Sine or Cosine.

When Sine is given. RULE.—If given sine is in Table, the degrees of it will be at top or bottom of page, and minutes in marginal column, at left or right side, according as sine corresponds to an angle less or greater than 45° .

If given sine is not in Table, take sine in Table which is next *less* than the one for which degrees, etc., are required, and note degrees, etc., for it. Subtract this sine from next *greater* tabular sine, and also from given sine.

Then, as tabular difference is to difference between given sine and tabular sine, so is 60 seconds to seconds for sine given.

EXAMPLE.—What are the degrees, minutes, and seconds for sine of .75?

Next less sine is .74992, arc for which is $48^{\circ} 35'$. Next greater sine is .75011, difference between which and next less is .75011 — .74992 = .00019. Difference between less tabular sine and one given is .75 — .74992 = 8.

Then $19 : 8 :: 60 : 25+$, which, added to $48^{\circ} 35' = 48^{\circ} 35' 25''$.

When Cosine is given. RULE.—If given cosine is found in Table, degrees of it will be found as in manner specified when sine is given.

If given cosine is not in Table, take cosine in Table which is next *greater* than one for which degrees, etc., are required, and note degrees, etc., for it. Subtract this cosine from next *less* tabular cosine, and also from given cosine.

Then, as tabular difference is to difference between given cosine and tabular cosine, so is 60 seconds to seconds for cosine given.

EXAMPLE.—What are the degrees, minutes, and seconds for cosine of .75?

Next greater cosine is .75011, arc for which is $41^{\circ} 24'$. Next less cosine is .74999, difference between which and next greater is .75011 — .74999 = .00012. Difference between greater tabular cosine and one given is .75011 — .75000 = 11.

Then $12 : 11 :: 60 : 35-$, which, added to $41^{\circ} 24' = 41^{\circ} 24' 35''$.

To Compute Versed Sine of an Angle.

EXAMPLE.—What is the versed sine of $21^{\circ} 30'$?

Is the versed sine of $21^{\circ} 30'$?

$30'$ is .93042, which, — 1 = .06958 versed sine.

To Compute the Versed Sine of an Angle.

EXAMPLE.—What is the versed sine of $21^{\circ} 30'$?

Is the versed sine of $21^{\circ} 30'$?

$30'$ is .93042, which, — 1 = .06958 versed sine.

Natural Secants and Co-secants.

| 0° | | | | 1° | | 2° | | 3° | |
|-----|------------|-----------|-----------|-----------|-----------|---------|-----------|----|--|
| NT. | CO-SECANT. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | | |
| | Infinite. | 1.0001 | 57.299 | 1.0006 | 28.654 | 1.0014 | 19.107 | 60 | |
| | 3437.7 | .0001 | 6.359 | .0006 | 8.417 | .0014 | 9.002 | 59 | |
| | 1718.9 | .0002 | 5.45 | .0006 | 8.184 | .0014 | 8.897 | 58 | |
| | 145.9 | .0002 | 4.57 | .0006 | 7.955 | .0014 | 8.794 | 57 | |
| | 859.44 | .0002 | 3.718 | .0006 | 7.73 | .0014 | 8.692 | 56 | |
| | 687.55 | 1.0002 | 52.891 | 1.0007 | 27.508 | 1.0014 | 18.591 | 55 | |
| | 572.96 | .0002 | 2.09 | .0007 | 7.29 | .0015 | 8.491 | 54 | |
| | 491.11 | .0002 | 1.313 | .0007 | 7.075 | .0015 | 8.393 | 53 | |
| | 29.72 | .0002 | 0.558 | .0007 | 6.864 | .0015 | 8.295 | 52 | |
| | 381.97 | .0002 | 49.826 | .0007 | 6.655 | .0015 | 8.198 | 51 | |
| | 343.77 | 1.0002 | 49.114 | 1.0007 | 26.45 | 1.0015 | 18.103 | 50 | |
| | 12.52 | .0002 | 8.422 | .0007 | 6.249 | .0015 | 8.008 | 49 | |
| | 286.48 | .0002 | 7.75 | .0007 | 6.05 | .0016 | 7.914 | 48 | |
| | 64.44 | .0002 | 7.096 | .0007 | 5.854 | .0016 | 7.821 | 47 | |
| | 45.55 | .0002 | 6.46 | .0008 | 5.661 | .0016 | 7.73 | 46 | |
| | 229.18 | 1.0002 | 45.84 | 1.0008 | 25.471 | 1.0016 | 17.639 | 45 | |
| | 14.86 | .0002 | 5.237 | .0008 | 5.284 | .0016 | 7.549 | 44 | |
| | 02.22 | .0002 | 4.65 | .0008 | 5.1 | .0016 | 7.46 | 43 | |
| | 190.99 | .0002 | 4.077 | .0008 | 4.918 | .0017 | 7.372 | 42 | |
| | 80.73 | .0003 | 3.52 | .0008 | 4.739 | .0017 | 7.285 | 41 | |
| | 171.89 | 1.0003 | 42.976 | 1.0008 | 24.562 | 1.0017 | 17.198 | 40 | |
| | 63.7 | .0003 | 2.445 | .0008 | 4.358 | .0017 | 7.113 | 39 | |
| | 56.26 | .0003 | 1.928 | .0008 | 4.216 | .0017 | 7.028 | 38 | |
| | 49.47 | .0003 | 1.423 | .0009 | 4.047 | .0017 | 6.944 | 37 | |
| | 43.24 | .0003 | 40.93 | .0009 | 3.88 | .0018 | 6.861 | 36 | |
| | 137.51 | 1.0003 | 40.448 | 1.0009 | 23.716 | 1.0018 | 16.779 | 35 | |
| | 32.22 | .0003 | 39.978 | .0009 | 3.553 | .0018 | 6.698 | 34 | |
| | 27.32 | .0003 | 9.518 | .0009 | 3.393 | .0018 | 6.617 | 33 | |
| | 22.78 | .0003 | 9.069 | .0009 | 3.235 | .0018 | 6.538 | 32 | |
| | 18.54 | .0003 | 8.631 | .0009 | 3.079 | .0018 | 6.459 | 31 | |
| | 114.59 | 1.0003 | 38.201 | 1.0009 | 22.925 | 1.0019 | 16.38 | 30 | |
| | 10.9 | .0003 | 7.782 | .001 | 2.774 | .0019 | 6.303 | 29 | |
| | 07.43 | .0003 | 7.371 | .001 | 2.624 | .0019 | 6.226 | 28 | |
| | 04.17 | .0004 | 6.969 | .001 | 2.476 | .0019 | 6.15 | 27 | |
| | 01.11 | .0004 | 6.576 | .001 | 2.33 | .0019 | 6.075 | 26 | |
| | 98.223 | 1.0004 | 36.191 | 1.001 | 22.186 | 1.0019 | 16 | 25 | |
| | 5.495 | .0004 | 5.814 | .001 | 2.044 | .002 | 5.926 | 24 | |
| | 2.914 | .0004 | 5.445 | .001 | 1.904 | .002 | 5.853 | 23 | |
| | 2.469 | .0004 | 5.084 | .001 | 1.765 | .002 | 5.78 | 22 | |
| 001 | 88.149 | .0004 | 4.729 | .0011 | 1.629 | .002 | 5.708 | 21 | |
| 001 | 85.946 | 1.0004 | 34.382 | 1.0011 | 21.494 | 1.002 | 15.637 | 20 | |
| 001 | 3.849 | .0004 | 4.042 | .0011 | 1.36 | .0021 | 5.566 | 19 | |
| 001 | 1.853 | .0004 | 3.708 | .0011 | 1.228 | .0021 | 5.496 | 18 | |
| 001 | 79.95 | .0004 | 3.381 | .0011 | 1.098 | .0021 | 5.427 | 17 | |
| 001 | 8.133 | .0004 | 3.06 | .0011 | 20.97 | .0021 | 5.358 | 16 | |
| 001 | 76.396 | 1.0005 | 32.745 | 1.0011 | 20.843 | 1.0021 | 15.29 | 15 | |
| 001 | 4.736 | .0005 | 2.437 | .0012 | 0.717 | .0022 | 5.222 | | |
| 001 | 3.146 | .0005 | 2.134 | .0012 | 0.593 | .0022 | 5.155 | | |
| 001 | 1.622 | .0005 | 1.836 | .0012 | 0.471 | .0022 | 5.089 | | |
| 001 | 1.16 | .0005 | 1.544 | .0012 | 0.35 | .0022 | 5.023 | | |
| 001 | 68.757 | 1.0005 | 31.257 | 1.0012 | 20.23 | 1.0022 | 14.958 | | |
| 001 | 7.409 | .0005 | 30.976 | .0012 | 0.112 | .0023 | 4.893 | | |
| 001 | 6.113 | .0005 | 0.699 | .0012 | 19.995 | .0023 | 4.829 | | |
| 001 | 4.866 | .0005 | 0.428 | .0013 | 9.88 | | 4.765 | | |
| 001 | 3.664 | .0005 | 0.161 | .0013 | 9.766 | | | | |
| 001 | 62.507 | 1.0005 | 29.899 | 1.0013 | 19.653 | | | | |
| 001 | 1.391 | .0006 | 9.641 | .0013 | 9.541 | | | | |
| 001 | 1.314 | .0006 | 9.388 | .0013 | 9.431 | | | | |
| 001 | 59.274 | .0006 | 9.139 | .0013 | 9.321 | | | | |
| 001 | 8.27 | .0006 | 8.894 | .0013 | 9.214 | | | | |
| 001 | 57.299 | 1.0006 | 28.654 | 1.0014 | 19.107 | | | | |
| 89° | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | | | | |
| 88° | | | | | | | | | |
| 87° | | | | | | | | | |

| | 4° | | 5° | | 6° | | 7° | |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. |
| 0 | 1.0024 | 14.335 | 1.0038 | 11.474 | 1.0055 | 9.5668 | 1.0075 | 8.2055 |
| 1 | .0025 | 4.276 | .0038 | 1.436 | .0055 | .5404 | .0075 | .1861 |
| 2 | .0025 | 4.217 | .0039 | 1.398 | .0056 | .5141 | .0076 | .1668 |
| 3 | .0025 | 4.159 | .0039 | 1.36 | .0056 | .488 | .0076 | .1476 |
| 4 | .0025 | 4.101 | .0039 | 1.323 | .0056 | .462 | .0076 | .1285 |
| 5 | 1.0025 | 14.043 | 1.0039 | 11.286 | 1.0057 | 9.4362 | 1.0077 | 8.1094 |
| 6 | .0026 | 3.986 | .004 | 1.249 | .0057 | .4105 | .0077 | .0905 |
| 7 | .0026 | 3.93 | .004 | 1.213 | .0057 | .385 | .0078 | .0717 |
| 8 | .0026 | 3.874 | .004 | 1.176 | .0057 | .3596 | .0078 | .0529 |
| 9 | .0026 | 3.818 | .004 | 1.14 | .0058 | .3343 | .0078 | .0342 |
| 10 | 1.0026 | 13.763 | 1.0041 | 11.104 | 1.0058 | 9.3092 | 1.0079 | 8.0156 |
| 11 | .0027 | 3.708 | .0041 | 1.069 | .0058 | .2842 | .0079 | .7.9971 |
| 12 | .0027 | 3.654 | .0041 | 1.033 | .0059 | .2593 | .0079 | .5787 |
| 13 | .0027 | 3.6 | .0041 | 0.988 | .0059 | .2346 | .008 | .5604 |
| 14 | .0027 | 3.547 | .0042 | 0.963 | .0059 | .21 | .008 | .5421 |
| 15 | 1.0027 | 13.494 | 1.0042 | 10.929 | 1.006 | 9.1855 | 1.008 | 7.924 |
| 16 | .0028 | 3.441 | .0042 | 0.894 | .006 | .1612 | .0081 | .9059 |
| 17 | .0028 | 3.389 | .0043 | 0.86 | .006 | .137 | .0081 | .8879 |
| 18 | .0028 | 3.337 | .0043 | 0.826 | .0061 | .1129 | .0082 | .87 |
| 19 | .0028 | 3.286 | .0043 | 0.792 | .0061 | .089 | .0082 | .8522 |
| 20 | 1.0029 | 13.235 | 1.0043 | 10.758 | 1.0061 | 9.0651 | 1.0082 | 7.8344 |
| 21 | .0029 | 3.184 | .0044 | 0.725 | .0062 | .0414 | .0083 | .8168 |
| 22 | .0029 | 3.134 | .0044 | 0.692 | .0062 | .0179 | .0083 | .7992 |
| 23 | .0029 | 3.084 | .0044 | 0.659 | .0062 | 8.9944 | .0084 | .7817 |
| 24 | .0029 | 3.034 | .0044 | 0.626 | .0063 | .9711 | .0084 | .7642 |
| 25 | 1.003 | 12.985 | 1.0045 | 10.593 | 1.0063 | 8.9479 | 1.0084 | 7.7409 |
| 26 | .003 | 2.937 | .0045 | 0.561 | .0063 | .9248 | .0085 | .7526 |
| 27 | .003 | 2.888 | .0045 | 0.529 | .0064 | .9018 | .0085 | .7342 |
| 28 | .003 | 2.84 | .0046 | 0.497 | .0064 | .879 | .0085 | .6953 |
| 29 | .0031 | 2.793 | .0046 | 0.465 | .0064 | .8563 | .0086 | .6783 |
| 30 | 1.0031 | 12.745 | 1.0046 | 10.433 | 1.0065 | 8.8337 | 1.0086 | 7.6613 |
| 31 | .0031 | 2.698 | .0046 | 0.402 | .0065 | .8112 | .0087 | .6444 |
| 32 | .0031 | 2.652 | .0047 | 0.371 | .0065 | .7888 | .0087 | .6276 |
| 33 | .0032 | 2.606 | .0047 | 0.34 | .0066 | .7665 | .0087 | .6108 |
| 34 | .0032 | 2.56 | .0047 | 0.309 | .0066 | .7444 | .0088 | .5942 |
| 35 | 1.0032 | 12.514 | 1.0048 | 10.278 | 1.0066 | 8.7223 | 1.0088 | 7.5776 |
| 36 | .0032 | 2.469 | .0048 | 0.248 | .0067 | .7004 | .0089 | .5611 |
| 37 | .0032 | 2.424 | .0048 | 0.217 | .0067 | .6786 | .0089 | .5446 |
| 38 | .0033 | 2.379 | .0048 | 0.187 | .0067 | .6569 | .0089 | .5282 |
| 39 | .0033 | 2.335 | .0049 | 0.157 | .0068 | .6353 | .009 | .5119 |
| 40 | 1.0033 | 12.291 | 1.0049 | 10.127 | 1.0068 | 8.6138 | 1.009 | 7.4957 |
| 41 | .0033 | 2.248 | .0049 | 0.098 | .0068 | .5924 | .009 | .4795 |
| 42 | .0034 | 2.204 | .005 | 0.068 | .0069 | .5711 | .0091 | .4634 |
| 43 | .0034 | 2.161 | .005 | 0.039 | .0069 | .5499 | .0091 | .4474 |
| 44 | .0034 | 2.118 | .005 | 0.01 | .0069 | .5289 | .0092 | .4315 |
| 45 | 1.0034 | 12.076 | 1.005 | 9.9812 | 1.007 | 8.5079 | 1.0092 | 7.4151 |
| 46 | .0035 | 2.034 | .0051 | .9525 | .007 | .4871 | .0092 | .3996 |
| 47 | .0035 | 1.992 | .0051 | .9239 | .007 | .4663 | .0093 | .384 |
| 48 | .0035 | 1.95 | .0051 | .8955 | .0071 | .4457 | .0093 | .3683 |
| 49 | .0035 | 1.909 | .0052 | .8672 | .0071 | .4251 | .0094 | .3527 |
| 50 | 1.0036 | 11.868 | 1.0052 | 9.8391 | 1.0071 | 8.4046 | 1.0094 | 7.3379 |
| 51 | .0036 | 1.828 | .0052 | .8112 | .0072 | .3843 | .0094 | .3377 |
| 52 | .0036 | 1.787 | .0053 | .7834 | .0072 | .3640 | .0095 | .3223 |
| 53 | .0036 | 1.747 | .0053 | .7558 | .0073 | .3439 | .0095 | .3069 |
| 54 | .0037 | 1.707 | .0053 | .7283 | .0073 | .3238 | .0096 | .2915 |
| 55 | 1.0037 | 11.668 | 1.0053 | 9.701 | 1.0073 | 8.3039 | 1.0096 | 7.2604 |
| 56 | .0037 | 1.628 | .0054 | .6739 | .0074 | .2840 | .0097 | .2453 |
| 57 | .0037 | 1.589 | .0054 | .6469 | .0074 | .2642 | .0097 | .2298 |
| 58 | .0038 | 1.55 | .0054 | .62 | .0074 | .2446 | .0097 | .2143 |
| 59 | .0038 | 1.512 | .0055 | .5933 | .0075 | .225 | .0098 | .2000 |
| 60 | 1.0038 | 11.474 | 1.0055 | 9.5668 | 1.0075 | 8.2055 | .0098 | 7.1853 |
| | 85° | | 84° | | 83° | | 82° | |
| | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. |

| 8° | 9° | | 10° | | 11° | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----|
| Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | ' |
| 7.1853 | 1.0125 | 6.3924 | 1.0154 | 5.7588 | 1.0187 | 5.2408 | 60 |
| .1704 | .0125 | .3807 | .0155 | .7493 | .0188 | .233 | 59 |
| .1557 | .0125 | .369 | .0155 | .7398 | .0188 | .2252 | 58 |
| .1409 | .0126 | .3574 | .0156 | .7304 | .0189 | .2174 | 57 |
| .1263 | .0126 | .3458 | .0156 | .721 | .0189 | .2097 | 56 |
| 7.1117 | 1.0127 | 6.3343 | 1.0157 | 5.7117 | 1.019 | 5.2019 | 55 |
| .0972 | .0127 | .3328 | .0157 | .7023 | .0191 | .1942 | 54 |
| .0827 | .0128 | .3213 | .0158 | .693 | .0191 | .1865 | 53 |
| .0683 | .0128 | .3099 | .0158 | .6838 | .0192 | .1788 | 52 |
| .0539 | .0129 | .2985 | .0159 | .6745 | .0192 | .1712 | 51 |
| 7.0396 | 1.0129 | 6.2772 | 1.0159 | 5.6653 | 1.0193 | 5.1636 | 50 |
| .0254 | .013 | .2869 | .016 | .6661 | .0193 | .165 | 49 |
| .0112 | .013 | .2746 | .016 | .647 | .0194 | .1484 | 48 |
| 6.9971 | .0131 | .2434 | .0161 | .6379 | .0195 | .1409 | 47 |
| .983 | .0131 | .2322 | .0162 | .6288 | .0195 | .1333 | 46 |
| 6.969 | 1.0132 | 6.2211 | 1.0162 | 5.6197 | 1.0196 | 5.1258 | 45 |
| .955 | .0132 | .21 | .0163 | .6107 | .0196 | .1183 | 44 |
| .9411 | .0133 | .199 | .0163 | .6017 | .0197 | .1109 | 43 |
| .9273 | .0133 | .188 | .0164 | .5928 | .0198 | .1034 | 42 |
| .9135 | .0134 | .177 | .0164 | .5838 | .0198 | .096 | 41 |
| 6.8998 | 1.0134 | 6.1661 | 1.0165 | 5.5749 | 1.0199 | 5.0886 | 40 |
| .8861 | .0135 | .1552 | .0165 | .566 | .0199 | .0812 | 39 |
| .8725 | .0135 | .1443 | .0166 | .5572 | .02 | .0739 | 38 |
| .8589 | .0136 | .1335 | .0166 | .5484 | .0201 | .0666 | 37 |
| .8454 | .0136 | .1227 | .0167 | .5396 | .0201 | .0593 | 36 |
| 6.832 | 1.0136 | 6.112 | 1.0167 | 5.5308 | 1.0202 | 5.052 | 35 |
| .8185 | .0137 | .1013 | .0168 | .5221 | .0202 | .0447 | 34 |
| .8052 | .0137 | .0906 | .0169 | .5134 | .0203 | .0375 | 33 |
| .7919 | .0138 | .08 | .0169 | .5047 | .0204 | .0302 | 32 |
| .7787 | .0138 | .0694 | .017 | .496 | .0204 | .023 | 31 |
| 6.7655 | 1.0139 | 6.0588 | 1.017 | 5.4874 | 1.0205 | 5.0158 | 30 |
| .7523 | .0139 | .0483 | .0171 | .4788 | .0205 | .0087 | 29 |
| .7392 | .014 | .0379 | .0171 | .4702 | .0206 | .0015 | 28 |
| .7262 | .014 | .0274 | .0172 | .4617 | .0207 | 4.9944 | 27 |
| .7132 | .0141 | .017 | .0172 | .4532 | .0207 | .9873 | 26 |
| 6.7003 | 1.0141 | 6.0066 | 1.0173 | 5.4447 | 1.0208 | 4.9802 | 25 |
| .6874 | .0142 | 5.9963 | .0174 | .4362 | .0208 | .9732 | 24 |
| .6745 | .0142 | .986 | .0174 | .4278 | .0209 | .9661 | 23 |
| .6617 | .0143 | .9758 | .0175 | .4194 | .021 | .9591 | 22 |
| .649 | .0143 | .9655 | .0175 | .411 | .021 | .9521 | 21 |
| 6.6363 | 1.0144 | 5.9554 | 1.0176 | 5.4026 | 1.0211 | 4.9452 | 20 |
| .6237 | .0144 | .9452 | .0176 | .3943 | .0211 | .9382 | 19 |
| .6111 | .0145 | .9351 | .0177 | .386 | .0212 | .9313 | 18 |
| .5985 | .0145 | .925 | .0177 | .3777 | .0213 | .9243 | 17 |
| .586 | .0146 | .915 | .0178 | .3695 | .0213 | .9175 | 16 |
| 6.5736 | 1.0146 | 5.9049 | 1.0179 | 5.3612 | 1.0214 | 4.9106 | 15 |
| .5612 | .0147 | .895 | .0179 | .353 | .0215 | .9037 | 14 |
| .5488 | .0147 | .885 | .018 | .3449 | .0215 | .8969 | 13 |
| .5365 | .0148 | .8751 | .018 | .3367 | .0216 | .8901 | 12 |
| .5243 | .0148 | .8652 | .0181 | .3286 | .0216 | .8833 | 11 |
| 6.5121 | 1.0149 | 5.8554 | 1.0181 | 5.3205 | 1.0217 | 4.8765 | 10 |
| .4999 | .015 | .8456 | .0182 | .3124 | .0218 | .8697 | 9 |
| .4878 | .015 | .8358 | .0182 | .3044 | .0218 | .863 | 8 |
| .4757 | .0151 | .8261 | .0183 | .2963 | .0219 | .8563 | 7 |
| .4637 | .0151 | .8163 | .0184 | .2883 | .022 | .8496 | 6 |
| 6.4517 | 1.0152 | 5.8067 | 1.0184 | 5.2803 | 1.022 | 4.8429 | 5 |
| .4398 | .0152 | .797 | .0185 | .2724 | .0221 | .8362 | 4 |
| .4279 | .0153 | .7874 | .0185 | .2645 | .0221 | .8296 | 3 |
| .416 | .0153 | .7778 | .0186 | .2566 | .0222 | .8229 | 2 |
| .4042 | .0154 | .7683 | .0186 | .2487 | .0223 | .8163 | 1 |
| 6.3924 | 1.0154 | 5.7588 | 1.0187 | 5.2408 | 1.0223 | 4.8097 | 0 |
| SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | |

1°

80°

79°

78°

| ° | 12° | | 13° | | 14° | | 15° | | ° |
|----|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.0223 | 4.8097 | 1.0263 | 4.4454 | 1.0306 | 4.1336 | 1.0353 | 3.8637 | 60 |
| 1 | .0224 | .8032 | .0264 | .4398 | .0307 | .1287 | .0353 | .8995 | 59 |
| 2 | .0225 | .7966 | .0264 | .4342 | .0308 | .1239 | .0354 | .8953 | 58 |
| 3 | .0225 | .7901 | .0265 | .4287 | .0308 | .1191 | .0355 | .8911 | 57 |
| 4 | .0226 | .7835 | .0266 | .4231 | .0309 | .1144 | .0356 | .8870 | 56 |
| 5 | 1.0226 | 4.777 | 1.0266 | 4.4176 | 1.031 | 4.1096 | 1.0357 | 3.8428 | 55 |
| 6 | .0227 | .7706 | .0267 | .4121 | .0311 | .1048 | .0358 | .8837 | 54 |
| 7 | .0228 | .7641 | .0268 | .4065 | .0311 | .1001 | .0358 | .8796 | 53 |
| 8 | .0228 | .7576 | .0268 | .4011 | .0312 | .0953 | .0359 | .8754 | 52 |
| 9 | .0229 | .7512 | .0269 | .3956 | .0313 | .0906 | .036 | .8713 | 51 |
| 10 | 1.023 | 4.7448 | 1.027 | 4.3901 | 1.0314 | 4.0859 | 1.0361 | 3.8222 | 50 |
| 11 | .023 | .7384 | .0271 | .3847 | .0314 | .0812 | .0362 | .8681 | 49 |
| 12 | .0231 | .732 | .0271 | .3792 | .0315 | .0765 | .0362 | .864 | 48 |
| 13 | .0232 | .7257 | .0272 | .3738 | .0316 | .0718 | .0363 | .86 | 47 |
| 14 | .0232 | .7193 | .0273 | .3684 | .0317 | .0672 | .0364 | .8559 | 46 |
| 15 | 1.0233 | 4.713 | 1.0273 | 4.363 | 1.0317 | 4.0625 | 1.0365 | 3.8018 | 45 |
| 16 | .0234 | .7067 | .0274 | .3576 | .0318 | .0579 | .0366 | .7978 | 44 |
| 17 | .0234 | .7004 | .0275 | .3522 | .0319 | .0532 | .0367 | .7937 | 43 |
| 18 | .0235 | .6942 | .0276 | .3469 | .032 | .0486 | .0367 | .7897 | 42 |
| 19 | .0235 | .6879 | .0276 | .3415 | .032 | .044 | .0368 | .7857 | 41 |
| 20 | 1.0236 | 4.6817 | 1.0277 | 4.3362 | 1.0321 | 4.0394 | 1.0369 | 3.7816 | 40 |
| 21 | .0237 | .6754 | .0278 | .3309 | .0322 | .0348 | .037 | .7776 | 39 |
| 22 | .0237 | .6692 | .0278 | .3256 | .0323 | .0302 | .0371 | .7736 | 38 |
| 23 | .0238 | .6631 | .0279 | .3203 | .0323 | .0256 | .0371 | .7697 | 37 |
| 24 | .0239 | .6569 | .028 | .315 | .0324 | .0211 | .0372 | .7657 | 36 |
| 25 | 1.0239 | 4.6507 | 1.028 | 4.3098 | 1.0325 | 4.0165 | 1.0373 | 3.7617 | 35 |
| 26 | .024 | .6446 | .0281 | .3045 | .0326 | .012 | .0374 | .7577 | 34 |
| 27 | .0241 | .6385 | .0282 | .2993 | .0327 | .0074 | .0375 | .7538 | 33 |
| 28 | .0241 | .6324 | .0283 | .2941 | .0327 | .0029 | .0376 | .7498 | 32 |
| 29 | .0242 | .6263 | .0283 | .2888 | .0328 | 3.9984 | .0376 | .7459 | 31 |
| 30 | 1.0243 | 4.6202 | 1.0284 | 4.2836 | 1.0329 | 3.9939 | 1.0377 | 3.742 | 30 |
| 31 | .0243 | .6142 | .0285 | .2785 | .033 | .9894 | .0378 | .738 | 29 |
| 32 | .0244 | .6081 | .0285 | .2733 | .033 | .985 | .0379 | .7341 | 28 |
| 33 | .0245 | .6021 | .0286 | .2681 | .0331 | .9805 | .038 | .7302 | 27 |
| 34 | .0245 | .5961 | .0287 | .263 | .0332 | .976 | .0381 | .7263 | 26 |
| 35 | 1.0246 | 4.5901 | 1.0288 | 4.2579 | 1.0333 | 3.9716 | 1.0382 | 3.7224 | 25 |
| 36 | .0247 | .5841 | .0288 | .2527 | .0334 | .9672 | .0382 | .7186 | 24 |
| 37 | .0247 | .5782 | .0289 | .2476 | .0334 | .9627 | .0383 | .7147 | 23 |
| 38 | .0248 | .5722 | .029 | .2425 | .0335 | .9583 | .0384 | .7108 | 22 |
| 39 | .0249 | .5663 | .0291 | .2375 | .0336 | .9539 | .0385 | .707 | 21 |
| 40 | 1.0249 | 4.5604 | 1.0291 | 4.2324 | 1.0337 | 3.9495 | 1.0386 | 3.7031 | 20 |
| 41 | .025 | .5545 | .0292 | .2273 | .0338 | .9451 | .0387 | .6993 | 19 |
| 42 | .0251 | .5486 | .0293 | .2223 | .0338 | .9408 | .0387 | .6955 | 18 |
| 43 | .0251 | .5428 | .0293 | .2173 | .0339 | .9364 | .0388 | .6917 | 17 |
| 44 | .0252 | .5369 | .0294 | .2122 | .034 | .932 | .0389 | .6878 | 16 |
| 45 | 1.0253 | 4.5311 | 1.0295 | 4.2072 | 1.0341 | 3.9277 | 1.039 | 3.684 | 15 |
| 46 | .0253 | .5253 | .0296 | .2022 | .0341 | .9234 | .0391 | .6802 | 14 |
| 47 | .0254 | .5195 | .0296 | .1972 | .0342 | .919 | .0392 | .6765 | 13 |
| 48 | .0255 | .5137 | .0297 | .1923 | .0343 | .9147 | .0393 | .6727 | 12 |
| 49 | .0255 | .5079 | .0298 | .1873 | .0344 | .9104 | .0393 | .6689 | 11 |
| 50 | 1.0256 | 4.5021 | 1.0299 | 4.1824 | 1.0345 | 3.9061 | 1.0394 | 3.6651 | 10 |
| 51 | .0257 | .4964 | .0299 | .1774 | .0345 | .9018 | .0395 | .6614 | 9 |
| 52 | .0257 | .4907 | .03 | .1725 | .0346 | .8976 | .0396 | .6576 | 8 |
| 53 | .0258 | .485 | .0301 | .1676 | .0347 | .8933 | .0397 | .6539 | 7 |
| 54 | .0259 | .4793 | .0302 | .1627 | .0348 | .889 | .0398 | .6502 | 6 |
| 55 | 1.026 | 4.4736 | 1.0302 | 4.1578 | 1.0349 | 3.8848 | 1.0399 | 3.6464 | 5 |
| 56 | .026 | .4679 | .0303 | .1529 | .0349 | .8805 | .0399 | .6427 | 4 |
| 57 | .0261 | .4623 | .0304 | .1481 | .035 | .8763 | .04 | .639 | 3 |
| 58 | .0262 | .4566 | .0305 | .1432 | .0351 | .8721 | .0401 | .6353 | 2 |
| 59 | .0262 | .451 | .0305 | .1384 | .0352 | .8679 | .0402 | .6316 | 1 |
| 60 | 1.0263 | 4.4454 | 1.0306 | 4.1336 | 1.0353 | 3.8637 | 1.0403 | 3.6279 | 0 |
| | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | |
| | 77° | | 76° | | 75° | | 74° | | |

| 16° | 17° | | 18° | | 19° | | |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----|
| Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | |
| 3.6279 | 1.0457 | 3.4203 | 1.0515 | 3.2361 | 1.0576 | 3.0715 | 60 |
| .6243 | .0458 | .427 | .0516 | .2332 | .0577 | .069 | 59 |
| .6206 | .0459 | .4138 | .0517 | .2303 | .0578 | .0664 | 58 |
| .6169 | .046 | .4106 | .0518 | .2274 | .0579 | .0638 | 57 |
| .6133 | .0461 | .4073 | .0519 | .2245 | .058 | .0612 | 56 |
| 3.6096 | 1.0461 | 3.4041 | 1.052 | 3.2216 | 1.0581 | 3.0586 | 55 |
| .606 | .0462 | .4009 | .0521 | .2188 | .0582 | .0561 | 54 |
| .6024 | .0463 | .3977 | .0522 | .2159 | .0584 | .0535 | 53 |
| .5987 | .0464 | .3945 | .0523 | .2131 | .0585 | .0509 | 52 |
| .5951 | .0465 | .3913 | .0524 | .2102 | .0586 | .0484 | 51 |
| 3.5915 | 1.0466 | 3.3881 | 1.0525 | 3.2074 | 1.0587 | 3.0458 | 50 |
| .5879 | .0467 | .3849 | .0526 | .2045 | .0588 | .0433 | 49 |
| .5843 | .0468 | .3817 | .0527 | .2017 | .0589 | .0407 | 48 |
| .5807 | .0469 | .3785 | .0528 | .1989 | .059 | .0382 | 47 |
| .5772 | .047 | .3754 | .0529 | .196 | .0591 | .0357 | 46 |
| 3.5736 | 1.0471 | 3.3722 | 1.053 | 3.1932 | 1.0592 | 3.0331 | 45 |
| .57 | .0472 | .369 | .0531 | .1904 | .0593 | .0306 | 44 |
| .5665 | .0473 | .3659 | .0532 | .1876 | .0594 | .0281 | 43 |
| .5629 | .0474 | .3627 | .0533 | .1848 | .0595 | .0256 | 42 |
| .5594 | .0475 | .3596 | .0534 | .182 | .0596 | .0231 | 41 |
| 3.5559 | 1.0476 | 3.3565 | 1.0535 | 3.1792 | 1.0598 | 3.0206 | 40 |
| .5523 | .0477 | .3534 | .0536 | .1764 | .0599 | .0181 | 39 |
| .5488 | .0478 | .3502 | .0537 | .1736 | .06 | .0156 | 38 |
| .5453 | .0478 | .3471 | .0538 | .1708 | .0601 | .0131 | 37 |
| .5418 | .0479 | .344 | .0539 | .1681 | .0602 | .0106 | 36 |
| 3.5383 | 1.048 | 3.3409 | 1.054 | 3.1653 | 1.0603 | 3.0081 | 35 |
| .5348 | .0481 | .3378 | .0541 | .1625 | .0604 | .0056 | 34 |
| .5313 | .0482 | .3347 | .0542 | .1598 | .0605 | .0031 | 33 |
| .5279 | .0483 | .3316 | .0543 | .157 | .0606 | .0007 | 32 |
| .5244 | .0484 | .3286 | .0544 | .1543 | .0607 | 2.9982 | 31 |
| 3.5209 | 1.0485 | 3.3255 | 1.0545 | 3.1515 | 1.0608 | 2.9957 | 30 |
| .5175 | .0486 | .3224 | .0546 | .1488 | .0609 | .9933 | 29 |
| .514 | .0487 | .3194 | .0547 | .1461 | .0611 | .9908 | 28 |
| .5106 | .0488 | .3163 | .0548 | .1433 | .0612 | .9884 | 27 |
| .5072 | .0489 | .3133 | .0549 | .1406 | .0613 | .9859 | 26 |
| 3.5037 | 1.049 | 3.3102 | 1.055 | 3.1379 | 1.0614 | 2.9835 | 25 |
| .5003 | .0491 | .3072 | .0551 | .1352 | .0615 | .981 | 24 |
| .4969 | .0492 | .3042 | .0552 | .1325 | .0616 | .9786 | 23 |
| .4935 | .0493 | .3011 | .0553 | .1298 | .0617 | .9762 | 22 |
| .4901 | .0494 | .2981 | .0554 | .1271 | .0618 | .9738 | 21 |
| 3.4867 | 1.0495 | 3.2951 | 1.0555 | 3.1244 | 1.0619 | 2.9713 | 20 |
| .4833 | .0496 | .2921 | .0556 | .1217 | .062 | .9689 | 19 |
| .4799 | .0497 | .2891 | .0557 | .119 | .0622 | .9665 | 18 |
| .4766 | .0498 | .2861 | .0558 | .1163 | .0623 | .9641 | 17 |
| .4732 | .0499 | .2831 | .0559 | .1137 | .0624 | .9617 | 16 |
| 3.4698 | 1.05 | 3.2801 | 1.056 | 3.111 | 1.0625 | 2.9593 | 15 |
| .4665 | .0501 | .2772 | .0561 | .1083 | .0626 | .9569 | 14 |
| .4632 | .0502 | .2742 | .0562 | .1057 | .0627 | .9545 | 13 |
| .4598 | .0503 | .2712 | .0563 | .103 | .0628 | .9521 | 12 |
| .4565 | .0504 | .2683 | .0565 | .1004 | .0629 | .9497 | 11 |
| 3.4532 | 1.0505 | 3.2653 | 1.0566 | 3.0977 | 1.063 | 2.9474 | 10 |
| .4498 | .0506 | .2624 | .0567 | .0951 | .0632 | .945 | 9 |
| .4465 | .0507 | .2594 | .0568 | .0925 | .0633 | .9426 | 8 |
| .4432 | .0508 | .2565 | .0569 | .0898 | .0634 | .9402 | 7 |
| .4399 | .0509 | .2535 | .057 | .0872 | .0635 | .9379 | 6 |
| 3.4366 | 1.051 | 3.2506 | 1.0571 | 3.0846 | 1.0636 | 2.9355 | 5 |
| .4334 | .0511 | .2477 | .0572 | .082 | .0637 | .9332 | 4 |
| .4301 | .0512 | .2448 | .0573 | .0793 | .0638 | .9308 | 3 |
| .4268 | .0513 | .2419 | .0574 | .0767 | .0639 | .9285 | 2 |
| .4236 | .0514 | .239 | .0575 | .0741 | .0641 | .9261 | 1 |
| 3.4203 | 1.0515 | 3.2361 | 1.0576 | 3.0715 | 1.0642 | 2.9238 | 0 |
| SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | |
| 30° | 72° | 71° | 70° | | | | |

| | 20° | | 21° | | 22° | | 23° | | |
|-----------|---------|-----------|-----------|-----------|---------|-----------|---------|-----------|-----|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.0642 | 2.9238 | 1.0711 | 2.7904 | 1.0785 | 2.6695 | 1.0864 | 2.5593 | 60 |
| 1 | .0643 | .9215 | .0713 | .7883 | .0787 | .6675 | .0865 | .5575 | 59 |
| 2 | .0644 | .9191 | .0714 | .7862 | .0788 | .6656 | .0866 | .5558 | 58 |
| 3 | .0645 | .9168 | .0715 | .7841 | .0789 | .6637 | .0868 | .554 | 57 |
| 4 | .0646 | .9145 | .0716 | .782 | .079 | .6618 | .0869 | .5523 | 56 |
| 5 | 1.0647 | 2.9122 | 1.0717 | 2.7799 | 1.0792 | 2.6599 | 1.087 | 2.5506 | 55 |
| 6 | .0648 | .9098 | .0719 | .7778 | .0793 | .658 | .0872 | .5488 | 54 |
| 7 | .065 | .9075 | .072 | .7757 | .0794 | .6561 | .0873 | .5471 | 53 |
| 8 | .0651 | .9052 | .0721 | .7736 | .0795 | .6542 | .0874 | .5453 | 52 |
| 9 | .0652 | .9029 | .0722 | .7715 | .0797 | .6523 | .0876 | .5436 | 51 |
| 10 | 1.0653 | 2.9006 | 1.0723 | 2.7694 | 1.0798 | 2.6504 | 1.0877 | 2.5419 | 50 |
| 11 | .0654 | .8983 | .0725 | .7674 | .0799 | .6485 | .0878 | .5402 | 49 |
| 12 | .0655 | .896 | .0726 | .7653 | .0801 | .6466 | .088 | .5384 | 48 |
| 13 | .0656 | .8937 | .0727 | .7632 | .0802 | .6447 | .0881 | .5367 | 47 |
| 14 | .0658 | .8915 | .0728 | .7611 | .0803 | .6428 | .0882 | .535 | 46 |
| 15 | 1.0659 | 2.8892 | 1.0729 | 2.7591 | 1.0804 | 2.641 | 1.0884 | 2.5333 | 45 |
| 16 | .066 | .8869 | .0731 | .757 | .0806 | .6391 | .0885 | .5316 | 44 |
| 17 | .0661 | .8846 | .0732 | .755 | .0807 | .6372 | .0886 | .5299 | 43 |
| 18 | .0662 | .8824 | .0733 | .7529 | .0808 | .6353 | .0888 | .5281 | 42 |
| 19 | .0663 | .8801 | .0734 | .7509 | .081 | .6335 | .0889 | .5264 | 41 |
| 20 | 1.0664 | 2.8778 | 1.0735 | 2.7488 | 1.0811 | 2.6316 | 1.0891 | 2.5247 | 40 |
| 21 | .0666 | .8756 | .0737 | .7468 | .0812 | .6297 | .0892 | .523 | 39 |
| 22 | .0667 | .8733 | .0738 | .7447 | .0813 | .6279 | .0893 | .5213 | 38 |
| 23 | .0668 | .8711 | .0739 | .7427 | .0815 | .626 | .0895 | .5196 | 37 |
| 24 | .0669 | .8688 | .074 | .7406 | .0816 | .6242 | .0896 | .5179 | 36 |
| 25 | 1.067 | 2.8666 | 1.0742 | 2.7386 | 1.0817 | 2.6223 | 1.0897 | 2.5163 | 35 |
| 26 | .0671 | .8644 | .0743 | .7366 | .0819 | .6205 | .0899 | .5146 | 34 |
| 27 | .0673 | .8621 | .0744 | .7346 | .082 | .6186 | .09 | .5129 | 33 |
| 28 | .0674 | .8599 | .0745 | .7325 | .0821 | .6168 | .0902 | .5112 | 32 |
| 29 | .0675 | .8577 | .0747 | .7305 | .0823 | .615 | .0903 | .5095 | 31 |
| 30 | 1.0676 | 2.8554 | 1.0748 | 2.7285 | 1.0824 | 2.6131 | 1.0904 | 2.5078 | 30 |
| 31 | .0677 | .8532 | .0749 | .7265 | .0825 | .6113 | .0906 | .5062 | 29 |
| 32 | .0678 | .851 | .075 | .7245 | .0826 | .6095 | .0907 | .5045 | 28 |
| 33 | .0679 | .8488 | .0751 | .7225 | .0828 | .6076 | .0908 | .5028 | 27 |
| 34 | .0681 | .8466 | .0753 | .7205 | .0829 | .6058 | .091 | .5011 | 26 |
| 35 | 1.0682 | 2.8444 | 1.0754 | 2.7185 | 1.083 | 2.604 | 1.0911 | 2.4995 | 25 |
| 36 | .0683 | .8422 | .0755 | .7165 | .0832 | .6022 | .0913 | .4978 | 24 |
| 37 | .0684 | .84 | .0756 | .7145 | .0833 | .6003 | .0914 | .4961 | 23 |
| 38 | .0685 | .8378 | .0758 | .7125 | .0834 | .5985 | .0915 | .4945 | 22 |
| 39 | .0686 | .8356 | .0759 | .7105 | .0836 | .5967 | .0917 | .4928 | 21 |
| 40 | 1.0688 | 2.8334 | 1.076 | 2.7085 | 1.0837 | 2.5949 | 1.0918 | 2.4912 | 20 |
| 41 | .0689 | .8312 | .0761 | .7065 | .0838 | .5931 | .092 | .4895 | 19 |
| 42 | .069 | .829 | .0763 | .7045 | .084 | .5913 | .0921 | .4879 | 18 |
| 43 | .0691 | .8269 | .0764 | .7026 | .0841 | .5895 | .0922 | .4862 | 17 |
| 44 | .0692 | .8247 | .0765 | .7006 | .0842 | .5877 | .0924 | .4846 | 16 |
| 45 | 1.0694 | 2.8225 | 1.0766 | 2.6986 | 1.0844 | 2.5859 | 1.0925 | 2.4829 | 15 |
| 46 | .0695 | .8204 | .0768 | .6967 | .0845 | .5841 | .0927 | .4813 | 14 |
| 47 | .0696 | .8182 | .0769 | .6947 | .0846 | .5823 | .0928 | .4797 | 13 |
| 48 | .0697 | .816 | .077 | .6927 | .0847 | .5805 | .0929 | .478 | 12 |
| 49 | .0698 | .8139 | .0771 | .6908 | .0849 | .5787 | .0931 | .4764 | 11 |
| 50 | 1.0699 | 2.8117 | 1.0773 | 2.6888 | 1.085 | 2.577 | 1.0932 | 2.4748 | 10 |
| 51 | .0701 | .8096 | .0774 | .6869 | .0851 | .5752 | .0934 | .4731 | 9 |
| 52 | .0702 | .8074 | .0775 | .6849 | .0853 | .5734 | .0935 | .4715 | 8 |
| 53 | .0703 | .8053 | .0776 | .683 | .0854 | .5716 | .0936 | .4699 | 7 |
| 54 | .0704 | .8032 | .0778 | .681 | .0855 | .5699 | .0938 | .4683 | 6 |
| 55 | 1.0705 | 2.801 | 1.0779 | 2.6791 | 1.0857 | 2.5681 | 1.0939 | 2.4666 | 5 |
| 56 | .0707 | .7989 | .078 | .6772 | .0858 | .5663 | .0941 | .465 | 4 |
| 57 | .0708 | .7968 | .0781 | .6752 | .0859 | .5646 | .0942 | .4634 | 3 |
| 58 | .0709 | .7947 | .0783 | .6733 | .0861 | .5628 | .0943 | .4618 | 2 |
| 59 | .071 | .7925 | .0784 | .6714 | .0862 | .561 | .0945 | .4602 | 1 |
| 60 | 1.0711 | 2.7904 | 1.0785 | 2.6695 | 1.0864 | 2.5593 | .0946 | 2.4586 | 0 |
| CO-SEC'T. | | SECANT. | CO-SEC'T. | | SECANT. | CO-SEC'T. | | SECANT. | |
| 69° | | | 68° | | | 67° | | | |
| | | | | | | | | | 66° |

| 8° | | 9° | | 10° | | 11° | | ° |
|---------|-----------|---------|-----------|---------|-----------|---------|-----------|----|
| SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | |
| 1.0098 | 7.1853 | 1.0125 | 6.3924 | 1.0154 | 5.7588 | 1.0187 | 5.2408 | 60 |
| .0099 | .1704 | .0125 | .3807 | .0155 | .7493 | .0188 | .233 | 59 |
| .0099 | .1557 | .0125 | .369 | .0155 | .7398 | .0188 | .2252 | 58 |
| .0099 | .1409 | .0126 | .3574 | .0156 | .7304 | .0189 | .2174 | 57 |
| .01 | .1263 | .0126 | .3458 | .0156 | .721 | .0189 | .2097 | 56 |
| 1.01 | 7.1117 | 1.0127 | 6.3343 | 1.0157 | 5.7117 | 1.019 | 5.2019 | 55 |
| .0101 | .0972 | .0127 | .3228 | .0157 | .7023 | .0191 | .1942 | 54 |
| .0101 | .0827 | .0128 | .3113 | .0158 | .693 | .0191 | .1865 | 53 |
| .0102 | .0683 | .0128 | .2999 | .0158 | .6838 | .0192 | .1788 | 52 |
| .0102 | .0539 | .0129 | .2885 | .0159 | .6745 | .0192 | .1712 | 51 |
| 1.0102 | 7.0396 | 1.0129 | 6.2772 | 1.0159 | 5.6653 | 1.0193 | 5.1636 | 50 |
| .0103 | .0254 | .013 | .2659 | .016 | .6561 | .0193 | .156 | 49 |
| .0103 | .0112 | .013 | .2546 | .016 | .647 | .0194 | .1484 | 48 |
| .0104 | 6.9971 | .0131 | .2434 | .0161 | .6379 | .0195 | .1409 | 47 |
| .0104 | .983 | .0131 | .2322 | .0162 | .6288 | .0195 | .1333 | 46 |
| 1.0104 | 6.969 | 1.0132 | 6.2211 | 1.0162 | 5.6197 | 1.0196 | 5.1258 | 45 |
| .0105 | .955 | .0132 | .21 | .0163 | .6107 | .0196 | .1183 | 44 |
| .0105 | .9411 | .0133 | .199 | .0163 | .6017 | .0197 | .1109 | 43 |
| .0106 | .9273 | .0133 | .188 | .0164 | .5928 | .0198 | .1034 | 42 |
| .0106 | .9135 | .0134 | .177 | .0164 | .5838 | .0198 | .096 | 41 |
| 1.0107 | 6.8998 | 1.0134 | 6.1661 | 1.0165 | 5.5749 | 1.0199 | 5.0886 | 40 |
| .0107 | .8861 | .0135 | .1552 | .0165 | .566 | .0199 | .0812 | 39 |
| .0107 | .8725 | .0135 | .1443 | .0166 | .5572 | .02 | .0739 | 38 |
| .0108 | .8589 | .0136 | .1335 | .0166 | .5484 | .0201 | .0666 | 37 |
| .0108 | .8454 | .0136 | .1227 | .0167 | .5396 | .0201 | .0593 | 36 |
| 1.0109 | 6.832 | 1.0136 | 6.112 | 1.0167 | 5.5308 | 1.0202 | 5.052 | 35 |
| .0109 | .8185 | .0137 | .1013 | .0168 | .5221 | .0202 | .0447 | 34 |
| .011 | .8052 | .0137 | .0906 | .0169 | .5134 | .0203 | .0375 | 33 |
| .011 | .7919 | .0138 | .08 | .0169 | .5047 | .0204 | .0302 | 32 |
| .0111 | .7787 | .0138 | .0694 | .017 | .496 | .0204 | .023 | 31 |
| 1.0111 | 6.7655 | 1.0139 | 6.0588 | 1.017 | 5.4874 | 1.0205 | 5.0158 | 30 |
| .0111 | .7523 | .0139 | .0483 | .0171 | .4788 | .0205 | .0087 | 29 |
| .0112 | .7392 | .014 | .0379 | .0171 | .4702 | .0206 | .0015 | 28 |
| .0112 | .7262 | .014 | .0274 | .0172 | .4617 | .0207 | 4.9944 | 27 |
| .0113 | .7132 | .0141 | .017 | .0172 | .4532 | .0207 | .9873 | 26 |
| 1.0113 | 6.7003 | 1.0141 | 6.0066 | 1.0173 | 5.4447 | 1.0208 | 4.9802 | 25 |
| .0114 | .6874 | .0142 | 5.9963 | .0174 | .4362 | .0208 | .9732 | 24 |
| .0114 | .6745 | .0142 | .986 | .0174 | .4278 | .0209 | .9661 | 23 |
| .0115 | .6617 | .0143 | .9758 | .0175 | .4194 | .021 | .9591 | 22 |
| .0115 | .649 | .0143 | .9655 | .0175 | .411 | .021 | .9521 | 21 |
| 1.0115 | 6.6363 | 1.0144 | 5.9554 | 1.0176 | 5.4026 | 1.0211 | 4.9452 | 20 |
| .0116 | .6237 | .0144 | .9452 | .0176 | .3943 | .0211 | .9382 | 19 |
| .0116 | .6111 | .0145 | .9351 | .0177 | .386 | .0212 | .9313 | 18 |
| .0117 | .5985 | .0145 | .925 | .0177 | .3777 | .0213 | .9243 | 17 |
| .0117 | .586 | .0146 | .915 | .0178 | .3695 | .0213 | .9175 | 16 |
| 1.0118 | 6.5736 | 1.0146 | 5.9049 | 1.0179 | 5.3612 | 1.0214 | 4.9106 | 15 |
| .0118 | .5612 | .0147 | .895 | .0179 | .353 | .0215 | .9037 | 14 |
| .0119 | .5488 | .0147 | .885 | .018 | .3449 | .0215 | .8969 | 13 |
| .0119 | .5365 | .0148 | .8751 | .018 | .3367 | .0216 | .8901 | |
| .0119 | .5243 | .0148 | .8652 | .0181 | .3286 | .0216 | .8832 | |
| 1.012 | 6.5121 | 1.0149 | 5.8554 | 1.0181 | 5.3205 | 1.0217 | 4.87 | |
| .012 | .4999 | .015 | .8456 | .0182 | .3124 | .0218 | .86 | |
| .0121 | .4878 | .015 | .8358 | .0182 | .3044 | .0218 | .85 | |
| .0121 | .4757 | .0151 | .8261 | .0183 | .2963 | .0219 | .84 | |
| .0122 | .4637 | .0151 | .8163 | .0184 | .2883 | . | . | |
| 1.0122 | 6.4517 | 1.0152 | 5.8067 | 1.0184 | 5.2803 | . | . | |
| .0123 | .4398 | .0152 | .797 | .0185 | .2724 | . | . | |
| .0123 | .4279 | .0153 | .7874 | .0185 | .2645 | . | . | |
| .0124 | .416 | .0153 | .7778 | .0186 | .2566 | . | . | |
| .0124 | .4042 | .0154 | .7683 | .0186 | .2487 | . | . | |
| 1.0124 | 6.3924 | 1.0154 | 5.7588 | 1.0187 | 5.2408 | . | . | |
| SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | SECANT. | Co-SEC'T. | |

| | 20° | | 21° | | 22° | | 23° | | |
|----|---------|----------|---------|-----------|-----------|-----------|-----------|-----------|----|
| | SECANT. | CO-SEC'T | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.0642 | 2.9238 | 1.0711 | 2.7904 | 1.0785 | 2.6695 | 1.0864 | 2.5593 | 60 |
| 1 | .0643 | .9215 | .0713 | .7883 | .0787 | .6675 | .0865 | .5575 | 59 |
| 2 | .0644 | .9191 | .0714 | .7862 | .0788 | .6656 | .0866 | .5558 | 58 |
| 3 | .0645 | .9168 | .0715 | .7841 | .0789 | .6637 | .0868 | .554 | 57 |
| 4 | .0646 | .9145 | .0716 | .782 | .079 | .6618 | .0869 | .5523 | 56 |
| 5 | 1.0647 | 2.9122 | 1.0717 | 2.7799 | 1.0792 | 2.6599 | 1.087 | 2.5506 | 55 |
| 6 | .0648 | .9098 | .0719 | .7778 | .0793 | .658 | .0872 | .5488 | 54 |
| 7 | .065 | .9075 | .072 | .7757 | .0794 | .6561 | .0873 | .5471 | 53 |
| 8 | .0651 | .9052 | .0721 | .7736 | .0795 | .6542 | .0874 | .5453 | 52 |
| 9 | .0652 | .9029 | .0722 | .7715 | .0797 | .6523 | .0876 | .5436 | 51 |
| 10 | 1.0653 | 2.9006 | 1.0723 | 2.7694 | 1.0798 | 2.6504 | 1.0877 | 2.5419 | 50 |
| 11 | .0654 | .8983 | .0725 | .7674 | .0799 | .6485 | .0878 | .5402 | 49 |
| 12 | .0655 | .896 | .0726 | .7653 | .0801 | .6466 | .088 | .5384 | 48 |
| 13 | .0656 | .8937 | .0727 | .7632 | .0802 | .6447 | .0881 | .5367 | 47 |
| 14 | .0658 | .8915 | .0728 | .7611 | .0803 | .6428 | .0882 | .535 | 46 |
| 15 | 1.0659 | 2.8892 | 1.0729 | 2.7591 | 1.0804 | 2.641 | 1.0884 | 2.5333 | 45 |
| 16 | .066 | .8869 | .0731 | .757 | .0806 | .6391 | .0885 | .5316 | 44 |
| 17 | .0661 | .8846 | .0732 | .755 | .0807 | .6372 | .0886 | .5299 | 43 |
| 18 | .0662 | .8824 | .0733 | .7529 | .0808 | .6353 | .0888 | .5281 | 42 |
| 19 | .0663 | .8801 | .0734 | .7509 | .081 | .6335 | .0889 | .5264 | 41 |
| 20 | 1.0664 | 2.8778 | 1.0736 | 2.7488 | 1.0811 | 2.6316 | 1.0891 | 2.5247 | 40 |
| 21 | .0666 | .8756 | .0737 | .7468 | .0812 | .6297 | .0892 | .523 | 39 |
| 22 | .0667 | .8733 | .0738 | .7447 | .0813 | .6279 | .0893 | .5213 | 38 |
| 23 | .0668 | .8711 | .0739 | .7427 | .0815 | .626 | .0895 | .5196 | 37 |
| 24 | .0669 | .8688 | .074 | .7406 | .0816 | .6242 | .0896 | .5179 | 36 |
| 25 | 1.067 | 2.8666 | 1.0742 | 2.7386 | 1.0817 | 2.6223 | 1.0897 | 2.5163 | 35 |
| 26 | .0671 | .8644 | .0743 | .7366 | .0819 | .6205 | .0899 | .5146 | 34 |
| 27 | .0673 | .8621 | .0744 | .7346 | .082 | .6186 | .09 | .5129 | 33 |
| 28 | .0674 | .8599 | .0745 | .7325 | .0821 | .6168 | .0902 | .5112 | 32 |
| 29 | .0675 | .8577 | .0747 | .7305 | .0823 | .615 | .0903 | .5095 | 31 |
| 30 | 1.0676 | 2.8554 | 1.0748 | 2.7285 | 1.0824 | 2.6131 | 1.0904 | 2.5078 | 30 |
| 31 | .0677 | .8532 | .0749 | .7265 | .0825 | .6113 | .0906 | .5062 | 29 |
| 32 | .0678 | .851 | .075 | .7245 | .0826 | .6095 | .0907 | .5045 | 28 |
| 33 | .0679 | .8488 | .0751 | .7225 | .0828 | .6076 | .0908 | .5028 | 27 |
| 34 | .0681 | .8466 | .0753 | .7205 | .0829 | .6058 | .091 | .5011 | 26 |
| 35 | 1.0682 | 2.8444 | 1.0754 | 2.7185 | 1.083 | 2.604 | 1.0911 | 2.4995 | 25 |
| 36 | .0683 | .8422 | .0755 | .7165 | .0832 | .6022 | .0913 | .4978 | 24 |
| 37 | .0684 | .84 | .0756 | .7145 | .0833 | .6003 | .0914 | .4961 | 23 |
| 38 | .0685 | .8378 | .0758 | .7125 | .0834 | .5985 | .0915 | .4945 | 22 |
| 39 | .0686 | .8356 | .0759 | .7105 | .0836 | .5967 | .0917 | .4928 | 21 |
| 40 | 1.0688 | 2.8334 | 1.076 | 2.7085 | 1.0837 | 2.5949 | 1.0918 | 2.4912 | 20 |
| 41 | .0689 | .8312 | .0761 | .7065 | .0838 | .5931 | .092 | .4895 | 19 |
| 42 | .069 | .829 | .0763 | .7045 | .084 | .5913 | .0921 | .4879 | 18 |
| 43 | .0691 | .8269 | .0764 | .7026 | .0841 | .5895 | .0922 | .4862 | 17 |
| 44 | .0692 | .8247 | .0765 | .7006 | .0842 | .5877 | .0924 | .4846 | 16 |
| 45 | 1.0694 | 2.8225 | 1.0766 | 2.6986 | 1.0844 | 2.5859 | 1.0925 | 2.4829 | 15 |
| | .0694 | .8204 | .0768 | .6967 | .0845 | .5841 | .0927 | .4813 | 14 |
| | | .8182 | .0769 | .6947 | .0846 | .5823 | .0928 | .4797 | 13 |
| | | | .077 | .6927 | .0847 | .5805 | .0929 | .478 | 12 |
| | | | .0771 | .6908 | .0849 | .5787 | .0931 | .4764 | 11 |
| | | | 1.0773 | 2.6888 | 1.085 | 2.577 | 1.0932 | 2.4748 | 10 |
| | | | .0774 | .6869 | .0851 | .5752 | .0934 | .4731 | 9 |
| | | | .0775 | .6849 | .0853 | .5734 | .0935 | .4715 | 8 |
| | | | .0776 | .683 | .0854 | .5716 | .0936 | .4699 | 7 |
| | | | .0778 | .681 | .0855 | .5699 | .0938 | .4683 | 6 |
| | | | 1.0779 | 2.6791 | 1.0857 | 2.5681 | 1.0939 | 2.4666 | 5 |
| | | | .078 | .6772 | .0858 | .5663 | .0941 | .465 | 4 |
| | | | | .6752 | .0859 | .5646 | .0942 | .4634 | 3 |
| | | | | | .0861 | .5628 | .0943 | .4618 | 2 |
| | | | | | .0862 | .561 | .0945 | .4602 | 1 |
| | | | | | 1.0864 | 2.5593 | .0946 | 2.4586 | 0 |
| | | | | | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | |
| | | | | | 67° | | 66° | | |

| 24° | | 25° | | 26° | | 27° | | |
|-----------|-----------|---------|-----------|-----------|-----------|---------|-----------|----|
| IN. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | SECANT. | Co-sec't. | |
| 16 | 2.4586 | 1.1034 | 2.3662 | 1.1126 | 2.2812 | 1.1223 | 2.2027 | 60 |
| 18 | .457 | .1035 | .3647 | .1127 | .2798 | .1225 | .2014 | 59 |
| 19 | .4554 | .1037 | .3632 | .1129 | .2784 | .1226 | .2002 | 58 |
| 21 | .4538 | .1038 | .3618 | .1131 | .2771 | .1228 | .1989 | 57 |
| 22 | .4522 | .104 | .3603 | .1132 | .2757 | .123 | .1977 | 56 |
| 23 | 2.4506 | 1.1041 | 2.3588 | 1.1134 | 2.2744 | 1.1231 | 2.1964 | 55 |
| 25 | .449 | .1043 | .3574 | .1135 | .273 | .1233 | .1952 | 54 |
| 26 | .4474 | .1044 | .3559 | .1137 | .2717 | .1235 | .1939 | 53 |
| 28 | .4458 | .1046 | .3544 | .1139 | .2703 | .1237 | .1927 | 52 |
| 29 | .4442 | .1047 | .353 | .114 | .269 | .1238 | .1914 | 51 |
| 31 | 2.4426 | 1.1049 | 2.3515 | 1.1142 | 2.2676 | 1.124 | 2.1902 | 50 |
| 32 | .4411 | .105 | .3501 | .1143 | .2663 | .1242 | .1889 | 49 |
| 33 | .4395 | .1052 | .3486 | .1145 | .265 | .1243 | .1877 | 48 |
| 35 | .4379 | .1053 | .3472 | .1147 | .2636 | .1245 | .1865 | 47 |
| 36 | .4363 | .1055 | .3457 | .1148 | .2623 | .1247 | .1852 | 46 |
| 38 | 2.4347 | 1.1056 | 2.3443 | 1.115 | 2.261 | 1.1248 | 2.184 | 45 |
| 39 | .4332 | .1058 | .3428 | .1151 | .2596 | .125 | .1828 | 44 |
| 41 | .4316 | .1059 | .3414 | .1153 | .2583 | .1252 | .1815 | 43 |
| 42 | .43 | .1061 | .3399 | .1155 | .257 | .1253 | .1803 | 42 |
| 43 | .4285 | .1062 | .3385 | .1156 | .2556 | .1255 | .1791 | 41 |
| 45 | 2.4269 | 1.1064 | 2.3371 | 1.1158 | 2.2543 | 1.1257 | 2.1778 | 40 |
| 46 | .4254 | .1065 | .3356 | .1159 | .253 | .1258 | .1766 | 39 |
| 48 | .4238 | .1067 | .3342 | .1161 | .2517 | .126 | .1754 | 38 |
| 49 | .4222 | .1068 | .3328 | .1163 | .2503 | .1262 | .1742 | 37 |
| 51 | .4207 | .107 | .3313 | .1164 | .249 | .1264 | .173 | 36 |
| 52 | 2.4191 | 1.1072 | 2.3299 | 1.1166 | 2.2477 | 1.1265 | 2.1717 | 35 |
| 54 | .4176 | .1073 | .3285 | .1167 | .2464 | .1267 | .1705 | 34 |
| 55 | .416 | .1075 | .3271 | .1169 | .2451 | .1269 | .1693 | 33 |
| 56 | .4145 | .1076 | .3256 | .1171 | .2438 | .127 | .1681 | 32 |
| 58 | .413 | .1078 | .3242 | .1172 | .2425 | .1272 | .1669 | 31 |
| 59 | 2.4114 | 1.1079 | 2.3228 | 1.1174 | 2.2411 | 1.1274 | 2.1657 | 30 |
| 61 | .4099 | .1081 | .3214 | .1176 | .2398 | .1275 | .1645 | 29 |
| 62 | .4083 | .1082 | .32 | .1177 | .2385 | .1277 | .1633 | 28 |
| 64 | .4068 | .1084 | .3186 | .1179 | .2372 | .1279 | .162 | 27 |
| 65 | .4053 | .1085 | .3172 | .118 | .2359 | .1281 | .1608 | 26 |
| 67 | 2.4037 | 1.1087 | 2.3158 | 1.1182 | 2.2340 | 1.1282 | 2.1596 | 25 |
| 68 | .4022 | .1088 | .3143 | .1184 | .2333 | .1284 | .1584 | 24 |
| 69 | .4007 | .109 | .3129 | .1185 | .232 | .1286 | .1572 | 23 |
| 71 | .3992 | .1092 | .3115 | .1187 | .2307 | .1287 | .156 | 22 |
| 73 | .3976 | .1093 | .3101 | .1189 | .2294 | .1289 | .1548 | 21 |
| 74 | 2.3961 | 1.1095 | 2.3087 | 1.119 | 2.2282 | 1.1291 | 2.1536 | 20 |
| 75 | .3946 | .1096 | .3073 | .1192 | .2269 | .1293 | .1525 | 19 |
| 76 | .3931 | .1098 | .3059 | .1193 | .2256 | .1294 | .1513 | 18 |
| 77 | .3916 | .1099 | .3046 | .1195 | .2243 | .1296 | .1501 | 17 |
| 78 | .3901 | .1101 | .3032 | .1197 | .223 | .1298 | .1489 | 16 |
| 79 | 2.3886 | 1.1102 | 2.3018 | 1.1198 | 2.2217 | 1.1299 | 2.1477 | 15 |
| 81 | .3871 | .1104 | .3004 | .12 | .2204 | .1301 | .1465 | 14 |
| 82 | .3856 | .1106 | .299 | .1202 | .2192 | .1303 | .1453 | 13 |
| 83 | .3841 | .1107 | .2976 | .1203 | .. | .. | .1441 | 12 |
| 84 | .3826 | .1109 | .2962 | .1205 | .. | .. | .143 | 11 |
| 85 | 2.3811 | 1.111 | 2.2949 | 1.1207 | 1 | .. | 2.1418 | 10 |
| 86 | .3796 | .1112 | .2935 | .1208 | .. | .. | .1406 | 9 |
| 87 | .3781 | .1113 | .2921 | .121 | .. | .. | .1394 | 8 |
| 88 | .3766 | .1115 | .2907 | .1212 | .. | .. | .1382 | 7 |
| 89 | .3751 | .1116 | .2894 | .1213 | .. | .. | .. | 6 |
| 90 | 2.3736 | 1.1118 | 2.288 | 1.1215 | 2 | .. | .. | 5 |
| 91 | .3721 | .112 | .2866 | .1217 | .1 | .. | .. | 4 |
| 92 | .3706 | .1121 | .2853 | .1218 | .2 | .. | .. | 3 |
| 93 | .3691 | .1123 | .2839 | .122 | .3 | .. | .. | 2 |
| 94 | .3677 | .1124 | .2825 | .1222 | .4 | .. | .. | 1 |
| 95 | 2.3662 | 1.1126 | 2.2812 | 1.1223 | 2.3662 | .. | .. | 0 |
| Co-sec't. | | SECANT. | | Co-sec't. | | SECANT. | | |
| 64° | | 63° | | | | | | |

| | 28° | | 29° | | 30° | | 31° | | |
|----|---------|-----------|---------|-----------|---------|-----------|---------|-----------|----|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.1326 | 2.13 | 1.1433 | 2.0627 | 1.1547 | 2 | 1.1666 | 1.9416 | 60 |
| 1 | 1.1327 | 1.280 | 1.1435 | 2.0616 | 1.1549 | 1.999 | 1.1668 | 9407 | 59 |
| 2 | 1.1329 | 1.277 | 1.1437 | 2.0605 | 1.1551 | 1.998 | 1.167 | 9397 | 58 |
| 3 | 1.1331 | 1.266 | 1.1439 | 2.0594 | 1.1553 | 1.997 | 1.1672 | 9388 | 57 |
| 4 | 1.1333 | 1.254 | 1.1441 | 2.0583 | 1.1555 | 1.996 | 1.1674 | 9378 | 56 |
| 5 | 1.1334 | 2.1242 | 1.1443 | 2.0573 | 1.1557 | 1.995 | 1.1676 | 1.9369 | 55 |
| 6 | 1.1336 | 1.231 | 1.1445 | 2.0562 | 1.1559 | 1.994 | 1.1678 | 936 | 54 |
| 7 | 1.1338 | 1.219 | 1.1446 | 2.0551 | 1.1561 | 1.993 | 1.1681 | 935 | 53 |
| 8 | 1.134 | 1.208 | 1.1448 | 2.054 | 1.1562 | 1.992 | 1.1683 | 9341 | 52 |
| 9 | 1.1341 | 1.196 | 1.145 | 2.053 | 1.1564 | 1.991 | 1.1685 | 9332 | 51 |
| 10 | 1.1343 | 2.1185 | 1.1452 | 2.0519 | 1.1566 | 1.99 | 1.1687 | 1.9322 | 50 |
| 11 | 1.1345 | 1.173 | 1.1454 | 2.0508 | 1.1568 | 1.989 | 1.1689 | 9313 | 49 |
| 12 | 1.1347 | 1.162 | 1.1456 | 2.0498 | 1.157 | 1.988 | 1.1691 | 9304 | 48 |
| 13 | 1.1349 | 1.15 | 1.1458 | 2.0487 | 1.1572 | 1.987 | 1.1693 | 9295 | 47 |
| 14 | 1.135 | 1.139 | 1.1459 | 2.0476 | 1.1574 | 1.986 | 1.1695 | 9285 | 46 |
| 15 | 1.1352 | 2.1127 | 1.1461 | 2.0466 | 1.1576 | 1.985 | 1.1697 | 1.9276 | 45 |
| 16 | 1.1354 | 1.116 | 1.1463 | 2.0455 | 1.1578 | 1.984 | 1.1699 | 9267 | 44 |
| 17 | 1.1356 | 1.104 | 1.1465 | 2.0444 | 1.158 | 1.983 | 1.1701 | 9258 | 43 |
| 18 | 1.1357 | 1.093 | 1.1467 | 2.0434 | 1.1582 | 1.982 | 1.1703 | 9248 | 42 |
| 19 | 1.1359 | 1.082 | 1.1469 | 2.0423 | 1.1584 | 1.9811 | 1.1705 | 9239 | 41 |
| 20 | 1.1361 | 2.107 | 1.1471 | 2.0413 | 1.1586 | 1.9801 | 1.1707 | 1.923 | 40 |
| 21 | 1.1363 | 1.059 | 1.1473 | 2.0402 | 1.1588 | 1.9791 | 1.1709 | 9221 | 39 |
| 22 | 1.1365 | 1.048 | 1.1474 | 2.0392 | 1.159 | 1.9781 | 1.1712 | 9212 | 38 |
| 23 | 1.1366 | 1.036 | 1.1476 | 2.0381 | 1.1592 | 1.9771 | 1.1714 | 9203 | 37 |
| 24 | 1.1368 | 1.025 | 1.1478 | 2.037 | 1.1594 | 1.9761 | 1.1716 | 9193 | 36 |
| 25 | 1.137 | 2.1014 | 1.148 | 2.036 | 1.1596 | 1.9752 | 1.1718 | 1.9184 | 35 |
| 26 | 1.1372 | 1.002 | 1.1482 | 2.0349 | 1.1598 | 1.9742 | 1.172 | 9175 | 34 |
| 27 | 1.1373 | 1.0991 | 1.1484 | 2.0339 | 1.16 | 1.9732 | 1.1722 | 9166 | 33 |
| 28 | 1.1375 | 1.098 | 1.1486 | 2.0329 | 1.1602 | 1.9722 | 1.1724 | 9157 | 32 |
| 29 | 1.1377 | 1.0969 | 1.1488 | 2.0318 | 1.1604 | 1.9713 | 1.1726 | 9148 | 31 |
| 30 | 1.1379 | 2.0957 | 1.1489 | 2.0308 | 1.1606 | 1.9703 | 1.1728 | 1.9139 | 30 |
| 31 | 1.1381 | 1.0946 | 1.1491 | 2.0297 | 1.1608 | 1.9693 | 1.173 | 913 | 29 |
| 32 | 1.1382 | 1.0935 | 1.1493 | 2.0287 | 1.161 | 1.9683 | 1.1732 | 9121 | 28 |
| 33 | 1.1384 | 1.0924 | 1.1495 | 2.0276 | 1.1612 | 1.9674 | 1.1734 | 9112 | 27 |
| 34 | 1.1386 | 1.0912 | 1.1497 | 2.0266 | 1.1614 | 1.9664 | 1.1737 | 9102 | 26 |
| 35 | 1.1388 | 2.0901 | 1.1499 | 2.0256 | 1.1616 | 1.9654 | 1.1739 | 1.9093 | 25 |
| 36 | 1.139 | 1.089 | 1.1501 | 2.0245 | 1.1618 | 1.9645 | 1.1741 | 9084 | 24 |
| 37 | 1.1391 | 1.0879 | 1.1503 | 2.0235 | 1.162 | 1.9635 | 1.1743 | 9075 | 23 |
| 38 | 1.1393 | 1.0868 | 1.1505 | 2.0224 | 1.1622 | 1.9625 | 1.1745 | 9066 | 22 |
| 39 | 1.1395 | 1.0857 | 1.1507 | 2.0214 | 1.1624 | 1.9616 | 1.1747 | 9057 | 21 |
| 40 | 1.1397 | 2.0846 | 1.1508 | 2.0204 | 1.1626 | 1.9606 | 1.1749 | 1.9048 | 20 |
| 41 | 1.1399 | 1.0835 | 1.151 | 2.0194 | 1.1628 | 1.9596 | 1.1751 | 9039 | 19 |
| 42 | 1.1401 | 1.0824 | 1.1512 | 2.0183 | 1.163 | 1.9587 | 1.1753 | 903 | 18 |
| 43 | 1.1402 | 1.0812 | 1.1514 | 2.0173 | 1.1632 | 1.9577 | 1.1755 | 9021 | 17 |
| 44 | 1.1404 | 1.0801 | 1.1516 | 2.0163 | 1.1634 | 1.9568 | 1.1758 | 9013 | 16 |
| 45 | 1.1406 | 2.079 | 1.1518 | 2.0152 | 1.1636 | 1.9558 | 1.176 | 1.9003 | 15 |
| 46 | 1.1408 | 1.0779 | 1.152 | 2.0142 | 1.1638 | 1.9549 | 1.1762 | 8995 | 14 |
| 47 | 1.141 | 1.0768 | 1.1522 | 2.0132 | 1.164 | 1.9539 | 1.1764 | 8986 | 13 |
| 48 | 1.1411 | 1.0757 | 1.1524 | 2.0122 | 1.1642 | 1.953 | 1.1766 | 8977 | 12 |
| 49 | 1.1413 | 2.0746 | 1.1526 | 2.0111 | 1.1644 | 1.952 | 1.1768 | 8968 | 11 |
| | 1.1415 | 2.0735 | 1.1528 | 2.0101 | 1.1646 | 1.951 | 1.177 | 1.8959 | 10 |
| | | 1.0725 | 1.153 | 2.0091 | 1.1648 | 1.9501 | 1.1772 | 895 | 9 |
| | | 1.0714 | 1.1531 | 2.0081 | 1.165 | 1.9491 | 1.1775 | 8941 | 8 |
| | | 1.0703 | 1.1533 | 2.0071 | 1.1652 | 1.9482 | 1.1777 | 8932 | 7 |
| | | | 1.1535 | 2.0061 | 1.1654 | 1.9473 | 1.1779 | 8924 | 6 |
| | | | 1.1537 | 2.005 | 1.1656 | 1.9463 | 1.1781 | 1.8915 | 5 |
| | | | 1.1539 | 2.004 | 1.1658 | 1.9454 | 1.1783 | 8906 | 4 |
| | | | | 2.003 | 1.166 | 1.9444 | 1.1785 | 8897 | 3 |
| | | | | 2.002 | 1.1662 | 1.9435 | 1.1787 | 8888 | 2 |
| | | | | 2.001 | 1.1664 | 1.9425 | 1.179 | 8879 | 1 |
| | | | | 2.000 | 1.1666 | 1.9416 | 1.1792 | 1.8871 | 0 |
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| ° | 32° | | 33° | | 34° | | 35° | | ° |
|----|-----------|-----------|-----------|-----------|-----------|-----------|---------|-----------|----|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.1792 | 1.8871 | 1.1924 | 1.8361 | 1.2062 | 1.7883 | 1.2208 | 1.7434 | 60 |
| 1 | 1.1794 | 1.8862 | 1.1926 | 1.8352 | 1.2064 | 1.7875 | 1.221 | 1.7427 | 59 |
| 2 | 1.1796 | 1.8853 | 1.1928 | 1.8344 | 1.2067 | 1.7867 | 1.2213 | 1.742 | 58 |
| 3 | 1.1798 | 1.8844 | 1.193 | 1.8336 | 1.2069 | 1.786 | 1.2215 | 1.7413 | 57 |
| 4 | 1.18 | 1.8836 | 1.1933 | 1.8328 | 1.2072 | 1.7852 | 1.2218 | 1.7405 | 56 |
| 5 | 1.1802 | 1.8827 | 1.1935 | 1.832 | 1.2074 | 1.7844 | 1.222 | 1.7398 | 55 |
| 6 | 1.1805 | 1.8818 | 1.1937 | 1.8311 | 1.2076 | 1.7837 | 1.2223 | 1.7391 | 54 |
| 7 | 1.1807 | 1.8809 | 1.1939 | 1.8303 | 1.2079 | 1.7829 | 1.2225 | 1.7384 | 53 |
| 8 | 1.1809 | 1.8801 | 1.1942 | 1.8295 | 1.2081 | 1.7821 | 1.2228 | 1.7377 | 52 |
| 9 | 1.1811 | 1.8792 | 1.1944 | 1.8287 | 1.2083 | 1.7814 | 1.223 | 1.7369 | 51 |
| 10 | 1.1813 | 1.8783 | 1.1946 | 1.8279 | 1.2086 | 1.7806 | 1.2233 | 1.7362 | 50 |
| 11 | 1.1815 | 1.8785 | 1.1948 | 1.8271 | 1.2088 | 1.7798 | 1.2235 | 1.7355 | 49 |
| 12 | 1.1818 | 1.8766 | 1.1951 | 1.8263 | 1.2091 | 1.7791 | 1.2238 | 1.7348 | 48 |
| 13 | 1.182 | 1.8757 | 1.1953 | 1.8255 | 1.2093 | 1.7783 | 1.224 | 1.7341 | 47 |
| 14 | 1.1822 | 1.8749 | 1.1955 | 1.8246 | 1.2095 | 1.7776 | 1.2243 | 1.7334 | 46 |
| 15 | 1.1824 | 1.874 | 1.1958 | 1.8238 | 1.2098 | 1.7768 | 1.2245 | 1.7327 | 45 |
| 16 | 1.1826 | 1.8731 | 1.196 | 1.823 | 1.21 | 1.776 | 1.2248 | 1.7319 | 44 |
| 17 | 1.1828 | 1.8723 | 1.1962 | 1.8222 | 1.2103 | 1.7753 | 1.225 | 1.7312 | 43 |
| 18 | 1.1831 | 1.8714 | 1.1964 | 1.8214 | 1.2105 | 1.7745 | 1.2253 | 1.7305 | 42 |
| 19 | 1.1833 | 1.8706 | 1.1967 | 1.8206 | 1.2107 | 1.7738 | 1.2255 | 1.7298 | 41 |
| 20 | 1.1835 | 1.8697 | 1.1969 | 1.8198 | 1.211 | 1.773 | 1.2258 | 1.7291 | 40 |
| 21 | 1.1837 | 1.8688 | 1.1971 | 1.819 | 1.2112 | 1.7723 | 1.226 | 1.7284 | 39 |
| 22 | 1.1839 | 1.868 | 1.1974 | 1.8182 | 1.2115 | 1.7715 | 1.2263 | 1.7277 | 38 |
| 23 | 1.1841 | 1.8671 | 1.1976 | 1.8174 | 1.2117 | 1.7708 | 1.2265 | 1.727 | 37 |
| 24 | 1.1844 | 1.8663 | 1.1978 | 1.8166 | 1.2119 | 1.77 | 1.2268 | 1.7263 | 36 |
| 25 | 1.1846 | 1.8654 | 1.198 | 1.8158 | 1.2122 | 1.7693 | 1.227 | 1.7256 | 35 |
| 26 | 1.1848 | 1.8646 | 1.1983 | 1.815 | 1.2124 | 1.7685 | 1.2273 | 1.7249 | 34 |
| 27 | 1.185 | 1.8637 | 1.1985 | 1.8142 | 1.2127 | 1.7678 | 1.2276 | 1.7242 | 33 |
| 28 | 1.1852 | 1.8629 | 1.1987 | 1.8134 | 1.2129 | 1.767 | 1.2278 | 1.7234 | 32 |
| 29 | 1.1855 | 1.862 | 1.199 | 1.8126 | 1.2132 | 1.7663 | 1.2281 | 1.7227 | 31 |
| 30 | 1.1857 | 1.8611 | 1.1992 | 1.8118 | 1.2134 | 1.7655 | 1.2283 | 1.722 | 30 |
| 31 | 1.1859 | 1.8603 | 1.1994 | 1.811 | 1.2136 | 1.7648 | 1.2286 | 1.7213 | 29 |
| 32 | 1.1861 | 1.8595 | 1.1997 | 1.8102 | 1.2139 | 1.764 | 1.2288 | 1.7206 | 28 |
| 33 | 1.1863 | 1.8586 | 1.1999 | 1.8094 | 1.2141 | 1.7633 | 1.2291 | 1.7199 | 27 |
| 34 | 1.1866 | 1.8578 | 1.2001 | 1.8086 | 1.2144 | 1.7625 | 1.2293 | 1.7192 | 26 |
| 35 | 1.1868 | 1.8569 | 1.2004 | 1.8078 | 1.2146 | 1.7618 | 1.2296 | 1.7185 | 25 |
| 36 | 1.187 | 1.8561 | 1.2006 | 1.807 | 1.2149 | 1.761 | 1.2298 | 1.7178 | 24 |
| 37 | 1.1872 | 1.8552 | 1.2008 | 1.8062 | 1.2151 | 1.7603 | 1.2301 | 1.7171 | 23 |
| 38 | 1.1874 | 1.8544 | 1.201 | 1.8054 | 1.2153 | 1.7596 | 1.2304 | 1.7164 | 22 |
| 39 | 1.1877 | 1.8535 | 1.2013 | 1.8047 | 1.2156 | 1.7588 | 1.2306 | 1.7157 | 21 |
| 40 | 1.1879 | 1.8527 | 1.2015 | 1.8039 | 1.2158 | 1.7581 | 1.2309 | 1.7151 | 20 |
| 41 | 1.1881 | 1.8519 | 1.2017 | 1.8031 | 1.2161 | 1.7573 | 1.2311 | 1.7144 | 19 |
| 42 | 1.1883 | 1.851 | 1.202 | 1.8023 | 1.2163 | 1.7566 | 1.2314 | 1.7137 | 18 |
| 43 | 1.1886 | 1.8502 | 1.2022 | 1.8015 | 1.2166 | 1.7559 | 1.2316 | 1.713 | 17 |
| 44 | 1.1888 | 1.8493 | 1.2024 | 1.8007 | 1.2168 | 1.7551 | 1.2319 | 1.7123 | 16 |
| 45 | 1.189 | 1.8485 | 1.2027 | 1.7999 | 1.2171 | 1.7544 | 1.2322 | 1.7116 | 15 |
| 46 | 1.1892 | 1.8477 | 1.2029 | 1.7992 | 1.2173 | 1.7537 | 1.2324 | 1.7109 | 14 |
| 47 | 1.1894 | 1.8468 | 1.2031 | 1.7984 | 1.2175 | 1.7529 | 1.2327 | 1.7102 | 13 |
| 48 | 1.1897 | 1.846 | 1.2034 | 1.7976 | 1.2178 | 1.7522 | 1.2329 | 1.7095 | 12 |
| 49 | 1.1899 | 1.8452 | 1.2036 | 1.7968 | 1.218 | 1.7514 | 1.2332 | 1.7088 | 11 |
| 50 | 1.1901 | 1.8443 | 1.2039 | 1.796 | 1.2183 | 1.7507 | 1.2335 | 1.7081 | 10 |
| 51 | 1.1903 | 1.8435 | 1.2041 | 1.7953 | 1.2185 | 1.75 | 1.2337 | 1.7075 | |
| 52 | 1.1906 | 1.8427 | 1.2043 | 1.7945 | 1.2188 | 1.7488 | | 1.7068 | |
| 53 | 1.1908 | 1.8418 | 1.2046 | 1.7937 | 1.219 | 1.748 | | 1.7061 | |
| 54 | 1.191 | 1.841 | 1.2048 | 1.7929 | 1.2193 | 1.7473 | | 1.7054 | |
| 55 | 1.1912 | 1.8402 | 1.205 | 1.7921 | 1.2195 | 1.7465 | | 1.7047 | |
| 56 | 1.1915 | 1.8394 | 1.2053 | 1.7914 | 1.2198 | 1.7458 | | 1.704 | |
| 57 | 1.1917 | 1.8385 | 1.2055 | 1.7906 | 1.22 | 1.745 | | 1.703 | |
| 58 | 1.1919 | 1.8377 | 1.2057 | 1.7898 | 1.2203 | 1.7443 | | 1.702 | |
| 59 | 1.1921 | 1.8369 | 1.206 | 1.7891 | 1.2205 | 1.7435 | | 1.701 | |
| 60 | 1.1922 | 1.8362 | 1.2062 | 1.7883 | 1.2208 | 1.7428 | | 1.700 | |
| | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | | | |

| | 36° | | 37° | | 38° | | 39° | | |
|----|---------|-----------|---------|-----------|---------|-----------|---------|-----------|----|
| | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | SECANT. | CO-SEC'T. | |
| 0 | 1.2361 | 1.7013 | 1.2521 | 1.6616 | 1.269 | 1.6243 | 1.2867 | 1.589 | 60 |
| 1 | .2363 | .7006 | .2524 | .661 | .2693 | .6237 | .2871 | .5884 | 59 |
| 2 | .2366 | .6999 | .2527 | .6603 | .2696 | .6231 | .2874 | .5879 | 58 |
| 3 | .2368 | .6993 | .253 | .6597 | .2699 | .6224 | .2877 | .5873 | 57 |
| 4 | .2371 | .6986 | .2532 | .6591 | .2702 | .6218 | .288 | .5867 | 56 |
| 5 | 1.2374 | 1.6979 | 1.2535 | 1.6584 | 1.2705 | 1.6212 | 1.2883 | 1.5862 | 55 |
| 6 | .2376 | .6972 | .2538 | .6578 | .2707 | .6206 | .2886 | .5856 | 54 |
| 7 | .2379 | .6965 | .2541 | .6572 | .271 | .62 | .2889 | .585 | 53 |
| 8 | .2382 | .6959 | .2543 | .6565 | .2713 | .6194 | .2892 | .5845 | 52 |
| 9 | .2384 | .6952 | .2546 | .6559 | .2716 | .6188 | .2895 | .5839 | 51 |
| 10 | 1.2387 | 1.6945 | 1.2549 | 1.6552 | 1.2719 | 1.6182 | 1.2898 | 1.5833 | 50 |
| 11 | .2389 | .6938 | .2552 | .6546 | .2722 | .6176 | .2901 | .5828 | 49 |
| 12 | .2392 | .6932 | .2554 | .654 | .2725 | .617 | .2904 | .5822 | 48 |
| 13 | .2395 | .6925 | .2557 | .6533 | .2728 | .6164 | .2907 | .5816 | 47 |
| 14 | .2397 | .6918 | .256 | .6527 | .2731 | .6159 | .291 | .5811 | 46 |
| 15 | 1.24 | 1.6912 | 1.2563 | 1.6521 | 1.2734 | 1.6153 | 1.2913 | 1.5805 | 45 |
| 16 | .2403 | .6905 | .2565 | .6514 | .2737 | .6147 | .2916 | .5799 | 44 |
| 17 | .2405 | .6898 | .2568 | .6508 | .2739 | .6141 | .2919 | .5794 | 43 |
| 18 | .2408 | .6891 | .2571 | .6502 | .2742 | .6135 | .2922 | .5788 | 42 |
| 19 | .2411 | .6885 | .2574 | .6496 | .2745 | .6129 | .2926 | .5783 | 41 |
| 20 | 1.2413 | 1.6878 | 1.2577 | 1.6489 | 1.2748 | 1.6123 | 1.2929 | 1.5777 | 40 |
| 21 | .2416 | .6871 | .2579 | .6483 | .2751 | .6117 | .2932 | .5771 | 39 |
| 22 | .2419 | .6865 | .2582 | .6477 | .2754 | .6111 | .2935 | .5766 | 38 |
| 23 | .2421 | .6858 | .2585 | .647 | .2757 | .6105 | .2938 | .576 | 37 |
| 24 | .2424 | .6851 | .2588 | .6464 | .276 | .6099 | .2941 | .5755 | 36 |
| 25 | 1.2427 | 1.6845 | 1.2591 | 1.6458 | 1.2763 | 1.6093 | 1.2944 | 1.5749 | 35 |
| 26 | .2429 | .6838 | .2593 | .6452 | .2766 | .6087 | .2947 | .5743 | 34 |
| 27 | .2432 | .6831 | .2596 | .6445 | .2769 | .6081 | .295 | .5738 | 33 |
| 28 | .2435 | .6825 | .2599 | .6439 | .2772 | .6077 | .2953 | .5732 | 32 |
| 29 | .2437 | .6818 | .2602 | .6433 | .2775 | .607 | .2956 | .5727 | 31 |
| 30 | 1.244 | 1.6812 | 1.2605 | 1.6427 | 1.2778 | 1.6064 | 1.296 | 1.5721 | 30 |
| 31 | .2443 | .6805 | .2607 | .642 | .2781 | .6058 | .2963 | .5716 | 29 |
| 32 | .2445 | .6798 | .261 | .6414 | .2784 | .6052 | .2966 | .571 | 28 |
| 33 | .2448 | .6792 | .2613 | .6408 | .2787 | .6046 | .2969 | .5705 | 27 |
| 34 | .2451 | .6785 | .2616 | .6402 | .279 | .604 | .2972 | .5699 | 26 |
| 35 | 1.2453 | 1.6779 | 1.2619 | 1.6396 | 1.2793 | 1.6034 | 1.2975 | 1.5694 | 25 |
| 36 | .2456 | .6772 | .2622 | .6389 | .2795 | .6029 | .2978 | .5688 | 24 |
| 37 | .2459 | .6766 | .2624 | .6383 | .2798 | .6023 | .2981 | .5683 | 23 |
| 38 | .2461 | .6759 | .2627 | .6377 | .2801 | .6017 | .2985 | .5677 | 22 |
| 39 | .2464 | .6752 | .263 | .6371 | .2804 | .6011 | .2988 | .5672 | 21 |
| 40 | 1.2467 | 1.6746 | 1.2633 | 1.6365 | 1.2807 | 1.6005 | 1.2991 | 1.5666 | 20 |
| 41 | .247 | .6739 | .2636 | .6359 | .281 | .6 | .2994 | .5661 | 19 |
| 42 | .2472 | .6733 | .2639 | .6352 | .2813 | .5994 | .2997 | .5655 | 18 |
| 43 | .2475 | .6726 | .2641 | .6346 | .2816 | .5988 | .3 | .565 | 17 |
| 44 | .2478 | .672 | .2644 | .634 | .2819 | .5982 | .3003 | .5644 | 16 |
| 45 | 1.248 | 1.6713 | 1.2647 | 1.6334 | 1.2822 | 1.5976 | 1.3006 | 1.5639 | 15 |
| 46 | .2483 | .6707 | .265 | .6328 | .2825 | .5971 | .301 | .5633 | 14 |
| 47 | .2486 | .67 | .2653 | .6322 | .2828 | .5965 | .3013 | .5628 | 13 |
| 48 | .2488 | .6694 | .2656 | .6316 | .2831 | .5959 | .3016 | .5622 | 12 |
| 49 | .249 | .6687 | .2659 | .6309 | .2834 | .5953 | .3019 | .5617 | 11 |
| 50 | 1.2494 | 1.6681 | 1.2661 | 1.6303 | 1.2837 | 1.5947 | 1.3022 | 1.5611 | 10 |
| 51 | .2497 | .6674 | .2664 | .6297 | .284 | .5942 | .3025 | .5606 | 9 |
| 52 | .2499 | .6667 | .2667 | .6291 | .2843 | .5936 | .3029 | .56 | 8 |
| 53 | .2502 | .666 | .267 | .6285 | .2846 | .593 | .3032 | .5595 | 7 |
| 54 | | | .2673 | .6279 | .2849 | .5924 | .3035 | .559 | 6 |
| 55 | | | .2676 | 1.6273 | 1.2852 | 1.5919 | 1.3038 | 1.5584 | 5 |
| | | | .2679 | .6267 | .2855 | .5913 | .3041 | .5579 | 4 |
| | | | .2682 | .6261 | .2858 | .5907 | .3044 | .5573 | 3 |
| | | | .2684 | .6255 | .2861 | .5901 | .3048 | .5568 | 2 |
| | | | | .6249 | .2864 | .5895 | .3051 | .5563 | 1 |
| | | | | | 1.2867 | 1.589 | 1.3054 | 1.5557 | 0 |

Natural Tangents and Co-tangents.

| 10 | | 20 | | 30 | |
|----------|----------|----------|----------|----------|----------|
| Co-TANG. | TANG. | Co-TANG. | TANG. | Co-TANG. | TANG. |
| Infinit. | .017 46 | 57.29 | .034 92 | 28.6363 | .052 41 |
| 3437.75 | .017 75 | 6.3506 | .035 21 | 8.3094 | .052 7 |
| 1718.87 | .018 04 | 5.4415 | .035 5 | 8.1664 | .052 99 |
| 145.94 | .018 33 | 4.5613 | .035 79 | 7.9372 | .053 28 |
| 859.436 | .018 62 | 3.7080 | .036 00 | 7.7117 | .053 57 |
| 667.549 | .018 91 | 52.8821 | .036 38 | 27.4899 | .053 87 |
| 572.957 | .019 2 | 2.0807 | .036 67 | 7.2715 | .054 16 |
| 491.106 | .019 49 | 1.3032 | .036 96 | 7.0566 | .054 45 |
| 29.778 | .019 78 | 0.5485 | .037 25 | 6.845 | .054 74 |
| 381.971 | .020 07 | 49.8157 | .037 54 | 6.6367 | .055 03 |
| 343.774 | .020 36 | 49.1039 | .037 83 | 26.4316 | .055 33 |
| 12.521 | .020 66 | 8.1121 | .038 12 | 6.2266 | .055 62 |
| 286.478 | .020 95 | 7.7393 | .038 42 | 6.0307 | .055 91 |
| 64.441 | .021 24 | 7.0853 | .038 71 | 5.8348 | .056 2 |
| 45.552 | .021 53 | 6.4489 | .039 | 5.6418 | .056 49 |
| 229.182 | .021 82 | 45.8264 | .039 29 | 25.4517 | .056 78 |
| 14.858 | .022 11 | 5.2861 | .039 58 | 5.2644 | .057 08 |
| 02.219 | .022 4 | 4.6386 | .039 87 | 5.0798 | .057 37 |
| 190.984 | .022 69 | 4.0661 | .040 16 | 4.8978 | .057 66 |
| 8.032 | .022 98 | 3.5081 | .040 46 | 4.7185 | .057 95 |
| 171.585 | .023 28 | 42.9641 | .040 75 | 24.5418 | .058 24 |
| 63.7 | .023 57 | 2.4335 | .041 04 | 4.3675 | .058 54 |
| 56.259 | .023 86 | 1.9158 | .041 33 | 4.1957 | .058 83 |
| 49.465 | .024 15 | 1.4106 | .041 62 | 4.0203 | .059 12 |
| 43.237 | .024 44 | 0.9174 | .041 91 | 3.8593 | .059 41 |
| 137.507 | .024 73 | 4.0358 | .042 2 | 23.6945 | .059 7 |
| 32.219 | .025 02 | 39.9655 | .042 5 | 3.5321 | .059 99 |
| 27.321 | .025 31 | 9.5059 | .042 79 | 3.3718 | .060 29 |
| 22.774 | .025 6 | 9.0568 | .043 08 | 3.2137 | .060 58 |
| 18.54 | .025 89 | 8.6177 | .043 37 | 3.0577 | .060 87 |
| 114.589 | .026 19 | 38.1885 | .043 66 | 22.9038 | .061 16 |
| 10.892 | .026 48 | 7.7686 | .043 95 | 2.7519 | .061 45 |
| 07.426 | .026 77 | 7.3579 | .044 24 | 2.602 | .061 75 |
| 04.171 | .027 06 | 6.956 | .044 54 | 2.4541 | .062 04 |
| 01.107 | .027 35 | 6.5627 | .044 83 | 2.3081 | .062 33 |
| 98.2179 | .027 64 | 36.1776 | .045 12 | 22.164 | .062 62 |
| 5.4895 | .027 93 | 5.8006 | .045 41 | 2.0217 | .062 91 |
| 2.9085 | .028 22 | 5.4313 | .045 7 | 1.8813 | .063 21 |
| 0.4633 | .028 51 | 5.0695 | .045 99 | 1.7426 | .063 5 |
| 88.1436 | .028 81 | 4.7151 | .046 28 | 1.6056 | .063 79 |
| 85.9398 | .029 1 | 34.3678 | .046 58 | 21.4704 | .064 08 |
| 3.8435 | .029 39 | 4.0273 | .046 87 | 1.3369 | .064 37 |
| 1.847 | .029 68 | 3.6935 | .047 16 | 1.2049 | .064 67 |
| 79.9434 | .029 97 | 3.3662 | .047 45 | 1.0747 | .064 96 |
| 8.1263 | .030 26 | 3.0452 | .047 74 | 0.946 | .065 25 |
| 76.39 | .030 55 | 32.7303 | .048 03 | 20.8188 | .065 54 |
| 4.7292 | .030 84 | 2.4213 | .048 32 | 0.6932 | .065 84 |
| 3.399 | .031 14 | 2.1181 | .048 62 | 0.5691 | .066 13 |
| 1.6151 | .031 43 | 1.8205 | .048 91 | 0.4465 | .066 42 |
| 0.1533 | .031 72 | 1.5284 | .049 2 | 0.3253 | .066 71 |
| 68.7501 | .032 01 | 31.2449 | .049 49 | 20.2056 | .067 |
| 7.4019 | .032 3 | 0.9599 | .049 78 | 0.0872 | .067 3 |
| 6.1055 | .032 59 | 0.6833 | .050 07 | 19.9702 | .067 59 |
| 4.858 | .032 88 | 0.4116 | .050 37 | 9.8546 | .067 88 |
| 3.6567 | .033 17 | 0.1446 | .050 66 | 7.7403 | .068 17 |
| 62.4992 | .033 46 | 20.8823 | .050 95 | 19.6273 | .068 47 |
| 1.3829 | .033 76 | 9.6245 | .051 24 | 9.5156 | .068 76 |
| 0.3058 | .034 05 | 9.3711 | .051 53 | 9.4051 | .069 05 |
| 59.2659 | .034 34 | 0.122 | .051 82 | 9.2959 | .069 34 |
| 8.2612 | .034 63 | 8.8771 | .052 12 | 1.8779 | .069 63 |
| 57.29 | .034 92 | 28.6363 | .052 41 | 19.0811 | .069 93 |
| TANG. | Co-TANG. | TANG. | Co-TANG. | TANG. | Co-TANG. |
| 880 | | 870 | | 860 | |
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| 44° | | | | 45° | | | | 46° | | | |
|-------------------|-----------|--------|----|-------------------|-----------|--------|----|-------------------|-----------|--------|----|
| SECANT. | CO-SEC'T. | | | SECANT. | CO-SEC'T. | | | SECANT. | CO-SEC'T. | | |
| 1 | 1.000 | 1.4595 | 50 | 21 | 1.3984 | 1.4305 | 39 | 41 | 1.4005 | 1.4288 | 21 |
| 2 | 1.005 | 1.4591 | 50 | 22 | 1.3988 | 1.4301 | 38 | 42 | 1.4009 | 1.4287 | 22 |
| 3 | 1.010 | 1.4587 | 50 | 23 | 1.3992 | 1.4297 | 37 | 43 | 1.4013 | 1.4286 | 23 |
| 4 | 1.015 | 1.4583 | 50 | 24 | 1.3996 | 1.4293 | 36 | 44 | 1.4017 | 1.4285 | 24 |
| 5 | 1.020 | 1.4579 | 50 | 25 | 1.4 | 1.4288 | 35 | 45 | 1.4021 | 1.4284 | 25 |
| 6 | 1.025 | 1.4575 | 50 | 26 | 1.4004 | 1.4284 | 34 | 46 | 1.4025 | 1.4283 | 26 |
| 7 | 1.030 | 1.4571 | 50 | 27 | 1.4008 | 1.428 | 33 | 47 | 1.4029 | 1.4282 | 27 |
| 8 | 1.035 | 1.4567 | 50 | 28 | 1.4012 | 1.4276 | 32 | 48 | 1.4033 | 1.4281 | 28 |
| 9 | 1.040 | 1.4563 | 50 | 29 | 1.4016 | 1.4271 | 31 | 49 | 1.4037 | 1.428 | 29 |
| 10 | 1.045 | 1.4559 | 50 | 30 | 1.402 | 1.4267 | 30 | 50 | 1.4041 | 1.4279 | 30 |
| 11 | 1.050 | 1.4555 | 50 | 31 | 1.4024 | 1.4263 | 29 | 51 | 1.4045 | 1.4278 | 31 |
| 12 | 1.055 | 1.4551 | 50 | 32 | 1.4028 | 1.4259 | 28 | 52 | 1.4049 | 1.4277 | 32 |
| 13 | 1.060 | 1.4547 | 50 | 33 | 1.4032 | 1.4254 | 27 | 53 | 1.4053 | 1.4276 | 33 |
| 14 | 1.065 | 1.4543 | 50 | 34 | 1.4036 | 1.425 | 26 | 54 | 1.4057 | 1.4275 | 34 |
| 15 | 1.070 | 1.4539 | 50 | 35 | 1.404 | 1.4246 | 25 | 55 | 1.4061 | 1.4274 | 35 |
| 16 | 1.075 | 1.4535 | 50 | 36 | 1.4044 | 1.4242 | 24 | 56 | 1.4065 | 1.4273 | 36 |
| 17 | 1.080 | 1.4531 | 50 | 37 | 1.4048 | 1.4238 | 23 | 57 | 1.4069 | 1.4272 | 37 |
| 18 | 1.085 | 1.4527 | 50 | 38 | 1.4052 | 1.4233 | 22 | 58 | 1.4073 | 1.4271 | 38 |
| 19 | 1.090 | 1.4523 | 50 | 39 | 1.4056 | 1.4229 | 21 | 59 | 1.4077 | 1.427 | 39 |
| 20 | 1.095 | 1.4519 | 50 | 40 | 1.406 | 1.4225 | 20 | 60 | 1.4081 | 1.4269 | 40 |
| CO-SEC'T. SECANT. | | | | CO-SEC'T. SECANT. | | | | CO-SEC'T. SECANT. | | | |
| 45° | | | | 45° | | | | 45° | | | |

Preceding Table contains Natural Secants and Co-secants for every minute of the Quadrant to Radius 1.

If Degrees are taken at head of column, Minutes, Secant, and Co-secant must be taken from head also; and if they are taken at foot of column, Minutes, etc., must be taken from foot also.

ILLUSTRATION.—1.05 is secant of $17^{\circ} 45'$ and co-secant of $72^{\circ} 15'$.

To Compute Secant or Co-secant of any Angle.

RULE.—Divide 1 by Cosine of angle for Secant, and by Sine for Co-secant.

EXAMPLE 1.—What is secant of $25^{\circ} 25'$?

Cosine of angle = .90321. Then $1 \div .90321 = 1.1072$, Secant.

2.—What is co-secant of $64^{\circ} 35'$?

Sine of angle = .90321. Then $1 \div .90321 = 1.1072$, Co-secant.

To Compute Degrees, Minutes, and Seconds of a Secant or Co-secant.

When Secant is given,

Proceed as by Rule, page 402, for Sines, substituting Secants for Sines.

EXAMPLE.—What is secant for 1.1607?

The secant is 1.1606, arc for which = $30^{\circ} 30'$.

In 1.1608, difference between which and next less is 1.1606—

1.1606 = .0002. One given is 1.1607—1.1606 = .0001.

And $10^{\circ} 30' = 30^{\circ} 30' 30'$.

secants for Cosines.

Natural Tangents and Co-tangents.

| ANG. | 0° | 1° | 2° | 3° | |
|-------|-----------|----------|----------|----------|----------|
| ANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 30 00 | Infinite. | .017 46 | 57.29 | .034 92 | 28.6363 |
| 30 20 | 3437.75 | .017 75 | 6.3506 | .035 21 | 8.3994 |
| 30 58 | 1718.87 | .018 04 | 5.4415 | .035 5 | 8.1064 |
| 30 87 | 145.92 | .018 33 | 4.5613 | .035 79 | 7.9372 |
| 01 16 | 859.436 | .018 62 | 3.7086 | .036 09 | 7.7117 |
| 01 45 | 687.549 | .018 91 | 52.8821 | .036 38 | 27.4899 |
| 01 75 | 572.957 | .019 2 | 2.0807 | .036 67 | 7.2715 |
| 02 04 | 491.106 | .019 49 | 1.3032 | .036 96 | 7.0566 |
| 02 33 | 29.718 | .019 78 | 0.5485 | .037 25 | 6.845 |
| 02 62 | 381.971 | .020 07 | 49.8157 | .037 54 | 6.6367 |
| 02 91 | 343.774 | .020 36 | 49.1039 | .037 83 | 26.4316 |
| 03 2 | 12.521 | .020 66 | 8.4121 | .038 12 | 6.2296 |
| 03 49 | 286.478 | .020 95 | 7.7395 | .038 42 | 6.0307 |
| 03 78 | 64.441 | .021 24 | 7.0853 | .038 71 | 5.8348 |
| 04 07 | 45.552 | .021 53 | 6.4489 | .039 | 5.6418 |
| 04 36 | 229.182 | .021 82 | 45.8294 | .039 29 | 25.4517 |
| 04 65 | 14.858 | .022 11 | 5.2261 | .039 58 | 5.2644 |
| 04 95 | 02.219 | .022 4 | 4.6386 | .039 87 | 5.0798 |
| 05 24 | 100.984 | .022 69 | 4.0661 | .040 16 | 4.8978 |
| 05 53 | 80.032 | .022 98 | 3.5081 | .040 46 | 4.7185 |
| 05 82 | 171.885 | .023 28 | 42.9641 | .040 75 | 24.5418 |
| 06 11 | 63.7 | .023 57 | 2.4335 | .041 04 | 4.3075 |
| 06 4 | 56.259 | .023 86 | 1.9158 | .041 33 | 4.1957 |
| 06 69 | 49.405 | .024 15 | 1.4106 | .041 62 | 4.0263 |
| 06 98 | 43.237 | .024 44 | 0.9174 | .041 91 | 3.8593 |
| 07 27 | 137.507 | .024 73 | 40.4358 | .042 2 | 23.0945 |
| 07 56 | 32.219 | .025 02 | 39.9655 | .042 5 | 3.5321 |
| 07 85 | 27.321 | .025 31 | 9.5059 | .042 79 | 3.3718 |
| 08 14 | 22.774 | .025 6 | 0.9568 | .043 08 | 3.2137 |
| 08 44 | 18.54 | .025 89 | 8.6177 | .043 37 | 3.0577 |
| 08 73 | 114.589 | .026 19 | 38.1885 | .043 66 | 22.9038 |
| 09 02 | 10.832 | .026 48 | 7.7686 | .043 95 | 2.7519 |
| 09 31 | 07.426 | .026 77 | 7.3579 | .044 24 | 2.602 |
| 09 6 | 04.171 | .027 06 | 6.956 | .044 54 | 2.4541 |
| 09 89 | 01.107 | .027 35 | 6.5627 | .044 83 | 2.3081 |
| 10 18 | 98.2179 | .027 64 | 36.1776 | .045 12 | 22.164 |
| 10 47 | 5.4895 | .027 93 | 5.8006 | .045 41 | 2.0217 |
| 10 76 | 2.9085 | .028 22 | 5.4313 | .045 7 | 1.8813 |
| 11 05 | 0.4633 | .028 51 | 5.0695 | .045 99 | 1.7426 |
| 11 35 | 88.1436 | .028 81 | 4.7151 | .046 28 | 1.6056 |
| 11 64 | 85.9398 | .029 1 | 34.3678 | .046 58 | 21.4704 |
| 11 93 | 3.8435 | .029 39 | 4.0273 | .046 87 | 1.3369 |
| 12 22 | 1.847 | .029 68 | 3.6935 | .047 16 | 1.2049 |
| 12 51 | 79.9434 | .029 97 | 3.3662 | .047 45 | 1.0747 |
| 12 8 | 8.1263 | .030 26 | 3.0452 | .047 74 | 0.946 |
| 13 09 | 76.39 | .030 55 | 32.7303 | .048 03 | 20.8188 |
| 13 38 | 4.7292 | .030 84 | 2.4213 | .048 32 | 0.6932 |
| 13 67 | 3.139 | .031 14 | 2.1181 | .048 62 | 0.5691 |
| 13 96 | 1.6151 | .031 43 | 1.8205 | .048 91 | 0.4465 |
| 14 25 | 0.1533 | .031 72 | 1.5284 | .049 2 | 0.3253 |
| 14 55 | 68.7501 | .032 01 | 31.2416 | .049 49 | 20.2056 |
| 14 84 | 7.4019 | .032 3 | 0.9599 | .049 78 | 0.0872 |
| 15 13 | 6.1055 | .032 59 | 0.6833 | .050 07 | 19.9702 |
| 15 42 | 4.858 | .032 88 | 0.4116 | .050 37 | 9.8541 |
| 15 71 | 3.6567 | .033 17 | 0.1446 | .050 66 | 9.7401 |
| 16 | 62.4992 | .033 46 | 29.8823 | .050 95 | 19.627 |
| 16 29 | 1.3829 | .033 76 | 0.6245 | .051 24 | 9.515 |
| 16 58 | 0.3058 | .034 05 | 0.3711 | .051 53 | 9.405 |
| 16 87 | 59.2659 | .034 34 | 0.122 | .051 82 | 9.295 |
| 17 16 | 8.2612 | .034 63 | 8.8771 | .052 12 | 9.187 |
| 17 46 | 57.29 | .034 92 | 28.6363 | .052 41 | 19.081 |
| ANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | |

| | 40 | | 50 | | 60 | | 70 | |
|----|---------|----------|---------|----------|---------|----------|----------|----------|
| | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 0 | .069 93 | 14.3007 | .087 49 | 11.430 1 | .105 1 | 9.514 36 | .122 78 | 8.144 35 |
| 1 | .070 22 | 4.2411 | .087 78 | 1.391 9 | .105 4 | .877 81 | .123 08 | .124 81 |
| 2 | .070 51 | 4.1821 | .088 07 | 1.354 | .105 69 | .461 41 | .123 38 | .105 36 |
| 3 | .070 8 | 4.1235 | .088 37 | 1.316 3 | .105 99 | .435 15 | .123 67 | .086 |
| 4 | .071 1 | 4.0655 | .088 66 | 1.278 9 | .106 28 | .409 04 | .123 97 | .066 74 |
| 5 | .071 39 | 14.0079 | .088 95 | 11.241 7 | .106 57 | 9.383 07 | .124 26 | 8.047 56 |
| 6 | .071 68 | 3.9507 | .089 25 | 1.204 8 | .106 87 | .357 24 | .124 56 | .028 48 |
| 7 | .071 97 | 3.894 | .089 54 | 1.168 1 | .107 16 | .331 54 | .124 85 | 8.009 48 |
| 8 | .072 27 | 3.8378 | .089 83 | 1.131 6 | .107 46 | .305 99 | .125 15 | 7.990 55 |
| 9 | .072 56 | 3.7821 | .090 13 | 1.095 4 | .107 75 | .280 58 | .125 44 | .971 76 |
| 10 | .072 85 | 13.7267 | .090 42 | 1.059 4 | .108 05 | 9.255 3 | .125 74 | 7.953 02 |
| 11 | .073 14 | 3.6719 | .090 71 | 1.023 7 | .108 34 | .230 16 | .126 03 | .934 38 |
| 12 | .073 44 | 3.6174 | .091 01 | 0.988 2 | .108 63 | .205 16 | .126 33 | .915 82 |
| 13 | .073 73 | 3.5634 | .091 31 | 0.952 9 | .108 93 | .180 28 | .126 62 | .897 34 |
| 14 | .074 02 | 3.5098 | .091 59 | 0.917 8 | .109 22 | .155 54 | .126 92 | .878 95 |
| 15 | .074 31 | 13.4566 | .091 89 | 0.882 9 | .109 52 | 9.130 93 | .127 22 | 7.860 64 |
| 16 | .074 61 | 3.4039 | .092 18 | 0.848 3 | .109 81 | .106 46 | .127 51 | .842 42 |
| 17 | .074 9 | 3.3515 | .092 47 | 0.813 9 | .110 11 | .082 11 | .127 81 | .824 28 |
| 18 | .075 19 | 3.2996 | .092 77 | 0.779 7 | .110 4 | .057 89 | .128 1 | .806 22 |
| 19 | .075 48 | 3.248 | .093 06 | 0.745 7 | .110 7 | .033 79 | .128 4 | .788 25 |
| 20 | .075 78 | 13.1969 | .093 35 | 0.711 9 | .110 99 | 9.009 83 | .128 69 | 7.770 35 |
| 21 | .076 07 | 3.1461 | .093 65 | 0.678 3 | .111 28 | 8.985 98 | .128 99 | .752 54 |
| 22 | .076 36 | 3.0958 | .093 94 | 0.645 | .111 58 | .962 27 | .129 29 | .734 8 |
| 23 | .076 65 | 3.0458 | .094 23 | 0.611 8 | .111 87 | .938 67 | .129 58 | .717 15 |
| 24 | .076 95 | 2.9962 | .094 53 | 0.578 9 | .112 17 | .915 2 | .129 88 | .699 57 |
| 25 | .077 24 | 12.9469 | .094 82 | 0.546 2 | .112 46 | 8.891 85 | .130 17 | 7.682 08 |
| 26 | .077 53 | 2.8981 | .095 11 | 0.513 6 | .112 76 | .888 62 | .130 47 | .664 66 |
| 27 | .077 82 | 2.8496 | .095 41 | 0.481 3 | .113 05 | .845 51 | .130 76 | .647 32 |
| 28 | .078 12 | 2.8014 | .095 7 | 0.449 1 | .113 35 | .822 52 | .131 06 | .630 05 |
| 29 | .078 41 | 2.7536 | .096 | 0.417 2 | .113 64 | .799 64 | .131 36 | .612 87 |
| 30 | .078 7 | 12.7602 | .096 29 | 0.385 4 | .113 94 | 8.776 89 | .131 65 | 7.595 75 |
| 31 | .078 99 | 2.6591 | .096 58 | 0.353 8 | .114 23 | .754 25 | .131 95 | .578 72 |
| 32 | .079 29 | 2.6124 | .096 88 | 0.322 4 | .114 52 | .731 72 | .132 24 | .561 76 |
| 33 | .079 58 | 2.566 | .097 17 | 0.291 3 | .114 82 | .709 31 | .132 54 | .544 87 |
| 34 | .079 87 | 2.5199 | .097 46 | 0.260 2 | .115 11 | .687 01 | .132 84 | .528 06 |
| 35 | .080 17 | 12.4742 | .097 76 | 0.229 4 | .115 41 | 8.664 82 | .133 13 | 7.511 32 |
| 36 | .080 46 | 2.4288 | .098 05 | 0.198 8 | .115 7 | .642 75 | .133 43 | .494 65 |
| 37 | .080 75 | 2.3838 | .098 34 | 0.168 3 | .116 | .620 78 | .133 72 | .478 06 |
| 38 | .081 04 | 2.339 | .098 64 | 0.138 1 | .116 29 | .598 93 | .134 02 | .461 54 |
| 39 | .081 34 | 2.2946 | .098 93 | 0.108 | .116 59 | .577 18 | .134 32 | .445 09 |
| 40 | .081 63 | 12.2505 | .099 23 | 0.078 | .116 88 | 8.555 55 | .134 61 | 7.428 71 |
| 41 | .081 92 | 2.2067 | .099 52 | 0.048 3 | .117 18 | .554 02 | .134 91 | .412 4 |
| 42 | .082 21 | 2.1632 | .099 81 | 0.018 7 | .117 47 | .512 59 | .135 21 | .396 16 |
| 43 | .082 51 | 2.1201 | .100 11 | 9.989 3 | .117 77 | .491 28 | .135 5 | .379 99 |
| 44 | .082 8 | 2.0772 | .100 4 | .960 07 | .118 06 | .470 07 | .135 8 | .363 89 |
| 45 | .083 09 | 12.0346 | .100 69 | 9.931 01 | .118 36 | 8.448 96 | .136 09 | 7.347 86 |
| 46 | .083 39 | 2.0923 | .100 99 | .902 11 | .118 65 | .427 95 | .136 39 | .331 9 |
| 47 | .083 68 | 1.9504 | .101 28 | .873 38 | .118 95 | .407 05 | .136 69 | .316 |
| 48 | .083 97 | 1.9087 | .101 58 | .844 82 | .119 24 | .386 25 | .136 98 | .300 18 |
| 49 | .084 27 | 1.8673 | .101 87 | .816 41 | .119 54 | .365 55 | .137 28 | .284 42 |
| 50 | .084 56 | 1.8262 | .102 16 | 9.788 17 | .119 83 | 8.344 96 | .137 58 | 7.268 75 |
| | .83 | | .102 46 | .760 09 | .120 13 | .324 46 | .137 87 | .253 1 |
| | .18 | | .102 75 | .732 17 | .120 42 | .304 06 | .138 17 | .237 54 |
| | .5 | | .103 05 | .704 41 | .120 72 | .283 76 | .138 46 | .222 04 |
| | .45 | | .103 34 | .676 8 | .121 01 | .263 55 | .138 76 | .206 61 |
| | .248 | | .103 63 | 9.649 35 | .121 31 | 8.243 45 | .139 06 | 7.191 25 |
| | .133 | | .103 93 | .622 05 | .121 6 | .223 44 | .139 35 | .175 94 |
| | .016 | | .104 22 | .594 9 | .121 9 | .203 52 | .139 65 | .160 71 |
| | | | .104 51 | .567 91 | .122 19 | .183 7 | .139 95 | .145 53 |
| | | | .104 80 | .541 06 | .122 49 | .163 98 | .140 24 | .130 42 |
| | | | | 4 36 | .122 79 | 8.144 35 | .140 54 | 7.115 51 |
| | | | | | | | CO-TANG. | TANG. |

| | 80° | | 90° | | 100° | | 110° | | |
|----------|---------|----------|---------|----------|---------|----------|----------|----------|----|
| | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | |
| 0 | .140 54 | 7.115 37 | .158 38 | 6.313 75 | .176 33 | 5.671 28 | .194 38 | 5.144 55 | 60 |
| 1 | .140 84 | .100 38 | .158 68 | .301 89 | .176 63 | .661 65 | .194 68 | .136 58 | 59 |
| 2 | .141 13 | .085 46 | .158 98 | .290 07 | .176 93 | .652 05 | .194 98 | .128 62 | 58 |
| 3 | .141 43 | .070 59 | .159 28 | .278 20 | .177 23 | .642 48 | .195 29 | .120 69 | 57 |
| 4 | .141 73 | .055 79 | .159 58 | .266 55 | .177 53 | .632 95 | .195 59 | .112 79 | 56 |
| 5 | .142 02 | 7.041 05 | .159 88 | 6.254 86 | .177 83 | 5.623 44 | .195 89 | 5.104 9 | 55 |
| 6 | .142 32 | .026 37 | .160 17 | .243 21 | .178 13 | .613 97 | .196 19 | .097 04 | 54 |
| 7 | .142 62 | .011 74 | .160 47 | .231 6 | .178 43 | .604 52 | .196 49 | .089 21 | 53 |
| 8 | .142 91 | 6.997 18 | .160 77 | .220 03 | .178 73 | .595 11 | .196 8 | .081 39 | 52 |
| 9 | .143 21 | .982 68 | .161 07 | .208 51 | .179 03 | .585 73 | .197 1 | .073 6 | 51 |
| 10 | .143 51 | 6.968 23 | .161 37 | 6.197 03 | .179 33 | 5.576 38 | .197 4 | 5.065 84 | 50 |
| 11 | .143 81 | .953 85 | .161 67 | .185 59 | .179 63 | .567 06 | .197 7 | .058 09 | 49 |
| 12 | .144 1 | .939 52 | .161 96 | .174 19 | .179 93 | .557 77 | .198 01 | .050 37 | 48 |
| 13 | .144 4 | .925 25 | .162 26 | .162 83 | .180 23 | .548 51 | .198 31 | .042 67 | 47 |
| 14 | .144 7 | .911 04 | .162 56 | .151 51 | .180 53 | .539 27 | .198 61 | .034 99 | 46 |
| 15 | .144 99 | 6.896 88 | .162 86 | 6.140 23 | .180 83 | 5.530 07 | .198 91 | 5.027 34 | 45 |
| 16 | .145 29 | .882 78 | .163 16 | .128 99 | .181 13 | .520 9 | .199 21 | .019 71 | 44 |
| 17 | .145 59 | .868 74 | .163 46 | .117 79 | .181 43 | .511 76 | .199 52 | .012 1 | 43 |
| 18 | .145 88 | .854 75 | .163 76 | .106 64 | .181 73 | .502 64 | .199 82 | .004 51 | 42 |
| 19 | .146 18 | .840 82 | .164 05 | .095 52 | .182 03 | .493 56 | .200 12 | 4.996 95 | 41 |
| 20 | .146 48 | 6.826 94 | .164 35 | 6.084 44 | .182 33 | 5.484 51 | .200 42 | 4.989 4 | 40 |
| 21 | .146 78 | .813 12 | .164 65 | .073 4 | .182 63 | .475 48 | .200 73 | .981 88 | 39 |
| 22 | .147 07 | .799 36 | .164 95 | .062 4 | .182 93 | .466 48 | .201 03 | .974 38 | 38 |
| 23 | .147 37 | .785 04 | .165 25 | .051 43 | .183 23 | .457 51 | .201 33 | .966 9 | 37 |
| 24 | .147 67 | .771 09 | .165 55 | .040 51 | .183 53 | .448 57 | .201 64 | .959 45 | 36 |
| 25 | .147 96 | 6.758 38 | .165 85 | 6.029 62 | .183 83 | 5.439 66 | .201 94 | 4.952 01 | 35 |
| 26 | .148 26 | .744 83 | .166 15 | .018 78 | .184 14 | .430 77 | .202 24 | .944 6 | 34 |
| 27 | .148 56 | .731 33 | .166 45 | .007 97 | .184 44 | .421 92 | .202 54 | .937 21 | 33 |
| 28 | .148 86 | .717 89 | .166 74 | 5.997 2 | .184 74 | .413 09 | .202 85 | .929 84 | 32 |
| 29 | .149 15 | .704 5 | .167 04 | .986 46 | .185 04 | .404 29 | .203 15 | .922 49 | 31 |
| 30 | .149 45 | 6.691 16 | .167 34 | 5.975 76 | .185 34 | 5.395 52 | .203 45 | 4.915 16 | 30 |
| 31 | .149 75 | .677 87 | .167 64 | .965 1 | .185 64 | .386 77 | .203 76 | .907 85 | 29 |
| 32 | .150 05 | .664 63 | .167 94 | .954 48 | .185 94 | .378 05 | .204 06 | .900 56 | 28 |
| 33 | .150 34 | .651 41 | .168 24 | .943 9 | .186 24 | .369 36 | .204 36 | .893 3 | 27 |
| 34 | .150 64 | .638 31 | .168 54 | .933 35 | .186 54 | .360 7 | .204 66 | .886 05 | 26 |
| 35 | .150 94 | 6.625 23 | .168 84 | 5.922 83 | .186 84 | 5.352 06 | .204 97 | 4.878 82 | 25 |
| 36 | .151 24 | .612 19 | .169 14 | .912 35 | .187 14 | .343 45 | .205 27 | .871 62 | 24 |
| 37 | .151 53 | .599 21 | .169 44 | .901 91 | .187 45 | .334 87 | .205 57 | .864 44 | 23 |
| 38 | .151 83 | .586 27 | .169 74 | .891 51 | .187 75 | .326 31 | .205 88 | .857 27 | 22 |
| 39 | .152 13 | .573 39 | .170 04 | .881 14 | .188 05 | .317 78 | .206 18 | .850 13 | 21 |
| 40 | .152 43 | 6.560 55 | .170 33 | 5.870 8 | .188 35 | 5.309 28 | .206 48 | 4.843 | 20 |
| 41 | .152 72 | .547 77 | .170 63 | .860 51 | .188 65 | .300 8 | .206 79 | .835 9 | 19 |
| 42 | .153 02 | .535 03 | .170 93 | .850 24 | .188 95 | .292 35 | .207 09 | .828 82 | 18 |
| 43 | .153 32 | .522 34 | .171 23 | .840 01 | .189 25 | .283 93 | .207 39 | .821 75 | 17 |
| 44 | .153 62 | .509 7 | .171 53 | .829 82 | .189 55 | .275 53 | .207 7 | .814 71 | 16 |
| 45 | .153 91 | 6.497 1 | .171 83 | 5.819 66 | .189 86 | 5.267 15 | .208 | 4.807 69 | 15 |
| 46 | .154 21 | .484 56 | .172 13 | .809 53 | .190 16 | .258 8 | .208 3 | .800 68 | 14 |
| 47 | .154 51 | .472 06 | .172 43 | .799 44 | .190 46 | .250 48 | .208 61 | .793 7 | 13 |
| 48 | .154 81 | .459 61 | .172 73 | .789 38 | .190 76 | .242 18 | .208 91 | .786 73 | 12 |
| 49 | .155 11 | .447 2 | .173 03 | .779 36 | .191 06 | .233 91 | .209 21 | .779 78 | 11 |
| 50 | .155 4 | 6.434 84 | .173 33 | 5.769 37 | .191 36 | 5.225 66 | .209 52 | 4.772 86 | 10 |
| 51 | .155 7 | .422 53 | .173 63 | .759 41 | .191 66 | .217 44 | .209 82 | .765 95 | 9 |
| 52 | .156 | .410 26 | .173 93 | .749 49 | .191 97 | .209 25 | .210 13 | .759 06 | 8 |
| 53 | .156 3 | .398 04 | .174 23 | .739 6 | .192 27 | .201 07 | .210 43 | .752 19 | 7 |
| 54 | .156 6 | .385 87 | .174 53 | .729 74 | .192 57 | .192 93 | | .745 34 | 6 |
| 55 | .156 89 | 6.373 74 | .174 83 | 5.719 92 | .192 87 | 5.184 8 | | .728 51 | 5 |
| 56 | .157 19 | .361 05 | .175 13 | .710 13 | .193 17 | .176 71 | | | 4 |
| 57 | .157 49 | .349 61 | .175 43 | .700 37 | .193 47 | .168 62 | | | 3 |
| 58 | .157 79 | .337 61 | .175 73 | .690 64 | .193 78 | .160 51 | | | 2 |
| 59 | .158 09 | .325 66 | .176 03 | .680 94 | .194 08 | .152 56 | | | 1 |
| 60 | .158 38 | 6.313 75 | .176 33 | 5.671 28 | .194 38 | 5.144 55 | | | 0 |
| CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | TANG. | CO-TANG. | | |
| | 81° | | 80° | | 79° | | | | |

420 NATURAL TANGENTS AND CO-TANGENTS.

| 20° | | 21° | | 22° | | 23° | |
|-------|----------|---------|----------|---------|----------|---------|----------|
| TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 0 | 371.97 | 2.74746 | 353.56 | 2.50509 | 424.03 | 2.47506 | 424.47 |
| 1 | 372.43 | 744.95 | 354.2 | 602.83 | 424.36 | 473.02 | 424.52 |
| 2 | 372.83 | 742.51 | 354.53 | 600.57 | 424.7 | 470.95 | 424.55 |
| 3 | 373.24 | 740.04 | 354.87 | 598.31 | 425.04 | 468.88 | 424.58 |
| 4 | 373.65 | 737.56 | 355.2 | 596.06 | 425.38 | 466.82 | 424.61 |
| 5 | 374.06 | 735.09 | 355.53 | 593.81 | 425.72 | 464.76 | 424.64 |
| 6 | 374.47 | 732.63 | 355.87 | 591.55 | 426.06 | 462.7 | 424.67 |
| 7 | 374.88 | 730.17 | 356.2 | 589.3 | 426.4 | 460.65 | 424.7 |
| 8 | 375.29 | 727.71 | 356.54 | 587.08 | 426.74 | 458.6 | 424.73 |
| 9 | 375.7 | 725.25 | 356.87 | 584.84 | 427.07 | 456.55 | 424.76 |
| 10 | 376.12 | 722.79 | 357.21 | 582.61 | 427.41 | 454.51 | 424.79 |
| 11 | 376.53 | 720.33 | 357.54 | 580.38 | 427.75 | 452.46 | 424.82 |
| 12 | 376.94 | 717.87 | 357.87 | 578.15 | 428.09 | 450.43 | 424.85 |
| 13 | 377.35 | 715.41 | 358.21 | 575.93 | 428.43 | 448.39 | 424.88 |
| 14 | 377.76 | 712.95 | 358.54 | 573.71 | 428.77 | 446.35 | 424.91 |
| 15 | 378.17 | 710.49 | 358.88 | 571.5 | 429.11 | 444.33 | 424.94 |
| 16 | 378.58 | 708.03 | 359.21 | 569.28 | 429.45 | 442.3 | 424.97 |
| 17 | 378.99 | 705.57 | 359.55 | 567.07 | 429.79 | 440.27 | 425.0 |
| 18 | 379.4 | 703.11 | 359.88 | 564.87 | 430.13 | 438.25 | 425.03 |
| 19 | 379.81 | 700.65 | 360.22 | 562.66 | 430.47 | 436.23 | 425.06 |
| 20 | 380.22 | 698.19 | 360.55 | 560.46 | 430.81 | 434.22 | 425.09 |
| 21 | 380.63 | 695.73 | 360.89 | 558.27 | 431.15 | 432.2 | 425.12 |
| 22 | 381.04 | 693.27 | 361.22 | 556.08 | 431.49 | 430.19 | 425.15 |
| 23 | 381.45 | 690.81 | 361.56 | 553.89 | 431.83 | 428.17 | 425.18 |
| 24 | 381.86 | 688.35 | 361.9 | 551.7 | 432.17 | 426.15 | 425.21 |
| 25 | 382.27 | 685.89 | 362.23 | 549.52 | 432.51 | 424.18 | 425.24 |
| 26 | 382.68 | 683.43 | 362.57 | 547.34 | 432.85 | 422.18 | 425.27 |
| 27 | 383.09 | 680.97 | 362.9 | 545.15 | 433.19 | 420.19 | 425.3 |
| 28 | 383.5 | 678.51 | 363.24 | 542.93 | 433.53 | 418.19 | 425.33 |
| 29 | 383.91 | 676.05 | 363.57 | 540.82 | 433.87 | 416.2 | 425.36 |
| 30 | 384.32 | 673.59 | 363.91 | 538.65 | 434.21 | 414.21 | 425.39 |
| 31 | 384.73 | 671.13 | 364.25 | 536.48 | 434.55 | 412.23 | 425.42 |
| 32 | 385.14 | 668.67 | 364.58 | 534.32 | 434.89 | 410.25 | 425.45 |
| 33 | 385.55 | 666.21 | 364.92 | 532.17 | 435.23 | 408.27 | 425.48 |
| 34 | 385.96 | 663.75 | 365.26 | 530.01 | 435.58 | 406.29 | 425.51 |
| 35 | 386.37 | 661.29 | 365.59 | 527.86 | 435.92 | 404.32 | 425.54 |
| 36 | 386.78 | 658.83 | 365.93 | 525.71 | 436.26 | 402.35 | 425.57 |
| 37 | 387.19 | 656.37 | 366.26 | 523.57 | 436.6 | 400.38 | 425.6 |
| 38 | 387.6 | 653.91 | 366.6 | 521.42 | 436.94 | 398.41 | 425.63 |
| 39 | 388.01 | 651.45 | 366.94 | 519.28 | 437.28 | 396.45 | 425.66 |
| 40 | 388.42 | 649.0 | 367.27 | 517.15 | 437.62 | 394.49 | 425.69 |
| 41 | 388.83 | 646.54 | 367.61 | 515.02 | 437.97 | 392.53 | 425.72 |
| 42 | 389.24 | 644.08 | 367.95 | 512.89 | 438.31 | 390.58 | 425.75 |
| 43 | 389.65 | 641.62 | 368.29 | 510.75 | 438.65 | 388.62 | 425.78 |
| 44 | 390.06 | 639.16 | 368.62 | 508.64 | 439.0 | 386.66 | 425.81 |
| 45 | 390.47 | 636.7 | 368.96 | 506.52 | 439.34 | 384.71 | 425.84 |
| 46 | 390.88 | 634.24 | 369.3 | 504.4 | 439.68 | 382.75 | 425.87 |
| 47 | 391.29 | 631.78 | 369.63 | 502.29 | 440.02 | 380.8 | 425.9 |
| 48 | 391.7 | 629.32 | 369.97 | 500.18 | 440.36 | 378.84 | 425.93 |
| 49 | 392.11 | 626.86 | 400.31 | 498.07 | 440.7 | 376.89 | 425.96 |
| 50 | 392.52 | 624.4 | 400.65 | 495.95 | 441.05 | 374.94 | 425.99 |
| 51 | 392.93 | 621.98 | 400.98 | 493.86 | 441.39 | 372.99 | 426.02 |
| 52 | 393.34 | 619.52 | 401.32 | 491.77 | 441.73 | 371.04 | 426.05 |
| 53 | 393.75 | 617.06 | 401.66 | 489.67 | 442.07 | 369.09 | 426.08 |
| 54 | 394.16 | 614.6 | 402.0 | 487.58 | 442.42 | 367.14 | 426.11 |
| 55 | 394.57 | 612.14 | 402.34 | 485.49 | 442.76 | 365.19 | 426.14 |
| 56 | 394.98 | 609.68 | 402.68 | 483.4 | 443.1 | 363.24 | 426.17 |
| 57 | 395.39 | 607.22 | 403.02 | 481.32 | 443.45 | 361.29 | 426.2 |
| 58 | 395.8 | 604.76 | 403.36 | 479.23 | 443.79 | 359.34 | 426.23 |
| 59 | 396.21 | 602.3 | 403.7 | 477.16 | 444.13 | 357.39 | 426.26 |
| 60 | 396.62 | 599.84 | 404.04 | 475.09 | 444.47 | 355.44 | 426.29 |

| 24° | | 25° | | 26° | | 27° | | |
|-----|----------|----------|----------|----------|----------|----------|----------|----|
| G. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | |
| 23 | 2.246 04 | .466 31 | 2.144 51 | .487 73 | 2.050 3 | .509 53 | 1.962 61 | 60 |
| 58 | .244 28 | .466 66 | .142 88 | .488 09 | .048 79 | .509 89 | .961 2 | 59 |
| 93 | .242 52 | .467 02 | .141 25 | .488 45 | .047 28 | .510 26 | .959 79 | 58 |
| 27 | .240 77 | .467 37 | .139 63 | .488 81 | .045 77 | .510 63 | .958 38 | 57 |
| 62 | .239 02 | .467 72 | .138 01 | .489 17 | .044 26 | .510 99 | .956 98 | 56 |
| 97 | 2.237 27 | .468 08 | 2.136 39 | .489 53 | 2.042 76 | .511 36 | 1.955 57 | 55 |
| 32 | .235 53 | .468 43 | .134 77 | .489 89 | .041 25 | .511 73 | .954 17 | 54 |
| 67 | .233 78 | .468 79 | .133 16 | .490 26 | .039 75 | .512 09 | .952 77 | 53 |
| 02 | .232 04 | .469 14 | .131 54 | .490 62 | .038 25 | .512 46 | .951 37 | 52 |
| 37 | .230 3 | .469 5 | .129 93 | .490 98 | .036 75 | .512 83 | .949 97 | 51 |
| 72 | 2.228 57 | .469 85 | 2.128 32 | .491 34 | 2.035 26 | .513 19 | 1.948 58 | 50 |
| 07 | .226 83 | .470 21 | .126 71 | .491 7 | .033 76 | .513 56 | .947 18 | 49 |
| 42 | .225 1 | .470 56 | .125 11 | .492 06 | .032 27 | .513 93 | .945 79 | 48 |
| 77 | .223 37 | .470 92 | .123 5 | .492 42 | .030 78 | .514 3 | .944 4 | 47 |
| 12 | .221 64 | .471 28 | .121 9 | .492 78 | .029 29 | .514 67 | .943 01 | 46 |
| 47 | 2.219 92 | .471 63 | 2.120 3 | .493 15 | 2.027 8 | .515 03 | 1.941 62 | 45 |
| 82 | .218 19 | .471 99 | .118 71 | .493 51 | .026 31 | .515 4 | .940 23 | 44 |
| 17 | .216 47 | .472 34 | .117 11 | .493 87 | .024 83 | .515 77 | .938 85 | 43 |
| 52 | .214 75 | .472 7 | .115 52 | .494 23 | .023 35 | .516 14 | .937 46 | 42 |
| 87 | .213 04 | .473 05 | .113 92 | .494 59 | .021 87 | .516 51 | .936 08 | 41 |
| 22 | 2.211 32 | .473 41 | 2.112 33 | .494 95 | 2.020 39 | .516 88 | 1.934 7 | 40 |
| 57 | .209 61 | .473 77 | .110 75 | .495 32 | .018 91 | .517 24 | .933 32 | 39 |
| 92 | .207 9 | .474 12 | .109 16 | .495 68 | .017 43 | .517 61 | .931 95 | 38 |
| 127 | .206 19 | .474 48 | .107 58 | .496 04 | .015 96 | .517 98 | .930 57 | 37 |
| 162 | .204 49 | .474 83 | .106 | .496 4 | .014 49 | .518 35 | .929 2 | 36 |
| 197 | 2.202 78 | .475 19 | 2.104 42 | .496 77 | 2.013 02 | .518 72 | 1.927 82 | 35 |
| 132 | .201 08 | .475 55 | .102 84 | .497 13 | .011 55 | .519 09 | .926 45 | 34 |
| 167 | .199 38 | .475 9 | .101 26 | .497 49 | .010 08 | .519 46 | .925 08 | 33 |
| 302 | .197 69 | .476 25 | .099 69 | .497 86 | .008 62 | .519 83 | .923 71 | 32 |
| 537 | .195 99 | .476 62 | .098 11 | .498 22 | .007 15 | .520 2 | .922 35 | 31 |
| 573 | 2.194 3 | .476 98 | 2.096 54 | .498 58 | 2.005 69 | .520 57 | 1.920 98 | 30 |
| 508 | .192 61 | .477 33 | .094 98 | .498 94 | .004 23 | .520 94 | .919 62 | 29 |
| 543 | .190 92 | .477 69 | .093 41 | .499 31 | .002 77 | .521 31 | .918 26 | 28 |
| 578 | .189 23 | .478 05 | .091 84 | .499 67 | .001 31 | .521 68 | .916 9 | 27 |
| 713 | .187 55 | .478 4 | .090 28 | .500 04 | 1.999 86 | .522 05 | .915 54 | 26 |
| 748 | 2.185 87 | .478 76 | 2.088 72 | .500 4 | 1.998 41 | .522 42 | 1.914 18 | 25 |
| 784 | .184 19 | .479 12 | .087 16 | .500 76 | .996 05 | .522 79 | .912 82 | 24 |
| 819 | .182 51 | .479 48 | .085 6 | .501 13 | .995 5 | .523 16 | .911 47 | 23 |
| 854 | .180 84 | .479 84 | .084 05 | .501 49 | .994 06 | .523 53 | .910 12 | 22 |
| 889 | .179 16 | .480 19 | .082 5 | .501 85 | .992 61 | .523 9 | .908 76 | 21 |
| 924 | 2.177 49 | .480 55 | 2.080 94 | .502 22 | 1.991 16 | .524 27 | 1.907 41 | 20 |
| 96 | .175 82 | .480 91 | .079 39 | .502 58 | .989 72 | .524 64 | .906 07 | 19 |
| 995 | .174 16 | .481 27 | .077 85 | .502 95 | .988 28 | .525 01 | .904 72 | 18 |
| 03 | .172 49 | .481 63 | .076 3 | .503 31 | .986 84 | .525 38 | .903 37 | 17 |
| 065 | .170 83 | .481 98 | .074 76 | .503 68 | .985 4 | .525 75 | .902 03 | 16 |
| 101 | 2.169 17 | .482 34 | 2.073 21 | .504 04 | 1.983 96 | .526 13 | 1.900 69 | 15 |
| 136 | .167 51 | .482 7 | .071 67 | .504 41 | .982 53 | .526 5 | .899 35 | 14 |
| 171 | .165 85 | .483 06 | .070 14 | .504 77 | .981 1 | .526 87 | .898 01 | 13 |
| 206 | .164 2 | .483 42 | .068 6 | .505 14 | .979 66 | .527 24 | .896 67 | 12 |
| 242 | .162 55 | .483 78 | .067 06 | .505 5 | .978 23 | .527 61 | .895 33 | 11 |
| 277 | 2.160 9 | .484 14 | 2.065 53 | .505 87 | 1.976 8 | .527 98 | 1.894 | |
| 312 | .159 25 | .484 5 | .064 | .506 23 | .975 38 | .528 36 | .893 | |
| 348 | .157 6 | .484 86 | .062 47 | .506 6 | .973 95 | .528 73 | .892 | |
| 383 | .155 96 | .485 21 | .060 94 | .506 96 | .972 53 | .529 1 | .891 | |
| 418 | .154 32 | .485 57 | .059 42 | .507 33 | .971 11 | .529 47 | .888 | |
| 454 | 2.152 68 | .485 93 | 2.057 9 | .507 69 | 1.969 69 | .529 84 | 1.887 | |
| 489 | .151 04 | .486 29 | .056 37 | .508 06 | .968 27 | .530 22 | .886 | |
| 525 | .149 4 | .486 65 | .054 85 | .508 43 | .966 85 | .530 59 | .885 | |
| 560 | .147 77 | .487 01 | .053 33 | .508 79 | .965 44 | .530 96 | .884 | |
| 595 | .146 14 | .487 37 | .051 82 | .509 16 | .964 02 | .531 34 | .883 | |
| 631 | 2.144 51 | .487 73 | 2.050 3 | .509 53 | 1.962 61 | .531 71 | 1.882 | |
| W. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | |
| 650 | | | 640 | | 630 | | 620 | |

| | 28° | | 29° | | 30° | | 31° | |
|-----|---------|----------|---------|----------|----------|----------|----------|----------|
| | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 0 | .531 71 | 1.880 73 | .554 31 | 1.804 05 | .577 35 | 1.732 05 | .600 86 | 1.664 28 |
| 1 | .532 08 | .879 41 | .554 69 | .802 81 | .577 74 | .730 89 | .601 26 | .663 18 |
| 2 | .532 46 | .878 09 | .555 07 | .801 58 | .578 13 | .729 73 | .601 65 | .662 09 |
| 3 | .532 83 | .876 77 | .555 45 | .800 34 | .578 51 | .728 57 | .602 05 | .660 99 |
| 4 | .533 2 | .875 46 | .555 83 | .799 11 | .578 9 | .727 41 | .602 45 | .659 9 |
| 5 | .533 58 | 1.874 15 | .556 21 | 1.797 88 | .579 29 | 1.726 25 | .602 84 | 1.658 81 |
| 6 | .533 95 | .872 83 | .556 59 | .796 65 | .579 68 | .725 09 | .603 24 | .657 72 |
| 7 | .534 32 | .871 52 | .556 97 | .795 42 | .580 07 | .723 93 | .603 64 | .656 63 |
| 8 | .534 7 | .870 21 | .557 36 | .794 19 | .580 46 | .722 78 | .604 03 | .655 54 |
| 9 | .535 07 | .868 91 | .557 74 | .792 96 | .580 85 | .721 63 | .604 43 | .654 45 |
| 10 | .535 45 | 1.867 6 | .558 12 | 1.791 74 | .581 24 | 1.720 47 | .604 83 | 1.653 37 |
| 11 | .535 82 | .866 3 | .558 5 | .790 51 | .581 62 | .719 32 | .605 22 | .652 28 |
| 12 | .536 2 | .864 99 | .558 88 | .789 29 | .582 01 | .718 17 | .605 62 | .651 2 |
| 13 | .536 57 | .863 69 | .559 26 | .788 07 | .582 4 | .717 02 | .606 02 | .650 11 |
| 14 | .536 94 | .862 39 | .559 64 | .786 85 | .582 79 | .715 88 | .606 42 | .649 03 |
| 15 | .537 32 | 1.861 09 | .560 03 | 1.785 63 | .583 18 | 1.714 73 | .606 81 | 1.647 95 |
| 16 | .537 69 | .859 79 | .560 41 | .784 41 | .583 57 | .713 58 | .607 21 | .646 87 |
| 17 | .538 07 | .858 5 | .560 79 | .783 19 | .583 96 | .712 44 | .607 61 | .645 79 |
| 18 | .538 44 | .857 2 | .561 17 | .781 98 | .584 35 | .711 29 | .608 01 | .644 71 |
| 19 | .538 82 | .855 91 | .561 56 | .780 77 | .584 74 | .710 15 | .608 41 | .643 63 |
| 20 | .539 2 | 1.854 62 | .561 94 | 1.779 55 | .585 13 | 1.709 01 | .608 81 | 1.642 56 |
| 21 | .539 57 | .853 33 | .562 32 | .778 34 | .585 52 | .707 87 | .609 21 | .641 48 |
| 22 | .539 95 | .852 04 | .562 7 | .777 13 | .585 91 | .706 73 | .609 6 | .640 41 |
| 23 | .540 32 | .850 75 | .563 09 | .775 92 | .586 31 | .705 6 | .61 | .639 34 |
| 24 | .540 7 | .849 46 | .563 47 | .774 71 | .586 7 | .704 46 | .610 4 | .638 26 |
| 25 | .541 07 | 1.848 18 | .563 85 | 1.773 51 | .587 09 | 1.703 32 | .610 8 | 1.637 19 |
| 26 | .541 45 | .846 89 | .564 24 | .772 3 | .587 48 | .702 19 | .611 2 | .636 11 |
| 27 | .541 83 | .845 61 | .564 62 | .771 1 | .587 87 | .701 06 | .611 6 | .635 03 |
| 28 | .542 2 | .844 33 | .565 | .769 9 | .588 26 | .699 92 | .612 | .633 95 |
| 29 | .542 58 | .843 05 | .565 39 | .768 69 | .588 65 | .698 79 | .612 4 | .632 87 |
| 30 | .542 96 | 1.841 77 | .565 77 | 1.767 49 | .589 04 | 1.697 66 | .612 8 | 1.631 8 |
| 31 | .543 33 | .840 49 | .566 16 | .766 3 | .589 44 | .696 53 | .613 2 | .630 79 |
| 32 | .543 71 | .839 22 | .566 54 | .765 1 | .589 83 | .695 41 | .613 6 | .629 72 |
| 33 | .544 09 | .837 94 | .566 93 | .763 9 | .590 22 | .694 28 | .614 | .628 66 |
| 34 | .544 46 | .836 67 | .567 31 | .762 71 | .590 61 | .693 16 | .614 4 | .627 6 |
| 35 | .544 84 | 1.835 4 | .567 69 | 1.761 51 | .591 01 | 1.692 03 | .614 8 | 1.626 54 |
| 36 | .545 22 | .834 13 | .568 08 | .760 32 | .591 4 | .690 91 | .615 2 | .625 48 |
| 37 | .545 6 | .832 86 | .568 46 | .759 13 | .591 79 | .689 79 | .615 61 | .624 42 |
| 38 | .545 97 | .831 59 | .568 85 | .757 94 | .592 18 | .688 66 | .616 01 | .623 36 |
| 39 | .546 35 | .830 33 | .569 23 | .756 75 | .592 58 | .687 54 | .616 41 | .622 3 |
| 40 | .546 73 | 1.829 06 | .569 62 | 1.755 56 | .592 97 | 1.686 43 | .616 81 | 1.621 25 |
| 41 | .547 11 | .827 8 | .57 | .754 37 | .593 36 | .685 31 | .617 21 | .620 19 |
| 42 | .547 48 | .826 54 | .570 39 | .753 19 | .593 76 | .684 19 | .617 61 | .619 14 |
| 43 | .547 86 | .825 28 | .570 78 | .752 | .594 15 | .683 08 | .618 01 | .618 08 |
| 44 | .548 24 | .824 02 | .571 16 | .750 82 | .594 54 | .681 96 | .618 42 | .617 03 |
| 45 | .548 62 | 1.822 76 | .571 55 | 1.749 64 | .594 94 | 1.680 85 | .618 82 | 1.615 98 |
| 46 | .549 | .821 5 | .571 93 | .748 46 | .595 33 | .679 74 | .619 22 | .614 93 |
| 47 | .549 38 | .820 25 | .572 32 | .747 28 | .595 73 | .678 63 | .619 62 | .613 88 |
| 48 | .549 75 | .818 99 | .572 71 | .746 1 | .596 12 | .677 52 | .620 03 | .612 83 |
| 49 | .550 13 | .817 74 | .573 09 | .744 92 | .596 51 | .676 41 | .620 43 | .611 79 |
| 50 | .550 51 | 1.816 49 | .573 48 | 1.743 75 | .596 91 | 1.675 3 | .620 83 | 1.610 74 |
| 51 | .815 24 | .573 86 | .742 57 | .597 3 | .674 19 | .621 24 | .609 7 | |
| 52 | .813 99 | .574 25 | .741 4 | .597 7 | .673 09 | .621 64 | .608 65 | |
| 53 | .812 74 | .574 64 | .740 22 | .598 09 | .671 98 | .622 04 | .607 61 | |
| 54 | .811 5 | .575 03 | .739 05 | .598 49 | .670 88 | .622 45 | .606 57 | |
| 55 | .575 41 | .575 41 | .737 88 | .598 88 | .669 78 | .622 85 | .605 53 | |
| 56 | | | | | .668 67 | .623 25 | .604 49 | |
| 57 | | | | | .667 57 | .623 66 | .603 45 | |
| 58 | | | | | .666 47 | .624 06 | .602 41 | |
| 59 | | | | | .665 38 | .624 46 | .601 37 | |
| 60 | | | | | 1.664 28 | .624 87 | 1.600 33 | |
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| 32° | | 33° | | 34° | | 35° | | |
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| TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | |
| 624 87 | 1.600 33 | .649 41 | 1.539 86 | .674 51 | 1.482 56 | .700 21 | 1.428 15 | 60 |
| 625 27 | .599 3 | .640 82 | .538 88 | .674 93 | .481 63 | .700 64 | .427 26 | 59 |
| 625 68 | .598 26 | .650 23 | .537 91 | .675 36 | .480 7 | .701 07 | .426 38 | 58 |
| 626 08 | .597 23 | .650 65 | .536 93 | .675 78 | .479 77 | .701 51 | .425 5 | 57 |
| 626 49 | .596 2 | .651 06 | .535 95 | .676 2 | .478 35 | .701 94 | .424 62 | 56 |
| 626 89 | 1.595 17 | .651 48 | 1.534 97 | .676 63 | 1.477 92 | .702 38 | 1.423 74 | 55 |
| 627 3 | .594 14 | .651 89 | .534 | .677 05 | .476 99 | .702 81 | .422 86 | 54 |
| 627 7 | .593 11 | .652 31 | .533 02 | .677 48 | .476 07 | .703 25 | .421 98 | 53 |
| 628 11 | .592 08 | .652 72 | .532 05 | .677 9 | .475 14 | .703 68 | .421 1 | 52 |
| 628 52 | .591 05 | .653 14 | .531 07 | .678 32 | .474 22 | .704 12 | .420 22 | 51 |
| 628 92 | 1.590 02 | .653 55 | 1.530 1 | .678 75 | 1.473 3 | .704 55 | 1.419 34 | 50 |
| 629 33 | .589 | .653 97 | .529 13 | .679 17 | .472 38 | .704 99 | .418 47 | 49 |
| 629 73 | .587 97 | .654 38 | .528 16 | .679 6 | .471 46 | .705 42 | .417 59 | 48 |
| 630 14 | .586 95 | .654 8 | .527 19 | .680 02 | .470 53 | .705 86 | .416 72 | 47 |
| 630 55 | .585 93 | .655 21 | .526 22 | .680 45 | .469 62 | .706 29 | .415 84 | 46 |
| 630 95 | 1.584 9 | .655 63 | 1.525 25 | .680 88 | 1.468 7 | .706 73 | 1.414 97 | 45 |
| 631 36 | .583 88 | .656 04 | .524 29 | .681 3 | .467 78 | .707 17 | .414 09 | 44 |
| 631 77 | .582 86 | .656 46 | .523 32 | .681 73 | .466 86 | .707 6 | .413 22 | 43 |
| 632 17 | .581 84 | .656 88 | .522 35 | .682 15 | .465 95 | .708 04 | .412 35 | 42 |
| 632 58 | .580 83 | .657 29 | .521 39 | .682 58 | .465 03 | .708 48 | .411 48 | 41 |
| 632 99 | 1.579 81 | .657 71 | 1.520 43 | .683 01 | 1.464 11 | .708 91 | 1.410 61 | 40 |
| 633 4 | .578 79 | .658 13 | .519 46 | .683 43 | .463 2 | .709 35 | .409 74 | 39 |
| 633 8 | .577 78 | .658 54 | .518 5 | .683 86 | .462 29 | .709 79 | .408 87 | 38 |
| 634 21 | .576 76 | .658 96 | .517 54 | .684 29 | .461 37 | .710 23 | .408 | 37 |
| 634 62 | .575 75 | .659 38 | .516 58 | .684 71 | .460 46 | .710 66 | .407 14 | 36 |
| 635 03 | 1.574 74 | .659 8 | 1.515 62 | .685 14 | 1.459 55 | .711 1 | 1.406 27 | 35 |
| 635 44 | .573 72 | .660 21 | .514 66 | .685 57 | .458 64 | .711 54 | .405 4 | 34 |
| 635 84 | .572 71 | .660 63 | .513 7 | .686 | .457 73 | .711 98 | .404 54 | 33 |
| 636 25 | .571 7 | .661 05 | .512 75 | .686 42 | .456 82 | .712 42 | .403 67 | 32 |
| 636 66 | .570 69 | .661 47 | .511 79 | .686 85 | .455 92 | .712 85 | .402 81 | 31 |
| 637 07 | 1.569 69 | .661 89 | 1.510 84 | .687 28 | 1.455 01 | .713 29 | 1.401 95 | 30 |
| 637 48 | .568 68 | .662 3 | .509 88 | .687 71 | .454 1 | .713 73 | .401 09 | 29 |
| 637 89 | .567 67 | .662 72 | .508 93 | .688 14 | .453 2 | .714 17 | .400 22 | 28 |
| 638 3 | .566 67 | .663 14 | .507 97 | .688 57 | .452 29 | .714 61 | .399 36 | 27 |
| 638 71 | .565 66 | .663 56 | .507 02 | .689 | .451 39 | .715 05 | .398 5 | 26 |
| 639 12 | 1.564 66 | .663 98 | 1.506 07 | .689 42 | 1.450 49 | .715 49 | 1.397 64 | 25 |
| 639 53 | .563 66 | .664 4 | .505 12 | .689 85 | .449 58 | .715 93 | .396 79 | 24 |
| 639 94 | .562 65 | .664 82 | .504 17 | .690 28 | .448 68 | .716 37 | .395 93 | 23 |
| 640 35 | .561 65 | .665 24 | .503 22 | .690 71 | .447 78 | .716 81 | .395 07 | 22 |
| 640 76 | .560 65 | .665 66 | .502 28 | .691 14 | .446 88 | .717 25 | .394 21 | 21 |
| 641 17 | 1.559 66 | .666 08 | 1.501 33 | .691 57 | 1.445 98 | .717 69 | 1.393 36 | 20 |
| 641 58 | .558 66 | .666 5 | .500 38 | .692 | .445 08 | .718 13 | .392 5 | 19 |
| 641 99 | .557 66 | .666 92 | .499 44 | .692 43 | .444 18 | .718 57 | .391 65 | 18 |
| 642 4 | .556 66 | .667 34 | .498 49 | .692 86 | .443 29 | .719 01 | .390 79 | 17 |
| 642 81 | .555 67 | .667 76 | .497 55 | .693 29 | .442 39 | .719 46 | .389 94 | 16 |
| 643 22 | 1.554 67 | .668 18 | 1.496 61 | .693 72 | 1.441 49 | .719 9 | 1.389 09 | 15 |
| 643 63 | .553 68 | .668 6 | .495 66 | .694 16 | .440 6 | .720 34 | .388 24 | 14 |
| 644 04 | .552 69 | .669 22 | .494 72 | .694 59 | .439 7 | .720 78 | .387 38 | 13 |
| 644 46 | .551 7 | .669 44 | .493 78 | .695 02 | .438 81 | .721 22 | .386 53 | 12 |
| 644 87 | .550 71 | .669 86 | .492 84 | .695 45 | .437 92 | .721 66 | .385 68 | 11 |
| 645 28 | 1.549 72 | .670 28 | 1.491 9 | .695 88 | 1.437 03 | .722 11 | 1.384 84 | 10 |
| 645 69 | .548 73 | .670 71 | .490 97 | .696 31 | .436 14 | .722 55 | .384 | 9 |
| 646 1 | .547 74 | .671 13 | .490 03 | .696 75 | .435 25 | .722 99 | .383 | 8 |
| 646 52 | .546 75 | .671 55 | .489 09 | .697 18 | .434 36 | .723 44 | .382 | 7 |
| 646 93 | .545 76 | .671 97 | .488 16 | .697 61 | .433 47 | .723 88 | .381 | 6 |
| 647 34 | 1.544 78 | .672 39 | 1.487 22 | .698 04 | 1.432 58 | .724 33 | .380 | 5 |
| 647 75 | .543 79 | .672 82 | .486 29 | .698 47 | .431 69 | .724 77 | .379 | 4 |
| 648 17 | .542 81 | .673 24 | .485 36 | .698 91 | .430 8 | .725 2 | .378 | 3 |
| 648 58 | .541 83 | .673 66 | .484 42 | .699 34 | .429 92 | .725 65 | .377 | 2 |
| 648 99 | .540 85 | .674 09 | .483 49 | .699 77 | .429 03 | .726 1 | .376 | 1 |
| 649 41 | 1.539 86 | .674 51 | 1.482 56 | .700 21 | 1.428 15 | .726 54 | .375 | 0 |
| DEGREES. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 67° | | | 56° | | 55° | | 54° | |

| | 86° | | 37° | | 38° | | 39° | |
|----|----------|----------|----------|----------|----------|----------|----------|----------|
| | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. |
| 0 | .726 54 | 1.316 38 | .753 55 | 1.327 04 | .781 29 | 1.279 94 | .809 78 | 1.234 9 |
| 1 | .726 99 | .375 54 | .754 01 | .326 24 | .781 75 | .279 17 | .810 27 | .234 16 |
| 2 | .727 43 | .374 7 | .754 47 | .325 44 | .782 22 | .278 41 | .810 75 | .233 43 |
| 3 | .727 88 | .373 86 | .754 92 | .324 64 | .782 69 | .277 64 | .811 23 | .232 7 |
| 4 | .728 32 | .373 02 | .755 38 | .323 84 | .783 16 | .276 88 | .811 71 | .231 96 |
| 5 | .728 77 | 1.372 18 | .755 84 | 1.323 04 | .783 63 | 1.276 11 | .812 2 | 1.231 43 |
| 6 | .729 21 | .371 34 | .756 29 | .322 24 | .784 1 | .275 35 | .812 68 | .230 5 |
| 7 | .729 66 | .370 5 | .756 75 | .321 44 | .784 57 | .274 58 | .813 16 | .229 77 |
| 8 | .730 1 | .369 67 | .757 21 | .320 64 | .785 04 | .273 82 | .813 64 | .229 04 |
| 9 | .730 55 | .368 83 | .757 67 | .319 84 | .785 51 | .273 06 | .814 13 | .228 31 |
| 10 | .731 | 1.368 | .758 12 | 1.319 04 | .785 98 | 1.272 3 | .814 61 | 1.227 58 |
| 11 | .731 44 | .367 16 | .758 58 | .318 25 | .786 45 | .271 53 | .815 1 | .226 85 |
| 12 | .731 89 | .366 33 | .759 04 | .317 45 | .786 92 | .270 77 | .815 58 | .226 12 |
| 13 | .732 34 | .365 49 | .759 5 | .316 66 | .787 39 | .270 01 | .816 06 | .225 39 |
| 14 | .732 78 | 1.364 66 | .759 96 | .315 86 | .787 86 | .269 25 | .816 55 | .224 67 |
| 15 | .733 23 | .363 83 | .760 42 | 1.315 07 | .788 34 | 1.268 49 | .817 03 | 1.223 94 |
| 16 | .733 68 | .363 | .760 88 | .314 27 | .788 81 | .267 74 | .817 52 | .223 21 |
| 17 | .734 13 | .362 17 | .761 34 | .313 48 | .789 28 | .266 98 | .818 | .222 49 |
| 18 | .734 57 | .361 33 | .761 8 | .312 69 | .789 75 | .266 22 | .818 49 | .221 76 |
| 19 | .735 02 | .360 51 | .762 26 | .311 9 | .790 22 | .265 46 | .818 98 | .221 04 |
| 20 | .735 47 | 1.359 68 | .762 72 | 1.311 1 | .790 7 | 1.264 71 | .819 46 | 1.220 31 |
| 21 | .735 92 | .358 85 | .763 18 | .310 31 | .791 17 | .263 95 | .819 95 | .219 59 |
| 22 | .736 37 | .358 02 | .763 64 | .309 52 | .791 64 | .263 19 | .820 44 | .218 86 |
| 23 | .736 81 | .357 19 | .764 1 | .308 73 | .792 12 | .262 44 | .820 92 | .218 14 |
| 24 | .737 26 | .356 37 | .764 56 | .307 95 | .792 59 | .261 69 | .821 41 | .217 41 |
| 25 | .737 71 | 1.355 54 | .765 02 | 1.307 16 | .793 06 | 1.260 93 | .821 9 | 1.217 7 |
| 26 | .738 16 | .354 72 | .765 48 | .306 37 | .793 54 | .260 18 | .822 38 | .215 96 |
| 27 | .738 61 | .353 89 | .765 94 | .305 58 | .794 01 | .259 43 | .822 87 | .215 26 |
| 28 | .739 06 | .353 07 | .766 4 | .304 8 | .794 49 | .258 67 | .823 36 | .214 54 |
| 29 | .739 51 | .352 24 | .766 86 | .304 01 | .794 96 | .257 92 | .823 85 | .213 82 |
| 30 | .739 96 | 1.351 42 | .767 33 | 1.303 23 | .795 44 | 1.257 17 | .824 34 | 1.213 1 |
| 31 | .740 41 | .350 6 | .767 79 | .302 44 | .795 91 | .256 42 | .824 83 | .212 38 |
| 32 | .740 86 | .349 78 | .768 25 | .301 66 | .796 39 | .255 67 | .825 31 | .211 66 |
| 33 | .741 31 | .348 96 | .768 71 | .300 87 | .796 86 | .254 92 | .825 8 | .210 94 |
| 34 | .741 76 | .348 14 | .769 18 | .300 09 | .797 34 | .254 17 | .826 29 | .210 23 |
| 35 | .742 21 | 1.347 32 | .769 64 | 1.299 31 | .797 81 | 1.253 43 | .826 78 | 1.209 51 |
| 36 | .742 67 | .346 5 | .770 1 | .298 53 | .798 29 | .252 68 | .827 27 | .208 79 |
| 37 | .743 12 | .345 68 | .770 57 | .297 75 | .798 77 | .251 93 | .827 76 | .208 08 |
| 38 | .743 57 | .344 87 | .771 03 | .296 96 | .799 24 | .251 18 | .828 25 | .207 36 |
| 39 | .744 02 | .344 05 | .771 49 | .296 18 | .799 72 | .250 44 | .828 74 | .206 65 |
| 40 | .744 47 | 1.343 23 | .771 96 | 1.295 41 | .800 2 | 1.249 69 | .829 23 | 1.205 93 |
| 41 | .744 92 | .342 42 | .772 42 | .294 63 | .800 67 | .248 95 | .829 72 | .205 22 |
| 42 | .745 38 | .341 6 | .772 89 | .293 85 | .801 15 | .248 2 | .830 22 | .204 51 |
| 43 | .745 83 | .340 79 | .773 35 | .293 07 | .801 63 | .247 46 | .830 71 | .203 79 |
| 44 | .746 28 | .339 98 | .773 82 | .292 29 | .802 11 | .246 72 | .831 2 | .203 08 |
| 45 | .746 74 | 1.339 16 | .774 28 | 1.291 52 | .802 58 | 1.245 97 | .831 69 | 1.202 37 |
| 46 | .747 19 | .338 35 | .774 75 | .290 74 | .803 06 | .245 23 | .832 18 | .201 66 |
| 47 | .747 64 | .337 54 | .775 21 | .289 97 | .803 54 | .244 49 | .832 68 | .200 95 |
| 48 | .748 1 | .336 73 | .775 68 | .289 19 | .804 02 | .243 75 | .833 17 | .200 24 |
| 49 | .748 55 | .335 92 | .776 15 | .288 42 | .804 5 | .243 01 | .833 66 | .199 53 |
| 50 | .749 | 1.335 11 | .776 61 | 1.287 64 | .804 98 | 1.242 27 | .834 15 | 1.198 82 |
| 51 | .749 46 | .334 3 | .777 08 | .286 87 | .805 46 | .241 53 | .834 65 | .198 11 |
| 52 | .749 91 | .333 49 | .777 54 | .286 1 | .805 94 | .240 79 | .835 14 | .197 4 |
| 53 | .750 37 | .332 68 | .778 01 | .285 33 | .806 42 | .240 05 | .835 64 | .196 69 |
| 54 | .750 82 | .331 87 | .778 48 | .284 56 | .806 9 | .239 31 | .836 13 | .195 99 |
| 55 | .751 28 | 1.331 07 | .778 95 | 1.283 79 | .807 38 | 1.238 58 | .836 62 | 1.195 28 |
| 56 | .751 73 | .330 26 | .779 41 | .283 02 | .807 86 | .237 84 | .837 12 | .194 57 |
| 57 | .752 19 | .329 46 | .779 88 | .282 25 | .808 34 | .237 1 | .837 61 | .193 87 |
| 58 | .752 64 | .328 65 | .780 35 | .281 48 | .808 82 | .236 37 | .838 11 | .193 16 |
| 59 | .753 1 | .327 85 | .780 82 | .280 71 | .809 3 | .235 63 | .838 6 | .192 46 |
| 60 | .753 55 | 1.327 04 | .781 29 | 1.279 94 | .809 78 | 1.234 9 | .839 1 | 1.191 75 |
| | 53° | | 52° | | 51° | | 50° | |
| | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. |

| 40° | | 41° | | 42° | | 43° | | |
|------|----------|---------|----------|---------|----------|---------|----------|-----|
| ANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | TANG. | CO-TANG. | |
| 39 1 | 1.191 75 | .869 29 | 1.150 37 | .900 4 | 1.110 61 | .932 52 | 1.072 37 | 60 |
| 39 6 | .191 05 | .869 8 | .149 69 | .900 93 | .109 90 | .933 06 | .071 74 | 59 |
| 40 0 | .190 35 | .870 31 | .149 02 | .901 46 | .109 31 | .933 6 | .071 12 | 58 |
| 40 5 | .189 64 | .870 82 | .148 34 | .901 99 | .108 67 | .934 15 | .070 49 | 57 |
| 41 0 | .188 94 | .871 33 | .147 67 | .902 51 | .108 02 | .934 69 | .069 87 | 56 |
| 41 5 | 1.188 24 | .871 84 | 1.146 99 | .903 04 | 1.107 37 | .935 24 | 1.069 25 | 55 |
| 42 0 | .187 54 | .872 36 | .146 32 | .903 57 | .106 72 | .935 78 | .068 62 | 54 |
| 42 5 | .186 84 | .872 87 | .145 65 | .904 1 | .106 07 | .936 33 | .068 | 53 |
| 43 0 | .186 14 | .873 38 | .144 98 | .904 63 | .105 43 | .936 88 | .067 38 | 52 |
| 43 5 | .185 44 | .873 89 | .144 3 | .905 16 | .104 78 | .937 42 | .066 76 | 51 |
| 44 0 | 1.184 74 | .874 41 | 1.143 63 | .905 69 | 1.104 14 | .937 97 | 1.066 13 | 50 |
| 44 5 | .184 04 | .874 92 | .142 96 | .906 21 | .103 49 | .938 52 | .065 51 | 49 |
| 45 0 | .183 34 | .875 43 | .142 29 | .906 74 | .102 85 | .939 06 | .064 89 | 48 |
| 45 5 | .182 64 | .875 95 | .141 62 | .907 27 | .102 2 | .939 61 | .064 27 | 47 |
| 46 0 | .181 94 | .876 46 | .140 95 | .907 81 | .101 56 | .940 16 | .063 65 | 46 |
| 46 5 | 1.181 25 | .876 98 | 1.140 28 | .908 34 | 1.100 91 | .940 71 | 1.063 03 | 45 |
| 47 0 | .180 55 | .877 49 | .139 61 | .908 87 | .100 27 | .941 25 | .062 41 | 44 |
| 47 5 | .179 86 | .878 01 | .138 94 | .909 4 | .099 63 | .941 8 | .061 79 | 43 |
| 48 0 | .179 16 | .878 52 | .138 28 | .909 93 | .099 09 | .942 35 | .061 17 | 42 |
| 48 5 | .178 46 | .879 04 | .137 61 | .910 46 | .098 39 | .942 9 | .060 56 | 41 |
| 49 0 | 1.177 77 | .879 55 | 1.136 94 | .910 99 | 1.097 7 | .943 45 | 1.059 94 | 40 |
| 49 5 | .177 08 | .880 07 | .136 27 | .911 53 | .097 06 | .944 | .059 32 | 39 |
| 50 0 | .176 38 | .880 59 | .135 61 | .912 06 | .096 42 | .944 55 | .058 7 | 38 |
| 50 5 | .175 69 | .881 1 | .134 94 | .912 59 | .095 78 | .945 1 | .058 09 | 37 |
| 51 0 | .175 | .881 62 | .134 28 | .913 13 | .095 14 | .945 65 | .057 47 | 36 |
| 51 5 | 1.174 3 | .882 14 | 1.133 61 | .913 66 | 1.094 5 | .946 2 | 1.056 85 | 35 |
| 52 0 | .173 61 | .882 65 | .132 95 | .914 19 | .093 86 | .946 76 | .056 24 | 34 |
| 52 5 | .172 92 | .883 17 | .132 28 | .914 73 | .093 22 | .947 31 | .055 62 | 33 |
| 53 0 | .172 23 | .883 69 | .131 62 | .915 26 | .092 58 | .947 86 | .055 01 | 32 |
| 53 5 | .171 54 | .884 21 | .130 96 | .915 8 | .091 95 | .948 41 | .054 39 | 31 |
| 54 0 | 1.170 85 | .884 73 | 1.130 29 | .916 37 | 1.091 31 | .948 96 | 1.053 78 | 30 |
| 54 5 | .170 16 | .885 24 | .129 63 | .916 87 | .090 67 | .949 52 | .053 17 | 29 |
| 55 0 | .169 47 | .885 76 | .128 97 | .917 4 | .090 03 | .950 07 | .052 55 | 28 |
| 55 5 | .168 78 | .886 28 | .128 31 | .917 94 | .089 4 | .950 62 | .051 94 | 27 |
| 56 0 | .168 09 | .886 8 | .127 65 | .918 47 | .088 76 | .951 18 | .051 33 | 26 |
| 56 5 | 1.167 41 | .887 32 | 1.126 99 | .919 01 | 1.088 13 | .951 73 | 1.050 72 | 25 |
| 57 0 | .166 72 | .887 84 | .126 33 | .919 55 | .087 49 | .952 29 | .050 1 | 24 |
| 57 5 | .166 03 | .888 36 | .125 67 | .920 08 | .086 86 | .952 84 | .049 49 | 23 |
| 58 0 | .165 35 | .888 88 | .125 01 | .920 62 | .086 22 | .953 4 | .048 88 | 22 |
| 58 5 | .164 66 | .889 4 | .124 35 | .921 16 | .085 59 | .953 95 | .048 27 | 21 |
| 59 0 | 1.163 98 | .889 92 | 1.123 69 | .921 7 | 1.084 96 | .954 51 | 1.047 66 | 20 |
| 59 5 | .163 29 | .890 45 | .123 03 | .922 23 | .084 32 | .955 06 | .047 05 | 19 |
| 60 0 | .162 61 | .890 97 | .122 38 | .922 77 | .083 69 | .955 62 | .046 44 | 18 |
| 60 5 | .161 92 | .891 49 | .121 72 | .923 31 | .083 06 | .956 18 | .045 83 | 17 |
| 61 0 | .161 24 | .892 01 | .121 06 | .923 85 | .082 43 | .956 73 | .045 22 | 16 |
| 61 5 | 1.160 56 | .892 53 | 1.120 41 | .924 39 | 1.081 79 | .957 29 | 1.044 61 | |
| 62 0 | .159 87 | .893 06 | .119 75 | .924 93 | .081 16 | .957 85 | .044 01 | |
| 62 5 | .159 19 | .893 58 | .119 09 | .925 47 | .080 53 | .958 41 | .043 4 | |
| 63 0 | .158 51 | .894 1 | .118 44 | .926 01 | .079 9 | .958 97 | .042 79 | |
| 63 5 | .157 83 | .894 63 | .117 78 | .926 55 | .079 27 | .959 52 | .042 18 | |
| 64 0 | 1.157 15 | .895 15 | 1.117 13 | .927 09 | 1.078 64 | .960 08 | 1.041 58 | |
| 64 5 | .156 47 | .895 67 | .116 48 | .927 63 | .078 01 | .960 64 | .040 97 | |
| 65 0 | .155 79 | .896 2 | .115 82 | .928 17 | .077 38 | .961 2 | .040 36 | |
| 65 5 | .155 11 | .896 72 | .115 17 | .928 72 | .076 76 | .961 76 | | |
| 66 0 | .154 43 | .897 25 | .114 52 | .929 26 | .076 13 | .962 32 | | |
| 66 5 | 1.153 75 | .897 77 | 1.113 87 | .929 8 | 1.075 5 | .962 88 | | |
| 67 0 | .153 08 | .898 3 | .113 21 | .930 34 | .074 87 | .963 44 | | |
| 67 5 | .152 4 | .898 83 | .112 56 | .930 88 | .074 25 | .964 | | |
| 68 0 | .151 72 | .899 35 | .111 91 | .931 43 | .073 62 | .964 57 | | |
| 68 5 | .151 04 | .899 88 | .111 26 | .931 97 | .072 99 | .965 13 | | |
| 69 0 | 1.150 37 | .900 4 | 1.110 61 | .932 52 | 1.072 37 | .965 69 | | |
| 48° | | TANG. | CO-TANG. | 47° | | TANG. | CO-TANG. | 46° |

| 44° | | | | 44° | | | | 44° | | | |
|----------|----------|---------|----|----------|----------|----|----|----------|----------|----|--|
| TANG. | CO-TANG. | | | TANG. | CO-TANG. | | | TANG. | CO-TANG. | | |
| 0 | .96569 | 1.03553 | 60 | .97756 | 1.02295 | 39 | 41 | .98901 | 1.01112 | 11 | |
| 1 | .96625 | 1.03493 | 59 | .97813 | 1.02236 | 38 | 42 | .98958 | 1.01053 | 10 | |
| 2 | .96681 | 1.03433 | 58 | .97870 | 1.02176 | 37 | 43 | .99016 | 1.00994 | 9 | |
| 3 | .96738 | 1.03372 | 57 | .97927 | 1.02117 | 36 | 44 | .99073 | 1.00935 | 8 | |
| 4 | .96794 | 1.03312 | 56 | .97984 | 1.02057 | 35 | 45 | .99131 | 1.00876 | 7 | |
| 5 | .9685 | 1.03252 | 55 | .98041 | 1.01998 | 34 | 46 | .99189 | 1.00818 | 6 | |
| 6 | .96907 | 1.03192 | 54 | .98098 | 1.01939 | 33 | 47 | .99247 | 1.00759 | 5 | |
| 7 | .96963 | 1.03132 | 53 | .98155 | 1.01879 | 32 | 48 | .99304 | 1.00701 | 4 | |
| 8 | .9702 | 1.03072 | 52 | .98213 | 1.0182 | 31 | 49 | .99362 | 1.00642 | 3 | |
| 9 | .97076 | 1.03012 | 51 | .9827 | 1.01761 | 30 | 50 | .9942 | 1.00583 | 2 | |
| 10 | .97133 | 1.02952 | 50 | .98327 | 1.01702 | 29 | 51 | .99478 | 1.00525 | 1 | |
| 11 | .97189 | 1.02892 | 49 | .98384 | 1.01642 | 28 | 52 | .99536 | 1.00467 | | |
| 12 | .97245 | 1.02832 | 48 | .98441 | 1.01583 | 27 | 53 | .99594 | 1.00408 | | |
| 13 | .97302 | 1.02772 | 47 | .98499 | 1.01524 | 26 | 54 | .99652 | 1.0035 | | |
| 14 | .97359 | 1.02713 | 46 | .98556 | 1.01465 | 25 | 55 | .9971 | 1.00291 | | |
| 15 | .97416 | 1.02653 | 45 | .98613 | 1.01406 | 24 | 56 | .99768 | 1.00233 | | |
| 16 | .97472 | 1.02593 | 44 | .98671 | 1.01347 | 23 | 57 | .99826 | 1.00175 | | |
| 17 | .97529 | 1.02533 | 43 | .98728 | 1.01288 | 22 | 58 | .99884 | 1.00116 | | |
| 18 | .97586 | 1.02474 | 42 | .98786 | 1.01229 | 21 | 59 | .99942 | 1.00058 | | |
| 19 | .97643 | 1.02414 | 41 | .98843 | 1.0117 | 20 | 60 | | | | |
| 20 | .977 | 1.02355 | 40 | | | | | | | | |
| CO-TANG. | TANG. | | | CO-TANG. | TANG. | | | CO-TANG. | TANG. | | |
| 45° | | | | 45° | | | | 45° | | | |

Preceding Table contains Natural Tangents and Co-tangents for every minute of the quadrant, to the radius of 1.

If Degrees are taken at head of columns, Minutes, Tangents, and Co-tangents must be taken from head also; and if they are taken at foot of columns, Minutes, etc., must be taken from foot also.

ILLUSTRATION.—1974 is tangent for $11^{\circ} 10'$, and co-tangent for $78^{\circ} 50'$.

To Compute Tangents and Co-tangents for Seconds.

Ascertain tangent or co-tangent of angle for degrees and minutes from Table; take difference between it and tangent or co-tangent next below it.

Then as 60 seconds is to difference, so are seconds given to result required, which is to be added to tangent and subtracted from co-tangent.

ILLUSTRATION.—What is the tangent and co-tangent of $54^{\circ} 40' 40''$?

Tangent of $54^{\circ} 40'$, per Table = 1.41061

Tangent of $54^{\circ} 41'$, " = 1.41148 .00087 difference.

Then $60 : .00087 :: 40 : .00058$, which, added to 1.41061 = 1.41119 tangent.

Co-tangent of $54^{\circ} 40'$, per Table = .70891

Co-tangent of $54^{\circ} 41'$, " = .70848 .00043 difference.

Then $60 : .00043 :: 40 : .00029$, which, subtracted from .70891 = .70862 co-tangent.

To Compute Tangent or Co-tangent of any Angle in Degrees, Minutes, and Seconds.

Use Sine by Cosine for Tangent, and Cosine by Sine for Co-tangent.

ILL.—What is tangent of $25^{\circ} 18'$?

Sine = .42736; cosine = .90408. Then $\frac{.42736}{.90408} = .4727$ tangent.

To Compute Number of Degrees, Minutes, and Seconds of a given Tangent or Co-tangent.

If it is given.—Proceed as by Rule, page 402, for Sines, substituting for Sines.

Tangent for 1.41119?

Ans. $54^{\circ} 40' 40''$

Next greatest tangent

$1.41119 - 1.41061 = .00058$
= $54^{\circ} 40' 40''$

page 402, in Col.

AEROSTATICS.

Atmospheric Air consists, by volume, of Oxygen 21, and Nitrogen 79 parts; and in 10000 parts there are 4.9 parts of Carbonic acid gas. y weight, it consists of 77 parts of Oxygen, and 23 of Nitrogen.

One cube foot of Atmospheric Air at surface of Earth, when barometer is at 30 ins., and at a temperature of 32° , weighs 565.0964 grains = 80.728 lbs. avoirdupois, being 773.19 times lighter than water.

Specific gravity compared with water, at $62.418 = .001\ 293\ 345$.

Mean weight of a column of air a foot square, and of an altitude equal to height of atmosphere (barometer 30 ins.), is 2124.6875 lbs. = 4.7548 lbs. per sq. inch = support of 34.0393 feet of water.

Standard pound is computed with a mercurial barometer at 30 ins.; hence, a cube inch of mercury at 60° weighs .490 776 9 lbs., pressure of atmosphere at $60^{\circ} = 14.723\ 307$ lbs. per square inch.

12.3873 cube feet of air weigh a pound, and its weight varies about gr. for each degree of heat.

Extreme height of barometer in latitude 30° to 35° N. = 30.21 ins.

Rate of expansion of Air, and all other Elastic Fluids for all temperatures, is essentially uniform. From 32° to 212° they expand from 1000 to 1376 volumes = .002 088 or $\frac{1}{475}$ th part of their bulk for every degree of heat. From 212° to 680° they expand from 1376 to 2322 = .002 021 for each degree of heat.

Thus, if volume of air at 132° is required. $132^{\circ} - 32^{\circ} = 100$, and $1000 \div 100 \times .002\ 088 = 1209$ volumes.

Height, at Equator is estimated at 300 feet greater than at Poles, its mean height at 45° latitude.

In like latitudes, air loses 1° for every 340 feet in height above level of sea.

Below surface of Earth, temperature increases.

Elasticity of air is inversely as space it occupies, and directly as its density.

When altitude of air is taken in arithmetical proportion, its Rarity will be in geometric proportion. Thus, at 7 miles above surface of Earth, air is 4 times rarer or lighter than at Earth's surface; at 14 miles, 16 times; at 21 miles, 64 times, and so on.

Density of an aeriform fluid mass at 32° and at t° will be to each other as $1 + .002\ 088 (t^{\circ} - 32^{\circ})$ is to 1.

For Volume, Pressure, and Density of Air, see Heat, page 521.

Altitude of Atmosphere at ordinary density is = a column of mercury 30 ins. in height, divided by specific gravity of air compared with mercury

Hence 30 ins. = 2.5 feet, which, divided by .000 094 987, specific gravity of air compared with mercury, = 26 319 feet = 4.985 miles.

Gay Lussac, Humboldt, and Boussingault estimated it at a minimum 30 miles, Sir John Herschell 83, Bravais 66 to 100, Dalton 280 or 204 miles.

The aqueous vapor always existing in air, in a greater quantity than air, diminishes its weight in mixing with it. At equal temperature, its quantity is greater the higher the temperature.

It is to be considered by increasing the multiplier

is

Bar and Coxwell, in 1862, ascended in a balloon to

At temperature of 32° , mean velocity of sound is 1089 feet per second. It is increased or diminished about one foot for each degree of temperature below or above 32° .

Velocity of sound in water is estimated at 4750 feet per second.

Velocity of Sound at Various Temperatures.

| ° | Per Second. | ° | Per Second. | ° | Per Second. | ° | Per Second. |
|----|-------------|----|-------------|----|-------------|-----|-------------|
| | Feet. | | Feet. | | Feet. | | Feet. |
| 5 | 1056 | 32 | 1089 | 68 | 1122 | 95 | 1152 |
| 14 | 1070 | 50 | 1102 | 77 | 1132 | 104 | 1161 |
| 23 | 1079 | 59 | 1112 | 86 | 1142 | 113 | 1171 |

Motions of air and all gases, by force of gravity, are precisely alike to those of fluids.

Sensation of hearing, or sound, cannot exist in an absolute vacuum. The human voice can be heard a distance of 3300 feet.

Echo.—At a less distance than 100 feet there is not a sufficient interval between the delivery of a sound and its reflection to render one perceptible.

To Compute Distances by Velocity of Sound in Air.

$1089 \times 13 \sqrt{1 + [.002008 (t - 32)]} = v$, distance in feet per second, t representing temperature of air.

ILLUSTRATION.—Flash of a cannon from a vessel was observed 13 seconds before report was heard; temperature of air 60° ; what was distance to vessel?

$1089 \times 13 \sqrt{1 + [.002088 (60^{\circ} - 32)]} = 1089 \times 13 \times 1.029 = 14567.55 \text{ feet} = 2.76 \text{ miles.}$

Theoretical velocity with which air will flow into a vacuum, if wholly unobstructed, is $\sqrt{2gh} = 1347.4 \text{ feet per second.}$ In operation, however, it is $1347.4 \times .707 = 952.61 \text{ feet.}$

To Compute Velocity of Air Flowing into a Vacuum.

$\sqrt{2gh} \times c = v$ in feet per second, c representing coefficient of efflux.

Coefficients for openings are as follows:

Circular aperture in a thin plate..... .65 to .7
Cylindrical adjutage..... .92 | Conical adjutage..... .93

Velocity of Sound in Several Solids.

Velocity in Air = 1.

Lead..... 3.9 | Zinc..... 9.8 | Pine..... 12.5 | Glass..... 11.9 | Steel..... 14.3
Gold..... 5.6 | Oak..... 9.9 | Copper..... 11.2 | Pine..... 12.5 | Iron..... 15.1

To Compute Elevations by a Barometer.

Approximately* $60000 (\log B - \log b) C = \text{height in feet;}$ B and b representing heights of barometer at lower and upper stations, and C correction due to $T + t$ or temperatures of lower and upper stations.

Values of C or $T + t$.

| ° | C | ° | C | ° | C | ° | C | ° | C | ° | C | ° | C |
|----|-------|----|-------|-----|-------|-----|-------|-----|-------|-----|-------|---|---|
| 60 | .996 | 80 | 1.018 | 100 | 1.04 | 120 | 1.062 | 140 | 1.084 | 160 | 1.106 | | |
| 62 | .998 | 82 | 1.02 | 102 | 1.042 | 122 | 1.064 | 142 | 1.087 | 162 | 1.108 | | |
| 64 | | 84 | 1.022 | 104 | 1.044 | 124 | 1.067 | 144 | 1.089 | 164 | 1.111 | | |
| 66 | 1.002 | 86 | 1.024 | 106 | 1.047 | 126 | 1.069 | 146 | 1.091 | 166 | 1.113 | | |
| 68 | 1.004 | 88 | 1.027 | 108 | 1.049 | 128 | 1.071 | 148 | 1.093 | 168 | 1.115 | | |
| 70 | 1.007 | 90 | 1.030 | 110 | 1.051 | 130 | 1.073 | 150 | 1.096 | 170 | 1.117 | | |
| 72 | 1.009 | 92 | | 112 | 1.053 | 132 | 1.076 | 152 | 1.098 | 172 | 1.12 | | |
| 74 | | 94 | | 114 | 1.055 | 134 | 1.078 | 154 | 1.1 | 174 | 1.122 | | |
| 76 | 1.011 | 96 | | 116 | 1.057 | 136 | 1.08 | 156 | 1.102 | 176 | 1.124 | | |
| 78 | | 98 | | 118 | 1.059 | 138 | 1.082 | 158 | 1.104 | 178 | 1.126 | | |

S. Lee, U. S. Top. Eng., 1855.

Their values vary approximately .0011 per degree.

| ILLUSTRATION.—Thermometer | Upper Station. | Lower Station. |
|---------------------------|----------------|----------------|
| Barometer | 70.4 | 77.6 |
| | 23.66 | 30.05 |

$$C = 77.6 + 70.4 = 1.093, \quad \log. B = 1.4778, \quad \log. b = 1.374$$

Then $60000 \times (1.4778 - 1.374) \times 1.093 = 6807.2$ feet.

To Compute Elevations by a Thermometer.

$520 B + B^2 \times C = \text{height in feet}$ B representing temperature of water boiling at elevated station deducted from 212° .

Correction for temperatures of air at lower and upper stations, or $T + t$, to be taken from table, page 428, as before.

ILLUSTRATION.—Temperature of water boiling at upper station 192° ; temperature of air 50° and 32° . $C = 1.02$.

Then $520 \times 212 - 192 + 212 - 192 \times 1.02 = 10808$ feet.

To Compute Capacity of a Balloon, etc., see page 218.

Barometer.

Elevations by Barometer Readings. (Astronomer Royal.)

Mean Temperature of Air 50° .

For correction for temperature, see note at foot.

| Height. | Barom. | Height. | Barom. | Height. | Barom. | Height. | Barom. | Height. | Barom. |
|---------|--------|---------|--------|---------|--------|---------|--------|---------|--------|
| Feet. | Ins. | Feet. | Ins. | Feet. | Ins. | Feet. | Ins. | Feet. | Ins. |
| 0 | 31 | 600 | 30.325 | 1500 | 29.34 | 4000 | 26.769 | 7000 | 23.979 |
| 50 | 30.943 | 650 | 30.269 | 1600 | 29.233 | 4250 | 26.524 | 7500 | 23.543 |
| 100 | 30.886 | 700 | 30.214 | 1750 | 29.072 | 4500 | 26.282 | 8000 | 23.115 |
| 150 | 30.83 | 750 | 30.159 | 1800 | 29.019 | 4750 | 26.042 | 8500 | 22.695 |
| 200 | 30.773 | 800 | 30.103 | 2000 | 28.807 | 5000 | 25.804 | 9000 | 22.282 |
| 250 | 30.717 | 850 | 30.048 | 2250 | 28.544 | 5250 | 25.569 | 9500 | 21.877 |
| 300 | 30.661 | 900 | 29.993 | 2500 | 28.283 | 5500 | 25.335 | 10000 | 21.479 |
| 350 | 30.604 | 1000 | 29.883 | 2750 | 28.025 | 5750 | 25.104 | 10500 | 21.089 |
| 400 | 30.548 | 1100 | 29.774 | 3000 | 27.769 | 6000 | 24.875 | 11000 | 20.706 |
| 450 | 30.492 | 1200 | 29.665 | 3250 | 27.515 | 6250 | 24.648 | 11500 | 20.329 |
| 500 | 30.436 | 1300 | 29.556 | 3500 | 27.264 | 6500 | 24.423 | 12000 | 19.959 |
| 550 | 30.381 | 1400 | 29.448 | 3750 | 27.015 | 6750 | 24.2 | 12500 | 19.582 |

Barometer.

Correction for Capillary Attraction to be added in Inches.

| | | | | | | | | | | |
|---------------------------|------|------|------|------|------|-----|------|-----|------|-----|
| Diameter of tube..... | .6 | .55 | .5 | .45 | .4 | .35 | .3 | .25 | .2 | .1 |
| Correction, unboiled..... | .004 | .005 | .007 | .01 | .014 | .02 | .025 | .04 | .059 | .08 |
| Correction, boiled..... | .002 | .003 | .004 | .005 | .007 | .01 | .014 | .02 | .029 | .04 |

To Compute Height.

RULE.—Subtract reading at lower station from reading at upper station, difference is height in feet.

Table assumes mean temperature of atmosphere to be 50° F. or 10° C. For all temperatures following correction must be applied.

Add together temperatures at upper and lower station. If this sum, in degrees F., is greater than 100° , increase height by $\frac{1}{1000}$ part for every degree of excess above 100° ; if sum is less than 100° , diminish height by $\frac{1}{1000}$ part for every degree of defect from 100° . Or if sum in $^\circ$ is greater than 20° , increase height by $\frac{1}{1000}$ part for every degree of excess above 20° ; if sum is less than 20° , diminish height by $\frac{1}{1000}$ part for every degree of defect from 20° .

Barometer Indications.

Increasing storm.—If mercury falls during a high wind from S. or S. E.

Violent but short.—If fall be rapid.

Less violent but of longer continuance.—If fall be slow.

Snow.—If mercury falls when thermometer is low.

Improved weather.—When a gradual continuous rise of mercury follows.

Heavy gales from N.—Soon after *first* rise of mercury from a very low point.

Unsettled weather.—With a *rapid* rise of mercury.

Settled weather.—With a *slow* rise of mercury.

Very fine weather.—With a continued *steadiness* of mercury with dry air.

Stormy weather with rain (or snow).—With a rapid and considerable fall of mercury.

Threatening, unsettled weather.—With an alternate rising and falling of mercury.

Lightning *only*.—When mercury is low, storm being beyond horizon.

Fine weather.—With a rosy sky at sunset.

Wind and rain.—When sky has a sickly greenish hue.

Rain.—When clouds are of a dark Indian red.

Foul weather or much wind.—When sky is red in morning.

Weather Glasses.

Explanatory Card. Vice-Admiral Fitzroy, F. R. S.

Barometer Rises for Northerly wind (including from N. W. by N. to E.), for dry, or less wet weather, for less wind, or for more than one of these changes—
Except on a few occasions when rain, hail, or snow comes from N. with *strong* wind.

Barometer Falls for Southerly wind (including from S. E. by S. to W.), for wet weather, for stronger wind, or for more than one of these changes—
Except on a few occasions when *moderate* wind with rain (or snow) comes from N.

For change of wind toward Northerly directions, a *Thermometer falls*.

For change of wind toward Southerly directions, a *Thermometer rises*.

Moisture or dampness in air (shown by a Hygrometer) increases *before* rain, fog, or dew.

Add one tenth of an inch to observed height for each hundred feet Barometer is above half-tide level.

Average height of Barometer, in England, at sea-level, is about 29.94 inches; and average temperature of air is nearly 50 degrees (latitude of London).

Thermometer falls about one degree for each 300 feet of elevation from ground, but varies with wind.

“When the wind shifts against the sun,
Trust it not, for back it will run.”

First rise after very low
Indicates a stronger blow.

Long foretold—long last,
Short notice—soon past.

Rarefaction of Air.

In consequence of rarefaction of air, gas loses of its illuminating power: cube inch for each 2.69 feet of elevation above the sea. (M. Bremond.)

Clouds.

Classification.—1. *Cirrus*—Like to a feather, commonly termed *Mare's*

2. *Cirro-cumulus*—Small round clouds, termed *mackerel sky*.

3. *Stratus*—Concave or undulated stratus. 4. *Cumulus*—Conical,

clusters, termed *wool-packs* and *cotton balls*. 5. *Cumulo-stratus*—

r mixed. 6. *Nimbus*—A cumulus spreading out in arms, and

ing rain beneath it. 7. *Stratus*—A level sheet.

Nimbus is most elevated.

—Clouds have been seen at a greater height than 37 000 feet.

—At apparent moderate speed, they attain a velocity of 80

Lightning.

—Developed with great rapidity.

Globular—When the electric
red at a comparatively lower
sh appears to rest upon the

WEATHER INDICATIONS.

| | <i>Clouds.</i> | <i>Sky.</i> |
|------|--|---|
| nd | Soft or delicate-looking and indefinite outlines. | Gray in morning and light, delicate tints and low dawn. |
| . | Hard-edged, oily-looking, and tawny or copper-colored, and the more hard, "greasy," and ragged, the more wind. | High dawn, and sunset of a bright yellow. |
| ily. | Light scud alone. | |
| . | Small and inky. | Sunset of a pale yellow. |
| nd | Light scud driving across heavy masses. | Orange or copper color. |
| nd | Hard defined outlines. | Gaudy unusual hues. |
| . | | |
| of | High upper, cross lower in a direction different to their course or that of wind. | |

General.

—When sea-birds fly early and far out, when dew is deposited, and when a confined in a bottle of water, will curl up at the bottom.

—Clear atmosphere near to horizon and light atmospheric pressure, or a "clearing day," as it is termed.

1.—When sea-birds remain near to shore or fly inland.

Snow, or Wind.—When a leech, confined in a bottle of water, will rise ex to the surface.

ler.—When a leech, confined as above, will be much excited and leave the

3 of Indications of Fair Weather, in Days, Compared to one of Rain.

From an extended series of observations. (Lowe.)

| | | | |
|--------------------------|------|------------------------------|-----|
| Dew..... | 4.5 | Mock Sun or Moon..... | 3.3 |
| stratus in a valley..... | 7.2 | Stars falling abundant..... | 3.2 |
| Clouds at sunset..... | 2.9 | Stars bright..... | 3.4 |
| alo..... | 1.9 | Stars dim..... | 1.5 |
| and rayless..... | 10.3 | Stars scintillated..... | 6 |
| e and sparkling..... | 1 | Aurora borealis..... | 1.8 |
| rost..... | 4.2 | Toads in evening..... | 2.4 |
| falo..... | 1 | Landrails noisy..... | 13 |
| urr, or rough-edged..... | 2.8 | Ducks and Geese noisy..... | 2.3 |
| lm..... | 2 | Fish rising..... | 1.5 |
| sing red..... | 7 | Smoke rising vertically..... | 5 |

weather-foretelling plants, see page 185.

ATMOSPHERIC AIR.

pure air contains Oxygen 20.96, Nitrogen 79, and Carbonic Acid. A man, respired by a human being in one hour is about 15 cube feet of air, and 15 grains of carbonic acid, corresponding to 137 grains of water. At this time about 200 grains of water will be exhaled by a man. During this period there would be consumed about 415 grains of air. In one hour, then, there would be vitiated 73 cube feet of pure air, weighing 150 lbs., requires 930 cube feet of air per hour. The air he breathes may not contain more than 1 per cent of carbonic acid, which proportion its impurity becomes sensible to a man. Therefore, to have 800 cube feet of well ventilated space.

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An adult human being consumes in food from 145 to 165 grains of carbon per hour, and gives off from 12 to 16 cube feet of carbonic acid gas.

An assemblage of 1000 persons will give off in two hours, in vapor, 85 gallons water, and nearly as much carbon as there is in 56 lbs. of bituminous coal.

Proportion of Oxygen and Carbonic Acid at following Locations.

Pure Air represented by Oxygen 20.96.

| | | | |
|----------------------------|--------|---------------------------------------|-------|
| Street in Glasgow..... | 20.895 | Metropolitan Railway (underground) .. | 20.6 |
| Regent Street, London..... | 20.865 | Pit of a Theatre..... | 20.74 |
| Centre Hyde Park..... | 21.005 | Gallery of a Theatre..... | 20.63 |

Carbonic Acid .04 Per cent.

| | | | |
|-------------------------------|-------|---------------------------------------|-------|
| Open field, Manchester..... | .0383 | Top of Monument, London..... | .0398 |
| Churchyard..... | .0323 | Hyde Park..... | .0334 |
| Market, Smithfield..... | .0446 | Metropolitan Railway (underground) .. | .0338 |
| Factory mills..... | .283 | Lake of Geneva..... | .046 |
| School-rooms..... | .097 | Boys' school..... | .31* |
| Pitt of theatre, 11 P. M..... | .32 | Girls' "..... | .723† |
| Boxes " 12 "..... | .218 | Horse stable..... | .7 |
| Gallery " 10 "..... | .101 | Convict prison..... | .045 |

* Roscoe.

† Peltenhoffer.

Consumption of Atmospheric Air. (Coathupe.)

One wax candle (three in a lb.) destroys, during its combustion, as much oxygen per hour as respiration of one adult.

A lighted taper, when confined within a given volume of atmospheric air, will become extinguished as soon as it has converted 3 per cent. of given volume of air into carbonic acid.

Carbonic Acid Exhaled per Minute by a Man. (Dr. Smith.)

During sleep 4.99 per cent., lying down 5.91, walking at rate of 2 miles per hour 18.1, at 3 miles 25.83, hard labor 44.97.

ANIMAL POWER.

Work.

Work is measured by product of the resistance and distance through which its point of application is moved. In performance of work by means of mechanism, work done upon weight is equal to work done by it.

Unit of Work is the moment or effect of 1 pound through a distance of 1 foot, and it is termed a foot-pound.

Force a kilogrammetre is the expression, or the pressure of a unit through a distance of 1 meter = 7.233 foot-pounds.

Observation upon animal power furnishes the following as maximum

to .33 of that which could be produced

by a man, and from .08 to .066 for a horse for a brief interval, and not invol-

untained without prejudice to health beyond 18 hours per day, however interrupted a constant attendance.

Men.

Mean effect of power of men working to best practicable advantage, is raising of 70 lbs. 1 foot high in a second, for 10 hours per day = 4200 foot-pounds per minute.

Windlass.—Two men, working at a windlass at right angles to each other, can raise 70 lbs. more easily than one man can 30 lbs.

Labor.—A man of ordinary strength can exert a force of 30 lbs. for 10 hours in a day, with a velocity of 2.5 feet in a second = 4500 lbs. raised one foot in a minute = .2 of work of a horse.

A man can travel, without a load, on level ground, during 8.5 hours a day, at a rate of 3.7 miles an hour, or 31.45 miles a day. He can carry 111 lbs. 1 mile in a day. Daily allowance of water, 1 gallon for all purposes; and he requires from 220 to 240 cube feet of fresh air per hour.

A porter going short distances, and returning unloaded, can carry 135 lbs. 1 mile a day, or he can transport, in a wheelbarrow, 150 lbs. 10 miles in a day.

Crane.—The maximum power of a man at a crane, as determined by Mr. Field, for constant operation, is 15 lbs., exclusive of frictional resistance, which, at a velocity of 220 feet per minute = 3300 foot-pounds, and when exerted for a period of 2.5 minutes was 17,329 foot-pounds per minute.

Pile-driving.—G. B. Bruce states that, in average work at a pile-driver, a laborer, for 10 hours, exerts a force of 16 lbs., plus resistance of gearing, and at a velocity of 270 feet per minute, making one blow every four minutes.

Rowing.—A man rowing a boat 1 mile in 7 minutes, performs the labor of 6 fully-worked laborers at ordinary occupations of 10 hours per day.

Drawing or Pushing.—A man drawing a boat in a canal can transport 10 000 lbs. for a distance of 7 miles, and produce 156 times the effect of a man weighing 154 lbs., and walking 31.25 miles in a day; and he can push on a horizontal plane 20 lbs. with a velocity of 2 feet per second for 10 hours per day.

Tread-mill.—A man either inside or outside of a tread-mill can raise 30 lbs. at a velocity of 1.3 feet per second for 10 hours, = 1 404 000 foot-pounds.

Pulley.—A man can raise by a single pulley 36 lbs., with a velocity of .8 foot per second, for 10 hours.

Walking.—A man can pass over 12.5 times the space horizontally that he can vertically, and, according to J. Robison, by walking in alternate directions upon a platform supported on a fulcrum in its centre, he can, weighing 167 lbs., produce an effect of 3 984 000 foot-pounds. for 10 hours per day.

Pump, Crank, Bell, and Rowing.—Mr. Buchanan ascertained that, in working a pump, turning a crank, ringing a bell, and rowing a boat, the effect of power of a man is as the numbers 100, 167, 227, and 248.

Pumping.—A practised laborer can raise, during 10 hours, 1 lb. water 1 foot in height, with a properly designed and constructed pump.

Crank.—A man can exert on the handle of a screw-jack a force of 25 lbs., and continuously of 20 lbs. Mr. J. Field's tests gave 11.5 lbs. as easily, and 27.6 with great difficulty.

Mowing.—A man can mow an acre of grass in 1 day.

Reaping.—A man can reap an acre of wheat in 2 days.

Ploughing.—A man and horse .8 of an acre per day.

Day's Work. (*D. K. Clark.*)

Laborer.—Carrying bricks or tiles, net load 106 lbs. = 600 lbs. 1 mile.

Carrying coal in a mine, net load 95 to 115 lbs. = 342 lbs. 1 mile.

Loading coke into a wagon, net load 100 lbs. = 270 lbs. 1 mile.

Loading a boat with coal, net load 190 lbs. = 1230 lbs. 1 mile, or 20 cube yards of earth in a wagon.

Digging stubble land .055 of an acre per day, or 2000 cube feet of superficial earth.

Breaking 1.5 cube yards hard stone into 2 inch cubes.

Quarrying.—A man can quarry from 5 to 8 tons of rock per day.

A foot-soldier travels in 1 minute, in common time, 90 steps = 70 yards.

He occupies in ranks a front of 20 inches, and a depth of 13, without a knapsack; interval between the ranks is 13 inches.

Average weight of men, 150 lbs. each, and five men can stand in a space of 1 square yard.

Effective Power of Men for a Short Period.

| Manner of Application. | Force. | Manner of Application. | Force. |
|-----------------------------|--------|-----------------------------|--------|
| | Lbs. | | Lbs. |
| Bench-vice or Chisel..... | 72 | Screw-driver, one-hand..... | 84 |
| Drawing-knife or Auger..... | 100 | Small screw-driver..... | 14 |
| Hand-plane..... | 50 | Thumb and fingers..... | 14 |
| Hand-saw..... | 36 | Windlass or Pincers..... | 60 |

The muscles of the human jaw exert a force of 534 lbs.

Mr. Smeaton estimated power of an ordinary laborer at ordinary work was equivalent to 3762 foot-pounds per minute. But, according to a particular case made by him in the pumping of water 4 feet high, by good English laborers, their power was equivalent to 3904 foot-pounds per minute; and this he assigned as twice that of ordinary persons promiscuously operated with.

Mr. J. Walker deduced from experiments that the power of an ordinary laborer, in turning a crank, was 13 lbs., at a velocity of 320 feet per minute for 8 hours per day.

Amount of Labor produced by a Man. (*Morin.*)

For 10 hours per day.

| MANNER OF APPLICATION. | Power. | Velocity per Second. | Weight raised, Feet per Minute. | HP for Period given. |
|--|--------|----------------------|---------------------------------|----------------------|
| | Lbs. | Feet. | Lbs. | No. |
| Throwing earth with a shovel, a height of 5 feet.. | 6 | 1.33 | 480 | 87 |
| Wheeling a loaded barrow up an inclined plane, 1 to 12..... | 132 | .625 | 4950 | 90 |
| Raising and pitching earth in a shovel 13 feet horizontally..... | 6 | 2.25 | 810 | 147 |
| Pushing and drawing alternately in a vertical direction..... | 13 | 2.5 | 1950 | 355 |
| Transporting weight upon a barrow, and returning unloaded..... | 132 | 1 | 7920 | 144 |
| FOR 8 HOURS PER DAY. | | | | |
| Raising a slight elevation, unloaded..... | 143 | .5 | 4290 | 62 |
| Pushing and drawing in a horizontal direction..... | 26 | 2 | 3120 | 45.8 |
| Turning a crank..... | 18 | 2.5 | 2790 | 33 |
| Turning a mill..... | 140 | .5 | 4200 | 61.1 |
| | 26 | 5 | 7800 | 113 |
| FOR 10 HOURS PER DAY. | | | | |
| Turning a crank..... | 88 | 2.5 | 13200 | 160.5 |
| Turning a mill..... | 140 | 1.75 | 14700 | 160.5 |
| Turning a crank..... | 140 | .2 | 1680 | 27 |
| Turning a mill..... | 44 | .5 | 1320 | 19 |

all, in an individual case, at 140 lbs., at 1 foot per second; hence 70 + 12

Compute Number of Men to Perform Work upon a Tread-mill or Pile-driver.

E.—To product of weight to be raised and radius of crank, add friction wheel, and divide sum by product of power and radius of wheel.

EXAMPLE.—How many men are required upon a tread-mill, 20 feet in diameter, a weight of 9233.33 lbs., crank 9 inches in length, weight of wheel and its rim estimated at 5000 lbs., and friction at .015.

Weight of a man assumed at 25 lbs. Radius of crank .75 feet.

Weight of a man on a tread-mill, page 433, 30 lbs. at a velocity of 1.3 feet per second, $< 60 = 78$ feet per minute.

$33 \times .75 + 5000 \times .015 = 7000$ lbs. resistance of load and wheel, and $7000 \div 1416 \times 10 \times 30 = 7000 =$ load and weight \div product of power increased by its over load, radius of wheel and power $= 7000 \div 1.241 \times 10 \times 30 = 18.8$ men.

Horse.

Amount of Labor produced by a Horse under different Circumstances. (Morin.)

For 10 hours per day.

| MANNER OF APPLICATION. | Power. | Velocity per Second. | Weight drawn. Feet per Minute. | HP for Period given. |
|--|--------|----------------------|--------------------------------|----------------------|
| | Lbs. | Feet. | Lbs. | No. |
| ing a 4-wheeled carriage at a walk | 154 | 3 | 27 720 | 504 |
| oad upon his back at a walk | 264 | 3.75 | 59 400 | 1080 |
| working a loaded wagon, and returning unloaded at a walk | 1540 | 2 | 184 800 | 3360 |
| ing a loaded wagon at a walk | 1540 | 3.75 | 346 500 | 6300 |
| FOR 8 HOURS PER DAY. | | | | |
| a revolving platform at a walk | 100 | 3 | 18 000 | 260.8 |
| FOR 4.5 HOURS PER DAY. | | | | |
| a revolving platform at a trot | 66 | 6.75 | 26 730 | 218.7 |
| ing an unloaded 4-wheeled carriage at a trot. | 97 | 7.25 | 43 195 | 353.5 |
| ing a loaded 4-wheeled carriage at a trot | 770 | 7.25 | 334 950 | 2742 |

Reaction power of a horse, when continuously at a walk, is equal to 120 lbs., made of road 1 in 30, resistance on a level being one thirtieth of load, he can a load of $120 \times 30 \div 2 = 1500$ lbs.

Street Rails or Tramways. (Henry Hughes.)

15, 26 lbs. per ton, or 1 to 86 as a mean.

Performance of Horses in France. (M. Charité-Marsailles.)

| SEASON. | Road. | Weight per Horse. | Speed per Hour. | Work per Hour, drawn One Mile. | Ratio of Pavement + Macadam |
|---------|-----------------------|-------------------|-----------------|--------------------------------|-----------------------------|
| | | Tons. | Miles. | Ton-miles. | |
| W..... | { Pavement Macadam | 1.306 .851 | 2.05 1.91 | 2.677 1.625 | 1.644 to |
| S..... | { Pavement Macadam | 1.395 1.141 | 2.17 2.16 | 3.027 2.464 | " |

average daily work of a Flemish horse in North of France, where roads heavy, is, on same authority, as follows:

Winter, 21.82 ton-miles per day. } Mean for the year,
Summer, 27.82 " " }

Example = 53.8 lbs., from which a deduction is to be made for excess of weight returned in 8 hours over 10. Or, as 10 : 8 :: 53.8 : 43.04 lbs., which does not differ 50 lbs. for that of an average performance.

Greatest mechanical effect of an ordinary horse is produced in operating gin or drawing a load on a railroad, when travelling at rate of 2.5 miles per hour, where he can exert a tractive force of 150 lbs. for 8 hours per day.

Horse upon Turnpike Road.

At a speed of 10 miles per hour, a horse will perform 13 miles per day for 3 years. In ordinary staging, a horse will perform 15 miles per day.

To Compute Tractive Power of a Horse Team, see Traction, page 848.

Assuming maximum load that a horse can draw on a gravel road as standard, he can draw,

| | |
|------------------------------------|---------------|
| On best-broken stone road..... | 2 to 3 times. |
| On a well-made stone pavement..... | 3 to 5 " |
| On a stone trackway..... | 7 to 8 " |
| On plank road..... | 4 to 12 " |
| On a railway..... | 18 to 20 " |

NOTE.—Track of an iron railway compared with a plank-road is as 27 to 10.

To Compute Power of Draught of a Horse at Different Elevations.



Let ABC represent an inclined plane, o weight of a horse which, being resolved into two component forces, one of which, n , is perpendicular to plane of inclination, and other, r , is parallel to it. Hence, r represents force which horse must overcome to move his own weight.

Then, by similar triangles, $A B$ or $l : B C$ or $h :: o : r$. Or, $\frac{h o}{l} = r$.

If t represents tractive power of horse, upon a level, of 100 lbs., t tractive power upon a plane of inclination, and r that part of force exerted by horse which is expended upon his own body, then $t = t - r$, or $t - \frac{h o}{l} = t$ in lbs.

ILLUSTRATION.—If inclination is 1 in 50.

Assume $t = 100$, weight of horse 900 lbs., and $l = 50.01$.

$$\text{Then, } 100 - \frac{1 \times 900}{50.01} = 100 - 17.99 = 82.01 \text{ lbs.}$$

Assuming load that a horse can draw on a level at 100, he can draw upon inclinations as follows:

| | | | | |
|------------------|-----------------|-----------------|-----------------|-----------------|
| 1 in 100..... 91 | 1 in 75..... 88 | 1 in 50..... 82 | 1 in 35..... 74 | 1 in 20..... 61 |
| 1 " 90..... 90 | 1 " 70..... 87 | 1 " 45..... 80 | 1 " 30..... 70 | 1 " 15..... 54 |
| 1 " 80..... 89 | 1 " 60..... 85 | 1 " 40..... 77 | 1 " 25..... 64 | 1 " 10..... 48 |

On his back a horse can carry from 220 to 390 lbs., or about 27.5 per cent of his weight.

Labor.—The work of a horse as assigned by Boulton & Watt, Tredgold, Rennie, Beardmore, and others, ranges from 20 000 to 39 320 foot-pounds per minute for 8 hours, a mean of 27 750 lbs.

A horse can travel, at a walk, 400 yards in 4.5 minutes; at a trot, 1000 yards in 10 minutes; and at a gallop, in 1 minute. He occupies in ranks, a front of 10 ins., and a depth of 10 feet; in a stall, from 3.5 to 4.5 feet front; and at a picket, 3 feet by 9; and his average weight = 1000 lbs.

Carrying a soldier and his equipments (225 lbs.) he can travel 25 miles in a day of 8 hours.

A draught-horse can draw 1600 lbs. 23 miles a day, weight of carriage included.

ry work of a horse may be stated at 22 500 lbs., raised 1 foot in a or 8 hours per day.

ll, he moves at rate of 3 feet in a second. Diameter of track should not in 25 feet.

ascertained that a horse weighing 1232 lbs. could draw a canal-boat d of 2.5 miles per hour, with a power of 108 lbs., 20 miles per day. quivalent to a work of 23 760 foot-lbs. per minute. He estimated average work of horses, strong and weak, is at the rate of 22 000 er minute.

esults of trials upon strength and endurance of horses at Bedford, Eng., it mined that average work of a horse = 20 000 foot-lbs. per minute. A good draw 1 ton at rate of 2.5 miles per hour, from 10 to 12 hours per day.

of conveying goods at 3 miles per hour, per horse teams being 1, expense es will be 1.33, and so on, expense being doubled when speed is 5.125 miles

1 of a horse is equivalent to that of 5 men, and his daily allowance of uld be 4 gallons.

at of Labor a Horse of average Strength is capa- of performing, at different Velocities, on Canal, road, and Turnpike.

Traction estimated at 83.3 lbs.

| Trac- ion of ork. | Useful Effect, drawn 1 Mile. | | | Veloci- ty per Hour. | Dura- tion of Work. | Useful Effect, drawn 1 Mile. | | |
|-------------------------|------------------------------|---------------------|---------------------|----------------------------|---------------------------|------------------------------|---------------------|---------------------|
| | On a Canal. | On a Rail- road. | On a Turn- pike. | | | On a Canal. | On a Rail- road. | On a Turn- pike. |
| ura. | Tons. | Tons. | Tons. | Miles. | Hours. | Tons. | Tons. | Tons. |
| 1.5 | 520 | 115 | 14 | 6 | 2 | 30 | 48 | 6 |
| 3 | 243 | 92 | 12 | 7 | 1.5 | 19 | 41 | 5.1 |
| 4.5 | 102 | 72 | 9 | 8 | 1.125 | 12.8 | 36 | 4.5 |
| 2.9 | 52 | 57 | 7.2 | 10 | .75 | 6.6 | 28.8 | 3.6 |

abor performed by horses is greater, but they are injured by it.

: Power of a horse decreases as his speed is increased, and within limits ed, or up to 4 miles per hour, it decreases nearly in an inverse ratio.

For 10 Hours per Day.

| Traction. | Miles. | Traction. | Miles. | Traction. | Miles. | Traction. |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Lbs. | Per Hour. | Lbs. | Per Hour. | Lbs. | Per Hour. | Lbs. |
| 330 | 1.5 | 165 | 2.25 | 110 | 3 | 82 |
| 250 | 1.75 | 140 | 2.5 | 100 | 3.5 | 70 |
| 200 | 2 | 125 | 2.75 | 90 | 4 | 62 |

For Ordinary or Short Periods. (Molesworth.)

| | | | | | | |
|---------------|-----|-----|-----|----|-----|----|
| per hour..... | 2 | 3 | 3.5 | 4 | 4.5 | 5 |
| r in lbs..... | 166 | 125 | 104 | 83 | 62 | 41 |

Mule. (*D. K. Clark.*)

m back, 170 to 220 lbs. day's work = 6400 lbs. 1 mile; 400 lbs. at 2.9 : hour = 5300 lbs. 1 mile, and 330 lbs. at 2 miles per hour = 5000 lbs.

revolving platform, at a velocity of 3 feet per second, = 11 880 lbs. raised er minute, or 172.2 HP for 8 hours per day

Ass.

m back, 176 lbs. carried 19 miles day's work = 3300 lbs. 1 mile.

a an ass carries 450 to 550 lbs. grain.

revolving platform, at a velocity of 2.75 feet per second, = 5280 lbs. raised r minute, or 76.5 HP for 8 hours per day.

O O*

Ox.

An Ox, walking at a velocity of 2 feet in a second (1.36 miles per hour), exerts a power of 154 lbs., = 18480 lbs. raised one foot per minute, or 268.8 HP for 8 hours per day.

A pair of well-conditioned bullocks in India have performed work = 8000 foot-lb. per minute.

Camel.

Load on back, 550 lbs. carried 30 miles per day for 4 days, 4 days' work 16500 lbs. 1 mile, for 5 days 13000 lbs. 1 mile = 44 HP for 10 hours per day.

Load of a Dromedary, 770 lbs.

Llama.

Load on back, 110 lbs., day's work 2000 to 3000 lbs. 1 mile = .5 to .75 HP for 10 hours per day.

Birds and Insects.

Area of their wing surface is in an inverse ratio to their weight.

Assuming weight of each of the following Birds to be one pound, and each insect one ounce, the relative area of their wing surface proportionate to that of their actual weight would be as follows (*M. De Lucy*):

| | Sq. ft. | | Sq. ft. | | Sq. ft. | | Sq. ft. |
|---------------|---------|-------------------|---------|--------------------|---------|----------------|---------|
| Swallow | 4.85 | Pigeon..... | 1.27 | Gnat..... | 3.05 | Cockchafer.... | 3 |
| Sparrow | 2.7 | Vulture..... | .82 | Dragon-fly, sm'll, | 1.83 | Lee..... | 33 |
| Turtle-dove.. | 2.13 | Crane, Australia, | .41 | Lady-bird..... | 1.66 | Meat-fly..... | 35 |

Crocodile and Dog.

The direct power of their jaws is estimated at 120 lbs. for the former and 44 for the latter, which, with the leverage, will give respectively 6000 and 1500 lbs.

PERFORMANCES OF MEN, HORSES, ETC.

Following are designed to furnish an authentic summary of the fastest or most successful recorded performances in each of the feats, etc., given.

MAN. Walking.

1874, *Wm. Perkins*, London, Eng., .5 mile, in 2 min. 56 sec.; 1, in 6 min. 23 sec.; 1877, 20, in 2 hours 39 min. 57 sec.

1881, *C. A. Harriman*, Chicago, Ill., 530 miles, in 5 days 20 hours 47 min.

1878, *W. Howes*, London, Eng., 50 miles, in 7 hours 57 min. 44 sec.; 1880, 57 miles, in 13 hours 7 min. 27 sec., and 100, in 18 hours 8 min. 15 sec.

1801, *Capt. R. Barclay*, Eng., country road, 90 miles, in 20 hours 22 min. 4 sec., including rests; 1803, .25 mile, in 56 sec., and Charing Cross to Newmarket, 64, in 10 hours, including rests; 1806, 100, in 19 hours, including 1 hour 30 min. in rests, 1800, 1000, in 1000 consecutive hours, walking a mile only at commencement of each hour.

1877, *D. O'Leary*, London, Eng., 200 miles, in 45 hours 21 min. 33 sec.

1818, *Jos. Eaton*, Stowmarket, Eng., 4032 quarter miles, in 4032 consecutive quarter hours.

1877, *Wm. Gale*, London, Eng., 1500 miles, in 1000 consecutive hours, 1.5 miles 1 hour; and 4000 quarter miles, in 4000 consecutive periods of 10 minutes.

- *Chas. Rowell*, New York, N. Y., and running, 89 miles 1640 yards, in 12 hours.

Geo. Hazael, New York, N. Y., and running, 600 miles 220 yards, in 6 days.

J. W. Raby, London, Eng., 2 miles in 13 min. 14 sec.; 3, in 20 min. 21.5 sec.; min. 28 sec.; 5, in 35 min. 10 sec.; and 10, in 1 hour 14 min. 45 sec.

New York, N. Y., 8 miles in 58 min. 37 sec.

London, Eng., 10 miles in 51 min. 6.6 sec.

tingham, Eng., 40 miles in 4 hours 46 min. 54 sec.

oklyn, N. Y., 50 miles in 7 hours 29 min. 47 sec.

Running.

S. S., Manchester, Eng., flying start, 100 yards, in 5.25 sec.

ster, Eng., 600 yards, in 1 min. 13 sec.

rk, N. Y., 1000 yards, in 2 min. 13 sec.

ket, Eng., 1 mile, in 4 min. 2 sec., descending ground; sec.; 1865, 11 miles 1660 yards, in 1 hour 2 min. 21.

- 1852, *Wm. Howitt*, "American Deer," London, Eng., 15 miles in 1 hour 22 min.
 1863, *L. Bennett*, "Deerfoot," Hackney Wick, Eng., 12 m., in 1 hour 2 min. 2.5 sec.
 1879, *Patrick Byrnes*, Halifax, N. S., 20 miles, in 1 hour 54 min.
 1880, *D. Donovan*, Providence, R. I., 40 miles, in 4 hours 48 min. 22 sec.
 17—, *A. Courier*, East Indies, 102 miles, in 24 hours.
 1889, *H. M. Johnson*, Denver, Col., 50 yards, in 5 sec.
 1884, *M. K. Kittleman*, Oakland, Cal., 150 yards (twice), in 14 min. 6 sec.
 1890, *James Grant*, Cambridge, Mass., 5 miles in 25 min. 22.25 sec.

Jumping, Leaping, etc.

- 1854, *J. Howard*, Chester, Eng., 1 jump, board raised 4 ins. in front, running start, with dumb-bells, 5 lbs., 29 feet 7 ins.
 1868, *Geo. M. Kelley*, Corinth, Mass., running, and from a spring board, leaped over 17 horses standing side by side.
 1879, *G. W. Hamilton*, Romeo, Mich., dumb-bells, 22 lbs., standing jump, 14 feet 5 ins.
 1886, *J. Purcell*, Dublin, running long jump, 23 feet 11.5 ins.
 1889, *J. Darby*, Ashton-under-Lyne, Eng., two standing jumps, with weights, 26 feet 8.5 ins.
H. M. Johnson, St. Louis, Mo., without weights, 22 feet 6.75 ins. 10 standing long jumps, without weights, 114 feet 8.5 ins.
J. F. Kearny, Walpole, Mass., 3 standing long jumps, with weights, 42 feet 3 ins.; without weights, at Boston, Mass., 35 feet 6 ins. Boston, Mass., running high jump, with weights, 6 feet 5.25 ins.; backward jump, with weights, heel to toe, 12 feet 1.25 ins. Oak Island, Mass., standing high leap, with weights, 5 feet 9.5 ins.

Lifting.

- 1825, *Thomas Gardner*, of New Brunswick, N. S., a barrel of pork, 320 lbs., under each arm; also transported across a pier an anchor, 1200 lbs.
 1868, *Wm. B. Curtis*, New York, N. Y., 3239 lbs., in harness.
 1883, *D. L. Dowd*, Springfield, Mass., by hands, 1442.25 lbs.

Throwing Weights.

- 1870, *D. Dinnie*, New York, N. Y., light stone, 18 lbs., 43 feet; heavy stone, 24 lbs., 4 feet 6 ins.; heavy hammer, 24 lbs., 83 feet 8 ins.; 1872, Aberdeen, Scotland, light hammer, 138 feet; run, 16 lbs., 162 feet.
 1887, *Peter Foley*, Milwaukee, Wis., 56 lbs., without follow, 31 feet 5 ins.

Swimming.

- 1835, *S. Bruck*, 15 miles, in rough sea, in 7 hours 30 min.
 1846, *A. Native*, off Sandwich Islands, 7 miles at sea, with a live pig under one arm.
 1870, *Pauline Rohn*, Milwaukee, Wis., 650 feet, still water, in 2 min. 43 sec.
 1872, *J. B. Johnson*, London, Eng., remained under water 3 min. 35 sec.
 1875, *Capt. M. Webb*, Dover, Eng., to Calais, France, 23 miles, crossing two full and two half tides = 35 miles, in 21 hours 45 min. 1880, Afloat 60 hours.
 1886, *J. Haggerty*, Blackburn Baths, Eng., 100 yards, 4 turns, in 1 min. 5.5 sec.
 1890, *J. Nuttall*, London, Eng., 1000 yards, 23 turns, in 13 min. 54.5 sec.
 1885, *J. J. Collier*, London, Eng., 1 mile in 26 min. 52 sec.

Skating.

- 1877, *John Ennis*, Chicago, Ill., 9 laps to a mile, 100 miles, in 11 hours 37 min. 45 sec.; and 145 inside of 19 hours.
 1887, *T. Donoghue, Jun.*, Newburgh, N. Y., 1 mile, with wind, in 2 min. 12.375 sec.
 1882, *S. J. Montgomery*, New York, N. Y., 50 miles, in 4 hours 14 min. 36 sec.
 NOTE.—The *Sporting Magazine*, London, vol. ix., page 135, reports a man in 1767 to have skated a mile upon the Serpentine, Hyde Park, London, in 57 seconds.

HORSE. Trotting.

- 1878, "Controller," San Francisco, Cal., 10 miles, harness, in 27 min. 27.25 sec., and 20 miles, wagon, in 58 min. 57 sec.
 1875, "Steel Grey," Yorkshire, Eng., 10 miles, saddle, in 27 min. 56.5 sec.
 1867, "John Stewart," Boston, Mass., half-mile track, 20 miles, harness, in 5.75 sec., and 20.5 miles in 59 min. 31 sec.
 1830, "Top Gallant," Philadelphia, Penn., 12 miles, harness, in 38 min.
 1829, "Tom Thumb," Sunbury Common, Eng., 16.5 miles, harness, 248 min. 45 sec.; and 100 miles, in 10 hours 7 min., including 37 min. in rest.
 1869, "Morning Star," Doncaster, Eng., 18 miles, harness (sulky 100 min. 27 sec.).
 1835, "Black Joke," Providence, R. I., 50 miles, saddle, 175 lbs., in 3 hours

1855, "Spangle," Long Island, N. Y., 50 miles, wagon and driver 400 l hours 59 min. 4 sec.

1837, "Mischief," Jersey City, N. J., to Philadelphia, Penn., 84.25 miles, very hot day and sandy road, in 8 hours 30 min.

1853, "Conqueror," Long Island, N. Y., 100 miles, harness, in 8 hours 5 sec., including 15 short rests.

1873, *M. Delaney's* mare, St. Paul's, Minn., 200 miles, race track, harne hours 20 min., including 15 hours 49 min. in rests.

1834, "Master Burke" and "Robin," Long Island, N. Y., 100 miles, wag hours, 17 min. 22 sec., including 28 min. 34 sec. in rests.

Stage-coaching.

1750, *By the Duke of Queensberry*, Newmarket, Eng., 19 miles, in 53 min.

1830, *London to Birmingham, Eng.*, "Tally-ho," 109 miles, in 7 hours including stop for breakfast of passengers.

Leaping.*

1821, A horse of Mr. Mane, at Loughborough, Leicestershire, Eng., 173 lb hedge 6 feet in height, 35 feet.

1821, A horse of Lieut. Green, Third Dragoon Guards, at Inchinnan, Eng by a heavy dragoon, over a wall 6 feet in height and 1 foot in width at top.

1839, "Lottery," Liverpool, Eng., over a wall, 33 feet.

1847, "Chandler," Warwick, Eng., over water, 37 feet.

NOTE.—The maximum stride of a horse is estimated to be 28 feet 9 ins.; "Eclipse" has feet. The maximum stride of an elk is 34 feet, and of an elephant 14 feet.

Running.

1701, *Mr. Sinclair*, on the Swift at Carlisle, a gelding, 1000 miles, in 1000 tive hours.

1731, *Geo. Osbaldeston*, Newmarket, 156 lbs., 100 miles, by 16 horses, in 4 min. 40 sec., and 200, by 28 horses, in 8 hours 39 min., including 1 hour 2 mi in rests; 1 horse, "Tranby," 16 miles, in 33 min. 15 sec.

1752, *Spedding's* mare, 100 miles, in 12 hours 30 min., for 2 consecutive c

1754, A Galloway mare of Daniel Corker's, Newmarket, 300 miles, by 0 67 lbs., in 64 hours 20 min.

1761, *John Woodcock*, Newmarket, 100 miles per day, by 14 horses, one c for 29 consecutive days.

1814, *An Officer of 14th Dragoons*, Blackwater, 12 miles. 1 horse, in 25 mi

1868, *N. H. Mowry*, San Francisco, Cal., race track, 160 lbs., 300 miles, by (Mexican), in 14 hours 9 min., including 40 minutes for rests; the first hours 2 min. 48 sec., and the fastest mile in 2 min. 8 sec.

1869, *Nell Coher*, San Pedro, Texas, 61 miles, in 2 hours 55 min. 15 sec., : rests.

1870, *John Faylor*, Carson City, Nevada, 50 miles, by 18 horses, in 1 hou: 33 sec.; and Omaha, Neb., 56 miles, in 2 hours 26 min., including rests.

1876, *John Murphy*, New York, N. Y., 155 miles, by 20 horses, in 6 hour: 7 sec.

1878, *Capt. Salvi*, Bergamo to Naples, Italy, 580 miles, in 10 days.

1880, "Mr. Brown," Rancocas, N. J., aged, 160 lbs., 10 miles, in 26 min. 1

1828, "Chapeau de Paille" (Arabian), India, 1.5 miles, 115 lbs., in 2 min.

183-, *Capt. Horne* (Arabians), Madras to Bungalow, India, 200 miles, in 10 hours.

DOGS. Coursing and Chasing.

Greyhound and Hare ran 12 miles in 30 min.

Fox, at Brende, Eng., ran 50 miles in 6.5 hours.

und, at Bushy Park, Eng., leaped over a brook 30 feet 6 ins.

BIRDS. Flying.

loose and Swallow, 90 miles; Crow, 25 miles]

h to Cologne, Germany, 600 miles, in 8 hours.

oe Lake to Paterson, N. J., 3 miles, in 3 min.

HORSE-POWER.

Horse-power.—HP is the principal measure of rate at which work is performed. One horse-power is computed to be equivalent to raising of 33 000 lbs. one foot high per minute, or 550 lbs. per second. Or, 33 000 foot-lbs. of work, and it is designated as being Nominal, Indicated, or Actual.

A HP in work is estimated at 33 000 lbs., raised 1 foot in a minute; but as a horse can exert that force for only 6 hours per day, one work HP is equivalent to that of .5 horses, at a rate of 3 miles per hour.

Cheval-vapeur of France is computed to be equivalent to 75 kilogram-meters of work per second, or 7.233 foot-lbs., or $75 \times 7.233 = 542.5$ foot-lbs., which is 1.37 per cent. less than American or English value.

BELTS AND BELTING.

Capacity of belts to transmit power is determined by extent of their adhesion to surface of pulley, and it is very limited in comparison with tensile strength of belt.

Resistance of a belt to slipping depends essentially upon character of surface of pulley, its degree of tension, and width, and as adhesion is in proportion to pressure on surface of pulley, long belts, by having greater weight, give greater adhesion.

Tensile strength of Belting per square inch of section ranges as follows:

Tanned Leather, .186 inch thick, from 2846 to 5000 lbs., or from 530 to 330 lbs. per inch of width; when *spliced* 385 lbs., and when *laced* 210 lbs.

Taking .3 as a factor of safety, 70 and 128 lbs. represent resistance per sq. inch that belts in operation may be subjected to, and they have been run successfully at these tensions.

Raw hide has a tensile strength of 1.5 times that of tanned.

By Experiments of H. R. Towne and Mr. Kirkaldy. (England.)

Tensile strength of Single leather belting per square inch of section.

Laced, 960 lbs. Riveted, 1740 lbs. Solid, 3080 lbs.

Norris & Co.—Double, 2 ins., 2942 lbs.; 6 ins., 5603 lbs.; 12 ins., 14 861 lbs. Single, 3.5 ins., 3007 lbs.; 5 ins., 4060 lbs.; 10 ins., 8846 lbs.

Spill's belting, from flax, saturated with an endurable substance, gave tensile strength per inch of width as follows:

No. 1, 5 ins. wide, 1254 lbs. No. 2, 5 ins. wide, 1489 lbs. No. 3, 10 ins. wide, 1663 lbs.

At a velocity of 1000 feet per minute, a width of leather belt of 1 inch will transmit power of 1 horse, and at a velocity of 1800 feet, .56 of an inch will transmit a like power, pulley being fully three feet in diameter, equal to a stress of 33 lbs. per inch of width of belt of ordinary thickness.

To Compute Width of a Leather Belt.

Assuming a well-defined case (where limit of adhesion was ascertained), a belt of ordinary construction (laced), and 9 inches in width, transmitted the power of 15 horses over a pulley 4 feet in diameter, at a velocity of 1800 feet per minute, with an arc of adhesion of 210° , or of .6 or 7.54 feet of circumference, and with an area of 95 square feet of belt per HP.

Hence, $\frac{4400 \text{ to } 5000 \text{ HP}}{d \cdot v} = w$; w representing width of belt in feet, and v velocity of belt in feet per minute.

NOTE.—Thickness of belt should be added to diameter of pulley in the formulas of 13 different authors, the result varies, mean of which is 10.675. For double belting width = .6 w .

ILLUSTRATION.—If HP 25, diameter of pulley 4 feet, and velocity 2250 feet; v should be width of belt?

$$\frac{4500 \times 25}{4 \times 2250} = 12.5 \text{ ins. for ordinary thickness of .1875 in.}$$

To Compute Elements of Belting.

$$\frac{v w}{1000} = HP; \quad \frac{HP \ 33000}{v w} = P; \quad \frac{33000 \ HP}{v} = W; \quad \frac{W}{l} = S; \quad \frac{A - a}{l t}$$

P representing power or stress transmitted, W weight or stress on t thickness of belt, S stress on belt per inch of width, A and a areas of end eye, and l length in feet.

NOTE.—70 square feet of good belting are capable of transmitting an indicate

India Rubber Belting. (Vulcanized.)

Results of Experiments upon Adhesion of India Rubber and Leather Belting
(J. H. Cheever).

| Rubber. | | | | Leather. | | | |
|---------------------------------------|--|------|----|-------------------------------------|--|--|--|
| Rubber belt slipped on iron pulley at | | Lbs. | 90 | Leather belt slipped on iron pulley | | | |
| " " leather " " | | 128 | | " " leather " | | | |
| " " rubber " " | | 183 | | " " rubber " | | | |

Hence it appears that a Rubber Belt for equal resistances with a Leather may be reduced respectively 46, 50, and 30 per cent.

Iron Wire.—A wire rope .375 inch in diameter, over a pulley 4 ft diameter, and running at a velocity of 1250 feet per minute, will transmit 4.5 HP.

Diameter of pulley should not be less than 140 times diameter of rope, in to avoid undue bending of wires.

A sheet-iron belt 7 inches in width proved more effective than one of leather like width.

General Notes.

Leather Belts.—Are best when oak tanned, should be frequently oiled,* and run with hair side over pulley will give greatest adhesion.

Ordinary thickness .1875 inch, and weight 60 lbs. per cube foot.

Relative effect of different pulleys and belts:

| | |
|--|------------------|
| Pulleys. —Leather surface..... 1. | Turned iron..... |
| Rough iron..... .41 | Turned wood..... |

Tensile strength of calf and sheep skins is about one half that of beaver and Morin assigns 50 lbs. as a proper stress per inch of width of good belting.

Presence of small holes in a belt will prevent its slipping or squealing.

Rubber Belts.—Best vulcanized rubber is stronger than leather, and its resistance is from 50 to 85 per cent. greater.

To increase adhesion, coat driving surface with boiled oil or cold tallow, and apply powdered chalk.

When new, cut them .1875 inch short for each foot in length required, to allow of the stretch that occurs in their early operation.

They should be kept free from contact with an animal oil.

Three ply, .1875 inch thick, has a tensile resistance of 600 lbs. per inch of width.

Relative slipping of a vulcanized belt, over smooth or turned leather or faced iron pulleys is as .5, .7, and 1.

Rubber, Gutta percha, and Canvas belts will stretch continuously.

Memoranda.

* θ be set as near horizontal as practicable, in order that the sag on pulley, and hence power should be communicated the

needed by belts is about 2 per cent., hence, to maintain power must be increased in diameter proportionally

A belt, 11 ins. in width, over a driver 4 feet in diameter, running from 1200 to 2250 feet per minute, will transmit the power from two steam cylinders, 6 ins. in diameter and 11 ins. stroke, averaging 125 revolutions per minute, with a pressure of 50 lbs. per sq. inch.

A double belt, 75 ins. in width and 153.5 feet in length, transmitted 650 IHP.

Pulleys should have a slight convexity of surface. Authorities differ, from .5 inch per foot of breadth to .1 of breadth. Belts run at a high speed are less liable to slip than at low speed.

The best speeds for economy are from 1200 to 1500 feet per minute, and the best for result not to exceed 1800.

| | | |
|---------------------------------|-----------------------------|----------------|
| Belts.—Leather, hair-side.... 1 | Leather, flesh-side.... .74 | Rubber..... .5 |
| Gutta percha..... .44 | Canvas..... .35 | |

Coefficient of Friction of a Belt in operation is assumed to be .423.

Smooth surface belts are most endurable and soft most adherent.

Round belts .25 and .5 inch in diameter are fully equal in operation to flat of 2 and 3 ins., and grooves in their pulleys should be angular or V shaped.

The neutral point of a rope belt is at .33 of diameter from inside surface.

Friction of driving and pulley bearings is about .025.

A fan-blower No. 6*, driven by a belt 3.875 ins. in width and .18 in thickness, at a velocity of 2820 revolutions per minute, requires power of 9.7 horses.

Area of belts per HP varies essentially, ranging from 25 to 100 square feet; the mean is 75.

BLASTING.

In *Blasting*, rock requires from .25 to 1.5 lbs. gunpowder per cube yard, according to its degree of hardness and position. In small blasts 2 cube yards have been rent and loosened, and in very large blasts 2 to 4 cube yards have been rent and loosened, by 1 lb. of powder.

Tunnels and shafts require 1.5 to 2 lbs. per cube yard of rock.

Gunpowder has an explosive force varying from 40000 to 90000 lbs. per sq. inch. That used for blasting is much inferior to that used for projectiles, the proportion being fully one third less.

Nitro-glycerine is an unctuous liquid, which explodes by concussion at an extreme pressure (2000 lbs. per sq. inch), or a temperature exceeding 600° if quickly applied to it; it will inflame, however, and burn gradually.

At a temperature below 40° it solidifies in crystals.

Its explosion is so instantaneous that in rock-blasting tamping is not necessary; its explosive power by weight is from 4 to 5 times that of gunpowder.

Dynamite is nitro-glycerine 75 parts, absorbed in 25 parts of a siliceous earth termed kieselguhr; it also explodes so instantaneously as to render tamping in blasting quite unnecessary.

It is insoluble in water, and may be used in wet holes; it congeals at 40° is rendered ineffective at 212°, and has an explosive force by weight of 5 times that of gunpowder, and by bulk 4.25 times.

Gun-cotton is insoluble in water, and has an explosive force by weight of from 2.75 to 3 times that of gunpowder, and by bulk 2.5 times. It may be detonated in a wet state with a small quantity of dry material.

Tonite is nitrated gun-cotton, and is known also as *cotton powder*. It is produced in a granulated form.

Litho-frauteur is a nitro-glycerine compound in the base or absorbent material is made explosive by the addition of nitrate of baryta and charcoal.

* For a table of Belts for Fan-blowers, &c., see J. H. Cooper, in "Jour. Fr."

Cellulose Dynamite is when gun-cotton is used as the absorbent for nitro-glycerine; it will explode frozen dynamite, and is more sensitive to percussion than it.

To Compute Charge of Gunpowder for Rock Blasting.

RULE.—Divide cube of line of least resistance by 25, as for limestone, 32 for granite, and quotient will give charge of powder in lbs.

$$\text{Or, } L^3 \div 32 = \text{lbs.}$$

EXAMPLE.—When line of least resistance is 6 feet, what is charge required?

$$6^3 \div 32 = 6.75 \text{ lbs.}$$

Line of least resistance should not exceed .5 depth of hole.

Tamping.—Dried clay is the most effective of all materials for tamping; Broken Brick the next, and Loose Sand the least.

Relative Costs of a Tunnel and Shaft in England. (Sir John Burgoyne.)

| | | | |
|----------------------|------|-------------|-------|
| Iron and steel..... | 8.98 | Powder..... | 20.04 |
| Smiths and coal..... | 6 | Labor..... | 48.8 |
| Fuses..... | 7.18 | | 100 |

Weight of Explosive Materials in Holes of Different Diameters.

Per Inch of Length.

| Diam. | Powder or Gun-cotton. | Dynamite. | Diam. | Powder or Gun-cotton. | Dynamite. | Diam. | Powder or Gun-cotton. | Dynamite. |
|-------|-----------------------|-----------|-------|-----------------------|-----------|-------|-----------------------|-----------|
| Ins. | Oz. | Oz. | Ins. | Oz. | Oz. | Ins. | Oz. | Oz. |
| 1 | .419 | .67 | 1.75 | 1.283 | 2.053 | 2.5 | 2.618 | 4.183 |
| 1.25 | .654 | 1.046 | 2 | 1.675 | 2.68 | 2.75 | 3.166 | 5.066 |
| 1.5 | .942 | 1.507 | 2.25 | 2.12 | 3.392 | 3 | 3.769 | 6.03 |

Boring Holes in Granite.

| Diam. of Juniper. | Depth of Hole. | Men. | Depth bored per Day. | Hammer. | Diam. of Juniper. | Depth of Hole. | Men. | Depth bored per Day. | Hammer. |
|-------------------|----------------|------|----------------------|---------|-------------------|----------------|------|----------------------|---------|
| Ins. | Ins. | No. | Feet. | Lbs. | Ins. | Ins. | No. | Feet. | Lbs. |
| 1 | 1 to 2 | 1 | 8 | 6 | 2.25 | 5 to 10 | 3 | 6 | 15 |
| 1.75 | 2.5 to 6 | 3 | 12 | 14 | 2.5 | 9 to 12 | 3 | 5 | 16 |
| 2 | 4 to 7 | 3 | 8 | 14 | 3 | 9 to 15 | 3 | 4 | 18 |

Drill.—Width of bit compared to stock .625.

Charges of Powder.

Usual practice of charging to one third depth of hole is erroneous, inasmuch as volume of charge increases as square of diameter of hole. Hence holes of 1.5 and 2 inches, although of equal depths, would require charges in proportion of 2.25 and 4.

| Line of least resistance. | Powder. | Line of least resistance. | Powder. | Line of least resistance. | Powder. | Line of least resistance. | Powder. |
|---------------------------|---------|---------------------------|----------|---------------------------|----------|---------------------------|----------|
| Feet. | Oz. | Feet. | Lbs. Oz. | Feet. | Lbs. Oz. | Feet. | Lbs. Oz. |
| .75 | 3 | 13.5 | 5 | 3 | 14.5 | 7 | 10 |
| 4 | 4 | 2 | 6 | 6 | 12 | 8 | 16 |

Effects.

under.—From its gradual combustion, rends and projects rather than

diameter and 10 feet 7 ins. in depth, filled to 8 feet 10 ins. with red and rent 1200 cube yards, equal to 2400 tons. The men for 14 days.
Explosion 4000°.

of its combustion, shatters.

of its combustion over gun-cotton, is not

Drilling.

Churn-drilling.—A churn-driller will drill, in ordinary hard rock, from 8 to 12 feet, 2 inch holes of 2.5 feet depth, per day, and at a cost of from 12 to 18 cents per foot, on a basis of ordinary labor at \$1 per day. Drillers receiving \$2.50.

One man can bore, with a bit 1 inch in diameter, from 50 to 100 inches per day of 10 hours in granite, or 300 to 400 inches per day in limestone.

Tamping.—Two strikers and a holder can bore, with a bit 2 inches in diameter, 10 feet in a day in rock of medium hardness.

Composition for waterproof charger or fuse consists by weight of Pitch, 8 parts; Beeswax and Tallow each 1 part.

Mining. (*Lefroy's Handbook.*)

In demolition of walls line of least resistance L = half thickness, and C is a coefficient depending on structure.

Charge in lbs. = $C \times L^3$.

In a wall without counterforts, where interval between the charge is 2 L , $C = .15$.

In a wall with counterforts the charge to be placed in centre of each counterfort at junction with wall, $C = .2$.

Where the charge is placed under a foundation, having equal support on both sides, $C = .4$.

A leather bag, containing 50 to 60 lbs. powder, hung or supported against a gate or like barrier, will demolish it.

For ordinary mines in average rock charge in ounces = $L^3 \div 160$.

BLOWING ENGINES.*For Smelting.*

Volume of oxygen in air is different at different temperatures. Thus, dry air at 85° contains 10 per cent. less oxygen than when it is at temperature of 32°; and when it is saturated with vapor, it contains 12 per cent. less. If an average supply of 1500 cube feet per minute is required in winter, 1650 feet will be required in summer.

Smelting of Iron Ore.

Coke or Anthracite Coal.—18 to 20 tons of air are required for each ton. Of Pig Iron, and with Charcoal 17 to 18 tons are required.

(1 ton of air at 34° = 29 751, and at 60° = 31 366 cube feet.)

Pressure.—Pressure ordinarily required for smelting purposes is equal to a column of mercury from 3 to 10 inches, or a pressure of 1.5 to 5 lbs. per square inch.

Reservoir.—Capacity of it, if dry, should be 15 to 20 times that of cylinder if single acting, and 10 times if double acting.

Pipes.—Their area, leading to reservoir, should be .2 that of blast cylinder and velocity of the air should not exceed 35 feet per second.

A smith's forge requires 150 cube feet of air per minute. A blast .25 to 2 lbs. per square inch. A ton of iron melted in a blast requires 3500 cube feet of air per minute. A fine blast requires 20000 cube feet of air for each ton of iron refined. A blast requires 20 cube feet per minute for each cube yard capacity.

A Ton of Pig Iron requires for its reduction from the blast of air, or 5.3 cube feet of air for each pound of pressure, .7 lb. per square inch.

To Compute Power Required to Drive a Blowing Engine.

$$\frac{.0000509}{c} V^3 \left(\frac{L}{d^5} + \frac{42}{d^4} \right) 60 \div 33000 = \text{HP.}$$

$$d' = \sqrt{\frac{V}{.93 \times .7854 \times v}} \quad v \text{ representing velocity of air in feet per sec.}$$

$$\text{ond, } d \text{ and } d' \text{ diameters of pipe and of nozzle in feet,} = \sqrt{\frac{35}{.93 \times .7854 \times 500}} = .309.$$

ILLUSTRATION.—What should be power of a steam-engine to drive 35 cube feet of air at a velocity of 500 feet per second, through a pipe 1 foot in diameter and 500 feet in length?

c = ratio between power employed and effect produced by it = in a well-constructed engine .5, and C = .93. d = .2974, assumed at .3.

$$\frac{.0000509}{.5} \times 35^3 \left(\frac{300}{.3^5} + \frac{42}{.3^4} \right) 60 \div 33000 = 22631.625 \times 60 \div 33000 = 41.15 \text{ HP.}$$

To Compute Required Power of a Blowing Engine.

$$\frac{P + f \times a v}{33000} = \text{HP.} \quad P \text{ representing pressure of blast in lbs. per sq. inch;}$$

a area of cylinder in sq. ins.; v velocity of piston in feet per minute; f friction of piston and from curvatures, etc., estimated at 1.25 per sq. inch of piston.

NOTE.—If cylinder is single acting, divide result by 2.

ILLUSTRATION.—Assume area of blast cylinder 5600 sq. ins., pressure of blast 2.25 lbs. per sq. inch, and velocity of piston 96 feet per second.

$$\frac{2.25 + 1.25 \times 5600 \times 96}{33000} = \frac{1881600}{33000} = 57 \text{ horses, the exact power developed in this case.}$$

To Compute Dimensions of a Driving Engine.

RULE 1.—Divide power in lbs. by product of mean effective pressure upon piston of steam cylinder in lbs. per sq. inch, and velocity of piston in feet per minute, and quotient will give area of cylinder in sq. ins.

2.—Divide velocity of piston by twice number of revolutions, and quotient will give stroke of piston in feet.

Volume of air at atmospheric density delivered into reservoir, in consequence of escape through valves, and partial vacuum necessary to produce a current, will be about .2 less than capacity of cylinder.

EXAMPLE.—Assume elements of preceding case, with a pressure of 50 lbs. steam cut off at .375, and with 12 revolutions of engine per minute, what should be area of cylinder of a non-condensing engine?

The pressure of steam with 5 per cent. clearance = 50 lbs., and $50 - 2.5 + 3.33 + 14.7 = 29.47$ lbs., and velocity of piston = 192 feet

$$\frac{1.25 \times 96}{192} = \frac{1881600}{5658} = 332.5 \text{ sq. ins., and } \frac{192}{12 \times 2} = 8 \text{ feet stroke}$$

area = 324 sq. ins.

city 1884, see Heat, page 521.

condensing engine, page 733.

To Compute Elements of a Blowing Engine.

Single Stroke.

$$\frac{V \overline{P+f}}{230} \text{ or } \frac{A s n \overline{P+f}}{33000} = \text{HP}; \quad \frac{\sqrt{V+10L}}{3} = d; \quad \frac{D^2 s n}{40 u} = v;$$

$$\frac{D^2 s n}{40 v} = a; \quad \frac{230 \text{ HP}}{P+f} = V; \quad \frac{D^2 s n}{92} = V; \text{ and } 34 P + 32 = t.$$

V representing volume of air in cube feet per minute, *P* pressure of air and frictional resistance in lbs. per sq. inch, *A* area of cylinder and a area of its valves in sq. ins., *s* stroke of piston in feet, *n* number of single strokes of piston per minute, *L* length of air-pipe from reservoir to discharge in feet, diameter of air or blast pipe and *D* diameter of cylinder in ins., *v* velocity of blast in feet per second, and *t* temperature of blast consequent upon compression in degrees.

ILLUSTRATIONS. — Assume blowing cylinder 50 ins. in diam., stroke of piston 10 ins., number of single strokes 10 per minute, pressure by mercurial manometer 12 ins., frictional resistance .4 lb., length of pipe 25.25 feet, and area of valves 5 sq. ins.

$$V = 1363.54 \text{ cube feet,} \quad P = 3 \text{ lbs.,} \quad A = 1963.5 \text{ sq. ins.}$$

$$\text{Then } \frac{1363.54 \times 3 + .4}{230} = 20.16 \text{ HP, and } \frac{1963.5 \times 10 \times 10 \times 3 + .4}{33000} = 20.23 \text{ HP.}$$

$$\frac{\sqrt{1363.54 + 10 \times 25.25}}{3} = 13.4 \text{ ins.} \quad \frac{50^2 \times 10 \times 10}{40 \times 95} = 65.8 \text{ feet.} \quad \frac{50^2 \times 10 \times 10}{40 \times 65.8} = 95 \text{ sq. ins.}$$

To Compute Volume of Air transmitted by an Engine.

When Pressure, Temperature, etc., are given.

$$4.5 \sqrt{h \left(\frac{1 + .004 t}{h + 11} \right)} C = v. \text{ Then } a v \times 60 = V \text{ in cube feet per minute.}$$

H and *h* representing height of barometer and pressure of blast in ins. of mercury; *t* temperature of blast; and *v* velocity in feet per second.

ILLUSTRATION. — A furnace having 2 tuyeres of 5 ins. diameter, pressure and temperature of blast 3 ins. and 350°, and barometer 30 ins.; what is volume of air transmitted per minute?

C for a conical opening = .94.

$$34.5 \sqrt{3 \left(\frac{1 + .004 \times 350}{3 + 30} \right)} \times .94 = 34.5 \sqrt{3 \left(\frac{2.4}{33} \right)} = 34.5 \times .467 \times .94 = 15.14$$

feet velocity per second.

Then, area 5 ins. = 19.635, which $\times 2 = 39.27$ ins., and $39.27 \times 15.14 \times 60 \div 144 = 47.73$ cube feet.

To Compute Pressure of Blast from Water or Mercurial Gauge.

RULE. — Divide Water and Mercurial Gauge in ins. by 27.67 and 2.04 respectively, and quotient will give pressure in lbs. per sq. in.

Fan-blowers.

Proportions of Parts. Blades. — Their width and length equal to .4 or .5 radius of fan.

Openings. — Inlet should be equal to radius of fan; also, should be in depth not less than .125 diameter, i. width of fan.

eccentricity. — .1 of diameter of fan. Journals, 4 diame.

By the experiments of Mr. Buckle, he deduced

1. That velocity of periphery of blades should be .9 that of their *theoretical* velocity; that is, velocity a body would acquire in falling height of a homogeneous column of air equivalent to required density.

2. That a diminution of inlet from proportions here given involved a greater expenditure of power to produce same density.

3. That greater the depth of blade, greater the density of air produced with same number of revolutions.

To Compute Elements of a Fan-blower.

$$\left(\frac{v}{8.02}\right)^2 \div 939.45 = d; \quad 244 \sqrt{d} = v; \quad \frac{av60}{160} = V; \quad \text{and} \quad \frac{d av}{400} = \text{HP.}$$

v representing velocity of periphery of fan in feet per second, *d* inches of mercury, *V* volume of air in cube feet, and *a* area of discharge in sq. ins.

ILLUSTRATION.—Assume velocity of periphery of fan 123 feet per second, density of blast .25 inch, volume of air 1845 cube feet, and area of discharge 40 sq. ins.

$$123 \div 8.02^2 \div 939.45 = .25 \text{ inch.} \quad 244 \sqrt{.25} = 122 \text{ feet.} \quad \frac{40 \times 123 \times 60}{160} = 1845 \text{ cub. ft.}$$

$$\frac{.242 \times 40 \times 123}{400} = 2.97 \text{ HP, independent of friction of blast in pipes and tuyeres.}$$

To Compute Power of a Centrifugal Fan.

$$V^2 \div 97300 = P. \quad V \text{ representing velocity of tips of fan in feet per second.}$$

Memoranda.

Operation of a blower requires about 2.5 per cent. of power of attached boiler.

An increase in number of blades renders operation of fan smoother, but ~~as~~ not increase its capacity.

Pressure or density of a blast is usually measured in ins. of mercury, a pressure of 1 lb. per sq. inch at 60° = 2.0376 ins.

When water is used, a pressure of 1 lb. = 27.671 ins.

Wool's blast .8 lbs., and *Smith's* forge .25 to .3 lbs. per sq. inch.

Ordinary Eccentric Fan, 4 feet in diameter, with 5 blades 10 ins. wide in length, set 1.5 ins. eccentric, with an inlet opening of 17.5 ins. in area, and an outlet of 12 ins. square, making 870 revolutions per minute supply air to 40 tuyeres, each of 1.625 ins. in diameter, and at a pressure per sq. inch of .5 inch of mercury.

Ordinary eccentric fan-blower, 50 ins. in diameter, running at 1000 revolutions per minute, will give a pressure of 15 ins. of water, and require for its operation 2 HP or 12 horses. Area of tuyere discharge 500 sq. ins.

Condensing engine, diameter of cylinder 8 ins., stroke of piston 1 foot, pressure 18 lbs. (mercurial gauge), and making 100 revolutions per minute, will supply an opening 2 feet by 2, 500 revolutions per minute.

Used as an exhausting draught to smoke-pipe of steamer 10 ins. by 8 feet, and evaporation was doubled over that

volume of air discharged 75 per cent. that of a smoke equal diameter of cylinder, and 1 minute.

.83 of that of cylinder for speeds to 111 for higher speeds. Area (M. Claudel.)

By some experiments lately concluded in England with boilers of two steamers, to determine relative effects of natural and forced draught furnaces the results were as follows (*R. J. Butler*):

Per Sq. Foot of Grate Surface.—*Natural Draught*, 10 to 10.87 IHP; *Steam Blast*, 12.5 to 13; *Forced or Blast Draught*, 15 to 16.

Heating Surface per IHP.—*Natural Draught*, 2.44 to 2.61; *Steam Blast* 1.71 to 2.86; *Forced or Blast Draught*, 1.56 to 2.5.

Tube Surface per IHP in Sq. Feet.—*Natural Draught*, 2.03 to 2.18; *Steam Blast*, 2.02 to 2.08; *Forced or Blast Draught*, 1.3 to 2.8.

IHP per Sq. Foot of Grate in these Trials.—*Natural Draught*, 10.15 to 10.87; *Steam Blast*, 12.76 to 13.1; *Forced or Blast Draught*, 10.6 to 16.9.

Root's Rotary Blower—Is constructed from .125 to 14 nominal HP, supplying from 150 to 10 800 cube feet of air per minute. Delivery pipe 2.5 to 19 ins. in diameter. Efficiency 65 to 80 per cent. of power.

For Ventilation of Mines—From 40 to 280 revolutions per minute, equal to discharge of 12 500 to 200 000 cube feet of air per minute. 15.5 to 189 HP.

Steam cylinder from 14 × 18 ins. to 28 × 48 ins.

For other details of Blowing Engines see page 838.

CENTRAL FORCES.

All bodies moving around a centre or fixed point have a tendency to fly off in a straight line: this is termed *Centrifugal Force*; it is opposed to a *Centripetal Force*, or that power which maintains a body in its curvilinear path.

Centrifugal Force of a body, moving with different velocities in same circle, is proportional to square of velocity. Thus, centrifugal force of a body making 10 revolutions in a minute is 4 times as great as centrifugal force of same body making 5 revolutions in a minute. Hence, in equal circles, the forces are inversely as squares of times of revolution.

If times are equal, velocities and forces are as radii of circle of revolution.

The squares of times are as cubes of distances of centrifugal force from axis of revolution.

Centrifugal forces of two unequal bodies, having same velocity, and at same distance from central body, are to one another as the respective quantities of matter in the two bodies.

Centrifugal forces of two bodies, which perform their revolutions in same time, the quantities of matter of which are inversely as their distances from centre, are equal to one another.

Centrifugal forces of two equal bodies, moving with equal velocities at different distances from centre, are inversely as their distances from centre.

Centrifugal forces of two unequal bodies, moving with equal velocities at different distances from centre, are to one another as their quantities of matter multiplied by their respective distances from centre.

Centrifugal forces of two unequal bodies, having unequal velocities at different distances from their axes are in compound ratio of their quantities multiplied by the squares of their velocities, and their distances from centre.

Centrifugal force is to weight of body, as double height due to rotation.

A Radius Vector is a line drawn from centre of force to moving body.

P P'

To Compute Centrifugal Force of any Body.

RULE 1.—Divide its velocity in feet per second by 4.01, also square of quotient by diameter of circle; this quotient is centrifugal force, assuming the weight of body as 1. Then this quotient, multiplied by weight of body, will give centrifugal force required.

EXAMPLE.—What is the centrifugal force of the rim of a fly-wheel having a diameter of 10 feet, and running with a velocity of 30 feet per second?

$$30 \div 4.01 = 7.48, \text{ and } 7.48^2 \div 10 = 5.59, \text{ or times weight of rim.}$$

$$\text{Or, } \frac{W n^2 \sqrt{R^2 + r^2}}{4100} = C, \quad r \text{ representing radius of inner diameter of ring.}$$

NOTE.—Diameter of a fly-wheel should be measured from centres of gravity of rim.

When great accuracy is required, ascertain centre of gyration of body, and take twice distance of it from axis for diameter.

RULE 2.—Multiply square of number of revolutions in a minute by diameter of circle of centre of gyration in feet, and divide product by constant number 5217; quotient is centrifugal force when weight of body is 1. Then, as in previous Rule, this quotient, multiplied by weight of body, is centrifugal force required.

Or, $\frac{n^2 d}{5217} \times W$, n representing number of revolutions per minute, d diameter of circle of gyration in feet, and W weight of revolving body in lbs.

EXAMPLE.—What is centrifugal force of a grindstone weighing 1200 lbs., 42 inches in diameter, and turning with a velocity of 400 revolutions in a minute?

Centre of gyration = rad. $(42 \div 2) \times .7071 = 14.85$ ins., which $\div 12$ and $\times 12$ = 2.475 feet = diameter of circle of gyration. Then $\frac{400^2 \times 2.475}{5217} \times 1200 = 9100$ lbs.

Formulas to Determine Various Elements.

$$C = \frac{W r^2}{32.166 R}; \quad = \frac{W R n^2}{2930}; \quad = W R v^2 1.225; \quad W = \frac{C 32.166 R}{v^2};$$

$$R = \frac{2930 C}{W n^2}; \quad = \frac{W r^2}{32.166 C}; \quad n = \sqrt{\frac{2930 C}{W R}}; \quad v = \sqrt{\frac{C R 32.166}{W}}; \quad = 6.28 \sqrt{\frac{C R}{W}}$$

C representing centrifugal force, W mass or weight of revolving body, both in lbs., R radius of circle of revolving body in feet, n number of revolutions per minute, v and v' linear or circumferential and a angular velocities of body in feet per second.

ILLUSTRATION.—What is centrifugal force of a sphere weighing 30 lbs., revolving around a centre at a distance of 5 feet, at 30 revolutions per second?

$$v = \frac{5 \times 2 \times 3.1416 \times 30}{60} = 15.71 \text{ feet. Then } \frac{C 30 \times 15.71^2}{32.166 \times 5} = 46.04 \text{ lbs.}$$

Centrifugal forces of two bodies are as radii of circles of revolution directly, and as squares of times inversely.

ILLUSTRATION.—If a fly-wheel, 10 feet in diameter and 3 tons in weight, revolves in 8 seconds, and another of like weight revolves in 5, what should be the diameter of the second when their centrifugal forces are equal?

$$\text{Then } 3 : 3.125 \frac{12}{8^2} : \frac{x}{5^2}; \text{ or } x = \frac{12 \times 6^2}{8^2} = 6.75 \text{ feet, } x = \text{unknown element.}$$

Centrifugal forces of two bodies, when weights are unequal, are directly as squares of times.

ILLUSTRATION.—What should be the ratio of the weights of the wheels in the previous case, their forces being equal?

$$3 : x :: 6^2 : 5^2, \text{ or } x = \frac{3 \times 5^2}{6^2} = 3.333 \text{ tons.}$$

which gives $400 \frac{34}{3} W R n^2 = C$.

FLY-WHEEL.

FLY-WHEEL by its inertia becomes a reservoir as well as a regulator of speed, and to be effective should have high velocity, and its diameter should be from 3 to 4 times that of stroke of driving engine.

Efficient of fluctuation of energy in a machine ranges from .015 to .05.

Weight of a fly-wheel in engines that are subjected to irregular motions in a cotton-press, rolling-mill, etc., must be greater than in others, so sudden a check is not experienced, and its diameter should be from 3.5 to 5 times length of the crank.

Single acting engine requires a weight of wheel about 2.5 times greater than for a double acting, and 5 times for double engines of double action.

To Compute Weight of Rim of a Fly-wheel.

1.—E.—Multiply mean effective pressure upon piston in lbs. by its stroke in feet, and divide product by product of square of number of revolutions per minute, and diameter of wheel, and .00023.

2.—If a light wheel is required, multiply by .0003; and if a heavy one, by .0004.

EXAMPLE 1.—A non-condensing engine (double acting), having a diameter of cylinder 14 ins., and a stroke of piston of 4 feet, working full stroke, at a pressure of 30 lbs. per sq. inch, and making 40 revolutions per minute, develops about what should be the weight of its fly-wheel, when adapted to ordinary work?

of cylinder 154 sq. ins. Mean pressure assumed 50 lbs. per sq. inch. Diameter of wheel 4 feet stroke $\times 3.5 = 14$ feet.

$$50 \times 154 \times 4 = 30800, \text{ which } \div 40^2 \times 14 \times .00023 = 5978 \text{ lbs.}$$

A fly-wheel, 16 feet in diameter and 4 tons in weight, is sufficient to regulate a single acting engine when it revolves in 4 seconds, what should be the weight of a fly-wheel, 12 feet in diameter, revolving in 2 seconds, so that it may have like centrifugal force?

1.—The centrifugal forces of two bodies are as the radii of the circles of revolution directly, and as squares of times inversely.

$$\frac{4 \times 16}{4^2} = \frac{x \times 12}{2^2}. \text{ Or, } x = \frac{4 \times 16 \times 2^2}{12 \times 4^2} = \frac{4 \times 16 \times 4}{12 \times 16} = 1.333 \text{ tons.}$$

are the elements of example 1.

$$5978 \times 1 \div 13.25 = 45.12 \text{ square ins.}$$

To Compute Dimensions of Rim.

1.—E.—Multiply weight of wheel in lbs. by .1, and divide product by diameter of rim in feet; quotient will give sectional area of rim in square inches of cast iron.

2.—S.— $\frac{P \times S}{10 \times D} = W$, and $\frac{W}{10 \times D} = A$. P representing pressure on piston and W weight of wheel in lbs., S stroke of piston and D mean diameter of wheel, both in feet, and A sectional area of rim in sq. ins.

3.— $\frac{16 \times n \times P \times S \times C}{60 \times D} = W$. C representing coefficient varying from 3 to 4 or less, according to the regularity of speed required, and n number of revolutions per minute.

4.—Maximum safe velocity for cast iron is assumed.

5.—Engines at high expansion of steam, or with irregular motion, multiply W by 1.5, or put W 100 lbs. for each IHP.

6.—In or like mills, the velocity of periphery of fly-wheel is assumed.

GOVERNORS.

A GOVERNOR OR CONICAL PENDULUM in its operation depends upon principles of Central Forces.

When in a *Ball Governor* the balls diverge, the ring on vertical is raised, and in proportion to the increase of velocity of the balls, square or the square roots of distances of ring from fixed point of arms, responding to two velocities, will be as these velocities.

Thus, if a governor makes 6 revolutions in 1 second when ring is ins. from fixed point or top, the distance of ring will be 5.76 ins. if speed is increased to 10 revolutions in same time.

For $10 : 6 :: \sqrt{10} : \sqrt{6}$, which, squared = 5.76 ins. distance of ring from top. Or, $6^2 : 10^2 :: 5.76 : 10$ ins.

A governor performs in one minute half as many revolutions as pendulum vibrates, the length of which is perpendicular distance between plane in which the balls move and the fixed point or centre suspension.

To Compute Number of Revolutions of a Ball Governor per Minute to maintain Balls at any given Height

$158 - \sqrt{H}$, H = vertical height between plane of balls and point of their suspension in ins.

Illustration.—If the rise of the balls of a centrifugal governor is 20 ins., are the number of revolutions per minute?

$$158 - \sqrt{20} = 154.35 \text{ revolutions}$$

To Compute Vertical Height between Plane of Balls and their Points of Suspension.

$158 - \sqrt{R}^2$ = vertical height in ins., R representing number of revolutions per minute.
Illustration.—If number of revolutions of a centrifugal governor is 100, what is the height of balls?

$$158 - 100 = 58^2 = 5.76 \text{ ins.}$$

To Compute Angle of Arms or Plane of Balls with Centre Shaft.

$\sin \angle = \frac{R}{158}$, R representing distance of balls from plane of centre shaft; \sin is sine of balls and point of suspension measured in plane of shaft.

Illustration.—If distance of balls from plane of centre shaft is 10 inches, and distance from point of suspension is 25, what is the angle?

$$10 \div 25 = .4, \text{ and } \sin .4 = 23^\circ 35'$$

When Number of Revolutions are given, $\frac{58.16 \div R^2}{1} = \cos \angle$.

Illustration.—Revolutions of a governor per minute are 50, and length of arms 10 ins. what is their angle with plane of shaft?

$$\frac{(58.16 \div 50^2)}{2} = \frac{1.173}{2} = .5865 = \cos 54^\circ 6'$$

PENDULUMS.

Pendulums are *Simple* or *Compound*, the former being a material weight suspended from a fixed point, about which, by a connection void of weight; and the latter of bodies suspended by a rod or connection have as many centres of oscillation as the motion to it, and when any one of these centres are readily ascertained.

$s \times g = \text{a constant product, and } s = \sqrt{\dots}$
 ing points of suspension, gravity, oscillation, and $g = \dots$

ny body, as a cone, a cylinder, or of any form, $g = \dots$
 ended as to be capable of vibrating, is a compound pendulum
 of its centre of oscillation from any assumed point of suspension
 ed as the length of an equivalent simple pendulum.

Amplitude of a simple pendulum is the distance $g = \dots$
 from its lowest position to its farthest on either side.

lete Period of a pendulum in motion is the time $g = \dots$
 rations.

ibrations of same pendulum, whether great or small $g = \dots$
 arly in same time.

ber of Oscillations of two different pendulums $g = \dots$
 lace are in inverse ratio of square roots of their lengths.

th of a Pendulum vibrating seconds is in a constant ratio $g = \dots$

of Vibration is half of a complete period, and it is $g = \dots$
 root of length of pendulum. Consequently, length $g = \dots$
 it vibrations are—

Latitude of Washington.

0958 ins. for one second. | 4.344 for third of a second.
 .774 ins. for half a second. | 2.4435 for quarter of a second.

ths of Pendulums vibrating Seconds at $g = \dots$
 the Sea in several Places.

| Ins. | | Ins. | |
|---------|----------|---------|--------|
| 39.0152 | New York | 39.1017 | Paris |
| 39.0958 | Lat. 45° | 39.127 | London |

ompute Length of a Simple Pendulum $g = \dots$
 Latitude.

$39.127 - .09982 \cos. 2 L = l$ L representing latitude.

TRATION.—Required the length of a simple pendulum vibrating $g = \dots$
 lude of $50^\circ 31'$.

$0^\circ 31' \cos. 2 L = 2 \times 50^\circ 31' = \cos. 180^\circ - 50^\circ 31' \times 2 = \cos. 70^\circ 2'$
 $7 + .19138 \times .09982$ (two — or negative = an affirmative or +) = $7.19138 \times .09982$

ompute Length of a Simple Pendulum for $g = \dots$
 Number of Vibrations.

= L L representing length for latitude, t time in seconds, and l length of pen-
 dulum in ins.

TRATION.—Required vibrations of a pendulum in a minute at New York $g = \dots$
 it should be its length?

$17 \times 12 = 39.1017$. Or, $\frac{L}{n^2} = l$ n representing number of vibrations per minute

ompute Number of Vibrations of a Simple Pendulum in a given Time.

$$\frac{\sqrt{L't}}{\sqrt{l}} = n, \frac{t}{n} \text{ representing time of one vibration in seconds.}$$

ompute Centre of Gravity of a Compound Pendulum
 of Two Weights connected in a Right Line.

When Weights are both on one Side of Point of

— = 0 = distance of centre of gravity from point of

When Weights are on Opposite Sides of Point of Suspension.

$$\frac{lW - l'w}{W + w} = 0 = \text{distance of centre of gravity of greater weight from point of suspension.}$$

NOTE.—To obtain strictly isochronous vibrations, the circular arc must be substituted for the cycloid curve, which possesses the property of having an inclination, the sine of which is simply proportional to distance measured on the arc from its lowest point.

For construction of a Cycloidal pendulum, see Deschanel's *Physics*, Part I, 71-2.

To Compute Length of a Simple Pendulum, Vibrations of which will be same in Number as Inches in Length.

$$\sqrt[3]{(\sqrt{L} \times 60)^2} = l \text{ in inches.}$$

ILLUSTRATION.—What will be length of a pendulum in New York, vibrations of which will be same number as the ins. in its length?

$$\sqrt[3]{(\sqrt{39.1017} \times 60)^2} = 7.2112 = 52 \text{ ins.}$$

To Compute Time of Vibration of a Simple Pendulum Length being given.

$$\sqrt{l \div L} = t \text{ in seconds.}$$

ILLUSTRATION.—Length of a pendulum is 156.4 ins.: what is the time of its vibration in New York?

$$\sqrt{\frac{156.4}{39.1017}} = 2 \text{ seconds.}$$

Or, $\sqrt{\frac{L}{g}} \times 3.1416 = t$, L representing length of a pendulum vibrating seconds, g measure of force of gravity, and t time of one oscillation.

ILLUSTRATION.—Length of a simple pendulum vibrating seconds, and measure of force of gravity at Washington, are 39.0958 ins., and 32.155 feet.

$$3.1416 \sqrt{\frac{39.0958}{32.155 \times 12}} = 3.1416 \times \sqrt{1.013} = 3.1416 \times .3183 = 1 \text{ second.}$$

To Compute Number of Vibrations of a Simple Pendulum in a given Time.

$$\sqrt{\frac{L}{l}} \times t = n. \quad n \text{ representing number of vibrations.}$$

ILLUSTRATION.—The length of a pendulum in New York is 156.4 ins., and its vibration is 2 seconds; what are number of its vibrations?

$$\sqrt{\frac{39.1017}{156.4}} \times 2 = \sqrt{\frac{6.253}{12.500}} \times 2 = .5 \times 2 = 1 \text{ vibration.} \quad \text{Hence, } 1 \times \frac{60}{2} = 30 \text{ vibrations per minute.}$$

To Compute Measure of Gravity, Length of Pendulum and Number of its Vibrations being given.

$$\frac{.82246 \, l \, n^2}{t^2} = g. \quad g \text{ representing measure of gravity in feet.}$$

To Compute Number of Revolutions of a Conical Pendulum per Minute.

$$\sqrt{\frac{2932}{h}} = \text{revol. per minute}$$

h representing distance between point of suspension and pendulum.

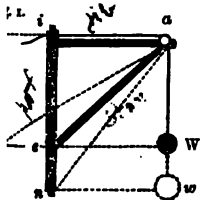
Revolutions per minute are constant for any given height, and are directly as square root of height.

CRANES.

Usual form of a Crane is that of a right-angled triangle, the sides being post or jib, and stay or strut, which is hypotenuse of triangle.

When jib and post are equal in length, and stay is diagonal of a square, a form is theoretically strongest, as the whole stress or weight is borne by y , tending to compress it in direction of its length; stress upon it, compared to weight supported, being as diagonal to side of square, or as 1.4142 x . Consequently, if weight borne by crane is 1000 lbs., thrust or compression upon stay will be 1414.2 lbs., or as $a e$ to $e W$, Fig. 1.

When Post is Supported at both Head and Foot, as Fig. 1.



Weight W is sustained by a rope or chain, and tension is equal upon both parts of it; that is, on two sides of square, $i a$ and $e W$. Consequently jib, $i a$, has no stress upon it, and serves merely to retain stay, $a e$.

If foot of stay is set at n , thrust upon it, as compared with weight, will be as $a n$ to $a w$; and if chain or rope from i to a is removed, and weight is suspended from a , tension on jib will be as $i a$ to $a W$.

If foot of stay is raised to o , thrust, as compared with weight, will be as $e a o$ is to $a W$, and tension on jib will be as line $a r$.

By dividing line representing weight, as $a W$ or $a w$, into equal parts, to represent tons or pounds, and using it as a scale, stress upon any other part may be measured upon described parallelogram.

Thus, as length of $a W$, compared to $a e$, is as 1 to 1.4142: if $a W$ is divided into 10 parts representing tons, $a e$ would measure 14.142 parts or tons.

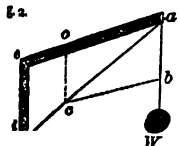
When Post is Supported at Foot only.

If post is wholly unsupported at head, and its foot is secured up to line N , then W , acting with leverage, $e W$, will tend to rupture post at e , with the effect as if twice that weight was laid upon middle of a beam equal to the length of $e W$, e being at middle of beam, which is assumed to be supported at both ends, and of like dimensions to those of post.

Or, force exerted to rupture post will be represented by stress, W , multiplied by 4 times length of lever, $e W$, divided by depth of post in line of axis, squared, and multiplied by breadth of it and Value* of its material.

Post of such a crane is in condition of half a beam supported at one end, right suspended from other; consequently, it must be estimated as a beam twice the length supported at both ends, stress applied in middle.

• Compute Stress on Jib, and on Stay or Strut.—Fig. 2.



On diagram of crane, Fig. 2, mark off on line of chain, $a W$, a distance, $a b$, representing weight on chain; from point b draw a line, $b c$, parallel to jib, $a e$, and where this intersects stay or strut, draw a vertical line, $c o$, extending to jib, and distances from a to points $b c$ and $o c$, measured upon a scale of equal parts, will represent proportionals.

ILLUSTRATION.—In figure, weight being x on stay or strut compressing, $a c$, will be on jib or tension-rods, $a o$, 26 tons.

* For Value of Materials, see page 779.

To Compute Dimensions of Post of a Crane.

When Post is Supported at Feet only. RULE.—Multiply weight or stress to be borne in lbs. by length of jib in feet measured upon a horizontal plane; divide product by *Value* of material to be used, and product, divided by breadth in ins., will give square of depth, also in ins.

EXAMPLE.—Stress upon a crane is to be 22 400 lbs., and distance of it from centre of post 20 feet; what should be dimension of post if of American white oak?

Value of American white oak 50. Assumed breadth 12 ins.

$$\frac{22\,400 \times 20}{50} = 8960, \text{ and } \frac{8960}{12} = 746.67. \text{ Then } \sqrt[3]{746.67} = 27.32 \text{ ins.}$$

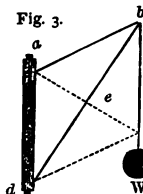
When Post is Supported at both Ends. RULE.—Multiply weight or stress to be borne in lbs. by twice length of jib in feet measured upon a horizontal plane; divide product by *Value* of material to be used, and product, divided by four times breadth in ins., will give square of depth, also in ins.

EXAMPLE.—Take same elements as in preceding case. Assumed breadth 10 ins.

$$\text{Then } \frac{22\,400 \times 20 \times 2}{50} = 17\,920, \frac{17\,920}{4 \times 10} = 448, \text{ and } \sqrt[3]{448} = 21.166 \text{ ins.}$$

In Fig. 3, angle abe and ebc being equal, chain or rope is represented by abc , and weight by W ; stress upon stay bd , as compared with weight, is as bd to ab or bc .

Fig. 3.



In practice, however, it is not prudent to consider chain as supporting stay; but it is proper to disregard chain or rope as forming part of system, and crane should be designed to support load independent of it. It is also proper that angles on each side of diagonal stay, in this case, should not be equal. If side ab is formed of tension-rods of wrought iron, point a should be depressed, so as to lengthen that side, and decrease angle abe ; but if it is of timber, point a should be raised, and angle abe increased.

Fig. 4.

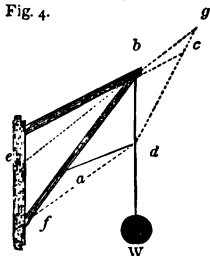


Fig. 4 shows a form of crane very generally used; angles are same as in Fig. 3, and weight suspended from it, being attached to point d , is represented by line bd . The tension, which is equal to weight, is shown by length of line bc , and thrust by length of line ba , measured by a scale of equal parts, into which line bd , representing weight, is supposed to be divided.

But if b be direction of jib, then bg will show tension, and bf the thrust (df being taken parallel to bd , both of them being now greater than before; line bd representing weight, and being same in both cases).

To Ascertain Stress on Jib, on Strut of a Crane.—Fig. 5.

Through a draw as , parallel to jib or tension-rod or , and also su parallel to strut ar ; then rs is diagonal of parallelogram, sides of which are equal to ra and ru .

If then rs represents a stress of 20 lbs., the two forces into which it is decomposed are shown by ru and ra ; or is equal to ru , as each of them is equal to as , and rs is equal to oa . Hence, as represented by a , stress on jib will be represented by or , and that on strut by ra .

Assuming then or 3 feet, ar 3 feet,

on jib will be 60 lbs., and on strut 70.

Thus, in all cases, stress on jib or tension-rod and on strut can be determined by relative proportions of sides of triangle formed.

To Compute Stress upon Strut of a Crane.

RULE.—Multiply length of strut in feet by weight to be borne in lbs.; divide product by height of jib from point of bearing of strut in feet, and quotient will give stress or thrust in lbs.

EXAMPLE.—Length of strut of a crane is 28.284 feet, height of post is 26.457 feet, and weight to be borne is 22 400 lbs.; what is stress?

$$\frac{28.284 \times 22\,400}{26.457} = \frac{633\,561.6}{26.457} = 23\,947 \text{ lbs.}$$

Chains and Ropes.

Chains for Cranes should be made of short oval links, and should not exceed 1 inch in diameter.

Short-linked Crane Chains and Ropes showing Dimensions and Weight of each, and Proof of Chain in Tons.

| Diam. of Chains. | Weight per Fathom. | Proof Strain. | Circumf. of Rope. | Weight of Rope per Fath. | Diam. of Chains. | Weight per Fathom. | Proof Strain. | Circumf. of Rope. | Weight of Rope per Fath. |
|------------------|--------------------|---------------|-------------------|--------------------------|------------------|--------------------|---------------|-------------------|--------------------------|
| Ina. | Lbs. | Tons. | Ina. | Lbs. | Ina. | Lbs. | Tons. | Ina. | Lbs. |
| .3125 | 6 | .75 | 2.5 | 1.5 | .6875 | 28 | 6.5 | 7 | 10.5 |
| .375 | 8.5 | 1.5 | 3.25 | 2.5 | .75 | 32 | 7.75 | 7.5 | 12 |
| .4375 | 11 | 2.5 | 4 | 3.75 | .8125 | 36 | 9.25 | 8.25 | 15 |
| .5 | 14 | 3.5 | 4.75 | 5 | .875 | 44 | 10.75 | 9 | 17.5 |
| .5625 | 18 | 4.5 | 5.5 | 7 | .9375 | 50 | 12.5 | 9.5 | 19.5 |
| .625 | 24 | 5.25 | 6.25 | 8.7 | 1 | 56 | 14 | 10 | 22 |

Ropes of circumferences given are considered to be of equal strength with the chains, which, being short-linked, are made without studs.

A crane chain will stretch, under a proof of 15 tons, half an inch per fathom.

Machinery of Cranes.

To attain greater effect of application of power to a crane, the wheel-work must be properly designed and executed.

If manual labor is employed, it should be exerted at a speed of 220 feet per minute.

Proportions.—Capacity of Crane, 5 tons.

Radius of winch or handle 15 to 18 ins. Height of axle from floor 36 to 39.

| | | |
|--|--|--------------------------------------|
| 1st pinion, 11 teeth, 1.25 ins. pitch. | | 2d pinion, 12 teeth, 1.5 ins. pitch. |
| 1st wheel, 89 " 1.25 " " | | 2d wheel, 96 " " " " |

Barrel 8 ins. \times 11 teeth \times 12 teeth \times 11 200 lbs. = 30 800
 Winch 17 ins. \times 89 teeth \times 96 teeth \times 4 men = 1513 = 20.35 lbs. = statical resistance to each of the 4 men at winches.

An experiment upon capacity of a crane, geared 1 to 105, developed that strong man for a period of 2.5 minutes exerted a power of 27 562 foot-pounds per minute, which, when friction of crane is considered, is five times the power of a horse for one minute.

In practice an ordinary man can develop a power of 15 lbs. weight moved at a velocity of 220 feet per minute, which is equal to 30 foot-pounds.

For Treatise on Cranes, see Weales' Series, No. 33.

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These particulars are embodied in following summary of condition of elements of 100 lbs. of average coal, after having been decomposed, and prior entering into combustion—

100 Lbs. of Average Coal in a Furnace.

| Composition | Lbs. | | Lbs. | Decomposition. |
|--------------------------|------|-------------|-------|---------------------------------|
| Carbon { Fixed | 60 | } forming { | 60 | fixed carbon. |
| Carbon { Volatilized.... | 20 | | 24 | hydrocarbons and free hydrogen. |
| Hydrogen | 5 | | 1.25 | sulphur. |
| Sulphur | 1.25 | | 85.25 | |
| Nitrogen | 8 | | 9 | water or steam. |
| Oxygen | 1.2 | | 1.2 | nitrogen. |
| Ash, etc. | 4.55 | | 4.55 | ash, etc. |
| | 100 | | 100 | |

giving a total useful combustible of 85.25 per cent., of which 25.25 per cent. is volatilized. While the decomposition proceeds, combustion proceeds, in the 25.25 per cent. of volatilized portions, and the 60 per cent. of fixed carbon, successively, are burned.

It may be added that the sulphur and a portion of the nitrogen are discharged in combination with hydrogen, as sulphuretted hydrogen and ammonia. But these compounds are small in quantity, and, for the sake of simplicity, they have not been indicated in the synopsis.

Volume of Air chemically consumed in complete Combustion of Coal.

Assume 100 lbs. of average coal. Then, by following

$$30 + 3 \left(5 - \frac{8}{8} \right) + .4 \times 1.25 \times 152 = 14060 \text{ cube feet of air at } 62^{\circ} \text{ for } 100 \text{ lbs. coal}$$

$$\begin{array}{lcl} \text{For volatilized portion, Hydrogen (H),} & 4 \text{ lbs.} & \times 457 = 1828 \text{ cube feet.} \\ \text{Carbon (C),} & 20 & \times 152 = 3040 \text{ " " } \\ \text{Sulphur (S),} & 1.25 & \times 57 = 71 \text{ " " } \end{array}$$

$$\begin{array}{lcl} \text{For fixed portion, Carbon,} & 60 \text{ lbs.} & \times 152 = 9120 \text{ " " } \\ \text{Total useful combustible,} & 85.25 & \times 14059 = 14059 \text{ " " for com-} \\ \text{plete combustion of } 100 \text{ lbs. coal of average composition at } 62^{\circ}. & & \end{array}$$

Compute Volume of Air at 62°, under One Atmosphere, chemically consumed in Complete Combustion of 1 Lb. of a given Fuel.

RULE.—Express constituent carbon, hydrogen, oxygen, and sulphur, as percentages of whole weight of fuel; divide oxygen by 8, deduct quotient from hydrogen, and multiply remainder by 3; multiply sulphur by .4; add products to the carbon, and multiply sum by 1.52. Final product is volume of air in cube feet.

To compute weight of air chemically consumed.—Divide volume thus found by 13.14; quotient is weight of air in lbs.

$$\text{Or, } 1.52 \left(C + 3 \left(H - \frac{O}{8} \right) + .4 S \right) = \text{Air. } O \text{ Oxygen.}$$

NOTE.—In ordinary or approximate computations, sulphur may be neglected.

EXAMPLE.—Assume 1 lb. Newcastle coal. C = 82.24, H = 5.42, O = 1.35.

$$\frac{6.44}{8} = .805, 5.42 - .805 = 4.615 \times 3 = 13.845, 1.35 \times .4 = .54, 13.845 + .54 = 14.385, \text{ and } 14.385 \times 1.52 = 21.87 \text{ cube feet.}$$

$$\text{Then } 21.87 \div 13.14 = 1.66 \text{ lbs.}$$

Labor.

For labor of a man, see Animal Power, pp. 433-34.

By Wheel-barrow.—A barrow-load may be assumed at 175 lbs. = 2 cube feet of space.

Blasting.—When labor is \$1 per day, hard rock in ordinary position may be blasted and loaded for 45 cents per cube yard.

The cost, however, in consequence of condition, position, etc., may vary from 5 cents to \$1.

See Blasting page 443.

17 cube yards of hard rock may be carted per day over a lead of 100 feet, at a cost of 7.29 cents per yard.

The preceding elements are essentially deduced from notes furnished by Elwell Morris, C.E., and the valuable treatise of John C. Trautwine, C.E., Phila., 1872.

Stone.

Hauling Stone.—A cart drawn by horses over an ordinary road will travel 1.5 miles per hour of trip = 2.3 miles per hour.

A four-horse team will haul from 25 to 36 cube feet of stone at each load.

Time expended in loading, unloading, etc., including delays, averages 35 minutes per trip. Cost of loading and unloading a cart, using a horse-crane at the quarry, and unloading by hand, when labor is \$1.25 per day, and a horse 75 cents, is 4 cents per perch = 24.75 cube feet = 1 cent per cube foot.

Work done by an animal is greatest when velocity with which he moves is less of greatest with which he can move when not impeded, and force then exerted is of utmost force the animal can exert at a dead pull.

Earthwork. (Molesworth.)

Proportion of Getters, Fillers, and Wheelers in different soils, Wheelers being calculated at 50 yards run.

| | Gett's. | Fill's. | Wheel's. | | Gett's. | Fill's. | Wheel's. |
|----------------------------|---------|---------|----------|-------------------|---------|---------|----------|
| In loose earth, sand, etc. | 1 | 1 | 1 | In Hard clay..... | 1 | 1.25 | 1.25 |
| " Compact..... | 1 | 2 | 2 | " Compact gravel | 1 | 1 | 1 |
| " Marl..... | 1 | 2 | 2 | " Rock, from..... | 3 | 1 | 1 |

Average Weight of Earths, Rocks, etc.

Per cube yard.

| | Lbs. | | Lbs. | | Lbs. | | Lbs. |
|-------------|------|------------|------|--------------|------|--------------|------|
| Sand..... | 3360 | Marl..... | 2912 | Sandstone... | 4368 | Granite..... | 4700 |
| Gravel..... | 3360 | Clay..... | 3472 | Shale..... | 4480 | Trap..... | 4700 |
| Mud..... | 2800 | Chalk..... | 4032 | Quartz..... | 4492 | Slate..... | 4710 |

Bulk of Rock, Earthwork, etc., Original Excavation assumed at 1.

When in Embankment.

| | | | | | |
|------------------|------|-----|-----|------------------------------------|------|
| Rock, large..... | 1.5 | 1.6 | 1.7 | Sand and gravel..... | 1.07 |
| Medium..... | 1.25 | to | 1.7 | Clay and earth after subsidence... | 1.08 |
| Metal..... | 1.2 | to | 1.8 | " " before " " | 1.2 |

In small stones, per cent. of interstices to total volume is 44 to 48, which is an increase in volume of solid rock to fragments of 79 and 92 per cent.

The relative proportions of Earth in Bank and Embankments, as given by different authorities, are so varied and so opposite that it is evident the difference is accidental, depending, primarily, upon the season, location, and character or condition of the earth, and then upon the height of the embankment, the manner and duration of its use.

This, ante p. 466, makes the embankment less, and Molesworth's error.

STRATION.—What is temperature of combustion of coal of average composi-

tion products as per preceding table 11.94, which $\times .246$ specific heat = 2.935 of heat at 1° .

ce, 14 133 units of combustion (from table, page 461) $\div 2.935 = 4812^{\circ}$ temperature of combustion of average coal.

plus air is mixed with products of combustion equal to volume of air chemically combined, total weight of gases for one lb. of this coal is increased to 22.64. following table, having a mean specific heat of .242.

n $22.64 \times .242 = 5.478$ units for 1° .

ce, 14 133 total heat of combustion $\div 5.478 = 2580^{\circ}$ temperature of combustion a little more than half that of undiluted products.

ing averages, it is seen that the evaporative efficiency of coal varies with volume of constituent carbon, and inversely with volume of fluent oxygen; and that it varies, not so much because there is more or less carbon, as, chiefly, because there is less or more oxygen. The percentage of constituent hydrogen, nitrogen, sulphur, and ash, taking averages, nearly constant, though there are individual exceptions, and their united, as a whole, appears to be nearly constant also.

Heat of Combustion.

number of times in combustion of a substance, its equivalent weight of water will be raised 1° , by heat evolved in combustion of substance.

| | | |
|-------------------|------------------------|---------------------------|
| al. 12 930 | Ether. 16 246 | Olefiant gas. 21 340 |
| coal. 14 545 | Olive oil. 17 750 | Hydrogen. 62 030 |

Combustion of Fuel.

Constituents of coal are Carbon, Hydrogen, Azote, and Oxygen.

Volatile products of combustion of coal are hydrogen and carbon, the names of which (relating to combustion in a furnace) are *Carburetted hydrogen* and *Bi-carburetted hydrogen* or *Olefiant gas*, which, upon coming in contact with atmospheric air, becomes *Carbonic acid* or *Carbonic oxide*, and uncombined *Nitrogen*.

Carbonic oxide is result of imperfect combustion, and Carbonic acid is result of perfect combustion.

Perfect combustion of carbon evolves heat as 15 to 4.55 compared to imperfect combustion of it, as when carbonic oxide is produced.

1 lb. carbon combines with 2.66 lbs. of oxygen, and produces 3.66 lbs. of carbonic acid.

Smoke is the combustible and incombustible products evolved in combustion of fuel which pass off by flues of a furnace, and it is composed of such portions of hydrogen and carbon of the fuel gas as have not been supplied or combined with oxygen, and consequently have not been converted either into steam or carbonic acid. The hydrogen so passing away is invisible, but the carbon, upon being separated from the hydrogen, loses its gaseous character, and returns to its elementary state of a black pulverulent body, and as such it becomes visible.

Auminous portion of coal is converted into gaseous state alone, carbonaceous portion only into solid state. It is partly combustible and partly incombustible.

For perfect combustion of 1 cube foot of coal gas, 2 cube feet of oxygen are required; and 10 cube feet of atmospheric air are necessary to supply this volume of gas. 1 cube foot of gas requires oxygen of 10 cube feet of air.

Furnaces with a natural draught, volume of air required exceeds the draught is produced artificially. Insufficient supply of air causes imperfect combustion; and, as a waste of heat.

14. When fibres of materials cross each other, friction is less than when they run in the same direction.

15. Friction is greater between surfaces of the same character than between those of different characters.

16. With hard substances, and within limits of abrasion, friction is as pressure, without regard to surfaces, time, or velocity.

17. The influence of duration of contact (friction of rest) varies with the nature of substances; thus, with hard bodies resting upon each other, the effect reaches a maximum very quickly; with soft bodies, very slowly; with wood upon wood, the limit is attained in a few minutes; and with metal on wood, the greatest effect is not attained for some days.

Coefficient of Friction of Journals.

Diameters from 2 to 4 ins. Speeds varied as 1 to 4. Pressure up to 2 tons

(From data of M. Morin.)

| SURFACES OF CONTACT. | | LUBRICATION. | Coefficient* Pressure = 1. Ordinary Lubrication. |
|----------------------------------|-----------|-----------------------------|---|
| Journals. | Bearings. | | |
| Cast iron on cast iron..... | | { Olive oil, or tallow..... | .07 to .08 |
| | | { Unctuous and wet..... | .14 |
| Cast iron on gun metal..... | | { Olive oil, or tallow..... | .07 to .08 |
| | | { Unctuous and wet..... | .16 |
| | | { Slightly unctuous..... | .18 |
| Cast iron on lignum-vitæ..... | | { Oil, or lard..... | — |
| | | { Lard and plumbago..... | .14 |
| Wrought iron on cast iron..... | | { Olive oil, or tallow..... | .07 to .08 |
| | | { Olive oil, or tallow..... | .07 to .08 |
| Wrought iron on gun metal..... | | { Unctuous and wet..... | .19 |
| | | { Slightly unctuous..... | .25 |
| Wrought iron on lignum-vitæ..... | | { Oil..... | .11 |
| | | { Unctuous..... | .19 |
| Gun metal on gun metal..... | | { Oil..... | .1 |
| Lignum-vitæ on cast iron..... | | { Unctuous..... | .15 |

* Continuous lubrication reduces the coefficients fully one half.

| SURFACES OF CONTACT. | | DISPOSITION OF FIBRES AND LUBRICATION. | Coefficient Pressure = 1. |
|----------------------|--|---|------------------------------|
| Oak..... | | Parallel and soaped | .16 |
| Iron on oak..... | | " " wet | .26 |
| " " "..... | | " " soaped | .21 |
| " " "..... | | " " wet | .22 |
| " " "..... | | " " soaped | .19 |
| " " "..... | | " " wet | .29 |
| " " "..... | | " " dry | .27 |
| " " "..... | | Perpendicular " " | .47 |

For bolts over 1 inch diameter, .47 of pressure, and over turned cast-iron pulleys, .47 of pressure.

Coefficient of Friction of Motion.

Condition of Surfaces and Unguents.

| | Dry. | Water. | Olive-oil. | Lard. | Tallow. | Dry Soap. | Olive-oil Soap. |
|----------------|------|--------|------------|-------|---------|-----------|-----------------|
| { On wood..... | .45 | .33 | — | — | — | — | — |
| { On iron..... | — | — | .15 | — | .19 | — | — |
| Mean..... | .18 | .31 | .07 | .09 | .09 | .2 | .2 |
| { Raw..... | .54 | .36 | .16 | — | .2 | — | — |
| { Dry..... | .34 | .31 | .14 | — | .14 | — | — |
| Mean..... | .42 | .24 | .09 | .07 | .08 | .2 | .2 |
| | .36 | .25 | — | .07 | .07 | .35 | .35 |

Relative Value of Unguents to Reduce Friction.

| UNGUENTS. | Wood upon Wood. | Wood upon Metals. | Metals upon Metals. | UNGUENTS. | Wood upon Wood. | Wood upon Metals. | Metals upon Metals. |
|--------------------|-----------------|-------------------|---------------------|----------------|-----------------|-------------------|---------------------|
| Try soap..... | .4 | .32 | .27 | Olive oil..... | — | 1 | 1 |
| lard..... | .82 | .85 | .7 | Tallow..... | 1 | .93 | .8 |
| lard and plumbago. | — | .67 | .96 | Water..... | .22 | .24 | .13 |

To Determine Coefficient of Friction of Bodies.

Place them upon a horizontal plane, attach a cord to them, and lead it in direction parallel to the plane over a pulley, and suspend from it a scale in which weights are to be placed until body moves.

Then weight that moves the body is numerator, and weight of body moved is denominator of a fraction, which represents coefficient required.

ILLUSTRATION.—If, by a pressure of 320 lbs. friction amounts to 80 lbs., its coefficient of friction in this case would be $80 \div 320 = .25$.

Hence, if coefficient of friction of a wagon over a gravel road was .25, and the load 400 lbs., the power required to draw it would be $400 \times .25 = 2100$ lbs.

Coefficients of Axle Friction. (*M. Morin.*)

| SUBSTANCES. | Condition of Surfaces and Unguents. | | | | |
|------------------------------------|-------------------------------------|----------------------------|-----------------------|---------------|---------------|
| | Dry and a little Greasy. | Greasy and wet with Water. | Oil, Tallow, or Lard. | In usual way. | Continuously. |
| Steel metal upon bell metal..... | | | .097 | | |
| Cast iron upon bell metal..... | .194 | .161 | .075 | .054 | .065 |
| Cast iron upon cast iron..... | | .079 | .075 | .054 | |
| Cast iron upon lignum-vitæ..... | .185 | | .1 | .092 | .109 |
| Wrought iron upon bell metal..... | .251 | .189 | .075 | .054 | .09 |
| Wrought iron upon cast iron..... | | | .075 | .054 | |
| Wrought iron upon lignum-vitæ..... | .188 | | .125 | | |

Friction of a journal of an axle which presses on one side only, as in a worn bearing, is less than when it presses at all points, the difference being about .005.

Friction of Axles.—With axles, friction of motion has alone been experimented upon. When weight upon axle and radius of its journal is given, mechanical effect of friction may be readily determined.

The mechanical effect absorbed by, or of friction, increases with pressure or weight upon journal of axle and number of revolutions.

Friction of an axle is greater the deeper it lies in its bearing.

If journal of an axle lies in a prismatic bearing, as in a triangle, etc., friction is greater, as there is more pressure on, and consequently greater friction in contact: in a triangular bearing it is about double that of a cylindrical bearing.

To Compute Mechanical Effect of Friction of an Axle.

$$\frac{p n f W r}{30} = F. \quad n \text{ representing number of revolutions, and } r \text{ in feet.}$$

ILLUSTRATION.—Weight of a wheel, with its axle or shaft resting on 360 lbs.; diameter of journals 2 ins.; and number of revolutions 30. Mechanical effect of the friction, the coefficient of it being .16?

$$\frac{3 \cdot 1416 \times 30 \times .16 \times 360 \times 1 \div 12}{30} = \frac{452.4}{30} = 15.08$$

By application of friction-wheels (rollers) friction is much reduced, and mechanical effect then becomes, when weights of friction-wheels are disregarded,

$$\frac{p n f W r}{30} \times \frac{r'}{a' \cos. a \div 2} = F. \quad r' \text{ representing radii of axles of friction-wheels,}$$

 a' radii of friction-wheels, and a angle of lines of direction between axis of roller and axis of friction-wheels.

When a single friction-wheel is used, $\frac{2 p r n}{60} \times f W = F$, and $\frac{F}{r' \div a} = F'. \quad F'$ representing mechanical effect.

ILLUSTRATION.—A wheel and its shaft, making 5 revolutions per minute, weighs 30 000 lbs.; its diameter and that of its journals are 32 feet and 10 ins. The journals rest upon a friction-wheel, the radius of which is 5 times greater than its axle.

1. What is the power at circumference of wheel necessary to overcome friction?
 2. What is mechanical effect of the friction? 3. What is reduction of friction by use of the friction-wheel?

1. $\frac{32 \div 2 \times 12}{10 \div 2} = 38.4$, circum. of wheel = 38.4 times that of axle.

Coefficient of friction assumed at .075. Hence $\frac{30000 \times .075}{38.4} = 58.59$ lbs. = power at circum. to overcome friction at axle. 2. $\frac{10 \times 3.1416}{12} = 2.618$ feet = distance passed by friction.

Consequently, $\frac{2.618 \times 5}{60} = .2181$ feet = distance passed by friction in one second.

Hence, $.2181 \times 2250$ ($30000 \times .075$) = 490.725. 3. $1 \div 5 = .2$ = radius of friction axle \div by radius of friction-wheel, and $38.4 \times .2 = 7.68$ = friction referred to circum. of wheel, and $\frac{490.725}{7.68} = 63.89$ = mechanical effect by application of friction-wheel = a reduction of four fifths.

Friction of Pivots.

Friction on Pivots is independent of their velocity, increases in a greater degree than their pressures, and approximates very near to that of sliding and axle friction.

Friction on Conical Bearings is greater than with like elements on plane surfaces.

Figure of point of a pivot, as to its acuteness, affects friction: with great pressure the most advantageous angle for the figure ranges from 30° to 45° ; with less pressure it may be reduced to 10° and 12° .

Relative Value of Angles of Pivots.

60° 1 | 15°66 | 45°39

Relative Values of different Materials for use as Pivots.

Agate..... .83 | Granite..... 1 | Tempered steel..... .44
 Glass..... .55 | Rock crystal..... .76

On and Rigidity of Cordage.

Amontons and Coulomb, with an apparatus of Amontons' deductions:

caused by stiffness of cords about the same or like the suspended weight.

caused by stiffness of cords increases not only in direct proportion of diameter, but also in direct proportion of diameter

Consequently, that resistance to motion over the same or like pulleys, arising from stiffness of cords, is in direct compound proportion of suspended weight and diameter of cords.

3. That resistance to bending varied inversely as diameter of sheave or drum.

4. That complete resistance is represented by expression $\frac{S+CT}{d}$. *S* representing constant for each rope and sheave, expressing stiffness of rope; *T* tension of rope which is being bent, expressed by *CT*; *C* constant for each rope and sheave; and *d* diameter of sheave, including diameter of rope.

5. That stiffness of tarred ropes is sensibly greater than that of white ropes.

Extending results obtained by Coulomb, Morin furnishes following formulas:

For White Ropes: $12 n \div d (.00215 + .00177 n + .0012 W) = R$. For Tarred Ropes: $12 n \div d (.01054 + .0025 n + .0014 W) = R$. *R* representing rigidity in lbs., *n* number of yarns, *d* diameter of sheave in ins. and rope combined, and *W* weight in lbs.

ILLUSTRATION.—What is value of stiffness or resistance of a dry white rope having a diameter of 60 yarns, which runs over a sheave 6 ins. in diameter in the groove, with an attached weight of 1000 lbs.?

Assume diameter for 60 yarns to be 7.2 ins. Then $\frac{12 \times 60}{7.2} (.00215 + .00177 \times 100 + .0012 \times 1000) = 100 \times 1.30835 = 130.835$ lbs.

Value of natural stiffness of ropes increases as the square of number of breads nearly, and value of stiffness proportional to tension is directly as number of threads, being a constant number. Hence, having the rigidity for any number of threads, the rigidity for a greater or lesser number is readily ascertained.

Wire Ropes.

Weisbach deduced from his experiments on wire ropes that their rigidity or diameters capable of supporting equal strains with hemp ropes is considerably less.

Wire ropes, newly tarred or greased, have about 40 per cent. less rigidity than untarred ropes.

Rolling Friction.

Rolling Friction increases with pressure, and is inversely as diameter of rolling body.

For rolling upon compressed wood, $f = .019$ to $.031$.

When a Body is moved upon Rollers and Power applied at the Base of the Body, $f + f' \frac{W}{r} = F$. *f* and *f'* representing coefficients of friction of two surfaces upon which rollers act.

When Power is applied at Circumference of Roller, $fW \div r = F$.

When Power is applied at Axis of Roller, $fW \div r \div 2 = F$.

Bearings for Propeller Shaft. (Mr. John Pe

| BEARINGS. | Pressure per Sq. Inch. | Time of Op- eration. | BEARINGS. |
|----------------------------|------------------------------|----------------------------|----------------------|
| | Lbs. | Min. | |
| Babbit's metal on iron*... | 1600 | 8 | Brass on iron† |
| Box on brass | 4480 | 5 | Brass on iron† |
| Box on iron | 448 | 30 | Lignum-vitæ on brass |
| Box on brass | 448 | 30 | Snake-wood on brass |
| Box on iron | 448 | 30 | Lignum-vitæ on iron |

* Rolled out.

† Abraded.
R R*

‡ Set fast.

Result of Experiments upon Friction of Several Instruments. (R. S. Ball.)

| INSTRUMENT. | Friction. | | Velocity ratio. | Mechanical efficiency. | Useful effect. |
|------------------------------------|-----------|--------|-----------------|------------------------|----------------|
| | F | L | | | Per Cent. |
| Pulley, single | 2.21 | +.5453 | 2 | 1.8 | 90 |
| " 3 sheaves | 2.36 | +.238 | 6 | 4 | 64 |
| " differential | 3.87 | +.151 | 16 | 6.1 | 36 |
| Screw | .0 | +.014 | 193 | 70 | 36 |
| Inclined plane, angle 17° 2' | .09 | +.55 | 3.4 | 1.72 | 51 |
| Screw Jack | .66 | +.007 | 414 | 116 | 28 |
| Wheel and Axle | .204 | +.043 | 31 | 22 | 70 |
| " " Barrel | .5 | +.169 | 5.95 | 5.55 | 93 |
| " " Pinion | 2.46 | +.21 | 8 | 4.1 | 51 |
| Crane | .0 | +.056 | 23 | 18 | 78 |
| " | .185 | +.008 | 137 | 87 | 63 |

F representing friction, and L load.

ILLUSTRATION I.—If it is required to ascertain power necessary to raise 200 lbs. 2 feet, by a single movable pulley, $200 \times .5453 + 2.21 = 111.27$ lbs., which must be applied as power to raise 200 lbs. 2 feet. $111.27 \times 2 = 222.54$ lbs. Hence, for application of 222.54 lbs., 200 or 89.87 per cent. are usefully or effectively employed.

2.—If it is required to raise 100 lbs. by a three-sheave pulley, then $100 \times .238 + 2.36 = 26.16$ lbs., which must be applied as power to raise 100 lbs. 6 feet ($3 \times 2 = 6$). $26.16 \times 6 = 156.96$ lbs. Hence, for application of 156.96 lbs., 100 or 63.71 per cent. are effectively employed.

3.—The velocity ratio of a crane being 137, and its mechanical efficiency 87, a man applying 26 lbs. to it can raise $87 \times 26 = 2262$ lbs.

Application of preceding Results.

ILLUSTRATION. — If a vessel, including cradle, weighing 1000 tons, is to be drawn upon an inclined plane having a rise of 10 feet in 100 of its length, what will be the resistance to be overcome, the cradle being supported on wrought-iron axles in cast-iron rollers, running on cast-iron rails?

$$\frac{1000 \times 10}{100} = 100 \text{ tons} = \text{power required to draw vessel independent of friction.}$$

Ratio of friction to pressure of wrought iron on cast, in an axle and its bearing, .075. Ratio of ditto of cast iron upon cast, say .005.

Hence $.075 + .005 = .08$ of 1000 tons = 80 tons, which, added to 100 tons before deducted, gives 180 tons, or resistance to be overcome.

Power or effect lost by friction in axles and their bearing may be expressed by formula

$$\frac{W f d r}{230} = P. \quad f \text{ representing coefficient of friction, } d \text{ diameter of axle in ins., and } r \text{ number of revolutions per minute.}$$

ILLUSTRATION.—Pressure on piston of a steam-engine is 20000 lbs., number of revolutions 20, and diameter of driving shaft of wrought iron in a brass journal 8 ins.; what is the effect of friction?

$$\frac{20000 \times .07 \times 8 \times 20}{230} = 973.91 \text{ lbs.}$$

Hence $P \div 33000 = HP.$ v representing circumference of shaft in feet \times by revolutions per minute.

The power or effect lost by friction in guides or slides may be expressed by following formula:

$$\frac{W f s r}{60 \times \sqrt{(5 l^2 - s^2)}} = P. \quad s \text{ representing stroke of cross-head, and } l \text{ length of connecting rod in feet.}$$

Frictional Resistances.

Friction of Steam-engines.

Friction of Condensing Engines in Lbs. per Sq. Inch of Piston.

| Order. | Oscillating and Trunk. | Beam and Geared. | Direct-acting and Vertical. | Diameter of Cylinder. | Oscillating and Trunk. | Beam and Geared. | Direct-acting and Vertical. |
|--------|------------------------|------------------|-----------------------------|-----------------------|------------------------|------------------|-----------------------------|
| | 5 | 6 | 7 | 50 | 2.5 | 2.7 | 3.3 |
| | 4 | 5 | 6 | 60 | 2.4 | 2.6 | 3 |
| | 3.5 | 4 | 5 | 70 | 2.3 | 2.5 | 2.7 |
| | 3 | 3.6 | 4.5 | 80 | 2 | 2.3 | 2.6 |
| | 3 | 3.5 | 4 | 100 | 1.6 | 2.2 | 2.5 |
| | 2.6 | 3 | 3.5 | 110 | 1.5 | 2 | 2.1 |

Experiments upon different steam-engines have determined that friction, pressure on piston is about 12 lbs. per sq. inch, does not exceed 1.5 lbs., about one tenth of power exerted.

Friction of double cylinder (50-inch diam.) direct-acting condensing proper engine is 1.25 lbs. per sq. inch of piston = 10.3 per cent. of total power expended; friction of load is .9 lbs. per sq. inch of piston = 7.5 per cent. of pressure; and friction of propeller is 1.3 lbs. per sq. inch of piston = per cent. of total power = 28.6 per cent.

Friction of double cylinder (70-inch diam.) inclined condensing water-wheel engine with its load is 15 per cent. of total power developed.

In general, when engines are in good order, their efficiency ranges from 80 per cent. for small engines to 93 per cent. for large.

Power required to work air-pumps is 5 per cent., and to work feed-pumps 1 per cent.

Results of Experiments upon Friction of Machinery.

(Davison.)

Steam-engine, vertical beam, one tenth its power; 190 feet horizontal, and feet vertical shafting, with 34 bearings, having an area of 3300 sq. ins., 11 pair of spur and bevel wheels; 7.65 HP.

Set of three-throw Pumps, 6 ins. in diam., delivering 5000 gallons per hour at an elevation of 165 feet; 4.7 HP, or about 13 per cent.

Two pair iron Rollers and an elevator, grinding and raising 320 bushels per hour; 8.5 HP.

Rolling Machine, 800 bushels malt at a time; 5.68 HP.

Archimedes Screw (ninety-five feet), 15 ins. in diameter, and an elevator conveying 320 bushels malt per hour to a height of 65 feet; 3.13 HP.

Friction Clutch.—Driven by a leather belt 14 ins. in width; face of clutch 8 in. deep; broke a cast-iron shaft 6.5 ins. in diameter.

Lax Mill (M. Cornut, 1872).—Two condensing engines, cylinders, 12.9 X 44.3 ins. stroke, and 22 ins. X 59.8 ins. stroke. Friction steam, 20 per cent. 47.

With vegetable oil and hand oiling a steam pressure was required, and with mineral oil and continuous only was required.

Continuous oiling, a saving of 44 per cent. was

Flax Mill.

Power required to Drive Engine, Shafting, and entire Machinery. (M. Cornut.)

| PARTS. | Total. | Indicated Horse-power. | | Effect of Machine. |
|---|--------|------------------------|--------|--------------------|
| | | One Machine at work. | empty. | |
| Engines, shafting, and belts..... | 30.41 | — | — | — |
| 4 cards..... | 8.42 | 2.105 | 1.423 | 32 |
| 14 drawing frames (29 heads or 156 } slivers)..... | 7.19 | .0934 | .0794 | 15 |
| 4 combing machines..... | 2.22 | .555 | .151 | 78 |
| 6 roving frames (330 spindles)..... | 7.78 | .02627* | 2.434 | 7.3 |
| 20 spinning frames. | | | | |
| Dry (1480 spindles)..... | 47.5 | .0321* | 2.515 | 21.6 |
| Wet (2080 ")..... | 46.59 | .0224* | 1.613 | 19 |
| Total 150.11 HP. | | | | |

* Per 100 spindles.

Estimate of Horse's Power.—2080 spindles, wet, 34.4 per HP, long fibre.
 640 " " " 20.1 " " " "
 840 " " " 14.5 " " " "
 3560 " " average, 23.7 " " "

The HP per 100 spindles varies inversely as sq. root of their number.

Winding Engine (G. H. Daglish).

Shafts 738 to 1740 feet in depth; cylinder 65 × 84 ins. stroke; pressure of steam 19 lbs. per sq. inch; revolutions 12.5 per minute; mean diameter of drum, 26 feet. HP 313.4; effect 235 = 75 per cent.

Tools. (Dr. Hartig).

Single shearing, $.1 + \frac{n t^2}{26.7} = \text{HP to drive tool}$. n representing number of cuts per minute, t thickness of plate, and $\frac{a F}{1980000} = \text{HP to shear}$. a representing area of surface cut or punched per hour in sq. ins., and F (1166 + 1691 t) a factor expressing work required to cut or shear a surface of 1 inch square.

ILLUSTRATION.—A shearing machine cutting 4648 sq. ins. of surface per hour. In plates .4 inch thick, required .68 HP to run and 4.3 to operate it, equal to 5 horses.

Iron Plate-bending. $\frac{85000 b t^2 l}{r} = P$ for cold plates, and $\frac{11300 b t^2 l}{r} = P$ for red-hot plates. b , t , and l representing breadth, thickness, and length of plate, r radius of curvature, all in ins., and P net power of bending.

Power for large rolls when running only .5 to .6 HP.

Ordinary Cutting Tools, in Metal.

Materials of a brittle nature, as cast iron, are reduced most economically in power consumed, by heavy cuts; while materials which yield tough curling shavings are more economically reduced by thinner cuttings. Following formulas apply to light cutting work:

Power required to plane cast iron is—

Planing Cast iron, $W \left(.0155 + \frac{1}{11000 s} \right) = \text{HP}$. W representing weight of cast iron, and s average sectional area of shavings, in sq. ins.

Wrought iron, and Gun-metal, with cuts of an average character—

$.2 W = \text{HP}$ | Wrought iron, $.052 W = \text{HP}$ | Gun-metal, $.0127 W = \text{HP}$

and Molding.—Run without cutting. $\frac{N}{2000} = \text{HP}$. N revolutions of all the shafts per minute.

Molding.—Pine, $.0566 + \frac{.02268}{h}$, and Red Beech, $.08895 + \frac{.00731}{h} = \text{HP}$. h representing depth of wood cut down to form molding.

Turning.—Steel, $.047 W = \text{HP}$; Wrought iron, $.0327 W = \text{HP}$; Cast iron, $.0314 W = \text{HP}$.

For turning off metals, power required is less than for planing, and it is ascertained that greater power is required for small diameters than large.

Light Lathes. $.05 + .0005 n = \text{HP}$; 1 or 2 shafts, $.05 + .0012 n = \text{HP}$; 3 or 4 shafts, $.05 + .05 n = \text{HP}$. **Heavy Lathes.** $.025 + .0031 n$; $.025 + .053 n$; $.025 + .18 n$. n representing number of revolutions of spindle per minute.

Drilling.—Power required to remove a given weight of metal is greater than planing. Volume being taken in place of weight.

Holes from .4 to 2 ins. in diameter.

Cast iron, dry. $V \left(.0168 + \frac{.00067}{d} \right) = \text{HP}$. Wrought iron, oil. $V \left(.0168 + \frac{.0269}{d} \right) = \text{HP}$.

V representing volume removed in cube ins. per hour, and d diameter of hole.

Without gearing, $.0006 n + .0005 n'$; with gearing, $.0006 n + .001 n'$; radial drills without gearing, $.0006 n + .004 n'$; radial drills with gearing, $.04 + .0006 n + .04 n'$. n representing number of revolutions per minute of gearing shaft, and n' drill.

Slotting.—Stroke 8 ins. $.045 + \frac{n s}{4000} = \text{HP}$. n representing number of strokes per minute, and s stroke in ins.

Wood-sawing, Circular.—A cube foot of soft wood and half a cube foot of hard, reduced to sawdust, requires 1 HP.

Hard wood, $\frac{A c}{6} = \text{HP}'$. Soft wood, $\frac{A c}{12} = \text{HP}'$. A representing area in sq. feet and HP' horse-power per sq. foot, both cut per hour, and c width of cut in ins.

From .4 to 4 ins. in diameter.—Pine. $V \left(.000125 + \frac{.00656}{d} \right) = \text{HP}$.

Dry pine timber. $.00428 + .0065 \frac{S c}{f} = \text{HP}'$. S representing stroke of saw in feet, and f feed per cut in ins.

$\frac{n d}{32000} = \text{HP}$ for horse-power to run only without cutting. d representing diameter of saw in ins., and n number of revolutions per minute.

Net power required to cut with a circular saw is proportional to volume of material removed. For a saw cutting hot iron, at a circumferential speed of 7875 feet per minute, and making a cut .14 inch wide, power is expressed by formulas—

$.702 A = \text{HP}$, for red-hot iron. $1.013 A = \text{HP}$, for red-hot steel.

A representing sectional area of surface cut through, in sq. feet.

Vertical Saw. $.00428 + .0065 \frac{S c}{f} = \text{HP}$ in dry pine timber per sq. foot per hour. S representing stroke of saw in feet, c width of cut in ins., and f feed of it in ins.

Band Saw. $.0034 + \frac{.758 c v}{10000 f} = \text{HP}'$ in Pine. $.00483 + \frac{.957 c v}{10000 f} = \text{HP}'$ in Oak. $.0576 + \frac{1.127 c v}{10000 f} = \text{HP}'$ in Beech. v representing velocity of rate of feed, feet per minute.

Screw Cutting. Screws, $\frac{5 l d^3}{64} = \text{HP}$. Taps, $\frac{l}{64}$. d in ins., and l length cut in feet per hour. Machine of medium dimensions, .2 HP.

FUEL.

Coal.

Anthracite.

Anthracite or Glance Coal, or Cullm—Is hard, compact, lustrous, and some times iridescent, most perfect being entirely free from bitumen; it ignites with difficulty, and breaks into fragments when heated.

Evaporative power, in furnace of a steam-boiler and under pressure, from 7.5 to 9.5 lbs. of fresh water per lb. of coal.

Coal from one pit will sometimes vary 6 per cent. in evaporative value.

Elements of Various American Coals.

| | Specific Gravity. | Fixed Carbon. | Volatile Matter. | Water. | Moisture. | Ash. | Earth Matter. |
|--------------------------------|-------------------|---------------|------------------|-----------|-----------|-----------|---------------|
| | | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
| Illinois, Warren Co. | 1.23 | 51.7 | 43.1 | — | — | — | 5.2 |
| Bureau " | 1.32 | 57.6 | 28.8 | — | 11.2 | 2.4 | — |
| Mercer " | 1.26 | 54.8 | 31.2 | — | 8.4 | 5.6 | — |
| Indiana, Clay " | 1.28 | 56.5 | 32.5 | 8.5 | — | 2.5 | — |
| Coopriders. | 1.28 | 50.5 | 42.5 | 3 | — | 4 | — |
| Pennsyl- } Connellsville. | 1.28 | 65 | 24 | 4.5 | — | 6.5 | — |
| vania } Youghiogheny ... | 1.3 | 58.4 | 35 | — | 1 | 5.6 | — |
| Fayette Co. | 1.29 | 58 | 34 | 3 | — | 5 | — |
| Kentucky, Sardinia. | 1.32 | 51 | 42.5 | 2 | — | 4.5 | — |
| Mud River. | 1.28 | 57 | 37 | 3.5 | — | 2.5 | — |
| Ohio, Nelsonville. | 1.27 | 58.4 | 33.05 | 0.65 | — | 1.9 | — |
| Colorado, Carbon City. | 1.21 | 56.8 | 34.2 | 4.5 | — | 4.5 | — |
| Washington Territory. | 1.32 | 58.25 | 31.75 | 7 | — | 3 | — |

Coke.

Coke.—Coking in a close oven will give an increase of yield of 40 per cent. over coking in heaps, gain in bulk being 22 per cent. Coals when coked in heaps will lose in bulk.

Cannel and Welsh (Cardiff) coals when coked in retorts will gain from 20 to 30 per cent. in bulk and lose 36.5 per cent. in weight.

Relative costs of coal and coke for like results, as developed by an experiment in a locomotive boiler, are as 1 to 2.4.

Evaporative power in furnace of a steam-boiler and under pressure, from 7.5 to 8.5 lbs. of fresh water per lb.

Bituminous coal will yield from 60 to 80 per cent. of coke. Averaging 66 per cent. It is capable of absorbing 15 to 20 per cent. of moisture.

Heat of combustion lost in coking of bituminous coal 40 per cent.

Charcoal.

Charcoal, properly termed, is not made below a temperature of 536°. The best quality is made from Oak, Maple, Beech, and Chestnut.

Wood will furnish, when properly burned, about 23 per cent. of coal.

Charcoal absorbs, upon an average of the various kinds, from .8 per cent. of water for Beech, to 16.3 for Black Poplar, Oak absorbing about 4.28, and Pine 8.9.

Evaporative power, in furnace of a boiler and under pressure, is 5.5 lbs. of fresh water per lb. of coal.

Volume of air chemically required for combustion of 1 lb. of charcoal is, when it consists of 79 carbon, 129 cubé feet at 62°.

128 bushels charcoal and 432 lbs. limestone, with 2612 lbs. of ore, will smelt 1 ton of pig iron.

Luce of Charcoal from Various Woods dried at 300° and Carbonized at 572°. (M. Violette.)

| Wood. | Weight. | Wood. | Weight. | Wood. | Weight. |
|-----------------------------|-----------|---------------|-----------|------------------|-----------|
| | Per Cent. | | Per Cent. | | Per Cent. |
| | 62.8 | Larch..... | 40.31 | Maple..... | 33.75 |
| | 46.09 | Chestnut..... | 36.06 | Willow..... | 33.74 |
| | 44.25 | Apple..... | 34.69 | Black elder..... | 33.61 |
| | 41.48 | Elm..... | 34.59 | Ash..... | 33.28 |
| ar roots..... | 40.9 | Birch..... | 34.17 | Pear..... | 31.88 |
| Poplar..... 31.12 per cent. | | | | | |

In a Green or Ordinary State. (Weight per cent.)

| | | | |
|--------------|-----------------|------------------|--------------------|
| le..... 23.8 | Birch..... 24.1 | Oak..... 22.85 | Red Pine..... 23 |
| 26.7 | Elm..... 25.1 | " young... 33.3 | White Pine... 23.5 |
| 21.1 | Maple..... 22.9 | Poplar..... 20.5 | Willow..... 18.6 |

appears from this that cork, the lightest of woods, yields largest per-centage charcoal, about 63 per cent.; and that poplar yields lowest, about 31 per cent. It does not appear to be any definite relation between density of wood and time of yield.

reduced by a slow process of charring is very nearly 50 per cent. greater than by quick process.

Lignite.

Lignite is an imperfect mineral coal. It is distinguished from coal by large proportion of oxygen, being from 13 to 29 per cent. Its specific gravity ranges from 1.12 to 1.35.

Elements of Various American Lignites. (W. M. Barr.)

| LOCATIONS. | Spec. Grav. | Fixed Carbon. | Volatile Matter. | Water. | Ash. | Total Volatile. | Coke. |
|-------------------------|-------------|---------------|------------------|-----------|-----------|-----------------|-----------|
| | | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. | Per Cent. |
| stucky..... | 1.2 | 40 | 23 | 30 | 7 | 53 | 47 |
| Blandville.... | 1.17 | 31 | 48 | 11.5 | 9.5 | 59.5 | 40.5 |
| ashington Terr'y..... | — | 58.25 | 31.75 | 7 | 3 | 38.75 | 61.25 |
| ancouver's Island..... | — | 62 | 31 | 4 | 3 | 35 | 65 |
| orado, Carbon City.... | 1.27 | 41.25 | 46 | 3.5 | 9.25 | 49.5 | 50.5 |
| Canon City.... | 1.28 | 56.8 | 34.2 | 4.5 | 4.5 | 38.7 | 61.3 |
| ansas..... | — | 34.5 | 28.5 | 32 | 5 | 62.5 | 39.5 |
| tas, Robertson Co. | 1.23 | 45 | 39.5 | 11 | 4.5 | 50.5 | 49.5 |

Asphalt.

Asphalt, alike to Lignite, contains a large proportion of oxygen.

Wood.

Wood, as a combustible, is divided into two classes, the hard, as Oak, Ash, Elm, Beech, Maple, and Hickory, and soft, as Pine, Cotton, Birch, Sycamore, and Chestnut.

Green wood subjected to a temperature ranging from 300° to 450° will lose 30 to 45 per cent. of its weight.

At a temperature of 300° Oak, Ash, Elm, and Walnut in a comparatively seasoned state, lost from 16 to 18 per cent.

Woods contain an average of 56 per cent. of immediate carbon.

From an analysis of M. Violette it appears that trunks and branches lose a proportion throughout the tree, and that of the bark less. That wood contains a large proportion of carbon, 45 per cent., and that bark contains 40 per cent. and roots 37 per cent. and small roots have less carbon, 34.5 per cent. and roots of 7 per cent.

when dried at 222° lost 60 per

branches of 100°

Evaporative power of 1 cube foot of pine wood is equal to that of 1 cube foot of fresh water; or, in the furnace of a steam-boiler and under pressure, it is 4.75 lbs. fresh water for 1 lb. of wood.

Northern Wood.—One cord of *hard* wood and one cord of *soft* wood, such as is used upon Lakes Ontario and Erie, is equal in evaporative effects to 2000 lbs. of anthracite coal.

Western Wood.—One cord of the description used by the river steamboats is equal in evaporative qualities to 12 bushels (960 lbs.) of Pittsburgh coal 9 cords cotton, ash, and cypress wood are equal to 7 cords of yellow pine.

Solid portion (*lignin*) of all woods, wherever and under whatever circumstances of growth, are nearly similar, specific gravity being as 1.46 to 1.53. Densest woods give greatest heat, as charcoal produces greater heat than flame.

For every 14 parts of an ordinary pile of wood there are 11 parts of space; or a cord of wood in pile has 71.68 feet of solid wood and 56.32 feet of void.

Trees in the early part of April contain 20 per cent. more water than they do in the end of January.

Ash.

Proportion of Ash in 100 Lbs. of several Woods.

| Woods. | Wood. | Leaves. | Woods. | Wood. | Leaves. |
|------------|-----------|-----------|-----------------|-----------|-----------|
| | Per Cent. | Per Cent. | | Per Cent. | Per Cent. |
| Ash..... | .5 | — | Elm..... | 1.88 | 11.8 |
| Beech..... | .35 | 5.4 | Oak..... | .21 | 4 |
| Birch..... | .34 | 5 | Pitch Pine..... | .25 | 3.15 |

Peat.

Peat is the organic matter, or soil, of bogs, swamps, and marshes—decayed moss, sedge, coarse grass, etc.—in beds varying from 1 to 40 feet in depth. That near the surface, and less advanced in transformation, is light, spongy, and fibrous, of reddish-brown color; lower down, it is more compact, of a darker brown color; and, in lowest strata, it is of a blackish brown, or almost black, of a pitchy or unctuous surface, the fibrous texture nearly or altogether transformed.

Peat, in its natural condition, contains from 75 to 80 per cent. of water. Occasionally its constituent water amounts to 85 or 90 per cent., in which case peat is of the consistency of mire. It shrinks very much in drying; and its specific gravity varies from .22 to 1.06, surface peat being lightest, and deepest peat densest.

When peat is milled, so that its fibre is broken up, its contraction in drying is much increased, and in this condition it is termed *condensed*.

When ordinarily air dried, it will contain 20 to 30 per cent. of moisture, and when effectively dried at least 15 per cent.

Products of Distillation of Peat.

Water 31.4. Tar 2.8. Gas 36.6. Charcoal 29.2.

The distillation of the tar will yield paraffine, oil, gas, water, and charcoal, and the water acetic acid, wood spirit, and chloride of ammonia.

Evaporative power, in furnace of a steam-boiler and under pressure, is from 3.5 to 5 lbs. of fresh water per lb. of fuel.

Tan.

Hamlock bark, after having been used in the process of tanning as a fuel. It consists of the fibre of the bark, and cleft, 5 parts of bark produce 4 parts of dry tan; and when perfectly dry, or containing but 15 per cent. of water, while that of tan in an ordinary state of dryness, contains 42.84. Weight of water evaporated at 212° of water, is 6.31 lbs. for dry, and 4.44 for moist.

Relative Values of different Fuels.

| DESCRIPTION. | Lbs. of Steam from Water at 212° by 1 lb. of Fuel. | Relative Evaporative Power for equal Weights. | Relative Evaporative Power for equal Volumes. | Relative Rapidities of Ignition. | Relative Freedom from Waste. | Relative Completeness of Combustion. | Relative Weights. |
|--------------------|--|---|---|----------------------------------|------------------------------|--------------------------------------|-------------------|
| Anthracites. | | | | | | | |
| Mountain, Pa. | 10.7 | 1 | 1 | .505 | .633 | .725 | .945 |
| Seaweed. | 9.88 | .923 | .982 | .207 | .748 | .6 | 1 |
| Luminous. | | | | | | | |
| | 8.66 | .809 | .776 | .595 | .887 | .346 | .904 |
| | 8.48 | .792 | .738 | .588 | .418 | 1 | .876 |
| | 7.84 | .733 | .663 | .581 | 1 | .333 | .852 |
| Ind. | 7.34 | .686 | .616 | 1 | .984 | .578 | .848 |
| | 6.95 | .649 | .625 | .521 | .499 | .649 | .909 |
| dry. | 4.69 | .436 | .175 | — | 16.417 | — | — |

ts, Evaporative Powers per Weight and Bulk, etc., of different Fuels: (W. R. Johnson and others.)

| FUEL. | Specific Gravity. | Weight per Cube Foot. | Steam from Water at 212° by 1 lb. of Fuel. | Clinker from 100 lbs. | Cube Feet in a Ton. |
|----------------------|-------------------|-----------------------|--|-----------------------|---------------------|
| BITUMINOUS. | | Lbs. | Lbs. | Lbs. | No. |
| and, maximum. | 1.313 | 52.92 | 10.7 | 2.13 | 42.3 |
| minimum. | 1.337 | 54.29 | 9.44 | 4.53 | 41.2 |
| | 1.326 | 53.22 | 10.14 | — | 42.09 |
| igan. | 1.23 | 48.3 | 7.7 | — | 46.37 |
| h. | 1.324 | 53.05 | 9.72 | 3.4 | 42.2 |
| n, screened. | 1.283 | 45.72 | 8.94 | 3.33 | 49 |
| average. | 1.294 | 54.04 | 8.39 | 8.82 | 41.4 |
| , Hartley. | 1.257 | 50.82 | 8.76 | 3.14 | 44 |
| | 1.318 | 49.25 | 8.41 | 6.13 | 45 |
| l. | 1.252 | 46.81 | 8.2 | .94 | 47.8 |
| | 1.338 | 47.44 | 7.99 | 2.25 | 47.2 |
| rtley. | 1.262 | 47.88 | 7.84 | 1.86 | 46.7 |
| ll, Va. | 1.285 | 45.49 | 7.67 | 3.86 | 49.2 |
| , Ind. | 1.273 | 47.65 | 7.34 | 1.64 | 47 |
| alkeith. | 1.519 | 51.09 | 7.08 | 5.63 | 43.8 |
| | — | — | 5.72 | — | — |
| | 1.231 | 48.3 | — | — | — |
| ANTHRACITE. | | | | | |
| Mountain. | 1.464 | 53.79 | 10.11 | 3.03 | 41.6 |
| Improvement. | 1.477 | 53.66 | 10.06 | .81 | 41.7 |
| Seaweed. | 1.554 | 56.19 | 9.88 | .6 | 39.8 |
| ina. | 1.421 | 48.89 | 9.79 | 1.24 | 45.8 |
| Seaweed, No. 3. | 1.61 | 54.93 | 9.21 | 1.01 | 40.7 |
| | 1.59 | 55.32 | 8.93 | 1.08 | 40.2 |
| COKE. | | | | | |
| Virginia. | 1.323 | 46.64 | 8.47 | 5.31 | — |
| n. | — | 32.7 | 8.63 | 10.51 | — |
| nd. | — | 31.6 | 8.99 | 3.55 | — |
| CELLULOSUS. | | | | Ash. | |
| Oak. | 1.5 | 24 | 5.5 | 3.06 | — |
| | .53 | 30 | 5 | — | — |
| bel. | 1.15 | 69.1 | 10.4 | 2.91 | — |
| | — | 65 | 8.9 | — | — |
| ry. | — | 21 | 4.7 | — | — |

Weights and Comparative Values of different Woods.

| Woods. | Cord. | Value. | Woods. | Cord. | Value. |
|------------------------|-------|--------|----------------------|-------|--------|
| | Lbs. | | | Lbs. | |
| Shell-bark Hickory ... | 4469 | 1 | New Jersey Pine..... | 2137 | .54 |
| Red-heart Hickory ... | 3705 | .81 | Yellow Pine..... | 1904 | .43 |
| White Oak..... | 3821 | .81 | White Pine..... | 1868 | .42 |
| Red Oak..... | 3254 | .69 | Beech..... | — | .7 |
| Virginia Pine..... | 2689 | .61 | Spruce..... | — | .52 |
| Southern Pine..... | 3375 | .73 | Hemlock..... | — | .44 |
| Hard Maple..... | 2878 | .6 | Cottonwood..... | — | .33 |

Liquid Fuels.

Petroleum.

Petroleum is a hydro-carbon liquid which is found in America and Europe. According to analysis of M. Sainte-Claire Deville, composition of 15 petroleum from different sources was found to be practically constant. Average specific gravity was .87. Extreme and average elementary composition was as follows:

| | | | | | |
|---------------|------|---------|-----------|---------------|-----------|
| Carbon..... | 82 | to 87.1 | per cent. | Average, 84.7 | per cent. |
| Hydrogen..... | 11.2 | to 14.8 | " | 13.1 | " |
| Oxygen..... | .5 | to 5.7 | " | 2.2 | " |
| 100 | | | | | |

Its heat of combustion is 20 240, and its evaporative power at 212° 20.3.

Petroleum Oils—Are obtained by distillation from petroleum, and are compounds of carbon and hydrogen, in average proportion of 72.6 and 27.4.

Boiling-point ranges from 86° to 495°.

Schist Oil—Consists of carbon 80.3 parts, hydrogen 11.5, and oxygen 8.2.

Pine Wood Oil—Consists of carbon 87.1 per cent., hydrogen 10.4, and oxygen 2.5.

Coal-gas.

Coal Gas—As furnished by Chartered Gas Co. of London is composed as follows:

| | Carbon. | Hydrogen. | | Oxygen. | Hydrogen. | Nitrogen. |
|--------------------|---------|-----------|---------------|------------|-----------|-----------|
| Olefant Gas, } .. | 3.096 | .434 | Hydrogen..... | — | 51.8 | — |
| Bi-carb. hyd. } | | | Oxygen..... | .68 | — | — |
| Marsh gas, } | 26.445 | 8.815 | Nitrogen..... | — | — | .38 |
| Carb. hyd. } | | | | | | |
| Carbonic oxide.... | 3.84 | 5.11 | Total..... | 100 parts. | | |

Heat of combustion at 212° 52 961 units, and evaporative power 47.51 lb.

Coal-gas. (V. Harcourt.)

| | Carb. | Hyd. | Oxy. | Nit. | | Carb. | Hyd. | Oxy. | Nit. |
|------------------|---------|---------|---------|---------|---------------|---------|---------|---------|---------|
| | Per ct. | Per ct. | Per ct. | Per ct. | | Per ct. | Per ct. | Per ct. | Per ct. |
| Coal gas..... | 10.5 | 1.7 | — | — | Hydrogen..... | — | 8.1 | — | — |
| Marsh gas..... | 39.7 | 13.2 | — | — | Nitrogen..... | — | — | — | 5.8 |
| Carbonic oxide.. | 5.9 | — | 7.9 | — | Oxygen..... | — | — | — | .3 |
| Carbonic oxide | 1.9 | — | 5 | — | Total..... | 58 | 23 | 13.2 | 5.1 |

this gas had a volume of 30 cube feet at 62°; heat of combustion: and of one cube foot 756 units, which is equivalent to .78 lb. of water from 62°, or of .78 lb. from 212° per cube foot.

Average Composition of Fuels.

| | Specific Grav. ity. | Carbon. | Hydrogen. | Nitrogen. | Oxygen. | Sulphur. | Ash. |
|----------------------------|---------------------|---------|-----------|-----------|---------|----------|---------|
| | | Per ct. | Per ct. | Per ct. | Per ct. | Per ct. | Per ct. |
| BITUMINOUS COALS. | | | | | | | |
| Australian | 1.31 | — | — | — | — | .5 | 8.38 |
| Neo | 1.28 | 64.52 | 4.74 | .8 | 20.75 | 1.45 | 7.74 |
| ish, lowest | — | 68.72 | 4.76 | — | 18.63 | 1.35 | — |
| head, dry, average | 1.18 | 63.94 | 8.86 | .96 | 4.7 | .32 | 21.22 |
| i, Conception Bay | 1.29 | 70.55 | 5.76 | .95 | 13.24 | 1.98 | 7.52 |
| Chiriqui | — | 38.98 | 4.01 | .58 | 13.38 | 6.14 | 36.91 |
| nel, Wigan | 1.23 | 79.23 | 6.08 | 1.18 | 7.24 | 1.43 | 4.84 |
| berland, Md. | 1.31 | 93.81 | 1.82 | — | 2.77 | — | 1.6 |
| e, Garesfield | — | 97.6 | — | — | — | .85 | 1.55 |
| Durham | — | 89.5 | — | — | — | 1.25 | 9.25 |
| Average | — | 93.44 | — | — | — | 1.22 | 5.34 |
| ryu. | 1.33 | 88.26 | 4.66 | 1.45 | .6 | 1.77 | 3.26 |
| mosa Island | 1.24 | 78.26 | 5.7 | .64 | 10.95 | .49 | 3.96 |
| nch, hard | 1.32 | 88.56 | 4.88 | (4.38) | — | — | 2.19 |
| " caking. | 1.29 | 87.73 | 5.08 | (5.65) | — | — | 1.54 |
| " long flame | 1.3 | 82.94 | 5.35 | (8.63) | — | — | 3.08 |
| " average* | 1.31 | 85 | 4.5 | (7) | — | — | 3.5 |
| ian, average | — | 47.3 | — | — | — | — | 22.9 |
| " Kotbec | — | 90 | — | — | — | — | 4 |
| agonia | — | 62.25 | 5.05 | .63 | 17.54 | 1.13 | 13.4 |
| asian, Miouchi† | — | 91.45 | 4.5 | (4.05) | — | — | — |
| ney, S. W. | — | 82.39 | 5.32 | 1.27 | 8.32 | .07 | 2.04 |
| int, Wylam | — | 74.82 | 6.18 | (5.09) | — | — | 13.91 |
| " Glasgow | — | 82.92 | 5.49 | (10.46) | — | — | 1.13 |
| " Cannel, Lancashire | — | 83.75 | 5.66 | (8.04) | — | — | 2.55 |
| " " Edinburgh | — | 67.6 | 5.4 | (12.43) | — | — | 14.57 |
| " Cherry, Newcastle | — | 84.85 | 5.05 | (8.43) | — | — | 1.67 |
| " Caking, Garesfield | — | 87.95 | 5.24 | (5.42) | — | — | 1.39 |
| " Ebbro Vale, Welsh | — | 89.78 | 5.15 | 2.16 | .39 | 1.02 | 1.5 |
| " Llangenneck " | — | 84.97 | 4.26 | 1.45 | 3.5 | .42 | 5.4 |
| icouver's Island. | — | 66.93 | 5.32 | 1.02 | 8.7 | 2.2 | 15.83 |
| ANTHRACITES. | | | | | | | |
| hracite | 1.5 | 88.54 | — | — | — | .52 | 8.67 |
| nch | 1.5 | 86.17 | 2.67 | (2.85) | — | — | 8.56 |
| asian | — | 96.66 | 1.35 | (1.99) | — | — | — |
| WOODS. | | | | | | | |
| ch | — | 50.17 | 6.12 | 1.05 | 40.38 | — | 1.77 |
| ch | — | 48.12 | 6.37 | 1.15 | 43.95 | — | .48 |
| t | — | 48.13 | 5.25 | .82 | 44.5 | — | 1.3 |
| lte Pine | — | 49.95 | 6.41 | — | 43.05 | — | .31 |
| ods, average | — | 49.7 | 6.06 | 1.05 | 41.3 | — | 1.8 |
| CHARCOAL. | | | | | | | |
| t | — | 87.68 | 2.83 | — | 6.43 | — | 3.06 |
| e | — | 71.36 | 5.95 | — | 22.19† | — | .4 |
| ple | — | 70.07 | 4.61 | — | 24.89† | — | .43 |
| MISCELLANEOUS. | | | | | | | |
| halt | 1.06 | 79.18 | 9.3 | (8.72) | — | — | 2.8 |
| nite, perfect | 1.20 | 69.02 | 5.05 | (20.12) | — | — | 5.82 |
| " imperfect | 1.25 | 60.18 | 5.29 | (29.03) | — | — | 5.57 |
| " bituminous | 1.18 | 74.82 | 7.36 | (13.38) | — | — | 4.45 |
| " Colorado | 1.28 | 56.8 | — | — | — | — | 4.5 |
| " Kentucky | 1.2 | 40 | — | — | — | — | 7 |
| " Arkansas | — | 34.5 | — | — | — | — | 5 |
| t, dense | — | 61.02 | 5.77 | .81 | 32.4 | — | — |
| Irish, average | .528 | 58.18 | — | — | 1.21 | — | 3.47 |
| nt, Warlich | 1.15 | 90.02 | — | — | — | 1.62 | 2.0 |
| Wylam's | 1.1 | 79.91 | — | — | — | — | 4 |

Test of Combustion of 1 Lb. 14723.
including Nitrogen.

Miscellaneous.

Experiments undertaken by Baltimore and Ohio R. R. Co. determined evaporating effect of 1 ton of Cumberland coal equal to 1.25 tons of anthracite, and 1 ton of anthracite to be equal to 1.75 cords of pine wood; also at 2000 lbs. of Lackawanna coal were equal to 4500 lbs. best pine wood.

One lb. of anthracite coal in a cupola furnace will melt from 5 to 10 lbs. of cast iron; 8 bushels bituminous coal in an air furnace will melt 1 ton of cast iron.

Small coal produces about .75 effect of large coal of same description.

Experiments by Messrs. Stevens, at Bordentown, N. J., gave following results:

Under a pressure of 30 lbs., 1 lb. pine wood evaporated 3.5 to 4.75 lbs. of water.
lb. Lehigh coal, 7.25 to 8.75 lbs.

Bituminous coal is 13 per cent. more effective than coke for equal weights; and England effects are alike for equal costs.

Radiation from Fuel.—Proportion which heat radiated from incandescent fuel bears to total heat of combustion is,

From Wood..... .29 | From Charcoal and Peat..... .5

Least consumption of coal yet attained is 1.5 lbs. per IHP. It usually varies in different engines from 2 to 8 lbs.

Volume of pine wood is about 5.5 times as great as its equivalent of bituminous coal.

GRAVITATION.

GRAVITY is an attraction common to all material substances, and they are affected by it directly, in exact proportion to their mass, and versely, as square of their distance apart.

This attraction is termed *terrestrial gravity*, and force with which a body is drawn toward centre of Earth is termed the *weight* of that body.

Force of gravity differs a little at different latitudes: the law of variation, however, is not accurately ascertained; but following theorems represent it very nearly:

$$\left. \begin{array}{l} g' (1 - .002837 \cos. 2 \text{ lat.}) \\ g' (1 + .002837), \text{ at the poles} \\ g' (1 - .002837), \text{ at the equator} \end{array} \right\} = g. \quad \begin{array}{l} g' \text{ representing force of gravity at latitude } 45^\circ, \text{ and } g \text{ force at other places.} \end{array}$$

Or, $32.171 (\text{lat. } 45^\circ) (1 + .005133 \sin. L) \left(1 - \frac{2H}{R}\right) = g.$ *L* representing latitude, height of elevation above level of sea, and *R* radius of Earth, both in feet.

NOTE.—If $2L$ exceeds 90° , put $\cos. 180 - 2L$, and *R* at Equator = 20926062, at poles 20853429, and mean 20889746.

ILLUSTRATION.—What is force of gravity at latitude 45° , at an elevation of 209 ft, and radius = 20900000 feet?

$$32.171 (1 + .005133 \sin. 45^\circ) \left(1 - \frac{2H}{20900000}\right) = 32.171 \times 1.00363 \times .99998 = 32.287.$$

Gravity at Various Locations at Level of Sea.

| | | | | | |
|-----------------|--------|-----------------------|--------|-------------|--------|
| Equator..... | 32.088 | New York..... | 32.161 | London..... | 32.189 |
| Washington..... | 32.155 | Lat. 45° | 32.171 | Poles..... | 32.253 |

In bodies descending freely by their own weight, their velocities are as *times* of their descent, and *spaces* passed through as square of the times.

Times, then, being 1, 2, 3, 4, etc., *Velocities* will be 1, 2, 3

spaces passed through will be as square of the velocities
same times, as 1, 4, 9, 16, etc.; and *spaces* for each time

A body falling freely will descend through 16.0833 feet in first second time, and will then have acquired a velocity which will carry it through 32.166 feet in next second.

If a body descends in a curved line, it suffers no loss of velocity, and the curve of a cycloid is that of quickest descent.

Motion of a falling body being uniformly accelerated by gravity, motion of a body projected vertically upwards is uniformly retarded in same manner.

A body projected perpendicularly upwards with a velocity equal to that which it would have acquired by falling from any height, will ascend to the same height before it loses its velocity. Hence, a body projected upwards is ascending for one half of time it is in motion, and descending for other half.

Various Formulas here given are for Bodies Projected Upwards or Falling Freely, in Vacuo.

When, however, weight of a body is great compared with its volume, and velocity of it is low, deductions given are sufficiently accurate for ordinary purposes.

In considering action of gravitation on bodies not far distant from surface of Earth, it is assumed, without sensible error, that the directions in which it acts are parallel, or perpendicular to the horizontal plane.

A distance of one mile only produces a deviation from parallelism less than one minute, or the 60th part of a degree.

Relation of Time, Space, and Velocities.

| Time from Beginning of Descent. | Velocity acquired at End of that Time. | Squares of Time. | Space fallen through in that Time. | Spaces for this Time. | Spaces fallen through in last Second of Time. |
|---------------------------------|--|------------------|------------------------------------|-----------------------|---|
| Seconds. | Feet. | Seconds. | Feet. | No. | Feet. |
| 1 | 32.166 | 1 | 16.083 | 1 | 16.08 |
| 2 | 64.333 | 4 | 64.333 | 3 | 48.25 |
| 3 | 96.5 | 9 | 144.75 | 5 | 80.41 |
| 4 | 128.665 | 16 | 257.33 | 7 | 112.56 |
| 5 | 160.832 | 25 | 402.08 | 9 | 144.75 |
| 6 | 193 | 36 | 579 | 11 | 176.91 |
| 7 | 225.166 | 49 | 788.08 | 13 | 209.08 |
| 8 | 257.333 | 64 | 1029.33 | 15 | 241.25 |
| 9 | 289.5 | 81 | 1302.75 | 17 | 273.41 |
| 10 | 321.666 | 100 | 1608.33 | 19 | 305.56 |

and in same manner this Table may be continued to any extent.

Velocity acquired to given Height of Fall and Height to given Velocity.

$$8.04\sqrt{h} = v; \quad .2t = v; \quad \frac{v^2}{64.4} = h; \text{ and } 16.083t^2 = h.$$

h representing height of fall in feet, *v* velocity acquired in feet per second, and *t* time of fall in seconds.

To Compute Action of Gravity. Time.

When Space is given. RULE.—Divide space by 16.083, and square root of quotient will give time.

EXAMPLE.—How long will a body be in falling through 402.08 feet?

$$\sqrt{402.08 \div 16.083} = 5 \text{ seconds.}$$

When Velocity is given. RULE.—Divide given velocity by 32.166, and square root of quotient will give time.

EXAMPLE.—How long will a body be in falling to acquire a velocity of 64.333 feet per second?

$$64.333 \div 32.166 = 2.00 \text{ seconds.}$$

Velocity.

When Space is given. RULE.—Multiply space in feet by 64.333, and are root of product will give velocity.

SAMPLE.—Required velocity a body acquires in descending through 579 feet.

$$\sqrt{579 \times 64.333} = 193 \text{ feet.}$$

Velocity acquired at any period is equal to twice the mean velocity during the period.

ILLUSTRATION.—If a ball fall through 2316 feet in 12 seconds, with what velocity it strike?

$$2316 \div 12 = 193, \text{ mean velocity, which } \times 2 = 386 \text{ feet} = \text{velocity.}$$

When Time is given. RULE.—Multiply time in seconds by 32.166, and duct will give velocity.

SAMPLE.—What is velocity acquired by a falling body in 6 seconds?

$$32.166 \times 6 = 192.996 \text{ feet.}$$

Space.

When Velocity is given. RULE.—Divide velocity by 8.04, and square of quotient will give distance fallen through to acquire that velocity.

Or, Divide square of velocity by 64.33.

SAMPLE.—If the velocity of a cannon-ball is 579 feet per second, from what height must a body fall to acquire the same velocity?

$$579 \div 8.04 = 72.014, \text{ and } 72.014^2 = 5186.02 \text{ feet.}$$

When Time is given. RULE.—Multiply square of time in seconds by .83, and it will give space in feet.

SAMPLE.—Required space fallen through in 5 seconds.

$$5^2 = 25, \text{ and } 25 \times .83 = 20.75 \text{ feet.}$$

Distance fallen through in feet is very nearly equal to square of time in fourths second.

ILLUSTRATION 1.—A bullet dropped from the spire of a church was 4 seconds in hitting the ground; what was height of the spire?

$$4 \times 4 = 16, \text{ and } 16^2 = 256 \text{ feet.}$$

Or Rule, $4 \times 4 \times 16.0833 = 257.33 \text{ feet.}$

—A bullet dropped into a well was 2 seconds in reaching bottom; what is the depth of the well?

$$\text{Then } 2 \times 4 = 8, \text{ and } 8^2 = 64 \text{ feet.}$$

Or Rule, $2 \times 2 \times 16.0833 = 64.33 \text{ feet.}$

Or Inversion.—In what time will a bullet fall through 256 feet?

$$\sqrt{256} = 16, \text{ and } 16 \div 4 = 4 \text{ seconds.}$$

Space fallen through in last Second of Fall.

When Time is given. RULE.—Subtract half of a second from time, and multiply remainder by 32.166.

SAMPLE.—What is space fallen through in last second of time, of a body falling 10 seconds?

$$10 - .5 \times 32.166 = 305.58 \text{ feet.}$$

Promiscuous Examples.

1. If a ball is 1 minute in falling, how far will it fall in last second?

Space fallen through = square of time, and 1 minute = 60 seconds.

$$60^2 \times 16.083 = 57898 \text{ feet for 60 seconds.}$$

$$59^2 \times 16.083 = 55984 \text{ " " } \frac{59}{1} \text{ " "}$$

Compute time of generating a velocity of 193 feet per second, and whole space fallen.

$$193 \div 32.166 = 6 \text{ seconds; } 6^2 \times 16.083 = 579 \text{ feet.}$$

3. If a body was to fall 579 feet, what time would it be in falling, and would it fall in the last second?

$$\sqrt{\frac{579 \times 2}{32.166}} = \sqrt{36} = 6 \text{ seconds, and } 6 - .5 \times 32.166 = 5.5 \times 32.166 = 176.9$$

Formulas to determine the various Element

$$\begin{aligned} 1. \quad T &= \sqrt{\frac{S}{.5g}}; = \frac{V}{g}; = \frac{2S}{V}; = \sqrt{\frac{2S}{g}}; = \frac{h}{g} + .5 \\ 2. \quad S &= \left(\frac{V}{.25g}\right)^2; = \frac{V^2}{2g}; = \frac{VT}{2}; = \frac{gT^2}{2}; = T^2 \cdot 5g. \quad 3. \quad h = (T \\ 4. \quad V &= \sqrt{S \times 2g}; = Tg; = 2\sqrt{.5gS}; = \frac{2S}{T}. \end{aligned}$$

T representing time of falling in seconds, *V* velocity acquired in feet per second, *S* space or vertical height in feet, *h* space fallen through in last second, *g* 32.5 *g* and .25 *g* representing 16.083 and 8.04.

Retarded Motion.

A body projected vertically upward is affected inversely to its when falling freely and directly downward, inasmuch as a like cause it in one case and accelerates it in the other.

In air a ball will not return with same velocity with which it started *vacuo* it would. Effect of the air is to lessen its velocity both ascending and descending. Difference of velocities will depend upon relative specificity of ball and density of medium through which it passes. Thus, weight of ball, greater its velocity.

To Compute Action of Gravity by a Body projected Upward or Downward with a given Velocity.

When projected Upward. **RULE.**—From the product of the given velocity and the time in seconds subtract the product of 32.166, and half the square of the time, and the remainder will give the space in feet.

Or, Square velocity, divide result by 64.33, and quotient will give the space in feet.

EXAMPLE.—If a body is projected upward with a velocity of 96.5 feet per second, through what space will it ascend before it stops?

$$96.5 \div 32.166 = 3 \text{ seconds} = \text{time to acquire this velocity.}$$

$$\text{Then, } 96.5 \times 3 - \left(32.166 \times \frac{3^2}{2}\right) = 289.5 - 144.75 = 144.75 \text{ feet.}$$

Time.

RULE.—Divide velocity in feet by 32.166, and quotient will give the time in seconds.

EXAMPLE.—Velocity as in preceding example.

$$96.5 \div 32.166 = 3 \text{ seconds.}$$

Velocity.

RULE.—Multiply time in seconds by 32.166, and product will give the velocity in feet per second.

EXAMPLE.—Time as in preceding example.

$$3 \times 32.166 = 96.5 \text{ feet velocity.}$$

Space fallen through in last Second.

RULE.—Multiply time in seconds by 32.166, and

subtract

the square of the

$$3 \times 32.166 = 96.5 \text{ feet.}$$

When projected Downward.
Space.

RULE.—Proceed as for projection upwards and take sum of products.

EXAMPLE 1.—If a body is projected downward with a velocity of 96.5 feet per second, through what space will it fall in 3 seconds?

$$96.5 \times 3 + \left(32.166 \times \frac{3^2}{2} \right) = 289.5 + 144.75 = 434.25 \text{ feet.}$$

$$\text{Or, } t^2 \times 16.083 + v \times t = s.$$

2. —If a body is projected downward with a velocity of 96.5 feet per second through what space must it descend to acquire a velocity of 193 feet per second?

$$96.5 \div 32.166 = 3 \text{ seconds, time to acquire this velocity.}$$

$$193 \div 32.166 = 6 \text{ seconds, time to acquire this velocity.}$$

Hence $6 - 3 = 3$ seconds, time of body falling.

Then $96.5 \times 3 = 289.5$ = product of velocity of projection and time.

$$16.083 \times 3^2 = 144.75 = \text{product of } 32.166, \text{ and half square of time.}$$

Therefore $289.5 + 144.75 = 434.25$ feet.

Time.

RULE.—Subtract space for velocity of projection from space given, and remainder, divided by velocity of projection, will give time.

EXAMPLE.—In what time will a body fall through 434.25 feet of space, when projected with a velocity of 96.5 feet?

Space for velocity of 96.5 = 144.75 feet.

$$\text{Then, } 434.25 - 144.75 \div 96.5 = 289.5 \div 96.5 = 3 \text{ seconds.}$$

Velocity.

RULE.—Divide twice space fallen through in feet by time in seconds.

EXAMPLE.—Elements as in preceding example.

Space fallen through when projected at velocity of 96.5 feet = 144.75 feet, and 434.25 feet = space fallen through in 3 seconds.

Then, $144.75 + 434.25 = 579$ feet space fallen through, and $\sqrt{579 \div 16.083} = 6$ seconds.

$$\text{Hence, } 579 \times 2 \div 6 = 1158 \div 6 = 193 \text{ feet.}$$

Space Fallen through in last Second.

RULE.—Subtract .5 from time, multiply remainder by 32.166, and product will give space in feet per second.

EXAMPLE.—Elements as in preceding example.

$$6 - .5 \times 32.166 = 5.5 \times 32.166 = 176.91 \text{ feet.}$$

Ascending bodies, as before stated, are retarded in same ratio that descending bodies are accelerated. Hence, a body projected upward is ascending for one half of the time it is in motion, and descending the other half.

ILLUSTRATION 1.—If a body projected vertically upwards return to earth in 12 seconds, how high did it ascend?

The body is half time in ascending. $12 \div 2 = 6$.

$$\text{Hence, by Rule, p. 489, } 6^2 \times 16.083 = 579 \text{ feet} = \text{product of square of time and } 16.083.$$

2.—If a body is projected upward with a velocity of 96.5 feet per second, it required to ascertain point of body at end of 10 seconds.

$$96.5 \div 32.166 = 3 \text{ seconds, time to acquire this velocity, and } 3^2 \times 16.083 = 144.75 \text{ feet, height body reached with its initial velocity.}$$

Then $10 - 3 = 7$ seconds left for body to fall in.

$$\text{Hence, by Rule, as in preceding example, } 7^2 \times 16.083 = 788.07, \text{ and } 788.07 + 144.75 = 932.82 \text{ feet} = \text{distance below point of projection.}$$

$$\text{Or, } 10^2 \times 16.083 = 1608.3 \text{ feet, space fallen through under the effect of gravity } 16.5 \times 10 = 965 \text{ feet, space if gravity did not act. Hence } 1608.3 - 965 = 643.3 \text{ feet.}$$

3.—A body is projected vertically with a velocity of 135 feet; what velocity will it have at 60 feet?

$135^2 \div 64.33 = 283.3$ feet space projected at that velocity, $135 \div 32.16 = 4.197$ seconds = time of projection, and $283.3 - 60 = 223.3$ = space to be passed through after attainment of 60 feet. Hence, $\sqrt{223.3 \times 64.33} = 119.85$ feet velocity, and $223.3 + 60 = 283.3$ feet.

By Inversion.—Velocity 119.85. Hence, $\frac{119.85^2}{64.33} = 223.3$ feet space, and $283.3 - 223.3 = 60$ feet.

Formulas to Determine Elements of Retarded Motion.

$$1. v = V - gt.$$

$$2. V = \frac{S}{t} - \frac{gt}{2}.$$

$$3. V = v + gt.$$

$$4. t' = \frac{V \div 2}{g}.$$

$$5. S = Vt - \frac{gt^2}{2}.$$

$$6. h = T - t - t' - .5t.$$

$$7. S = tv + \frac{gt^2}{2}.$$

$$8. t = \frac{V - v}{g}.$$

$$9. t = \frac{V}{g} - \sqrt{\frac{V^2 - 2gS}{g^2}}.$$

v representing velocity at expiration of time, t any less time than T , t' less time than t , s space through which a body ascends in time t , V , T , S , and h as in previous formulas, page 490.

ILLUSTRATION.—A body projected upwards with a velocity of 193 feet per second, was arrested in 5 seconds.

$$T = 6, t' = 1.$$

1. What was its velocity when arrested? (1.)
2. What was the time of its passing through 562.92 feet of space? (8.)
3. What space had it passed through? (5.)
4. What was the time of its projection, when it had a velocity of 96.5 feet? (4.)
5. What was the height it was projected in the last second of time? (6.)

$$1. 193 - 32.166 \times 5 = 32.17 \text{ feet.}$$

$$3. 32.17 + 32.166 \times 5 = 193 \text{ velocity.}$$

$$2. \frac{562.92}{5} + \frac{32.166 \times 5}{2} = 193 \text{ velocity.}$$

$$4. \frac{193 \div 2}{32.166} = \frac{96.5}{32.166} = 3 \text{ seconds.}$$

$$5. 193 \times 5 - \frac{32.166 \times 5^2}{2} = 562.92 \text{ feet.}$$

$$6. 6 - 5 - 1 - .5 \times 32.166 = 48.5 \text{ feet.}$$

$$7. S = tv + \frac{gt^2}{2} = 562.92 \text{ feet.}$$

$$8. \frac{193 - 32.17}{32.166} = 5 \text{ seconds.}$$

$$9. \frac{193}{32.166} - \sqrt{\frac{193^2}{32.166^2} - \frac{2 \times 562.92}{32.166}} = 6 - \sqrt{36 - 35} = 5 \text{ seconds.}$$

Gravity and Motion at an Inclination.

If a body freely descend at an inclination, as upon an inclined plane, by force of gravity alone, the velocity acquired by it when it arrives at termination of inclination is that which it would acquire by falling freely to the vertical height thereof. Or, velocity is that due to height of inclination of the plane.

occupied in making descent is greater than that due to height, in proportion to the length of its inclination, or distance passed, to its height.

Consequently, times of descending different inclinations or planes of like height are to one another as lengths of the inclinations or planes.

Which a body descends upon an inclination, when descending by gravity alone, will freely fall in same time as height of inclination, and being same, times will be inversely in this proportion.

it suffers no loss of velocity.

from rest, from same point, one upon another, and freely, their velocities at all equal heights will be the same.

ILLUSTRATION.—What distance will a body roll down an inclined plane 300 feet and 25 feet high in one second, by force of gravity alone?

As $300 : 25 :: 16.083 : 1.34025$ feet.

ence, if proportion of height to length of above plane is reduced from 25 to 300 to 600, the time required for body to fall 1.34025 feet would be determined as follows:

$3 : 25 : 600 :: 1.34025 : 32.166$, and $32.166 = 16.083 \times 2 =$ *twice time or space in which it would fall freely required for one half proportion of height to length.*

Or, as $\frac{300}{25} : \frac{600}{25} :: 1.34025 : 32.166$, as above.

Impelling or accelerating force by gravitation acting in a direction parallel to an inclination, is less than weight of body, in ratio of height of inclination to its length. It is, therefore, inversely in proportion to length of inclination, when height is the same.

Time of descent, under this condition, is inversely in proportion to accelerating force.

If, for instance, length of inclination is five times height, time of making freely descent at inclination by gravitation is five times that in which a body would freely fall vertically through height; and impelling force down inclination is .2 of weight of body.

When bodies move down inclined planes, the accelerating force is expressed by $h \div l$, *quotient of height \div length of plane*; or, what is equivalent thereto, sine of inclination of plane, i. e., $\sin. a$.

ILLUSTRATION.—An inclined plane having a height of one half its length, the space in through in any time would be *one half of that which it would fall freely.*

velocity which a body rolling down such a plane would acquire in 5 seconds is 16 feet.

thus, $32.166 \times 5 = 160.833$ feet, and an inclined plane, having a height one half its length, has an angle or sine of 30° . Hence, $\sin. 30^\circ = .5$, and $160.833 \times .5 = 80.416$ feet.

Formulas to Determine various Elements of Gravitation on an Inclined Plane.

1. $S = .5 g T^2 \sin. a$; $= \frac{V^2}{2 g \sin. a}$; $= .5 T V$.
2. $V = g T \sin. a$; $= \sqrt{(2 g S \sin. a)}$; $= \frac{2 S}{T}$.
3. $T = \sqrt{\left(\frac{2 S}{g \sin. a}\right)}$; $= \frac{2 S}{V}$; $= .25 \sqrt{\frac{l^2}{H}}$; $= \frac{l}{4 \sqrt{H}}$.
4. $V = \sqrt{g T \sin. a}$.
5. $S = V T \div \frac{V^2}{2 g \sin. a}$. Or, $\frac{V^2}{2 g \sin. a}$.
6. $H = \frac{l^2}{.5 g T^2}$.
7. $l = 4 T \sqrt{H}$.

representing velocity of projection in feet per second, S space or vertical height of inclination, a angle of inclination of plane, l length, and H height of plane.

ILLUSTRATION.—Assume elements of preceding illustration. $V = 80.416$, $T = 5$, $H = 201.04$.

$$.5 \times 32.166 \times 5^2 \times .5 = 201.04 \text{ feet.}$$

$$2. 32.166 \times 5 \times .5 = 80.416 \text{ feet.}$$

$$3. \sqrt{\left(\frac{2 \times 201.04}{32.166 \times .5}\right)} = \sqrt{\left(\frac{402.08}{16.083}\right)} \sqrt{25} = 5 \text{ seconds.}$$

$$\frac{283.42^2}{.5 \times 16.083 \times 5^2} = 201.04 \text{ feet.}$$

$$7. 4 \times 5 \times \sqrt{201.04} = 283.42$$

projected downward with an initial velocity of 16.083 feet per sec

$$16.083 + 32.166 \times 5 \times .5 = 96.5 \text{ feet.}$$

$$80.416 + 16.083 \times 5 - \frac{.5 \times 32.166 \times 5^2 \times .5}{T} = 281.46 \text{ feet.}$$

ILLUSTRATION.—What time will it take for a ball to roll 38 feet down an inclined plane, the angle $\alpha = 12^\circ 20'$, and what velocity will it attain at 38 feet from its starting-point?

$$T = \sqrt{\frac{2S}{g \sin \alpha}} = \sqrt{\frac{2 \times 38}{32.166 \times .2136}} = 3.33 \text{ seconds. } V = g T \sin \alpha = 32.166 \times 3.33 \times .2136 = 22.88 \text{ feet per second.}$$

When a body is projected upward it is retarded in the same ratio that a descending body is accelerated.

ILLUSTRATION.—If a body is projected up an inclined plane having a length of twice its height, at a velocity of 96.5 feet per second,

$$\text{Then, } T = 96.5 \div 32.166 = 3 \text{ seconds. } S = .5 \times 32.166 \times 3^2 \times .5 = 72.375 \text{ feet. } v = 32.166 \times 3 \times .5 = 48.25 \text{ feet.}$$

Inclined Plane.

Problems on descent of bodies on inclined planes are soluble by formulas 1 to 9, page 495, for relations of accelerating forces. As a preliminary step, however, accelerating force is to be determined by multiplying weight of descending body by height of plane, and dividing product by length of plane.

ILLUSTRATION.—If a body of 15 lbs. weight gravitate freely down an inclined plane, length of which is five times height, accelerating force is $15 \div 5 = 3$ lbs. If length of plane is 100 feet and height 20, velocity acquired in falling freely from top to bottom of plane would be

$$v = 8 \sqrt{\frac{3 \times 100}{15}} = 8 \sqrt{20} = 35.776 \text{ feet.}$$

Time occupied in making descent,

$$t = .25 \sqrt{\frac{15 \times 100}{3}} = .25 \sqrt{500} = 5.59 \text{ seconds.}$$

Whereas, for a free vertical fall through height of 20 feet, time would be,

$$t = \frac{35.776}{32.166} = 1.118 \text{ seconds,}$$

which is .2 of time of making descent on inclined plane.

Velocities acquired by bodies in falling down planes of like height will all be equal when arriving at base of plane.

When Length of an Inclined Plane and Time of Free Descent are given.

RULE.—Divide square of length by square of time in seconds and by 16; the quotient is height of inclined plane.

EXAMPLE.—Length of plane is 100 feet, and time of descent is 5.59 seconds; then vertical height of descent is

$$\frac{100^2}{5.59^2 \times 16.68} = 20 \text{ feet.}$$

Accelerated and Retarded Motion.

If an Accelerating or Retarding force is greater than gravity, that is weight of the body, the constant, g , or 32.166, is to be varied in proportion thereto, so that it is to be multiplied by the accelerating force, and product is the height of body.

The accelerating force is equal to weight of body.

$$\text{or } \frac{32.166}{16.68} \times 16.68 \text{ become the constants.}$$

When given for action of gravity is uniformly accelerating or retarding, adapted to the force.

Average Velocity of a Moving Body uniformly Accelerated or Retarded.

Average velocity of a moving body uniformly accelerated or retarded, during a given time or in a given space, is equal to half sum of initial and final velocities; and if body begin from a state of rest or arrive at a state of rest, its average speed is half the final or initial velocity, as the case may be.

Thus, in example of a ball rolling, initial speed or velocity is, in either case, 60 feet per second, and terminal speed is nothing; average speed is therefore $\frac{60+0}{2}$, namely, one half of that, or 30 feet per second.

When a cannon-ball is projected at an angle to horizon, there are two forces acting on it at same time—viz., force of charge, which propels it uniformly in a right line, and force of gravity, which causes it to fall from a right line with an accelerated motion; these two motions (uniform and accelerated) cause the ball to move in the curved line of a Parabola.

Formulas for Flight of a Cannon-ball.

$$V = 2800 \sqrt{\frac{P}{w}}; \quad P = \frac{w V^2}{7840000};$$

$$b = \frac{V^2 \sin. a. \cos. a}{g}; \quad t = \frac{V \sin. a}{g}; \quad h = \frac{V^2 \sin.^2 a}{2g}.$$

w representing weight of ball and *P* of powder in lbs.; *t* time of flight in seconds; *b* horizontal range, and *h* vertical height of range of projection of ball in feet.

ILLUSTRATION.—A cannon loaded to give a ball a velocity of 900 feet per second, the angle $a = 45^\circ$; what is horizontal range, the time *t* and height of range *h*?

$$b = \frac{900^2 \times \sin. 45^\circ \times \cos. 45^\circ}{32.166} = \frac{900^2 \times .5}{32.166} = 12590 \text{ feet.}$$

$$t = \frac{900 \times .7071}{32.166} = 19.78 \text{ seconds}; \quad h = \frac{900^2 \times .7071^2}{2 \times 32.166} = 6295 \text{ feet.}$$

NOTE.—As distance *b* will be greatest when angle $a = 45^\circ$, product of sine and cosine is greatest for that angle. $\sin. 45^\circ \times \cos. 45^\circ = .5$.

24 lb. ball with a velocity of 2000 feet per second at 45° range 7300 feet.

General Formulas for Accelerating and Retarding Forces.

$$\begin{array}{llll} 1. \quad V = \frac{gft}{w}. & 2. \quad S = \frac{.5 gft^2}{w}. & 3. \quad t = \frac{wV}{gf}. & 4. \quad S = \frac{wV^2}{2gf} \\ 5. \quad t = .25 \sqrt{\frac{wS}{f}}. & 6. \quad V = 8 \sqrt{\frac{fS}{w}}. & & \\ 7. \quad f = \frac{wV^2}{2gS}. & 8. \quad f = \frac{wV}{t32.2}. & 9. \quad w = \frac{gft}{V}. & \end{array}$$

NOTE 1.—When accelerating or retarding force bears a simple ratio to weight of body, the ratio may, for facility of calculation, be substituted in the quantities representing modified constants, for force and weight. Thus, if accelerating force is a tenth part of weight, then ratio is 1 to 10, and $\frac{32.166}{10} = 3.2166$; or, $\frac{16.083}{10} = 1.6083$,

and $\frac{64.333}{10} = 6.4333$; and these quotients may be substituted for 16.083, 3.2166, and 64.333 respectively, in formulas for action of gravity 1 to 9, to fit them for action in an accelerating or retarding force one-tenth of gravity.

2.—Table, page 488, giving relations of velocity and height of fall to be employed in solving questions of accelerating force general.

EXAMPLE.—A ball weighing 10 lbs. is projected with an initial velocity of 60 feet per second on a level plane, and frictional resistance to its motion is 1 lb. How long will it traverse before it comes to a state of rest? By formula.

$$\frac{10 \text{ lbs.} \times 60^2}{64.333 \times 1 \text{ lb.}} = 559.59 \text{ feet.}$$

Again, same result may be arrived at, according to Note 1, by multiplying 64.333, in Rule, page 494, for gravity, by ratio of force and weight, in this case is $\frac{1}{10}$, and $64.333 \times \frac{1}{10} = 6.4333$. Substituting 6.4333 for 64.333 in rule, formula becomes

$$S = \frac{V^2}{6.4333} = \frac{60^2}{6.4333} = 559.59 \text{ feet.}$$

The question may be answered more directly by aid of table for falling page 488. Height due to a velocity of 60 feet per second, is 55.9 feet; which be multiplied by inverse ratio of accelerating force and weight of body, or $\frac{1}{10}$, that is,

$$55.9 \times 10 = 559 \text{ feet.}$$

If the question is put otherwise—What space will a weight move over if it comes to a state of rest, with an initial velocity of 60 feet per second, allowing it to be one tenth weight? The answer is that friction, which is retardation being one tenth of weight, or of gravity, space described will be 10 times as much as necessary for gravity, supposing the weight to be projected vertically upwards to bring it to a state of rest. The height due to velocity being 55.9 feet; then

$$55.9 \times 10 = 559 \text{ feet.}$$

Average velocity of a moving body, uniformly accelerated or retarded in a given period or space, is equal to half sum of initial and final velocities.

To Compute Velocity of a Falling Stream of Water at End of any given Time.

When Perpendicular Distance is given.

EXAMPLE.—What is the distance a stream of water will descend on an inclined plane 10 feet high, and 100 feet long at base, in 5 seconds?

$$5^2 \times 16.083 = 402.08 \text{ feet} = \text{space a body will freely fall in this time.}$$

Then, as $100 : 10 :: 402.08 : 40.21 \text{ feet} = \text{proportionate velocity on a plane inclined to velocity when falling freely.}$

Miscellaneous Illustrations.

1.—What is the space descended vertically by a falling body in 7 seconds?

$$S = .5 g \times t^2. \text{ Then } 16.083 \times 7^2 = 788.067 \text{ feet.}$$

2.—What is the time of a falling body descending 400 feet, and velocity at end of that time?

$$t = \frac{v}{g}. \text{ Then } \frac{160.4}{32.166} = 4.98 \text{ sec. } v = \sqrt{2 g \times S}. \text{ Then } \sqrt{64.333 \times 400} = 160.4$$

3.—If a drop of rain fall through 176 feet in last second of its fall, how high was the cloud from which it fell?

$$S = \frac{h^2}{2g}. \text{ Then } \frac{176^2}{64.166} = 482.75 \text{ feet.}$$

4.—If two weights, one of 5 lbs. and one of 3, hanging freely over a pulley, set free, how far will heavier one descend or lighter one rise in 4 seconds?

$$\frac{5-3}{5+3} \times 16.083 \times 4^2 = \frac{2}{8} \times 257.328 = 64.33 \text{ feet.}$$

5.—If length of an inclined plane is 100 feet, and time of descent of a body is 6 seconds, what is vertical height of plane or space fallen through?

$$\frac{100^2}{6^2 \times .5 g} = \frac{10000}{579} = 17.27 \text{ feet.}$$

6.—If a bullet is projected vertically with a velocity of 135 feet per second, what velocity will it have at 60 feet?

$$\text{Formula 9, page 492. } \frac{135}{32.166} - \sqrt{\frac{135^2}{32.166^2} - \frac{2 \times 60}{32.166}} = 47 \text{ feet.}$$

GUNNERY.

heavy body impelled by a force of projection describes in its flight a parabola, *parameter* of which is four times height due to velocity of projection.

Velocity of a shot projected from a gun varies as square root of velocity directly, and as square root of weight of shot reciprocally.

To Compute Velocity of a Shot or Shell.

LE.—Multiply square root of triple weight of powder in lbs. by 1600; product by square root of weight of shot; and quotient will give velocity in feet per second.

MPLE.—What is velocity of a shot of 196 lbs., projected with a charge of 9 lbs. powder?

$$\sqrt{9 \times 3} \times 1600 \div \sqrt{196} = 8320 \div 14 = 594.3 \text{ feet.}$$

Compute Range for a Charge, or Charge for a Range.

When Range for a Charge is given.—Ranges have same proportion as weight of powder; that is, as one range is to its charge, so is any other to its charge, elevation of gun being same in both cases. *Consequently,*

To Compute Range.

LE.—Multiply range determined by charge in lbs. for range required, by product by given charge, and quotient will give range required.

MPLE.—If, with a charge of 9 lbs. of powder, a shot ranges 4000 feet, how far charge of 6.75 lbs. project same shot at same elevation?

$$4000 \times 6.75 \div 9 = 3000 \text{ feet.}$$

To Compute Charge.

LE.—Multiply given range by charge in lbs. for range determined, by product by range determined, and quotient will give charge required.

MPLE.—If required range of a shot is 3000 feet, and charge for a range of 4000 has been determined to be 9 lbs. of powder, what is charge required to project shot at same elevation?

$$3000 \times 9 \div 4000 = 6.75 \text{ lbs.}$$

Compute Range at one Elevation, when Range for another is given.

LE.—As sine of double first elevation in degrees is to its range, so is sine of double another elevation to its range.

MPLE.—If a shot range 1000 yards when projected at an elevation of 45° , how far will it range when elevation is $30^\circ 16'$, charge of powder being same?

$$\text{Sine of } 45^\circ \times 2 = 100000; \text{ sine of } 30^\circ 16' \times 2 = 87064.$$

$$n, \text{ as } 100000 : 1000 :: 87064 : 870.64 \text{ feet.}$$

Compute Elevation at one Range, when Elevation for another is given.

LE.—As range for first elevation is to sine of double its elevation, so is range for elevation required to sine for double its elevation.

MPLE.—If range of a shell at 45° elevation is 3750 feet, at what elevation a gun be set for a shell to range 2810 feet with a like charge of powder?

$$\text{Sine of } 45^\circ \times 2 = 100000.$$

$$n, \text{ as } 3750 : 100000 :: 2810 : 74933 = \text{sine for double} \quad = 24^\circ 16'.$$

Approximate Rule for Time of

flight under 4000 yards, velocity of projectile 900 feet per second; and over 6000 yards, velocity 800 feet; and

and Howitzers take their denomination from round numbers, up to the 42-pounder; larger from diameter of their bore.

Vertical Velocity and Range of Shot and Shells.

The *Range* of a shot or shell is the distance it is first seen upon a horizontal plane the place required upon its regular passage.

| Calibre of Gun. | Velocity. | Time. | Initial Velocity. | Time of Flight. | Range. | Height. |
|-----------------|-----------|-------|-------------------|-----------------|--------|---------|
| 12 lb. Cannon. | 1,200 ft. | 1.00 | 1,200 | 1.00 | 1,200 | 1.00 |
| 12 lb. Cannon. | 1,200 ft. | 1.10 | 1,200 | 1.10 | 1,320 | 1.21 |
| 12 lb. Cannon. | 1,200 ft. | 1.20 | 1,200 | 1.20 | 1,440 | 1.44 |
| 12 lb. Cannon. | 1,200 ft. | 1.30 | 1,200 | 1.30 | 1,560 | 1.69 |
| 12 lb. Cannon. | 1,200 ft. | 1.40 | 1,200 | 1.40 | 1,680 | 1.96 |
| 12 lb. Cannon. | 1,200 ft. | 1.50 | 1,200 | 1.50 | 1,800 | 2.25 |
| 12 lb. Cannon. | 1,200 ft. | 1.60 | 1,200 | 1.60 | 1,920 | 2.56 |
| 12 lb. Cannon. | 1,200 ft. | 1.70 | 1,200 | 1.70 | 2,040 | 2.89 |
| 12 lb. Cannon. | 1,200 ft. | 1.80 | 1,200 | 1.80 | 2,160 | 3.24 |
| 12 lb. Cannon. | 1,200 ft. | 1.90 | 1,200 | 1.90 | 2,280 | 3.61 |
| 12 lb. Cannon. | 1,200 ft. | 2.00 | 1,200 | 2.00 | 2,400 | 4.00 |
| 12 lb. Cannon. | 1,200 ft. | 2.10 | 1,200 | 2.10 | 2,520 | 4.41 |
| 12 lb. Cannon. | 1,200 ft. | 2.20 | 1,200 | 2.20 | 2,640 | 4.84 |
| 12 lb. Cannon. | 1,200 ft. | 2.30 | 1,200 | 2.30 | 2,760 | 5.29 |
| 12 lb. Cannon. | 1,200 ft. | 2.40 | 1,200 | 2.40 | 2,880 | 5.76 |
| 12 lb. Cannon. | 1,200 ft. | 2.50 | 1,200 | 2.50 | 3,000 | 6.25 |
| 12 lb. Cannon. | 1,200 ft. | 2.60 | 1,200 | 2.60 | 3,120 | 6.76 |
| 12 lb. Cannon. | 1,200 ft. | 2.70 | 1,200 | 2.70 | 3,240 | 7.29 |
| 12 lb. Cannon. | 1,200 ft. | 2.80 | 1,200 | 2.80 | 3,360 | 7.84 |
| 12 lb. Cannon. | 1,200 ft. | 2.90 | 1,200 | 2.90 | 3,480 | 8.41 |
| 12 lb. Cannon. | 1,200 ft. | 3.00 | 1,200 | 3.00 | 3,600 | 9.00 |
| 12 lb. Cannon. | 1,200 ft. | 3.10 | 1,200 | 3.10 | 3,720 | 9.61 |
| 12 lb. Cannon. | 1,200 ft. | 3.20 | 1,200 | 3.20 | 3,840 | 10.24 |
| 12 lb. Cannon. | 1,200 ft. | 3.30 | 1,200 | 3.30 | 3,960 | 10.89 |
| 12 lb. Cannon. | 1,200 ft. | 3.40 | 1,200 | 3.40 | 4,080 | 11.56 |
| 12 lb. Cannon. | 1,200 ft. | 3.50 | 1,200 | 3.50 | 4,200 | 12.25 |
| 12 lb. Cannon. | 1,200 ft. | 3.60 | 1,200 | 3.60 | 4,320 | 12.96 |
| 12 lb. Cannon. | 1,200 ft. | 3.70 | 1,200 | 3.70 | 4,440 | 13.69 |
| 12 lb. Cannon. | 1,200 ft. | 3.80 | 1,200 | 3.80 | 4,560 | 14.44 |
| 12 lb. Cannon. | 1,200 ft. | 3.90 | 1,200 | 3.90 | 4,680 | 15.21 |
| 12 lb. Cannon. | 1,200 ft. | 4.00 | 1,200 | 4.00 | 4,800 | 16.00 |
| 12 lb. Cannon. | 1,200 ft. | 4.10 | 1,200 | 4.10 | 4,920 | 16.81 |
| 12 lb. Cannon. | 1,200 ft. | 4.20 | 1,200 | 4.20 | 5,040 | 17.64 |
| 12 lb. Cannon. | 1,200 ft. | 4.30 | 1,200 | 4.30 | 5,160 | 18.49 |
| 12 lb. Cannon. | 1,200 ft. | 4.40 | 1,200 | 4.40 | 5,280 | 19.36 |
| 12 lb. Cannon. | 1,200 ft. | 4.50 | 1,200 | 4.50 | 5,400 | 20.25 |
| 12 lb. Cannon. | 1,200 ft. | 4.60 | 1,200 | 4.60 | 5,520 | 21.16 |
| 12 lb. Cannon. | 1,200 ft. | 4.70 | 1,200 | 4.70 | 5,640 | 22.09 |
| 12 lb. Cannon. | 1,200 ft. | 4.80 | 1,200 | 4.80 | 5,760 | 23.04 |
| 12 lb. Cannon. | 1,200 ft. | 4.90 | 1,200 | 4.90 | 5,880 | 24.01 |
| 12 lb. Cannon. | 1,200 ft. | 5.00 | 1,200 | 5.00 | 6,000 | 25.00 |
| 12 lb. Cannon. | 1,200 ft. | 5.10 | 1,200 | 5.10 | 6,120 | 26.01 |
| 12 lb. Cannon. | 1,200 ft. | 5.20 | 1,200 | 5.20 | 6,240 | 27.04 |
| 12 lb. Cannon. | 1,200 ft. | 5.30 | 1,200 | 5.30 | 6,360 | 28.09 |
| 12 lb. Cannon. | 1,200 ft. | 5.40 | 1,200 | 5.40 | 6,480 | 29.16 |
| 12 lb. Cannon. | 1,200 ft. | 5.50 | 1,200 | 5.50 | 6,600 | 30.25 |
| 12 lb. Cannon. | 1,200 ft. | 5.60 | 1,200 | 5.60 | 6,720 | 31.36 |
| 12 lb. Cannon. | 1,200 ft. | 5.70 | 1,200 | 5.70 | 6,840 | 32.49 |
| 12 lb. Cannon. | 1,200 ft. | 5.80 | 1,200 | 5.80 | 6,960 | 33.64 |
| 12 lb. Cannon. | 1,200 ft. | 5.90 | 1,200 | 5.90 | 7,080 | 34.81 |
| 12 lb. Cannon. | 1,200 ft. | 6.00 | 1,200 | 6.00 | 7,200 | 36.00 |
| 12 lb. Cannon. | 1,200 ft. | 6.10 | 1,200 | 6.10 | 7,320 | 37.21 |
| 12 lb. Cannon. | 1,200 ft. | 6.20 | 1,200 | 6.20 | 7,440 | 38.44 |
| 12 lb. Cannon. | 1,200 ft. | 6.30 | 1,200 | 6.30 | 7,560 | 39.69 |
| 12 lb. Cannon. | 1,200 ft. | 6.40 | 1,200 | 6.40 | 7,680 | 40.96 |
| 12 lb. Cannon. | 1,200 ft. | 6.50 | 1,200 | 6.50 | 7,800 | 42.25 |
| 12 lb. Cannon. | 1,200 ft. | 6.60 | 1,200 | 6.60 | 7,920 | 43.56 |
| 12 lb. Cannon. | 1,200 ft. | 6.70 | 1,200 | 6.70 | 8,040 | 44.89 |
| 12 lb. Cannon. | 1,200 ft. | 6.80 | 1,200 | 6.80 | 8,160 | 46.24 |
| 12 lb. Cannon. | 1,200 ft. | 6.90 | 1,200 | 6.90 | 8,280 | 47.61 |
| 12 lb. Cannon. | 1,200 ft. | 7.00 | 1,200 | 7.00 | 8,400 | 49.00 |
| 12 lb. Cannon. | 1,200 ft. | 7.10 | 1,200 | 7.10 | 8,520 | 50.41 |
| 12 lb. Cannon. | 1,200 ft. | 7.20 | 1,200 | 7.20 | 8,640 | 51.84 |
| 12 lb. Cannon. | 1,200 ft. | 7.30 | 1,200 | 7.30 | 8,760 | 53.29 |
| 12 lb. Cannon. | 1,200 ft. | 7.40 | 1,200 | 7.40 | 8,880 | 54.76 |
| 12 lb. Cannon. | 1,200 ft. | 7.50 | 1,200 | 7.50 | 9,000 | 56.25 |
| 12 lb. Cannon. | 1,200 ft. | 7.60 | 1,200 | 7.60 | 9,120 | 57.76 |
| 12 lb. Cannon. | 1,200 ft. | 7.70 | 1,200 | 7.70 | 9,240 | 59.29 |
| 12 lb. Cannon. | 1,200 ft. | 7.80 | 1,200 | 7.80 | 9,360 | 60.84 |
| 12 lb. Cannon. | 1,200 ft. | 7.90 | 1,200 | 7.90 | 9,480 | 62.41 |
| 12 lb. Cannon. | 1,200 ft. | 8.00 | 1,200 | 8.00 | 9,600 | 64.00 |
| 12 lb. Cannon. | 1,200 ft. | 8.10 | 1,200 | 8.10 | 9,720 | 65.61 |
| 12 lb. Cannon. | 1,200 ft. | 8.20 | 1,200 | 8.20 | 9,840 | 67.24 |
| 12 lb. Cannon. | 1,200 ft. | 8.30 | 1,200 | 8.30 | 9,960 | 68.89 |
| 12 lb. Cannon. | 1,200 ft. | 8.40 | 1,200 | 8.40 | 10,080 | 70.56 |
| 12 lb. Cannon. | 1,200 ft. | 8.50 | 1,200 | 8.50 | 10,200 | 72.25 |
| 12 lb. Cannon. | 1,200 ft. | 8.60 | 1,200 | 8.60 | 10,320 | 73.96 |
| 12 lb. Cannon. | 1,200 ft. | 8.70 | 1,200 | 8.70 | 10,440 | 75.69 |
| 12 lb. Cannon. | 1,200 ft. | 8.80 | 1,200 | 8.80 | 10,560 | 77.44 |
| 12 lb. Cannon. | 1,200 ft. | 8.90 | 1,200 | 8.90 | 10,680 | 79.21 |
| 12 lb. Cannon. | 1,200 ft. | 9.00 | 1,200 | 9.00 | 10,800 | 81.00 |
| 12 lb. Cannon. | 1,200 ft. | 9.10 | 1,200 | 9.10 | 10,920 | 82.81 |
| 12 lb. Cannon. | 1,200 ft. | 9.20 | 1,200 | 9.20 | 11,040 | 84.64 |
| 12 lb. Cannon. | 1,200 ft. | 9.30 | 1,200 | 9.30 | 11,160 | 86.49 |
| 12 lb. Cannon. | 1,200 ft. | 9.40 | 1,200 | 9.40 | 11,280 | 88.36 |
| 12 lb. Cannon. | 1,200 ft. | 9.50 | 1,200 | 9.50 | 11,400 | 90.25 |
| 12 lb. Cannon. | 1,200 ft. | 9.60 | 1,200 | 9.60 | 11,520 | 92.16 |
| 12 lb. Cannon. | 1,200 ft. | 9.70 | 1,200 | 9.70 | 11,640 | 94.09 |
| 12 lb. Cannon. | 1,200 ft. | 9.80 | 1,200 | 9.80 | 11,760 | 96.04 |
| 12 lb. Cannon. | 1,200 ft. | 9.90 | 1,200 | 9.90 | 11,880 | 98.01 |
| 12 lb. Cannon. | 1,200 ft. | 10.00 | 1,200 | 10.00 | 12,000 | 100.00 |

Vertical Velocity and Range of Shot and Shells.

The *Range* of a shot or shell is the distance it is first seen upon a horizontal plane the place required upon its regular passage.

| Calibre of Gun. | Velocity. | Time. | Initial Velocity. | Time of Flight. | Range. | Height. |
|-----------------|-----------|-------|-------------------|-----------------|--------|---------|
| 12 lb. Cannon. | 1,200 ft. | 1.00 | 1,200 | 1.00 | 1,200 | 1.00 |
| 12 lb. Cannon. | 1,200 ft. | 1.10 | 1,200 | 1.10 | 1,320 | 1.21 |
| 12 lb. Cannon. | 1,200 ft. | 1.20 | 1,200 | 1.20 | 1,440 | 1.44 |
| 12 lb. Cannon. | 1,200 ft. | 1.30 | 1,200 | 1.30 | 1,560 | 1.69 |
| 12 lb. Cannon. | 1,200 ft. | 1.40 | 1,200 | 1.40 | 1,680 | 1.96 |
| 12 lb. Cannon. | 1,200 ft. | 1.50 | 1,200 | 1.50 | 1,800 | 2.25 |
| 12 lb. Cannon. | 1,200 ft. | 1.60 | 1,200 | 1.60 | 1,920 | 2.56 |
| 12 lb. Cannon. | 1,200 ft. | 1.70 | 1,200 | 1.70 | 2,040 | 2.89 |
| 12 lb. Cannon. | 1,200 ft. | 1.80 | 1,200 | 1.80 | 2,160 | 3.24 |
| 12 lb. Cannon. | 1,200 ft. | 1.90 | 1,200 | 1.90 | 2,280 | 3.61 |
| 12 lb. Cannon. | 1,200 ft. | 2.00 | 1,200 | 2.00 | 2,400 | 4.00 |
| 12 lb. Cannon. | 1,200 ft. | 2.10 | 1,200 | 2.10 | 2,520 | 4.41 |
| 12 lb. Cannon. | 1,200 ft. | 2.20 | 1,200 | 2.20 | 2,640 | 4.84 |
| 12 lb. Cannon. | 1,200 ft. | 2.30 | 1,200 | 2.30 | 2,760 | 5.29 |
| 12 lb. Cannon. | 1,200 ft. | 2.40 | 1,200 | 2.40 | 2,880 | 5.76 |
| 12 lb. Cannon. | 1,200 ft. | 2.50 | 1,200 | 2.50 | 3,000 | 6.25 |
| 12 lb. Cannon. | 1,200 ft. | 2.60 | 1,200 | 2.60 | 3,120 | 6.76 |
| 12 lb. Cannon. | 1,200 ft. | 2.70 | 1,200 | 2.70 | 3,240 | 7.29 |
| 12 lb. Cannon. | 1,200 ft. | 2.80 | 1,200 | 2.80 | 3,360 | 7.84 |
| 12 lb. Cannon. | 1,200 ft. | 2.90 | 1,200 | 2.90 | 3,480 | 8.41 |
| 12 lb. Cannon. | 1,200 ft. | 3.00 | 1,200 | 3.00 | 3,600 | 9.00 |
| 12 lb. Cannon. | 1,200 ft. | 3.10 | 1,200 | 3.10 | 3,720 | 9.61 |
| 12 lb. Cannon. | 1,200 ft. | 3.20 | 1,200 | 3.20 | 3,840 | 10.24 |
| 12 lb. Cannon. | 1,200 ft. | 3.30 | 1,200 | 3.30 | 3,960 | 10.89 |
| 12 lb. Cannon. | 1,200 ft. | 3.40 | 1,200 | 3.40 | 4,080 | 11.56 |
| 12 lb. Cannon. | 1,200 ft. | 3.50 | 1,200 | 3.50 | 4,200 | 12.25 |
| 12 lb. Cannon. | 1,200 ft. | 3.60 | 1,200 | 3.60 | 4,320 | 12.96 |
| 12 lb. Cannon. | 1,200 ft. | 3.70 | 1,200 | 3.70 | 4,440 | 13.69 |
| 12 lb. Cannon. | 1,200 ft. | 3.80 | 1,200 | 3.80 | 4,560 | 14.44 |
| 12 lb. Cannon. | 1,200 ft. | 3.90 | 1,200 | 3.90 | 4,680 | 15.21 |
| 12 lb. Cannon. | 1,200 ft. | 4.00 | 1,200 | 4.00 | 4,800 | 16.00 |
| 12 lb. Cannon. | 1,200 ft. | 4.10 | 1,200 | 4.10 | 4,920 | 16.81 |
| 12 lb. Cannon. | 1,200 ft. | 4.20 | 1,200 | 4.20 | 5,040 | 17.64 |
| 12 lb. Cannon. | 1,200 ft. | 4.30 | 1,200 | 4.30 | 5,160 | 18.49 |
| 12 lb. Cannon. | 1,200 ft. | 4.40 | 1,200 | 4.40 | 5,280 | 19.36 |
| 12 lb. Cannon. | 1,200 ft. | 4.50 | 1,200 | 4.50 | 5,400 | 20.25 |
| 12 lb. Cannon. | 1,200 ft. | 4.60 | 1,200 | 4.60 | 5,520 | 21.16 |
| 12 lb. Cannon. | 1,200 ft. | 4.70 | 1,200 | 4.70 | 5,640 | 22.09 |
| 12 lb. Cannon. | 1,200 ft. | 4.80 | 1,200 | 4.80 | 5,760 | 23.04 |
| 12 lb. Cannon. | 1,200 ft. | 4.90 | 1,200 | 4.90 | 5,880 | 24.01 |
| 12 lb. Cannon. | 1,200 ft. | 5.00 | 1,200 | 5.00 | 6,000 | 25.00 |
| 12 lb. Cannon. | 1,200 ft. | 5.10 | 1,200 | 5.10 | 6,120 | 26.01 |
| 12 lb. Cannon. | 1,200 ft. | 5.20 | 1,200 | 5.20 | 6,240 | 27.04 |
| 12 lb. Cannon. | 1,200 ft. | 5.30 | 1,200 | 5.30 | 6,360 | 28.09 |
| 12 lb. Cannon. | 1,200 ft. | 5.40 | 1,200 | 5.40 | 6,480 | 29.16 |
| 12 lb. Cannon. | 1,200 ft. | 5.50 | 1,200 | 5.50 | 6,600 | 30.25 |
| 12 lb. Cannon. | 1,200 ft. | 5.60 | 1,200 | 5.60 | 6,720 | 31.36 |
| 12 lb. Cannon. | 1,200 ft. | 5.70 | 1,200 | 5.70 | 6,840 | 32.49 |
| 12 lb. Cannon. | 1,200 ft. | 5.80 | 1,200 | 5.80 | 6,960 | 33.64 |
| 12 lb. Cannon. | 1,200 ft. | 5.90 | 1,200 | 5.90 | 7,080 | 34.81 |
| 12 lb. Cannon. | 1,200 ft. | 6.00 | 1,200 | 6.00 | 7,200 | 36.00 |
| 12 lb. Cannon. | 1,200 ft. | 6.10 | 1,200 | 6.10 | 7,320 | 37.21 |
| 12 lb. Cannon. | 1,200 ft. | 6.20 | 1,200 | 6.20 | 7,440 | 38.44 |
| 12 lb. Cannon. | 1,200 ft. | 6.30 | 1,200 | 6.30 | 7,560 | 39.69 |
| 12 lb. Cannon. | 1,200 ft. | 6.40 | 1,200 | 6.40 | 7,680 | 40.96 |
| 12 lb. Cannon. | 1,200 ft. | 6.50 | 1,200 | 6.50 | 7,800 | 42.25 |
| 12 lb. Cannon. | 1,200 ft. | 6.60 | 1,200 | 6.60 | 7,920 | 43.56 |
| 12 lb. Cannon. | 1,200 ft. | 6.70 | 1,200 | 6.70 | 8,040 | 44.89 |
| 12 lb. Cannon. | 1,200 ft. | 6.80 | 1,200 | 6.80 | 8,160 | 46.24 |
| 12 lb. Cannon. | 1,200 ft. | 6.90 | 1,200 | 6.90 | 8,280 | 47.61 |
| 12 lb. Cannon. | 1,200 ft. | 7.00 | 1,200 | 7.00 | 8,400 | 49.00 |
| 12 lb. Cannon. | 1,200 ft. | 7.10 | 1,200 | 7.10 | 8,520 | 50.41 |
| 12 lb. Cannon. | 1,200 ft. | 7.20 | 1,200 | 7.20 | 8,640 | 51.84 |
| 12 lb. Cannon. | 1,200 ft. | 7.30 | 1,200 | 7.30 | 8,760 | 53.29 |
| 12 lb. Cannon. | 1,200 ft. | 7.40 | 1,200 | 7.40 | 8,880 | 54.76 |
| 12 lb. Cannon. | 1,200 ft. | 7.50 | 1,200 | 7.50 | 9,000 | 56.25 |
| 12 lb. Cannon. | 1,200 ft. | 7.60 | 1,200 | 7.60 | 9,120 | 57.76 |
| 12 lb. Cannon. | 1,200 ft. | 7.70 | 1,200 | 7.70 | 9 | |

ments of Report of Board of Engineers for Fortifications, U. S. A. Professional Papers No. 25. (Brev. Maj.-Gen. Z. B. Tower.)

Experimental firings for penetration during the past twenty years have ascertained that wrought iron and cast iron, unless chilled, are unsuitable for tiles to be used against iron armor; that the best material for that use is hammered steel or Whitworth's compressed steel.

That cast-iron and cast-steel armor-plates will break up under the impact of the heaviest projectiles now in service, unless made so thick as to preclude their use in ship-protection.

That wrought-iron plates have been so perfected that they do not break when penetrated by displacement or crowding aside of the material in front of the shot, the rate of penetration bearing an approximately determined ratio to the striking energy of the projectile, measured per inch of circumference, as expressed by the following formula:

$$\sqrt{\frac{V^2 P}{2g \times 2 \pi r \times 2240 \times .86}} = \text{penetration in ins. } V \text{ representing velocity in ft. per second, } P \text{ weight of shot in lbs., and } r \text{ radius of shot in ins.}$$

That such plates can therefore be safely used in ship construction, their thickness being determined by the limit of flotation and the protection required.

That, though experiments with wrought-iron plates, faced with steel, have not been sufficiently extended to determine the best combination of two materials, we may nevertheless assume that they give a resistance at one fourth greater than those of homogenous iron.

That hammered steel in the late Spezzia trials proved superior to any material hitherto tested for armor-plates. The 10-inch plate resisted penetration, and was only partially broken up by 4 shots, three of which had striking energy of between 33 000 and 34 000 foot-tons each. Not one shot penetrated the plate. Those of chilled iron were broken up, and the steel tile, though of excellent quality, was set up to about two thirds of its original thickness.

Velocity and Ranges of Shot. (Krupp's Ballistic Tables.)

Penetration in Wrought Iron.

$$\sqrt{\frac{V^2 P}{2g \times 2 \pi r \times 2240 \times C}} = \text{penetration in ins. } C = 2.53.$$

| GUN. | Caliber. | Powder. | Shot. | Velocity | | | Penetration | | |
|---------------|----------|---------|-------|--------------------|--------|------|-------------|-------|-------|
| | | | | at Muzzle per Sec. | Range. | | at Muzzle | Range | |
| | Tons. | Ins. | Lbs. | Feet. | Yds. | Yds. | Ins. | Ins. | Ins. |
| Long, 100.. | 17.75 | 550 | 2022 | 1715 | 1424 | 1191 | 34.76 | 33.2 | 27.55 |
| " " " " | 17.75 | 776 | 2000 | 1832 | 1518 | 1259 | 37.52 | 35.81 | 29.66 |
| 12-inch, 81.. | 16 | 445 | 1760 | 1657 | 1393 | 1181 | 32.6 | 31.23 | 26.24 |
| " " " " | 71.. | 15.75 | 485 | 1715 | 1434 | 1211 | 33.52 | 32.12 | 27.04 |
| " " " " | 18.. | 9.45 | 165 | 474 | 1688 | 1351 | 1113 | 20.42 | 19.31 |
| 8-inch.... | 8 | 35 | 180 | 1450 | 1036 | 840 | 10.23 | 9.22 | 6.72 |

* Unchambered.

2. — For 100-ton gun, steel plate 22 ins. thick, backed with 28.8 ins. of wrought-iron plates 1.5 ins. thick, and the frame of a vessel.

— Total destruction of steel plate, and backing entered to a depth of 22 in perforated.

Summary of Record of Practice in Europe with Heavy Armstrong, Woolwich, and Krupp Guns.

Board of Engineers for Fortifications, U. S. A., Professional Papers No. 25.

| Gcn. | Powder. | Projectile. | Charge of Powder. | Weight of Projectile. | Initial Velocity per Second, V. | Initial V^2 $\frac{1}{2} V^2$ $\frac{1}{2} V^2$ | Energy per inch of diameter of ball. |
|--|------------------|-------------|-------------------|-----------------------|---------------------------------|---|--------------------------------------|
| | | | Lbs. | Lbs. | Feet. | Ft.-tons. | Foot-ton. |
| ARMSTRONG, 100 Tons, caliber 17 ins., bore 30.5 feet. | 1.5-inch cubes.. | Shot..... | 330 | 2000 | 1446 | 28 990 | 544.05 |
| | Waltham Abbey | "..... | 375 | 2000 | 1543 | 33 000 | 623 |
| | Fossano..... | "..... | 400 | 2000 | 1502 | 31 282 | 585.74 |
| | "..... | "..... | 776 | 2000 | 1832 | 46 580 | 835.38 |
| WOOLWICH, 81 Tons, caliber 14.5 ins., bore 24 feet. caliber 16 ins..... | .75-inch cubes. | "..... | 170 | 1258 | 1393 | 16 922 | 371.5 |
| | 1.5 " " " | "..... | 230 | 1450 | 1440 | 20 842 | 457.57 |
| | 1.5 " " " | "..... | 250 | 1260 | 1523 | 20 259 | 444.70 |
| | 1.5 " " " | "..... | 310 | 1466 | 1553 | 24 508 | 520.4 |
| 38 Tons, caliber 12.5 ins., bore 16.5 feet. | 1.5 " " " | Ball shell | 130 | 800 | 1451 | 11 668 | 267.64 |
| | 1.5 " " " | "..... | 200 | 800 | 1421 | 11 210 | 265.4 |
| | 1.5 " " " | "..... | 180 | 800 | 1504 | 12 545 | 319.4 |
| | 1.5 " " " | "..... | 180 | 800 | 1504 | 12 545 | 319.4 |
| KRUPP, 71 Tons, caliber 15.75 ins., bore 28.58 feet. | Prism A..... | Plain ... | 298 | 1707 | 1184 | 16 602 | 335.42 |
| | " H..... | Shrapnel | 485 | 1725 | 1703 | 34 503 | 697.91 |
| | " 2 inch... | Shell ... | 441 | 1419 | 1761 | 30 484 | 616.14 |
| | " 1 hole... | Plain ... | 132 | 300 | 1873 | 7 298 | 246.09 |
| 18 Tons, caliber 9.45 ins., bore 17.5 feet. | " 2 inch... | Shrapnel | 145 | 474 | 1688 | 9 397 | 315.60 |
| | "..... | Shell ... | 165 | 300 | 1991 | 8 244 | 277.69 |

Penetration in Ball Cartridge Paper, No. 1.

| | |
|---|-------------|
| Musket, with 134 grains, at 13.3 yards..... | 653 sheets. |
| Common rifle, 92 grains, at 13.3 yards..... | 500 sheets. |

Penetration of Lead Balls in Small Arms.

Experiments at Washington Arsenal in 1839, and at West Point in 1837.

| Arm. | Diameter of Ball. | Charge Powder. | Distance. | Weight of Ball. | Penetration. | |
|-------------------------|-------------------|----------------|-----------|-----------------|--------------|-------------|
| | | | | | White Oak. | White Pine. |
| | Inch. | Grains. | Yards. | Grains. | Inch. | Inch. |
| Musket..... | .64 | 134 | 9 | 397.5 | 1.6 | — |
| | .64 | 144 | 5 | 397.5 | 3 | — |
| Common Rifle..... | — | 100 | 5 | 219 | 2.05 | — |
| | — | 92 | 9 | — | 1.8 | — |
| Hall's rifle..... | — | 100 | 5 | 219 | 2 | — |
| | — | 70 | 9 | 219 | .6 | — |
| | — | 70 | 5 | — | 1.7 | — |
| 1's carbine, musket | .5775 | 80 | 5 | 219 | .8 | — |
| liber..... | | 90* | 5 | — | 1.1 | — |
| | | 100* | 5 | — | 1.2 | — |
| Pistol..... | — | 51 | 5 | 219 | .725 | — |
| 6 musket..... | .5775 | — | 200 | 500 | — | 11 |
| 7 musket..... | .685 | 60 | 200 | 730 | — | 10.5 |
| Rifle, Harper's Ferry.. | .5775 | 70 | 200 | 500 | — | 9.33 |
| Pistol carbine..... | .5775 | 40 | 200 | 450 | — | 5.75 |
| Sharpe's carbine..... | .55 | 60 | 30 | 463 | — | 7.17 |
| Burr..... | .55 | 55 | 30 | 350 | — | 6.15 |

* Charges too great for service.

at 9 yards distance, with a charge of 134 grains, 1 ball will penetrate 1.15 ins., buckshot, .41 inch.

Loss of Force by Windage.

parison of results shows that 4 lbs. of powder give to a ball without wind-
 urly as great a velocity as is given by 6 lbs. having .14 inch windage, which
 windage of a 24-lb. ball; or, in other words, this windage causes a loss of
one third of force of charge.

5.—Experiments show that loss of force by escape of gas from vent un is altogether inconsiderable when compared with whole force of

meter of *Vent* in U. S. Ordnance is in all cases .2 inch.

Effect of different Waddings with a Charge of 77 Grains of Powder.

| WAD. | Velocity of Ball per Second. |
|--|---------------------------------|
| wrapped in cartridge paper and crumpled..... | Feet. 1377 |
| ; wad upon powder and 1 upon ball..... | 1346 |
| ; wads upon powder and 1 upon ball..... | 1482 |
| stic wad upon powder and 1 upon ball..... | 1132 |
| teboard wads upon powder..... | 1200 |
| stic wads upon powder..... | 1100 |

wads cut from body of a hat, weight 3 grains.

board wads .1 of an inch thick, weight 8 grains.

ridge paper 3 X 4.5 ins., weight 12.82 grains.

tic wads, "Baldwin's indented," a little more than .1 of an inch thick,
5.127 grains.

the most advantageous wads are those made of thick pasteboard, or of ordinary cartridge paper.

service of *cannon*, heavy wads over ball are in all respects injurious.

purpose of retaining the ball in its place, light *grommets* should be used.

the other hand, it is of great importance, and especially so in use of small that there should be a good wad over powder for developing full force of unless, as in the rifle, the ball has but very little windage. (Capt. Mordecai.)

Weight and Dimensions of Lead Balls.

Number of Balls in a Lb., from 1.67 to .237 of an Inch Diameter.

| No. | Diam. | No. | Diam. | No. | Diam. | No. | Diam. | No. | Diam. | No. |
|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|
| | Inch. | | Inch. | | Inch. | | Inch. | | Inch. | |
| 1 | .75 | 11 | .57 | 25 | .388 | 80 | .301 | 170 | .259 | 270 |
| 2 | .73 | 12 | .537 | 30 | .375 | 88 | .295 | 180 | .256 | 280 |
| 3 | .71 | 13 | .51 | 35 | .372 | 90 | .29 | 190 | .252 | 290 |
| 4 | .693 | 14 | .505 | 36 | .359 | 100 | .285 | 200 | .249 | 300 |
| 5 | .677 | 15 | .488 | 40 | .348 | 110 | .281 | 210 | .247 | 310 |
| 6 | .662 | 16 | .469 | 45 | .338 | 120 | .276 | 220 | .244 | 320 |
| 7 | .65 | 17 | .453 | 50 | .329 | 130 | .272 | 230 | .242 | 330 |
| 8 | .637 | 18 | .426 | 60 | .321 | 140 | .268 | 240 | .239 | 340 |
| 9 | .625 | 19 | .405 | 70 | .314 | 150 | .265 | 250 | .237 | 350 |
| 10 | .615 | 20 | .395 | 75 | .307 | 160 | .262 | 260 | | |

led shot do not return to their original dimensions upon cooling, but retain a permanent enlargement of about .02 per cent. in volume.

Number of Pellets in an Ounce of Lead Shot of the different Sizes.

| | | | | | | | | |
|-------------|------------|-----|------------|-----|------------|-----|------------|---|
| 40 | B..... | 75 | No. 3..... | 135 | No. 6..... | 280 | No. 9..... | 6 |
| 50 | No. 1..... | 82 | 4..... | 177 | 7..... | 341 | 10..... | 1 |
| 58 | 2..... | 112 | 5..... | 218 | 8..... | 600 | 12..... | |
| No. 14..... | | | 3150 | | | | | |

Proportion of Powder to Shot for following Numbers of Shot.

| No. | Shot. | Powder. | No. | Shot. | Powder. | No. | Shot. | Powder. |
|-----|-------|---------|-----|-------|---------|-----|-------|---------|
| | Oz. | Drams. | | Oz. | Drams. | | Oz. | Drams. |
| 2 | 1.5 | 1.5 | 4 | 1.5 | 1.875 | 6 | 1.25 | 2.375 |
| 3 | 1.75 | 1.625 | 5 | 1.375 | 2.125 | 7 | 1.125 | 2.625 |

NOTE.—2 oz. of No. 2 shot, with 1.5 drams of powder, produced greatest effect.

Increase of powder for greater number of pellets is in consequence of increased friction of their projection.

Numbers of Percussion Caps corresponding with Birmingham Numbers.

| Eley's..... | 5 | 6 | 7 | 8 | 9 | 24 | 10 | 11 | 12 | 13 | 14 |
|--------------|----|----|----|----|----|----|-----------|-----------|-----------|----|----|
| Birmingham.. | 43 | 44 | 46 | 48 | 49 | 50 | 51 and 52 | 53 and 54 | 55 and 56 | 57 | 58 |

Where there are two numbers of Birmingham sizes corresponding with only one of Eley's, it is in consequence of two numbers being of *same size*, varying only in length of caps.

Comparison of Force of a Charge in various Arms.

| ARM. | Lock. | Powder, A 5. | Windage. | Weight of Ball. | Velocity. |
|----------------------|-------------|--------------|------------|-----------------|-----------|
| Ordinary rifle..... | Percussion. | Grains. 100 | Inch. .015 | Grains. 219 | Ft. 200 |
| " " | " | 70 | .015 | 219 | 175 |
| Hall's rifle..... | Flint. | 70 | .0 | 219 | 149 |
| Hall's carbine..... | Percussion. | 70 | .0 | 219 | 149 |
| Jenks's carbine..... | " | 70 | .0 | 219 | 149 |
| Cadet's musket..... | Flint. | 70 | .045 | 219 | 149 |
| Pistol..... | Percussion. | 35 | .015 | 218.5 | 94 |

Ranges for Small Arms.

Musket.—With a ball of 17 to pound, and a charge of 110 grains of powder, etc., an elevation of 36° is required for a range of 200 yards; and for a range of 500 yards, an elevation of 30° 30' is necessary, and at this distance a ball will pass through a pine board 1 inch in thickness.

Rifle.—With a charge of 70 grains, an effective range of from 300 to 350 yards is obtained; but as 75 grains can be used without *stripping* the ball, it is deemed better to use it, to allow for accidental loss, deterioration of powder, etc.

Pistol.—With a charge of 30 grains, the ball is projected through a pine board 1 inch in thickness at a distance of 80 yards.

Gunpowder.

Gunpowder is distinguished as *Musket*, *Mortar*, *Common*, *Mammoth*, and *Sporting* powder; it is all made in same manner, of same proportions of materials, and differs only in size of its grain.

Pressure or Explosive Energy.—By the experiments of Captain Rodman, U. S. Ordnance Corps, a pressure of 45 000 lbs. per square inch was obtained with 10 lbs. of powder, and a ball of 45 lbs.

Also, a pressure of 185 000 lbs. per sq. inch was obtained when the powder was burned in its own volume, in a cast-iron shell having diameters of 3.65 and 12 in.

Proof of Powder. (U. S. Ordnance Manual.)

Powder in magazines that does not range over 150 yards is held to be unserviceable.

Good powder averages from 250 to 300 yards; small grain, from 300 to 500 yards.

Restoring Deteriorated Powder.—When powder has been damaged by being stored in damp places, it loses its strength, and requires to be worked over. A quantity of moisture absorbed does not exceed 7 per cent., it is sufficient to render it unfit for service. This is done by exposing it to the sun.

—It has absorbed more than 7 per cent. of water it should be *worked over*.

Properties and Results of Gunpowder, determined by Experiments. (Captain A. Mordecai, U. S. A.)

| 24-POUNDER GUN. | | | MUSKET PENDULUM. | | |
|----------------------------|------------|--|----------------------|--------------|--|
| Height of ball and wad.... | 24.25 lbs. | | Weight of ball..... | 397.5 grains | |
| " powder..... | 6 " | | " powder..... | 120 " | |
| Age of ball..... | .135 inch. | | Windage of ball..... | .09 inch | |

| GRAIN. | Composition. | | | Manufacture. Where from. | Number of Grains in 10 Troy Grains. | Relative Quickness of Burning. | Water ab- sorbed by ex- posure to Air. | Relative Force. |
|---------------|-----------------|----------------|---------------|----------------------------------|---|--------------------------------------|--|--------------------|
| | Salt- petre. | Char- coal. | Sul- phur. | | | | | |
| on, large... | 76 | 14 | 12 | * Dupont's Mills, Del. | 77 | 275 | 2.77 | .677 |
| small... | | | | | 569 | 314 | 3.35 | .72 |
| et..... | | | | | 1134 | 214 | — | .808 |
| | | | | | 6174 | 142 | — | .907 |
| et..... | | | | | 5344 | 282 | 3.55 | .728 |
| | 75 | 12.5 | 12.5 | † Dupont's Mills, Del. | 1642 | — | — | .834 |
| on, uneven... | | | | | 13152 | — | — | .943 |
| large... | | | | | 166 | 183 | 2.09 | .788 |
| ing..... | | | | | 103 | 182 | 1.91 | .756 |
| ing, uneven | | | | | 72808 | 100 | 4.42 | — |
| | 77 | 13 | 10 | * Dupont's Mills, Del. | 295 | 212 | — | .82 |
| ing..... | 70 | 15 | 15 | | 2378 | 204 | — | .888 |
| | 76 | 15 | 9 | Loomis, Hazard, & Co., Conn.* | — | — | — | .865 |
| ing..... | 75 | 15 | 10 | Waltham Abbey, England.* | 11600 | — | — | — |

* Glazed.

† Rough.

Manufacture of Powder.—Powder of greatest force, whether for cannon or small, is produced by incorporation in the "cylinder mill."

Effect of Size of Grain.—Within limits of difference in size of grain, which occurs in ordinary cannon powder, the granulation appears to exercise but little influence on the force of it, unless grain be exceedingly dense and hard.

Effect of Glazing.—Glazing is favorable to production of greatest force, and to the combustion of grains, by affording a rapid transmission of flame through the powder.

Effect of using Percussion Primers.—Increase of force by use of primers, which close vent, is constant and appreciable in amount, yet not of sufficient value to authorize a reduction of charge.

Ratio of Relative Strength of different Powders for use under water differs but little from the reciprocal of the ratio between the sizes of the grains, owing that the strength is nearly inversely proportional thereto.*

Relative Force of different Powders.—Mortar, .08; Oliver, .09; Cannon, .18; Mortar, 1; Musket, 1.57; Shot, 2.61, and Safety Compound 30.62.

Qualities of different Powders.—Qualin is nitro-glycerine absorbed by Schultz's powder.

Other powders and explosive materials see Blasting, p. 443.

Heat and Explosive Power. (Capt. Noble and F. A. Abel.)

Heat of explosion.—A gram of fired powder evolves a mean temperature of 730°. Temperature of explosion 3970°. Volume of permanent gas (which is in an inverse ratio to units of heat evolved) at 32° = 250.

Explosive power of powder, as tested in Ordnance, ranges, for volumes of expansion of 1.5 to 50 times, from 36 to 170 foot-tons per lb. burned.

Charge of 70 lbs. gave to an 180 lbs. shot a velocity of 1694 feet per second, equal to a total energy of 3637 foot-tons, and a charge of 100 lbs. a velocity of 2182 feet, and an energy of 5940 foot-tons.

* of Experiments and Investigations to develop a system of submarine mines. Prof. S. E., No. 23.

H E A T.

Heat, alike to gravity, is a universal force, and is referred to both as cause and effect.

Caloric is usually treated of as a material substance, though its claims to this distinction are not decided; the strongest argument in favor of this position is that of its power of radiation. Upon touching a body having a higher temperature than our own, caloric passes from it, and excites the feeling of warmth; and when we touch a body having a lower temperature than our own, caloric passes from our body to it, and thus arises the sensation of cold.

To avoid any ambiguity that may arise from use of the same expression, it is usual and proper to employ the word *Caloric* to signify the principle or cause of sensation of heat.

Heat Unit.—For purpose of expressing and comparing quantities of heat, it is convenient and customary to adopt a *Unit of heat* or *Thermal unit*, being that quantity of heat which is raised or lost in a defined period of temperature in a defined weight of a particular substance.

Thus, a Thermal unit, *Is quantity of heat which corresponds to an interval of 1° in temperature of 1 lb. of pure liquid water, at and near its temperature of greatest density.*

Thermal unit in France, termed *Caloric*, *Is quantity of heat which corresponds to an interval of 1° C. in temperature of 1 kilogramme of pure liquid water, at and near its temperature of greatest density.*

Thermal unit to Caloric, .396832; Caloric to Thermal unit, .251996.

One Thermal unit or 1° in 1 lb. of water, 772 foot-lbs.

One Caloric or 1° C. in 1 kilogramme of water, 423.55 kilogrammetres.

1° C. in 1 lb. water, 1389.6 foot-lbs.

Ratio of Fahrenheit to Centigrade, 1.8; of Centigrade to Fahrenheit, .555.

Absolute Temperature, Is a temperature assigned by deduction, as an opportunity of observing it cannot occur, it being the temperature corresponding to entire absence of gaseous elasticity, or when pressure and volume = 0. By Fahrenheit it is -461.2° , by Reaumur -229.2° , and by Centigrade -274° .

Heat is termed *Sensible* when it diffuses itself to all surrounding bodies; hence it is free and uncombined, passing from one substance to another, affecting the senses in its passage, determining the height of the thermometer, etc.

Temperature of a body, is the quantity of sensible heat in it, present at any moment.

Heat is developed by water when it is violently agitated.

Heat is developed by percussion of a metal, and it is greatest at the first blow.

Quantities of heat evolved are nearly the same for same substance, without reference to temperature of its combustion.

Mechanical power in
tion or compression,
tion to quantity of,
necessary to raise 1
the mechanical equi

ed in production of heat either by fric-
heat produced bears the same propor-
tr expended, being 1 unit for power
this number of 772 is termed

Specific Heat.

Specific Heat of a body signifies its capacity for heat, or quantity required to raise temperature of a body 1° , or it is that which is absorbed by different bodies of equal weights or volumes when their temperature is equal, based upon the law, *That similar quantities of different bodies require unequal quantities of heat at any given temperature.* It is also the *quantity of heat* requisite to change the temperature of a body any stated number of degrees compared with that which would produce same effect upon water at 32° .

Quantity of heat, therefore, is the quantity necessary to change the temperature of a body by any given amount (as 1°), divided by quantity of heat necessary to change an equal weight or volume of water 32° by same amount.

NOTE.—Water has greater specific heat than any known body.

Every substance has a specific heat peculiar to itself, whence a change of position will be attended by a change of its capacity for heat.

Specific heat of a body varies with its form. A solid has a less capacity for heat than same substance when in state of a liquid; specific heat of water, for instance, being .5 in solid state (ice), .622 in gaseous (steam), and 1 in liquid.

Specific heat of equal weights of same gas increases as density decreases; exact rate of increase is not known, but ratio is less rapid than diminution of density.

Change of capacity for heat always occasions a change of temperature. Increase in former is attended by diminution of latter, and contrariwise.

Specific heat multiplied by atomic weight of a substance will give a constant 37.5 as an average, which shows that the atoms of all substances have equal capacity for heat. This is a result for which as no reason has been assigned.

Thus: atomic weights of lead and copper are respectively 1294.5 and 395.7, and their specific heats are .031 and .095. Hence $1294.5 \times .031 = 40.129$, and $395.7 \times .095 = 37.591$.

It is important to know the relative Specific Heat of bodies. The most convenient method of discovering it is by mixing different substances together at different temperatures, and noting temperature of mixture; and by experiments it appears that the same quantity of heat imparts twice as high a temperature to mercury as to an equal quantity of water; thus, when water at 100° and mercury at 40° are mixed together, the mixture will be at 80° , the 20° lost by the water being a rise of 40° in the mercury; and when weights are substituted for measures, the fact is strikingly illustrated; for instance, on mixing a pound of mercury at 100° with a pound of water at 160° , a thermometer placed in it will fall to 155° . It appears that same quantity of heat imparts twice as high a temperature to mercury as to an equal volume of water, and that the heat which gives 5° to water will raise an equal weight of mercury 115° , being the ratio of 1 to 23. Hence, if equal quantities of heat be added to equal weights of water and mercury, their temperatures will be expressed in relation to each other by numbers 1 and 23; or, in order to increase the temperature of equal weights of the two substances to the same extent, the water will require 23 times as much heat as the mercury.

Capacity for Heat is relative power of a body in receiving heat in being raised to any given temperature, or the quantity of heat so received and retained.

Specific Heat of Air and other permanent gases. The specific heat, or capacity for heat, of permanent gases is the same at all temperatures, and for all densities. Capacity for heat

same for each degree of temperature. M. Regnault proved this for heat for air was uniform for temperatures varying from $+437^{\circ}$; consequently, specific heat for equal weights of air, at pressure, averaged .2377.

Specific Heat. Water at $32^{\circ} = 1$.

| <i>Metals from 32° to 212°.</i> | | <i>Woods.</i> | <i>Sulphur</i> |
|---|-------|--------------------------|----------------|
| Silver..... | .056 | | <i>L</i> |
| Steel..... | .1165 | Oak..... | .57 |
| Tin..... | .0562 | Pear..... | .5 |
| Wrought iron..... | .1138 | Pine..... | .65 |
| Zinc..... | .0955 | | Alcohol |
| | | <i>Min'l Substances.</i> | Ether.. |
| | | Charcoal.... | .2415 |
| | | Coal..... | .2411 |
| | | Coke..... | .203 |
| | | Glass..... | .1977 |
| | | Gypsum..... | .1966 |
| | | Phosphorus.. | .2503 |
| | | | Ice..... |
| | | | Linseed |
| | | | Olive oil |
| | | | Steam. |
| | | | Turpen |
| | | | Vinegar |

Stones.

| | |
|----------------|-------|
| Chalk..... | .2149 |
| Limestone.... | .2174 |
| Masonry..... | .2 |
| Marble, gray.. | .2694 |
| " white..... | .2158 |

Gases.

| | | | |
|-------------|-------|--------------------|--|
| Air..... | .2377 | Hydrogen..... | |
| Oxygen..... | .2412 | Carbonic Acid..... | |

For Equal Weights.

| | | | |
|-------------|-------|--------------------|--|
| Air..... | .1688 | Hydrogen..... | |
| Oxygen..... | .1559 | Carbonic Acid..... | |

Metals have least, ranging from Bismuth .0308 to Cast Iron .1298. Mineral Substances have .2 that of water, and Woods about .5. Liquids, except of Bromine, are less than water, Olive oil being lowest and Vine

ILLUSTRATION.—If 1 lb. of coal will heat 1 lb. of water to 100° , $\frac{1}{.033}$ heat 1 lb. of mercury to 100° .

To Compute Temperature of a Mixture of liquids.

$\frac{W T + w t}{W + w} = t'$; $\frac{w (t' - t)}{T - t'} = W$; $\frac{w (t' - t)}{W} + t' = T$. *W represents weight or volume of a substance of temperature T, w weight or volume of a like substance of temperature t, and t' temperature of mixture W + w.*

ILLUSTRATION 1.—When 5 cube feet of water (W) at a temperature of mixed with 7.5 cube feet (w) at 50° (t), what is the resultant temperature of mixture?

$$\frac{5 \times 150^{\circ} + 7.5 \times 50^{\circ}}{5 + 7.5} = \frac{1125}{12.5} = 90^{\circ}.$$

2.—How much water at $(T) 100^{\circ}$ should be mixed with 30 gallons (w) at a required temperature of 80° ?

$$\frac{30 (80^{\circ} - 60^{\circ})}{100^{\circ} - 80^{\circ}} = \frac{600}{20} = 30 \text{ gallons.}$$

To Compute Temperature of a Mixture of Solids.

$$\frac{W S T + w s t}{W S + w s} = t'; \quad \frac{w s (t - t')}{S (T - t')} = W; \quad \frac{t' (W S + w s) \sim w s t}{W S} = T.$$

W and w representing weights, and S and s specific heat of substances.

What temperature should 20 lbs. cast iron (W) be heated to a temperature (t) of 50° to 60° ?

$$\frac{0 \times .1298 + 150 \times 1}{20 \times .1298} \sim 150 \times 1 \times 50^{\circ} =$$

To Compute Specific Heat at Constant Volume.

When Specific Heat at Constant Pressure is known. $\frac{S p}{H} = s$. S representing specific heat at constant pressure, p proportion of heat absorbed at constant volume, H total heat absorbed at constant pressure, and s specific heat at constant volume.

Or, $\frac{S(t' - t) - 2.742(V - v)}{t' - t} = s$. t and t' representing initial and final temperature of the gas and that to which it is raised, and V and v initial and final volumes of the gas under 14.7 lbs. per sq. inch, and of it heated under constant pressure in cubic feet.

ILLUSTRATION.—Assume 1 lb. air at atmospheric pressure and at 32° , doubled in volume by heat. $S = .2377^{\circ}$, $t - t' = 32^{\circ} \sim 525^{\circ} = 493^{\circ}$ and $V - v = 12.387^{\circ}$ cubic feet.

$$\frac{.2377 \times 493 - (2.742 \times 12.387)}{493} = .1688 \text{ specific heat.}$$

For comparative volumes of other gases, see Table, page 506.

To Compute Specific Heat for Equal Volume of Gas and Air.

RULE.—Multiply specific heat of the gas for equal weights of gas and air by specific gravity of gas, and product is specific heat for equal volume.

EXAMPLE.—What is specific heat of air at equal volume with hydrogen?

Specific heat of hydrogen for equal weights at constant volume, 2.4096, and specific gravity of the gas, .0692. (See Table, page 506.)

Then, $2.4096 \times .0692 = .1667$ specific heat for equal volumes at constant volume.

Specific heat of steam, air at unity = 1.281.

Capacity for Heat.

When a body has its density increased, its capacity for heat is diminished. The rapid reduction of air to .2 of its volume evolves heat sufficient to inflame tinder, which requires 550° .

Relative Capacity for Heat of Various Bodies. (Water at $32^{\circ} = 1$.)

| BODIES. | Equal Weights. | Equal Volumes. | BODIES. | Equal Weights. | Equal Volumes. | BODIES. | Equal Weights. | Equal Volumes. |
|---------------|----------------|----------------|----------|----------------|----------------|----------|----------------|----------------|
| Water.. | 1 | 1 | Gold.... | .05 | .966 | Mercury | .036 | — |
| Brass.. | .116 | .971 | Ice..... | .9 | — | Silver.. | .082 | .833 |
| Copper.. | .114 | 1.027 | Iron.... | .126 | .993 | Tin..... | .06 | — |
| Glass... .187 | .448 | .448 | Lead... | .043 | .487 | Zinc.... | .102 | — |

To Ascertain Relative Capacities of Different Bodies, combined with experiment.

RULE.—Multiply weight of each body by number of degrees of heat lost or gained by mixture, and capacities of bodies will be inversely as products.

Or, if bodies be mingled in unequal quantities, capacities of the bodies will be reciprocally as quantities of matter, multiplied into their respective changes of temperature.

ILLUSTRATION.—If 1 lb. of water at 156° is mixed with 1 lb. of mercury at 40° , resultant temperature is 152° .

Thus, $1 \times 156^{\circ} - 152^{\circ} = 4^{\circ}$, and $1 \times 40^{\circ} \sim 152^{\circ} = 112^{\circ}$. Hence capacity of water or heat is to capacity of mercury as 112° to 4° , or 28 to 1.

Sensible Heat

Sensible heat or temperature to raise water at units.

$$180^{\circ} = 180.9$$

Latent Heat.

Latent Heat is that which is insensible to the touch of our hand and is incapable of being detected by a thermometer.

When a solid body is exposed to heat, and ultimately passes into liquid state under its influence, its temperature rises until it attains point of fusion, or melting point. The temperature of the body at point remains stationary until the whole of it is melted; and the heat is time absorbed, without affecting the temperature or being sensible to touch or to the indications of a thermometer, is said to become *latent*. In fact, the *latent heat of fusion*, or the *latent heat of liquidity*, and its action is to separate the particles of the body, hitherto solid, and change condition into that of a liquid. When, on the contrary, a liquid is solid the latent heat is disengaged.

If to a pound of newly-fallen snow were added a pound of water at the snow would be melted, and 32° would be resulting temperature.

When a body is *fusing*, no rise in its temperature occurs, however the additional quantity of heat may be imparted to it, as the increased is absorbed in the operation of fusion. The quantity of heat thus latent varies in different bodies.

A pound of water, in passing from a liquid at 212° to steam at 212° ceives as much heat as would be sufficient to raise it through 966.6 mometric degrees, if that heat, instead of becoming *latent*, had been *sensible*.

If 5.5 lbs. of water, at temperature of 32° , be placed in a vessel, commun with another one (in which water is kept constantly boiling at temperature of until former reaches temperature of latter quantity, then let it be weighed it will be found to weigh 6.5 lbs., showing that one lb. of water has been raised in form of steam through communication, and reconverted into water by temperature in vessel. Now this pound of water, received in the form of steam, had, when in that form, a temperature of 212° . It is now converted into form, and still retains same temperature of 212° ; but it has caused 5.5 lbs. of to rise from the temperature of 32° to 212° , and this without losing any temperature of itself. Now this heat was combined with the steam, but as it is not as to a thermometer, it is termed *Latent*.

Quantity of heat necessary to enable ice to resume the fluid state is to that which would raise temperature of same weight of water 140° ; a equal quantity of heat is set free from water when it assumes the solid.

Sum of Sensible and Latent Heats.

From Water at 32° .

| Pressure. | Latent. | Sum. | Pressure. | Latent. | Sum. | Pressure. | Latent. | Sum. | Pressure. | Latent. |
|-----------|---------|--------|-----------|---------|--------|-----------|---------|--------|-----------|---------|
| Lbs. | ° | ° | Lbs. | ° | ° | Lbs. | ° | ° | Lbs. | ° |
| 14.7 | 964.3 | 1146.1 | 26 | 943.7 | 1155.3 | 55 | 912 | 1169 | 120 | 873.7 |
| 15 | 962.1 | 1147.4 | 27 | 942.2 | 1155.8 | 60 | 908 | 1170.7 | 130 | 869.4 |
| 17 | 959.8 | 1148.3 | 28 | 940.8 | 1156.4 | 65 | 904.2 | 1172.3 | 140 | 865.4 |
| 18 | 957.7 | 1149.2 | 29 | 939.4 | 1157.1 | 70 | 900.8 | 1173.8 | 150 | 861.5 |
| 19 | 955.7 | 1150.1 | 30 | 937.9 | 1157.8 | 75 | 897.5 | 1175.2 | 160 | 857.9 |
| 20 | 952.8 | 1150.9 | 32 | 935.3 | 1158.9 | 80 | 894.3 | 1176.5 | 170 | 854.5 |
| 21 | 951.3 | 1151.7 | 35 | 931.6 | 1160.5 | 85 | 891.4 | 1177.9 | 180 | 851.3 |
| 22 | 949.9 | 1152.5 | 37 | 929.3 | 1161.5 | 90 | 888.5 | 1179.1 | 190 | 848 |
| 23 | 948.5 | 1153.2 | 40 | 926 | 1162.9 | 95 | 885.8 | 1180.3 | 200 | 845 |
| 24 | 946.9 | 1153.9 | 45 | 920.9 | 1164.6 | 100 | 883.1 | 1181.4 | 220 | 839.2 |
| 25 | 945.3 | 1154.6 | 50 | 916.3 | 1167.1 | 110 | 878.3 | 1183.5 | 250 | 831.2 |

Latent Heat of Vaporization, or Number of Degrees of Heat required to raise substances from their Liquidities to Vapor at Pressure.

| | |
|-----------------------|------------------------|
| 142.6° | Water..... |
| 157° | Zinc..... |
| 298° | Oil of Turpentine..... |

Latent Heat of Fusion of Solids. (Person.)

| Substances. | Melt- ing Point. | Specific Heat. Liquid. | Solid. | In Heat- units of 1 lb. | Substances. | Melt- ing Point. | Specific Heat. Liquid. | Solid. | In Heat- units of 1 lb. |
|-------------|------------------------|---------------------------|--------|-------------------------------|--------------------|------------------------|---------------------------|--------|-------------------------------|
| Tin..... | 442 | .0637 | .0562 | 25.6 | Ice..... | 32 | — | .504 | 142.85 |
| Bismuth.. | 507 | .0363 | .0308 | 22.7 | Phosphorus.... | 112 | .2045 | .1788 | 9 |
| Lead..... | 617 | .0402 | .0314 | 9.86 | Spermaceti..... | 120 | — | — | 148 |
| Zinc..... | 773 | — | .0956 | 50.6 | Wax..... | 142 | — | — | 175 |
| Silver..... | 1873 | — | .057 | 37.9 | Sulphur..... | 239 | .234 | .2026 | 17 |
| Mercury... | 39 | .0333 | .0319 | 5 | Nitrate of soda.. | 591 | .413 | .2782 | 113 |
| Cast Iron.. | 3400 | — | .129 | 233 | Nit. of potassia.. | 642 | .3319 | .2388 | 85 |

To Compute Latent Heat of Fusion of a Non-metallic Substance.

$C \sim c(t + 256^\circ) = L$ C and c representing specific heats of substance in solid and liquid state, t temperature of fusion, and L latent heat.

ILLUSTRATION.—What is latent heat of fusion of ice?

$C = .504$; $c = 1$; and $t = 32^\circ$.

$$.504 \sim 1 \times 32 + 256 = 142.85^\circ \text{ units.}$$

NOTE.—For Latent Heat of Fusion of some substances, see Deschanel's, New York 1872, Heat, part 2.

Radiation of Heat.

Radiation of Heat is diffusion of heat by projection of it in diverging right lines into space, from a body having a higher temperature than space surrounding it, or body or bodies enveloping it.

Radiation is affected by nature of surface of body; thus, black and rough surfaces radiate and absorb more heat than light and polished surfaces. Bodies which radiate heat best absorb it best.

Radiant heat passes through moderate thicknesses of air and gas without suffering any appreciable loss or heating them. When a polished surface receives a ray of heat, it absorbs a portion of it and reflects the rest. The quantity of heat absorbed by the body from its surface is the measure of its *absorbing power*, and the heat reflected, that of its *reflecting power*.

When temperature of a body remains constant it is in consequence of quantity of heat emitted being equal to quantity of heat absorbed by body. Reflecting power of a body is complement of its absorbing power; or, sum of absorbing and reflecting powers of all bodies is the same.

Thus, if quantity of heat which strikes a body = 100, and radiating and reflecting powers each 50, the absorbent would be 10.

Radiating or Absorbent and Reflecting Powers of Substances.

| SUBSTANCES. | Radiating or Ab- sorbing. | Reflect- ing. | SUBSTANCES. | Radiating or Ab- sorbing. | Reflect- ing. |
|-----------------------------|---------------------------------|------------------|-------------------------------------|---------------------------------|------------------|
| Lamp Black..... | 100 | — | Wrought Iron, polished.. | 23 | 77 |
| Water..... | 100 | — | Lead, polished..... | 19 | 81 |
| Carbonate of Lead.... | 100 | — | Zinc, polished..... | 19 | 81 |
| Lead, white..... | 100 | — | Steel, polished..... | 17 | 83 |
| Writing Paper..... | 98 | 2 | Platinum, in sheet..... | 17 | 83 |
| Ivory, Jet, Marble.... | 93 to 98 | 7 to 2 | Tin..... | 15 | 85 |
| Resin..... | 96 | 4 | Copper, varnished..... | 14 | 86 |
| Glass..... | 90 | 10 | Brass, dead polished.... | 11 | 89 |
| India Ink..... | 85 | 15 | “ bright polishes” | — | 90 |
| Ice..... | 85 | 15 | Copper, ham'ered or deposited on | — | — |
| Shellac..... | 72 | 28 | Gold, plated..... | — | — |
| Lead..... | 45 | 55 | “ polished.... | — | — |
| Cast Iron, bright polished | 25 | 75 | Silver, polished.... | — | — |
| Platinum, a little polish'd | 24 | 76 | “ cast, polish | — | — |
| Mercury..... | 23 | 77 | | | |

Radiating and Absorbing Power of various Bodies
Units of Heat per Sq. Foot per Hour for a Differ
of 1°. (*Pecet.*)

| | Unit. | | Unit. | |
|-----------------------|-------|-----------------------|-------|--------------------|
| Silver, polished..... | .0266 | Iron, ordinary..... | .5662 | Woollen stuff..... |
| Copper..... | .0327 | Glass..... | .5948 | Oil paint..... |
| Tin..... | .0439 | Iron, cast..... | .648 | Paper..... |
| Brass, polished..... | .0491 | Wood sawdust..... | .7225 | Lamp-black..... |
| Iron, sheet..... | .092 | Stone, Brick, etc.... | .7358 | Water..... |

To Compute Loss of Heat by Radiation per Sq. f.

$\frac{1.7 \, l \, (T - t)}{d \, v} = R$. *T* representing temperature of pipe, which is assumed to be less than that of steam; *t* temperature of air; *l* length of pipe, and *v* velocity in feet per second; *d* diameter in ins., and *R* radiation in degrees per second.

ILLUSTRATION.—Assume temperatures of a steam-pipe, steam, 212°, 200°, 60°, length of pipe 20 feet, velocity of heat (steam) 15 feet per second, and *d* of pipe 16 ins.; what will be loss of heat by radiation?

$$\frac{1.7 \times 20 \, (200 - 60)}{16 \times 15} = 15.66^\circ.$$

Reflection.

Reflection of Heat is passage of heat from surface of one substance to another or into space, and it is the converse of radiation.

Heat is reflected from surface upon which its rays fall in same material, angle of reflection being opposite and equal to that of incidence. Metals are the strongest reflectors.

Reflecting Power of various Substances.

| | | | | | |
|-------------|-----|---------------------|-----|-----------|--|
| Silver..... | .97 | Specular metal..... | .86 | Zinc..... | |
| Gold..... | .95 | Tin..... | .85 | Iron..... | |
| Brass..... | .93 | Steel..... | .83 | Lead..... | |

Communication and Transmission of Heat.

Communication of Heat is passage of heat through different substances with different degrees of velocity. This has led to division of substances into *Conductors* and *Non-conductors*; former includes such as metals which allow caloric to pass freely through their substance, and comprise those that do not give an easy passage to it, such as glass, wood, charcoal, etc.

Velocity of cooling, other things being equal, increases with extent of surface compared with volume of substance; and of two bodies of same material, temperature, and form, but differing in volume.

Transmission of Heat is passage of heat through different bodies with different degrees of intensity. Gaseous bodies and a vacuum are high order of transmittents.

Relative Power of various Substances to Transmit Heat.

All bodies capable of transmitting heat are more or less transmitters though their powers of transmitting heat and light are not in same relative proportions.

| | | | | | | | |
|---------------|-----|------------------|-----|------------------|-----|---------------------|--|
| Air..... | 1 | Flint-glass..... | .67 | Nitric acid..... | .15 | Sulphuric acid..... | |
| Alcohol..... | .15 | Gypsum..... | .2 | Rock-crystal.. | .62 | Turpentine..... | |
| Crown-glass.. | .49 | Ice..... | .06 | Rape-seed oil.. | .3 | Water..... | |

Heat which passes through one plate of glass is less subject to absorption in passing through a second and a third plate. Of 1000 rays, 457 were intercepted by 4 plates as follows:

1st. 381. 2d. 43. 3d. 18. 4th. 9.

Average Results of Heating and Evaporating Water by Steam in Copper Pipes and Boilers. (D. K. Clark.)

| | Steam condensed | | Heat transmitted | |
|---------------------------|--|--------------|--|--------------|
| | Per sq. foot for 1° difference per hour. | | Per sq. foot for 1° difference per hour. | |
| | Heating. | Evaporating. | Heating. | Evaporating. |
| | Lbs. | Lbs. | Units. | Units. |
| Iron plate surface..... | .077 | .105 | 82 | 100 |
| Copper-plate surface..... | .248 | .483 | 276 | 534 |
| Copper-pipe surface..... | .291 | 1.07 | 312 | 1034 |

hence.—Efficiency of copper-plate surface for evaporation of water is 10 times its efficiency for heating; for copper-pipe surface efficiency is more than three times as much; and for cast-iron-plate surface, a fourth more.

Efficiency of pipe surface is a fifth more than that of plate surface for heating, and more than twice as much for evaporation.

Generally, copper-plate surface condenses .5 lb. of steam, copper-pipe surface .1 lb. per sq. foot per 1° of temperature per hour, for evaporation.

Quantity of heat transmitted is at rate of about 1000 units per lb. of steam condensed.

Transmission of Heat through Glass of different Colors.

Direct = 100.

| | | |
|--------------------|--------------------|----------------|
| 65.5 | Blue, deep..... 19 | Yellow..... 40 |
| Low..... 52 | " light..... 42 | Orange..... 44 |
| High, deep..... 53 | Green..... 26 | Red..... 53 |

Peclet defines law of transmission of heat as: The flow of heat which crosses an element of a body in a unit of time is proportional to its surface and to difference of temperature of the two faces perpendicular to direction of flow, and is in inverse of thickness of element.

$(t - t') \frac{C}{T} = H$ t and t' representing temperatures of surfaces, C constant for material 1 inch thick, or quantity of heat transmitted per hour for 1° difference of temperature through 1 unit of thickness, T thickness, and H quantity of heat in units transmitted through plate per sq. foot per hour.

Quantities of Heat Transmitted from Water to Water through Plates or Beds of Metals and other Solid Bodies, 1 Inch thick, per Sq. Foot.

For 1° Difference of Temperature between the two Faces per Hour.

Selected from M. Peclet's tables. (D. K. Clark.)

| SUBSTANCE. | C or Quantity of Heat. | SUBSTANCE. | C or Quantity of Heat. | SUBSTANCE. | C or Quantity of Heat. |
|---------------|------------------------|------------|------------------------|--------------|------------------------|
| | Units. | | Units. | | Units. |
| Aluminum..... | 620 | Iron..... | 225 | Marble..... | 24 |
| Copper..... | 604 | Zinc..... | 225 | Plaster..... | 2.6 |
| Flue..... | 596 | Tin..... | 177 | Glass..... | 6.56 |
| Steel..... | 555 | Lead..... | 112 | Sand..... | 2.16 |

The conditions are, that the surfaces of conducting material must be perfectly clean, that they be in contact with water at both faces of different temperatures, and that the water in contact with surfaces be thoroughly and constantly changed. M. Peclet found that when metallic surfaces became rusty, the rate of transmission of heat through all metals became very nearly the same.

To Compute Units of Heat Transmitted

ILLUSTRATION I.—If 2000 lbs. beet root juice at 40° are cooled with a double bottom, and heated to 212°, with a heating surface subjected to steam at a temperature of 275°, for a period of 1 hour, the total heat, and heat per degree of difference transmitted

$212^{\circ} - 40^{\circ} \times 60 \div 15 = 688^{\circ}$ per hour, and $2000 \times 688 \div 25 = 55\ 040$ units per hour.

$(212^{\circ} + 40^{\circ}) \div 2 = 126^{\circ}$ mean temperature of juice, and $275^{\circ} - 126^{\circ} = 149^{\circ}$ difference of temperature.

Hence, $55\ 040 \div 149 = 369.4$ units per sq. foot per degree of difference per

2.—If 48.2 sq. feet of iron pipe 1.36 ins. in diameter, is supplied with steam and it raises temperature of 880 lbs. water from 45° to 212° in 4 minutes, the total heat per sq. foot per hour, total heat per sq. foot per degree, and condensed per sq. foot per degree per hour?

$212^{\circ} - 45^{\circ} \times 60 \div 4 = 2490^{\circ}$ in an hour; $45^{\circ} + 212^{\circ} \div 2 = 129^{\circ}$ mean temperature, and $275^{\circ} - 129^{\circ} = 146^{\circ}$ difference of temperature.

$\frac{2490^{\circ} \times 880}{48.2} = 45\ 563$ units per sq. foot per hour, $45\ 563 \div 146 = 312.1$ units per degree, and total heat of steam above $129^{\circ} = 1068^{\circ}$.

Hence $\frac{312.1}{1068} = .292$ lbs. steam condensed per sq. foot per degree per hour.

Evaporation.

Evaporation or *Vaporization* is conversion of a fluid into vapor. It produces cold in consequence of heat being absorbed to form it.

It proceeds only from surface of fluids, and therefore, *other things being equal*, must depend upon extent of surface exposed.

When a liquid is covered by a stratum of dry air, evaporation is retarded, even when temperature is low.

As a large quantity of heat passes from a sensible to a latent state in the formation of vapor, it follows that cold is generated by evaporation.

Fluids evaporate in vacuo at from 120° to 125° below their boiling point.

Heat required to Evaporate 1 lb. Water at Temperature below 212° from a Vessel in open air at 32°

(Thomas Box.)

| TEMPERATURE. | HEAT | | | | Total lost per hour. | TEMPERATURE. | HEAT | | | | Total lost per hour. |
|--------------|---|---------------------------------|--------------|------------------------------|----------------------|--------------|---|---------------------------------|--------------|------------------------------|----------------------|
| | Water evaporated per sq. foot of surface p ^r hour. | lost by radiation from surface. | lost in air. | to evaporate 1 lb. of water. | | | Water evaporated per sq. foot of surface p ^r hour. | lost by radiation from surface. | lost in air. | to evaporate 1 lb. of water. | |
| 0 | | | | | | 0 | | | | | |
| 32 | .027 | — | — | 1091 | 29 | 132 | .706 | 182 | 202 | 1091 | 29 |
| 42 | .04 | 270 | 424 | 1788 | 71 | 142 | .916 | 158 | 162 | 1788 | 71 |
| 52 | .058 | 375 | 581 | 2052 | 119 | 152 | 1.178 | 137 | 127 | 2052 | 119 |
| 62 | .083 | 405 | 605 | 2110 | 174 | 162 | 1.505 | 118 | 97 | 2110 | 174 |
| 72 | .117 | 386 | 566 | 2055 | 239 | 172 | 1.895 | 106 | 72 | 2055 | 172 |
| 82 | .162 | 358 | 504 | 1968 | 319 | 182 | 2.373 | 92 | 50 | 1968 | 182 |
| 92 | .223 | 319 | 434 | 1862 | 415 | 192 | 2.947 | 81 | 32 | 1862 | 192 |
| 102 | .303 | 280 | 366 | 1758 | 533 | 202 | 3.633 | 71 | 14 | 1758 | 202 |
| 112 | .406 | 245 | 304 | 1664 | 671 | 212 | 4.471 | 63 | — | 1664 | 212 |
| 122 | .528 | 211 | 250 | 1580 | 849 | — | — | — | — | — | — |

To Compute Surface of a Refrigerator.

Illustration of Table.—If it is required to cool 20 barrels, of 42 gallon beer, from 202° to 82° in an hour.

Result to be attained is to dissipate 42×8.33 (lbs. U. S. gallons) $\times 202 = 840\ 000$ units of heat per hour.

At 202° , 4465 units are lost, and at 82° , 319, hence, average loss for entire surface between extremes $= 1850$ units per sq. foot per hour.

Then $\frac{840\ 000}{1850} = 454$ sq. feet in a still air.

The volume of air required per hour in this case would be about 100

Compute Area of Grate and Consumption of Fuel for Evaporation.

Question of Table.—If it is required to evaporate 6 Beer gallons (122 lbs.) per hour, at a temperature not exceeding 152° .

$s = 50$ lbs. At 152° , water evaporated as per table is 1.17% per hour.

42 sq. feet. Heat required to effect this = $1392 \times 50 = 69,600$ units.

At 6000 units as average economic value of coal, then grate of 1 sq. foot.

It is practicable to evaporate at a high temperature, but at an economic rate.

Water requires only 1209 units per lb. if surface is exposed to the atmosphere (1209 - 63) to 1146 units.

Five Powers of Different Tubes per Degree of Heat per Sq. Foot of Surface.—In Units.

1 tube, 230; Double-bottomed vessel, 330; Horizontal tube in tank, 440.

Compute Volume of Water Evaporated in a Given Time.

Question.—What is volume evaporated at 212° in a double-bottomed vessel having an area of 1000 sq. feet, exposed to steam at a pressure of 25 lbs.?

Temperature of steam at 25 = 14.7 lbs. = 17.7% per hour.

$$\text{Then } \frac{330 \times 57 \times 17 \times 15}{927 \times 60} = 4.2 \text{ lbs. water}$$

When Water is at a Lower Temperature than 212° .

6 gallons or 1000 lbs. of water were to be evaporated in the same vessel and under like pressure the following

would be required $1000 \times (212^{\circ} - 42^{\circ}) = 170,000$ units of heat. Water while being heated = $\frac{170,000}{1209} = 140.6$ lbs.

Since between temperature of steam and water is 14.7 lbs.

$$\frac{170,000}{330 \times 140 \times 17} = .216 \text{ hour} = 13 \text{ min. } 36 \text{ sec.}$$

17 left for evaporation, and quantity evaporated = $\frac{17 \times 1000}{1146} = 14.8$ lbs. or 32.44 gallons.

Domestication.

Domestication, or the drying of a substance, is a process, and it is imperative that it should be admitted at highest point of temperature and at its lowest.

It is submitted to an average temperature of 50° for a period of 2.5 days, with a view to the removal of 10.5 lbs. of water from the substance. The quantity of water per lb. of material is 10.5.

Temperature for the purpose of domestication is 50° .

Evaporation of Water per Sq. Foot of Surface per Hour (Dr. Dalton.)

| Temperature of Water. | Evaporation. | | | Temperature of Water. | Evaporation. | | |
|--------------------------|--------------|---------------|----------------|--------------------------|--------------|---------------|----------------|
| | Calm. | Light Air. | Brisk Wind. | | Calm. | Light Air. | Brisk Wind. |
| 0 | Lbs. | Lbs. | Lbs. | 0 | Lbs. | Lbs. | Lbs. |
| 32 | .0349 | .0448 | .055 | 100 | .3248 | .4169 | .5 |
| 40 | .0459 | .0589 | .0723 | 125 | .6619 | .8494 | 1.0 |
| 50 | .0655 | .0841 | .1032 | 150 | 1.296 | 1.663 | 2.0 |
| 60 | .0917 | .1175 | .1441 | 175 | 2.378 | 3.053 | 3.5 |
| 70 | .1257 | .1616 | .1983 | 200 | 4.128 | 5.298 | 6.0 |
| 80 | .1746 | .2241 | .2751 | 212 | 5.239 | 6.724 | 8.0 |

The rates of evaporation for these conditions of the air when perfectly dry at 1, 1.28, and 1.5°.

To Compute Quantity of Water exposed to Air that would be evaporated above.—Subtract tabulated weight of water corresponding to dew-point weight of water corresponding to temperature of dry air, and remainder weight of water that would be evaporated per sq. foot of surface per hour.

Distillation.

Distillation is depriving vapor of its latent heat, and, though it be effected in a vacuum with very little heat, no advantage in regard to a saving of fuel is gained, as latent heat of vapor is increased proportionately to diminution of sensible heat.

A temperature of 70° is sufficient for distillation of water in a vessel exhausted of air.

Conduction or Convection of Heat.

Air and gases are very imperfect conductors. Heat appears to be transmitted through them almost entirely by conveyance, the portions of air becoming lighter, and diffusing the heat through the mass in their ascent. Hence, in heating a room with air, the heat should be introduced at lowest part. The advantage of double windows for retention of heat depends, in a great measure, upon sheets of glass confined between them, through which heat is very slowly transmitted.

Convection of heat refers to transfer and diffusion of heat in a fluid by means of the motion of the particles of the mass.

Relative Internal Conducting Powers of Various Substances.

Metals.

| | | | | | | | |
|---------------------|------|--------------------|-----|-----------------------|------|------------------------|--|
| Brass | .76 | Gold | 1 | Porcelain | .012 | Tin | |
| Cast Iron | .517 | Lead | .18 | Silver | .97 | Wrought Iron | |
| Copper | .89 | Platinum | .98 | Terra Cotta | .011 | Zinc | |

Minerals.

| | | | | | | | |
|--------------------|-----|---------------------------|------|----------------------|-----|------------------|--|
| Cement | .21 | Coal, anth'cite | 1.92 | Fire brick | .61 | Gypsum | |
| Chalk | .6 | " bitumin. | 1.68 | Fire clay | .76 | Lime | |
| Charcoal | .07 | Coke | 1.98 | Glass | .96 | Marble | |
| Slate | 1 | | | Wood ash | .08 | | |

Woods with Birch = .41 with Silver.

| | | | | | | | |
|-----------------|-----|--------------------|----|-----------------|-----|----------------|--|
| Apple | .68 | Birch | 1 | Ebony | .5 | Oak | |
| Ash | .73 | Chestnut | .7 | Elm | .73 | Pine | |

Hair and Fur with Air = 1.

| | | | | | | | |
|----------------------|-----|-----------------------|------|----------------------|-----|----------------|--|
| Cotton | .55 | Flannel | 2.44 | Hair | 2 | Silk | |
| Eider down | .44 | Hemp Canvas | .28 | Hare's fur | .43 | Wool | |

Liquids with Water.

| | | | | | | | |
|-------------------|-----|--------------------------|-----|----------------------|--|--|--|
| Alcohol | .93 | Proof spirit | .85 | Turpentine | | | |
| Mercury | 2.8 | Sulphuric acid | 1.7 | Water | | | |

Practical Deductions from preceding Results.

Asphalt is best composition for resisting moisture, and, being a slow conductor of heat, it is best adapted for economy of heat and dryness.

Slate is a very dry material, but, from its quick conducting power, it is not adapted for retention of heat.

Cements.—*Plaster of Paris* and *Woods* are well adapted for lining of rooms, having low conductive powers, while *Hair and Lime*, being a quick conductor, is one of the coldest compositions.

Fire-brick absorbs much heat, and is well adapted for lining of fire-places, etc.; while *Iron*, being a high conductor of heat, is one of the worst of substances for this purpose. *Common brick* is not a very slow conductor of heat.

Steam Pipe.—A wrought-iron pipe, 4 ins. in internal diameter, conveying steam at a pressure of 35 lbs. per sq. inch (280°) and 100 feet in length, will lose .84 HP.

Casing to Pipes.—A like pipe with the above, cased with following materials and covered with canvas, to give like radiating power to the outer surface, gave loss of heat in units per hour, and for the thickness given, as follows:

| CASING. | .5 Inch. | 1 Inch. | 2 Inches. | 4 Inches. | 6 Inches. |
|-------------------|----------|---------|-----------|-----------|-----------|
| Woollen Felt..... | 71 | 36 | 16 | 7 | 4.3 |
| Sawdust..... | 100 | 55 | 26 | 11 | 7 |
| Coal-ashes..... | 172 | 110 | 60 | 27 | 16.6 |

Condensation.

Tredgold ascertained by experiment that steam at pressure (absolute) of 17.5 lbs. per sq. inch, 221° , produced 1 cube foot of water per hour by condensation in 182 sq. feet of cast-iron pipe, at a uniform and quiescent temperature of 60° . Hence, condensation .352 lb. water per hour, or .0022 lbs. per degree of difference of temperature ($221-60$).

From experiments of Mr. B. G. Nichol in England, 1875, it was deduced:

That rates of transmission of heat, between temperature of steam and hat of water of condensation at its exit, at the rate of 150 feet per minute, may be taken as 380 units for vertical tubes and 520 for horizontal.

Condensation of Steam in Cast-iron Pipes. (M. Burnat.)

| Average Press. per Sq. Inch. | Temperature. | | | Condensation per sq. foot of external surface of pipe per hour. | | | | |
|------------------------------|--------------|------|-------------|---|--------|-------|--------|----------|
| | Steam. | Air. | Difference. | Bare. | Straw. | Pipe. | Waste. | Plaster. |
| Lbs. | 0 | 0 | 0 | Lb. | Lb. | Lb. | Lb. | Lb. |
| 22 | 233 | 36.5 | 196.5 | .581 | .2 | .229 | .286 | .324 |

From these data, following constants are deduced for an absolute pressure of 2 lbs. per sq. inch of steam condensed, and heat passed off per sq. foot of external surface of pipe per hour of 1° difference of temperature.

| SURFACE OF PIPE. | Steam condensed per Sq. Foot. | Heat passed off. | SURFACE OF PIPE. | Steam condensed per Sq. Foot. | Heat passed off. |
|------------------------|-------------------------------|------------------|-----------------------|-------------------------------|------------------|
| | Lb. | Units. | | Lb. | Units. |
| Iron pipe..... | .003 | 2.812 | Cotton waste 1 inch.. | .00146 | 1.57 |
| Iron coat..... | .00102 | .968 | Earth and hair..... | .00165 | 1.85 |
| Iron with clay pipe... | .00115 | 1.108 | White paint..... | .00156 | 1.72 |

Pipes were 4.72 ins. diameter, .25 inch thick, and had area of 58.5 sq. ft. Bare—rough surface as cast. Straw coat—laid lengthwise .6 inch thick and bound. Pipe—laid in clay pipe with an air space between them, the whole covered with loam and straw. Waste cotton—1 inch thick and bound with twine. Plaster—laid in clay and hair 2.36 ins. thick.

A wrought-iron pipe 3.75 ins. in external diameter, .25 inch thick, and lagged with felt and spun yarn .5 inch thick, condensed steam at 245° at rate of .56 lb. per sq. foot per hour, in an external temperature of 60° .

Steam Condensed per Sq. Foot and per Degree per Hour.
Mean Results of several Experiments with bare Cast-iron Pipes, with Steam at Absolute Pressure of 20 lbs. per Sq. Inch.

.4 lb. per sq. foot, and .00239 lb. per degree.

Hence, to ascertain quantity of heat lost by condensation of .00239 lb. = $\frac{1}{420}$ of a lb.

Difference of total and sensible heats of 1 lb. steam at 20 lbs. absolute pressure = $1151^{\circ} + 32^{\circ} - 228^{\circ} = 955$ units, and $955 \div 420 = 2.274$ units = heat condensed.

The loss of heat from a naked boiler in air at 62° , under an absolute pressure of 9 lbs. per sq. inch, was 5.8 units.

Congelation and Liquefaction.

Freezing water gives out 140° of heat. All solids absorb heat when becoming fluid.

Particular quantity of heat which renders a substance fluid is termed its caloric of fluidity, or latent heat.

Temperature of Solidification of Several Gases. (Faraday.)

Cyanogen..... 31° | Ammonia..... 103° | Sulphuretted Hydrogen, 177°
 Carbonic Acid..... 72° | Sulphurous Acid... 105° | Protoxide of Nitrogen... 145°

Frigorific Mixtures.

| MIXTURES. | Parts. | Fall of Temperature. | MIXTURES. | Parts. | Fall of Temperature. |
|-------------------------|--------|----------------------|-----------------------|--------|----------------------|
| Sea salt..... | 5 | 0 0 | Nitrate of ammonia. | 1 | 0 |
| Nitrate of ammonia... | 5 | -18 to -25 | Water..... | 1 | +50 to +74 |
| Snow, or pounded ice.. | 12 | | Snow..... | 1 | -10 to -60 |
| Muriate of ammonia } | 5 | -5 to -18 | Dilute sulphuric acid | 1 | |
| Nitrate of potash } | 1 | | Sulphate of soda.... | 3 | +50 to -3 |
| Snow, or pounded ice.. | 1 | | Diluted nitric acid.. | 2 | |
| Phosphate of soda.... | 3 | | Nitrate of ammonia. | 1 | |
| Nitrate of ammonia... | 2 | -34 to -50 | Carbonate of soda... | 1 | +50 to -7 |
| Dilute mixed acids.... | 4 | | Water..... | 1 | |
| Snow..... | 1 | | Sulphate of soda.... | 6 | |
| Crystallized muriate } | 3 | -40 to -73 | Muriate of ammonia. | 2 | +50 to -20 |
| of lime..... | 3 | | Nitrate of potash... | 4 | |
| Snow..... | 8 | | Dilute nitric acid... | 4 | |
| Dilute sulphuric acid.. | 10 | -68 to -91 | Phosphate of soda... | 9 | +50 to -13 |
| Phosphate of soda.... | 5 | | Dilute nitric acid... | 4 | |
| Nitrate of ammonia... | 3 | 0 to -34 | Snow..... | 3 | +20 to -43 |
| Dilute nitric acid..... | 4 | | Muriate of lime.... | 4 | |
| | 3 | 0 to -46 | Potash..... | 1 | +32 to -2 |
| | 2 | | Snow..... | 3 | |

Carbonic Acid and Sulphuric Ether, under receiver of an 18 lbs. to 14 lbs., exhibited a temperature ranging to the most intense cold as yet known. (Faraday.)

Pressure of Saturated Vapors under Various Temperatures. (Regnault)

| Temperature. | Water. | Alcohol. | Ether. | Chloroform. | Temperature. | Water. | Alcohol. | Ether. |
|--------------|--------|----------|--------|-------------|--------------|--------|----------|--------|
| 0 | Lbs. | Lbs. | Lbs. | Lbs. | 0 | Lbs. | Lbs. | Lbs. |
| 32 | .089 | .246 | 3.53 | — | 212 | 14.7 | 32.6 | 95.17 |
| 50 | .178 | .466 | 5.54 | 2.52 | 230 | 20.8 | 45.5 | 120.9 |
| 68 | .337 | .851 | 8.6 | 3.68 | 240.8 | 25.37 | — | 137 |
| 86 | .609 | 1.52 | 12.32 | 5.34 | 248 | 29.88 | 62.05 | — |
| 104 | 1.06 | 2.59 | 17.67 | 7.04 | 266 | 39.27 | 83.8 | — |
| 122 | 1.78 | 4.26 | 24.53 | 10.14 | 276.8 | 46.87 | — | — |
| 140 | 2.88 | 6.77 | 33.47 | 14.27 | 284 | 52.56 | 109.1 | — |
| 158 | 4.51 | 10.43 | 44.67 | 18.88 | 302 | 69.27 | 140.4 | — |
| 176 | 6.86 | 15.72 | 57.01 | 26.46 | 305.6 | 73.07 | 147.3 | — |
| 194 | 10.16 | 23.02 | 75.41 | 35.03 | 320 | 89.97 | — | — |

Boiling-points of Water corresponding to Altitudes of Barometer 62 and 31 Ins.

| Barom. | Boiling-point. | Barom. | Boiling-point. | Barom. | Boiling-point. | Barom. | Boiling-point. |
|--------|----------------|--------|----------------|--------|----------------|--------|----------------|
| 26 | 204.91 | 27.5 | 207.55 | 29 | 210.19 | 30.5 | — |
| 26.5 | 205.79 | 28 | 208.43 | 29.5 | 211.07 | 32 | — |
| 27 | 206.67 | 28.5 | 209.31 | 30 | 212 | — | — |

Boiling-point of Salt water, 213.2°. Water may be heated in a to 400° without boiling.

Fluids boil in a vacuum with less heat than under pressure of atm. On Mont Blanc water boils at 187°; and in a vacuum water boils at 100°, according as it is more or less perfect.

Water may be reduced to 5° if confined in tubes of from .003 to .005 inch eter: this is in consequence of adhesion of water to surface of tube, interfere a change in its state. It may also be reduced in its temperature below 3 kept perfectly quiescent.

Effect upon Various Bodies by Heat.

Wedgewood's zero is 1077° (Fahrenheit), and each degree = 130°.

In designation of degrees of temperature, symbol + is omitted when temperature is above 0; but when below it, symbol — must be prefixed.

| Degrees. | Degrees. | Degrees. |
|------------------------------------|--|--|
| Acetification ends.... 88 | Highest natural temperature, Egypt... 117 | Sea-water freezes |
| Acetous fermentation begins... 78 | India-rubber and Gutta-percha vulcanize..... 293 | Snow and Salt, equal parts..... |
| Air Furnace..... 3300 | Iron, bright red in the dark..... 752 | Spirits Turpen. freeze |
| Ammonia (liq.) freezes — 46 | Iron, red hot in twilight..... 884 | Steel, faint yellow. |
| Blood (hum.), heat of, freezes. 25 | Iron, wrought, welds. 2700 | “ full “ |
| Brandy freezes..... — 7 | Ignition of bodies.... 750 | “ purple..... |
| Charcoal burns..... 800 | Combustion of do.... 800 | “ blue..... |
| Cold, greatest artificial — 166 | Mercury volatilizes... 680 | “ full blue.... |
| “ natural — 56 | Milk freezes..... 30 | “ dark “ |
| Common fire..... 790 | Nitric Acid (sp. grav. 1.424) freezes... — 45 | “ polished, blue |
| Fire brick..... 4000 to 5000 | Nitrous Oxide freezes — 150 | “ strawcol. |
| Gutta-percha softens. 145 | Olive-oil freezes..... 36 | Strong Wines freeze |
| Heat, cherry red..... 1500 | Petroleum boils..... 306 | Sulph. Acid (sp. grav. 1.641) freezes... |
| “ (Daniell) 1141 | Proof Spirit freezes... — 1 | Sulph. Ether freeze |
| “ bright red..... 1860 | | Vinegar freezes... |
| “ visible by } 1077 | | Vinous ferment... 6 |
| “ } 2900 | | Zinc boils..... |
| | | Wood, dried..... |

Liquids at their Boiling

| Steam. | Steam. |
|----------|-------------------|
| 528 | 1 Ether 298 |

ght corresponding to Boiling-point of Pure Water.

Boiling-point at Level of Sea = 212°.

| Feet. | Degree. | Feet. | Degree. | Feet. | Degree. | Feet. | Degree. | Feet. |
|-------|---------|-------|---------|-------|---------|-------|---------|-------|
| 521 | 207 | 2625 | 203 | 4761 | 199 | 6929 | 195 | 9129 |
| 1044 | 206 | 3156 | 202 | 5300 | 198 | 7476 | 194 | 9684 |
| 1569 | 205 | 3689 | 201 | 5841 | 197 | 8025 | 193 | 10241 |
| 2096 | 204 | 4224 | 200 | 6384 | 196 | 8576 | 192 | 10800 |

rection for temperature of air same as given at page 428 for Elevation Barometer by multiplying by C.

ISTRATION.—If water boils at a temperature of 200° and C = 136°,

Then $6384 \times 1.08 = 6894.72$ feet.

Underground Temperature.

an increase of underground temperature per foot, from observations in nes in various and extended localities, is .01565° = 1° in 64 feet.

Linear Expansion or Dilatation of a Bar or Prism by Heat.

For 1° in a Length of 100 Feet.

METALS, MINERALS, ETC.

| | Inch. | | Inch. |
|-------------------------------|--------|----------------------------------|---------|
| ony..... | .00722 | Iron, from 32° to 572°..... | .00326 |
| th..... | .00928 | Iron wire..... | .00823 |
| | .0125 | Lead..... | .019 |
| yellow..... | .0126 | Marble..... | .00566 |
| | .00144 | Palladium..... | .00667 |
| ron..... | .0074 | Platinum..... | .00571 |
| it..... | .00956 | “ from 32° to 572°..... | .00204 |
| r from 0° to 212°..... | .0115 | Sandstone..... | .013 |
| from 32° to 572°..... | .00418 | “..... | .00814 |
| rick..... | .0033 | Silver..... | .0127 |
| | .00574 | Speculum metal..... | .013 |
| lint..... | .00541 | Steel, rod..... | .00763 |
| ube..... | .0612 | “ cast..... | .0072 |
| -Paris standard annealed..... | .0101 | “ tempered..... | .00826 |
| “ “ unannealed..... | .0103 | “ not tempered..... | .00719 |
| | .00525 | Tin..... | .0145 |
| etal—16 copper + 1 tin..... | .0127 | Water..... | .000229 |
| “ 8 copper + 1 tin..... | .0121 | White Solder—tin 1 + 2 lead..... | .0167 |
| | .0333 | Zinc, forged..... | .0207 |
| forged..... | .00814 | “ sheet..... | .0196 |
| from 0° to 212°..... | .00788 | “ 8 + 1 tin..... | .0179 |

perficial expansion is twice linear, and cubical, three times linear.

To Compute Linear Expansion of a Substance.

vide 1 by decimal given in above Table, and quotient will give pro-
n.

ISTRATION I.—A rod of copper 100 feet in length will expand between tem-
res of 32° and 212°. $212 - 32 = 180 \times .0115 = 2.07$ ins.

A cube of cast iron of 1 foot will expand in volume between temperatures of
212°. $212 - 62 = 150$, and $150 \times .0074 = 1.11$, which $\div 100$ for 1 foot =
inch, and $12 + .0111 \times 3 = 12.0333$ ins.

solids, as ice, cast iron, etc., have more volume when near to their melting-
than when melted. This is illustrated in floating of solid metal in the liquid

Expansion of Water.

ter expands from temperature of maximum density (see page
to 46°, at which degree it regains its initial volume of 32°, and
it expands under one atmosphere to 212°; and its cubical exp.
that is, its volume is dilated from 1 at 32° to 1.0466 at 212°.
pansion increases in a greater ratio than that of temperature.

To Compute Density of Water at a given Temperature

$$\frac{62.5 \times 2}{1 + 46t + \frac{500}{1 + 46t}} = \text{approximate density, } t \text{ representing temperature of water.}$$

ILLUSTRATION.—What is density of pure water at 29.8°?

$$\frac{62.5 \times 2}{\frac{500}{29.8 + 46t} + \frac{500}{29.8 + 46t}} = 57.42 \text{ lbs. or weight of 1 cube foot.}$$

Expansion of Water. (Dalton.)

| Temp. | Expansion. | Temp. | Expansion. | Temp. | Expansion. | Temp. | Expansion. |
|-------|------------|-------|------------|-------|------------|-------|------------|
| 0 | | 0 | | 0 | | 0 | |
| 22 | 1.0009 | 52 | 1.00021 | 112 | 1.0088 | 172 | 1.02575 |
| 32 | 1 | 72 | 1.0018 | 132 | 1.01367 | 192 | 1.03265 |
| 40 | 1 | 92 | 1.00477 | 152 | 1.01934 | 212 | 1.0466 |

* Greatest density 39.1°.

Hence, at 72°, water expands $\frac{1}{.0018} = 555.55$ th part of its original bulk.

Expansion of Liquids from 32° to 212°. Volume at 32°=1

| Liquids. | Volume at 212°. | Liquids. | Volume at 212°. |
|--------------------|-----------------|---------------------------|-----------------|
| Alcohol..... | 1.11 | Olive oil..... | 1.08 |
| Lined oil..... | 1.08 | Sulphuric acid..... | 1.06 |
| Mercury..... | 1.0254 | " ether..... | 1.07 |
| " 212° to 32°..... | 1.0284331 | Turpentine..... | 1.07 |
| " 32° to 72°..... | 1.028679 | Water..... | 1.0466 |
| Nitric acid..... | 1.11 | Water sat. with salt..... | 1.05 |

Expansion of Gases from 32° to 212°. Volume at 32°=1

| Gases. | Volume at 212°. | Gases. | Volume at 212°. |
|----------------------------|-----------------|--------------------------------------|-----------------|
| Air..... 1 Atmosphere..... | 1.36000 | Nitrous oxide..... 1 Atmosphere..... | 1.3779 |
| "..... 243 " | 1.36004 | Sulphurous acid..... 1 " | 1.3993 |
| Oxygen..... 1 " | 1.36013 | " 1.15 " | 1.398 |
| "..... 1.15 " | 1.36015 | Carbonic oxide..... 1 " | 1.3669 |
| Carbonic acid..... 1 " | 1.36000 | Cyanogen..... 1 " | 1.3771 |
| "..... 1.15 " | 1.36000 | | |

Expansion of Gases is uniform for all temperatures.

Volume of the Several Gases at 32° under one Atmosphere.

| Gases. | Volume. | Gases. | Volume. | Gases. | Volume. |
|--------------------------------------|---------|---------------------------------|---------|--------------|---------|
| Air..... 1 Atmosphere..... | 1.36000 | Hydrogen..... 1 Atmosphere..... | 1.3779 | Oxygen..... | 1.3993 |
| Carbonic acid..... 1 Atmosphere..... | 1.36000 | Nitrogen..... 1 Atmosphere..... | 1.3779 | Mercury..... | 1.3779 |
| Water vapor..... 1 Atmosphere..... | 1.36000 | Chlorine..... 1 Atmosphere..... | 1.3779 | Sulphur..... | 1.3779 |

Expansion of Air. (Dalton.)

| Temp. | Expansion. | Temp. | Expansion. | Temp. | Expansion. | Temp. | Expansion. | Temp. | Expansion. |
|-------|------------|-------|------------|-------|------------|-------|------------|-------|------------|
| 0 | | 0 | | 0 | | 0 | | 0 | |
| 22 | 1.0009 | 22 | 1.0009 | 22 | 1.0009 | 22 | 1.0009 | 22 | 1.0009 |
| 32 | 1.0018 | 32 | 1.0018 | 32 | 1.0018 | 32 | 1.0018 | 32 | 1.0018 |
| 40 | 1.0027 | 40 | 1.0027 | 40 | 1.0027 | 40 | 1.0027 | 40 | 1.0027 |
| 50 | 1.0045 | 50 | 1.0045 | 50 | 1.0045 | 50 | 1.0045 | 50 | 1.0045 |
| 60 | 1.0072 | 60 | 1.0072 | 60 | 1.0072 | 60 | 1.0072 | 60 | 1.0072 |
| 70 | 1.0109 | 70 | 1.0109 | 70 | 1.0109 | 70 | 1.0109 | 70 | 1.0109 |
| 80 | 1.0156 | 80 | 1.0156 | 80 | 1.0156 | 80 | 1.0156 | 80 | 1.0156 |
| 90 | 1.0213 | 90 | 1.0213 | 90 | 1.0213 | 90 | 1.0213 | 90 | 1.0213 |
| 100 | 1.0280 | 100 | 1.0280 | 100 | 1.0280 | 100 | 1.0280 | 100 | 1.0280 |
| 110 | 1.0357 | 110 | 1.0357 | 110 | 1.0357 | 110 | 1.0357 | 110 | 1.0357 |
| 120 | 1.0444 | 120 | 1.0444 | 120 | 1.0444 | 120 | 1.0444 | 120 | 1.0444 |
| 130 | 1.0541 | 130 | 1.0541 | 130 | 1.0541 | 130 | 1.0541 | 130 | 1.0541 |
| 140 | 1.0648 | 140 | 1.0648 | 140 | 1.0648 | 140 | 1.0648 | 140 | 1.0648 |
| 150 | 1.0765 | 150 | 1.0765 | 150 | 1.0765 | 150 | 1.0765 | 150 | 1.0765 |
| 160 | 1.0892 | 160 | 1.0892 | 160 | 1.0892 | 160 | 1.0892 | 160 | 1.0892 |
| 170 | 1.1029 | 170 | 1.1029 | 170 | 1.1029 | 170 | 1.1029 | 170 | 1.1029 |
| 180 | 1.1176 | 180 | 1.1176 | 180 | 1.1176 | 180 | 1.1176 | 180 | 1.1176 |
| 190 | 1.1333 | 190 | 1.1333 | 190 | 1.1333 | 190 | 1.1333 | 190 | 1.1333 |
| 200 | 1.1499 | 200 | 1.1499 | 200 | 1.1499 | 200 | 1.1499 | 200 | 1.1499 |
| 210 | 1.1675 | 210 | 1.1675 | 210 | 1.1675 | 210 | 1.1675 | 210 | 1.1675 |
| 220 | 1.1861 | 220 | 1.1861 | 220 | 1.1861 | 220 | 1.1861 | 220 | 1.1861 |
| 230 | 1.2057 | 230 | 1.2057 | 230 | 1.2057 | 230 | 1.2057 | 230 | 1.2057 |
| 240 | 1.2263 | 240 | 1.2263 | 240 | 1.2263 | 240 | 1.2263 | 240 | 1.2263 |
| 250 | 1.2479 | 250 | 1.2479 | 250 | 1.2479 | 250 | 1.2479 | 250 | 1.2479 |
| 260 | 1.2705 | 260 | 1.2705 | 260 | 1.2705 | 260 | 1.2705 | 260 | 1.2705 |
| 270 | 1.2941 | 270 | 1.2941 | 270 | 1.2941 | 270 | 1.2941 | 270 | 1.2941 |
| 280 | 1.3187 | 280 | 1.3187 | 280 | 1.3187 | 280 | 1.3187 | 280 | 1.3187 |
| 290 | 1.3443 | 290 | 1.3443 | 290 | 1.3443 | 290 | 1.3443 | 290 | 1.3443 |
| 300 | 1.3709 | 300 | 1.3709 | 300 | 1.3709 | 300 | 1.3709 | 300 | 1.3709 |
| 310 | 1.3985 | 310 | 1.3985 | 310 | 1.3985 | 310 | 1.3985 | 310 | 1.3985 |
| 320 | 1.4271 | 320 | 1.4271 | 320 | 1.4271 | 320 | 1.4271 | 320 | 1.4271 |
| 330 | 1.4567 | 330 | 1.4567 | 330 | 1.4567 | 330 | 1.4567 | 330 | 1.4567 |
| 340 | 1.4873 | 340 | 1.4873 | 340 | 1.4873 | 340 | 1.4873 | 340 | 1.4873 |
| 350 | 1.5189 | 350 | 1.5189 | 350 | 1.5189 | 350 | 1.5189 | 350 | 1.5189 |
| 360 | 1.5515 | 360 | 1.5515 | 360 | 1.5515 | 360 | 1.5515 | 360 | 1.5515 |
| 370 | 1.5851 | 370 | 1.5851 | 370 | 1.5851 | 370 | 1.5851 | 370 | 1.5851 |
| 380 | 1.6197 | 380 | 1.6197 | 380 | 1.6197 | 380 | 1.6197 | 380 | 1.6197 |
| 390 | 1.6553 | 390 | 1.6553 | 390 | 1.6553 | 390 | 1.6553 | 390 | 1.6553 |
| 400 | 1.6919 | 400 | 1.6919 | 400 | 1.6919 | 400 | 1.6919 | 400 | 1.6919 |
| 410 | 1.7295 | 410 | 1.7295 | 410 | 1.7295 | 410 | 1.7295 | 410 | 1.7295 |
| 420 | 1.7681 | 420 | 1.7681 | 420 | 1.7681 | 420 | 1.7681 | 420 | 1.7681 |
| 430 | 1.8077 | 430 | 1.8077 | 430 | 1.8077 | 430 | 1.8077 | 430 | 1.8077 |
| 440 | 1.8483 | 440 | 1.8483 | 440 | 1.8483 | 440 | 1.8483 | 440 | 1.8483 |
| 450 | 1.8899 | 450 | 1.8899 | 450 | 1.8899 | 450 | 1.8899 | 450 | 1.8899 |
| 460 | 1.9325 | 460 | 1.9325 | 460 | 1.9325 | 460 | 1.9325 | 460 | 1.9325 |
| 470 | 1.9761 | 470 | 1.9761 | 470 | 1.9761 | 470 | 1.9761 | 470 | 1.9761 |
| 480 | 2.0207 | 480 | 2.0207 | 480 | 2.0207 | 480 | 2.0207 | 480 | 2.0207 |
| 490 | 2.0663 | 490 | 2.0663 | 490 | 2.0663 | 490 | 2.0663 | 490 | 2.0663 |
| 500 | 2.1129 | 500 | 2.1129 | 500 | 2.1129 | 500 | 2.1129 | 500 | 2.1129 |
| 510 | 2.1605 | 510 | 2.1605 | 510 | 2.1605 | 510 | 2.1605 | 510 | 2.1605 |
| 520 | 2.2091 | 520 | 2.2091 | 520 | 2.2091 | 520 | 2.2091 | 520 | 2.2091 |
| 530 | 2.2587 | 530 | 2.2587 | 530 | 2.2587 | 530 | 2.2587 | 530 | 2.2587 |
| 540 | 2.3093 | 540 | 2.3093 | 540 | 2.3093 | 540 | 2.3093 | 540 | 2.3093 |
| 550 | 2.3609 | 550 | 2.3609 | 550 | 2.3609 | 550 | 2.3609 | 550 | 2.3609 |
| 560 | 2.4135 | 560 | 2.4135 | 560 | 2.4135 | 560 | 2.4135 | 560 | 2.4135 |
| 570 | 2.4671 | 570 | 2.4671 | 570 | 2.4671 | 570 | 2.4671 | 570 | 2.4671 |
| 580 | 2.5217 | 580 | 2.5217 | 580 | 2.5217 | 580 | 2.5217 | 580 | 2.5217 |
| 590 | 2.5773 | 590 | 2.5773 | 590 | 2.5773 | 590 | 2.5773 | 590 | 2.5773 |
| 600 | 2.6339 | 600 | 2.6339 | 600 | 2.6339 | 600 | 2.6339 | 600 | 2.6339 |
| 610 | 2.6915 | 610 | 2.6915 | 610 | 2.6915 | 610 | 2.6915 | 610 | 2.6915 |
| 620 | 2.7501 | 620 | 2.7501 | 620 | 2.7501 | 620 | 2.7501 | 620 | 2.7501 |
| 630 | 2.8097 | 630 | 2.8097 | 630 | 2.8097 | 630 | 2.8097 | 630 | 2.8097 |
| 640 | 2.8703 | 640 | 2.8703 | 640 | 2.8703 | 640 | 2.8703 | 640 | 2.8703 |
| 650 | 2.9319 | 650 | 2.9319 | 650 | 2.9319 | 650 | 2.9319 | 650 | 2.9319 |
| 660 | 2.9945 | 660 | 2.9945 | 660 | 2.9945 | 660 | 2.9945 | 660 | 2.9945 |
| 670 | 3.0581 | 670 | 3.0581 | 670 | 3.0581 | 670 | 3.0581 | 670 | 3.0581 |
| 680 | 3.1227 | 680 | 3.1227 | 680 | 3.1227 | 680 | 3.1227 | 680 | 3.1227 |
| 690 | 3.1883 | 690 | 3.1883 | 690 | 3.1883 | 690 | 3.1883 | 690 | 3.1883 |
| 700 | 3.2549 | 700 | 3.2549 | 700 | 3.2549 | 700 | 3.2549 | 700 | 3.2549 |
| 710 | 3.3225 | 710 | 3.3225 | 710 | 3.3225 | 710 | 3.3225 | 710 | 3.3225 |
| 720 | 3.3911 | 720 | 3.3911 | 720 | 3.3911 | 720 | 3.3911 | 720 | 3.3911 |
| 730 | 3.4607 | 730 | 3.4607 | 730 | 3.4607 | 730 | 3.4607 | 730 | 3.4607 |
| 740 | 3.5313 | 740 | 3.5313 | 740 | 3.5313 | 740 | 3.5313 | 740 | 3.5313 |
| 750 | 3.6029 | 750 | 3.6029 | 750 | 3.6029 | 750 | 3.6029 | 750 | 3.6029 |
| 760 | 3.6755 | 760 | 3.6755 | 760 | 3.6755 | 760 | 3.6755 | 760 | 3.6755 |
| 770 | 3.7491 | 770 | 3.7491 | 770 | 3.7491 | 770 | 3.7491 | 770 | 3.7491 |
| 780 | 3.8237 | 780 | 3.8237 | 780 | 3.8237 | 780 | 3.8237 | 780 | 3.8237 |
| 790 | 3.8993 | 790 | 3.8993 | 790 | 3.8993 | 790 | 3.8993 | 790 | 3.8993 |
| 800 | 3.9759 | 800 | 3.9759 | 800 | 3.9759 | 800 | 3.9759 | 800 | 3.9759 |
| 810 | 4.0535 | 810 | 4.0535 | 810 | 4.0535 | 810 | 4.0535 | 810 | 4.0535 |
| 820 | 4.1321 | 820 | 4.1321 | 820 | 4.1321 | 820 | 4.1321 | 820 | 4.1321 |
| 830 | 4.2117 | 830 | 4.2117 | 830 | 4.2117 | 830 | 4.2117 | 830 | 4.2117 |
| 840 | 4.2923 | 840 | 4.2923 | 840 | 4.2923 | 840 | 4.2923 | 840 | 4.2923 |
| 850 | 4.3739 | 850 | 4.3739 | 850 | 4.3739 | 850 | 4.3739 | 850 | 4.3739 |
| 860 | 4.4565 | 860 | 4.4565 | 860 | 4.4565 | 860 | 4.4565 | 860 | 4.4565 |
| 870 | 4.5401 | 870 | 4.5401 | 870 | 4.5401 | 870 | 4.5401 | 870 | 4.5401 |
| 880 | 4.6247 | 880 | 4.6247 | 880 | 4.6247 | 880 | 4.6247 | 880 | 4.6247 |
| 890 | 4.7103 | 890 | 4.7103 | 890 | 4.7103 | 890 | 4.7103 | 890 | 4.7103 |
| 900 | 4.7969 | 900 | 4.7969 | 900 | 4.7969 | 900 | 4.7969 | 900 | 4.7969 |
| 910 | 4.8845 | 910 | 4.8845 | 910 | 4.8845 | 910 | 4.8845 | 910 | 4.8845 |
| 920 | 4.9731 | 920 | 4.9731 | 920 | 4.9731 | 920 | 4.9731 | 920 | 4.9731 |
| 930 | 5.0627 | 930 | 5.0627 | 930 | 5.0627 | 930 | 5.0627 | 930 | 5.0627 |
| 940 | 5.1533 | 940 | 5.1533 | 940 | 5.1533 | 940 | 5.1533 | 940 | 5.1533 |
| 950 | 5.2449 | 950 | 5.2449 | 950 | 5.2449 | 950 | 5.2449 | 950 | 5.2449 |
| 960 | 5.3375 | 960 | 5.3375 | 960 | 5.3375 | 960 | 5.3375 | 960 | 5.3375 |
| 970 | 5.4311 | 970 | 5.4311 | 970 | 5.4311 | 970 | 5.4311 | 970 | 5.4311 |
| 980 | 5.5257 | 980 | 5.5257 | 980 | 5.5257 | 980 | 5.5257 | 980 | 5.5257 |
| 990 | 5.6213 | 990 | 5.6213 | 990 | 5.6213 | 990 | 5.6213 | 990 | 5.6213 |
| 1000 | 5.7179 | 1000 | 5.7179 | 1000 | 5.7179 | 1000 | 5.7179 | 1000 | 5.7179 |

Compute Volume of a Constant Weight

Relative Densities of some Vapors.

Water 1. Alcohol 2.59. Ether 4.16. Spirits of Turpentine 8.06. Sulphur 3.59.

Volume, Pressure, and Density of Air at Various Temperatures.

Volume and Atmospheric Pressure at 62° = 1.

| per- re. | Volume of 1 lb. of air at atmospheric pressure of 14.7 lbs. | Pressure of a given weight of air. | Density, or weight of one cube foot of air at 14.7 lbs. | Temper- ature. | Volume of 1 lb. of air at atmospheric pressure of 14.7 lbs. | Pressure of a given weight of air. | Density, or weight of one cube foot of air at 14.7 lbs. |
|-------------|---|---|---|-------------------|---|---|---|
| | Cube feet. | Lbs. per Sq. Inch. | Lbs. | ° | Cube feet. | Lbs. per Sq. Inch. | Lbs. |
| 0 | 11.583 | 12.06 | .086 331 | 360 | 20.63 | 23.08 | .048 476 |
| 2 | 12.387 | 13.86 | .080 728 | 380 | 21.131 | 23.64 | .047 323 |
| 0 | 12.586 | 14.08 | .079 439 | 400 | 21.634 | 24.2 | .046 223 |
| 0 | 12.84 | 14.36 | .077 884 | 425 | 22.262 | 24.9 | .044 92 |
| 2 | 13.141 | 14.7 | .076 097 | 450 | 22.89 | 25.61 | .043 686 |
| 0 | 13.342 | 14.92 | .074 95 | 475 | 23.518 | 26.31 | .042 52 |
| 0 | 13.593 | 15.21 | .073 565 | 500 | 24.146 | 27.01 | .041 414 |
| 0 | 13.845 | 15.49 | .072 23 | 525 | 24.775 | 27.71 | .040 364 |
| 0 | 14.096 | 15.77 | .070 942 | 550 | 25.403 | 28.42 | .039 365 |
| 0 | 14.592 | 16.33 | .068 5 | 575 | 26.031 | 29.12 | .038 415 |
| 0 | 15.1 | 16.89 | .066 221 | 600 | 26.659 | 29.82 | .037 51 |
| 0 | 15.603 | 17.5 | .064 088 | 650 | 27.915 | 31.23 | .035 822 |
| 0 | 16.106 | 18.02 | .062 09 | 700 | 29.171 | 32.635 | .034 28 |
| 0 | 16.606 | 18.58 | .060 21 | 750 | 30.428 | 34.04 | .032 865 |
| 0 | 16.86 | 18.86 | .059 313 | 800 | 31.684 | 35.445 | .031 561 |
| 2 | 16.91 | 18.92 | .059 135 | 850 | 32.941 | 36.85 | .030 358 |
| 0 | 17.111 | 19.14 | .058 442 | 900 | 34.197 | 38.255 | .029 242 |
| 0 | 17.612 | 19.7 | .056 774 | 950 | 35.454 | 39.66 | .028 206 |
| 0 | 18.116 | 20.27 | .055 2 | 1000 | 36.811 | 41.065 | .027 241 |
| 0 | 18.621 | 20.83 | .053 71 | 1500 | 49.375 | 55.115 | .020 295 |
| 0 | 19.121 | 21.39 | .052 297 | 2000 | 61.94 | 69.165 | .016 172 |
| 0 | 19.624 | 21.95 | .050 959 | 2500 | 74.505 | 83.215 | .013 441 |
| 0 | 20.126 | 22.51 | .049 686 | 3000 | 87.13 | 97.265 | .011 499 |

Compute Volume of a Constant Weight of Air or other Permanent Gas for any other Pressure and Temperature.

Then volume is known at a given pressure and temperature. **RULE.**—Multiply given volume by given pressure, and by new absolute temperature, divide by new pressure, and by given absolute temperature.

SAMPLE.—Given volume 16.91 cube feet, pressure 13.86 lbs., and temperature; what is volume at this temperature?
temperature for volume 16.91 = 212°.

$$16.91 \times 13.86 \times 32 + 461 \div 13.86 \times 212 + 461 = 12.39 \text{ cube feet.}$$

Compute Pressure of a Constant Weight of Air or other Permanent Gas for any other Volume and Temperature.

Then pressure is known for a given volume and temperature. **RULE.**—Multiply given pressure by new absolute temperature, and divide by old temperature.

EX.—Absolute temperature is found by adding 461° to temperature.

SAMPLE.—Given pressure 13.86 lbs., and temperature at this volume 32°; what is pressure at temperature of 212°?

$$13.86 \times \frac{212 + 461}{32 + 461} = 18.92 \text{ lbs.}$$

To Compute Volume of a Constant Weight of other Permanent Gas at any Temperature

When volume at a given temperature is known, pressure being
RULE.—Multiply given volume by new absolute temperature, as by given absolute temperature.

Absolute zero-point by different thermometrical scales is: Fahrenheit Reaumur -219.2° ; Centigrade -274° .

EXAMPLE.—Volume of 1 lb. air at $32^{\circ} = 12.387$ cube feet; what is its 212° ?

$$12.387 \times \frac{212 + 461}{32 + 461} = 16.91 \text{ cube feet.}$$

To Compute Increased Volume of a Constant Weight of Air.

When initial volume at $62^{\circ} = 1$ under 1 atmosphere. **RULE.**—Temperature add 461, and divide sum by 523 ($62 + 461$).

EXAMPLE.—Assume elements of preceding case.

$$212^{\circ} + 461 + 523 = 1.287 \text{ comparative volume to 1.}$$

To Compute Pressure of a Constant Weight of other Gas at 62° , and at 14.7 lbs. Pressure per Sq. with Constant Volume, for a given Temperature

RULE.—Add 461 to given temperature, and divide sum by 35.58.

EXAMPLE.—Temperature is 212° ; what is pressure?

$$212 + 461 + 35.58 = 18.92 \text{ lbs.}$$

To Compute Volume, Pressure, Temperature, Density of Air.

$$\frac{t + 461}{p \cdot 2.71} = V; \quad \frac{t + 461}{39.8} = V; \quad \frac{t + 461}{V \cdot 2.71} = p; \quad V \cdot 2.7074 p - 461 =$$

$2.71 \frac{p}{t + 461} = D.$ t representing temperature, p pressure in lbs. per sq. inc. in cube feet, and D weight of 1 cube foot at 14.7 lbs. per sq. inch.

Product of volume and pressure of a constant weight of air, or an permanent gas, is equal to product of absolute temperature and a coe determined for each gas by its density.

$$\text{Or, } Vp = C \frac{t + 461}{1}.$$

Coefficients, as determined by volumes and consequent densities.*

| | | | | | |
|--------------------|------|---------------|-------|--------------|--|
| Air..... | 2.71 | Hydrogen..... | .1875 | Oxygen..... | |
| Carbonic acid..... | 4.14 | Nitrogen..... | 2.63 | Mercury..... | |
| Ether, vapor..... | 7.02 | Olefant..... | 2.67 | Steam..... | |

* See D. K. Clark, London, 1877, page 349.

Decrease of Temperature by Altitudes.

| From 1 to | 1000 feet..... | In clear sky. | 10° in 139 feet..... | With cloud | 10° in 22 |
|-----------|----------------|---------------|----------------------|------------|-----------|
| 1 " | 10000 " | | 10° " 288 " | | 10° " 33 |
| 1 " | 20000 " | | 10° " 365 " | | 10° " 46 |

Compute Temperature to which a Substance of any Length or Dimension must be Submitted, to give it a Greater or Less Length of Expansion or Contraction.

$$n \text{ Length is to be increased. } \frac{L - l}{C} + t = T. \text{ L and l same substance in like denomination}$$

108.—A copper rod at 32° is 100 feet in length; to what temperature subjected to increase its length 1.1633 ins.?

for a length of 100 feet of copper for $1^{\circ} = .0115$.

$$\frac{100 \times 12 + 1.1633 - 100 \times 12}{.0115} + 32 = \frac{1.1633}{.0115} + 32 = 133.16^{\circ}.$$

length is to be reduced. $\frac{L-l}{C} - T = t$.

109.—Take elements of preceding case.

$$\frac{1201.1633 - 1200}{.0115} - 133.16^{\circ} = 101.16 - 133.16 = 32^{\circ}.$$

100 Degrees of Fahrenheit to Reaumur and Centigrade, and Contrariwise.

Reaumur to Fahrenheit. *If above zero.*—Multiply difference number of degrees and 32 by 4, and divide product by 9.

$1^{\circ} - 32^{\circ} = 180^{\circ}$, and $180^{\circ} \times 4 \div 9 = 80^{\circ}$.

zero.—Add 32 to number of degrees; multiply sum by 4, and divide by 9.

$0^{\circ} + 32^{\circ} = 72^{\circ}$, and $72^{\circ} \times 4 \div 9 = 32^{\circ}$.

Fahrenheit to Reaumur. *If above freezing-point.*—Multiply degrees by 9, divide by 4, and add 32 to quotient.

$9 \div 4 = 2.25$, and $180^{\circ} + 32 = 212^{\circ}$.

freezing-point.—Multiply number of degrees by 9, divide by 4, and subtract 32 from product.

$9 \times 9 \div 4 = 20.25$, and $72^{\circ} - 32 = 40^{\circ}$.

Reaumur to Centigrade. *If above zero.*—Multiply difference number of degrees and 32 by 5, and divide product by 9.

$1^{\circ} - 32^{\circ} \times 5 \div 9 = 180^{\circ} \times 5 \div 9 = 100^{\circ}$.

zero.—Add 32 to number of degrees, multiply sum by 5, and divide by 9.

$0^{\circ} + 32^{\circ} \times 5 \div 9 = 72^{\circ} \times 5 \div 9 = 40^{\circ}$.

Centigrade to Fahrenheit. *If above freezing-point.*—Multiply degrees by 9, divide product by 5, and add 32 to quotient.

$9 \times 9 \div 5 = 16.2$, and $180^{\circ} + 32 = 212^{\circ}$.

freezing-point.—Multiply number of degrees by 9, divide product by 5, and subtract 32 from quotient.

$9 \times 9 \div 5 = 16.2$, and $180^{\circ} - 32 = 148^{\circ}$.

Fahrenheit to Centigrade.—Divide by 4, and add product.

$4 \div 4 = 1^{\circ}$, and $20^{\circ} + 80^{\circ} = 100^{\circ}$.

Centigrade to Reaumur.—Divide by 5, and subtract product.

$5 \div 5 = 1^{\circ}$, and $100^{\circ} - 20 = 80^{\circ}$.

Corresponding Degrees upon the Three Scales.

| Fahr. | Reaum. | Fahr. | Cent. | Reaum. | Fahr. | Cent. | Reaum. |
|-------|--------|-------|-------|--------|-------|-------|--------|
| 100 | 80 | 32 | 0 | 0 | -40 | -40 | -32 |

Compute Expansion of Fluids in Volume.

Proceed by preceding formulas for computing length of a substitute V and v for volume, instead of L and l , the lengths.

ing length of pipe for any given space it is proper to include the character and occupancy of the space. Thus, a ; hours of service, or a dwelling-room, will require less service gth of pipe than a hallway or a public building.

f Heat by Surfaces of Glass or Metal.—In addition to the to be heated per minute for each occupant, 1.25 cube feet for of glass or metal the space is enclosed with must be added. cating power of the glass and metal being directly proportion- ce of external and internal temperature of the air. Thus, 80 ill reduce 100 feet of air per minute.

are laid in Trenches in the Earth.—The loss of heat is es- r. Hood at from 5 to 7 per cent.

of Water in Pipes.—In consequence of the complex forms of and the roughness of their internal surface, it is impracticable e to determine the velocity of circulation, as consequent upon reights of ascending and descending columns of the water.

ence of temperature in the two columns of 30° (190° — 160°) of 20 feet, the velocity due to the height would be 3.74 feet. t .3, and in some cases but .1, would be attained.

and Large Public Rooms, with ordinary area of doors and windows ventilation, a large amount of heat is generated by the respiration assembled therein.

is it is not necessary to heat the air above 55°, and a rule that will ary ranges of temperature from 10° is to divide volume in cube l quotient will give area of plate in sq. feet or length of 4-inch pipe

r required per Hour for each Occupant in an Enclosed Space.
(General Morin.)

| Cube Feet. | Cube Feet. | Cube Feet. |
|--------------|----------------------------|--------------------------|
| 1100 to 3700 | Lecture-rooms 1000 to 2100 | Prisons..... 1800 |
| 1100 " 3500 | Theatres..... 1400 " 1800 | Schools..... 424 to 1060 |

ite Length of Iron Pipe required to Heat Air in an Enclosed Space.

By Hot Water.

ultiply volume of air to be heated per minute in cube feet by mperatures in space and external air, divide product by differ- ratures of surface of pipe and space, multiply result by follow- s, and product will give length of pipe in feet.

er of 4 ins. multiply by .5 to .55, for 3 ins. by .7 to .75, and for 1.1.

18 in diameter, .375 inch thick, and 1 foot in length has an al surface of 1.05 sq. feet.

Volume of a room of a protected dwelling is 4000 cube feet; what . pipe, at 200°, is necessary to maintain a temperature of 70°, when at 0°?

$$\frac{4000 \times 70 - 0}{200 - 70} \times .4 = 862 \text{ feet.}$$

ng length of pipe or surface .
ients here given and comp
tion or change of air or
m 5 to 10 cube feet in t
is restricted the coeffic

to be borne in mind
table are base
cube feet r
ce. W

th temperature, 1 sq. meter (10.76 sq. feet) of
 (2472 cube feet) = 4.35 sq. feet or 5.11 lineal

transverse section of 260 sq. feet, with a window-
 it is customary in France to assign 1.33 sq. feet
 of shop = 5.2 sq. feet per 1000 cube feet.

Heating by Steam. (R. Briggs, M. I. C. E.)

Heating halls and vaults..... 286
 137 370 sq. feet.
 1 923 500 cube feet.
 650
 at any time..... 1300
 ing vaults..... 1443 cube feet.
 grate surface and 5000 sq. feet of heating surface.
 the above, to operate the elevators and electric
 applying steam to heat a distant building, requiring

By Steam.

of Iron Pipe required to Heat Air
 space, with Steam at 5 lbs. per Sq.
 sure of Atmosphere.

one of air in cube feet to be heated per minute, by
 in space and external air, divide product by coeffi-
 , and quotient will give length of 4-inch pipe in
 to-surface in sq. feet.

at 5 lbs. + pressure = 228°. Hence, if temperature of space
 or 120°, the differences will be 168°, 158°, 148°, and 108°,
 f. 5, as given in rule for hot water, would be 336, 316, 296,
 in diameter, and for

| | 60° | 70° | 80° | 120° |
|--------------|-----|-----|-----|------|
| ch pipe..... | 252 | 237 | 222 | 162 |
| "..... | 168 | 158 | 148 | 108 |
| "..... | 84 | 79 | 74 | 54 |

me of combined spaces of a factory is 50000 cube feet; what
 plate at 200° is necessary to maintain a temperature of 50°
 0°?

$$\frac{0000 \times 50 - 0}{200 - 50} \times .4 = 6666 \text{ square feet.}$$

d per Hour to Heat 100 Feet of Pipe.
 (Chas. Hood.)

ference of Temperature of Pipe and Air in Space, in Degrees.

| 135 | 130 | 125 | 120 | 115 | 110 | 105 | 100 | 95 | 90 | 85 | 80 |
|------|------|------|------|------|------|------|------|------|------|------|------|
| Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 1 | 1 | .9 | .9 | .8 | .8 | .7 | .7 | .7 | .7 | .6 | .6 |
| 2.1 | 2 | 1.9 | 1.8 | 1.7 | 1.6 | 1.5 | 1.4 | 1.4 | 1.3 | 1.2 | 1.2 |
| 3.1 | 3 | 2.9 | 2.8 | 2.7 | 2.5 | 2.4 | 2.3 | 2.2 | 2.1 | 2 | 1.8 |
| 4.2 | 4.1 | 3.9 | 3.7 | 3.6 | 3.4 | 3.2 | 3.1 | 2.9 | 2.8 | 2.6 | 2.5 |

ording to M. Claudel, 43 feet in width by 10.5 high, a single
 ins. in diameter per foot of length of room, appears to be
 being from 170° to 180°.

ifference of 120°, it is conve
 ed surface as equivalent to
 to 9 sq. feet of hot-water

Sq. feet of cast-iron pipe-
 104°, will warm 1000 cu.
 Steam is condensed at 7.

| | | |
|--|-----------|------------|
| (<i>U. S. Green.</i>) Length of fronts of buildings..... | 2 000 | feet |
| Total volume of rooms..... | 2 574 084 | cubic feet |
| Radiating surfaces, direct, 10 804..... | 34 100 | sq. feet |
| indirect, 23 290..... | | |
| <i>Boilers.</i> Grate-surface..... | 130 | - |
| Heating surface..... | 5 863 | - |

Volume of Air Heated by Radiators; Consumption of Fuel; Areas of Grate and Heating-surface of Boilers.

(*Robt Briggs.*)

Per 100 Sq. Feet of Warming-surface of Radiator.

| Pressure of Steam per sq. inch — | — | 3 | 10 | 30 | 60 |
|--|--------|--------|--------|--------|--------|
| Atmosphere of air..... | — | — | — | — | — |
| Heat from radiators per minute..... | 450 | 486 | 537 | 642 | 792 |
| Volume of air heated per minute..... | 25 110 | 26 772 | 29 570 | 35 352 | 40 840 |
| Volume of air heated in 24 hours..... | — | 1 006 | 1 178 | 1 408 | 1 660 |
| Coal consumed per hour in 100 sq. ft..... | 3.04 | 3.24 | 3.53 | 4.28 | 4.91 |
| Area of grate consuming 1 lb. of coal..... | 33 | 405 | 448 | — | — |
| Coal consumed per sq. ft..... | — | — | 298 | 357 | 491 |
| Heating surface of boiler, coal..... | 5.512 | — | 10.02 | 11.98 | 13.15 |
| Coal consumed per hour in 100 sq. ft..... | 22.4 | 22.4 | 22.4 | — | — |
| Coal consumed per sq. ft..... | — | — | 33.6 | 33.6 | 33.6 |

Hot-Air Furnaces or Stoves.

A square foot of heating surface in a hot-air furnace or stove is held to be equivalent to 1 sq. foot of hot water pipe.

M. Pelet reduced that when the true-ripe of a stove radiated its heat into a room of 4 space, the heat radiated per sq. foot per hour, for difference of temperature, were, for: Cast iron, 3.05 units; Wrought iron, 1.6 units; and "terra cotta" 1 inch thick, 1.12 units.

In ordinary practice, 1 sq. foot of cast iron is assigned to 328 cubic feet of space.

Open Fires.

According to M. Claudel, the quantity of heat radiated into an apartment from an ordinary fireplace is .25 of total heat radiated by combustible.

For wood the heat utilized is but from 6 to 7 per cent., and for coal 13 per cent.

In combustion of wood, chimney of an ordinary open fireplace draws from 1000 to 1600 cubic feet of air per pound of fuel, and a sectional area of from 50 to 60 sq. ins. is sufficient for an ordinary apartment.

Proportions of fuel required to heat an apartment are: For ordinary fireplaces, 100; metal stoves, 63; and open fires, 13 to 16.

Furnaces.

By D. K. Clark, from investigations of Mr. J. Lothian Bell.

Cupola.—M. Pelet estimates that in melting pig-iron by combustion of 20 per cent. of its weight of coke, 14 per cent. only of the heat of combustion is utilized.

Open.—According to Dr. Siemens, 1 ton of coal is enough to melt 10 tons of wrought iron to welding-point of 2700°, and melting up 30 tons of iron; from which it appears that 100 per cent. of the heat is appropriated by the iron. 814,000 Btu. of heat generated is utilized in 1 ton of iron.

ordinary furnaces, whilst, in his regenerative furnace, 1 ton of steel is melted by combustion of 1344 lbs. of small coal, showing that 6 per cent. heat is utilized.

Blast-furnace.—Mr. Bell has formed detailed estimates of approximation of the heat of Durham coke in a blast-furnace; from which is the following abstract:

Ham coke consists of 92.5 per cent. of carbon, 2.5 of water, and 5 of sulphur. To produce 1 ton of pig-iron, there are required 1232 lbs. of iron-stone, and 5388 lbs. of calcined iron-stone; the iron-stone consists of 68 lbs. of iron, 1008 lbs. of oxygen, and 2509 lbs. of earths. There is 1813 lbs. of slag, of which 123 lbs. is formed with ash of the coke, 690 lbs. with the limestone. There are 2397 lbs. of earths from the iron-ore, less 93 lbs. of bases taken up by the pig-iron and dissipated in fume, 14 lbs. Total of slag and earths, 3127 lbs.

Bell assumes that 30.4 per cent. of the carbon of the fuel, which escapes in a gaseous form, is carbonic acid; and that, therefore, only 51.27 per cent. of heating power of fuel is developed, and remaining 48.73 per cent. leaves tunnel-head undeveloped. He adopts, as a unit of heat, the quantity required to raise the temperature of 112 lbs. of water 33.8°.

HYDRAULICS.

Descending Fluids are actuated by same laws as *Falling Bodies*.

Fluid will fall through 1 foot in .25 of a second, 4 feet in .5 of a second, and through 9 feet in .75 of a second, and so on.

Velocity of a fluid, flowing through an aperture in side of a vessel, or, or bulkhead, is same that a heavy body would acquire by falling freely from a height equal to that between surface of fluid and centre of aperture.

Velocity of a fluid flowing out of an aperture is as square root of head of fluid. *Theoretical* velocity, therefore, in feet per second, is as square root of product of space fallen through in feet and g ; $v = \sqrt{2gh}$; consequently, for one foot it is $\sqrt{64.333} = 8.02$ feet. Actual velocity, however, of a number of experiments gives 5.4 feet, or of theoretical velocity.

Short ajutages accurately rounded, and of form of contracted vein, (*vena contracta*), coefficient of discharge = .974 of theoretical.

A surface subsiding to a natural level, or curve similar to Earth's convexity; apparent level taken by any instrument for that purpose, is only a tangent to Earth's surface; hence, in leveling for canals, etc., difference caused by Earth's curvature must be deducted from apparent level, to obtain true level.

Conclusions from Experiments on Discharge of Fluids from Reservoirs.

That volumes of a fluid discharged in equal times by same apertures under same head are nearly as areas of apertures.

That volumes of a fluid discharged in equal times by similar apertures under different heads, are nearly as square roots of corresponding heads above surface of apertures.

That, on account of friction, small-lipped or thin orifices discharge more fluid than those which are larger and of similar height of fluid.

4. That in consequence of a slight augmentation which contraction of the fluid vein undergoes, in proportion as the height of a fluid increases, the flow is a little diminished.

5. That if a cylindrical horizontal tube is of greater length than its diameter, discharge of a fluid is much increased, and may be increased with advantage, up to a length of tube of four times diameter of aperture.

6. That discharge of a fluid by a vertical pipe is augmented, on the principle of gravitation of falling bodies; consequently, greater the length of a pipe, greater the discharge of the fluid.

7. That discharge of a fluid is inversely as square root of its density.

8. That velocity of a fluid line passing from a reservoir at any point is equal to ordinate of a parabola, of which twice the action of gravity (2g) is parameter, the distance of this point below surface of reservoir being the abscissa.* Or, velocity of a jet being ascertained, its curve is a parabola, parameter of which $= 4h$, due to velocity of projection.†

9. Volume of water discharged through an aperture from a prismatic vessel which empties itself, is only half of what it would have been during the time of emptying, if flow had taken place constantly under same head, and corresponding velocity as at commencement of discharge; consequently the time in which such a vessel empties itself is double the time in which all its fluid would have run out if the head had remained uniform.

10. Mean velocity of a fluid flowing from a rectangular slit in side of a reservoir is two thirds of that due to velocity at sill or lowest point, or it is that due to a point four ninths of whole height from surface of reservoir.

11. When a fluid issues through a short tube, the vein is less contracted than in preceding case, in proportion of 16 to 13; and if it issues through an aperture which is alike to frustum of a cone, base of which is the aperture, the height of frustum half diameter of aperture, and area of small end 10 area of large end as 10 to 16, there will be no contraction of the vein. Hence this form of aperture will give greatest attainable discharge of a fluid.

12. Velocity of efflux increases as square root of pressure on surface of fluid.

13. In efflux under water, difference of levels between the surfaces must be taken as head of the flowing water.

14. To attain greatest mechanical effect, or *vis viva*, of water flowing through an opening, it should flow through a circular aperture in a plate, as it has less frictional surface.

From Conduits or Pipes. (Bossut.)

1. Less diameter of pipe, the less is proportional discharge of fluid.

2. Discharges made in equal times by horizontal pipes of different lengths, but of same diameter, and under same altitude of fluid, are to one another in inverse ratio of sq. roots of their lengths.

3. In order to have a perceptible and continuous discharge of fluid, the altitude of it in a reservoir, above plane of conduit pipe, must not be less than .082 ins. for every 100 feet of length of pipe.

4. In construction of hydraulic machines, it is not enough that obstructions be avoided, but also any intermediate enlargements, the injurious effects of which are proportionate, as in following Table, to the volumes of fluid, under like heads in pipes, having a different number of enlarged parts.

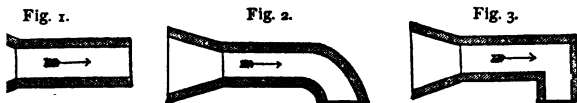
| No. of Parts. | Velocity. | No. of Parts. | Velocity. | No. of Parts. | Velocity. | No. of Parts. | Velocity. |
|---------------|-----------|---------------|-----------|---------------|-----------|---------------|-----------|
| 0 | 1 | 1 | .741 | 3 | .569 | 5 | .471 |

* See D'Aubulsson, page 66.

Friction.

Flow of liquids through pipes or in natural channels is materially affected by friction.

Equal volumes of water were to be discharged through pipes of equal diameters and lengths, but of following figures:



| | Fig. 1. | Fig. 2. | Fig. 3. |
|-----------------------|---------|---------|---------|
| Time would be as..... | 1, | 1.11, | 1.55. |
| Velocities as..... | 1, | .72, | .64. |

Discharges from Compound or Divided Reservoirs.

Velocity in each may be considered as generated by difference of heights of contiguous reservoirs; consequently, square root of difference will represent velocities, which, if there are several apertures, must be inversely as respective areas.

1.—When water flows into a vacuum, 32.166 feet must be added to height of fall when into a rarefied space only, height due to difference of external and internal pressure must be added.

VELOCITY OF WATER OR OF FLUIDS.

Coefficients of Discharge.

Coefficient of Discharge or Efflux is product of coefficients of *Contraction* and *Velocity*.

It is ascertained in practice that water issuing from a *Circular Aperture* in plate contracts its section at a distance of .5 its diameter from the orifice to very nearly .8 diameter of aperture, so as to reduce its area to about .61.* Velocity at this point is also ascertained to be about .97 of theoretical velocity due to a body falling from a height equal to the head of water. Mean velocity in aperture is therefore .974, which, multiplied by .594, theoretical discharge; and in this case .594 becomes *coefficient of velocity*, which, if expressed generally by C, will give for discharge itself

$g h \times C = V$. *a* representing area of aperture, and *V* volume discharged per second. Or, $4.97 a \sqrt{h} = V$. Or, $3.91 d^2 \sqrt{h} = V$. *d* representing diameter in feet. *e*, for cube feet per second, $4.97 a \sqrt{h}$, or $3.91 d^2 \sqrt{h}$.

EXAMPLE.—Assume head of water 10 feet, diameter of opening 1.127 feet, 1 sq. foot, and *C* = .62.

$1 \sqrt{2} g 10 \times .62 = 15.72$ cube feet. $4.97 \times 1 \times \sqrt{10} = 15.72$ cube feet, and $1.127^2 \times \sqrt{10} = 15.7$ cube feet.

For square aperture it is .615, and for rectangular .621.

Time of water or a fluid discharged in a given time from an aperture of given area depends on head, form of aperture, and nature of approaches.

$\bar{h} = v^2$, and $\frac{v^2}{64.333} = \bar{h}$. *h* representing height to centre of opening in feet.

1.—*Head*, or *height*, *h*, may be measured from surface of water to centre of orifice without practical error, for it has been proved by Mr. Neville that for circular apertures, having their centre at the depth of their radius below the surface, the error circumference touching the surface, the error cannot exceed 4 per cent excess of the true theoretical discharge, and that for depths exceeding three

61. Observed discharges of water coincide nearer to unit of Bayer than that of all other

times the diameter, the error is practically immaterial. For rectangular apertures it is also shown that, when their upper side is at surface of the water, as in notches, the extreme error cannot exceed 4.17 per cent. in excess; and when the upper is three times depth of aperture below the surface, the excess is inappreciable.

For notches, weirs, slits, etc., however, it is usual to take full depth for head, when .666 only of above equation must be taken to ascertain the discharge.

Experiments show that coefficient for similar apertures in thin plates, for small apertures and low velocities, is greater than for large apertures and high velocities, and that for elongated and small apertures it is greater than for apertures which have a regular form, and which approximate to the circle.

When Discharge of a Fluid is under the Surface of another body of a like Fluid.—The difference of levels between the two surfaces must be taken as the head of the fluid.

$$\text{Or, } \sqrt{2g(h-h')} = v.$$

When Outer Side of opening of a discharging Vessel is pressed by a Force.—The difference of height of head of fluid and quotient of pressures on two sides of vessel, divided by density of fluid, must be taken as heads of fluid.

$$\text{Or, } \sqrt{2g\left(h - \frac{(p-p') \times 144}{S}\right)} = v. \quad S \text{ representing density of fluid.}$$

ILLUSTRATION.—Assume head of water in open reservoir is 12 feet above water-line in boiler, and pressures of atmosphere and steam are 14.7 and 19.7 lbs.

$$\text{Then } \sqrt{2g\left(12 - \frac{(19.7-14.7) \times 144}{62.5}\right)} = \sqrt{64.333 \times \left(12 - \frac{5 \times 144}{62.5}\right)} = 5.56 \text{ feet.}$$

When Water flows into a rarefied Space, as into Condenser of a Steam-engine, and is either pressed upon or open to Atmosphere.—The height due to mean pressure of atmosphere within condenser, added to height of water above internal surface of it, must be taken as head of the water.

$$\text{Or, } \sqrt{2g(h+h')} = v.$$

ILLUSTRATION.—Assume head of water external to condenser of a steam-engine be 3 feet, vacuum gauge to indicate a column of mercury of 26.467 ins. (=13 lbs.), and a column of water of 13 lbs.=29.9 feet.

$$\text{Then } \sqrt{2g(3+29.9)} = \sqrt{64.333 \times 32.9} = \sqrt{2116.57} = 46 \text{ feet.}$$

Relative Velocity of Discharge of Water through different Apertures and under like Heads.

Velocity that would result from direct, unretarded action of the column of water which produces it, being a constant, or.....

| | |
|---|------|
| Through a cylindrical aperture in a thin plate..... | 1 |
| A tube from 2 to 3 diameters in length, projecting outward..... | 1.05 |
| A tube of the same length, projecting inward..... | 1.05 |
| A conical tube of form of contracted vein..... | 1.05 |
| Wide opening, bottom of which is on a level with that of reservoir; sluice with walls in a line with orifice; or bridge with pointed piers..... | 1.05 |
| Narrow opening, bottom of which is on a level with that of reservoir; abrupt projections and square piers of bridges..... | 1.05 |
| Sluice without side walls..... | 1.05 |

Discharge or Efflux of Water for various Openings Apertures.

Rectangular Weir.

Weirs are designated *Perfect* when their sill is above surface of stream, and *Imperfect*, *Submerged*, or *Drowned* when it is below 0

Height measured from Surface of Water to Sill. (Jas. B. Francis.)

| Mean Head. | Length of Opening. | Mean Discharge per Second. | Mean Coefficient. |
|-------------------|--------------------|----------------------------|-------------------|
| .62 to 1.55 feet. | 10 feet. | 32.9 cube feet. | .623 |

Principal causes for variation in coefficients derived from most experiments giving discharge of water over weirs arises from,

1. Depth being taken from only one part of surface, for it has been proved at heads on, at, and above a weir should be taken in order to determine the discharge.

2. Nature of the approaches, including ratio of the water-way in channel over, to water-way on weir.

When a weir extends from side to side of a channel, the contraction is as when it forms a notch, or Poncelet weir, and coefficient sometimes as high as .667.

When weir or notch extends only one fourth, or a less portion of width, coefficient has been found to vary from .584 to .6.

When wing-boards are added at an angle of about 64° , coefficient is greater even when head is less.

Computation of Volume of Discharge.

Mean velocity of a fluid issuing through a rectangular opening in the side of a vessel is two thirds of that due to velocity at sill or lower edge of opening, or it is that due to a point four ninths of whole height on surface of fluid.

Height measured from Surface of Head of Water to Sill of Opening.

RULE.—Multiply square root of product of 64.333 and height or whole depth of the fluid in feet, by area in feet, and by coefficient for opening, and two thirds of product will give volume in cube feet per second.

$$\text{Or, } \frac{2}{3} b h \sqrt{2 g h} C = V; \quad \frac{V}{\frac{2}{3} b h \sqrt{2 g h} C} = t; \text{ and } \left(\frac{V}{\frac{2}{3} C b h} \right)^2 \div 2 g = h.$$

t representing time in seconds and *V* volume in cube feet.

EXAMPLE.—Sill of a weir is 1 foot below surface of water, and its breadth is 10 ft; what volume of water will it discharge in one second?

$$C = .623, \quad \sqrt{64.33 \times 1 \times 10 \times 1} = 80.2, \text{ and } \frac{2}{3} 80.2 \times .623 = 33.32 \text{ cube feet.}$$

NOTE.—Mean coefficient of discharge of weirs, breadth of which is no more than half part of breadth of stream, is two thirds of .6 = .4; and for weirs which extend whole width of stream it is two thirds of .666 = .444.

$$\text{Or, } 214 \sqrt{h^3} = V \text{ in cube feet per minute. When } h \text{ is in ins., put } 5.15 \text{ for } 214.$$

$$\text{Or, } C b h \sqrt{2 g h} = V. \quad C \text{ for a depth .1 of length} = .417, \text{ and for .33 of length} = .4.$$

$$\text{Or, by formula of Jas. B. Francis: } 3.33 (L - .1 n H) H^{\frac{3}{2}} = V.$$

L representing length of weir and *H* depth of water in canal, sufficiently far from ends to be unaffected by depression caused by the current, both in feet, and *n* number of end contractions.

NOTE.—When contraction exists at each end of weir, *n* = 2; and when weir is of width of canal or conduit, end contraction does not exist, and *n* = 0.

This formula is applicable only to rectangular and horizontal weirs in side of a canal, vertical on water-side, with sharp edges to current; for if bevelled or rounded in any perceptible degree, a material effect will be produced in the discharge essential also that the stream, after passing the edges, should in no wise be disturbed in its flow and descent.

In cases in which depth exceeds one third of length of weir, this form applicable. In the observations from which it was deduced, the depth v 7 to nearly 19 ins.

With end contraction, a distance from side of canal to weir equal to weir is least admissible, in order that formula may apply correctly.

Depth of water in canal should not be less than three times that on v curate computation of flow.

ILLUSTRATION.—If an overflow weir has a length of 7.94 feet and a d (as determined by a hook gauge), what volume will it discharge in 24 ho

$$3.33 (7.94 - .2 \times .986) .986^{\frac{3}{2}} = 3.33 \times 7.94 - .1972 \times .97907 = 3.33 \times 7.7428 = 25.243875, \text{ which } \times 60 \times 60 \times 24 = 2181061 \text{ cube feet.}$$

$$\text{By Logarithms.—Log. } 3.33 = .522444$$

$$7.7428 = .888898$$

$$.986^{\frac{3}{2}} = 1.993877$$

$$\frac{3}{2} = 1.5$$

$$2) 1.981631$$

$$1.990815 = 1.990815$$

$$\text{Log. } 24 \text{ hours} = 86400 \text{ seconds.}$$

$$1.402157$$

$$4.936514$$

$$6.338671$$

$$\text{Log. } 6.33867 = 2181073 \text{ cube feet. } C \text{ in this case} = .615.$$

Or, $214\sqrt{H^3}$ and $5.15\sqrt{h^3} = V$, if stream above the sill is not in representing height of surface of water above sill in feet, h in in $214\sqrt{H^3} + .035v^2H^3 = V$, if in motion. v representing velocity of a water in feet per second, and V volume in cube feet discharged over each of sill per minute.

In gauging, waste-board must have a thin edge. Height measured to face not affected by the current of overfall. (Molesworth.)

To Compute Depth of Flow over a Sill that w charge a given Volume of Water.

$$\left(\frac{3V}{2Cb\sqrt{2g}} + k^{\frac{3}{2}} \right)^{\frac{2}{3}} - k = d. \quad k = \frac{v^2}{2g} \text{ representing height due to velo flows to the weir.}$$

NOTE.—When back-water is raised considerably, say 2 feet, velocity c proaching weir (k) may be neglected.

Rectangular Notches, or Vertical Apertures or

A Notch is an opening, either vertical or oblique, in side of a v voir, etc., alike to a narrow and deep weir.

Vertical Apertures or Slits are narrow notches or weirs, run near to bottom of vessel or reservoir.

Coefficient for opening, 8 ins. by 5, mean .606 (Poncelet and Les

Coefficient increases as depth decreases, or as ratio of length o its depth increases.

When sides and under edge of a notch increase in thickness, so as to be into a short open channel, coefficients reduce considerably, and to an ext what increased resistance from friction, particularly for small depths, in

Poncelet and Lesbros found, for apertures 8×8 ins., that addition of a shoot 21 ins. long reduced coefficient from .604 to .601, with a head of ab but for a head of 4.5 ins. coefficient fell from .572 to .483.

For Rule and Formulas, see preceding page.

gular Openings or Sluices, or Horizontal Slits.
asured from Surface of Head of Water to Upper Side and to Sill of Opening.

ient for { Opening, 1 inch by 1 inch. Head, 7 to 23 feet. = .621.
 " 3 " " 3 " " 7 " 23 " = .614.
 " 2 feet " 1 foot. " 1 " 2 " = .641.

and Lesbros deduced that coefficient of discharge increases with small long apertures as they approach the surface, and decreases with large apertures under like circumstances.

is ranged, in square apertures of 8 by 8 ins., under a head of 6 ins. to apertures, 8 by 4 ins.; under a head of 10 feet, from .572 to .745.

Plate, $C = .616$ (Bossut); $C = .61$ (Michelotti).

To Compute Discharge.

Multiply square root of 64.333 and breadth of opening in feet, by for opening, and by difference of products of heights of water and the roots, and two thirds of whole product will give discharge in per second.

$$\sqrt{2g(h\sqrt{h}-h'\sqrt{h'})} C = V; \quad \frac{V}{\frac{2}{3} b \sqrt{2g}(h\sqrt{h}-h'\sqrt{h'})} C = t; \quad \text{and}$$

v. h and h' representing depth to sill and opening in feet, and v velocity per second.

—Sill of a rectangular sluice, 6 feet in width by 5 feet in depth, is 9 feet above of water; what is discharge in cube feet per second?

$$9-5=4, \text{ and } \frac{2}{3} \sqrt{2g} \times 6 \times .625 \times (9\sqrt{9}-4\sqrt{4}) = 380.95 \text{ cube feet.}$$

$$\frac{2}{3} g d a C = V. \quad d \text{ representing depth to centre of opening in feet.}$$

$$= 6.5, \quad a = 6 \times 5 = 30, \text{ and } \sqrt{64.33 \times 6.5} \times 30 \times .625 = 383.44 \text{ cube ft.}$$

Sluice Weirs or Sluices.

Flow of water by Sluices occurs under three forms—viz., *Unimpeded, Partly Unimpeded.*

To Compute Discharge when Unimpeded.

$\sqrt{h} = V.$ d representing depth of opening and h taken from centre of surface of water.

y, k , with which water flows to sluice is considered,

$$\left(\sqrt{h} - k \right)^2 = h; \quad \frac{V}{C b \sqrt{2g} h} = d; \quad \text{and} \quad \frac{V}{C b \sqrt{2g} \left(h' - \frac{d}{2} \right)} = d.$$

Height to which water is raised by dam above sill.

Prob.—How high must the gate of a sluice weir be raised, to discharge 1000 cu ft of water per second, its breadth being 24 feet and height, h' , 5 feet?

Assessment = .6. d approximately = 1.

$$\frac{250}{.6 \times 24 \sqrt{64.33} (5 - \frac{1}{2})} = \frac{250}{14.4 \times 17.014} = 1.0204 \text{ feet.}$$

To Compute Discharge when Impeded.

$$C d b \sqrt{2g} h = V, \quad \text{and} \quad \frac{V}{C b \sqrt{2g} h} = d.$$

Height difference of level between supply and back-water.

To Compute Discharge when partly Impeded.

$Cb\sqrt{2g}\left(d\sqrt{h-\frac{d}{2}}+d'\sqrt{h}\right)=V$. d' representing depth of back-water above upper edge of sill.

ILLUSTRATION.—Dimensions of a sluice are 18 feet in breadth by .5 in depth; height of opening above surface of water .7 feet, and difference between levels of supply and surface water is 2 feet; what is discharge per second?

$$.6 \times 18 \times 8.02 \left(.7 \sqrt{2 - \frac{.7}{2}} + .5 \sqrt{2} \right) = 86.62 \times .896 + .707 = 138.85 \text{ cube feet}$$

Coefficients of Circular Openings or Sluices.

Height measured from Surface of Head of Water to Centre of Opening.

Contraction of section from 1 to .633, and reduction of velocity to .974; hence $.633 \times .974 = .617$ (Neville).

In a Thin Plate, $C = .666$ (Bossut); .631 (Venturi); .64 (Eytelwein).

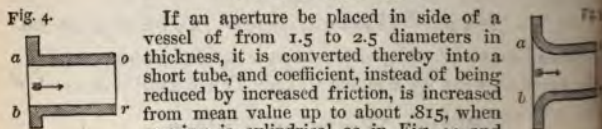
Cylindrical Ajutages, or Additional Tubes, give a greater discharge than apertures in a thin side, head and area of opening being the same; but it is necessary that the flowing water should entirely fill mouth of ajutage.

Mean coefficient, as deduced by Castel, Bossut, and Eytelwein, is .82.

Short Tubes, Mouth-pieces, and Cylindrical Prolongations or Ajutages.

Fig. 4.

If an aperture be placed in side of a vessel of from 1.5 to 2.5 diameters in thickness, it is converted thereby into a short tube, and coefficient, instead of being reduced by increased friction, is increased from mean value up to about .815, when opening is cylindrical, as in Fig. 4; and when junction is rounded, as in Fig. 5, to form of contracted vein, coefficient increases to .958, .959, and .975 for heads of 1, 10, and 15 feet.



Conically Convergent and Divergent Tubes.

Fig. 6.

In conically divergent tube, Fig. 6, coefficient of discharge is greater than for short tube placed convergent, fluid filling in both cases, and the smaller diameters, or those at same distance from centres, O O, being used in the computations.

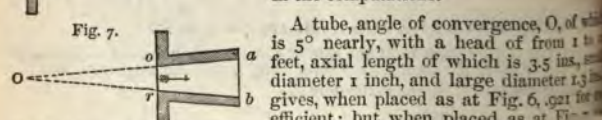


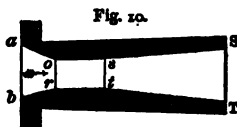
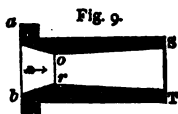
Fig. 7.

A tube, angle of convergence, O, of which is 5° nearly, with a head of from 1 to 10 feet, axial length of which is 3.5 ins., small diameter 1 inch, and large diameter 1.3 ins. gives, when placed as at Fig. 6, .921 for coefficient; but when placed as at Fig. 7, efficient increases up to .948. Coefficient of velocity is, however, 1 in Fig. 6 than for Fig. 7, and discharging jet has greater amplitude in Fig. 6. If a prismatic tube project beyond sides into a vessel, coefficient was deduced to .715 nearly.

Form of tube which gives greatest discharge is that of a truncated lesser base being fitted to reservoir, Fig. 7. Venturi concluded that

nents that tube of greatest discharge has a length 9 times diameter of r opening base, and a diverging angle of $5^\circ 8'$ —discharge being 2.5 times that through a thin plate, 1.9 times greater than through a cylindrical tube, and 1.46 greater than theoretic discharge.

Compound Mouth-pieces and Ajutages.



Coefficients for Mouth-pieces, Short Tubes, and Cylindrical Prolongations.

Computed and reduced by Mr. Neville, from Venturi's Experiments.

| Description of Aperture, Mouth-piece, or Tube. | C. for Diam. o b. | C. for Diam. o r. |
|---|------------------------|------------------------|
| A aperture 1.5 ins. diameter, in a thin plate | .622 | .974 |
| A tube 1.5 ins. diameter, and 4.5 ins. long, Fig. 4..... | .823 | .823 |
| A tube, Fig. 5, having junction rounded to form of contracted vein | .611 | .956 |
| A short conical convergent mouth-piece, Fig. 6 | .607 | .934 |
| A like tube divergent, with smaller diameter at junction with reservoir; length 3.5 ins., $or = 1$ in., and $ab = 1.3$ ins. ... | .561 | .948 |
| A double conical tube, $ao, ST, r b$, Fig. 9, when $ab = ST = 1.5$ ins., $or = 1.21$ ins., $ao = .92$ in., and $oS = 4.1$ ins. | .928 | 1.428 |
| A like tube when, as in Fig. 8, $ao r b = o S T r$, and $ao S =$ 1.84 ins. | .823 | 1.266 |
| A like tube when $ST = 1.46$ ins., and $oS = 2.17$ ins. | .823 | 1.266 |
| A like tube when $ST = 3$ ins., and $oS = 9.5$ ins. | .911 | 1.4 |
| A like tube when $oS = 6.5$ ins., and $ST = 1.92$ ins. | 1.02 | 1.569 |
| A like tube when $ST = 2.25$ ins., and $oS = 12.125$ ins. | 1.215 | 1.855 |
| A tube, Fig. 10, when $os = r t = 3$ ins., $or = s t = 1.21$ ins., and tube $o S T r$, as in No. 6, $ST = 1.5$ ins., and $s S = 4.1$ ins. | .895 | 1.377 |

Mean of various experiments with tubes of .5 to 3 ins. in diameter, and a head of fluid of from 3 to 20 feet, gave a coefficient of .813; and as for circular apertures in a thin plate is .63, it follows that under similar circumstances, $.813 \div .63 = 1.29$ times as much fluid flows through as through a like aperture in a thin plate.

The preceding Table gives coefficients of discharge for figures given, and it be found of great value, as coefficients are calculated for large as well as small diameters, and the necessity for taking into consideration form of section of a pipe with a reservoir will be understood from the results.

Circular Sluices, etc.

To Compute Discharge.

Height measured from Surface of Head of Water to Centre of Opening.

RULE.—Multiply square root of product of 64.333 and depth of centre of opening from surface of water, by area of opening in square feet, and this product by coefficient for the opening, and whole product will give discharge in cubic feet per second.

$\sqrt{2gd}$, $aC = V$, a representing area in sq. feet, and d depth of surface of fluid from centre of opening in feet.

EXAMPLE.—Diameter of a circular sluice is 1 foot, and its centre is 1.5 feet below surface of the water; what is discharge in cube feet per second?

Area of 1 foot = .7854; $C = .64$, and $\sqrt{64.333 \times 1.5 \times .7854 \times .64} = 4.938$ cube

When Circumference reaches Surface of Water. $\sqrt{2gr}$, .9604 a $C = r$ representing radius of circle in feet.

ILLUSTRATION.—In what time will 800 cube feet of water be discharged through circular opening of .025 sq. foot, centre of which is 8 feet below surface of water

$$C = .63. \quad \frac{800}{\sqrt{2gd} \times .025 \times .63} = \frac{800}{22.68 \times .025 \times .63} = 2239.58 = 37 \text{ min. } 19.6$$

NOTE.—For circular orifices, the formula $\sqrt{2gd} a C = V$ is sufficiently exact all depths exceeding 3 times diameter; the finish of openings being of more than extreme accuracy in coefficient.

Semicircular Sluices.

When Diameter is either Upward or Downward. $\sqrt{2gd} a C = V$, d representing depth of centre of gravity of figure from surface.

When Diameter as above is at Depth d , below Surface. $\sqrt{2gd} 1.188 a C$:

Circular, Semicircular, Triangular, Trapezoidal, Prismatic Wedges, Sluices, Slits, etc.

See Neville, London, 1860, pp. 51–63, and Weisbach, vol. 1. p. 456.

For greater number of apertures at any depth below surface of water product of area, and velocity of depth of centre, or centre of gravity if practicable to obtain it, will give discharge with sufficient accuracy.

Discharge from Vessels not Receiving any Supply

For prismatic vessels the general law applies, that twice as much water be discharged from like apertures if the vessels were kept full during time which is required for emptying them.

$$\text{To Compute Time.} \quad \frac{2A\sqrt{h}}{Ca\sqrt{2g}} = \frac{2Ah}{V} = t.$$

ILLUSTRATION.—A rectangular cistern has a transverse horizontal section of 2 feet, a depth of 4 feet, and a circular opening in its bottom of 2 ins. in diameter; what time will it discharge its volume of water, when supply to it is cut off; cistern allowed to be emptied of its contents?

$h = 4$ feet, $a = 2^2 \times .7854 \div 144 = .0218$, $C = .613$, and $\sqrt{2gh} \times a \times C = .1$ cube foot per second. Then $\frac{2 \times 14 \times 4}{.2143} = 522.6$ seconds.

To Compute Time and Fall.

When or subsidence of surface of water in a vessel, corresponding to the fall of efflux, is $h - h'$, h' representing lesser depth.

$$\frac{A}{\sqrt{2g}} (\sqrt{h} - \sqrt{h'}) = t. \quad \text{Inversely, } \left(\sqrt{h} - \frac{Ca\sqrt{2g}}{2A} t \right)^2 = h'.$$

What time will the water in cistern, as given in preceding illustration, subside in that time?

$$\sqrt{2g} = 8.02, \quad h = 4, \quad h' = 4 - 1.6 = 2.4$$

$$\sqrt{2.4} = \frac{28}{.1049} \times (2 - 1.55) = 120.1 \text{ seconds}$$

$$\sqrt{2} = 1.414, \quad 4 - 2.4 = 1.6 \text{ feet; hence, } 4 - 2.4 =$$

Divide result obtained as preceding

charge, when Form and Dimensions of Vessel of Efflux are not known.

lume discharged may be estimated by observing heads of the water at intervals of time; and at end of half time of discharge, head of water is .25 of whole height from surface to delivery.

en t = such interval. For openings in bottom or side, $C a t \sqrt{2g} \left(\frac{\sqrt{h} + \sqrt{h_1}}{2} \right)$ for 1 depth; $C a t \sqrt{2g} \left(\frac{\sqrt{h} + 4\sqrt{h_1} + \sqrt{h_2}}{3} \right) = V$ for 2 depths; and $\frac{1}{2} g \left(\frac{\sqrt{h} + 4\sqrt{h_1} + 2\sqrt{h_2} + 4\sqrt{h_3} + \sqrt{h_4}}{3} \right) = V$ for 4 depths.

At end of half time of discharge, head of water will be .25 of whole height surface to delivery.

Weirs or Notches.

$b t \sqrt{2g} (\sqrt{h^3} + 4\sqrt{h_1^3} + \sqrt{h_2^3}) = V$. b representing breadth in feet.

EXAMPLE.—A prismatic reservoir 9 feet in depth is discharged through a 2.222 feet wide, surface subsiding 6.75 feet in 935 seconds; what is volume discharged?

.6, $h_1 = 9 - 6.75 = 2.25$ feet, and $\frac{2}{9} .6 \times 2.222 \times 935 \times 8.02 (\sqrt{9^3} + 4\sqrt{2.25^3} + \sqrt{0^3}) = 2221.6 \times 40.5 = 89974.8$ cube feet.

When there is an Influx and Efflux.

A reservoir during an efflux from it has an influx into it, determination is in which surface of water rises or falls a certain height becomes so indicated that an approximate determination is here alone essayed.

Rate of permanency or constant height occurs whenever head of water is increased or decreased by $\frac{I}{2g} \left(\frac{I}{C a} \right) = k$. I representing influx in cube feet per second.

Time t in which variable head (x) increases by volume $(v) = \frac{A_1 v}{I - C a \sqrt{2g} x}$; time in which it sinks height, k , by $\frac{A_1 v}{C a \sqrt{2g} x - I}$. Time of efflux, in which surface falls from A to A_1 , etc., and head of water from h to h_1 , when represented by $\frac{I}{C a \sqrt{2g}} = \sqrt{k}$, is

$$\frac{h_4}{\sqrt{2g}} \left(\frac{A}{\sqrt{h} - \sqrt{k}} + \frac{4A_1}{\sqrt{h_1} - \sqrt{k}} + \frac{2A_2}{\sqrt{h_2} - \sqrt{k}} + \frac{4A_3}{\sqrt{h_3} - \sqrt{k}} + \frac{A_4}{\sqrt{h_4} - \sqrt{k}} \right) = t$$

EXAMPLE.—In what time will surface of water in a pond, as in a previous example, fall 6 feet, if there is an influx into it of 3.0444 cube feet per second?

$$\sqrt{k} = \frac{3.044}{.537 \times .8836 \times 8.02} = .8. \quad C = .537 \text{ and } a = .8836.$$

$$\frac{20 - 14}{.537 \times .8836 \times 8.02} \times \left(\frac{600000}{4.472 - .8} + \frac{4 \times 495000}{4.301 - .8} + \frac{2 \times 410000}{4.123 - .8} + \frac{4 \times 325000}{3.937 - .8} \right) = \frac{6}{45.665} \times 1480201 = 194486 \text{ seconds} = 54 \text{ h., } 1 \text{ min., } 26 \text{ sec.}$$

Prismatic Vessels.

Vessel has a uniform transverse section, A .

$$\frac{2A}{C a \sqrt{2g}} \left[\sqrt{h} - \sqrt{h_1} + \sqrt{k} \times \text{hyp. log.} \left(\frac{\sqrt{h} - \sqrt{k}}{\sqrt{h_1} - \sqrt{k}} \right) \right] = t = \text{time water flows from } h \text{ to } h_1.$$

ILLUSTRATION.—A reservoir has a surface of 500 000 sq. feet, a depth of 20 feet; it is fed by a stream affording a supply of 3.0444 cube feet per second, and outlet has an area of .8836 sq. foot; in what time will it subside 6 feet?

$$\sqrt{k}, \text{ as before, } = .8, \quad C = .537, \quad \text{and} \quad \frac{2 \times 500\,000}{C a \sqrt{2} g} \times \left[\sqrt{20} - \sqrt{14} + .8 \times \text{hyp. log.} \left(\frac{\sqrt{20} - .8}{\sqrt{14} - .8} \right) \times 2.303 \right] = 238.414 \text{ seconds} = 66 \text{ h. } 13 \text{ min. } 34 \text{ sec.}$$

To Compute Fall in a given Time.

This is determining head h_1 at end of that time, and it should be subtracted from head h at commencement of discharge. Put into preceding equation several values of h_1 , until one is found to meet the condition.

ILLUSTRATION.—Take a prismatic pond having a surface of 38 750 sq. feet, a depth to centre of opening of sluice of 10.5 feet, a supply of 33.6 cube feet, and a discharge of 40 cube feet per second.

$$\sqrt{k} = .84.$$

Putting these numerical values into the equation, and assuming different values for h_1 , a value which nearly satisfies the equation is 4. Consequently, 10.5—4 = 6.5 feet, fall.

$$\frac{A k}{3 I} \left[\text{hyp. log. } \frac{h_1 + \sqrt{h_1 k} + k}{(\sqrt{h_1} - \sqrt{k})^2} + \sqrt{12} \text{ arc } \left(\text{tang. } = \frac{-\sqrt{3 h_1}}{2 \sqrt{k} + \sqrt{h_1}} \right) \right] = t;$$

$$\left(\frac{I}{\frac{2}{3} C b \sqrt{2} g} \right)^{\frac{2}{3}} = k; \text{ arc } (\text{tang. } = y, \text{ arc tangent of which } = y, \text{ and } I \text{ as preceding.})$$

According as k is $\leq h$, and influx of water, $I \geq \frac{2}{3} C I \sqrt{2 g h^3}$, there is a rise or fall of fluid surface, the condition of permanency occurring when $h_1 = k$, and time corresponding becomes ∞ .

ILLUSTRATION.—In what time will water in a rectangular tank, 12 feet in length by 6 feet in breadth, rise from sill of a weir or notch, 6 inches broad, to 2 feet above it, when 5 cube feet of water flow into the tank per second?

$$h_1 = 2, \quad h = 0, \quad A = 12 \times 6 = 72, \quad I = 5, \quad b = .5, \quad C = .6.$$

$$k = \left(\frac{5}{\frac{2}{3} \cdot 6 \times .5 \times 8.02} \right)^{\frac{2}{3}} = \sqrt[3]{3.117^2} = 2.1338.$$

$$\text{Then } \frac{72 \times 2.1338}{3 \times 5} \left[\text{hyp. logarithm } \frac{2 + \sqrt{2 \times 2.1338} + 2.1338}{(\sqrt{2} - \sqrt{2.1338})^2} + \sqrt{12} \text{ arc } \left(\text{tang. } = \frac{-\sqrt{3 \times 2}}{2 \sqrt{2.1338} + \sqrt{2}} \right) \right] = 10.2423 \times \text{hyp. log. } \frac{6.1996}{.002162} - 3.4641 \times \text{arc } \left(\text{tang. } \frac{\sqrt{6}}{4.335} \right) = 10.2423 \times [7.961 - (3.461 \times \text{arc, tangent of which } = .56497, \text{ or } 29^\circ 28' = 29.46^\circ \text{ length of which } = .5143) = 1.781] = 10.2423 - 7.961 - 1.781 = 10.2423 \times 6.11 = 63.297 \text{ seconds.}$$

Discharge of Water under Variable Pressures.

To Compute Time, Rise and Fall, and Volume.

$\frac{a}{A} \sqrt{2 g x} = v$. x representing variable head, A and a areas of transverse horizontal section of vessel and discharge, and v theoretical velocity of efflux.

To Compute Volume.

$A y = V$. y representing extent of fall, and V volume of water discharged $h - h_1$.

ILLUSTRATION.—Assume elements of preceding case.

$$A = 14, \quad y = 4 \text{ feet.} \quad \text{Then } 56 \times 4 = 224 \text{ cube feet.}$$

Discharge from Vessels of Communication.
When Reservoir of Supply is maintained at a uniform Height.—Fig. 11.

To Compute Time. $\frac{2 A \sqrt{h}}{C a \sqrt{2} g} = t.$

ILLUSTRATION 1.—In what time will level of water in a receiving vessel having a section of 14 sq. feet attain height of that in supply, through a pipe 2 ins. in diameter, placed 4 feet below level of supply?

$$C = .613. \quad \frac{2 \times 14 \times \sqrt{4}}{.613 \times .0218 \times 8.02} = \frac{56}{.1072} = 522.3 \text{ seconds.}$$

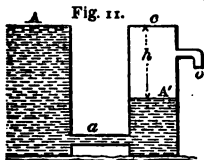


Fig. 11.

2.—Assume C, vessel, Fig. 11, to be a cylinder 18 ins. in diameter, head of water in A = 4 feet, at A' 1 foot, and 2 feet below outlet o; in what time will water in vessel run out and over at o through a pipe, a, 1.5 ins. diameter?

$$h - h' = 4 - 1 - 2 = 1 \text{ foot.} \quad C = .8.$$

$$\frac{A}{a} = \left(\frac{18}{1.5}\right)^2 = 144.$$

$$\text{Then } \frac{2 \times 144}{.8 \times 8.03} (\sqrt{3} - \sqrt{1}) = \frac{288}{6.424} \times 1.73 - 1 = 32.73 \text{ seconds.}$$

When Vessel of Supply has no Influx, and is not indefinitely great compared with Receiving Vessel.

$$\frac{2 A A' \sqrt{h}}{C a (A + A') \sqrt{2} g} = t. \quad A' \text{ representing section of receiving vessel, } t \text{ time in which}$$

the two surfaces of water attain same level; and $\frac{2 A A' (\sqrt{h} - \sqrt{h'})}{C a (A + A') \sqrt{2} g} = t$, time within which level falls from h to h' .

ILLUSTRATION.—Section of a cistern from which water is to be drawn is 10 sq. feet, and section of receiving cistern is 4 sq. feet; initial difference of level is 3 feet, and diameter of communicating pipe is 1 inch; in what time will surfaces of water in both vessels attain like levels?

$$C = .82. \quad 1'' = .7854. \quad \frac{2 \times 10 \times 4 \sqrt{3}}{.82 \times .7854 \times \frac{14}{144} \times 8.02} = \frac{138.56}{.502} = 276 \text{ seconds.}$$

Discharge from a Notch* in Side of a Vessel.

When it has no Influx. $\frac{3 A}{C b \times \sqrt{2} g} \left(\frac{1}{\sqrt{h'}} - \frac{1}{\sqrt{h}} \right) = t. \quad b \text{ breadth of notch in feet.}$

ILLUSTRATION.—If a reservoir of water, 110 feet in length by 40 in breadth, has a notch in end of 9 ins. in width; in what time will head of water of 15 ins. fall to 6?

$$C = .6. \quad 9'' = .75 \text{ foot.} \quad h' = .5. \quad h = 1.25.$$

$$\frac{3 \times 110 \times 40}{.6 \times .75 \times 8.02} \times \left(\frac{1}{\sqrt{.5}} - \frac{1}{\sqrt{1.25}} \right) = \frac{13200}{3.61} \times 1.414 - .894 = 1901 \text{ seconds.}$$

NOTE.—For discharge of vessels in motion, see Weisbach, vol. 1, pp. 394–396.

Reservoirs or Cisterns.

To Compute Time of Filling and of Emptying a Reservoir under Operation of both Supply and Discharge.

$\frac{V}{S - D} = T$, and $\frac{V}{D - S} = t$. V representing volume of vessel, S supply of and D discharge of water, both per minute, and in cube feet. T time of full and t time of discharging it, both in minutes.

* When the notch extends to the bottom of the reservoir, etc., the time for the water definite, as $h' = 0$.

Irregular-Shaped Vessels, as a Pond, Lake, etc. To Compute Time and Volume Discharged.

Operation.—Divide whole mass of water into four or six strata of equal depths.

Then, for 4 Strata, $\frac{h-h^4}{12 C a \sqrt{2g}} \times \left(\frac{a}{\sqrt{h}} + \frac{4a^2}{\sqrt{h^2}} + \frac{2a^3}{\sqrt{h^3}} + \frac{4a^4}{\sqrt{h^4}} + \frac{a^5}{\sqrt{h^5}} \right) = t; A, A_1$, etc., representing depths of strata at a, a_1 , etc., commencing at surface; a, a_1 , etc., being areas of first, second, etc., transverse sections of pond, etc.; and $\frac{h-h^4}{12}$
 $\times a + 4a^2 + 2a^3 + 4a^4 + a^5 = V$.

Fig. 12.

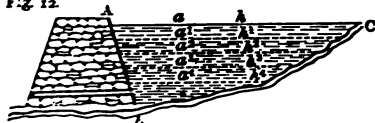


ILLUSTRATION.—In what time will depth of water in a lake, A B C, Fig. 12, subside 6 feet, surfaces of its strata having following areas, outline of sluice being a semicircle, 15 ins. wide, 9 deep, and 60 feet in length?

| | | | | |
|------------------------|----------------|-----------|---------|-----------|
| a at 20 feet (h) | depth of water | = area of | 600 000 | Sq. feet. |
| a_1 " 13.5 " | (A_1) | " " | 495 000 | " |
| a_2 " 17 " | (A_2) | " " | 410 000 | " |
| a_3 " 15.5 " | (A_3) | " " | 325 000 | " |
| a_4 " 14 " | (A_4) | " " | 265 000 | " |

a = area of 18 ÷ 2 = .8836 sq. feet; C = .537.

Then $\frac{20-14}{12 \times .537 \times .8836 \times 8.02} \times \left(\frac{600000}{4.472} + \frac{4 \times 495000}{4.371} + \frac{2 \times 410000}{4.123} + \frac{4 \times 325000}{3.937} + \frac{265000}{3.742} \right) = \frac{6}{45.665} \times 1194431 = 156933 \text{ sec.} = 43 \text{ h., } 35 \text{ min. } 38 \text{ sec.}$

And discharge = $\frac{6}{12} \times (600000 + 4 \times 495000 + 2 \times 410000 + 4 \times 325000 + 265000) = .5 \times 4965000 = 2482500 \text{ cube feet.}$

For 6 Strata, put 2 a_4 , instead of a_4 , and 4 a_5 and a^5 additional, and divide by 18 instead of 12.

Flow of Water in Beds.

Flow of water in beds is either *Uniform* or *Variable*. It is uniform when mean velocity at all transverse sections is the same, and consequently when areas of sections are equal; it is variable when mean velocities, and therefore areas of sections, vary.

To Compute Fall of Flow.

$C \frac{1}{a} \times \frac{v^2}{2g} = h$. C representing coefficient of friction, l length of flow, p perimeter of sides and bottom of bed, and h fall in feet.

ILLUSTRATION.—A canal 2600 feet in length has breadths of 3 and 7 feet, a depth of 3 feet, with a flow of 40 cube feet per second; what is its fall?

C as per table below .007565; $p = \sqrt{3^2 + 2^2} \times 2 + 3 = 10.2$; $a = 15$; and $v = 40 \div 15 = 2.66$. Hence .007565 $\times \frac{2600 \times 10.2}{15} \times \frac{2.66^2}{64.33} = 1.47 \text{ feet.}$

To Compute Velocity of Flow. $\sqrt{\frac{a}{C \times l p} 2g h} = v$

canal 5800 feet in length has breadths of 4 and 12 feet, a depth of 3 feet, what is velocity and volume of flow?

$= \sqrt{5^2 + 4^2} \times 2 + 4 = 16.8$, and $a = 40$

$= 4.4 \times 3 = \sqrt{.0542 \times 193} = 3.23 \text{ feet.}$

Elements of Friction of Flow of Water in Beds, as in Rivers, Canals, Streams, etc.

In Feet per Second.

| C. | Velocity. | C. | Velocity. | C. | Velocity. | C. |
|---------|-----------|---------|-----------|---------|-----------|---------|
| .008 15 | .7 | .007 73 | 1.5 | .007 59 | 5 | .007 45 |
| .007 97 | .8 | .007 69 | 2 | .007 52 | 8 | .007 44 |
| .007 85 | .9 | .007 66 | 2.5 | .007 51 | 10 | .007 43 |
| .007 78 | 1 | .007 63 | 3 | .007 49 | 12 | .007 42 |

Forms of Transverse Sections of Canals, etc.

Friction or friction which bed of a stream, etc., opposes to flow of water, hence of its adhesion or viscosity, increases with surface of contact bed and water, and therefore with the perimeter of water profile, or portion of transverse section which comprises the bed.

Rate of flow of water in a bed is inversely as area of it.

regular figures, that which has greatest number of sides has for a least perimeter; hence, for enclosed conduits, nearer its transverse profile approaches to a regular figure, less the coefficient of its friction; finally, a circle has the profile which presents minimum of friction.

a canal is cut in earth or sand and not walled up, the slope of its bed should not exceed 45°.

Variable Motion.

The motion of water in beds of rivers or streams may be reduced to uniform motion when resistance of friction for an observed length can be taken as constant.

Compute Volume of Water flowing in a River.

$$\frac{\sqrt{2gh}}{-\frac{1}{A^2} + C \frac{1p}{A_1 + A} \left(\frac{1}{A_1^2} + \frac{1}{A^2} \right)} = V. \quad \text{A and } A_1 \text{ representing areas of upper and lower transverse sections of flow.}$$

EXAMPLE.—A stream having a mean perimeter of water profile of 40 feet for 300 feet has a fall of 9.6 ins.; area of its upper section is 70 sq. feet, and is 60; what is volume of its discharge?

$$\text{Find } C \text{ for velocity due to this case, } 92.35 \sqrt{\frac{70 + 60 \times \frac{9.6}{12}}{40 \times 300}} = 8.59 \text{ feet,}$$

for which, see Table above, = .007 44.

$$\frac{\sqrt{64.33 \times (9.6 \div 12)}}{-\frac{1}{60^2} + .007 44 \frac{300 \times 40}{70 + 60} \left(\frac{1}{70^2} + \frac{1}{60^2} \right)} = \frac{7.174}{\sqrt{.000 330 89}} = 394.6 \text{ cube feet;}$$

$$\text{velocity} = \frac{394.6 \times 2}{70 + 60} = 6.07 \text{ feet, } C \text{ for which is .007 45.}$$

FRICION IN PIPES AND SEWERS.

Rate of flow of water through pipes, etc., of a uniform diameter is in direct ratio to the square root of pressure, and increases directly as length, very nearly as square of flow, and inversely as diameter of pipe.

Friction in wooden pipes is 1.75 times greater than in metallic.

Resistance to flow of an equal quantity of water through Pipes of equal lengths, and with equal heads, is proportionally as follows:

in a Straight Line as 90, in a True Curve as 100, and in a Right Angle as

To Compute Head necessary to overcome Friction of Pipe. (Weisbach.)

$\left(.0144 + \frac{.01746}{\sqrt{v}}\right) \times \frac{l}{d} \times \frac{v^2}{5.4} = h'$. h' representing head to overcome friction of pipe in feet, l length of pipe, and v velocity of water per second, all in feet, and d internal diameter of pipe in ins.

ILLUSTRATION.—Length of a conduit-pipe is 1000 feet, its diameter 3 ins., and the required velocity of its discharge 4 feet per second; what is required head of water to overcome friction of flow in pipe?

$$\left(.0144 + \frac{.01746}{\sqrt{4}}\right) \times \frac{1000}{3} \times \frac{16}{5.4} = .02313 \times 333.333 \times 2.963 = 22.845 \text{ feet}$$

Head here deduced is height necessary to overcome friction of water in pipe alone.

Whole or entire head or fall includes, in addition to above, height between surface of supply and centre of opening of pipe at its upper end. Consequently, it is whole height or vertical distance between supply and centre of outlet.

To Compute whole Head, or Height from Surface of Supply to Centre of Discharge.

$$(C \times \frac{l}{d} + 1.5) \times \frac{v^2}{2g} = h.$$

1.5 is taken as a mean, and is coefficient of friction for interior orifice, or that of upper portion of pipe.

To obtain C or coefficient. $\left(.0144 + \frac{.01746}{\sqrt{v}}\right) = C.$

For facilitating computation, following Table of coefficients of resistance is introduced, being a reduction of preceding formula:

Coefficients of Friction of Water.

In Pipes at Different Velocities.

| V. | C. | V. | C. | V. | C. | V. | C. | V. | C. |
|----------|-------|----------|-------|----------|-------|----------|-------|----------|-------|
| Ft. Ins. | | Ft. Ins. | | Ft. Ins. | | Ft. Ins. | | Ft. Ins. | |
| 4 | .0443 | 2 8 | .025 | 5 | .0221 | 7 4 | .0208 | 11 6 | .0195 |
| 8 | .0350 | 3 | .0244 | 5 4 | .0219 | 7 8 | .0206 | 12 | .0192 |
| 1 | .0317 | 3 4 | .0239 | 5 8 | .0217 | 8 | .0205 | 12 6 | .0190 |
| 1 4 | .0294 | 3 8 | .0234 | 6 | .0215 | 8 6 | .0204 | 13 | .0188 |
| 1 8 | .0278 | 4 | .0231 | 6 4 | .0213 | 9 | .0202 | 14 | .0186 |
| 2 | .0266 | 4 4 | .0227 | 6 8 | .0211 | 10 | .0199 | 15 | .0182 |
| 2 4 | .0257 | 4 8 | .0224 | 7 | .0209 | 11 | .0196 | 16 | .0180 |

ILLUSTRATION 1.—Coefficient due to a velocity of 4 feet per second is .0231.

2.—Take elements of preceding case.

$$(.0231 \times \frac{1000 \times 12}{3} + 1.5) \times \frac{4^2}{64.33} = 93.9 \times \frac{16}{64.33} = 23.35 \text{ feet}$$

NOTE.—In preceding formula l was taken in feet, as the multiplier of 12 for ins. was cancelled by taking 5.4 for g , but in above formula it is necessary to retain this multiplier.

Radii of Curvatures.

When Pipes branch off from Mains, or when they are deflected at angles, radius of curvature should be proportionate to their diameter.

| | Ins. | Ins. | Ins. | Ins. |
|----------------|--------|--------|------|------|
| Diameter | 2 to 3 | 3 to 4 | 6 | 8 |
| Radius | 18 | 20 | 30 | 40 |

Curves and Bends.

Resistance or loss of head due to curves and bends, alike to that of friction increases as square of velocity; when, however, curves have a long radius and bends are obtuse, the loss is small.

Curved Circular Pipe. (Weisbach). $\frac{a}{180} \times \left[.131 + 1.847 \left(\frac{d}{2r} \right)^{\frac{7}{2}} \right] \times \frac{v^2}{2g} = h$.

a representing angle of curve, d diameter of pipe, r radius of curve, and h height due to friction or resistance of curve, all in feet.

For facility of computations, following values of $.131 + 1.847 \left(\frac{d}{2r} \right)^{\frac{7}{2}}$ are introduced.

Coefficients of Resistance.

In Curved Pipes with Section of a Circle.

| | | | | | | | | | | | | |
|----------------|-----|------|-----|------|-----|------|-----|------|-----|-------|-----|-------|
| $\frac{d}{2r}$ | .1 | .131 | .25 | .145 | .4 | .206 | .6 | .44 | .75 | .806 | .9 | 1.408 |
| $\frac{d}{2r}$ | .15 | .133 | .3 | .158 | .45 | .244 | .65 | .54 | .8 | .977 | .95 | 1.674 |
| $\frac{d}{2r}$ | .2 | .138 | .35 | .178 | .5 | .294 | .7 | .661 | .85 | 1.177 | 1 | 1.978 |

ILLUSTRATION. — If in a pipe 18 ins. in diameter and 1 mile in length there is right-angled curve of 5 feet radius, what additional head of flow should be given to attain velocity due to a head of 20 feet?

$\alpha = 90^\circ$, v for such a pipe and head = 4 feet per second; $18 = 1.5$ and $\frac{1.5}{2 \times} = .15$, and .15 by table = .133.

$$\text{Hence, } \frac{90}{180} \times .133 \times \frac{4^2}{64.33} = .5 \times .133 \times \frac{16}{64.33} = .01653 \text{ foot.}$$

NOTE.—If angle is greater than 90° , head should be proportionately increased.

Bent or Angular Circular Pipes.

Coefficient for angle of bend = $.9457 \sin^2 \alpha + 2.047 \sin^4 \alpha$. Hence,

| | | | | | | | | | | |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| α | 10° | 20° | 30° | 40° | 45° | 50° | 55° | 60° | 65° | 70° |
| C | .046 | .139 | .364 | .74 | .984 | 1.26 | 1.556 | 1.861 | 2.158 | 2.43 |

and $\frac{v^2}{2g} \times C = h$. α representing half angle of bend.

ILLUSTRATION. — Assume $v = 4$ feet, and angle = 90° ; $\alpha = \frac{90^\circ}{2} = 45^\circ$.

Then $\frac{4^2}{64.33} \times .984 = .2447$ foot additional head required.

In Valve Gates or Slide Valves.

In Rectangular Pipes.

| | | | | | | | | | | |
|-----|----|-----|-----|-----|------|------|------|------|------|-----|
| r | 1 | .9 | .8 | .7 | .6 | .5 | .4 | .3 | .2 | .1 |
| C | .0 | .09 | .39 | .95 | 2.08 | 4.02 | 8.12 | 17.8 | 44.5 | 193 |

r = ratio of cross section.

In Cylindrical Pipes.

| | | | | | | | | |
|---------------|----|------|------|------|------|------|------|------|
| h | 0 | .125 | .25 | .375 | .5 | .625 | .75 | .875 |
| $\frac{r}{C}$ | 1 | .948 | .856 | .74 | .609 | .466 | .315 | .156 |
| C | .0 | .07 | .26 | .81 | 2.06 | 5.52 | 17 | 97.8 |

h = relative height of opening.

In a Throttle Valve. In Cylindrical Pipes.

| | | | | | | | | | | | |
|---------------|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| A | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 60° |
| $\frac{r}{C}$ | .913 | .826 | .741 | .658 | .577 | .5 | .426 | .357 | .293 | .234 | .134 |
| C | .24 | .52 | .9 | 1.54 | 2.51 | 3.91 | 6.22 | 10.8 | 18.7 | 32.6 | 118 |

A = angle of position.

In a Clack or Trap Valve.

| Angle of opening..... | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° | 70° |
|-----------------------|-----|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|-----|
| C | .90 | .62 | .42 | .30 | .20 | .14 | .9.5 | 6.6 | 4.6 | 3.2 | 2.3 | 1.4 |

In a Cook. In Cylindrical Pipes.

| A | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° | 70° |
|---|------|-----|------|------|------|------|------|------|------|------|-----|------|-----|-----|
| r | .926 | .85 | .772 | .692 | .613 | .535 | .458 | .385 | .315 | .25 | .19 | .137 | .08 | .04 |
| C | .05 | .29 | .75 | 1.56 | 3.1 | 5.47 | 9.68 | 17.3 | 31.2 | 52.6 | 106 | 206 | 406 | 806 |

In a Conical Valve. $\left(1.645 \frac{a}{a'} - 1\right)^2 = C$. a and a' = areas of pipe and opening.

In Imperfect Contractions. $\left(\frac{a}{c} - 1\right)^2 = C$. c = a factor, ranging from .624 for $\frac{a}{c} = .1$ to 1 for $\frac{a}{c} = 1$, being greater the greater the ratio.

ILLUSTRATION.—If a slide valve is set in a cylindrical pipe 3 ins. in diameter 500 feet in length, is opened to .375 of diameter of pipe (hence, .625 diameter closed) what volume of water will it discharge under a head of 100 feet, coefficient of contraction of pipe assumed at .5?

$$C, \text{ by table, p. 545, pipe being .625 closed} = 5.52. \quad \frac{\sqrt{2g} \sqrt{h}}{\sqrt{\left(1.5 + C + 0.5 \frac{A \times n}{a}\right)}} = 1$$

C = from table, p. 544, for an assumed velocity of 11 feet 6 ins. = .0195.

$$\text{Then } \frac{\sqrt{64.33} \times \sqrt{100}}{\sqrt{\left(1.5 + 5.52 + .0195 \frac{500 \times 12}{3}\right)}} = \frac{8.03 \times 10.}{\sqrt{(7.02 + 39)}} = \frac{80.3}{6.78} = 11.85 \text{ feet.}$$

Hence, area of 3 ins. = 7.07, and $7.07 \times 12 \times 11.85 = 1005.4$ cube feet per second.

Valves. (Conical, Spherical, or Flap.)

Conical or Spherical Valve Puppet.

Height due to resistance or loss of head of water = $11 \frac{v^2}{2g}$. v represents velocity of water in full diameter of pipe or vessel.

$\left(\frac{A}{A'} - 1\right)^2 = C$. A and A' representing transverse areas of vessel and of opening, and $\left(1.645 \frac{A}{A'} - 1\right)^2 = C$ of contraction in general.

ILLUSTRATION.—If $A' = .5$ of vessel, $C = \left(1.645 \times \frac{1}{.5} - 1\right)^2 = 2.29^2 = 5.24$

Clack or Trap Valve.— C decreases with diameter of vessel.

ILLUSTRATION.—If a single-acting force-pump, 6 ins. in diameter, delivers at 5 stroke 5 cube feet of water in 4 seconds, diameter of valve seat 3.5 ins., and at 4.5; what resistance has water in its passage, and what is loss of mechanical energy?

$a = .196$. $\left(\frac{3.5}{6}\right)^2 = .34$ ratio of transverse area of opening. $1 - \left(\frac{4.5}{6}\right)^2 = .44$ ratio of annular contraction to transverse area of vessel.

Hence, $\frac{.34 + .44}{2} = .39$ mean ratio, and coefficient of resistance same thereto = $\left(\frac{1.645}{.39} - 1\right)^2 = 3.22^2 = 10.37$. $\frac{5}{4 \times .196} = 6.37$ velocity per

= .63 height due to velocity. Consequently, $10.37 \times .63 = 6.53$ height due to
 nce of valve, and $\frac{5}{4} \times 62.5 \times 6.53 = 510.15$ lbs. mechanical effect lost.

Discharge of Water in Pipes.

any Length and Head, and for Diameters from
 1 Inch to 10 Feet. In Cubic Feet per Minute. (Beardmore.)

| Tab. No. | Diam. | Tab. No. | Diam. | Tab. No. | Diam. | Tab. No. | Diam. | Tab. No. | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----|--------|
| | Ft. Ins. | | Ft. Ins. | | Ft. Ins. | | Ft. Ins. | | | |
| 4.71 | 9 | 1147.6 | 1 | 11983 | 3 | 1 | 39329 | 4 | 9 | 115854 |
| 8.48 | 10 | 1493.5 | 2 | 13328 | 3 | 2 | 42040 | 5 | 1 | 131703 |
| 13.02 | 11 | 1894.9 | 2 | 14758 | 3 | 3 | 44803 | 5 | 3 | 148791 |
| 19.15 | 1 | 2356 | 2 | 16278 | 3 | 4 | 47794 | 5 | 6 | 167132 |
| 26.69 | 1 | 2876.7 | 2 | 17889 | 3 | 5 | 50835 | 5 | 9 | 186786 |
| 46.67 | 1 | 3463.3 | 2 | 19592 | 3 | 6 | 53995 | 6 | 12 | 207754 |
| 73.5 | 1 | 4115.9 | 2 | 21390 | 3 | 7 | 57265 | 6 | 15 | 233781 |
| 108.14 | 1 | 4836.9 | 2 | 23282 | 3 | 8 | 60648 | 7 | 18 | 265437 |
| 151.02 | 1 | 5628.5 | 2 | 25270 | 3 | 9 | 64156 | 7 | 21 | 302935 |
| 194.84 | 1 | 6493.1 | 2 | 27358 | 3 | 10 | 67782 | 8 | 24 | 345431 |
| 263.87 | 1 | 7433 | 2 | 29547 | 3 | 11 | 71526 | 8 | 27 | 393275 |
| 416.54 | 1 | 8449 | 2 | 31834 | 4 | 1 | 75392 | 9 | 30 | 446508 |
| 612.32 | 1 | 9544 | 2 | 34228 | 4 | 2 | 79370 | 9 | 33 | 505369 |
| 854.99 | 1 | 10722 | 3 | 36725 | 4 | 3 | 83407 | 10 | 36 | 570333 |

s Table is applicable to Sewers and Drains by taking same proportion
 ular numbers that area of cross-section of water in sewer or drain
 to whole area of sewer or drain.

mula upon which the table is constructed is, $2356 \sqrt{\frac{h}{l}} \times d^5 = V$ in
 et per minute, and $39.27 \sqrt{\frac{h}{l}} \times d^5 = V$ in cubic feet per second. h represents
 ight of fall of water and d diameter of pipe and l length, all in feet.

To Compute Discharge.

elwein.) $\sqrt{\frac{d^5 h}{l}} 4.71 = V$, and $\sqrt{\frac{h V^2}{d^5}} .538 = d$. d = diameter of pipe in
 length of pipe and h head of water, both in feet.

wsley.) $\sqrt[5]{\frac{d^5 l}{h}} \frac{1}{15} = d$, and $\sqrt[5]{\frac{15 d^5 h}{l}} = G$. G = number of Imperial
 s per hour, and l length of pipe in yards.

ille.) $140 \sqrt{r s} - 11 \sqrt[3]{r s} = v$ in feet per second. r = hydraulic mean dia-
 and s sine of the inclination or total fall divided by total length.
 .124 $d^2 = V$, and $v 233.786 d^2 =$ Imperial gallons per minute. d = diam-
 e in feet.

To Compute Volume discharged.

ien Length of Pipe, Height or Fall, and Diameter.
 vide tabular number, opposite to diameter
 of inclination, and quotient will give volume
 te.

AMPLE.—A pipe has a diameter of 9 ins., and a head of 27.5 feet.
 charge per minute under a head of 27.5 feet?

Tab. No. 9 ins. = 1147.6, and $\frac{1147.6}{\sqrt{27.5}} = 214.7$ = discharge per minute.

To Compute Diameter.

When Length, Head, and Volume are given. **RULE.**—Multiply discharge per minute by square root of ratio of inclination; take nearest corresponding number in Table, and opposite to it is diameter required.

EXAMPLE.—Take elements of preceding case.

$$69.67 \times \sqrt{\frac{4750}{17.5}} = 1147.61, \text{ and opposite to this is } 9 \text{ ins.}$$

Or, $\sqrt{\frac{v l}{1542 h}} = d \text{ in feet. } v \text{ representing velocity in feet per second and } l \text{ length in feet.}$

To Compute Head.

When Length, Discharge, and Diameter are given. **RULE.**—Divide tabular number for diameter by discharge per minute, square quotient, and divide length of pipe by it; quotient will give head necessary to force given volume of water through pipe in one minute.

EXAMPLE.—Take elements of preceding cases.

$$\frac{1147.61}{69.67} = 16.47; 16.47^2 = 271.3; 4750 \div 271.3 = 17.5 \text{ feet.}$$

To Compute whole Head necessary to furnish requisite Discharge.

See Formula and Illustration, page 544.

To Compute Velocity.

When Volume and Diameter alone are given. **RULE.**—Divide volume when in feet per minute by area in feet, and quotient, divided by 60, will give velocity in feet per second.

EXAMPLE.—Take elements of preceding case.

$$\frac{69.67}{.75^2 \times .7854} \div 60 = 2.63 \text{ feet.}$$

When Volume is not given. **RULE.**—Multiply square root of product of height of pipe by diameter in feet, divided by length in feet, by 50, and product will give velocity in feet per second. (*Beardmore.*)

To Compute Inclination of a Pipe.

When Volume, Diameter, and Length are given. $\left(\frac{V}{2356}\right)^2 \frac{1}{d^5} = \frac{h}{l}$

ILLUSTRATION.—Take elements of preceding case.

$$\left(\frac{69.67}{2356}\right)^2 \times \frac{1}{.75^5} = .000874 \times 4.214 = .00368, \text{ and } \frac{17.5}{4750} = .00368, \text{ or } 4750 \times .00368 = 17.49 \text{ feet head.}$$

To Compute Elements of Long Pipes.

$$\frac{r}{x d^2} = 1.2732 \frac{V}{d^2} = v; \quad \left(1 + c + C \frac{l}{d}\right) \frac{v^2}{2g} = h; \quad \frac{\sqrt{2gh}}{\sqrt{1 + c + C \frac{l}{d}}} = v$$

$$(1.505 \times d + c l) \frac{V^2}{h} = d \text{ in ins.}$$

Or, only give an approximate dimension in consequence of

$$C, \text{ as } v = \frac{4V}{3.1416 \times d^2}$$

see Illustration, page 556.

To Compute Vertical Height of a Stream projected from Pipe of a Fire-engine or Pump.

RULE.—Ascertain velocity of stream by computing volume of water running or forced through opening in a second; then, by Rule in Gravitation, page 488, ascertain height to which stream would be elevated if wholly unobstructed, which multiply by a coefficient for particular case.

In great heights and with small apertures, coefficients should be reduced. In consequence of the varying elements and conditions of operation of fire-engines, it is difficult to assign a coefficient for them. Difference between actual discharge and that as computed by capacity and stroke of cylinder, as ascertained by Mr. Larned, 1859, was 18 per cent. = a coefficient of .82.

A steam fire-engine of the Portland Company, discharging a stream 1.125 ins. in diameter, through 100 feet 2.5 inch hose, gave a theoretical head, computed from actual discharge, of 225 feet, and stream vertically projected was 200 feet; hence coefficient in this case was .88.

EXAMPLE.—If a fire-engine discharges 14 cube feet of water vertically through a pipe .75 inch in diameter in one minute, how high will the water be projected?

$14 \times 1728 \div .4417$ area of pipe, $\div 12$ ins. in a foot, $\div 60$ seconds = 76.07 feet velocity; and as coefficient of such a stream = at .85, then $114.1 \times .85 = 96.98$ feet.

Or, $H = \frac{.0022 H^2}{d} = h$. H representing head at nozzle, and d height of jet, both in feet, and d diameter of nozzle in ins. (R. F. Hartford.)

ILLUSTRATION.—Assume head of 110 feet and diameter of nozzle .75 inch.

$$110 - \frac{.0022 \times 110^2}{.75} = 110 - 35.5 = 74.5 \text{ feet.}$$

NOTE.—The loss of head is greater with ring than with smooth nozzles. E. B. Weston, Am. Soc. C. E., puts the difference at .000171 v^2 .

The loss of head increases with the absolute height of the jet, and is less with an increase of its diameter. This loss increases nearly in ratio of square of height of jet, and varies nearly in inverse ratio to its diameter.

Cylindrical Ajutage.

Mean coefficient as determined by Mariotte and Bossut = .003066 square effective head for cylindrical ajutages; hence, for conical, alike to that of an engine pipe, coefficient ranges from .72 to .9, or a mean of .81.

By formula of D'Aubuisson, .003047 $h^2 = h'$.

Effective head, or h , in preceding example = 114.1. Then $114.1 - .003047 \times 4.1^2 = 114.1 - 39.67 = 74.43$ feet height of jet.

Hence, for a conical or engine pipe, $74.43 \times .81 = 60.29$ feet, or a coefficient of .535.

To Compute Distance a Jet of Water will be projected from a Vessel through an Opening in its Side.

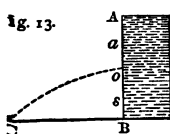


Fig. 13.

BC, Fig. 13, is equal to twice square root of $A \times o \times B$.

If s is 4 times as deep below A as a is, s will discharge twice volume of water that will flow from a in same time, as 2 is $\sqrt{4}$ of $A s$ and 1 is $\sqrt{1}$ of $A a$.

NOTE.—Water will spout farthest when o is equidistant from A and B ; and if vessel is raised above a plane, B must be taken upon plane.

Volumes of water passing through equal apertures in same time are as square roots of their depths from surface.

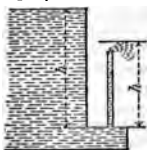
RULE.—Multiply square root of product of distance of opening from surface of water, and its height from plane upon which water falls, by product will give distance in feet.

EXAMPLE.—A vessel 20 feet deep is raised 5 feet above a plane, what is 5 feet from bottom of vessel?

$$20 - 5 \times 5 + 5 = 150, \text{ and } \sqrt{150} \times 2 = 24.41$$

Velocity of a jet of water flowing from a cylindrical tube is determined to be .974 to .98 of actual to theoretic velocity, or $\approx .82$ of that due to height of reservoir. Hence volume of discharge through a cylindrical opening $\approx .82 a \sqrt{2gh}$.

Fig. 14.



Jets d'Eau. (Fig. 14.)

That a jet may ascend to greatest practicable height, communication with supply should be perfectly free.

Short tubes shaped alike to contracted fluid vein, and conically convergent pipes, are those which give greatest velocities of efflux. Hence, to attain greatest effect, as in fire-engines, long and slightly conically convergent tubes or pipes should be applied.

In order to diminish resistance of descending water, jet must be directed with a slight inclination from vertical.

Effect of combined causes which diminish height of a jet from that due to elevation of its supply can only be determined by experiments. Great jets rise higher than small ones.

With cylindrical tubes, velocity being reduced in ratio of 1 to .82, and as heights of jets are as squares of these coefficients or ratios, or as 1 to .67, height of a jet through a cylindrical tube is two thirds that of head of water from which it flows.

$H C = h$. H representing head of water, C coefficient, and h height of jet. (Molworth.)

| |
|---------------------------------------|
| When $d = H \div 300$, $C = .96$. |
| " " " $\div 450$, " $\approx .93$. |
| " " " $\div 600$, " $\approx .9$. |
| " " " $\div 800$, " $\approx .87$. |
| " " " $\div 1000$, " $\approx .85$. |

| |
|---------------------------------------|
| When $d = H \div 1500$, $C = .8$. |
| " " " $\div 1800$, " $\approx .7$. |
| " " " $\div 2800$, " $\approx .6$. |
| " " " $\div 3500$, " $\approx .5$. |
| " " " $\div 4500$, " $\approx .25$. |

FLOW OF WATER IN RIVERS, CANALS, AND STREAMS.

Running Water.—Water flows either in a natural or artificial bed or course. In first case it forms Streams, Brooks, and Rivers; in second, Drains, Cuts, and Canals.

Bed of a water-course is formed of a *Bottom* and two *Banks* or *Shores*.

Transverse Section is a vertical plane at right angles to course of the flowing water; **Perimeter** is length of this section in its bed.

Longitudinal Section or **Profile** is a vertical plane in the course or *thread* of current of flowing water.

Slope or **Declivity** is the mean angle of inclination of surface of the water to the horizon.

Fall is vertical distance of the two extreme points of a defined length of flowing course, measured upon a horizontal plane, and this fall assigns to defined length of the course.

Line or **Thread of Current** is the point where flowing water attains its minimum velocity.

Channel is deepest point of the bed in thread of current. **Velocity** is at surface and in middle of current; and surface of flowing water is in current, and lowest at banks or shore.

Velocity, **Current**, **is** in a state of permanency when an equal quantity of its transverse sections in an equal time of section, and mean velocity through whole extent of number.

To Compute Mean Depth of Flowing Water.

RULE.—Set off breadth of the stream, etc., into any convenient number of divisions; ascertain mean depths of these divisions; then divide their sum by number of divisions, and quotient is the mean depth.

To Compute Mean Area of Flowing Water.

RULE 1.—Multiply breadth or breadths of the stream, etc., by the mean depth or depths, and product is the area.

2.—Divide the volume flowing in cube feet per second by mean velocity in feet per second, and quotient is area in sq. feet.

To Compute Volume of Flowing Water.

RULE.—Multiply area of the stream, etc., in sq. feet, by the mean velocity of its flow in feet, and product is volume in cube feet.

To Compute Mean Velocity of Flowing Water.

RULE.—Divide surface velocity of flow in feet per second by area of the stream, etc., and quotient, multiplied by coefficient of velocity, will give mean velocity in feet.

Mean velocity at half depth of a stream has been ascertained to be as .915 to 1, and at bottom of it as .83 to 1, compared with velocity at surface. Again, the velocity diminishes from line of current toward banks, and, to obtain mean superficial velocity,

$$\frac{v_1 + v_2 + v_3}{n} = .915 v; \text{ hence,}$$

To Compute Mean Velocity in whole Profile of a Navigable River, etc.,

$\sqrt{V+1} - 2\sqrt{V} = \text{velocity at bottom, and } \sqrt{V+.5} - \sqrt{V} = \text{mean velocity.}$

In rivers of low velocities multiply mean velocity by .8.

Obstruction in Rivers. (Molesworth.)

$\frac{v_2}{8.6} + .05 \times \left(\frac{A}{a}\right)^2 - 1 = R.$ *v* representing velocity in ins. per second previous to obstruction, *A* and *a* areas of river unobstructed and at obstruction in sq. feet, and *R* rise in feet.

ILLUSTRATION.—Velocity of obstructed flow of a river is 6 feet per second, and area of section before and after obstruction are 100 and 90 sq. feet; what would be *R* in feet?

$$\frac{6^2}{58.6} + .05 \times \left(\frac{100}{90}\right)^2 - 1 = .664 \times .234 = .155 \text{ feet.}$$

Flow of Water in Lined Channels. (Bazin.)

$\sqrt{\frac{C D}{F}} = V;$ $\frac{1}{x \left(y + \frac{1}{D}\right)} = C.$ *D* representing mean hydraulic depth in feet, *F* fall, or length of channel to fall of 1, *x* and *y* as per table, and *C* as per table p. 543.

| | x | y | | x | y |
|-------------|----------|-------|---------------------|--------|-------|
| Stoned..... | .0000045 | 10.16 | Rubble Masonry..... | .00006 | 1.219 |
| Stone..... | .000013 | 4.354 | Earth..... | .00035 | .214 |

For Sections of Uniform Area, as Canals, Sewers, etc. $\sqrt{\frac{A}{P}} 2 D = v.$ *A* = area of flow in sq. feet, *P* wet perimeter of section, and *D* fall of stream per mile feet.

ILLUSTRATION.—Area of transverse section of a sewer is 50 sq. feet, its wet perimeter 20 feet, and its fall 5 feet per mile.

$\sqrt{\left(\frac{50}{20} \times 2 \times 5\right)} = \sqrt{25} = 5 \text{ feet.}$ **For Sections of Rivers.** $12 \sqrt{\frac{A}{P}} = v.$

ILLUSTRATION.—Assume area 500 sq. feet, wet perimeter 200, and fall 5 feet.

$$12 \sqrt{5 \times \frac{500}{200}} = 12 \sqrt{12.5} = 42.4 \text{ feet.}$$

Hydraulic Radius or Mean Depth is obtained by dividing area of transverse section by wet perimeter, both in feet.

To Compute Fall per Mile for a required Mean Velocity.

$$\left(\frac{v \times 12}{12}\right)^2 \div 2 r = D. \quad r \text{ representing hydraulic radius in feet.}$$

Upper surface of flowing water is not exactly horizontal, as water at its surface flows with different velocities with respect to each other, and consequently on each other different pressures.

If v and v_1 are velocities at line of current and bank of a stream, the difference of the two levels is $\frac{v^2 - v_1^2}{2g} = h$.

ILLUSTRATION.—If $v = 5$ feet, and $v_1 = .9$ v ; then $\frac{5^2 - .9 \times 5}{2g} = \frac{4.75}{64.33} = .0738$ feet.

A velocity of 7 to 8 ins. per second is necessary to prevent deposit of slime and growth of grass, and 15 ins. is necessary to prevent deposit of sand.

Maximum velocity of water in a canal should depend on character of bed of the channel.

Thus, *Mean Velocity* should not exceed per second over

| | | | | | |
|------------------|--------|--------------------|-------|--------------------|-------|
| Fine clay..... | 6 ins. | River sand..... | 1 ft. | Broken stones..... | 1 ft. |
| A slimy bed..... | 8 " | Small gravel..... | 1 " | Stones..... | 1 ft. |
| Common clay..... | 6 " | Large shingle..... | 3 " | Loose rocks..... | 1 ft. |

To Compute Velocity of Flow or Discharge of Water in Streams, Pipes, Canals, etc.

1. *When Volume discharged per Minute is given in Cube Feet, and Area of Canal, etc., in Sq. Feet.* RULE.—Divide volume by area, and quotient divided by 60, will give velocity in feet per second.

2. *When Volume is given in Cube Feet, and Area in Sq. Ins.* RULE.—Divide volume by area; multiply quotient by 144, and divide product by 60.

3. *When Volume is given in Cube Ins., and Area in Sq. Ins.* RULE.—Divide volume by area, and again by 12 and by 60.

To Compute Flow or Volume of Discharge.

1. *When Area is given in Sq. Feet.* RULE.—Multiply area of flow by its velocity in feet per second, and product, multiplied by 60, will give volume in cube feet per minute.

2. *When Area is given in Sq. Ins.* RULE.—Multiply area by its velocity, and again by 60, and divide product by 144.

NOTE 1.—Velocities and discharges here deduced are theoretical, actual depending upon coefficient of efflux used. Mean velocity, however, as before, page 529, may be taken at $\sqrt{2g} \cdot .673 = 5.4$ feet, instead of 8.02 feet.

2.—As a rule, with large bodies, as vessels, etc., their floating velocity is somewhat greater than that of flow of water, not only because in floating they are on an inclined plane, formed by surface of the water, but because they are but slightly affected by the irregular intimate motion of water: the variation for small bodies is so slight that it may be neglected.

To Compute Height of Head of Flowing Water.

When Volume and Area of Flow are given in Feet. RULE.—Divide volume in feet per second by product of area, and $\frac{2}{3}$ coefficient for opening, square of quotient, divided by 64.33, will give height in feet.

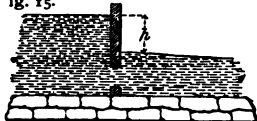
EXAMPLE.—Assume volume 266.48 cube feet, area 40 sq. feet, and $C = .673$.

$$\text{Then } \left(\frac{266.48}{40 \times \frac{2}{3} \cdot 673}\right)^2 \div 64.33 = \frac{257.28}{64.33} = 4 \text{ feet.}$$

Submerged or Drowned Orifices and Weirs.

When wholly submerged (Fig. 15).—Available pressure at any point in depth of orifice is equal to difference of pressure on each side.

Fig. 15.



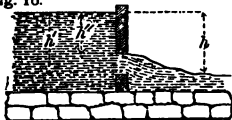
Whence, $C\sqrt{2gh} = v$, and $Ca\sqrt{2gh} = V$,
a representing area of sluice in sq. feet.

ILLUSTRATION.— Assume opening 3 feet by 5,
 $h = 4$ feet, and $C = .5$.

Then, $.5 \times 3 \times 5 \sqrt{64.33 \times 4} = 7.5 \times 16.04 = 120.3$ cube feet per second.

When partly submerged (Fig. 16). $h' - h = d =$ submerged depth, and $h - h'' = d' =$ remaining portion of depth; whence $d' + d =$ entire depth, and

Fig. 16.



$C\sqrt{2g} (d\sqrt{h} + \frac{2}{3}h\sqrt{h} - h''\sqrt{h''}) = V$.

ILLUSTRATION.— Assume opening as above, $h = 4$ feet, $h' = 6$, $h'' = 3$, and $C = .5$. Then $d = 6 - 4 = 2$ feet.

Then $.5 \times 5 \times 8.02 (2\sqrt{4} + \frac{2}{3} \times 6\sqrt{6} - 3\sqrt{3}) = 20.05 \times 5.869 = 117.67$ cube feet per second.

Fig. 17.



When drowned (Fig. 17).

$C\sqrt{2gh} (d + \frac{2}{3}h) = V$.

ILLUSTRATION.— Assume opening as above, $h = 4$ feet, $d = 2$, and $C = .52$.

Then, $.52 \times 5 \times \sqrt{64.33 \times 4} \times (2 + \frac{2}{3} \times 4) = 2.6 \times 16.04 \times 4.66 = 194.34$ cube feet per second.

CANAL LOCKS.**Single Locks.**

When a fluid passes from one level or reservoir to another, through an aperture covered by the fluid in the latter, effective head on each point of aperture, and consequently head due to velocity of efflux at each instant, is the difference of levels of the two reservoirs at that instant.

Hence $Ca\sqrt{2gh'} = V$ per second. h' representing difference of levels.

To Compute Time of Filling and Discharging a Single Lock.—Fig. 18.

When Sluice in Upper Gate is entirely under Water, and above Lower Level.

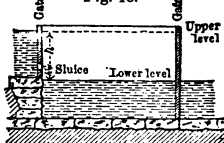
$\frac{Ah'}{Ca\sqrt{2gh}}$ = time of filling up to centre of sluice.

h representing height of centre of sluice in upper gate from surface of canal or reservoir, and h' height of centre of sluice in upper gate from lower surface, or water in the lock or river, all in feet; and

$\frac{2Ah}{2a\sqrt{2gh}}$ = time of filling the remaining space, where a gradual diminution of head of water occurs.

Consequently, $\frac{(h' + 2h)A}{Ca\sqrt{2gh}} = t$ time of filling a single lock.

Fig. 18.



When Aperture or Sluice in Lower Gate is entirely under Water Level. $\frac{2A\sqrt{h+h'}}{Ca\sqrt{2g}} =$ time of emptying or discharging it of lower sluice.

ILLUSTRATION.—Mean dimensions of a lock, Fig. 18, are 200 feet in length by 11 in breadth; height of centre of aperture of sluice from upper and lower surfaces 5 feet; breadth of both upper and lower sluices is 2.5 feet; height of upper is 4 feet, and of lower—entirely under water—5 feet; required the times of filling and discharging.

$h = 5, h' = 5, A = 200 \times 24 = 4800, C = .545, a = 4 \times 2.5 = 10, a' = 5 \times 2.5 = 12.5$
 $\frac{4800 \times 5}{.545 \times 10 \times \sqrt{2g}} = \frac{24000}{97.72} = 245.59 \text{ seconds} = \text{time of filling lock up to centre of sluice; and}$
 $\frac{2 \times 4800 \times 5}{.545 \times 10 \times \sqrt{2g}} = \frac{48000}{97.72} = 491.18 \text{ seconds} = \text{time of filling remaining space, or lock above centre of sluice, and } 245.59 + 491.18 = 736.77 \text{ seconds, whole time.}$

Or, $\frac{(5 + 2 \times 5) \times 4800}{.545 \times 10 \times \sqrt{2g}} = \frac{72000}{97.72} = 736.77 \text{ sec.} = \text{time of filling.}$ $\frac{2 \times 4800 \sqrt{5+3}}{.545 \times 12.5 \times \sqrt{11}} = \frac{30358.08}{54.7} = 554.9 \text{ seconds} = \text{time of discharging.}$

When Aperture or Sluice in Upper Gate is entirely under Water and below Lower Level. $\frac{2A\sqrt{h-h'}}{Ca\sqrt{2g}} = \text{time of filling lock.}$

When Sluice in the Lower Gate is in part above Surface of Lower Level and in part below it. $\frac{2A(h+h')}{Cb\sqrt{2g} \left(d\sqrt{h+h'} - \frac{d}{2} + d'\sqrt{h+h'} \right)} = \text{time of discharging.}$

d and d' representing distances of part of aperture above and of below surface of lower water, b breadth of aperture, and h and h' as before.

ILLUSTRATION.—Assume sluice in preceding example to be 1 foot above lower level of water, or that of lower canal; what is time of discharge of lock, distance of part of aperture 1 foot and of that below surface of water 4 feet?

$\frac{2 \times 4800 (5+5)}{.545 \times 2.5 \times 8.02 [1 \times \sqrt{5+5} - (1 \div 2) + 4 \times \sqrt{5+5}]} = \frac{96000}{10.93 \times [3.082 + 12.65]} = \frac{96000}{171.95} = 558.3 \text{ seconds.}$

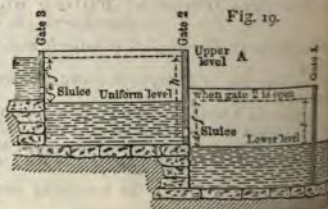
Double Lock. (J. D. Van Buren, Jr.)

A double lock is not a duplication of a single lock in its operation, for in lower chamber supply of water is from upper one, having no influx, instead of a uniform supply flowing directly from surface level of canal or feeder.

Operation, therefore, of a double lock is complex, addition to formula for a single lock being that of discharging of water in upper lock to fill lower, the head of water gradually decreasing in the chamber, which is closed from upper reach during discharge into lower.

To Compute Time required for Water to Fall from Upper to Uniform Water Level.

1. $\frac{A}{Ca\sqrt{g}} (\sqrt{f} + \sqrt{2h} - \sqrt{2h-2d}) = t.$ A representing horizontal area and a area of sluice opening, both in sq. feet, C coefficient of discharge = openings with square arrises, g acceleration of gravity, f depth of centre



below uniform level, h depth of centre sluice opening below upper water level, and height of centre of sluice above lower water level, all in feet, and t time for water to fall from upper to uniform water level, in seconds.

ILLUSTRATION.— $A=2000$ sq. feet; $C=.545$; $a=5$; $f=6$; $h=14$; and $d=2$ feet. (Fig. 19.)

$$\text{Then, } \frac{2000}{.545 \times 5 \times 5.67} = \frac{2000}{15.45} \times 7.74 - 4.9 = 367.6 \text{ seconds.}$$

$$2. \text{ If } d=0; \frac{A\sqrt{f}}{Ca\sqrt{g}} = t; = \frac{2000 \times \sqrt{8}}{.545 \times 5 \times 5.67} = \frac{5660}{15.45} = 366.34 \text{ seconds.}$$

NOTE.— f is never greater than l (lift in feet); it is equal to l when $d=0$; f_2 equal to l when $f_1=0$, never greater. In each case it is the unbalanced head above sluice, however far below the lowest water level the sluice is.

To Fill Upper Lock or Empty Lower.

To fill upper lock or empty lower, when the sluice is below the lowest water-line in either case, takes the same time; for the head diminishes at the same rate, or from the upper surface, the other from the bottom.

$$3. \frac{A\sqrt{2f}}{Ca\sqrt{g}} = t. \text{ Here, } f \text{ being below lowest water level of lock} = 8 \text{ feet, as } d=$$

$$\text{and } f = \text{whole lift} = \frac{2000 \sqrt{2 \times 8}}{.545 \times 5 \times 5.67} = \frac{8000}{15.45} = 517.8 \text{ seconds.}$$

To Discharge a like Volume under a Constant Head.

$$4. \frac{A\sqrt{f}}{Ca\sqrt{2g}} = \frac{A}{Ca} \sqrt{\frac{f}{2g}} = t. = \frac{2000}{.545 \times 5} \sqrt{\frac{8}{64.33}} = 258.9 \text{ seconds,}$$

Or, one half the time given by preceding case.

The times deduced by preceding formulas are in the following proportions :

$$\text{order, as } 1 : \sqrt{2} : \frac{\sqrt{2}}{2}, \text{ or } 1 : \sqrt{2} : \frac{1}{\sqrt{2}}.$$

If sluice of upper lock, through which it is filled, is above lowest water level then, by combining formulas 3 and 4, the time is thus deduced.

To fill from Lowest Water Level of said Lock to Level of Centre of Sluice

$$5. \frac{A\sqrt{f'}}{Ca\sqrt{2g}} = t'. \text{ } f' \text{ representing height of centre of sluice above said lowest water level.}$$

To fill remaining Portion of Lock above Sluice.

$$6. \frac{2A\sqrt{f''}}{Ca\sqrt{2g}} = t''. \text{ } f'' \text{ representing depth below upper water level of centre of}$$

$$\text{sluice or remaining portion of lift. Hence, } t' + t'' = \frac{A}{Ca\sqrt{2g}} (\sqrt{f'} + 2\sqrt{f''}) =$$

To fill Lower Lock under Constant Head from Upper Canal Level.

$$7. \frac{A\sqrt{h}}{Ca\sqrt{2g}} \left(2 + \frac{d}{h} - \frac{2\sqrt{h-f}}{\sqrt{h}} \right) = t.$$

$$8. \text{ If both lifts are the same, } h-f=l, \text{ and } \frac{A\sqrt{h}}{Ca\sqrt{2g}} \left(2 + \frac{d}{h} - 2\sqrt{\frac{l}{h}} \right)$$

If lower lock is filled from upper one under a constant head, when h is down to lowest level, formula 7 will apply by making $h=f$, and

$$\frac{A}{Ca\sqrt{2g}} \left(2\sqrt{f} + \frac{d}{\sqrt{f}} \right), \text{ which is identical with 7, for } f=f_2 \text{ and } d=.$$

being the same.

What volume will a pipe 48 feet in length and 2 ins. in diameter, under a head of 5 feet, deliver per second? (Formula page 547.)

Formula number for diameter 2 ins., page 547, = 26.69.

$$\sqrt{\frac{48}{5}} = 3.1. \text{ Then } \frac{26.69}{3.1} = 8.61, \text{ which } \div 60 = .143 \text{ cube feet.}$$

This pipe had 5 curves of 90°, with radii $\frac{d}{2r} = \frac{2}{4} = .5$; what would be its discharge per second?

$$= .143; a = 2 \div 144 = .0139; C \text{ per table} = \frac{d}{2r} = .294; v = \frac{.143}{.0139} = 10.29 \text{ feet.}$$

$$\text{en } .294 \times \frac{90^\circ}{180^\circ} \times \frac{10.29^2}{64.33} = .147 \times 1.64 = .241, \text{ which } \times 5 \text{ for 5 curves} = 1.2 = t \text{ due to resistance of curves. } h = 5 - 1.2 = 3.8.$$

$$\text{now, if } \sqrt{2g} \, h = .143; \sqrt{2g} \, 3.8 = .125 \text{ cube feet.}$$

If a slide stop valve, set in a cylindrical conduit 500 feet in length and 3 ins. in diameter, is raised so as to close .625 of conduit; what volume will it discharge under a head of 4 feet? (Formula page 546.)

For conduit = .5, for friction .025, and for slide valve .375 open, table, page 545, d = .25, and a = 7.07 sq. ins.

$$\text{en } \frac{2g \, h}{\sqrt{\left(1 + .5 + 5.52 + .025 \frac{500}{.25}\right)}} = \frac{16.06}{\sqrt{(7.02 + 50)}} = 2.13 \text{ feet velocity, and } \times 12 \times 7.07 = 180.71 \text{ cube ins.}$$

If a single lock chamber is 200 feet in length by 24 in breadth, with a depth of 5 feet, centre of upper gate, which is 4 feet in depth by 2.5 in breadth, is at the middle of depth of chamber, lower gate, 5 feet in depth by 2.5 in breadth and wholly closed; what is time required for filling and discharging it? (Formula p. 553.)

$$= .615, h = 5, h' = 5, A = 200 \times 24 = 4800, a = 4 \times 2.5 = 10, \text{ and } a' = 5$$

$$5 = 12.5$$

$$\frac{(2 \times 5 + 5) 4800}{.615 \times 10 \sqrt{64.33} \times 5} = \frac{72000}{110.27} = 652.8 \text{ seconds time of filling.}$$

$$\frac{2 \times 4800 \times \sqrt{5 + 5}}{.615 \times 12.5 \sqrt{2g}} = \frac{30336}{61.73} = 491.4 \text{ seconds time of emptying.}$$

In a moderately direct and uniform course of a river, the depths and velocities as follows; what is the volume of its flow and what its mean velocity? (p. 551.)

| | Feet. | Feet. | Feet. | Feet. | Feet. | |
|---|-------|-------|-------|-------|-------|---|
| nces..... | 5 | 12 | 20 | 15 | 7 | Area of profiles = $5 \times 3 + 12 \times 6 + 20 \times 11 + 15 \times 8 + 7 \times 4 = 455 \text{ sq. feet.}$ |
| hs..... | 3 | 6 | 11 | 8 | 4 | |
| velocity..... | 1.9 | 2.3 | 2.8 | 2.4 | 2.1 | |
| $\times 1.9 + 72 \times 2.3 + 220 \times 2.8 + 120 \times 2.4 + 28 \times 2.1 = 1156.9 \text{ cube feet volume,}$ | | | | | | |
| $\frac{1156.9}{455} = 2.54 \text{ feet velocity.}$ | | | | | | |

Miner's Inch.

"Miner's inch" is a measure for flow of water, and is an opening one square through a plank two inches in thickness, under a head of six inches of water to upper edge of opening.

will discharge 11.625 U. S. gallons water in one minute

Theoretical HP under different Heads.

| in feet. | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 1 |
|----------|------|------|------|------|------|-----|------|------|------|---|
| HP... | 3.25 | 3.61 | 4.06 | 4.64 | 5.41 | 6.5 | 8.12 | 10.8 | 16.2 | 2 |

Miner's Inch (Pouce d'eau).—Circular opening of 1 inch in diameter discharges 19.1953 cube meters per 24 hours.

HYDRODYNAMICS.

Hydrodynamics treats of the force of action of Liquids on Fluids, and it embraces *Hydraulics* and *Hydrostatics*: the latter treats of liquids in motion, as flow of water in pipes and of pressure, weight, and equilibrium of liquids in a state of rest.

Fluids are of two kinds, aeriform and liquid, or elastic and inelastic, and they press equally in all directions, and any pressure communicated to a fluid at rest is equally transmitted throughout the whole.

Pressure of a fluid at any depth is as depth or vertical height. Pressure upon bottom of a containing vessel is as base and perpendicular height, whatever may be the figure of vessel. Pressure of a fluid, upon any surface, whether *Vertical*, *Oblique*, or *Horizontal*, is equal to weight of a column of the fluid, base of which is equal to area of surface pressed, and height equal to distance of centre of gravity of surface pressed, below surface of the fluid.

Side of any vessel sustains a pressure equal to its area, multiplied by half depth of fluid, and whole pressure upon bottom and against sides of a cubical vessel is equal to three times weight of fluid.

Pressure upon a number of surfaces is ascertained by multiplying sum of surfaces into depth of their common centre of gravity below surface of fluid.

When a body is partly or wholly immersed in a fluid, vertical pressure of the fluid tends to raise the body with a force equal to weight of fluid displaced; hence weight of any quantity of a fluid displaced by a buoyant body equals weight of that body.

Centre of Pressure is that point of a surface against which a fluid presses, to which, if a force equal to whole pressure were applied, the surface would be at rest. Hence distance of centre of pressure from surface of fluid is same as *Centre of Pressure* of a solid of the same figure.

Centres of Pressure.

Centre of Pressure of a Triangle, Side, Base, Tangent, or Vertex of Figure at Surface of Fluid, is that point (determined by the intersection of two lines, one running downward) that joins centres of two horizontal surfaces of the figure, the uppermost, is at centre of a line raised from lower apex to the centre of base; and *Vertex uppermost*, it is at .75 of a line joining it with centre of base.

Centre of Pressure of a Triangle, Base uppermost, is at intersection of a line extended from apex to extremity of triangle by a line running horizontally from the centre of the base to the vertex, by a line running horizontally from the apex to the vertex, measured from base.

Centre of Pressure of a Triangle, either of parallel Sides at Surface, $\frac{b+3b'}{2b+4b'} \times a = d$. b and b' are the widths of figure, d distance from surface of fluid, and a a length of the triangle.

Centre of Pressure of a Circle, .45 of its radius, measured from upper edge.

Centre of Pressure of a Hemisphere, $\frac{3pr}{16} = d$. r representing radius.

Centre of Pressure of a Paraboloid, $\frac{r-32r}{p-16} = d$.

3, Base, or Tangent of Figure below Surface of Fluid.

angle or Parallelog'm. $\frac{2}{3} \times \frac{h'^3 - h^3}{h'^2 - h^2} = d$; or, $\frac{3m o + m^2}{3 o} = d$; and $\frac{m^2}{3 o} = d''$.

d h' representing depths of upper and lower surfaces of figure and d depth, o surface of fluid, m half depth of figure, o depth of centre of gravity of figure from surface of fluid, d' distance from upper side of figure, and d'' distance from centre of gravity.

ngle. — Vertex Uppermost. $\frac{l^2 + 18 o^2}{18 o} = d$; $\frac{l^2}{18 o} = d'$. Base Uppermost.

$o^2 = d$. l representing depth of figure, d distance from surface of fluid upon from vertex to centre of base, and d' distance from centre of gravity of figure.

e. $\frac{4 o^2 + r^2}{4 o} = d$, or $\frac{r^2}{4 o} = \text{distance from centre of circle}$.

circle. — Diam. Horizontal and Upward or Downward. $\frac{l^2}{4 o} - \frac{16 l^2}{9 p o} + o = d$;

$\frac{4 l}{3 p} = d'$; $\frac{4 l}{3 p} = d''$, and $\frac{l^2}{4 o} - \frac{16 l^2}{9 p o} = c$. d representing distance from of fluid, d' distance of centre of gravity from centre of arc, d'' distance of gravity from diameter when it is uppermost, and c centre of pressure.

Pressure.

Compute Pressure of a Fluid upon Bottom of its Containing Vessel.

E. — Multiply area of base by height of fluid in feet, and product by of a cube foot of fluid.

Compute Pressure of a Fluid upon a Vertical, Inclined, Curved, or any Surface.

E. — Multiply area of surface by height of centre of gravity of fluid, and product by weight of a cube foot of fluid.

EXAMPLE 1. — What is pressure upon a sloping side of a pond of fresh water 10 feet and 8 feet in depth?

of gravity, $8 \div 2 = 4$ feet from surface. Then $10^2 \times 4 \times 62.5 = 25000$ lbs.

What is pressure upon staves of a cylindrical reservoir when filled with fresh water being 6 feet, and diameter of base 5 feet?

$.1416 = 15.708$ feet curved surface of reservoir, which is considered as a plane.

$15.708 \times 6 \times 6 \div 2 = 282.744$, which $\times 62.5 = 17671.5$ lbs.

A rectangular flood-gate in fresh water is 25 feet in length by 12 feet deep; pressure upon it?

$25 \times 12 \times 12 \div 2 = 1800$, which $\times 62.5 = 112500$ lbs.

When water presses against both sides of a plane surface, there arises from it forces, corresponding to the two sides, a new resultant, which is found by subtraction of former, as they are opposed to each other.

EXAMPLE 2. — Depth of water in a canal is 7 feet; in its adjoining lock it is 4 feet; breadth of gates is 15 feet; what mean pressure have they to sustain, and depth of point of its application below surface?

$5 = 105$, and $4 \times 15 = 60$ sq. feet. $(105 \times \frac{7}{2} - 60 \times 2) \times 62.5 = 1$;

pressure.

$15468.75 \div 62.5 = 247.5 = \text{cube feet pressing upon gates upon } h$

$247.5 \times 7 = 1732.5 \text{ feet} = \text{depth of centre of gravity of mean pressure}$

To Compute Pressure on a Sluice.

$= P$, and $CP = P'$. A representing area of sluice in sq. feet, h cube foot, d mean depth of sluice below surface, in feet, P pressure required to operate it, both in lbs.

$C = .68$ when sluice is of wood, and $.31$ when of iron.

EXAMPLE.—What is pressure on a sluice-gate 3 feet square, its centre of gravity being 30 feet below surface of a pond of fresh water?

$$3 \times 3 \times 30 = 270, \text{ which } \times 62.5 = 16875 \text{ lbs.}$$

To Compute Pressure of a Column of a Fluid per Sq. Inch.

RULE.—Multiply height of column in feet by weight of a cube foot fluid, and divide product by 144; quotient will give weight or pressure sq. inch in lbs.

NOTE.—When height is given in ins., omit division by 144.

PIPES.

To Compute required Thickness of a Pipe.

RULE.—Multiply pressure in lbs. per sq. inch by diameter of pipe in inches and divide product by twice assumed tensile resistance or *value* of a inch of material of which pipe is constructed.

By experiment, it has been found that a cast-iron pipe 15 ins. in diameter, .75 of an inch thick, will support a head of water of 600 feet; and that one of same diameter, and 2 ins. thick, will support a head of 180 feet?

EXAMPLE 1.—Pressure upon a cast-iron pipe 15 ins. in diameter is 300 lbs. per inch; what is required thickness of metal?

$$300 \times 15 = 4500, \text{ which } \div 3000 \times 2 = .75 \text{ inch.}$$

NOTE.—Here 3000 is taken as *value* of tensile strength of cast iron in ordinary small water-pipes. This is in consequence of liability of such castings to be imperfect from honey-combs, springing of core, etc.

2.—Pressure upon a lead pipe 1 inch in diameter is 150 lbs. per sq. inch; what required thickness of metal?

Here 500 is taken as *value* of tensile strength.

$$150 \times 1 = 150, \text{ which } \div 500 \times 2 = .15 \text{ inch.}$$

Cast-iron Pipes.

To Compute Thickness, etc., of Flanged Pipes.

For 75 lbs. Pressure.

$$\begin{aligned} .085 D + .25 &= T \\ D + .3 &= t \\ D + 1.15 &= l \\ D + .35 &= f \\ D + 4.25 \overline{d} + 1.25 &= o \\ D + 2 \times \overline{d} + 1 &= o' \end{aligned}$$

For 100 lbs. Pressure.

$$\begin{aligned} .03 D + .3 &= T \\ .035 D + .45 &= t \\ .05 D + 1.15 &= l \\ .04 D + .6 &= f \\ 1.1 D + 5 \times \overline{d} + 1.5 &= o \\ 1.1 D + 2.5 \times \overline{d} + 1.4 &= o' \end{aligned}$$

$$7 D + 2.2 = n; \quad \frac{A \times p \div n}{4000} = a, \text{ and } \sqrt{\frac{a}{.7854}} + C = d.$$

representing diam. of pipe, T thickness of metal, t thickness and l length of flange, o diam. of flange, o' diam. of centres at bolt holes, and d diam. all in ins.; A area of pipe and a area of bolt at base of its thread, in sq. ins. pressure in lbs. per sq. inch, and C a coefficient due to diam. of bolt.

Illustration.—What should be dimensions of a flanged pipe, 10 ins. in diameter, pressure of 100 lbs. per sq. inch?

$$10 \times 10 + 2.2 = 9.2 = 10 \text{ number of bolts, and diam. } 10 \text{ ins.} = 78.54 \text{ ins. area} =$$

$$.54 \times 10 = .54, \text{ and } \sqrt{\frac{.19635}{.7854}} + C = \sqrt{.25} = .5; \text{ hence, } .5 + .107 =$$

$$.03 \times 10 + .3 = .6 = \text{thickness of metal; } .035 \times 10 +$$

$$.05 \times 10 + 1.15 = 1.65 = \text{length of bolt; } 1.1 \times$$

$$10 + 5 \times .625 + 1.5 = 15.625 = \text{diam. of bolt hole}$$

$$1.4 = 1.4 = \text{diam. of bolt hole}$$

To Compute Elements of Water-pipes.

$14.5 P d + C = t$; or, $.000054 H d + C = t$; $.4336 H = P$; and $\times 2.45 = W$. P representing pressure of water in lbs. per sq. inch, D and d and internal diameters of pipe, and t thickness of metal, all in ins., C coefficient of pipe, and H head of water in feet.
 .7 for pipes less than 12 ins. in diameter, .5 from 12 to 30, and .6 from 30 to 50

To Compute Weight of Pipes.

Diameter add thickness of metal, multiply sum by 10 times thickness, duct will give weight in lbs. per foot of length.
 ght of Faucet end is equal to 8 ins. of length of pipe.

Hydrostatic Press.**o Compute Elements of a Hydrostatic Press.**

$= W$; $\frac{W l' a}{P l} = A$; $\frac{W l' a}{l A} = P$; $\frac{P A l}{W l'} = a$. P representing power or pressure, W weight or resistance in lbs., l and l' lengths of lever and fulcrum in feet, and A and a areas of ram and piston in sq. ins.

EXAMPLE.—Areas of a ram and piston are 86.6 and 1 sq. ins., lengths of lever arm 4 feet and 9 ins., and power applied 20 lbs.; what is weight that may be lifted?

$$\frac{20 \times 4 \times 12 \times 86.6}{9 \times 1} = \frac{83136}{9} = 9237.3 \text{ lbs.}$$

Compute Thickness of Metal to Resist a given Pressure.

EX.—Multiply pressure per sq. inch in lbs. by diameter of cylinder in ins. and divide product by twice estimated tensile resistance or value of n lbs. per sq. inch, and quotient will give thickness of metal required.

EXAMPLE.—Pressure required is 9000 lbs. per sq. inch, and diameter of cylinder is 12 ins.; what is required thickness of metal of cast iron?

$$\text{Thickness of metal is taken at } 6000. \quad \frac{9000 \times 5.3}{6000 \times 2} = \frac{47700}{12000} = 3.975 \text{ ins.}$$

Values of Different Metals in Tons. (Molesworth.)

n41 | Gun metal..... .22 | Wrought iron.. .14 | Steel..... .06

Hydraulic Ram.

Effect of an Hydraulic Ram, as determined by Eytelwein, varied as the square root of the power expended. When height to which water is raised is low, effect is greater than with any other machine; but diminishes as height increases.

Height of supply pipe should not be less than .75 of height to which water is to be raised, or 5 times height of supply; it may be much longer.

To Compute Elements.

$$3 \sqrt{h} = \text{HP}; \quad \frac{881 \text{ HP}}{h} = V; \quad 1.45 \sqrt{V} = D; \quad .75 \sqrt{V} = d; \quad \text{and}$$

h , h' and v representing volumes expended and raised, in cu. ft., h and h' heights from which water is drawn and elevated in feet, and v volume of supply and discharging pipes in ins., and HP effective horse power.

EXAMPLE.—Heights of a fall and of elevation are 10 and 26 feet, and power expended per minute are 1.71 and .543 cube feet.

$$\times 1.71 \times 10 = .0193 \text{ HP}; \quad \frac{881 \times .0193}{10} = 1.71 \text{ cube feet; and}$$

$$\sqrt{1.71} = .975 \text{ ins.; and } \frac{5}{6} \times \frac{.543 \times 26.3}{1.71 \times 10} = .696 \text{ efficiency.}$$

WATER-WHEELS.

WHEELS are divided into two classes, Vertical and Horizontal, comprises *Overshot*, *Breast*, and *Undershot*; and Horizontal, *Impact*, or *Reaction* wheels.

Wheels are limited by construction to falls of less than 60 feet, are applicable to falls of any height from 1 foot upward.

Wheels applied to a fall of from 20 to 40 feet give a greater effect than a Turbine, and for very low falls Turbines give a greater effect.

Methods of admitting water to an Overshot or Breast wheel, various, consisting of *Overfall*, *Guide-bucket*, and *Penstock*.

Sluice is a saddle-beam with a curved surface, so as to direct the water tangentially to buckets; a *Guide-bucket* is an apron by which water is directed in a course tangential to buckets; and a *Penstock* is sluice-board or as close to wheel as practicable, and of such thickness at its lower edge as to prevent a contraction of current. Bottom surface of penstock is formed with a curve.

The rim of a wheel consists of plates at its periphery, which form the sides of the bucket.

The fall of a water-wheel is measured between surfaces of water in penstock race, and, ordinarily, two thirds of height between level of reservoir and level at which water strikes a wheel is lost for all effective operation.

The velocity of a wheel at centre of percussion of fluid should be from .5 to .6 that of the water.

The effect in a fall of water is expressed by product of its weight and the fall.

Ratio of Effective Power of Water Motors.

| | | | | |
|-----------|----------------|-------|--------------------------------------|-----------------|
| Head high | from .68 to .6 | to 1 | Undershot, Poncelet's, from .6 to .4 | to 1 |
| | | | Undershot..... | .27 to .45 to 1 |
| | .6 to .8 | to 1 | Impact and Reaction..... | .3 to .5 to 1 |
| | .45 to .65 | to 1 | Water-pressure engine | .8 to 1 |
| | .6 | to 1 | | |

Overshot-wheel.

OT-WHEEL.—The flow of water acts in some degree by impact, and by its weight.

The speed of wheel at its circumference, the greater will be mechanical effect of the water, in some cases rising to 80 per cent.; with velocities to 6.5 feet, efficiency ranges from 70 to 75 per cent. Proper velocity about 5 feet per second.

Number of buckets should be as great, and should retain water as long, as possible. Maximum effect is attained when the buckets are so numerous that water surface in the bucket commencing to be empty is in contact with the under side of the bucket next above it. Buckets should be 12 ins. apart.

Radial buckets give greatest effect, and Radial give but .78 effect. Tangential buckets. Wheel 40 feet in diameter should have 152 buckets.

Small wheels give a less effect than large, in consequence of the small action, and discharging water from the buckets at a distance from the wheel with larger wheels, or when their velocity is lower.

Head of water bears to fall or height of wheel a proportion of .5, ratio of effect to power is reduced. The general law is, ratio of effect to power decreases as proportion of head to fall increases.

Wheel with shallow *Shrouding* acts more efficiently than on deep, and depth is usually made 10 or 12 ins., but in some cases increased to 15.

Breadth of a wheel depends upon capacity necessary to give to receive required volume of water.

Form of Buckets.—Radial buckets—that is, when the bottom is a involve so great a loss of mechanical effect as to render their use inconeconomy; and when a bucket is formed of two pieces, lower or i termed *bottom* or *floor*, and outer piece *arm* or *wrist*. Former is usu a line with radius of wheel.

Line of a circle passing through *elbow*, made by junction of *floor* termed division circle, or *bucket pitch*, and it is usual to put this at o of shrouding.

When *arm* of a bucket is included in division angle of buckets, th representing number of buckets, the cells are not sufficiently covered, a shallow shrouding; hence it is best to extend arm of a bucket over 1 angle, so as to cover or overlap *elbow* of bucket next in advance of it.

Construction of Buckets (Fig. 1).—Capacity of bucket should be 3 of water.

Fig. 1.



Fairbairn gives area of opening of a wheel of great diameter, compared to the as 5 to 24.

Buckets having a bottom of two planes, two bottoms, and two division circles or b and an arm, give a greater effect than with

When an opening is made in base of b to afford an escape of air contained with loss of water admitted, the buckets are *tilted*, and effective power of wheel is n than with closed buckets.

D = distance apart at periphery = d , shrouding, s length of radial start = .33 d bucket curve = 1.25 d in large wheels, and under 25 feet, a angle of radius of curv with radial line of wheel at points of b (Molesworth.)

pute Radius and Revolutions of an O wheel, and Height of Fall of Water.

whole Fall and Velocity of Flow, etc., are given.

$$= n, \quad \frac{v^2}{2g} 1.1 = h', \quad \text{and} \quad \frac{3.1416 \pi r}{h} = c. \quad h \text{ representing height}$$

Height between the centre of gravity of discharge and half depth of water flows, v velocity of flow in feet per second, a angle of water into a bucket makes with summit of wheel, n number of minute, v velocity of wheel at its circumference per second, and r its

Height of whole fall is distance between surface of water at which lower buckets are emptied of water, and as a proportion is lost, it is proper to assume height h' as above given.

ACCELERATION.—A fall of 30 feet, velocity of its flow is 16 feet per second, and required velocity of wheel is 120, and number of revolutions, and height of fall is 16 feet.

$$h = \frac{16^2}{2 \times 32} \times 1.1 = 4.16 \text{ feet}$$

$$= .978; \quad \frac{30 - 4.33}{1 + .978} = \frac{25.67}{1.978} = 12.98 \text{ (revs.)}$$

HYDRODYNAMICS.

n Number of Revolutions and Ratio between Velocities of Flow & Circumference of Wheel are given.

$$\frac{.000772 (2n)^2 h + (1 + \cos. a)^2 - 1 + \cos. a}{.000386 (2n)^2} = r, x = \frac{v}{c}, \text{ and } \frac{3.1416 \pi r}{30} =$$

ILLUSTRATION.—If number of revolutions are 5, $x = 2$, and fall, etc., as in pre what is radius of wheel, velocity of flow, and height of fall?

$$\frac{\sqrt{.000772 (2 \times 5)^2 \times 30 + (1.978)^2} - 1.978}{.000386 (2 \times 5)^2} = \frac{.518}{.0386} = 13.41 \text{ feet.}$$

$$\frac{416 \times 5 \times 13.41}{30} = 7.03 \text{ feet. Hence } 7.03 \times 2 = 14.06 \text{ velocity of flow, and } \frac{14}{64} = 3.38 \text{ feet.}$$

To Compute Width of an Overshot-wheel.

C = w . C representing a coefficient = 3, when buckets are filled to an excess, a n they are deficiently filled, V volume of water in cube feet per second, s depth of buckets, w width of buckets, both in feet, and c' velocity of wheel at centre of wheel, in feet per second.

ILLUSTRATION.—A wheel is to be 31 feet in diameter, with a depth of shrouding of 1, and is required to make 5 revolutions per minute under a discharge of 11 feet of water per second, what should be width of buckets?

$$\text{same } C = 4, \text{ and } c' = \frac{31 - 1 \times 3.1416 \times 5}{60} = 7.854. \text{ Then } \frac{4 \times 10}{1 \times 7.854} = 5.09 \text{ feet.}$$

To Compute Number of Buckets.

$1 + \frac{s}{.83} \div 12 = d$, and $\frac{D p s}{d} = n$. D representing diameter of wheel, d distance between centres of buckets, in feet, and n number of buckets.

ILLUSTRATION.—Take elements of preceding case.

$$17 \left(1 + \frac{1}{.83} \right) = 7 \times 2.2 \div 12 = 1.283, \text{ and } \frac{31 - 1 \times 3.1416 \times 1}{1.283} = 73.4, \text{ say } 72$$

; hence $\frac{360}{72} = 5^\circ$, angle of subdivision of buckets.

To Compute Effect of an Overshot-wheel.

$$1 - \left(\frac{v'^2}{2g} V w + f \right) = P. \quad w \text{ representing weight of cube foot of water in lbs., } v' \text{ of it discharged at tail of wheel, in feet per second, } V \text{ volume of flow in and } f \text{ friction of wheel in lbs.}$$

ILLUSTRATION.—A volume of 12 cube feet per second has a fall of 10 feet, wheel 8.5 feet of it, and velocity of water discharged is 9 feet per second; what fall?

of wheel is assumed to be 750 lbs.

$$\times 62.5 - \left(\frac{9^2}{64.33} \times 12 \times 62.5 + 750 \right) = \frac{6375 - (1.26 \times 750 + 750)}{7500} = .60 -$$

of effect to power; and $4680 \times 60 \text{ seconds} \div 33000 = 8.51 \text{ HP.}$

Compute Power of an Overshot-wheel

Multiply weight of water in lbs. discharged upon wheel or distance in feet from centre of opening in race; divide product by 33000, and multiply quotient by 50000, and quotient is HP.

$$H = \text{HP, and } \frac{H \times 7}{h} = V \text{ per second; or, } \frac{771 \text{ HP}}{h} = V \text{ per min.}$$

Poncelet's Wheel.

PONCELET'S WHEEL.—Buckets are curved, so that flow of water is in se of their concave side, pressing upon them without impact; and effect eater than when water impinges at nearly right angles to a plane sur-or blade.

his wheel is advantageous for application to falls under 6 feet, as its t is greater than that of other undershot wheels with a curb, and for from 3 to 6 feet its effect is equal to that of a Turbine.

r falls of 4 feet and less, efficiency is 65 per cent., for 4.25 to 5 feet, 60 cent., and from 6 to 6.5 feet, 55 to 50 per cent.

its arrangement, aperture of sluice should be brought close to face of sl. First part of course should be inclined from 4° to 6° ; remainder of se, which should cover or embrace at least three buckets, should be car-concentric to wheel, and at end of it a quick fall of 6 ins. made, to guard nst effect of back-water. Sluice should not be opened over 1 foot in any and 6 ins. is a suitable height for falls of 5 and 6 feet.

istance between two buckets should not exceed 8 or 10 ins., and radius heel should not be less than 40 ins., or more than 8 feet.

ane of stream or head of water should meet periphery of wheel at an e of from 24° to 30° . Space between wheel and its curb should not ex-.4 of an inch.

epth of shrouding should be at least .25 depth of head of water, or such o prevent water from flowing through it and over the buckets, and width heel should be equal to that of stream of impinging water.

fect of this wheel increases with depth of water flow, and, therefore, r elements being equal, as filling of buckets, to obtain maximum effect, r should flow to buckets without impact, and velocity of wheel should ly a little less than half that of velocity of water flowing upon wheel.

To Compute Proportions of a Poncelet Wheel.

RE.—As it is impracticable to arrive at the results by a direct formula, they be obtained by gradual approximation.

AMPLR.—Height of fall is 4.5 feet; volume of water 40 cube feet per second; s of wheel = 2 h, or 9 feet; depth of the stream = .75 feet; and C assumed at .9.

representing volume of water in cube feet per second, h height of fall, d depth of dling = $\frac{1}{4} \cdot \frac{v^2}{2g} + d'$; d' opening of and e width of sluice, r radius of curva-

f buckets = $\frac{d}{\cos. z}$, and a of wheel, all in feet; n number of revolutions = $\frac{30^{\circ} C}{p a}$

minute; c velocity of circumference of wheel and v velocity of water, $d = \frac{1}{2} a$ second; C coefficient of resistance of flow of water; z angle between

ng water and that of circumference of wheel at point of contact.

. z; z angle made by circumference of wheel with end of bucket

angle of direction of water from circumference of wheel = $\frac{p c}{2 a}$

$$v = .9 \sqrt{2g \left(h - \frac{d'}{2} \right)} = .9 \times 16.29 = 14.66 \text{ feet } \therefore \text{velocity of}$$

h *Breast-wheel* is used when level of water in *tail-race* and *penstock* *bay* are subject to variation of heights, as wheel revolves in direction in which water flows from blades, and *back-water* is therefore less disadvantageous, added to which, penstocks can be so constructed as to admit of unstable point of opening for the water to flow upon the wheel.

Effect of this wheel is equal to that of the overshot, and in some instances, the advantageous manner in which water is admitted to it, it is greater than both wheels have same general proportions.

Under circumstances of a variable supply of water, *Breast-wheel* is better adapted for effective duty than *Overshot*, as it can be made of a greater diameter; whereby it affords an increased facility for reception of water in buckets, also for its discharge at bottom; and further, its buckets easily overcome retardation of back-water, enabling it to be worked longer period in back-water consequent upon a flood.

A well-constructed wheel an efficiency of 93 per cent. was observed by M. and Sir Wm. Fairbairn gives, at a velocity of circumference of wheel of an efficiency of 75 per cent. Velocity usually adopted by him was from 4 ft. per second, both for high and low falls; a minimum of 3.5 feet for a fall of a maximum of 7 feet for a fall of 5 to 6 feet.

In water flows at from 10° to 12° above horizontal centre of wheel, Fairbairn area of opening of buckets, compared with their volume, as 8 to 24.

Capacity between two buckets or blades should be very nearly double that of volume of water expended.

Compute Proportions and Effect of a Breast-wheel.

STATION.—Flow of water is 15 cube feet per second; height of fall, measured centre of pressure of opening to tail-race, is 8.5 feet; velocity of circumference wheel 5 feet per second; and depth of buckets or blades 1 foot, filled to .5 of their diameter.

Height of wheel = $\frac{V}{s d}$, d representing depth, and v velocity of buckets; $\frac{15}{1 \times 5} = 3$; buckets are but .5 filled, $3 \div .5 = 6$ feet. Assume water is to flow with double velocity of circumference of wheel; $v = 5 \times 2 = 10$ feet; and fall required to generate this velocity = $\frac{v^2}{2g} \times 1.1 = h' = \frac{100}{64.32} \times 1.1 = 1.71$ feet.

Subtracting this height from total fall, there remains for height of curb or shroud-fall during which weight of water alone acts, $h - h' = 8.5 - 1.71 = 6.79$ feet. Assuming radius of wheel 12 feet, and radius of bucket circle 11 feet, whole mechanical effect of flow of water = $15 \times 62.5 \times 8.5 = 7968.75$ lbs., from which is to be deducted from 10 to 15 per cent. for loss of water by escape.

Theoretical effect, as determined by M. Morin, velocity of circumference about 10 ft. of water, and within velocities of 1.66 to 6 feet.

$\cos. \frac{\alpha - v}{g} + h''$ V 62.5. α representing angle of direction of velocity with which water flows to wheel at centre of thread of flow and direction of velocity of wheel at this time, and $h'' = h - h'$ in feet.

Here assumed at 20° . See Weisbach, London, 1848, vol. II. page 197, and for necessarily small value of α , its cosine may be taken at 1. *Cor.* 20° —

$\left(\frac{(10 \times .94 - 5)}{32.16} + 6.79 \right) \times 15 \times 62.5 = 7.474 \times 15 \times 62.5 = 708$
is reduced by a coefficient of .77 for a penstock sluice, and .9

Theoretical effect, as determined by Weisbach, 7273 lbs..

Deducted losses, which he computes as follows:

by escape of water between wheel and curb.....
by escape at sides of wheel and curb.....
friction and resistance of water = 2.5 per cent.....

Flutter-wheel.

r or *Saw-mill Wheel*.—Is a small, low breast-wheel operating under head of water; the design of its construction, water being plenty, is inment of a simple application to high-speed connections, as a gang lar saw. In effect it is from .6 to .7 that of an overshot-wheel of d of fall.

$$\frac{Vs}{150} (v - s) = \text{HP. } v \text{ and } s \text{ as preceding.}$$

Friction of Journals or Gudgeons.

ry considerable portion of mechanical effect of a wheel is lost in ef-
orbed by friction of its gudgeons.

Compute Friction of Journals or Gudgeons of a Water-wheel.

$C .0086 = f$. *W* representing weight of wheel in lbs., *r* radius of gudgeon in : *n* number of revolutions of wheel per minute.

ell-turned surfaces and good bearings, $C = .075$ with oil or tallow; when it is well supplied $= .054$; and, as in ordinary circumstances, when a black-
uent is alone applied $= .11$.

RATION.—A wheel weighing 25 000 lbs. has gudgeons 6 ins. in diameter, and revolutions per minute; what is loss of effect?

Assume $C = .08$. Then $25000 \times \frac{6}{2} \times 6 \times .08 \times .0086 = 309.6 \text{ lbs.}$

ghts.—Iron wheels of 18 to 20 feet in diameter will weigh from 800 to
per HP

wheels of 30 feet in diameter, 2000 to 2500 lbs. per HP.

Compute Diameter and Journals of a Shaft, Stress laid uniformly along its Length.

ron, $\frac{\sqrt[3]{Wl}}{9.6} = d$. *W*on't, $6.12 \sqrt[3]{\frac{HP}{4}} = d$. *W* representing weight or load in
gth of shaft between journals in feet, and *d* diameter of shaft in its body

als or Gudgeons.—Cast Iron, $.048 \sqrt{\frac{W}{2}} = d$.

Shaft has to resist both Lateral and Torsional Stress.—Ascertain
meter for each stress, and cube root of sum of their cubes will give
r.

To Compute Dimensions of Arms.

ron, $\frac{1.7d}{\sqrt[3]{n}} = w$. *d* representing diameter of shaft, and *w* width of arm, both

number of arms, $\frac{w}{5} = t$, and *t* thickness of arm.

Arm is of Oak, *w* should be 1.4 times that of iron, and thickness .7 that

Memoranda.

ime of water of 17.5 cube feet per second, with a fall of 25 feet, applied to an
ot-wheel, will drive a hammer of 1500 lbs. in weight from 100 to 150 blows
ute, with a lift of from 1 to 1.5 feet.*

ime of water of 21.5 cube feet per second, with a fall of 12
having a great height of water above its summit, being
drive a hammer of 500 lbs. in weight 100 blows per min.

1 Estimate of power 31.5 horses.

f water required for a hammer increases in a much greater ratio than
nearly as cube of velocity.

A Stream and Overshot Wheel of following dimensions—viz., height of head to centre of opening, 24.875 ins.; opening, 1.75 by 80 ins.; wheel, 22 feet in diameter by 8 feet face; 32 buckets, each 1 foot in depth, making 3.5 revolutions per minute—drove 3 run of 4.5 feet stones 130 revolutions per minute, with all attendant machinery, and ground and dressed 25 bushels of wheat per hour.

4.5 bushels Southern and 5 bushels Northern wheat are required to make 1 barrel of flour.

A Breast-wheel and Stream of following dimensions—viz., head, 20 feet; height of water upon wheel, 16 feet; opening, 18 feet by 2 ins.; diameter of wheel, 26 feet 4 ins.; face of wheel, 20 feet 9 ins.; depth of buckets, 15.75 ins.; number of buckets, 70; revolutions, 4.5 per minute—drove 6144 self-acting mule spindles; 26 looms, weaving printing-cloths 27 ins. wide of No. 33 yarn (33 hanks to a lb.), and producing 24000 hanks in a day of 11 hours.

Horizontal Wheels.

In horizontal water-wheels, water produces its effect either by *Impact, Pressure, or Reaction*, but never directly by its weight.

These wheels are therefore classed as *Impact, Pressure, and Reaction*, but are now designated by the generic term of *Turbine*.

Turbines.

TURBINES, being operated at a higher number of revolutions than Vertical Wheels, are more generally applicable to mechanical purposes; but in operations requiring low velocities, Vertical Wheel is preferred.

For variable resistances, as rolling-mills, etc., Vertical Wheel is far preferable, as its mass serves to regulate motion better than a small wheel.

In economy of construction there is no essential difference between a Vertical Wheel and a Turbine. When, however, fall of water and volume of it are great, the Turbine is least expensive. Variations in supply of water affect vertical wheels less than Turbines.

Durability of a Turbine is less than that of a Vertical Wheel; and it is indispensable to its operation that the water should be free from sand, silt, branches, leaves, etc.

With Overshot and Breast Wheels, when only a small quantity of water is available, or when it is required or becomes necessary to produce only a portion of the power of the fall, their efficiency is relatively increased, from the blades being but proportionately filled; but with Turbines the effect is contrary, as when the sluice is lowered or supply decreased water enters the wheel under circumstances involving greater loss of effect. To produce maximum effect of a stream of water upon a wheel, it must flow without impact upon it, and leave it without velocity; and distance between point at which the water flows upon a wheel and level of water in reservoir should be as short as practicable.

Small wheels give less effect than large, in consequence of their making a greater number of revolutions and having a smaller water arc.

In *High-pressure Turbines* reservoir (of wheel) is enclosed at top, and water is admitted through a pipe at its side. In *Low-pressure*, water flows into reservoir, which is open.

Turbines working under water, height is measured from surface of in supply to surface of discharged water or race; and when they work height is measured from surface in supply to centre of wheel.

To obtain maximum effect from water, velocity of it, when less should be the least practicable.

Efficiency is greater when sluice or supply is wide open, and it is less affected by head than by variations in supply of water. It varies but little with velocity, as it was ascertained by experiment that when 35 revolution gave an effect of .64, 55 gave but .66.

When Turbines operate under water, the flow is always full through them hence they become *Reaction-wheels*, which are the most efficient.

Experiments of Morin gave efficiency of Turbines as high as .75 of power.

Angle of plane of water entering a Turbine, with inner periphery of it should be greater than 90° , and angle which plane of water leaving reservoir makes with inner circumference of Turbine should be less than 90° .

When Turbines are constructed without a *guide curve**, angle of plane of flowing water and inner circumference of wheel = 90° .

Great curvature involves greater resistance to efflux of water; and hence it is advisable to make angle of plane of entering water rather obtuse than acute, say 100° ; angle of plane of water leaving, then, should be 50° , if internal pressure is to balance the external; and if wheel operates free of water, it may be reduced to 25° and 30° .

If blades are given increased length, and formed to such a hollow curve that the water leaves wheel in nearly a horizontal direction, water then both impinges on blades and exerts a pressure upon them; therefore effect is greater than with an impact-wheel alone.

Turbines are of three descriptions: Outward, Downward, and Inward flow

Outward-flow Turbines.

FOURNEYRON TURBINE, as recently constructed, may be considered as one of the most perfect of horizontal wheels; it operates both in and out of back-water, is applicable to high or low falls, and is either a high or low pressure turbine.

In high-pressure, the reservoir is closed at top and the water is led to through a pipe. In low-pressure, the water flows directly into an open reservoir. Pressure upon the step is confined to weight of wheel alone.

Fourneyron makes angle of plane of water entering = 90° , and angle of plane of water leaving = 30° .

Efficiency is reduced in proportion as sluice is lowered, for action of water on wheel is less favorably exerted. M. Morin tested a Fourneyron turbine 6.56 feet in diameter, and he found that efficiency varied from a minimum of 24, to 79 per cent., when supply of water was reduced to .25 of full supply. In practice, radial length of blades of wheel is .25 of radius, for falls not exceeding 6.5 feet, .3 for falls of from 6.5 to 19 feet, and .66 for higher falls.

To Compute Elements and Results.

High Pressure, $6.6 \sqrt{h} = v$; $\frac{V}{v} = A$; $\frac{\sqrt{1.77 V}}{\sqrt{h}} = D \dagger$; $12.6 \frac{HP}{h} = V$; and $.079 V h = HP$. *h* representing head of water, *v* velocity of turbine at periphery per minute, and *D* internal diameter of turbine, all in feet. *V* volume of water in cube feet per second, *A* sum of area of orifices in sq. feet, and *HP* effective horse-power.

1.2 *D* = external diameter of turbine in feet, when it is more than 6 feet, and 1 when it is less than 6 feet. Number of guides = number of blades \ddagger when less than 24, and number $\div 3$ when greater than 24. Area of section of supply-pipe = .4

For construction of blades and guides, see Molesworth, London, 1882, page 5

* Guide curves are plates upon centre body of a Turbine, which give direction to flow or to blades of wheel which surround them.

† In extreme cases of very high falls diameter given by this formula may be increased.

‡ Fourneyron's rule for the number of blades is constant number 36, irrespective of size

Operation of High-Pressure Turbines.

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|------|------|-----|-----|-----|-----|
| h | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 120 | 140 | 160 | 180 | 200 |
| V | 4.2 | 3.1 | 2.5 | 2.1 | 1.8 | 1.6 | 1.4 | 1.27 | 1.05 | .9 | .8 | .7 | .6 |
| v | 36 | 42 | 47 | 51 | 55 | 59 | 63 | 66 | 73 | 78 | 84 | 89 | 94 |

h = head of water in feet, V volume of water in cube feet required for each revolution, and v velocity of periphery of turbine in feet per second.

BOYDEN TURBINE.—Mr. — Boyden, of Massachusetts, designed outward-flow turbine of 75 HP, which realized an efficiency of 88 per cent. Peculiar features, as compared with a Fourneyron turbine, are, 1st, and most important, the conduction of the water to turbine through a vertical tapered cone, concentric with the shaft. The water, as it descends, acquires gradually increasing velocity, together with a spiral movement in direction of motion of wheel. The spiral movement is, in fact, a continuation of motion of the water as it enters cone.—2d. Guide-plates at base are inclined so as to meet tangentially the approaching water.—3d. A “diffuser,” or a spiral chamber surrounding wheel, into which water from wheel is discharged. This chamber expands outwardly, and, thus escaping velocity of water is eased off and reduced to a fourth when outside of diffuser is reached. Loss of diffuser is to accelerate velocity of water through machine; and gain in efficiency is 3 per cent. Diffuser must be entirely submerged. (D. K. C.)

PONCELET TURBINE.—This wheel is alike to one of his undershot-wheel set horizontally, and it is the most simple of all horizontal wheels.

To Compute Elements of General Proportion and Results. (Lt. F. A. Mahan, U. S. A.)

$.0425 D^2 h \sqrt{h} = \text{HP}$; $4.85 \sqrt{\frac{P}{h \sqrt{h}}} = D$; $.5 D^2 \sqrt{h} = V$; $.1 D = H$; $4.49 \sqrt{3(D + 10)} = N$; $\frac{D}{N} = w$; $\frac{4D}{N} = W$; $D - \frac{8D}{N} = d$; $.5 N \text{ to } .75 N = n$; $\frac{d}{n}$ and C coefficient for V' in terms of $V = \frac{V'}{\sqrt{C}}$. D and d representing exterior and interior diameters of wheel, H and h heights of orifices of discharge at outer circumference and of fall acting on wheel, w and w' shortest distances between two adjacent blades and two adjacent guides, all in feet, V , V' , and v velocities due to fall of water passing through narrowest section of wheel, and of interior circumference of wheel, all in feet per second, N and n numbers of blades and guides, and HP actual power.

For falls of from 5 feet to 40, and diameters not less than 2 feet, n is equal to diameter of wheel. H equal to $.1 D$, $n w' = d$, and $4 w = \text{width of } C$. For falls exceeding this, H should be smaller, in proportion to diameter of wheel.

Downward-flow Turbines.

In turbines with downward flow, wheel is placed below an annular space of guide-blades, by which water is conducted to wheel. The water strikes curved blades, and falls vertically, or nearly so, into tail-race; consequently centrifugal action is avoided, and downward flow is more compact.

FOXTAINE TURBINE yields an efficiency of 70 per cent., when the character of the supply of water is shut off to .75, by sluice, efficiency is 57 per cent. Best velocity at mean circumference of wheel is equal to .57 of that due to height of fall. It may vary .25 of this either way, materially affecting efficiency.

In Turbine, the water in race is in immediate contact with wheel and in air, height of water in race is greatest when sluice is fully opened. Its efficiency is also affected by variations of head of flow than in volume of water supplied to a Turbine, adapted for Tide-mills.

JONVAL TURBINE.—This wheel is essentially alike in its principal proportions to Fontaine's, and in principle of operation it is the same. Water in race must be at a certain depth below wheel.

For convenience, it is placed at some height above level of tail-race, within an air-tight cylinder, or "draft-tube," so that a partial vacuum or reduction of pressure is induced under wheel, and effect of wheel is by so much increased. Resulting efficiency is same as if wheel was placed at level of tail race; and thus, while it may be placed at any level, advantage is taken of whole height of fall, and its efficiency decreases as volume of water is diminished or as sluice is contracted.

To Compute Elements and Results.

Low Pressure.—For falls of 30 feet and less.

$$6\sqrt{h} = v; \quad \frac{V}{v} = A; \quad \frac{\sqrt{1.77V}}{\sqrt{h}} = D^*; \quad 12.7 \frac{HP}{h} = V; \quad \text{and } .079 V h = HP.$$

h representing head of water, *v* velocity of turbine at periphery per minute, and *D* internal diameter of turbine, all in feet, *V* volume of water in cube feet per second, *A* sum of area of orifices in sq. feet, and *HP* effective horse-power.

1.2 *D* = external diameter of turbine in feet, when it is more than 6 feet, and 1.1 when it is less than 6 feet. Number of guides = number of blades† when less than 24, and number ÷ 3 when greater than 24. Area of section of supply-pipe = .4 *V*.

For construction of blades and guides, see Molesworth, London, 1882, page 540.

Low-Pressure Turbines. (Molesworth.)

| Head. | <i>v</i> | 5 HP | | 10 HP | | 15 HP | | 20 HP | | 30 HP | | 40 HP | | 50 HP | |
|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> | <i>V</i> | <i>R</i> |
| 5 | 9.48 | 25 | 34 | 50 | 24 | 75 | 20 | 100 | 17 | — | — | — | — | — | — |
| 10 | 13.38 | 12.5 | 81 | 25 | 57 | 38 | 47 | 50 | 41 | 75 | 33 | 100 | 28 | 126 | 2 |
| 15 | 16.38 | 8.5 | 136 | 17 | 97 | 25 | 79 | 33 | 68 | 51 | 56 | 68 | 48 | 85 | 4 |
| 20 | 18.96 | 6.3 | 180 | 12.6 | 128 | 19 | 105 | 25 | 90 | 38 | 75 | 50 | 64 | 63 | 5 |
| 25 | 23.22 | 4.2 | 319 | 8.4 | 226 | 12.6 | 185 | 17 | 160 | 25 | 131 | 33 | 113 | 42 | 10 |
| 30 | 26.82 | — | — | 6.3 | 329 | 9.3 | 273 | 12.6 | 232 | 18.9 | 194 | 25 | 164 | 31 | 14 |
| 35 | 30 | — | — | — | — | 7.5 | 358 | 10 | 310 | 15 | 253 | 20 | 220 | 25 | 19 |
| 40 | 32.88 | — | — | — | — | — | — | 8.4 | 380 | 12.6 | 310 | 17 | 263 | 21 | 24 |

v representing velocity of centre of blades in feet and *V* volume of water, in cube feet, both per second, *R* revolutions per minute, and *HP* effective horse-power.

$$\text{Vertical Shaft. } \sqrt[3]{\frac{230 HP}{R}} = \text{diameter of shaft in ins.}$$

Inward-flow Turbine.

INWARD-FLOW TURBINE.—Inward-flow or vortex wheel is made with radiating blades, and is surrounded by an annular case, closed externally and open internally to wheel, having its inner circumference fitted with four curved guide-passages. The water is admitted by one or more pipes to the case, and it issues centripetally through the guide-passages upon circumference of wheel. The water acting against the curved blades, wheel is driven at a velocity dependent on height of fall, and water having expended its force, passes out at centre. This wheel has realized an efficiency as high as 77.5 per cent. It was originally designed by Prof. James Thomson.

SWAIN TURBINE.—Combines an inward and a downward discharge. Receiving edges of buckets of wheel are vertical opposite guide-blades, and lower portions of the edges are bent into form of a quadrant. Each bucket thus forms, with the surface of adjoining bucket, an outlet which combines inward and a downward discharge. One, 72 ins. in diameter, was

* In extreme cases of very high falls diameter given by this formula may be increased.

† Fourneyron's rule for the number of blades is constant number 36, irrespective of size of

by Mr. J. B. Francis, for several heights of gate or sluice, from 2 to 136 ins., and circumferential velocities of wheel ranging from 60 to 80 per cent. of respective velocities due to heads acting on wheel.

For a velocity of 60 per cent., and for heights of gate varying within limits already stated, efficiency ranged from 47.5 to 76.5 per cent., and for a velocity of 80 per cent. it ranged from 37.5 to 83 per cent. Maximum efficiency attained was 84 per cent., with a 12-inch gate and a velocity-ratio of 76 per cent.; but from 9-inch to 13-inch gate, or from .66 gate to full gate, maximum efficiency varied within very narrow limits—from 83 to 84 per cent.,—velocity-ratios being 72 per cent. for 9-inch gate, and 76.5 per cent. for full gate. At half-gate, maximum efficiency was 78 per cent., when velocity-ratio was 68 per cent. At quarter-gate, maximum efficiency was 61 per cent., and velocity-ratio 66 per cent.

TREMONT TURBINE, as observed by Mr. Francis, in his experiments at Lowell, Mass., gave a ratio of effect to power as .793 to 1.

VICTOR TURBINE is alleged to have given an effect of .88 per cent. under a head of 18.34 feet, with a discharge of 977 cube feet of water per minute, and with 343.5 revolutions.

Tangential Wheel.

Wheels to which water is applied at a portion only of the circumference are termed tangential. They are suited for very high falls, where diameter and high tangential velocity may be combined with moderate revolutions. The Girard turbine belongs to this class. It is employed at Goeschman station for St. Gothard tunnel; it operates under a head of 279 feet. The wheels are 7 feet 10.5 ins. in diam., having 80 blades, and their speed is 120 revolutions per minute, with a maximum charge of water of 67 gallons per second. An efficiency of 87 per cent. is claimed for them at the Paris water-works; ordinarily it is from 75 to 80 per cent. (D. K. Clark.)

Impact and Reaction Wheel.

IMPACT-WHEEL.—Impact Turbine is most simple but least efficient form of impact-wheel. It consists of a series of rectangular buckets or blades, set upon a wheel at an angle of 50° to 70° to horizon; the water flows to blades through a pyramidal trough set at an angle of 20° to 40° , so that the water impinges nearly at right angles to blades. Effect is .5 entire mechanical effect, which is increased by enclosing blades in a border or frame.

If buckets are given increased length, and formed to such a hollow curve that the water leaves wheel in nearly a horizontal direction, the water then impinges on buckets and exerts a pressure upon them; effect therefore is greater than with the force of impact alone.

By deductions of Weisbach it appears that effect of impact is only half available effect under most favorable circumstances.

REACTION-WHEEL.—Reaction of water issuing from an orifice of less capacity than section of vessel of supply, is equal to weight of a column of water, basis of which is area of orifice or of stream, and height of which is twice height due to velocity of water discharged.

Hence, the expression is $2 \cdot \frac{v^2}{2g} \cdot a \cdot w = R$, w representing weight of a cube foot of water in lbs., and a area of opening in sq. feet.

WHITE-LAW'S is a modification of Barker's; the arms taper from centre towards circumference and are curved in such a manner as to enable the water to pass from central openings to orifices in a line nearly right and radial, when instrument is operating at a proper velocity; in order that very little centrifugal force may be imparted to the water by the revolution of the arms. Consequently a minimum of frictional resistance is of

Turbine 9.55 feet in diameter, with orifices 4.944 ins. in diameter, operating by a fall of 25 feet, gave an efficiency of 75 per cent., including friction arising of an inclined plane.

When a reaction wheel is loaded, so that height due to velocity, corresponding to velocity of rotation v , is equal to fall, or $\frac{v^2}{2g} = h$, or $v = \sqrt{2gh}$, there is a loss of 17 per cent. of available effect; and when $\frac{v^2}{2g} = 2h$, there is a loss of but 10 per cent.;

when $\frac{v^2}{2g} = 4h$, there is a loss of but 6 per cent. Consequently, for moderate fall, and when a velocity of rotation exceeding velocity due to height of fall may be adopted, this wheel works very effectively.

Efficiency of wheel is but one half that of an undershot-wheel.

When sluice is lowered, so that only a portion of wheel is opened, efficiency of Reaction-wheel is less than that of a Pressure Turbine.

Ratio of Effect to Power of several Turbines is as follows:

| | | | |
|---------------|-----------------|----------------|---------------|
| Let | .65 to .75 to 1 | Jonval | .6 to .7 to 1 |
| Reynold | .6 to .75 to 1 | Fontaine | .6 to .7 to 1 |

ARKER'S MILL.—Effect of this mill is considerably greater than that of the same quantity of water would produce if applied to an undershot-wheel, but less than that which it would produce if properly applied to an undershot-wheel.

For a description of it, see *Grier's Mechanics' Calculator*, page 234; and for its uses, see *London Artisan*, 1845, page 229.

IMPULSE AND RESISTANCE OF FLUIDS.

Impulse and Resistance of Water.—Water or any other fluid, flowing against a body, imparts a force to it by which its condition of motion is altered. Resistance which a fluid opposes to motion of a body does not essentially differ from Impulse.

Impulse of one and same mass of fluid under otherwise similar circumstances is proportional to relative velocities $c \mp v$ of fluid.

For an equal transverse section of a stream, the impulse against a surface does not increase as square of velocity of water.

Impulse against Plane Surfaces.—The impulse of a stream of water depends principally upon angle under which, after impulse, it leaves the water; nothing if the angle is 0, and a maximum if it is deflected back in a direction parallel to that of its flow, or 180° , $2 \frac{c \mp v}{g} V w = P^*$.

When Surface of Resistance is a Plane, and $= 90^\circ$, then $\frac{c \mp v}{g} V w = P$, and for a surface at rest, $2ahw = P$. a representing area of opening in sq. feet. $= 2Ahw$; c and v representing velocities of water and of surface upon which it strikes in feet per second, w weight of fluid per cube foot in lbs., A transverse section of stream in sq. ins., and $c \mp v$ relative motions of water and surface.

Normal impulse of water against a plane surface is equal to weight of column which has for its base transverse section

under twice height due to its velocity, $2h = 2 \frac{c^2}{2g}$.

Resistance of a fluid to a body in motion is same as resistance of fluid against a body at rest.

* Weisbach, New York, 1870, vol. i. page 1008.

To Compute Velocities of Elastic Bodies after Im-

When Impelled in One Direction. $\frac{B-b}{B+b} V + \frac{2}{B+b} b v = R$, and $\frac{2}{B+b} B V - \frac{B-b}{B+b} v = r$.

ILLUSTRATION.—Assume elements as preceding.

$$\frac{50-30}{50+30} \times 7 + \frac{2}{50+30} \times 30 \times 3 = \frac{320}{80} = 4 \text{ feet, and } \frac{2}{50+30} \times 50 \times 7 - \frac{50-30}{50+30} \times 3 = \frac{640}{80} = 8 \text{ feet.}$$

Or, $V - \frac{2}{B+b} b v = \text{velocity of } R$, and $v + \frac{2}{B+b} B V - v = \text{velocity of } r$.

When Impelled in Opposite Directions.

$$\frac{B-b}{B+b} V + \frac{2}{B+b} b v = R, \text{ and } \frac{2}{B+b} B V + \frac{B-b}{B+b} v = r.$$

ILLUSTRATION.—Assume elements as preceding.

$$\frac{50-30}{50+30} \times 7 + \frac{2}{50+30} \times 30 \times 3 = \frac{140 + 180}{80} = -.5 \text{ feet, and } \frac{2}{50+30} \times 50 \times 7 + \frac{50-30}{50+30} \times 3 = \frac{700+60}{80} = 9.5 \text{ feet.}$$

Or, $\frac{2}{B+b} (V+v) = \text{velocity lost by } B$. As $\frac{2}{50+30} \times 30 \times 7 + 3 = 7.5 \text{ feet.}$

When One Body is at Rest. $\frac{V(B-b)}{B+b} = R$, and $\frac{2}{B+b} B V = r$.

ILLUSTRATION.—Assume elements as preceding.

$$\frac{7 \times 50 - 30}{50 + 30} = \frac{140}{80} = 1.75 \text{ feet, and } \frac{2 \times 50 \times 7}{50 + 30} = \frac{700}{80} = 8.75 \text{ feet.}$$

To Compute Velocities of Imperfect Elastic Bodies after Impact.

Effect of Collision is increased over that of perfectly inelastic bodies not doubled, as in case of perfectly elastic bodies; it must be multiple $1 + \frac{n}{m}$ or $\frac{m+n}{m}$, when $\frac{n}{m}$ represents degree of elasticity relative to both perfect inelasticity and elasticity.

Moving in same Direction. $V - \frac{m+n}{m} \times \frac{b}{B+b} (V-v) = R$; and $v + \frac{B}{B+b} (V-v) = r$. m and n representing ratio of perfect to imperfect elastic

ILLUSTRATION.—Assume elements as preceding. m and $n = 2$ and 1 .

$$7 - \frac{2+1}{2} \times \frac{30}{50+30} \times 7 - 3 = 7 - 1.5 \times \frac{30}{80} \times 4 = 7 - 2.25 = 4.75 \text{ feet, and}$$

$$\frac{2+1}{2} \times \frac{50}{50+30} \times 7 - 3 = 3 + 3.75 = 6.75 \text{ feet.}$$

When Moving in Opposite Directions.

$$V - \frac{m+n}{m} \times \frac{b(V+v)}{B+b} = R, \text{ and } \frac{m+n}{m} \times \frac{B}{B+b} \times (V+v) - v = r.$$

When One Body is at Rest. $\frac{V(B - \frac{n}{m}b)}{B+b} = R$, and $\frac{B V (1 + \frac{n}{m})}{B+b} = r$.

ILLUSTRATION.—Assume elements of preceding case.

$$\frac{7}{2} \times 30 = \frac{210}{2} = 105 \text{ feet, and } \frac{50 \times 7 \times (1 + \frac{1}{2})}{50 + 30} = \frac{50 \times 7 \times 1.5}{80} = 5.625 \text{ feet,}$$

LIGHT.

It is similar to Heat in many of its qualities, being emitted in rays, and subject to same laws of reflection.

of two kinds, *Natural* and *Artificial*; one proceeding from Sun and stars, the other from heated bodies.

stars shine in dark only at a temperature from 600° to 700° , and in day at 1000° .

Intensity of Light is inversely as square of distance from luminous body.

Velocity of Light of Sun is 185 000 miles per second.

Standard of Intensity or of comparison of light between different methods of illumination is a Sperm Candle "short 6," burning 120 grains per hour.

Candles.

A Sperm candle .85 of an inch in diameter consumes an inch in length in one hour.

Decomposition of Light.

| Maximum Ray. | Contrasts. | | | Combinations. | | |
|--------------|------------|-----------|-----------|---------------|------------|-----------|
| | Primary. | Second'y. | Tertiary. | Primary. | Secondary. | Tertiary. |
| Chemical. | — | — | — | Blue... | Green.. | — |
| Electrical. | Blue. | — | Brown. | Yellow. | Purple. | Dark. |
| Light. | — | Green. | Green. | Blue... | Orange. | Green. |
| Heat. | Yellow. | — | — | Red... | Green.. | Gray. |
| | — | Orange. | Broken. | Yellow. | Purple. | — |
| | Red. | Purple. | Green. | Red... | Orange. | Brown. |

Colors of spectrum, when combined, are white.

Consumption and Comparative Intensity of Light of Candles.

| CANDLE. | No. in a Lb. | Diameter. | | Length. | Consumption per Hour. | Light comp'd with Carcel. |
|---------|--------------|-----------|-------|---------|-----------------------|---------------------------|
| | | Inch. | Ins. | | Grains. | |
| | 3 | 1 | 12 | } | 135 | .09 |
| | 3 | .875 | 15 | | | |
| | 3 | .9 | 15 | | | |
| | 4 | .8 | 13.5 | } | 156 | .09 |
| | 6 | .84 | 8.5 | | | |
| | 3 | 1 | 12.5 | | | |
| | 3 | .9 | 15 | } | 135 | .09 |
| | 4 | .8 | 13.75 | | | |

Compared with 1000 Cube Feet of Gas.

| Gas=1. | Consumption. | Light. | Consumption for equal Light. | CANDLE. | Gas=1. | Consumption. |
|--------|--------------|--------|------------------------------|-------------|--------|--------------|
| | Lbs. | Lbs. | | | | Lbs. |
| .098 | 3.5 | 35.5 | 103 | Adamantine. | .108 | 5.1 |
| .095 | 3.9 | 41.1 | 120 | Tallow..... | .074 | 5.1 |

Consumption of oil in an ordinary lamp, a straight or horizontal, is more economical than one with an irregularly cut.

Relative Intensity, Consumption, Illumination, and Cost of various Modes of Illumination.

Oil at 11 cents, Tallow at 14 cents, Wax at 52 cents, and Stearine at 32 cents per lb. 100 cube feet coal gas at 14 cents, and 100 cube feet of oil gas at 52 cents.

| ILLUMINATOR. | Illumination. Carcel Lamp = 100. | Actual Cost per Hour. | Cost for equal Intensity. | ILLUMINATOR. | Illumination. Carcel Lamp = 100. | Actual Cost per Hour. | Cost for equal Intensity. |
|--------------------------------|----------------------------------|-----------------------|---------------------------|--------------------------|----------------------------------|-----------------------|---------------------------|
| Carcel Lamp..... | 100 | Cents. .87 | Per H'r. .87 | Stearine Candle 5 to lb. | 66.6 | Cents. .59 | Per H'r. 4.11 |
| Lamp with inverted reserv'r. } | 57.8 | .89 | .99 | Tallow " 6 " | 54 | .25 | 2.28 |
| Astral Lamp..... | 48.7 | .56 | 1.78 | Sperm " 6 " | 67.5 | .89 | .57 |
| Wax Candle 6 to lb. | 61.6 | .92 | 6.31 | Coal Gas..... | — | 1.22 | — |
| | | | | Oil Gas..... | — | 1.25 | — |

1000 cube feet of 13-candle coal gas is equal to 7.5 gallons sperm oil, 52.9 lbs. tallow and 44.6 lbs. sperm candles.

Candles, Lamps, Fluids, and Gas.

Comparison of several Varieties of Candles, Lamps, and Fluids, with Coal* Gas, deduced from Reports of Com. of Franklin Institute, and of A. Frye, M.D., etc.

| CANDLE. | Intensity of Light.† | Light at Equal Cost. | Cost compared with Gas for Equal Light. | CANDLE. | Intensity of Light.† | Light at Equal Cost. | Cost compared with Gas for Equal Light. |
|-----------------------|----------------------|----------------------|---|----------------------|----------------------|----------------------|---|
| Diaphane..... | .7 | .5 | 15.1 | Tallow, short 6's, } | 1 | 1 | 7 |
| Spermacet, short 6's. | .8 | .54 | 16.2 | double wick.. } | | | |
| Tallow, short 6's, } | | | | Wax, short 6's.... | .8 | .61 | 14 |
| single wick... } | .58 | .85 | 7.5 | Palm oil..... | 7 | .77 | 10 |

* City of Philadelphia. † Compared with a fish-tail jet of Edinburgh gas, containing 12 per cent of condensable matter and consuming 1 cube foot per hour.

| LAMP AND FLUID. | Intensity of Light. | Light at Equal Cost. | Time of Burning 1 Pint of Oil. | LAMP AND FLUID. | Intensity of Light. | Light at Equal Cost. | Time of Burning 1 Pint of Oil. |
|------------------|---------------------|----------------------|--------------------------------|-----------------------|---------------------|----------------------|--------------------------------|
| Carcel. | | | Hours. | Gas..... | 1 | 1 | 1 |
| Sperm oil, max'm | 2.15 | 1.8 | 6.32 | Semi-solar, Sperm oil | 1.15 | .93 | 6 |
| " mean. | 1.22 | 1.35 | 9.87 | Solar, Sperm oil..... | 1.76 | 1.55 | 8 |
| " min'm | .69 | 1.2 | 14.6 | Camphene..... | 1.75 | 3.08 | 9 |
| Lard oil..... | .77 | .97 | 11.3 | | | | |

Loss of Light by Use of Glass Globes.

Clear Glass, 12 per cent. | Half ground, 35 per cent. | Full ground, 40 per cent.

Refraction.

Relative Index of Refraction—Is. Ratio of sine of angle of incidence to sine angle of refraction, when a ray of light passes from one medium into another.

Absolute Index or Index of Refraction—Is. When a ray passes from a vacuum any medium, the ratio is greater than unity.

Relative index of refraction from any medium, as A, into another, as B, is, is equal to absolute index of B, divided by absolute index of A.

Absolute index of air is so small, that it may be neglected when compared with liquids or solids; strictly, however, relative index for a ray passing from air is given substance must be multiplied by absolute index for air, in order to obtain like index of refraction for the substance.

Mean Indices of Refraction.

| | | | |
|------------|-------------------|------|-------------------|
| 1 | Glass, fluid..... | 1.53 | Humors of eye.... |
| 1.37 | " crown..... | 1.64 | Salt, rock..... |
| ... 1.54 | | 1.53 | Water, fresh.. |
| 1.34 | | 1.56 | " sea.. |

Gas.

ret.—A retort produces about 600 cube feet of gas in 5 hours with a of about 1.5 cwt. of coal, or 2800 cube feet in 24 hours.

estimating number of retorts required, one fourth should be added for under repairs, etc.

sure with which gas is forced through pipes should seldom exceed 2.5 water at the Works, or leakage will exceed advantages to be obtained increased pressure.

average mean pressure in street mains is equal to that of 1 inch of

in pipes are laid at an inclination either above or below horizon, a cor- will have to be made in estimating supply, by adding or deducting h from initial pressure for every foot of rise or fall in the length of pipe.

customary to locate a governor at each change of level of 30 feet.

ninating power of coal-gas varies from 1.6 to 4.4 times that of a tallow 6 to a lb.; consumption being from 1.5 to 2.3 cube feet per hour, and : gravity from .42 to .58.

her the flame from a burner greater the intensity of the light, the ffective height being 5 ins.

dard of gas burning is a 15-hole Argand lamp, internal diameter .44 himney 7 ins. in height, and consumption 5 cube feet per hour, giving from ordinary coal-gas of from 10 to 12 candles, with Cannel coal o to 24 candles, and with rich coals of Virginia and Pennsylvania of 4 to 16 candles.

hadelphia, with a fish-tail burner, consuming 4.26 cube feet per hour, ating power was equal to 17.9 candles, and with an Argand burner, ing 5.28 cube feet per hour, illuminating power was 20.4 candles.

which at level of sea would have a *Value* of 100, would have but 60 of Mexico.

nal lights require 4 cube feet, and external lights about 5 per hour. large or Argand burners are used, from 6 to 10 are required.

rdinary single-jet house burner consumes 5 to 6 cube feet per hour.

st-lamps in city of New York consume 3 cube feet per hour. In some 4 and 5 cube feet are consumed. Fish-tail burners for ordinary coal ume from 4 to 5 cube feet of gas per hour.

the foot of good gas, from a jet .033 inch in diameter and height of of 4 ins., will burn for 65 minutes.

n Gas.—Jet .033, flame 5 ins., 1.25 cube feet per hour.

fiers.—Wet purifiers require 1 bushel of lime mixed with 48 bushels er for 10000 cube feet of gas.

purifiers require 1 bushel of lime to 10000 cube feet of gas, and 1 cial foot for every 400 cube feet of gas.

Intensity of Light with Equal Volumes of Gas from different Burners.

Equal to Spermaceti Candle burning 120 Grains per Hour.

| BURNERS. | Expenditure in Cube Feet per Hour. | | | | BURNERS. | Expenditure in Cube Feet per Hour. |
|--------------------------|------------------------------------|-----|-----|-----|----------------------|------------------------------------|
| | 1 | 2 | 3 | 4 | | 1 |
| $\frac{1}{2}$ 1 foot.... | 2.6 | — | — | — | Argand, 16 holes.... | .35 |
| No. 3..... | 3.5 | 4 | 4.2 | — | Argand, 24 holes.... | .3 |
| | 3 | 4.1 | 4.3 | 4.5 | Argand, 28 holes.... | .3 |

Volume of Gas obtained from a Ton of Coal, Resi

| Material. | Cube Feet. | Material. | Cube Feet. | Material. |
|----------------------------------|------------|---------------------|------------|------------------|
| Boghead Cannel... | 13 334 | Cumberland..... | 9 800 | Pittsburgh |
| Wigan Cannel..... | 15 426 | English, mean..... | 11 000 | Resin..... |
| Cannel..... | 8 960 | Newcastle..... | 9 500 | Scotch..... |
| Cape Breton, "Cow Bay," etc..... | 15 000 | Oil and Grease..... | 10 000 | Virginia..... |
| | | Pictou and Sidney.. | 23 000 | West'n..... |
| | | Pine wood..... | 8 000 | Walls-end..... |
| | | | 11 800 | |

1 Chaldron Newcastle coal, 3136 lbs., will furnish 8600 cube feet of a specific gravity of .4, 1454 lbs. coke, 14.1 gallons tar, and 15 gallons moniacal liquor.

Australian coal is superior to Welsh in producing of gas.

Wigan Cannel, 1 ton, has produced coke, 1326 lbs.; gas, 338 lb 250 lbs.; loss, 326 lbs.

Peat, 1 lb. will produce gas for a light of one hour.

Fuel, required for a retort 18 lbs. per 100 lbs. of coal.

In distilling 56 lbs. of coal, volume of gas produced in cube feet distillation was effected in 3 hours was 41.3, in 7, 37.5, in 20, 33.5, 25, 31.7.

Flow of Gas in Pipes.

Flow of Gas is determined by same rules as govern that of flow of Pressure applied is indicated and estimated in inches of water, usually .5 to 1 inch.

Volumes of gases of like specific gravities discharged in equal time horizontal pipe, under same pressure and for different lengths, are in as square roots of lengths.

Velocity of gases of different specific gravities, under like pressure, is inversely as square roots of their gravities.

By experiment, 30 000 cube feet of gas, specific gravity of .42, was charged in an hour through a main 6 ins. in diameter and 22.5 feet in length.

Loss of volume of discharge by friction, in a pipe 6 ins. in diameter mile in length, is estimated at 95 per cent.

Diameter and Length of Gas-pipes to transmit given Volumes of Gas to Branch-pipes. (Dr. Ure.)

| Volume per Hour. | Diameter. | Length. | Volume per Hour. | Diameter. | Length. | Volume per Hour. | Diameter. | Length. |
|------------------|-----------|---------|------------------|-----------|---------|------------------|-----------|---------|
| Cube Feet. | Ins. | Feet. | Cube Feet. | Ins. | Feet. | Cube Feet. | Ins. | Feet. |
| 50 | .4 | 100 | 1000 | 3.16 | 1000 | 2000 | 7 | 1000 |
| 250 | 1 | 200 | 1500 | 3.87 | 1000 | 6000 | 7.75 | 1000 |
| 500 | 1.97 | 600 | 2000 | 5.32 | 2000 | 6000 | 9.21 | 1000 |
| 700 | 2.65 | 1000 | 2000 | 6.33 | 4000 | 8000 | 8.95 | 1000 |

Regulation of Diameter and Extreme Length of Gas-pipe, and Number of Burners permitted.

| Diameter of | Length. | Capacity of Meters. | Burners. | Diameter of Tubing. | Length. | Capacity of Meters. | Burners. |
|-------------|---------|---------------------|----------|---------------------|---------|---------------------|----------|
| | | Light. | No. | Ins. | | Light. | No. |
| | | 3 | 9 | .75 | 50 | 30 | |
| | | 5 | 15 | 1 | 70 | 45 | |
| | | 10 | 30 | 1.25 | 100 | 60 | |
| | | | 60 | 1.5 | 150 | 100 | |

Temperature of Gases.—Combustion of a cube foot of common gas will
50 lbs. of water 1°.

Services for Lamps.

| Length from Main. | Diameter of Pipe. | Lamps. | Length from Main. | Diameter of Pipe. | Lamps. | Length from Main. | Diameter of Pipe. |
|----------------------|----------------------|--------|----------------------|----------------------|--------|----------------------|----------------------|
| Feet. | Ins. | No. | Feet. | Ins. | No. | Feet. | Ins. |
| 40 | .375 | 10 | 100 | .75 | 25 | 180 | 1.5 |
| 40 | .5 | 15 | 130 | 1 | 30 | 200 | 1.75 |
| 50 | .625 | 20 | 150 | 1.25 | | | |

**Times of Gas Discharged per Hour under a Pressure
of Half an Inch of Water.**

Specific Gravity .42.

| of g. | Volume. | Diam. of Opening. | Volume. | Diam. of Opening. | Volume. | Diam. of Opening. | Volume. |
|----------|------------|----------------------|------------|----------------------|------------|----------------------|------------|
| | Cube Feet. | Ins. | Cube Feet. | Ins. | Cube Feet. | Ins. | Cube Feet. |
| | 80 | .75 | 723 | 1.125 | 1625 | 1.5 | 2885 |
| | 321 | 1 | 1287 | 1.25 | 2010 | 5 | 46150 |

Compute Volume of Gas Discharged through a Pipe.

$\sqrt{\frac{d^5 h}{G l}} = V$, and $.0635 \sqrt{\frac{V^2 G l}{h}} = d$. d representing diameter of pipe, and
h of water in ins., denoting pressure upon gas, l length of pipe in yards, G
specific gravity of gas, and V volume in cube feet per hour.

may be assumed for ordinary computation at .42, and h .5 to 1 inch.

STRATION.—Assume diameter of pipe 1 inch, pressure 1.68 ins., and length
1 yard.

$$1000 \times \sqrt{\frac{1 \times 1.68}{.42 \times 1}} = 1000 \times \sqrt{\frac{1.68}{.42}} = 2000 \text{ cube feet,}$$

$$\text{and } .0635 \times \sqrt{\frac{4000000 \times .42 \times 1}{1.68}} = \sqrt{\frac{1680000}{1.68}} = 1.05 \text{ ins.}$$

2.—For tables deduced by above formulas see Molesworth, 1878, page 226.

Dimensions of Mains, with Weight of One Length.

| | | | | | | | | |
|------------------|------|------|-----|-----|-----|------|------|------|
| ter in ins. | 4 | 6 | 8 | 9 | 10 | 14 | 18 | 20 |
| 1 in feet. | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| ess in ins. ... | .375 | .375 | .5 | .5 | .5 | .625 | .75 | .75 |
| t in lbs. | 288 | 224 | 400 | 454 | 489 | 868 | 1316 | 1484 |

GAS ENGINES.

he Lenoir engine, the best proportions of air and gas are, for common
volumes of air to 1 of gas, and for cannel gas, 11 of air to 1 of gas.

time of explosion is about the 27th part of a second.

engine, having a cylinder 4.625 ins. in diameter and 8.75 ins. stroke of
making 185 revolutions per minute, develops a half horse-power.

Distribution of Heat Generated in the Cylinder. (*M. Tresca.*)

| | Per cent. | Per cent. |
|--|-----------|-----------|
| ated by the water and prod- of combustion. | 69 | 27 |
| rted into work. | 4 | 100 |

ce efficiency as determined by the brake = 4 per cent.

Atmospheric Gas Engine.

le-acting cylinder 6 ins. in diameter, making 81 strokes per m'
EP, and the gas consumed per minute for cylinder 20 cube fe.
cube feet. (*M. Tresca.*)

LIMES, CEMENTS, MORTARS, AND CONCRETES.

*Essentially from a Treatise by Brig.-Gen'l Q. A. Gillmore, U.S.A.**

Lime.

Calcination of marble or any pure limestone produces lime (quicklime). Pure limestones burn white, and give richest limes.

Finest calcareous minerals are rhombohedral prisms of calcareous spar, the transparent double-reflecting Iceland spar, and white or statuary marble.

Property of hardening under water, or when excluded from air, conferred upon a paste of lime, is effected by presence of foreign substances—as silicium, alumina, iron, etc.—when their aggregate present amounts to .1 of whole.

Limes are classed; 1. Common or Fat limes, which do not set in water. 2. Poor or Meagre, mixed with sand, which does not alter its condition. 3. Hydraulic Lime, containing 8 to 12 per cent. of silica, alumina, iron, etc., and slowly in water. 4. Hydraulic, containing 12 to 20 per cent. of similar ingredients, sets in water in 6 or 8 days. 5. Eminently Hydraulic, containing 20 to 30 per cent. of similar ingredients, sets in water in 2 to 3 days. 6. Hydraulic Cement, containing 30 to 50 per cent. of argil, sets in a few minutes, and attains the hardness of stone in a few months. 7. Natural Pozzuolanas, including pozzuolana properly so called, Trass or Terras, and Ocherous earths, Basaltic sands, and a variety of similar substances.

Indications of Limestones. They dissolve wholly or partly in weak acids with brisk effervescence, and are nearly insoluble in water.

Rich Limes are fully dissolved in water frequently renewed, and they maintain a long time without hardening; they also increase greatly in volume, from a 2 to 3.5 times their original bulk, and will not harden without the action of air. They are rendered *Hydraulic* by admixture of pozzuolana or trass.

Rich, fat, or common Limes usually contain less than 10 per cent. of pozzuolana.

Hydraulic Limestones are those which contain iron and clay, so as to enable them to produce cements which become solid when under water.

Poor Limes have all the defects of rich limes, and increase but slightly in bulk, the poorer limes are invariably basis of the most rapidly-setting and most durable cements and mortars, and they are also the only limes which have the property, when in combination with silica, etc., of hardening under water, and are therefore applicable for admixture of hydraulic cements or mortars. Altho to rich limes, they will not harden if in a state of contact under water or in wet soil, or if excluded from contact with the atmosphere or carbonic acid gas. They should be employed for mortar only—hardening is impossible to procure common or hydraulic lime or cement, in which case it is recommended to reduce them to powder by grinding.

Hydraulic Limes are those which readily harden under water. They are valuable or eminently hydraulic set from the 21 to the 4th day after setting; at end of a month they become hard and insoluble, and at end of 6 months they are capable of being worked like the hard, natural stone. They absorb less water than pure limes, and only increase in bulk from 2 to 2.5 times their original volume.

* See also the Treatise on Limes, Hydraulic Cements, and Mortars, by Brig.-Gen'l Q. A. Gillmore, U.S.A.

inferior grades, or *moderately hydraulic*, require a period of from 15 to 30 days' immersion, and continue to harden for a period of 6 months. The resistance of hydraulic limes increase if sand is mixed in proportion of 50 to 180 per cent. of the part in volume; from thence it decreases.

Vicat declares that lime is rendered hydraulic by admixture with it of from 40 per cent. of clay and silica, and that a lime is obtained which does not set, and which quickly sets under water.

Artificial Hydraulic Limes do not attain, even under favorable circumstances, the same degree of hardness and power of resistance to compression as natural limes of same class.

Coarse-grained and densest limestones furnish best limes.

Hydraulic limes lose or depreciate in value by exposure to the air.

Tests of fat limes shrink, in hardening, to such a degree that they cannot be used as mortar without a large proportion of sand.

Arènes is a species of ochreous sand. It is found in France. On account of the large proportion of clay it contains, sometimes as great as .7, it can be made into a paste with water without any addition of lime; hence it is sometimes used in that state for walls constructed *en pisé*, as well as for mortar. Mixed with rich lime it gives excellent mortar, which attains great hardness under water, and possesses great hydraulic energy.

Azzulana is of volcanic origin. It comprises Trass or Terras, the Arènes, and the ochreous earths, and the sand of certain graywackes, granites, gneisses, and basalts; their principal elements are silica and alumina, the former preponderating. None contain more than 10 per cent. of lime.

When finely pulverized, without previous calcination, and combined with paste of lime in proportions suitable to supply its deficiency in that element, it possesses hydraulic energy to a valuable degree. It is used in combination with rich lime and may be made by slightly calcining clay and driving off the water of combination at a temperature of 1200°.

Brick or Tile Dust combined with rich lime possesses hydraulic energy.

Trass or Terras is a blue-black trap, and is also of volcanic origin. It requires to be pulverized and combined with rich lime to render it fit for use and to develop any of its hydraulic properties.

General Gillmore designates the varieties of hydraulic limes as follows: If, after slaking, they harden under water in periods varying from 15 to 20 days after exposure, *slightly hydraulic*; if from 6 to 8 days, *hydraulic*; and if from 1 to 4 days, *eminently hydraulic*.

Pulverized silica burned with rich lime produces hydraulic lime of excellent quality. Hydraulic limes are injured by air-slaking in a ratio varying directly with their hydraulicity, and they deteriorate by age.

For foundations in a damp soil or exposure, hydraulic limes must be exclusively employed.

Dracal Lime of Teil is a silicious hydraulic lime; it is slow in setting, requiring a period of from 18 to 24 hours.

Cements.

Dracal Cements contain a larger proportion of silica, alumina, and iron than any of preceding varieties of lime; they do not slake so fast, and are superior to the very best of hydraulic limes, as they require less water at a moderate temperature (65°) in from 3 to 4 days, and require as many hours. They do not shrink in hardening, and form an excellent mortar without any admixture of sand.

When exposed to air, they absorb moisture and carbonic acid gas, and rapidly deteriorated thereby.

Roman Cement is made from a lime of a peculiar character, found in England and France, derived from argillo-calcareous kidney-shaped stones termed *Septaria*.

It is about .33 strength of Portland, and is not adapted for use with sand. *Rosendale Cement* is from Rosendale, New York.

Portland Cement is made in England and France. It requires less water (cement 1, water .29) than Roman cement, sets slowly, and can be removed with additional water after an interval of 12 or even 24 hours from its first mixture.

Property of setting slow may be an obstacle to use of some designations of the cement, as the Boulogne, when required for localities having to contend against immediate causes of destruction, as in sea constructions, having to be executed under water and between tides. On the other hand, a quick-setting cement is also difficult of use; it requires special workmen and an active supervision. A slow setting cement, however, like natural Portland, possesses the advantage of being managed by ordinary workmen, and it can also be remixed with additional water after an interval of 12 or even 24 hours from its first mixing.

Conclusions derived from Mr. Grant's Experiments.

1. Portland cement improves by age, if kept from moisture.
2. Longer it is in setting, stronger it will be.
3. At end of a year, 1 of cement to 1 sand is about .75 strength of neat cement; 1 to 2, .5 strength; 1 to 3, .33; 1 to 4, .25; 1 to 5, .16.
4. Cleaner and sharper the sand, greater the strength.
5. Strong cement is heavy; blue gray, slow-setting. Quick-setting has generally too much clay in its composition—is brownish and weak.
6. Less water used in mixing cement the better.
7. Bricks, stones, etc., used with cement should be well wetted before use.
8. Cement setting under still water will be stronger than if kept dry.
9. Bricks of neat Portland cement in a few months are equal to Blue brick.
10. Bricks of 1 cement to 4 or 5 of sand are equal to picked stock bricks.
11. When concrete is being used, a current of water will wash away the cement.

Artificial Cement is made by a combination of slaked lime with unbaked clay in suitable proportions.

Artificial Pozzuolana is made by subjecting clay to a slight calcination.

Salt water has a tendency to decompose cements of all kinds, and their strength is considerably impaired by their mixture with it.

Mortar.

Lime or Cement paste is the cementing substance in mortar, and its proportion should be determined by the rule that *Volume of cementing substance should be somewhat in excess of volume of voids or spaces in sand or concrete material to be united*, the excess being added to meet imperfect manipulation of the mass.

Hydraulic Mortar, if re-pulverized and formed into a paste after having once set, immediately loses a great portion of its hydraulicity, and descends to the level of moderate hydraulic limes.

The retarding influence of sea-water upon initial hydraulic induces is not very great, if the cement is mixed with fresh water. The strength of mortars, however, is considerably impaired by being mixed with sea-water.

Pointing Mortar is composed of a paste of finely-ground cement and sharp siliceous sand, in such proportions that the volume of cement is slightly in excess of the volume of voids or spaces in the sand. The

nd varies from 2.5 to 2.75 that of the cement paste, or by weight, 1 of nt powder to 3 to 3.33 of sand. The mixture should be made under er, and in small quantities.

mortars are much improved by being worked or manipulated; and as rich gain somewhat by exposure to the air, it is advisable to work mortar in quantities, and then render it fit for use by a second manipulation. The lime will take a larger proportion of sand than brown lime.

of salt-water in the composition of mortar injures adhesion of it.

hen a small quantity of water is mixed with slaked lime, a stiff paste ide, which, upon becoming dry or hard, has but very little tenacity, but, eing mixed with sand or like substance, it acquires the properties of a nt or mortar.

oportion of sand that can be incorporated with mortar depends partly the degree of fineness of the sand itself, and partly upon character of ime. For rich limes, the resistance is increased if the sand is in proportions varying from 50 to 240 per cent. of the paste in volume; beyond proportion the resistance decreases.

me, 1, clean sharp sand, 2.5. An excess of water in slaking the lime s the mortar, which remains light and porous, or shrinks in drying; an ss of sand destroys the cohesive properties of the mass.

s indispensable that the sand should be sharp and clean.

me Mortar.—8 parts cement, 3 parts lime, and 31 parts of sand; or 1 cement, 325 lbs., .5 cask of lime, 120 lbs., and 14.7 cube feet of sand== cube feet of mortar.

ick Mortar.—8 parts cement, 3 parts lime, and 27 parts of sand; or 1 cement, 325 lbs., .5 cask of lime, 120 lbs., and 12 cube feet of sand== 12 cube feet of mortar.

own Mortar.—Lime 1 part, sand 2 parts, and a small quantity of hair. ne and sand, and cement and sand, lessen about .33 in volume when mixed her.

lcareous Mortar, being composed of one or more of the varieties of lime ment, natural or artificial, mixed with sand, will vary in its properties quality of the lime or cement used, the nature and quality of sand, and od of manipulation.

Turkish Plaster, or Hydraulic Cement.

5 lbs. fresh lime reduced to powder, 10 quarts linseed-oil, and 1 to 2 as cotton. Manipulate the lime, gradually mixing the oil and cotton, in oden vessel, until mixture becomes of the consistency of bread-dough.

7, and when required for use, mix with linseed-oil to the consistency of paste, hen lay on in coats. Water-pipes of clay or metal, joined or coated with it, the effect of humidity for very long periods.

Stucco.

ucco or Exterior Plaster is term given to a certain mor- rior plastering; it is sometimes manipulated to res- le, and consists of 1 volume of cement powder to 2 vol-

India, to water for mixing the plaster is added 1 lb. of 8 Imperial gallons of water, for the first coat; and for 1 lb. sugar to 2 gallons of water.

dered slaked lime and Smith's forge scales, mixed w- oportions, make a moderate hydraulic mortar, which v previously coated with boiled oil.

ed for
rated

592 LIMES, CEMENTS, MORTARS, AND CONCRETES.

Plaster should be applied in two coats laid on in one operation, first coat being thinner than second. Second coat is applied upon first while latter is yet soft.

The two coats should form one of about 1.5 inches in thickness, and when finished it should be kept moist for several days.

When the cement is of too dark a color for desired shade, it may be mixed with white sand in whole or in part, or lime paste may be added until its volume equals that of the cement paste.

Khorassar, or Turkish Mortar,

Used for the construction of buildings requiring great solidity, .33 powdered brick and tiles, .66 fine sifted lime. Mix with water to required consistency, and lay between the courses of brick or stones.

Mortars.

Mortars used for inside plastering are termed Coarse, Fine, Gauge or hard finish, and Stucco.

Plastering.—1 bushel, or 1.25 cube feet of cement, mortar, etc., will cover 15 square yards .75 inch thick. 75 volumes are required upon brick work for 70 square laths.

When full time for hardening cannot be allowed, substitute from 15 to 20 per cent. of the lime by an equal proportion of hydraulic cement.

For the second or *brown coat* the proportion of hair may be slightly diminished.

Coarse Stuff.—Common lime mortar, as made for brick masonry, with a small quantity of hair; or by volumes, lime paste (30 lbs. lime) 1 part, sand 2 to 2.25 parts, hair .16 part.

Fine Stuff (lime putty).—Lump lime slaked to a paste with a moderate volume of water, and afterwards diluted to consistency of cream, and then to harden by evaporation to required consistency for working.

In this state it is used for a *stipped coat*, and when mixed with sand or plaster of Paris, it is used for *finishing coat*.

Gauge, or Hard Finish, is composed of from 3 to 4 volumes fine stuff and 1 volume plaster of Paris, in proportions regulated by rapidity required in hardening; for cornices, etc., proportions are equal volumes of each, fine stuff and plaster.

Scratch Coat.—First of three coats when laid upon laths, and is from .25 to .375 of an inch in thickness.

One-coat Work.—Plastering in one coat without finish, either on masonry or laths—that is, *rendered or laid*.

Two-coat Work.—Plastering in two coats is done either in a *laid coat and set*, or in a *screed coat and set*.

Red coat is also termed a *Float coat*. Laid first coat in two-coat is resorted to in common work instead of *screeding*, when finished and not required to be exact to a straight-edge. It is laid in a coat of .5 inch in thickness.

Second coat, except for very common work, should be *hand-floated*. Ease and tenacity of plastering is very much increased by hand-floating.

Floats are strips of mortar 6 to 8 inches in width, and of required thickness. First coat, applied to the angles of a room, or edge of a wall and partition, at intervals of 3 to 5 feet over surface to be covered. When these have become sufficiently hard to withstand pressure of a straight-edge, the intervals between the screeds are filled out flush with them.

Smoothing off of a brown coat with a small quantity of 3 per cent. of white sand, so as to make a smooth

to be finished in distemper, or w

Concrete or Beton

mixture of mortar (generally hydraulic) with coarse materials, as pebbles, stones, shells, broken bricks, etc. Two or more of these, or all of them, may be used together. As lime or cement paste is the cementing substance in mortar, so is mortar the cementing substance in concrete or beton. The original distinction between cement and beton was, that the former possessed hydraulic energy, while the latter did not.

Concrete.—1.5 parts unslaked hydraulic lime, 1.5 parts sand, 1 part gravel, and 2 parts of a hard broken limestone. Concrete has contracts one fifth in volume. Fat lime may be mixed with concrete, but it is a serious prejudice to its hydraulic energy.

Various Compositions of Concrete.

Concrete.—308 lbs. cement = 3.65 to 3.7 cube feet of stiff paste. 12 cube feet loose sand = 9.75 cube feet of dense.

Superstructure.—11.75 cube feet of mortar as above, and 16 cube feet of gravel fragments.

Wall.—*Boston Harbor*.—*Hydraulic*.—308 lbs. cement, 8 cube feet of gravel, and 30 cube feet of gravel. Whole producing 32.3 cube feet.

Structure.—308 lbs. cement, 85 lbs. lime, and 14.6 cube feet dense gravel. Whole producing 12.825 cube feet.

It is made of clay or earth rammed in layers of from 3 to 4 ins. in depth. In climates, it is necessary to protect the external surface of a wall constructed in this manner with a coat of mortar.

Asphalt Composition.

General pitch 1 part, bitumen 11, powdered stone, or wood ashes, 7 parts. These 2 parts, clay 3 parts, and sand 1 part, mixed with a little oil, makes a good and durable cement, suitable for external use.

Ting.—8 lbs. of composition will cover 1 sup. foot, .75 inch thick.

Utum 55 lbs. and gravel 28.7 lbs. will cover an area of 10.75 sq. feet.

stic.—Pulverized burnt clay 93 parts, litharge, ground very fine, 7 parts, with a sufficient quantity of pure linseed oil. Coarse sand 14, pulverized calcareous stone 14, litharge 2, and linseed oil 4 weight.

powders to be well dried in an oven, and the surface upon which it is to be used must be saturated with oil.

Roads.—Bitumen 16.875 parts, asphaltum 225 parts, oil of resin 6.25 parts, and gravel 135 parts. Thickness, from 1.25 to 1.375 ins.

cial Mastic.—Composition of 1 square yard .9 inch thick:

| | | | |
|---------------|---------------|------------------|----------------|
| eral tar..... | 205 cube ins. | Gravel..... | 275 cube ins. |
| h..... | 165 " " | Slaked lime..... | 55 " " |
| d..... | 549 " " | | 1249 cube ins. |

ral Efflorescence.—White alkaline efflorescence upon the surface of walls laid in mortar, of which natural hydraulic lime or cement is the basis, is mixed with animal fat in the proportion of .025 of its weight for prevention.

alkalization of these salts within the pores of bricks, into which they are sorbed from the mortar, causes disintegration.

imper is term for all coloring mixed with water and size.

ting.—Mortar composed of lime and fine sand, in a section into the upper beds and internal joints of masonry.

nce is the pulpy and gelatinous fluid, of a milky hue, that is present upon its being deposited in water. It is produced in the sea water than in fresh; it sets very imperfectly, and it lessens the strength of the concrete.

Slaking.

Slaked Lime is a hydrate of lime, and it absorbs a mean of 2.5 times its volume, and 2.25 times its weight of water.

Lime (*quicklime*) must be slaked before it can be used as a matrix in mortar.

Ordinary method of slaking is by submitting the lime to its full proportion of water (previously known or attained by trial) in order to reduce its consistency of a thick pulp. The volume of water required for this purpose will vary with different limes, and will range from 2.5 to 3 volumes that of the lime, and it is imperative that it should all be poured upon it nearly at one time as to be in advance of the elevation of the temperature consequent upon its reduction.

This process, when the water used is in an excessive quantity, is termed "drowning," and when the volume of lime has increased by the absorption of water it is termed its "growth."

If too much water is used, the binding qualities of the lime is injured by its semi-fluidity; and if too little, it is injurious to add after the reduction of the lime has commenced, as it reduces its temperature and renders it granular and lumpy.

While lime is in progress of slaking it should be covered with a tarpaulin or canvas (a layer of sand will suffice), in order to concentrate its evolved heat.

The essential point in slaking is to attain the complete reduction of the lime, and the greater the hydraulic energy of a lime, the more difficult it becomes to effect it.

Whitewash or Grouting.—When lime is required for a whitewash or grouting, it should be thoroughly "drowned," and then run off into tight vessels and closed.

Slaking by Immersion is the method of suspending lime in a suitable vessel in water for a very brief period, and withdrawing it before reduction commences. The lime is then transferred to casks or like suitable receptacles and tightly enclosed, until it is reduced to a fine powder, in which condition if secured from absorption of air, it may be preserved for several months without essential deterioration.

Spontaneous or Air Slaking.—When lime is not wholly secured from exposure to the air, it absorbs moisture therefrom, slakes, and falls into a powder.

Limes and Cements.—A Cask of Lime = 240 lbs., will make from 7.5 to 8.15 cube feet of stiff paste.

A Cask of Cement = 300 * lbs., will make from 3.7 to 3.75 cube feet of stiff paste.

A Cask of Portland Cement = 4 bushels or 5 cube feet = 420 lbs.

A Cask of Roman Cement = 3 bushels or 3.75 cube feet = 364 lbs.

A Bushel of cement will cover.....

| | | |
|------------|-----------|------------|
| .5 inch. | .75 inch. | 1 inch. |
| 2.25 yards | 1.5 yards | 1.24 yards |

From experiments of General Totten, it appeared that

| | | | |
|---|--------------------------------|-------------------------------|------------------|
| 1 | volume of lime slaked with .33 | its volume of water gave 2.27 | volumes of paste |
| 1 | " " " .66 | " " " 1.74 | " " |
| 1 | " " " 1 | " " " 2.06 | " " |

One cube foot of dry cement, mixed with .33 cube foot of water, will make .635 cube foot of stiff paste.

Lime should be slaked at least one day before it is incorporated with the sand when they are thoroughly mixed, the mortar should be brought to a uniform mass, for use as required.

* 300 lbs. net is standard; it usually overruns 8 lbs.

Mortar, Cement, &c. (Molesworth.)

Mortar.—1 of lime to 2 to 3 of sharp river sand.

Brick Mortar.—1 of lime to 2 sand and 1 blacksmith's ashes, or coarsely ground coke.

Coarse Mortar.—1 of lime to 4 of coarse gravelly sand.

Concrete.—1 of lime to 4 of gravel and 2 of sand.

Hydraulic Mortar.—1 of blue lias lime to 2.5 of burnt clay, ground together.

Brick Mortar.—1 of blue lias lime to 6 of sharp sand, 1 of pozzuolana and 1 of calcined stone.

Concrete.—1 of hydraulic mortar to 1.5 of angular stones.

Cement.—1 of sand to 1 of cement.—If great tenacity is required, the cement should be used without sand.

Portland Cement

Composed of clayey mud and chalk ground together, and afterwards cald at a high temperature—after calcining it is ground to a fine powder.

Strength of Mortars, Cements, and Concretes.

Deduced from Experiments of Vicat, Paisley, Treussart, and Voisin.

Tensile

Weight or Power required to Tear asunder One Sq. Inch.

Cement Mortar. (42 days old.)

| | | Proportion of Sand to 1 of Cement. | | | | | | | | | | |
|-------------|--|------------------------------------|-----|-----|-----|-----|-----|-----|-----|----|----|---------|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Weight..... | | 284 | 284 | 199 | 166 | 142 | 128 | 116 | 106 | 99 | 92 | 95 lbs. |
| Power..... | | 142 | 142 | 113 | 92 | 79 | 67 | 57 | 42 | 35 | 25 | — " |

Brick, Stone, and Granite Masonry. (320 days old.)

Experiments of General Gillmore, U. S. A.

| Cement on Bricks. | | Cement on Granite. | |
|-------------------|------|----------------------------|-------|
| | Lbs. | | Lbs. |
| Weight, average | 30.8 | Pure | 27.5 |
| Weight 1 | 15.7 | Sand 1 | 7.9 |
| Weight 2 | 12.3 | Cement 1 | 20.5 |
| Weight 3 | 6.8 | Sand 1 | 37.25 |
| | | Cement 2 | 29.15 |
| | | Sand 1 | |
| | | Cement 3 | |
| Field and Baxter. | Lbs. | James River. | Lbs. |
| Weight cement | 68 | Pure cement | 87 |
| Weight 4 | 68 | Cement 4 | 62 |
| Weight 1 | 80 | Sand 1 | |
| Weight 2 | 82 | Newark Lime and Cement Co. | |
| Weight 3 | 74 | Pure cement | 93 |
| Weight cement Co. | 87 | Cement 1 | 40 |
| Weight | 54 | Sand 2 | |
| | | Newark and Rosendale. | |
| | | Pure cement | 75 |
| | | Cement 1 | 16 |
| | | Sand 1 | |
| | | Newark and Rosendale. | |
| | | Pure, without | 45 |
| | | mortar, mean | |
| | | Mort | |
| | | Lime paste 1. | |
| | | " " 1 | |
| | | " " " | |
| | | " " " | |
| | | cement pa | |

Pure Cement.

| | Lbs. | | |
|-----------------------------------|------|-------------------------------------|--|
| Boulogne 100, water 50..... | 112 | Portland, in sea-water, 45 days.... | |
| Portland, natural, 1 year..... | 675 | " English, 6 months..... | |
| " artificial, Eng., 1 year.... | 462 | Roman "Septaria," 1 year..... | |
| " English, 320 days..... | 1152 | " masonry, 5 months..... | |
| " " 1 month..... | 393 | Rosendale, 9 months..... | |
| Newark and Rosendale..... | 339 | Lawrence Cement Co..... | |

Transverse.

Reduced to a uniform Measure of One Inch Square and One Foot in Length Supported at Both Ends.

Experiments of General Gillmore.

Formed in molds under a pressure of 32 lbs. per sq. inch, applied until hard set. Exposed to moisture for 24 hours, and then immersed in sea-water.

Prisms 2 by 2 by 8 ins. between supports.

Reduced by Formula $\frac{2}{3} \frac{l W}{4 b d^2} - \frac{a}{2} = C$. *C coefficient of rupture, and a weight portion of prism l.*

Cement.

| MATERIAL. | Age. | Pure. |
|---------------------------------------|-------|-------|
| | Days. | Lbs. |
| James River. | | |
| Thick cream..... | 59 | 3.9 |
| Thin paste..... | 320 | 5.8 |
| Stiff paste..... | 59 | 6.9 |
| Rosendale "Hoffman."..... | | |
| Thin paste..... | 320 | 9 |
| Stiff paste..... | 320 | 8.9 |
| "Delafield and Baxter."..... | | |
| Thin paste..... | 320 | 8.5 |
| Stiff paste..... | 320 | 12 |
| English. | | |
| Portland, pure..... | 320 | 16 |
| Stiff paste..... | 320 | 13 |
| Cumberland, Md., pure.... | 320 | 13.2 |
| High Falls, Ul-ster Co., N. Y. }..... | 95 | 8.4 |
| Complete calcination.... | 95 | 4.2 |

Mortar.

| MATERIAL. | Age. | Cement % Based on Sand. |
|--|-------|-------------------------|
| | Days. | Lbs. |
| Portland, Eng., stiff paste | 320 | 13 |
| Roman, " " " " | 20 | 5 |
| " " " " " " | 100 | 6 |
| Cumberland, Md..... | 320 | 12.8 |
| Akron, N. Y..... | 320 | 8.8 |
| James River, Va..... | 320 | 8.6 |
| Pulverized and re-mixed after set. . . } | 3 | 3.6 |
| Fresh..... | 320 | 9 |
| Kingston and Rosendale. | 320 | 7.6 |
| High Falls, Ul-ster Co., N.Y. }..... | 95 | — |
| Fresh water to a stiff paste..... | 95 | — |
| Sea-water to a stiff paste | 95 | — |
| Lawrence Cement Co. | | |
| Fresh..... | 320 | 10.2 |

Crushing.

Cements, Stones, etc. (Crystal Palace, London.)

Reduced to a uniform Measure of One Sq. Inch.

| MATERIAL. | Destructive Pressure. | MATERIAL. | Destructive Pressure. |
|----------------------------------|-----------------------|-------------------------|-----------------------|
| | Lbs. | | Lbs. |
| Portl'd cem't, area 1, height 1. | 1680 | Portland cement 1 } | |
| " cement } | 1244 | " sand 4 } | 114 |
| " sand... } | | " cement 1 } | 60 |
| " stone..... | 1144 | " sand 7 } | 50 |
| | | Roman cement, pure..... | 70 |

General Deductions.

1. Particles of unground cement exceeding .0125 of an inch in diameter not allowed in cement paste without sand, to extent of 50 per cent. of whole value detriment to its properties, while a corresponding proportion of sand imparts strength of mortar about 40 per cent.

When these unground particles exist in cement paste to extent of 66 per cent., adhesive strength is diminished about 28 per cent. For a corresponding proportion of sand the diminution is 68 per cent.

Proportion of siftings exercises a less injurious effect upon the cohesive than upon adhesive property of cement. The converse is true when sand, instead of sifted, is used.

All mixtures with siftings, even when the latter amounted to 66 per cent. of cohesive strength of mortars exceeded their adhesion to bricks. Same reappears to exist when siftings are replaced by sand, until volume of the latter is 20 per cent. of whole, after which adhesion exceeds cohesion.

Age of 320 days (and perhaps considerably within that period) cohesive strength of pure cement mortar exceeds that of Croton front bricks. The converse when the mortar contains 50 per cent. or more of sand.

When cement is to be used without sand, as may be the case when *grouting* is resorted to, or when old walls are to be repaired by injections of thin paste, there is advantage in having it ground to an impalpable powder.

For economy it is customary to add lime to cement mortars, and this may be to a considerable extent when in positions where hydraulic activity and adhesion are not required in an eminent degree.

Setting of concrete under water is held to be injurious.

Mortars of common lime, when suitably made, set in a very few days, and with rapidity that there is no need of awaiting its hardening in the prosecution of

Clay.—The fusibility of clay arises from the presence of impurities, such as lime, iron, and manganese. These may be removed by steeping the clay in hydrofluoric acid, then washing it with water. Crucibles from common clay may be made in this manner.

As by General Gillmore, U. S. A.—Recent experiments have developed that American cements will sustain, without any great loss of strength, a dose of water equal to that of the cement paste, while a dose equal to 5 to 75 the volume of cement paste may be safely added to any Rosendale cement without producing any essential deterioration of the quality of the mortar. Neither is the hydraulic activity of the mortars so far impaired by this limited addition of lime as to render them unsuited for concrete under water, or other submarine work. By the use of lime is secured the double advantages of slow setting and strength.

As by General Totten, U. S. A.—240 lbs. lime = 1 cask, will make from 7.8 to 10 feet of stiff paste.

One foot of dry cement powder, measured when loose, will measure .78 to .8 foot when packed, as at a manufactory.

Composition of Concretes, at Toulon, Marseilles, Cherbourg, Dover, Alderney, &c. Treatise of General Gillmore, pp. 253-256.

MASONRY.

Brickwork.

Bed is an arrangement of bricks or stones, laid side of and above another, so that the vertical joint between any two bricks or stones does not coincide with that between any other two.

It is termed "breaking joints."

Header is a brick or stone laid with an end to face of wall.

Stretcher is a brick or stone laid parallel to face of wall.

Header Course or Bond is a course or courses of headers alone.

Stretcher Course or Bond is a course or courses of stretchers alone.

Half Bricks are pieces of bricks inserted in alternate courses, in order to prevent two headers from being exactly over each other.

English Bond is laying of headers and stretchers in alternate courses.

Flemish Bond is laying of headers and stretchers alternately in each course. **Gauged Work**.—Bricks cut and rubbed to exact shape required.

String Course is a horizontal and projecting course around a building.

Corbelling is projection of some courses of a wall beyond its face, in order to support wall-plates or floor-beams, etc.

Wood Bricks, Pallets, Plugs, or Slips are pieces of wood laid in a wall in order the better to secure any woodwork that it may be necessary to fasten to it.

Reveals are portions of sides of an opening in a wall in front of the recess for a door or window frame.

Brick Ashlar.—Walls with ashlar-facing backed with brick.

Grouting is pouring liquid mortar over last course for the purpose of filling all vacuities.

Larrying is filling in of interior of thick walls or piers, after exterior has been laid, with a bed of soft mortar and floating bricks or spawls in it.

Rendering (Eng.) is application of first coat on masonry, **Laying it on** or two coats on laths, and "Pricking up" if three-coat work on laths.

Bricks should be well wetted before use. *Sea sand* should not be used in the composition of mortar, as it contains salt and its grains are round, being worn by attrition, and consequently having less tenacity than sharp-edged grains.

A common burned brick will absorb 1 pint or about one sixth of its weight of water to saturate it. The volume of water a brick will absorb is inversely a test of its quality.

A good brick should not absorb to exceed .067 of its weight of water.

The courses of brick walls should be of same height in front and rear, whether front is laid with stretchers and thin joints or not.

In ashlar-facing the stones should have a width or depth of bed at least equal to the height of stone.

Hard bricks set in cement and 3 months set will sustain a pressure of 40 tons per sq. foot.

The compression to which a stone should be subjected should not exceed .1 of its crushing resistance.

The extreme stress upon any part of the masonry of St. Peter's at Rome is computed at 15.5 tons per sq. foot; of St. Paul's, London, 14 tons; and of piers of New York and Brooklyn Bridge, 5.5 tons.

The absorption of water in 24 hours by granites, sandstones, and limestones of durable description is 1, 8, and 12 per cent. of volume of the stone.

Color of Bricks depends upon composition of the clay, the molding sand, temperature of burning, and volume of air admitted to kiln.

Pure clay free of iron will burn *white*, and mixing of chalk with the clay will produce a like effect.

Presence of iron produces a tint ranging from *red* and *orange* to *light yellow*, according to proportion of iron.

A large proportion of oxide of iron, mixed with a pure clay, will produce a *light red*, and when there is from 8 to 10 per cent., and the brick is exposed to an intense heat, the oxide fuses and produces a *dark blue* or *purple*, and with a small volume of manganese and an increased proportion of the oxide the color is darkened, even to a *black*.

Small volume of lime and iron produces a *cream color*, an increase of iron produces *red*, and an increase of lime *brown*.

Magnesia in presence of iron produces *yellow*.

Clay containing alkalis and burned at a high temperature produces a *bluish green*.

For other notes on materials of masonry, their manipulation, etc., see "Lime Cements, Mortars, and Concretes," pp. 588-597.

Pointing.—Before pointing, the joints should be reamed, and in case they must be open to 2 of an inch, then thoroughly saturated with mortar, and maintained in a condition that they will neither absorb water from the mortar nor to it. Masonry should not be allowed to dry rapidly after pointing.

It is driven in by the aid of a caulking iron and hammer.

The same general directions are to be observed

One end of brick measure (Fig. 1) is 14 in. wide and weighs 25 lbs. conventional feet by 14 in. thick, averaging 200 bricks, measuring 2 cube yard and 100 gallons water.

Bricklayers' feet will contain 11 bricks or 2 cubic feet masonry.

Fire-Bricks.

Fire-brick contains Silica, Alumina, Oxide of Iron, and a small part of Lime, Magnesia, Potash, and Soda. Its fire-resisting properties depend upon the relative proportions of these constituents and character of grain.

A good clay should be of a uniform structure, a coarse open grain to the hand, and free from any alkaline surface.

The Bournebridge clay is black and is composed as follows:

Silica..... 65.2 | Alumina..... 17.5 | Lime..... 1.2 | Protoxide of iron..... 1.5
Water and organic matter..... 14.3

Sevenside clay is very similar.

Thickness of Brick Walls for Warehouses in (Birmingham.)

| Height in Feet..... | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|-----------------------|----|----|----|----|-----|-----|-----|-----|
| Length in Feet..... | — | — | — | — | — | — | — | — |
| Thickness in Ins..... | 24 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| Length in Feet..... | 70 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| Thickness in Ins..... | 30 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |
| Length in Feet..... | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| Thickness in Ins..... | 26 | 26 | 32 | 38 | 44 | 50 | 56 | 62 |

Stone Masonry.

Masonry is classed as *Ashlar* or *Rubble*.

Ashlar is composed of blocks of stone dressed square and laid in close joints.

Coursed Ashlar consists of blocks of same height throughout each course.

Rubble Ashlar

Is ashlar faced stone with rubble backing.

Fig. 1.



Fig. 1.—Coursed, with chamfered and rusticated quoins and plinth.

Fig. 2.

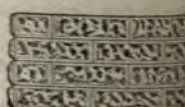


Fig. 2.—Coursed, with rock draft edges.

Fig. 3.



Fig. 3.—Coursed, with rock face.

Fig. 4.

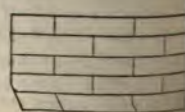


Fig. 4.—Regular Coursed.

Rubble Masonry
composed of small stones irregular in form, and rough.

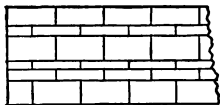
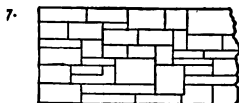


Fig. 5.—Irregular Coursed.



7.—Ranged Random, level, and in courses.



9.—Block Coursed.—Large blocks in courses (regular or irregular), Bed joints roughly dressed.



11.—Ranged Random.—Squared stones laid in level and broken.

Fig. 6.

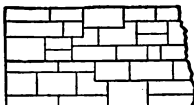


Fig. 6.—Random Coursed.

Fig. 8.

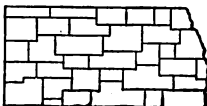


Fig. 8.—Random, level, and broken.

Fig. 10.



Fig. 10.—Coursed and Ranged Random.

Fig. 12.



Fig. 12.—Coursed Random.—Stones laid in courses at intervals of from 12 to 18 ins. in height.

Dry Rubble

wall laid without cement or mortar.



13.—Dry Rubble.—Without mortar.

Fig. 14.



Fig. 14.—Rustic or Rag.—Stones of irregular form, and dressed to make close joints.



15.—Uncoursed or Random.—Bed joints undressed, projections chipped off, and laid at random. In-filled with spalls and mortar.

Fig. 16.



Fig. 16.—Laced bands of stone or give stability.

—Rustic or Rag work is frequently laid in mortar.

Terra Cotta.

Terra Cotta in blocks should not exceed 4 cube feet in volume. properly burned, it is unaffected by the atmosphere or by fumes of any

Arches and Walls.

Springing.—Point *s*, Fig. 15, on each side, from which arch springs.

Crown.—Highest point of arch.

Haunches.—Sides of arch, from springing half-way up to crown.

Spandrel.—Space between extrados, a horizontal line drawn through crown and a vertical line through upper end of skewback.

Skewback is upper surface of an abutment or pier from which an arch springs, and its face is on a line radiating from centre of arch.

Abutment is outer body that supports arch and from which it springs.

Pier is the intermediate support for two or more arches.

Jambs are sides of abutments or piers.

Voussoirs are the blocks forming an arch.

Key-stone is centre voussoir at crown.

Span is horizontal distance from springing to springing of arch.

Rise.—Height from springing line to under side of arch at key-stone.

Length is that of springing line or span.

Ring-course of a wall or arch is parallel to face of it, and in direction of its span.

String and Collar courses are projecting ashlar dressed broad stones at right angles to face of a wall or arch, and in direction of its length.

Camber is a slight rise of an arch as .125 to .25 of an inch per span.

Quoin is the external angle or course of a wall.

Plinth is a projecting base to a wall.

Footing is projecting course at bottom of a wall, in order to distribute weight over an increased area. Its width should be double that of wall, diminishing in regular offsets .5 width of their height.

Blocking Course.—A course placed on top of a cornice.

Parapet is a low wall, over edge of a roof or terrace.

Extrados.—Back or upper and outer surface of an arch.

Intrados or *Soffit* is underside of lower surface of arch or an opening.

Grained is when arches intersect one another.

Invert.—An inverted arch, an arch with its intrados below axis or springing line.

Ashlar masonry requires .125 of its volume of mortar. Rubble, 1 yard stone and .25 cube yard mortar for each cube yard.

Rubble masonry in cement, 160 feet in height, will stand and bear lbs. per sq. inch.

Stones should be laid with their strata horizontal.

When "through" or "thorough bonds" are not introduced, headers overlap one another from opposite sides, known as *dogs' tooth bond*.

Aggregate surface of ends of bond stones should be from .125 to area of each face of wall.

Weak stones, as sandstone and granular limestone, should not be length over 3 times their depth. Strong or hard stones may have length from 4 to 5 times their depth.



Gallets are small and sharp pieces of stone stuck into mortar joints, in which case the work is termed *galleted*.

Snapped work is when stones are split and roughly squared.

Quarry or Rock-faced.—Quarried stones with their faces undressed.

Pitch-faced.—Stones on which the arris or angles of their face, with their sides and ends, is defined by a chisel, in order to show a right-lined edge.

Drafted or Drafted Margin is a narrow border chiselled around edges or faces of a block of rough stone.

Diamond-faced is when planes are either sunk or raised from each edge and meet in the centre.

Squared Stones.—Stones roughly squared and dressed.

Rubble.—Unsquared stones, as taken from a quarry or elsewhere, in their natural form, or their extreme projections removed.

Cut Stones.—Stones squared and with dressed sides and ends.

Dressed Stones.

The following are the modes of dressing the faces of ashlar in engineering

Rough Pointed.—Rough dressing with a pick or heavy point.

Fine Pointed.—Rough dressing, followed by dressing with a fine point.

Crandalled.—Fine pointing in right lines with a hammer, the face of which is close serried with sharp edges.

Cross Crandalled.—When the operation of crandalling is right angled.

Hammered.—The surface of stone may be finished or smooth dressed by being *Axed* or *Bushed*; the former is a finish by a heavy hammer alike to *Crundall*, the latter is a final finish by a heavy hammer with a face serried with sharp points at right angles.

Thickness of Brick Walls for Warehouses. (Molesworth.)

| Length. | Height. | Thickness. | Length. | Height. | Thickness. | Length. | Height. | Thickness. |
|------------|---------|------------|------------|---------|------------|---------|---------|------------|
| Feet. | Feet. | Ins. | Feet. | Feet. | Ins. | Feet. | Feet. | Ins. |
| Unlimited. | 25 | 13 | Unlimited. | 100 | 34 | 45 | 30 | 13 |
| do. | 30 | 17.5 | do. | 40 | 17.5 | 30 | 40 | 13 |
| do. | 40 | 21.5 | do. | 50 | 21.5 | 40 | 50 | 17.5 |
| do. | 50 | 26 | do. | 60 | 21.5 | 35 | 60 | 17.5 |
| do. | 60 | 26 | do. | 70 | 21.5 | 30 | 70 | 17.5 |
| do. | 70 | 26 | do. | 80 | 26 | 45 | 80 | 21.5 |
| do. | 80 | 30 | do. | 90 | 30 | 60 | 90 | 26 |
| do. | 90 | 34 | do. | 100 | 30 | 55 | 100 | 26 |

For drawings and a description of stone-dressing tools, see a paper by J. R. Cross, W. E. Merrill, and E. B. Van Winkle, "A. S. Civil Engineer Trans." 1877

Walls not exceeding 30 feet in height, upper story walls may be

From 16 feet below top of wall to base of it, it should not be defined by two right lines drawn from each side of wall at its top.

Thickness not to be less in any case than one fourteenth of height.

Laths.

Laths are 1.25 to 1.5 ins. by 4 feet in length, are usually square, and a bundle contains 100.

Plastering.

Volumes required for Various Thickness.

| MATERIAL. | Square Yards. | | | MATERIAL. | Square Yards. | | |
|---------------------|---------------|------|------|--------------------|----------------------|------|------|
| | .5 | .75 | 1 | | .5 | .75 | 1 |
| Cube Feet. | Ins. | Ins. | Ins. | Cube Feet. | Ins. | Ins. | Ins. |
| Cement 1..... | 2.25 | 1.5 | 1.15 | Lime 1, sand 2, .. | 75 yards, sup'li- | | |
| Cement 1, sand 1... | 4.5 | 3 | 2.25 | hair 3-75..... | dered and set on | | |
| Cement 1, sand 2... | 6.75 | 4.5 | 3.33 | | brick or 700 on laid | | |

Estimate of Materials and Labor for 100 Sq. Yards of Lath and Plaster.

| Materials and Labor. | Three Coats Hard Finish. | Two Coats Slipped. | Materials and Labor. | Three Coats Hard Finish. | Two Coats Slipped. |
|----------------------|--------------------------|--------------------|----------------------|--------------------------|--------------------|
| Lime..... | 4 casks. | 3.5 casks. | White sand.... | 2.5 bushels. | |
| Lump lime..... | .66 " | | Nails..... | 13 lbs. | 13 lbs. |
| Plaster of Paris.. | .5 " | | Masons..... | 4 days. | 3.5 days |
| Laths..... | 2000. | 2000. | Laborer..... | 3 " | 2 " |
| Hair..... | 4 bushels. | 3 bushels. | Cartage..... | 1 " | .75 " |
| Sand..... | 7 loads. | 6 loads. | | | |

Rough Cast is washed gravel mixed with hot hydraulic lime and water and applied in a semi-fluid condition.

Arches and Abutments.

To Compute Depth of Keystone of Circular or Elliptic Arch.

$$\frac{\sqrt{R+s+2}}{4} + .25 = d. \quad R \text{ representing radius, } s \text{ span, and } d \text{ depth, all in feet.}$$

This is for a rise of about .25 of span; when it is reduced, as to .125, add .5 instead of .25.

ILLUSTRATION.—Arch of Washington aqueduct at "Cabin John" has a span of 220 feet, a rise of 57.25, and a radius of 134.25; what should be depth of its keystone?

$$\frac{\sqrt{134.25 + 220 + 2}}{4} + .25 = \frac{15.63}{4} + .25 = 4.16 \text{ feet.} \quad \text{Depth is 4.16 feet.}$$

Viaducts of several arches increase results as determined above by adding .125 to .15 to depth.

For arches of 2d class materials and work, and for spans exceeding 20 feet, add .125 to depth of keystone, and for good rubble or brick-work add .25.

NOTE.—It is customary to make the keystones of elliptic arches of greater depth than that obtained by above formula. Trautwine, however, who is high authority in this case, declares it is unnecessary.

To Compute Radius of an Arch, Circular or Elliptic.

$$\left(\frac{s}{2}\right)^2 + r^2 \div 2r = R. \quad r \text{ representing rise.}$$

Railway Arches.

For Spans between 25 and 70 feet. Rise .2 of span. Depth of arch .055 of span. Thickness of abutments .2 to .25 of span, and of pier .14 to .16 of span.

Abutments.

Height does not exceed 1.5 times base. $R \div 5 + .1r + 2 = \text{thickness of pier}$ (Trautwine.)

.1.5 ins. per foot of height of wall.

To Compute Depth of Arch. (Hurst.)

$c \sqrt{R} = D$. c = Stone (block) .3. Brick = .4. Rubble = .45.
 in there are a series of arches, put .3 = .35, .4 = .45, and .45 = .5.

Minimum Thickness of Abutments for Bridge and similar Arches of 120°. (Hurst.)

en depth of crown does not exceed 3 feet. Computed from formula.

$$+ \left(\frac{3R}{2H} \right)^2 - \frac{3R}{2H} = T. \quad H \text{ representing height of abutment to springing in feet.}$$

| Height of Abutment to Springing. | | | | | Radius of Arch. | Height of Abutment to Springing. | | | | |
|----------------------------------|-------|-------|-------|-------|-----------------|----------------------------------|-------|-------|-------|-------|
| 5 | 7.5 | 10 | 20 | 30 | | 5 | 7.5 | 10 | 20 | 30 |
| Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. |
| 3.7 | 4.2 | 4.3 | 4.6 | 4.7 | 12 | 5.6 | 6.4 | 6.9 | 7.6 | 7.9 |
| 3.9 | 4.4 | 4.6 | 4.9 | 5 | 15 | 6 | 7 | 7.5 | 8.4 | 8.8 |
| 4.2 | 4.6 | 4.8 | 5.1 | 5.2 | 20 | 6.5 | 7.7 | 8.4 | 9.6 | 10 |
| 4.5 | 4.7 | 5.2 | 5.6 | 5.7 | 25 | 6.9 | 8.2 | 9.1 | 10.5 | 11.1 |
| 4.7 | 5.2 | 5.5 | 6 | 6.1 | 30 | 7.2 | 8.7 | 9.7 | 11.4 | 12 |
| 4.9 | 5.5 | 5.8 | 6.4 | 6.5 | 35 | 7.4 | 9.1 | 10.2 | 11.8 | 12.9 |
| 5.1 | 5.8 | 6.1 | 6.7 | 6.9 | 40 | 7.6 | 9.4 | 10.6 | 12.8 | 13.6 |
| 5.3 | 6 | 6.4 | 7.1 | 7.3 | 45 | 7.8 | 9.7 | 11 | 13.4 | 14.3 |
| 5.5 | 6.2 | 6.6 | 7.3 | 7.6 | 50 | 7.9 | 10 | 11.4 | 14 | 15 |

1.—Abutments in Table are assumed to be without counterforts or wing.
 A sufficient margin of safety must be allowed beyond dimensions here

verte for a road having double tracks are not necessarily twice the
 for a single track.

other and full notes, tables, etc., see Trautwine's Pocket Book, pp. 693-710.

MECHANICAL CENTRES.

There are four Mechanical centres of force in bodies, namely, Centre of Gravity, Centre of Gyration, Centre of Oscillation, and Centre of Suspension.

Centre of Gravity.

Centre of Gravity of a body, or any system of bodies rigidly connected together, is point about which, if suspended, all parts will be in equilibrium.

A body or system of bodies, suspended at a point *out* of centre of gravity, will be at rest with its centre of gravity vertical under point of suspension.

A body or system of bodies, suspended at a point *out* of centre of gravity, successively suspended at two or more such points, the vertical lines through these points of suspension will intersect each other at centre of gravity of body or bodies.

The centre of gravity of a body is not always within the body.
 Centres of gravity of two bodies, as B C, be connected by a line. Draw lines of B and C from their common centre of gravity, e , to the bodies. Thus, $B : C :: C e : e B$.

2. Ascertain Centre of Gravity of any Plane Figure M

Find the figure by any point near its edge, and mark a vertical line hung from that point; then suspend it from another point and again mark direction of plumb-line. Then centre of gravity will be at point of intersection of the two marks of plumb-line.

Centre of gravity of parallel-sided objects may readily be found in this way. For instance, to ascertain centre of gravity of an arch of a bridge, draw elevation upon paper to a scale, cut out figure, and proceed with it as above directed, in order to find position of centre of gravity in elevation of the model. In actual arch, centre of gravity will have same relative position as in paper model.

In regular figures or solids, centre of gravity is same as their geometrical centres.

Line.

Circular Arc. $\frac{r^2 c}{l} = \text{distance from centre, } r \text{ representing radius, } c \text{ chord, and } l \text{ length of arc.}$

Surfaces.

Square, Rectangle, Rhombus, Rhomboid, Gnomon, Cube, Regular Polygon, Circle, Sphere, Spheroid or Ellipsoid, Spheroidal Zone, Cylinder, Circular Ring, Cylindrical Ring, Link, Helix, Plain Spiral, Spindle, all Regular Figures, and Middle Frusta of all Spheroids, Spindles, etc.

The centre of gravity of the surfaces of these figures is in their geometrical centre.

Triangle.—On a line drawn from any angle to the middle of opposite side, at two thirds of the distance from angle.

Trapezium.—Draw two diagonals, and ascertain centres of gravity of each of four triangles thus formed; join each opposite pair of these centres, and centre is at intersection of the lines.

Trapezoid. $\left(\frac{B+2b}{B+b}\right) \times \frac{m}{3} = \text{distance from B on a line joining middle of parallel sides B b, } m \text{ representing middle line.}$

Circular Arc. $\frac{c^2 r}{l} = \text{distance from centre of circle.}$

Sector of a Circle. $.4244 r = \text{distance from centre of circle, } c \text{ representing chord.}$

Semicircle. $.4244 r = \text{distance from centre.}$

Semi-semicircle. $.4244 r = \text{distance from both base and height and at their intersection.}$

Segment of a Circle. $\frac{c^3}{12a} = \text{distance from centre, } a \text{ representing area of segment.}$

Sector of a Circular Ring. $\frac{4}{3} \times \frac{\sin .5 \angle}{\text{arc } \angle} \times \frac{r^3 - r'^3}{r^2 - r'^2} = \text{distance from centre of arcs, } r \text{ and } r' \text{ representing the radii.}$

ILLUSTRATION.—Radii of surfaces of a dome are 5 and 3.5 feet, and angle (\angle) at centre = 130° .

$$\frac{4}{3} \times \frac{\sin .65^\circ}{\text{arc } 130^\circ} \times \frac{125 - 42.875}{25 - 12.25} = \frac{4}{3} \times \frac{.9063}{2.2689} \times \frac{82.125}{6.8067 \times 12.75} = 3.43 \text{ feet}$$

here, *Spherical Segment, and Spherical Zone, At centre of their*

Zone.—Ascertain centres of gravity of trapezoid and segmental zone; draw a line (equally dividing zone) perpendicular to meet centres of segments by a line cutting perpendicular to

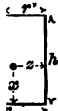
of gravity of figure will be on perpendicular, toward base, in proportionate distance of difference between centres of gravity of segments, as area of zone of trapezoid.

m and Wedge.—When end is a Parallelogram, in their geometrical centre; when the end is a Triangle, Trapezium, etc., it is in middle of its at same distance from base, as that of triangle or trapezoid of which section.

abola in its axis = .6 distance from vertex.

moid.—At same distance from its base as that of the trapezoid or ium, which is a section of it.

e.—On a line connecting centres of gravity of arcs at a proportionate point to respective areas of arcs.



$$\text{Co-ordinates. } \left(r + r' - \frac{r r'}{r + r'} \right) \frac{1}{3} = z,$$

$$\text{and } \left(\frac{2 r' + r}{r + r'} \right) \frac{h}{3} = x.$$

Solids.

2. *Parallelopipedon, Hexahedron, Octahedron, Dodecahedron, Icosahedron, Sphere, Right Spherical Zone, Spheroid or Ellipsoid, Cylinder, Link, Spindle, all Regular Bodies, and Middle Frusta of all nids and Spindles, etc.* Centre of gravity of these figures is in their 'rical centre.

hedron.—In common centre of centres of gravity of the triangles made by a through centre of each side of the figures.

and Pyramid. .25 of line joining vertex and centre of gravity of base = distance from base.

tum of a Cone or Pyramid. $\frac{(r + r')^2 + 2 r^2}{(r + r')^2 - r^2} \times \frac{1}{4} h$ = distance from centre r end, r and r', in a cone representing radii, and in a pyramid sides, and h

3. *Frustum of a Cone, Pyramid, Frustum of a Pyramid, and Ungula.*—ie distance from base as in that of triangle, parallelogram, or semicircle which is a right section of them.

isphere. .375 r = distance from centre.

rical Segment. $3.1416 \, v s^2 \left(r - \frac{v s}{2} \right)^2 \div v$ = distance from centre, vs represented sine, and v volume of segment. $\left(\frac{8 r - 3 h}{12 r - 4 h} \right) \times h$ = distance from

rical Sector. .75 (r - .5 h) = distance from centre. $\frac{2 r + 3 h}{8}$ = distance from vertex.

als.—Plane, in its geometrical centre. Conical, at a distance from the .25 of line joining vertex and centre of gravity of base.

tum of a Circular Spindle. $\frac{r^2 - r'^2}{2 (h - D.z)}$ = distance from centre of spindle, sending distance between two bases, D distance of centre of spindle from centre e, and z generating arc, expressed in units of radius.

ent of a Circular Spindle. $\frac{r^2}{2 (h - D.z)}$ = distance from centre of spindle.

-spheroids.—Prolate. .375 a.—Oblate. .375 a = distance from spheroid or Ellipsoid and its Segment.—See Haswell's M 1282.

2 of *Spheroids or Ellipsoids.* Prolate. $.75 \frac{h (2 a^2 - h^2)}{3 a^2 - h^2}$ spheroid, a representing semi-transverse diameter in a prolate in an oblate frustum.

Centre of gravity of way. For instance, to draw elevation upon above directed, in order the model. In actual as in paper model.

In regular figures or centres.

Circular Arc. $\frac{r c}{l} = \text{dis}$
length of arc.

Square, Rectangle, Rhombus, Circle, Sphere, Spheroid Ring, Cylindrical Ring, Lenses, and Middle Frusta.

The centre of gravity of each centre.

Triangle.—On a line drawn at two thirds of the distance.

Trapezium.—Draw two of four triangles thus formed is at intersection of the line.

Trapezoid. $\left(\frac{B + b}{2} \right) \times \frac{h}{3}$
parallel sides B b, h represents

Circular Arc. $\frac{c r}{l} = \text{distan}$

Sector of a Circle. $.4244 r =$

Semicircle. $.4244 r = \text{distan}$

Semi-semicircle. $.4244 r = \text{dis}$
section.

Segment of a Circle. $\frac{c^3}{12 a} =$

Sector of a Circular Ring. $\frac{r^2 - r'^2}{2} \theta$
arcs, r and r' representing the radii

ILLUSTRATION.—Radii of surface centre = 130° .

$$\frac{4}{3} \times \frac{\sin 65^\circ}{\text{arc } 130^\circ} \times \frac{125 - 42.8}{25 - 12.8}$$

Hemisphere, Spherical Segment heights.

Circular Zone.—Ascertain the comprising zone; draw a line of chords; connect centres of segments.

Then centre of gravity of the chord, at such proportionate distance of trapezoid and line connecting bears to area of trapezoid.

Centre of Gyration.

CENTRE OF GYRATION is that point in any revolving body or system of bodies in which, if the whole quantity of matter were collected, the *angular velocity* would be the same; that is, the *Momentum* of the body system of bodies is centred at this point, and the position of it is a mean proportional between the centres of Oscillation and Gravity.

If a straight bar of uniform dimensions was struck at this point, the stroke would communicate the same *angular velocity* to the bar as if the whole bar was collected at that point.

The *Angular velocity* of a body or system of bodies is the motion of a line connecting any point and the centre or axis of motion: it is the same in all parts of the same revolving body.

In different unconnected bodies, each oscillating about a common centre, their *angular velocity* is as the velocity directly, and as the distance from the centre inversely. Hence, if their velocities are as their radii, or distances from the axis of motion, their angular velocities will be equal.

When a body revolves on an axis, and a force is impressed upon it sufficient to cause it to revolve on another, it will revolve on neither, but on a line in the plane of the axes, dividing the angle which they contain; so that the sine of each part will be in the inverse ratio of the angular velocities with which the bodies would have revolved about these axes separately.

Weight of revolving body, multiplied into height due to the velocity with which centre of gyration moves in its circle, is energy of body, or mechanical power, which must be communicated to it to give it that motion.

Distance of centre of gyration from axis of motion is termed the *Radius of gyration*; and the moment of inertia is equal to product of square of radius of gyration and mass or weight of body.

The moment of inertia of a revolving body is ascertained exactly by ascertaining the moments of inertia of every particle separately, and adding them together; or, approximately, by adding together the moments of the small parts arrived at by a subdivision of the body.

○ Compute Moment of Inertia of a Revolving Body.

RULE.—Divide body into small parts of regular figure. Multiply mass weight of each part by square of distance of its centre of gravity from axis of revolution. The sum of products is moment of inertia of body.

NOTE.—The value of moment of inertia obtained by this process will be more exact, the smaller and more numerous the parts into which body is divided.

○ Compute Radius of Gyration of a Revolving Body about its Axis of Revolution.

RULE.—Divide moment of inertia of body by its mass, or its weight, and square root of quotient is length of radius of gyration.

NOTE.—When the parts into which body is divided are equal, radius of gyration may be determined by taking mean of all squares of distances of parts from axis of revolution, and taking square root of their sum.

Or, $\sqrt{R^2 + r^2} \div 2 = G$. R and r representing radii.

EXAMPLE.—A straight rod of uniform diameter and 4 feet in length, weighs 4 lbs.; what is its inertia, and where is its radius or centre of gyration?

Each foot of length weighs 1 lb., and if divided into 4 parts, centre of gyration of each is respectively .5, 1.5, 2.5, and 3.5 feet. Hence,

$$\left. \begin{array}{l} 1 \times .5^2 = .25 \\ 1 \times 1.5^2 = 2.25 \\ 1 \times 2.5^2 = 6.25 \\ 1 \times 3.5^2 = 12.25 \end{array} \right\} 21 = \text{inertia, which } \div 4 = 5.25, \text{ and } \sqrt{5.25} = 2.29 \text{ feet radius.}$$

Body is a Compound one. RULE.—Multiply weight of several bodies by squares of their distances in feet from centre of motion, and divide sum of their products by weight of entire mass; root of quotient will give distance of centre of gyration from motion or rotation.

—If two weights, of 3 and 4 lbs. respectively, be laid upon a lever (which need not be without weight) at the respective distances of 1 and 2 feet, whence of centre of gyration from centre of motion (the fulcrum)?

$$= 3; \quad 4 \times 2^2 = 16; \quad \frac{3+16}{3+4} = \frac{19}{7} = 2.71, \text{ and } \sqrt{2.71} = 1.64 \text{ feet.}$$

A single weight of 7 lbs., placed at 1.64 feet from centre of motion, and the same time, would have same *momentum* as the two weights in their places.

Centre of Gravity is given. RULE.—Multiply distance of centre of mass from centre or point of suspension, by distance of centre of gravity from same point, and square root of product will give distance of centre of gravity.

—Centre of oscillation of a body is 9 feet, and that of its gravity 4 feet from centre or point of suspension; at what distance from this point is the centre of gravity?

$$9 \times 4 = 36, \text{ and } \sqrt{36} = 6 \text{ feet.}$$

Compute Centre of Gyration of a Water-wheel.

Multiply severally twice weight of rim, as composed of buckets, etc., and twice that of arms and that of water in the buckets (when in operation) by square of radius of wheel in feet; divide the sum of these several weights, and square root of quotient will give distance in feet.

—In a wheel 20 feet in diameter, weight of rim is 3 tons, weight of buckets 2 tons, and weight of water in buckets 1 ton; what is distance of centre of gyration from axis of motion?

$$\begin{aligned} &= 3 \text{ tons} \times 10^2 \times 2 = 600 & 3 + 2 + 1 \times 2 = 12 \text{ sum of weights.} \\ &= 2 \text{ tons} \times 10^2 \times 2 = 400 \\ &= 1 \text{ ton} \times 10^2 = 100 & \text{Hence } \sqrt{\frac{1100}{12}} = \sqrt{91.67} = 9.57 \text{ feet.} \end{aligned}$$

FORMULAS.—P representing power, H horses' power, F force applied to in lbs., M mass of revolving body in lbs., r radius upon which F acts in feet, t time force is applied, n number of revolutions in time t, ω angular velocity, or number of revolutions per minute at end of time t, and $G = \frac{32.166 F r^2}{M d^2}$.

$$\begin{aligned} t &= \frac{2 P r^2 \omega}{60 G} = t; & \frac{M \omega d^2}{153.5 t r} &= F; & \frac{M n d^2}{2.56 t^2 F} &= r; & \frac{2.56 t^2 F r}{M d^2} &= n; \\ \frac{t F r}{t^2} &= \omega; & \frac{244 t P}{\omega^2 d^2} &= M; & \frac{\omega^2 M d^2}{244 t} &= P; & \frac{\omega^2 M d^2}{134 100 t} &= H. \end{aligned}$$

PROBLEM.—Rim of a fly-wheel weighing 7000 lbs., what is its centre of gyration, and what force must be applied to give it an angular velocity of 130 r.p.m. in 40 s.

Ans. 5.75 feet in 40 s.

$$\begin{aligned} \frac{130^2 \times 7000 \times 6.14^2}{134 100 \times 40} &= \frac{4 459 862 680}{5 364 000} = 829.7 \text{ lbs.} \\ \text{gyration} &= \sqrt{\frac{6.5^2 + 5.75^2}{2}} = 6.14 \text{ feet. Then } F = \\ 793.7 \text{ lbs., and } \frac{2.56 \times 40^2 \times 2793.7 \times 2}{7000 \times 6.14^2} &= 86.67 \text{ rev.} \end{aligned}$$

$\frac{l^2}{3} \div W \times g = \text{distance from axis}$. Or, square radius of gyration of body
 3
 ide by distance of centre of gravity from axis of suspension.

PLK.—Where is centre of oscillation in a rod 9 feet in length from its point
 ension, and weighing 9 lbs.?

$\frac{9}{2} = 4.5 = \text{product of weight and its centre of gravity}$; $\frac{9 \times 9^2}{3} = 243 = \text{quo-}$
 $\frac{243}{40.5} = 6 \text{ feet}$.
 product of weight of body and square of its length $\div 3$; $\frac{243}{40.5} = 6 \text{ feet}$.

n Point of Suspension is not at End of Rod. RULE.—To cube of
 e of point of suspension from top of rod or bar, add cube of its dis-
 from lower end, and multiply sum by 2.

ide product by three times difference of squares of these distances, and
 it is distance of point of oscillation from point of suspension.

PLK.—A homogeneous rod of uniform dimensions, 6 feet in length, is sus-
 1.5 feet from its upper end; what is distance of point of oscillation from
 suspension?

$$6 - 1.5 = 4.5. \quad \frac{2(4.5^3 + 1.5^3)}{3(4.5^2 - 1.5^2)} = \frac{189}{54} = 3.5 \text{ feet.}$$

Centres of Oscillation and Percussion in Bodies of Various Figures.

Axis of Motion is in Vertex of Figure, and when Oscillation or Motion is Facewise.

t Line, or any figure of uniform shape and density = .66 l.

les Triangle = .75 h.

Circle = 1.25 r.

bola = .714 h.

Cone = .8 h.

s Axis of Motion is in Centre of Body. *Wheel* = .75 radius.

n Oscillation or Motion is Sidewise. *Right Line*, or any figure of uni-
 hape and density = .66 l. *Rectangle*, suspended at one angle = .66 of di-

bola, if suspended by its vertex = .714 of axis + .33 parameter; if suspended
 dle of its base = .57 of axis + .5 parameter.

r of a Circle = $\frac{3 \text{ arc } r}{4 c}$, *c* representing chord of arc, and *r* radius of base.

$$e = .75 d. \quad \text{Cone} = \frac{4}{5} \text{ axis} + \frac{r^2}{5 \text{ axis}}.$$

$$re = \frac{2 r^2}{5(c+r)} + r + c, \text{ c representing length of cord by which it is suspended.}$$

Ascertain Centres of Oscillation and Percussion experimentally.

end body very freely from a fixed point, and make it vibrate in small arcs,
 number of vibrations it makes in a minute, and let number made in a min-
 represented by *n*; then will distance of centre of oscillation from point of

$$\text{sion be} = \frac{140850}{n^2} = \text{ins.}$$

length of a pendulum vibrating seconds, or 60 times in a min
 ins., and lengths of pendulums being reciprocally as the squares

$$\text{tions made in same time, therefore } n^2:60^2 :: 39.125: \frac{60^2 \times 39.125}{n^2}$$

th of pendulum which vibrates *n* times in a minute, or dista:
 ion below axis of motion.

To Compute Centres of Oscillation or Percussion of System of Particles or Bodies.

RULE.—Multiply weight of each particle or body by square of its distance from point of suspension, and divide sum of their products by sum of weights multiplied by distance of centre of gravity from point of suspension, and quotient will give centre required, measured from point of suspension.

$$\text{Or, } \frac{W d^2 + W' d'^2}{W g + W' g'} = \text{distance of centre.}$$

EXAMPLE 1.—Length of a suspended rod being 20 feet, and weight of a foot in length of it equal 100 oz., has a ball attached at under end weighing 100 oz.; at what point of rod from point of suspension is centre of percussion?

$100 \times 20 = 2000 = \text{weight of rod}; 2000 \times \frac{20}{2} = 20000 = \text{momentum of rod, or product of its weight, and distance of its centre of gravity}; \frac{2000 \times 20^2}{3} = 266666.66 =$

$\text{force of rod}; 1000 \times 20^2 = 400000 = \text{force of ball.}$

$$\text{Then } \frac{266666.66 + 400000}{20000 + 20000} = 16.66 \text{ feet.}$$

2.—Assume a rod 12 feet in length, and weighing 2 lbs. for each foot of its length with 2 balls of 3 lbs. each—one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$$\begin{array}{rcl} 12 \times 2 \times \frac{1}{2} & = 144 = \text{momentum of rod.} & 24 \times 12^2 = 3456 = 1152 = \text{force of rod.} \\ 3 \times 6 & = 18 = \text{“ of 1st ball.} & 3 \times 6^2 = 3 \times 36 = 108 = \text{“ of 1st ball.} \\ 3 \times 12 & = 36 = \text{“ of 2d ball.} & 3 \times 12^2 = 3 \times 144 = 432 = \text{“ of 2d ball.} \\ & 198 \text{ sum of moments.} & 1692 \text{ sum of forces.} \end{array}$$

$$\text{Then } 1692 \div 198 = 8.545 \text{ feet.}$$

MECHANICS.

MECHANICS is the science which treats of and investigates effects of forces, motion and resistance of material bodies, and of equilibrium; it is divided into two parts—**STATICS** and **DYNAMICS**.

STATICS treats of equilibrium of forces or bodies at rest. **DYNAMICS** treats of forces that produce motion, or bodies in motion.

These bodies are further divided into *Mechanics of Solid, Fluid, and Aeriform bodies*; hence the following combinations:

1. *Statics of Solid Bodies, or Geostatics.*
2. *Dynamics of Solid Bodies, or Geodynamics.*
3. *Statics of Fluids, or Hydrostatics.*
4. *Dynamics of Fluids, or Hydrodynamics.*
5. *Statics of Aeriform Bodies, or Aerostatics.*
6. *Dynamics of Aeriform Bodies, Pneumatics or Aerodynamics.*

Forces are various, and are divided into moving forces or resistances:

| | | |
|-------------------------------|------------------|-----------------|
| Gravity, | Heat or Caloric, | Inertia, |
| Muscular, | Magnetism, | Cohesion, |
| Elasticity and Contractility, | Percussion, | Adhesion, |
| Chemical, | Expansion, | and Explosions. |

Two forces of equal magnitude applied to or operating upon parallel and opposite directions, but not in same line of action, form a **couple**, and its force is sum of magnitude of the two equal forces.

Quantity of motion in a moving body, which is always equal to quantity of matter and its velocity.

Quantities of two moving bodies are inversely as their quantities of motion are equal.

STATICS.

Composition and Resolution of Forces.

When two forces act upon a body in same or in an opposite direction, effect is same as if only one force acted upon it, being sum or difference of the forces. Hence, when a body is drawn or projected in actions immediately opposite, by two or more unequal forces, it is affected as if it were drawn or projected by a single force equal to difference between two or more forces, and acting in direction of greater force.

This single force, derived from the combined action of two or more forces, is termed their *Resultant*.

The process by which the *resultant* of two or more forces, or a single force equivalent in its effect to two or more forces, is determined, is termed

Composition of Forces, and the inverse operation; or, when combined effects of two or more forces are equivalent to that of a single given force, the process by which they are determined is termed the *Decomposition or Resolution of Forces*. Two or more forces which are equivalent to a single force are termed *Components*.

When two forces act on same point their intensities are represented by sides of a parallelogram, and their combined effect will be equivalent to that of a single force acting on point in direction of diagonal of parallelogram, the intensity of which is proportional to diagonal.

ILLUSTRATION.—Attach three cords to a fixed point, *c*, Fig. 1; let *c a* and *c b* pass over fixed rollers, and suspend weights *A* and *B* therefrom.

Point *c* will be drawn by the forces *A* and *B* in directions *c a* and *c b*. Now, in order to ascertain which single force, *P*, would produce the same effect upon it, set off the distances *c m* and *c n* on the cords in the same proportion of length as weights of *A* and *B*; that is, so that $cm : cn :: A : B$; then draw parallelogram *c m o n* and diagonal *o c*, and it will represent a single force, *P*, acting in its direction, and having same ratio to weights *A* or *B* as it has to sides *c m* or *c n* of parallelogram. Consequently, it will produce same effect on point *c* as combined actions of *A* and *B*.

A parallelogram, constructed from lateral forces, and diagonal of which is mean force, is termed a *Parallelogram of Forces*.

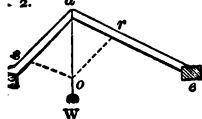


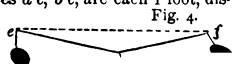
ILLUSTRATION.—Assume a weight, *W*, Fig. 2, to be suspended from *a*; then, if any distance, *a o*, is set off in numerical value upon the vertical line, *a W*, and the parallelogram, *o r a s*, is completed, *a s* and *a r*, measured upon the scale, *a o*, will represent strain upon *a c* and *a e* in same proportion that *a o* bears to weight *W*.

If several forces act upon same point, and their intensities taken in order be represented by sides of a polygon, except one, a single force proportioned and acting in direction of that one side will be their resultant.

To Resolve a Single Force into a Pair of Forces.—Figs. 3 and 4.

The ends of a cord, Fig. 3, are led over two points, *a* and *b*, and in centre of *d* at *c* a weight of 4 lbs. is suspended. If distances *a c*, *b c*, are each 1 foot, distance *a b* should be 18 ins.

Fig. 3. When cord is in this position, weight at *c* draws upon *c a* and *c b* with a force of 3 lbs.; hence *c* of 4 lbs. is equal to two forces of 3 lbs. each in direction of *a c* and *b c*.



ply ends of cord to *e f*, Fig. 4, distance being 22 ins., then the weight of 5 lbs.; hence one force of 4 lbs. is equal to two of 5 lbs.

Equilibrium of Forces.

Two bodies which act directly against each other in same line are in equilibrium when their quantities of motion are equal; that is, when product of mass of one, into velocity with which it moves or tends to move, is equal to product of mass of other, into its actual or virtual* velocity.

When the velocities with which bodies are moved are same, their forces are proportional to their masses or quantities of matter. Hence, when equal masses are in motion, their forces are proportional to their velocities.

Relative magnitudes and directions of any two forces may be represented by two right lines, which shall bear to each other the relations of the forces and which shall be inclined to each other in an angle equal to that made by direction of the forces.

Fig. 4.

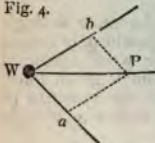


ILLUSTRATION.—Assume a body, W, to weigh 150 lbs., and resting upon a smooth surface, to be drawn by two forces, a and b, Fig. 5, = 24 and 30 lbs., which make with each other an angle, $\angle W b = 105^\circ$, in which direction and with what acceleration will motion occur?

$\angle a W b = 105^\circ$, and $\cos. 180^\circ - 105^\circ = \cos. 75^\circ$, $\cos.$

$$P = \sqrt{30^2 + 24^2 - 2 \times 30 \times 24 \cos. 75^\circ} = \sqrt{900 + 576 - 1440 \cos. 75^\circ} \\ = \sqrt{1476 - (1440 \times .26882)} = \sqrt{1103.3} = 33.21 \text{ lbs.}$$

The acceleration is $\frac{P g}{W} = \frac{33.21 \times 32.166}{150} = 7.1215 \text{ feet.}$

Angle of Repose is greatest inclination of a plane to horizon at which a body will remain in equilibrium upon it.

Hence greatest angle of obliquity of pressure between two planes, coincident with stability, is the angle tangent of which is equal to coefficient of friction of the two planes.

Inertia is resistance which a body at rest offers to an external power to be put in motion or to change its velocity or direction when in motion.

To Compute Inertia of a Revolving Body.

Divide it into small parts of a regular figure, multiply weight of each part by square of its distance of its centre of gravity from axis of revolution, and sum of products will give moment of inertia of body.

DYNAMICS.

DYNAMICS is the investigation of the laws of *Motion of Solid Bodies or of Matter, Force, Velocity, Space, and Time.*

Mass of a body is the quantity of matter of which it is composed.

Force is divided into Motive, Accelerative, or Retardative.

Motive Force, or *Momentum*, of a body, is the product of its mass and its velocity, and is its quantity of motion. This force can, therefore, be ascertained and compared in any number of bodies when these two quantities are known.†

Accelerative or Retardative Force is that which respects velocity of motion only, accelerating or retarding it; and it is denoted by quotient of motive force, divided by mass or weight of body. Thus, if a body

* Virtual velocity is the velocity which a body in equilibrium would acquire were the equilibrium to be disturbed.

† It is compared, because it is not referable to any standard, as a ton, pound, &c. Thus, if a cannon-ball weighing 15 lbs., projected with a velocity of 1500 feet per second, the momentum of the body, according to the above rule, would be $15 \times 1500 = 22,500$, not pounds, for it is a pressure with which it cannot be compared.

lbs. is impelled by a force of 40 lbs., accelerating force is 8 lbs.; if a force of 40 lbs. act upon a body of 10 lbs., accelerating force is 4 lbs., or half former, and will produce only half velocity.

With equal masses, velocities are proportional to their forces.

With equal forces, velocities are inversely as the masses.

With equal velocities, forces are proportional to the masses.

Work is product of force, velocity, and time.

Motion.—The succession of positions which a body in its motion progressively occupies forms a line which is termed the trajectory, or path of the moving body.

Motion is *Uniform* when equal spaces are described by it in equal times, and *Variable* when this equality does not occur. When spaces described in equal times increase continuously with the time, a variable motion is termed *accelerated*, when spaces decrease, *retarded*, and when equal spaces are described within certain intervals only, the motion is termed *periodic*, and intervals periods. Uniform motion is illustrated by the progressive motion of hands of a watch; variable in progressive velocity of falling and upwardly projected bodies; and periodic by oscillation of a pendulum or strokes of a piston of a steam-engine.

Uniform Motion.

$$\text{FORMULAS. } f v, \frac{f s}{t}, H \text{ 550, and } \frac{W}{t} = P; \quad \frac{P}{v}, \frac{W}{s}, \text{ and } \frac{H \text{ 550}}{v} = f; \quad \frac{s}{t}, \frac{H \text{ 550}}{f}, \text{ and } \frac{W}{f t} = v; \quad v t, \frac{P t}{f}, \frac{W}{f}, \text{ and } \frac{H \text{ 550 } t}{f} = s; \quad \frac{s f}{P}, \frac{s}{v}, \frac{W}{f v}, \frac{f s}{H \text{ 550}} = t; \quad f s, H \text{ 550 } t, P t, \text{ and } f v t = W; \quad \frac{P}{550}, \frac{f v}{550}, \frac{f s}{550 t}, \text{ and}$$

$= H$. P representing power in effect, body, or momentum, f force in lbs., v and s velocity and space in feet per second, t time in seconds, H horse-power, and W work in foot-lbs.

When two or more bodies, etc., are compared, two or more corresponding letters, p, p', V, v, v' , etc., are employed.

ILLUSTRATION 1.—Two bodies, one of 20, the other of 10 lbs., are impelled by same momentum, say 60. They move uniformly, first for 8 seconds, second for 6; what spaces described by both?

$$60 \div 20 = 3 = V, \text{ and } 60 \div 10 = 6 = v.$$

$$\text{Hence } T V = 3 \times 8 = 24 = S, \text{ and } t v = 6 \times 6 = 36 = s, \text{ spaces respectively.}$$

—If a power of 12 800 effects has a velocity of 10 feet per second, what is its force?

$$12\,800 \div 10 = 1280 \text{ lbs.}$$

Uniform Variable Motion.

Motion described by a body having uniform variable motion is represented by the sum or difference of velocity, and product of acceleration and time, according as the motion is accelerated or retarded.

ILLUSTRATION 1.—A sphere rolling down an inclined plane with an initial velocity of 25 feet, acquires in its course an additional velocity at each second of time of 5 feet; what will be its velocity after 3 seconds?

$$25 + 5 \times 3 = 40 \text{ feet.}$$

—A locomotive having an initial velocity of 30 feet per second is so retarded that each second it loses 4 feet; what is its velocity after 6 seconds?

$$30 - 4 \times 6 = 6 \text{ feet.}$$

3 F*

Uniform Motion Accelerated.

In this motion, velocity acquired at end of any time whatever is equal to product of accelerating force into time, and space described is equal to product of half accelerating force into square of time, or half product of velocity and time of acquiring the velocity.

Spaces described in successive seconds of time are as the odd numbers, 1, 3, 5, 7, 9, etc.

Gravity is a constant force, and its effect upon a body falling freely in a vertical line is represented by g , and the motion of such body is uniformly accelerated.

The following theorems are applicable to all cases of motion uniformly accelerated by any constant force, F :

$$.5 t v = .5 g F t^2 = \frac{v^2}{2 g F} = s.$$

$$\frac{2 s}{v} = \frac{v}{g F} = \sqrt{.5 g F} = t.$$

$$\frac{2 s}{t} = g F t = \sqrt{2 g F s} = v.$$

$$\frac{v}{g t} = \frac{2 s}{g t^2} = \frac{v^2}{2 g s} = F.$$

When gravity acts alone, as when a body falls in a vertical line, F is omitted. Thus,

$$.5 g t^2 = \frac{v^2}{2 g} = s.$$

$$g t = \sqrt{2 g s} = v.$$

$$\frac{v}{g} = \sqrt{\frac{2 s}{g}} = t.$$

$$\frac{v}{t} = \frac{2 s}{t^2} = \frac{v^2}{2 s} = g$$

t representing time in seconds, and s velocity in feet per second.

If, instead of a heavy body falling freely, it be projected vertically upward or downward with a given velocity, v , then $s = t v \mp .5 g t^2$; an expression in which — must be taken when the projection is upward, and + when it is downward.

ILLUSTRATION I. — If a body in 10 seconds has acquired a velocity by uniformly accelerated motion of 26 feet, what is accelerating force, and what space described in that time?

$$26 \div 10 = 2.6 = \text{accelerating force}; \quad \frac{2.6}{2} \times 10^2 = 130 \text{ feet} = \text{space described}$$

$$2. \text{—A body moving with an acceleration of } 15.625 \text{ feet describes in } 1.5 \text{ seconds a space} = \frac{15.625 \times (1.5)^2}{2} = 17.578 \text{ feet.}$$

$$3. \text{—A body propelled with an initial velocity of } 3 \text{ feet, and with an acceleration of } 5 \text{ feet, describes in } 7 \text{ seconds a space} = 3 \times 7 + 5 \times \frac{7^2}{2} = 143.5 \text{ feet.}$$

$$4. \text{—A body which in } 180 \text{ seconds changes its velocity from } 2.5 \text{ to } 7.5 \text{ feet, traverses in that time a distance of } \frac{2.5 + 7.5}{2} \times 180 = 900 \text{ feet.}$$

$$5. \text{—A body which rolls up an inclined plane with an initial velocity of } 40 \text{ feet per second, by which it suffers a retardation of } 8 \text{ feet, ascends only } \frac{40}{8} = 5 \text{ seconds, and } 40^2 \div 2 \times 8 = 100 \text{ feet in height, then rolls back, and returns, after } 10 \text{ seconds with a velocity of } 40 \text{ feet, to its initial point; and after } 12 \text{ seconds arrives at a distance of } 40 \times 12 - 4 \times 12^2 = 96 \text{ feet below point, assuming plane to be extended backward.}$$

Circular Motion.

$$\frac{2 p r n}{60} = \frac{2 p r n'}{t} = v;$$

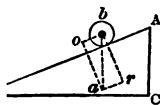
$$\frac{5500 \text{ HP}}{r n} = \frac{W}{2 p r n'} = f;$$

$$\frac{f r n}{5500} = \frac{f 2 p r n'}{5500 \times 60} = \text{HP}$$

$f 2 p r n' = \frac{f t 2 p r n}{60} = W.$ r representing radius in feet, n number of revolutions of circle per minute, n' total revolutions, f force in lbs., t time in seconds, or horse-power.

Motion on an Inclined Plane.

To Ascertain Conditions of Motion by Gravity.



Assume A B, Fig. 6, an inclined plane, B C its base, A C its height, and *b* a body descending the plane; from dot, centre of gravity of body, draw *b a* perpendicular to B C, representing pressure of *b* by gravity; draw *b o* parallel and *b r* perpendicular to A B, and complete parallelogram; then force *b a* is equal to both *b o*, *b r*, of which *b r* is sustained by reaction of plane, and *o* is wholly effective in accelerating motion of body.

This force be represented by *f*, and *b a*, by *g* or force of gravity, then by similar $e, f : g :: b o : b a : A C : A B$. Hence, $\frac{A C \times g}{A B} = f$.

A B = *l*, A C = *h* and $\angle A B C = a$, then force which produces motion on the incline becomes $g \frac{h}{l}$, and $g \sin. a$.

Therefore, accelerating force on an inclined plane is constant, and equations of it will be obtained by substituting its value of *f* for *g* in equations 1, 2, and 618.

$$\frac{g h t^2}{2 l}, \quad \frac{l v^2}{2 g h}, \quad .5 t v, \quad .5 g t^2 \sin. a, \quad \text{and} \quad \frac{v^2}{2 g \sin. a} = s$$

$$\frac{2 s}{t}, \quad \frac{g h t}{l}, \quad \sqrt{\frac{2 g h s}{l}}, \quad g t \sin. a, \quad \text{and} \quad \sqrt{2 g s \sin. a} = v.$$

$$\frac{l v}{g h}, \quad \sqrt{\frac{2 l s}{g h}}, \quad \frac{v}{g \sin. a}, \quad \text{and} \quad \sqrt{\frac{2 s}{g \sin. a}} = t. \quad a \text{ representing } \angle A B C.$$

When a Body is projected down or up an Inclined Plane, with a given Velocity.—The distance which it will be from point of projection in a given time will be

$$t v \pm \frac{g h t^2}{2 l}, \quad \text{and} \quad \frac{t}{2 l} (2 l v \pm g h t) = s.$$

EXAMPLE 1.—Length of an inclined plane is 100 feet, and its angle of inclination 60° ; what is time of a body rolling down it, and velocity acquired?

$$\sin. 60^\circ = .866.$$

$$\frac{2 \times 100}{32.16 \times .866} = \sqrt{7.18} = 2.68 \text{ seconds, and } 32.16 \times 2.68 \times .866 = 74.64 \text{ feet.}$$

If a body is projected up an inclined plane, which rises 1 in 6, with a velocity of 10 feet per second, what will be its place and velocity at end of 6 seconds?

$$50 - \frac{32.16 \times 1 \times 6^2}{2 \times 6} = 203.52 \text{ feet from bottom, and } 50 - \left(32.16 \times 6 \times \frac{1}{6} \right) = 17.84 \text{ feet.}$$

Effect an ascent up an inclined plane in least time, its length, to its height, as twice weight to power.

Work Accumulated in Moving Bodies.

Quantity of work stored in a body in motion is same as that which would be accumulated in it by gravity if it fell from the height due to the velocity. Accumulated work expressed in foot-lbs. is equal to product of height so in feet, and weight of body in lbs. Height due to velocity is equal to square of velocity divided by 64.4, and work and velocity may be determined from each other by following rules:

To Compute Accumulated Work.

RULE.—Multiply weight in lbs. by square of velocity in feet per second, and divide by 64.4, and quotient is accumulated work in foot-lbs.

$$W = \frac{v^2 \times w}{64.4}, \quad \text{or,} \quad W = w \times h. \quad W \text{ representing work, } w \text{ weight in lbs., and } h \text{ due to velocity in feet per second.}$$

Work by Percussive Force.

If a wedge is driven by strokes of a hammer or other heavy mass of percussive force is measured by quantity of work accumulated in a body. This work is computed by preceding rules, from weight of body and velocity with which a stroke is delivered, or directly from height of fall, if gravity be percussive power.

Useful work done through a wedge is equal to work expended assuming that there is no elastic or vibrating reaction from the stroke; the work had been exerted by a constant pressure equal to weight of driving body, exerted through a space equal to height of fall, or height of its final velocity.

If elastic action intervenes, a portion of work exerted is absorbed in elastic stress to resisting body; and the elastic action may be, in some cases, so great as to absorb the work expended.

The principle of action of a blow on a wedge is alike applicable to the stroke of a monkey of a pile-driver upon a pile.

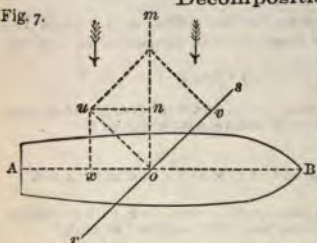
If there be no elastic action, the work expended being product of weight of monkey by height of its fall, is equal to work performed in driving pile: that is, to product of resistance to its descent by depth through which it is driven by each blow of monkey.

ILLUSTRATION.—If a horse draws 200 lbs. out of a mine, at a speed of 2 miles an hour, how many units of work does he perform in a minute, coefficient of friction .05?

$$\frac{2 \times 5280}{60} = 176 \text{ feet per minute. Hence, } 176 \times 200 + .05 \times 200 = 35210 \text{ ft. lbs.}$$

Decomposition of Force.

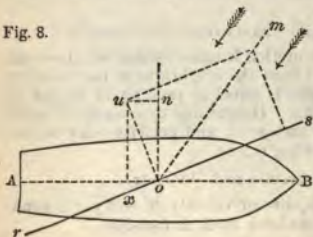
Fig. 7.



and ou , since they are the sides of parallelogram of which mo represents force of wind, is diagonal. Force of wind, therefore, is measured by ou , both in magnitude and direction, and represents actual pressure on sail.

Draw un and ux parallel to oa and om , thus forming parallelogram $ounx$.

Fig. 8.



By parallelogram of force illustrated above a vessel is enabled to be sailed with a free wind and one.

Assume wind to be free or in the direction of arrows, Fig. 7, and perpendicular line AB , the course of vessel.

Let line mo represent direction of wind, and rs plane of sail. From o draw ou perpendicular to mo , and or perpendicular to rs .

By principle of parallelogram of force mo may be decomposed into ou and or .

Hence force ou is equal to the force or . Force or acts in a direction perpendicular to vessel's course, and ou is to drive vessel onward.

It can thus be shown that the direction of sail bisects angle mor , the effect of or is greater than when the sail is in any other position.

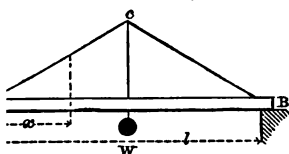
Assume wind to be ahead as shown in arrows, Fig. 8. Let mo represent direction and force of wind, and rs direction of sail; from o draw ou perpendicular to mo , and or perpendicular to rs . ou represents effective force that acts upon the vessel, and or that which drives her to leeward.

For full treatises on this subject, see John C. Trautwine's *Engineer's Pocket-book*, 2nd ed., Experimental Mechanics, London, 1871; and *Dynamics, Construction of Machinery*, etc., Warr, London, 1851.

MOMENTS OF STRESS.

Describe and Compute Moments of Stress on Girders or Beams.

Supported at Both Ends.



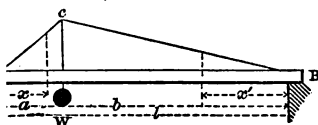
Loaded in Centre, Fig. 1.—Assume A B, a beam. At centre erect $Wc = \frac{Wl}{4}$. Connect A c and c B, and any vertical distance between them and A B will give moment required at that point.

$$\frac{Wx}{2} = M \text{ at any point. } W \text{ represents}$$

weight or load, l length of span, x horizontal distance from nearest support at M, the moment of stress, is required.

ILLUSTRATION.—Assume $l = 10$ feet, $W = 10$ lbs., and $x = 3$ feet.

$$n, Wc = \frac{10 \times 10}{4} = 25 \text{ lbs. at centre of span; and } \frac{10 \times 3}{2} = 15 \text{ lbs. at } x.$$



Loaded at Any Point, Fig. 2.—Proceed as for previous figure.

$$\frac{Wab}{l} \text{ or } Wc = \text{maximum load.}$$

$$\frac{Wxb}{l} = M \text{ between A and W.}$$

$$\frac{Wxa}{l} = M \text{ between W and B.}$$

presenting least distance of W to support, greatest distance.

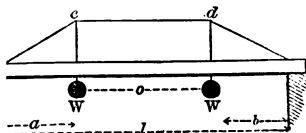
ILLUSTRATION.—Take elements as before with $a = 3$ feet, $x = 1.5$, and $a' = 3.5$ feet.

$$n, Wc = \frac{10 \times 3 \times 7}{10} = 21 \text{ lbs. at point of stress; } \frac{10 \times 1.5 \times 7}{10} = 10.5 \text{ lbs. at } x$$

$$n \text{ A and W, and } \frac{10 \times 3.5 \times 3}{10} = 10.5 \text{ lbs. at } x \text{ between W and B.}$$

x and x' must be taken from the pier, which is on the same side of W as the stress desired.

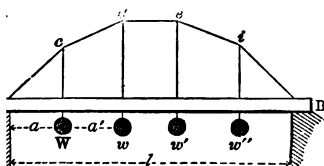
d with Two Equal Weights at Equal Distances from Supports, alike to a Transverse Girder in a Single Line of Railway.—Fig. 3.



At point of stress of weights erect Wc and Wd , each $= Wa$. Connect A c d and B, and vertical distances between them and A B will give moments required.

$$\frac{W(l-a)}{2} = Wa = Wb = M \text{ at any point between weights.}$$

d with Four Equal Weights, symmetrically bearing from Centre, alike to a Transverse Girder in a Double Line of Railway.—Fig. 4.



At W and w'' erect Wc , and $w''i = 2Wa$, and at w and w' erect Wd , $w'e$, each $= W(2a + a')$.

Connect A c d e i and B, and ordinates from them to A B will give moments required.

$$W(2a + a') = M \text{ at } w \text{ and } w';$$

$$2Wa = M \text{ at } W \text{ and } w''.$$

ILLUSTRATION.—Assume W each

10 lbs. 2 feet apart, and $l = 10$ feet.

$$l, 10(2 \times 2 + 2) = 60 \text{ at } w \text{ or } w', \text{ and } 2 \times 10 \times 2 = 40 \text{ at } W \text{ or } w''.$$

Equilibrium of Forces.

Two bodies which act directly against each other in same line are in equilibrium when their quantities of motion are equal: that is, when product mass of one, into velocity with which it moves or tends to move, is equal product of mass of other, into its actual or virtual* velocity.

When the velocities with which bodies are moved are same, their forces are proportional to their masses or quantities of matter. Hence, when masses are in motion, their forces are proportional to their velocities.

Relative magnitudes and directions of any two forces may be represented by two right lines, which shall bear to each other the relations of the forces and which shall be inclined to each other in an angle equal to that made by direction of the forces.

Fig. 4.

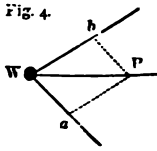


ILLUSTRATION.—Assume a body, W, to weigh 150 lbs., resting upon a smooth surface, to be drawn by two forces, a and b, Fig. 5 = 24 and 30 lbs., which make with each other an angle, $\angle W b = 105^\circ$, in which direction and with what acceleration will motion occur?

$\angle a W b = 105^\circ$, and $\cos 180^\circ - 105^\circ = \cos 75^\circ$, and force.

$$P = \sqrt{30^2 + 24^2 - 2 \times 30 \times 24 \cos 75^\circ} = \sqrt{900 + 576 - 1440 \cos 75^\circ} \\ = \sqrt{1476 - (1440 \times .25882)} = \sqrt{1123.3} = 33.21 \text{ lbs.}$$

$$\text{The acceleration is } \frac{P \cdot g}{W} = \frac{33.21 \times 32.166}{150} = 7.1215 \text{ feet.}$$

Angle of Repose is greatest inclination of a plane to horizon at which body will remain in equilibrium upon it.

Hence greatest angle of obliquity of pressure between two planes, consistent with stability, is the angle tangent of which is equal to coefficient of friction of the two planes.

Inertia is resistance which a body at rest offers to an external power be put in motion or to change its velocity or direction when in motion.

To Compute Inertia of a Revolving Body.

Divide it into small parts of a regular figure, multiply weight of each part by square of its distance of its centre of gravity from axis of revolution, and sum of products will give moment of inertia of body.

DYNAMICS.

DYNAMICS is the investigation of the laws of Motion of Solid Bodies of Matter, Force, Velocity, Space, and Time.

Mass of a body is the quantity of matter of which it is composed.

Force is divided into Motive, Accelerative, or Retardative.

Motive Force, or Momentum, of a body, is the product of its mass and velocity, and is its quantity of motion. This force can, therefore, be ascertained and compared in any number of bodies when these two quantities are known.†

Accelerative or Retardative Force is that which respects velocity of motion only, accelerating or retarding it; and it is denoted by quotient of motive force, divided by mass or weight of body. Thus, if a body

in the velocity which a body is equilibrium would acquire were the action

is applicable to any standard, as a ton, pound, etc. Thus, if a body with a velocity of 150 feet per second, strikes a mass of 100 lbs., it would be 150 x 150 = 22,500, and the product is compared.

is impelled by a force of 40 lbs., accelerating force is 8 lbs.; force of 40 lbs. act upon a body of 10 lbs., accelerating force 8 lbs., or half former, and will produce only half velocity.

Equal masses, velocities are proportional to their forces.

Equal forces, velocities are inversely as the masses.

Equal velocities, forces are proportional to the masses.

is product of force, velocity, and time.

—The succession of positions which a body in its motion occupies forms a line which is termed the trajectory, or path moving body.

Motion is *Uniform* when equal spaces are described by it in equal time; and *Variable* when this equality does not occur. When spaces

are described in equal times increase continuously with the time, a variable motion is termed *accelerated*, when spaces decrease, *retarded*, and when

spaces are described within certain intervals only, the motion is termed *periodic*, and intervals periods. Uniform motion is illustrated

by the successive motion of hands of a watch; variable in progressive

receding and upwardly projected bodies; and periodic by oscillation of a pendulum or strokes of a piston of a steam-engine.

Uniform Motion.

As $f v$, $\frac{f s}{t}$, $H 550$, and $\frac{W}{t} = P$; $\frac{P}{v}$, $\frac{W}{s}$, and $\frac{H 550}{v} = f$; $\frac{s}{t}$, $\frac{v}{f}$, and $\frac{W}{f t} = v$; $v t$, $\frac{P t}{f}$, $\frac{W}{f}$, and $\frac{H 550 t}{f} = s$; $\frac{s f}{P}$, $\frac{s}{v}$, $\frac{W}{f v}$, t ; $f s$, $H 550 t$, $P t$, and $f v t = W$; $\frac{P}{550}$, $\frac{f v}{550}$, $\frac{f s}{550 t}$, and

P representing power in effect, body, or momentum, f force in lbs., v and s space in feet per second, t time in seconds, H horse-power, and W work

for more bodies, etc., are compared, two or more corresponding letters, V, v, v' , etc., are employed.

EXAMPLE 1.—Two bodies, one of 20, the other of 10 lbs., are impelled by same power, say 60. They move uniformly, first for 8 seconds, second for 6; what spaces described by both?

$$60 \div 20 = 3 = V, \text{ and } 60 \div 10 = 6 = v.$$

$$V = 3 \times 8 = 24 = S, \text{ and } t v = 6 \times 6 = 36 = s, \text{ spaces respectively.}$$

power of 12800 effects has a velocity of 10 feet per second, what is its velocity after 3 seconds?

$$12800 \div 10 = 1280 \text{ lbs.}$$

Uniform Variable Motion.

described by a body having uniform variable motion is represented by the difference of velocity, and product of acceleration and time, as the motion is accelerated or retarded.

EXAMPLE 1.—A sphere rolling down an inclined plane with an initial velocity of 30 feet per second acquires in its course an additional velocity at each second of time of 3; what will be its velocity after 3 seconds?

$$25 + 5 \times 3 = 40 \text{ feet.}$$

EXAMPLE 2.—A sphere having an initial velocity of 30 feet per second and second it loses 4 feet; what is its velocity after 6 seconds?

$$30 - 4 \times 6 = 6 \text{ feet.}$$

3 F*

Fig. 5.

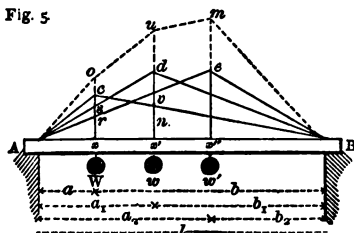


ILLUSTRATION.—Take $a = 2$ feet, $a_1 = 4$, $a_2 = 6$, $b = 8$, $b_1 = 6$, $b_2 = 4$, W, w, w' each 10 lbs., and $l = 10$ feet, carefully observing Note to Fig. 2.

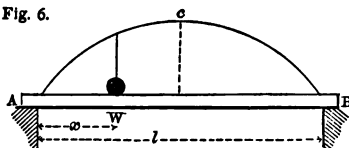
Then $\frac{1}{l} (W b + w b_1 + w' b_2) = M$ at x .

Take $x = 2$. Then $\frac{1}{10} (10 \times 8 \times 2 + 10 \times 6 \times 2 + 10 \times 4 \times 2) = \frac{360}{10} = 36$ lbs.

$x' = 4$. $\frac{1}{10} (10 \times 2 \times 6 + 10 \times 6 \times 4 + 10 \times 4 \times 4) = \frac{520}{10} = 52$ lbs.

$x'' = 6$. $\frac{1}{10} (10 \times 2 \times 4 + 10 \times 4 \times 4 + 10 \times 4 \times 6) = \frac{480}{10} = 48$ lbs.

Fig. 6.



Loaded with a Rolling Weight.—Fig. 6.

Define parabola $A c B$ as determined by $\frac{W x}{l} =$ the ordinate at x and vertical distances between A will give moments.

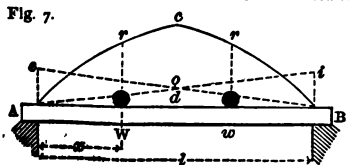
$$\frac{W x (l - x)}{l} = M \text{ at any point}$$

Loaded Uniformly its Entire Length.—Define parabola as at Fig. 6, ordinate which at $c = \frac{w l^2}{8}$. L representing stationary or dead load per unit of length.

$$\frac{L x}{2} (l - x) = M \text{ at any point, and } \frac{w l^2}{8} = M \text{ at centre.}$$

Loaded with Two Connected Weights, moving in either Direction, alike to a Locomotive or Car on a Railway.—Fig. 7.

Fig. 7.



Define parabola $A c B$ as determined by $\frac{(W + w) x}{l} = c$.

At A and B erect $A c$, $B i$ = vertical distances between A and B will give moments.

$$\frac{x}{l} [(W + w) (l - x) - w d] =$$

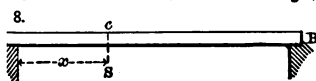
at $x = \frac{l}{2} \pm \frac{w d}{2(W + w)}$. Or if W and w

$w = 3$, $d = 4$, and W each 10 lbs., and $l = 10$ feet.

$$\frac{10 \times 4}{10 \times 4} = M \text{ at any point, as at } W r, w r.$$

Shearing Stress.

Determine Shearing Stress at any Part of a Girder or Beam and under any Distribution of Load.



Required to determine stress of a beam at any point as c, Fig. 8.

Assume W = load between A and c, and w that between B and c.

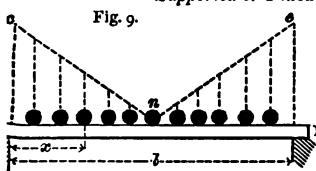
Then Sx at $c = P - W$, or $P' - w$.

be greater of the two values to be taken.

x representing shearing stress at any point x , P and P' the reaction on supports to total load on beam between supports, W and w loads or stress concentrated at point.

To Describe and Ascertain Shearing Stress in a Girder or Beam.

Supported or Fixed at Both Ends.



Loaded Uniformly. Fig. 9.

At A and B, erect Ac , Be , each equal to $\frac{Wl}{2}$. Connect c and e at middle of span as at n , and vertical distances between AB and cn will give shearing stresses as determined by the ordinates to cn .

$L\left(\frac{l}{2} - x\right) = S$. Sign of result to

be regarded. L representing distributed load per unit of length.

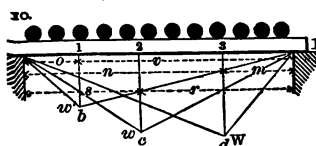
ILLUSTRATION.—Assume $L = 10$ lbs. per foot, $l = 10$, and $x = 2.5$ feet.

$$\text{Then } 10\left(\frac{10}{2} - 2.5\right) = 25 \text{ lbs.}$$

NOTE.—The moment of rupture at any point, produced by several loads acting simultaneously on a beam, is equal to the sum of the moments produced by the several loads acting separately.

For other Formulas and Diagrams see Strains in Girders, by William Humber, J.E., London, 1872.

Illustration deduced by Graphic Delineation of Greatest Stress, with a Uniformly Distributed Load of 14 000 Lbs.—Fig. 10.



Determine moment of weights by formulas $\frac{Wmn}{l}$, $\frac{wrs}{l}$, and $\frac{w'ov}{l}$.

Assume $W = 7000$ lbs., $w = 4000$, and $w' = 3000$, $m = 7$ feet, $n = 13$, $r = 13$, $s = 7$, $o = 3$, $v = 17$, and $l = 20$.

$$\text{Then } W = \frac{7000 \times 13 \times 7}{20} = 31850,$$

$$: \frac{4000 \times 13 \times 7}{20} = 18200, \text{ and } w' = \frac{3000 \times 3 \times 17}{20} = 7650, \text{ and let fall perpendicular}$$

thereto, as $3d$, $2c$, and $1b$.

Connect d , c , and b with A , B , and sum of distances of intersections of the perpendiculars, from 3 , 2 , and 1 respectively, will give stress upon the points.

To determine Greatest Stress at Greatest Load.

$$\text{Stress at } 3d = 31850$$

$$\text{" " } 2c = 13 : 18200 : 7 = 9800$$

$$\text{Stress at } 1b = 17 : 7650 : 3$$

$$9800 + \frac{7 \times 13 \times 4000 \times 5}{20} = 52200 \text{ lbs., concentrated load at } W, \text{ and } P$$

on a beam of length l of 4000 lbs.

MECHANICAL POWERS.

MECHANICAL POWER is a compound of *Weight*, or *Force* and *Fulcrum* it cannot be increased by mechanical means.

The Powers are three in number—viz., LEVER, INCLINED PLANE, PULLEY.

NOTE.—A Wheel and Axle is a *continuous* or *revolving lever*, a Wedge a *declined plane*, and a Screw a *revolving inclined plane*.

LEVER.

Levers are straight, bent, curved, single, or compound.

To Compute Length of a Lever.

When Weight and Power are given. **RULE.**—Divide weight by *p* and quotient is leverage, or distance from fulcrum at which power sup weight.

Or, $\frac{W}{P} = p$. *W* representing weight, *P* power, and *p* distance of power from fulcrum.

EXAMPLE.—A weight of 1600 lbs. is to be raised by a power or force of 8 lbs. required length of longest arm of lever, shortest being 1 foot.

$$1600 \div 80 = 20 \text{ feet.}$$

To Compute Weight that can be raised by a Lever.

When its Length, Power, and Position of its Fulcrum are given. **RULE.**—Multiply power by its distance from fulcrum, and divide product by distance of weight from fulcrum.

Or, $\frac{P \cdot p}{w} = W$. *w* representing distance of weight from fulcrum.

EXAMPLE.—What weight can be raised by 375 lbs. suspended from end of 8 feet from fulcrum, distance of weight from fulcrum being 2 feet?

$$375 \times 8 \div 2 = 1500 \text{ lbs.}$$

To Compute Position of Fulcrum.

When Weight and Power and Length of Lever are given, and when fulcrum is between Weight and Power. **RULE.**—Divide weight by power *p* to quotient, and divide length by sum thus obtained.

Or, $L \div \left(\frac{W}{P} + 1 \right) = w$. *L* representing entire length of lever.

EXAMPLE.—A weight of 2460 lbs. is to be raised with a lever 7 feet long; power of 300; at what part of lever must fulcrum be placed?

$$2460 \div 300 = 8.2, \text{ and } 8.2 + 1 = 9.2. \text{ Then } 7 \times 12 \div 9.2 = 9.13 \text{ ins.}$$

When Weight is between Fulcrum and Power. **RULE.**—Divide *w* by quotient of weight, divided by power.

$$\text{Or, } L \div \frac{W}{P} = w.$$

NOTE.—To Compute Length of Arm of Lever to which Weight is attached.

RULE.—To Compute Length of Arm of Lever to which Power is attached.

— Multiply power by length of arm to which weight is attached.

$$\text{Or, } \frac{P \cdot p}{W} = w.$$

EXPL.—A weight of 1600 lbs., suspended from a lever, is supported by a power applied at other end of arm, 20 feet in length; what is length of arm?

$$80 \times 20 \div 1600 = 1 \text{ foot.}$$

1.—These rules apply equally *When fulcrum (or support) of lever is between and power; * when fulcrum is at one extremity of lever, and power, or weight, at other; † and when arms of lever are equally or unequally bent or curved.*

Compute Power Required to Raise a given Weight.

When Length of Lever and Position of Fulcrum are given. **RULE.**—Multiply weight to be raised by its distance from fulcrum, and divide product by distance of power from fulcrum.

$$\text{Or, } \frac{W w}{P} = P.$$

EXPL.—Length of a lever is 10 feet, weight to be raised is 3000 lbs., and its distance from fulcrum is 2 feet; what is power required?

$$\frac{3000 \times 2}{10 - 2} = \frac{6000}{8} = 750 \text{ lbs.}$$

Compute Length of Arm of Lever to which Power is applied.

When Weight, Power, and Distance of Fulcrum are given. **RULE.**—Multiply weight by its distance from fulcrum, and divide product by power.

$$\text{Or, } \frac{W w}{P} = p.$$

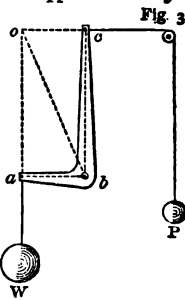
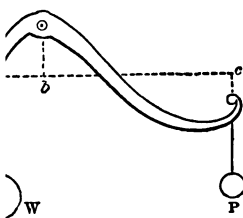
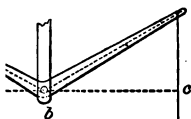
EXPL.—A weight of 400 lbs., suspended 15 ins. from fulcrum, is supported by a power of 50, applied at other; what is length of the arm?

$$400 \times 15 \div 50 = 120 \text{ ins.}$$

When Arms of a Lever are bent or curved, Distances taken from perpendiculars, drawn from lines of direction of weight and power, must be measured on a line running horizontally through fulcrum, as $a b c$, Figs. 1 and 2.

When Arms of a Lever are at Right Angles, and Power and Weight are applied at a Right Angle to each other, Fig. 3; The moments are computed directly as $a b$ to $b c$.

Thrust, or pressure on fulcrum, is in this case less than sum of power and weight; and it may be determined by drawing a parallelogram upon the two arms of lever, arms representing inversely their respective forces. That is, $a b$ represents magnitude and direction of power P . Diagonal $o b$ of parallelogram represents direction of third force, or thrust upon fulcrum.



* Pressure upon fulcrum is equal to sum of weight and power.

† Pressure upon fulcrum is equal to difference of weight and power.

Wheel and Pinion Combinations, or Complex Wheel-work.

Power, multiplied by product of radii or circumferences, or number of teeth of wheels, is equal to weight, multiplied by product of radii or circumferences, or number of teeth or leaves of pinions.

$$\text{Or, } P R R' R'', \text{ etc.} = W r r' r'', \text{ etc.}$$

NOTE.—Cogs on face of wheel are termed *teeth*, and those on surface of axle are termed *leaves*; the axle itself in this case is termed a *pinion*.

Rack and Pinion.

To Compute Power of a Rack and Pinion.

RULE.—Multiply weight to be sustained by quotient of radius of pinion, divided by radius of crank, and product is power required.

$$\text{Or, } W \frac{r}{R} = P.$$

When Pinion on Crank Axle communicates with a Wheel and Pinion. RULE.—Multiply weight to be sustained by quotient of product of radii of pinions, divided by radii of crank and wheel, and product is power required.

$$\text{Or, } W \frac{r r'}{R R'} = P.$$

EXAMPLE.—If radii of pinions of a jack-screw are each one inch; of crank wheel 10 and 5 ins.; what power will sustain a weight of 750 lbs.?

$$750 \times \frac{1 \times 1}{10 \times 5} = \frac{750}{50} = 15 \text{ lbs.}$$

INCLINED PLANE.

To Compute Length of Base, Height, or Length.

When any Two of them are given, and when Line of Direction of Power or Traction is Parallel to Face of Plane.—Proceed as in Mensuration or Trigonometry to determine side of a right-angled triangle, any two of them being given.

To Compute Power necessary to Support a Weight on an Inclined Plane.

When Height and Length are given. RULE.—Multiply weight by height of plane, and divide product by length.

$$\text{Or, } \frac{W h}{l} = P. \quad h \text{ and } l \text{ representing height and length of plane.}$$

EXAMPLE.—What is power necessary to support 1000 lbs. on an inclined plane 4 feet in height and 6 feet in length?

$$1000 \times 4 \div 6 = 666.67 \text{ lbs.}$$

To Compute Weight that may be Sustained by a given Power on an Inclined Plane.

When Height and Length of Plane are given. RULE.—Multiply power by length of plane, and divide product by height.

$$\text{Or, } \frac{P l}{h} = W.$$

EXAMPLE.—What is weight that can be sustained on an inclined plane 5 feet in height and 7 feet in length by a power of 700 lbs.?

$$700 \times 7 \div 5 = 980 \text{ lbs.}$$

Estimating power required to overcome resistance of a body held or acted upon an inclined plane, and contrariwise, if body is supported by power, in proportion of power of plane (i. e., as its length is to its height, so is the power to resistance, if being drawn up or supported, or to the weight, if being drawn down).

Compute Height or Length of an Inclined Plane.

Then Weight and Power and one of required Elements are given, and Height is required. RULE.—Multiply power by length, and divide luct by weight.

Then Length is required. RULE.—Multiply weight by height, and divide luct by power.

$$\text{Or, } \frac{P l}{W} = h, \text{ and } \frac{W h}{P} = l.$$

To Compute Pressure on an Inclined Plane.

ULE.—Multiply weight by length of base of plane, and divide product engh of face.

$$\text{Or, } \frac{W b}{l} = \text{pressure. } b \text{ representing length of base of plane.}$$

SAMPLE.—Weight on an inclined plane is 100 lbs., base of plane is 4 feet, and th of it 5; required pressure on plane.

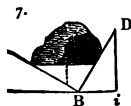
$$100 \times 4 \div 5 = 80 \text{ lbs.}$$

en Two Bodies on Two Inclined Planes sustain each other, as by Connection a Cord over a Pulley, their Weights are directly as Lengths of Planes.

ILLUSTRATION.—If a weight of 50 lbs. upon an inclined plane, of 10 feet rise in 100 n inclination, is sustained by a weight on another plane of 10 feet rise in 90, t is the weight of the latter?

$$50 : 90 :: 50 : 45 = \text{weight that on shortest plane would sustain that on largest.}$$

Then a Body is Supported by Two Planes, as Fig. 7, pressure upon them will be reciprocally as sines of inclinations of planes.



Thus, weight is as sin. A B D.

Pressure on A B as sin. D B i.

Pressure on B D as sin. A B h.

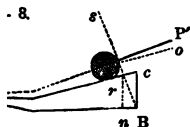
Assume angle A B D to be 90°, and D B i, 60°; then angle A B h will be 30°; and as sines of 90°, 60°, and 30° are respectively .1, .866, and .5, if weight = 100 lbs., then pressures on and B D will be 86.6 and 50 lbs., centre of gravity of weight assumed to be in its re.

When Line of Direction of Power is parallel to Base of Plane, power is Weight as height of plane to length of its base.

$$\text{Or, } P : W :: h : b.$$

$$\text{Hence, } P = \frac{W h}{b}; \quad W = \frac{P b}{h}; \quad h = \frac{P b}{W}; \quad b = \frac{W h}{P}.$$

When Line of Direction of Power is neither parallel to Face of Plane, is Base, but in some other Direction, as P', Fig. 8, power is to weight as sine of angle of plane's elevation to cosine of angle which line of power o ction describes with face of plane.



Thus, P' : W :: sin. A : cos. P' e c.

Sin. A : cos. P' e c :: P' : W.

Cos. P' e c : sin. A :: W : P'.

ILLUSTRATION.—A weight of 500 lbs. is required to be sustained on a plane, angle of elevation of which, c A B, is 10°; line of direction of power or traction, P' e c, is 5°; what is sustaining power required?

$$\text{Cos. P' e c (5°) = .99619 : sin. A (10°) = .17365}$$

r, draw a line, B s, perpendicular to direction of the line (at back of plane), and intersection. e Determine length and height (n r) of the plane.

Differential Screw.

When a hollow screw revolves upon one of less diameter and pitch (as designed by Mr. Hunter), effect is same as that of a single screw, in which the distance between threads is equal to difference of distances between threads of the two screws.

Therefore power, to effect or weight sustained, is as difference between distances of threads of the two screws to circumference described by power.

ILLUSTRATION.—If external screw has 20 threads, and internal one 21 threads, pitch of 1 inch, and power applied describes a circumference of 35 ins., the result of power is as $\frac{1}{21} \propto \frac{1}{20} = \frac{1}{420}$, or .00238. Hence $\frac{35}{.00238} = 14706$.

PULLEY.

PULLEYS are designated as *Fixed* and *Movable*, according as cord is passed over a fixed or a movable pulley. A *movable* pulley is when cord passes through a second pulley or block in suspension; a single movable pulley is termed a *runner*; and a combination of pulleys is termed a *system of pulleys*.

A *Whip* is a single cord over a fixed pulley.

To Compute Power Required to Raise a given Weight.

When Number of Parts of Cord supporting Lower Block are given, and when only one Cord or Rope is used. RULE.—Divide weight to be raised by number of parts of cord supporting lower or movable block.

Or, $W \div n = P$. Or, $n P = W$. n representing number of parts of cord sustaining lower block.

EXAMPLE.—What power is required to raise 600 lbs. when lower block contains six sheaves?

When Cord is attached to Upper or Fixed Block.

$$\frac{600}{6 \times 2} = 50 \text{ lbs.} = \text{weight} \div \text{number of parts of rope sustaining lower block.}$$

When Cord is attached to Lower or Movable Block.

$$\frac{600}{6 \times 2 + 1} = 46.15 \text{ lbs.} = \text{weight} \div \text{number of parts of rope sustaining lower block.}$$

To Compute Weight a given Power will Raise.

When Number of Parts of Cord supporting Lower Block are given. RULE.—Multiply power by number of parts of cord supporting lower block.

$$\text{Or, } P n = W.$$

To Compute Number of Cords necessary to Sustain Lower Block.

When Weight and Power are given. RULE.—Divide weight by power.

$$\text{Or, } W \div P = n.$$

When more than one Cord is used.

In a *Spanish Burton*, Fig. 10, where ends of one cord, a P , are fastened to support and power, and ends of the other, c o , to lower and upper blocks, weight is to power as 4 to 1.

In another, Fig. 11, where there are two cords, a and o , two movable pulleys, and one fixed pulley, with ends of one rope fastened to support and upper movable pulley, and ends of other fastened to lower block and power, weight is to power as 5 to 1.

Fig. 10.

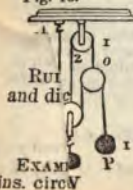
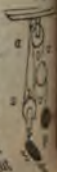
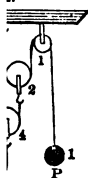


Fig. 11.



Compound or Fast and Loose Pulleys.

2. When Cord is attached to Fixed Block, Fig. 12. RULE.—Multiply power by the power of 2, of which the index is number of movable pulleys.



$$\text{Or, } P 2^n = W.$$

Or, Multiply power successively by 2 for each pulley.

EXAMPLE 1.—What weight will one pound support in a system of three movable pulleys, the cords being connected to a fixed block on Fig. 12.

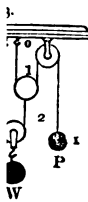
$$1 \times 2^3 = 8 \text{ lbs.}$$

EXAMPLE 2.—What would a like power support, fixed block being made movable and cord attached thereto?

$$1 \times 2^4 - 1 = 15 \text{ lbs.}$$

If fixed pulleys were substituted for hooks *a b c*, Fig. 12, power would be increased threefold; hence $1 \times 3^3 = 27$.

System of Pulleys, Figs. 13 and 14, with any Number of Cords, *o o e e*, Ends being fastened to Support.

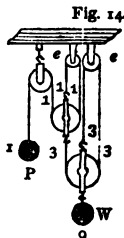


$$W \div 2^n = P; \quad 2^n \times P = W; \quad \frac{W}{P} = 2^n. \quad n \text{ representing number of distinct cords.}$$

ILLUSTRATION.—What weight will a power of 1 lb. sustain in a system of two movable pulleys and two cords?

$$1 \times 2 \times 2 = 4 \text{ lbs.}$$

When fixed Pulleys, *e e*, are used in Place of Hooks, to Attach Ends of Rope to Support.—Fig. 14.



$$W \div 3^n = P; \quad 3^n \times P = W; \quad \frac{W}{P} = 3^n.$$

ILLUSTRATION.—What weight will a power of 5 lbs. sustain with two movable and fixed pulleys, and two cords? $5 \times 3 \times 3 = 45 \text{ lbs.}$

Ends of Cord or Fixed Pulleys are fastened to Weight, as by an Inversion of the last Figures, putting Supports for Weights, and contrariwise.—Figs. 13 and 14.

$$\text{Fig. 13. } \frac{W}{(2^n - 1)} = P; \quad (2^n - 1) P = W; \quad \frac{W}{P} = (2^n - 1).$$

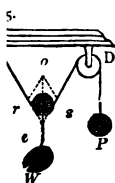
$$\text{Fig. 14. } \frac{W}{(3^n - 1)} = P; \quad (3^n - 1) P = W; \quad \frac{W}{P} = (3^n - 1).$$

ILLUSTRATION.—What weight will a power of 1 lb. sustain in a system of two movable pulleys and two cords, and one of two movable and two fixed pulleys and two cords?

$$1 \times 2 \times 2 - 1 = 3 \text{ lbs.}$$

$$1 \times 3 \times 3 - 1 = 8 \text{ lbs.}$$

When Cords sustaining Pulleys are not in a Vertical Direction.—Fig. 15.



e o, Fig. 15, is vertical line through which weight bears, and from *o* draw *o r*, *o s* parallel to *D e* and *A e*.

Forces acting at *e* are represented by lines *e s*, *e r*, and *e o*; and as tension of every part of cord is same, and equal to power *P*, sides *o s* and *o r* of parallelogram must be equal, and therefore diagonal *e o* divides the angle *r o s* into two equal portions. Hence the weight will always fall into the position in which the two parts of cord *A e* and *e D* will be equally inclined to vertical line, and it will bear to power same as *e o* to *e s*.

Therefore $W : P :: 2 \cos .5 e : 1$. *e* representing angle *A e o* $\times \cos .5 e = W$. That is, twice power, multiplied by cosine of half the point of suspension of weight, is equal to weight.

ILLUSTRATION.—What weight will be sustained by a power of 5 lbs., with a movable pulley, Fig. 15, having an angle, $A \text{ } \angle D$, of 30° ?

$$5 \times 2 \times .96593 = 9.6593 \text{ lbs.} = \text{twice power} \times \cos. 15^\circ.$$

When Direction of Cord is Irregular, Weight not resting in Centre

$$\frac{P}{W} = \frac{\sin. a}{\sin. (a+b)}; \quad \frac{P \sin. (a+b)}{\sin. a} = W; \quad \frac{W \sin. a}{\sin. (a+b)} = P. \quad a \text{ and } b \text{ req. greater and lesser angles of cord at } c.$$

METALS.

ALLOYS AND COMPOSITIONS.

Alloy is the proportion of a baser metal mixed with a finer one, as copper is mixed with gold, etc.

Amalgam is a compound of Mercury and a metal—a soft

Compositions of copper contract in admixture, and all *Amalgams* expand.

In manufacture of Alloys and Compositions, the less fusible should be melted first.

In Compositions of Brass, as proportion of Zinc is increased, malleability decreased.

Tenacity of Brass is impaired by addition of Lead or Tin.

Steel alloyed with one five-hundredth part of Platinum, or Silver rendered harder, more malleable, and better adapted for cutting instruments.

Specific gravity of alloys* does not follow the ratios of those of components; it is sometimes greater and sometimes less than the

Composition for Welding Cast Steel.

Borax, 91 parts; Sal-ammoniac, 9 parts. Grind or pound them roughly together in a metal-pot over a clear fire, continuing heat until all spume has appeared from surface. When liquid is clear, pour composition out to cool a little, and grind to a fine powder; then it is ready for use.

To use this composition, the steel to be welded should be raised to a bright heat; then dip it in the welding powder, and again raise it to a like heat as before; it is then ready to be submitted to the hammer.

Fusible Compounds.

| COMPOUNDS. | Zinc. | Tin. | Lead. | Bismuth. | Cadm. |
|--|-------|------|-------|----------|-------|
| Rose's, fusing at 200° | — | 25 | 25 | 50 | — |
| Fusing at less than 200° | 33.3 | — | 33.3 | 33.4 | — |
| Newton's, fusing at less than 212° .. | — | 19 | 31 | 50 | — |
| Fusing at 150° to 160° | — | 12 | 25 | 50 | 13 |

Solders.

Solder is an alloy used to make joints between metals, and it must be more fusible than the metals it is designed to unite, and it is distinguished as *hard* and *soft*, according to the temperature of its fusing.

The addition of a small portion of Bismuth increases its fusibility.

* For a table of Alloys, having densities different from a mean of their components, see *Annual*, London, 1877, page 201.

Alloys and Compositions.

| | Copper. | Zinc. | Tin. | Nickel. | Lead. | Antimony. | Bismuth. | Aluminum. |
|-------------------------------------|---------|-------|------|---------|-------|-----------|----------|-----------|
| tan..... | 55 | 24 | — | 21 | — | — | — | — |
| num, brown..... | 95 | — | — | — | — | — | — | 5 |
| it's metal*..... | 3.7 | — | 89 | — | — | 7.3 | — | — |
| common..... | 84.3 | 5.2 | 10.5 | — | — | — | — | — |
| “ hard..... | 75 | 25 | — | — | — | — | — | — |
| instruments..... | 79.3 | 6.4 | 14.3 | — | — | — | — | — |
| locomot. bearings..... | 92.2 | — | 7.8 | — | — | — | — | — |
| Pinchbeck..... | 90 | 1 | 9 | — | — | — | — | — |
| red Tombac..... | 80 | 20 | — | — | — | — | — | — |
| rolled..... | 88.8 | 11.2 | — | — | — | — | — | — |
| Tutenag..... | 74.3 | 22.3 | 3.4 | — | — | — | — | — |
| very tenacious..... | 50 | 31 | — | 19 | — | — | — | — |
| wheels, valves..... | 88.9 | 2.8 | 8.3 | — | — | — | — | — |
| white..... | 90 | — | 10 | — | — | — | — | — |
| “..... | 10 | 80 | 10 | — | — | — | — | — |
| “..... | 3 | 90 | — | — | — | 7 | — | — |
| wire..... | 7 | — | — | — | 46 | 47 | — | — |
| yellow, fine..... | 67 | 33 | — | — | — | — | — | — |
| nia metal..... | 66 | 34 | — | — | — | — | — | — |
| When fused add..... | — | — | 25 | — | — | 25 | — | — |
| e, red..... | — | — | — | — | — | 25 | 25 | — |
| “..... | 87 | 13 | — | — | — | — | — | — |
| yellow..... | 86 | 11.1 | 2.9 | — | — | — | — | — |
| Gun metal, large..... | 67.2 | 31.2 | 1.6 | — | — | — | — | — |
| “ small..... | 90 | — | 10 | — | — | — | — | — |
| “ soft..... | 93 | — | 7 | — | — | — | — | — |
| Cymbals..... | 95 | — | 5 | — | — | — | — | — |
| Medals..... | 80 | — | 20 | — | — | — | — | — |
| Statuary..... | 93 | — | 7 | — | — | — | — | — |
| se silver..... | 91.4 | 5.5 | 1.4 | — | 1.7 | — | — | — |
| white copper..... | 58.1 | 17.2 | — | 11.6 | — | — | — | 11.1 |
| h bells..... | 40.4 | 25.4 | 2.6 | 31.6 | — | — | — | — |
| “..... | 80 | 5.6 | 10.1 | — | 4.3 | — | — | — |
| “ Musical bells..... | 69 | — | 31 | — | — | — | — | — |
| bells..... | 87.5 | — | 12.5 | — | — | — | — | — |
| in silver..... | 72 | — | 26.5 | — | — | — | — | 1.5 |
| “..... | 33.3 | 33.4 | — | 33.3 | — | — | — | — |
| “ fine..... | 40.4 | 25.4 | — | 31.6 | — | — | — | 2.6 |
| “..... | 49.5 | 24 | — | 24 | — | — | — | 2.5 |
| “..... | 81.6 | — | 18.4 | — | — | — | — | — |
| bells..... | 77 | — | 23 | — | — | — | — | — |
| bushes..... | 80 | — | 20 | — | — | — | — | — |
| nery bearings..... | 87.5 | — | 12.5 | — | — | — | — | — |
| “ hard..... | 77.4 | 7 | 15.6 | — | — | — | — | — |
| that expands in..... | — | — | — | — | 75 | 16.7 | 8.3 | — |
| ing..... | — | — | — | — | — | — | — | — |
| : metal, 10 oz. lead..... | 60 | 40 | — | — | — | — | — | — |
| r, best..... | — | — | 86 | — | — | 14 | — | — |
| “..... | — | — | 80 | — | 20 | — | — | — |
| ing metal..... | 56 | 45 | — | — | — | — | — | — |
| lum..... | 66 | — | 22 | — | — | — | — | — |
| “..... | 50 | 21 | 29 | — | — | — | — | 12 |
| opic mirrors..... | 66.6 | — | 33.4 | — | — | — | — | — |
| rt..... | 33.4 | — | 66.6 | — | — | — | — | — |
| metal and stereo..... | — | — | — | — | 75 | 25 | — | — |
| plates..... | — | — | — | — | 87.5 | 12.5 | — | — |
| metal..... | — | — | — | — | — | 50.8 | — | — |
| “ hard..... | 7.4 | 7.4 | 28.4 | — | — | — | — | — |
| “..... | 69.8 | 25.8 | 4.4 | — | — | — | — | — |
| | 73 | 12.3 | — | — | — | — | — | — |
| { Magnesia..... 4.4 Cream of tartar | | | | | | | | |
| { Sal-ammoniac..... 2.5 Quicklime. | | | | | | | | |

* 636 for directions.

† For adding small quantities of copper.

Solders.

| | Copper. | Tin. | Lead. | Zinc. | Silver. | Bismuth. | Gold. | Cadmium. | Antimony. |
|------------------------------|---------|------|-------|-------|---------|----------|-------|----------|-----------|
| Tin..... | — | 25 | 75 | — | — | — | — | — | — |
| " coarse, melts | — | 58 | 16 | — | — | 16 | — | — | 10 |
| at 500°.... | — | 33 | 67 | — | — | — | — | — | — |
| " ordi'y, melts | — | 67 | 33 | — | — | — | — | — | — |
| at 360°.... | — | — | — | — | — | — | — | — | — |
| Spelter, soft..... | 50 | — | — | 50 | — | — | — | — | — |
| " hard..... | 65 | — | — | 35 | — | — | — | — | — |
| Lead..... | — | 33 | 67 | — | — | — | — | — | — |
| Steel..... | 13 | — | — | 5 | 82 | — | — | — | — |
| Brass or Copper... | 50 | — | — | 50 | — | — | — | — | — |
| Fine brass..... | 47 | — | — | 47 | 6 | — | — | — | — |
| Pewterers' or Soft. | — | 33 | 45 | — | — | 22 | — | — | — |
| "..... | — | 50 | 25 | — | — | 25 | — | — | — |
| Plumbers' pot- metal..... | — | 33 | 67 | — | — | — | — | — | — |
| " coarse..... | — | 25 | 75 | — | — | — | — | — | — |
| " fine..... | — | 67 | 33 | — | — | — | — | — | — |
| " fusible..... | — | 50 | 50 | — | — | — | — | — | — |
| " very "..... | — | 25 | 25 | — | — | 50 | — | — | — |
| Gold..... | 4 | — | — | — | 7 | — | 89 | — | — |
| " hard..... | 66 | — | — | 34 | — | — | — | — | — |
| " soft..... | — | 66 | 34 | — | — | — | — | — | — |
| Silver, hard..... | 20 | — | — | — | 80 | — | — | — | — |
| " soft..... | 12 | — | — | — | 67 | — | — | 21 | — |
| Pewter..... | — | 40 | 20 | — | — | 40 | — | — | — |
| Iron..... | 66 | — | — | 33 | — | — | — | — | — |
| Copper..... | 53 | 47 | — | — | — | — | — | — | — |

A Plastic Metallic Alloy.—See Journal of Franklin Institute, vol. xxxix., page 5 for its composition and manufacture.

Soldering Fluid for use with Soft Solder.

To 2 fluid oz. of Muriatic acid add small pieces of Zinc until bubbles cease to rise. Add .5 a teaspoonful of Sal-ammoniac and two fluid oz. of Water.

By the application of this to Iron or Steel, they may be soldered without their surfaces being previously tinned.

Fluxes for Soldering or Welding.

| | | | |
|-----------------------|---------------|-------------------|----------------------|
| Iron..... | Borax. | Zinc..... | Chloride of zinc. |
| Tinned iron..... | Resin. | Lead..... | Tallow or resin. |
| Copper and Brass..... | Sal-ammoniac. | Lead and tin..... | Resin and sweet oil. |

Babbitt's Anti-attrition Metal.

Melt 4 lbs. Copper; add by degrees 12 lbs. best Banca tin, 8 lbs. Regulus of antimony, and 12 lbs. more of Tin. After 4 or 5 lbs. Tin have been added, reduce heat to a dull red, then add remainder of metal as above.

This composition is termed *hardening*; for *lining*, take 1 lb. of this *hardening* melt with it 2 lbs. Banca tin, which produces the lining metal for use. Hence, the proportions for lining metal are 4 lbs. of copper, 8 of regulus of antimony, and 1 of tin.

Brass.

Brass is an alloy of copper and zinc, in proportions varying with purpose of metal required, its color depending upon the proportions.

It is rendered brittle by continued impacts, more malleable than copper when cold, but is impracticable of being forged, as its zinc melts at a low temperature.

Its fusibility is governed by its proportion of zinc; a small quantity of zinc gives it fluidity.

Bronze.

ze is an alloy of copper and tin; it is harder, more fusible, and stronger than copper. It is usually known as *Gun-metal*.

Minum Bronze contains 90 to 95 per cent. of copper, and 5 to 10 per cent. of tin.

Phosphor Bronze contains copper and tin and a small proportion of phosphorus. It wears better than bronze.

IRON.

several substances which iron contains modify its essential properties.

Carbon adds to its hardness, but destroys some of its qualities, and reduces Cast Iron or Steel, according to proportion it contains. .25 per cent. renders it malleable, .5 steel, 1.75 is limit of weldability, and 2 is lowest limit of cast iron. *Sulphur* renders it fusible, but to weld, and brittle when heated, or "hot short." *Phosphorus* renders it "cold short," but may be present in proportion of .002 to .01 without affecting injuriously its tenacity. *Antimony*, *Arsenic*, and *Mercury* have same effect as sulphur, the last in a greater degree. *Silicon* renders it hard and brittle. *Manganese*, in proportion of .02, renders it "cold short," and *Vanadium* adds to its ductility.

Cast Iron.

The success of making Cast Iron depends much upon description of fuel used; whether charcoal, coke, bituminous, or anthracite coals. A larger yield from the furnace, and a great economy in fuel, are effected by use of a *hot blast*. Greater heat thus produced causes the iron to combine with a larger quantity of foreign substances.

Cast Iron for purposes requiring great strength should be smelted with *hot blast*. *Pig-iron*, according to proportion of carbon which it contains, is divided into *Foundry Iron* and *Forge Iron*, latter adapted only to convert into malleable iron; while former, containing largest proportion of carbon, can be used either for castings or bars.

High temperature in melting injures gun-metal.

There are many varieties of Cast Iron, differing by almost insensible degrees; the two principal divisions are *gray* and *white*, so termed from the color of their fracture. Their properties are very different.

Gray Iron is softer and less brittle than white; it is in a slight degree malleable and flexible, and is insonorous; it can easily be drilled or turned, and does not resist the file. It has a brilliant fracture, of a gray, or somewhat bluish-gray, color; color is lighter as grain becomes closer, and its strength increases. It melts at a lower heat than white, and preserves its strength longer. Color of the fluid metal is red, and deeper in proportion as the heat is lower; it does not adhere to the ladle; it fills molds well, congeals, and contains fewer cavities than white; edges of its castings are sharp, and surfaces smooth and convex. It is used for machinery and castings where the pieces are to be bored or fitted. Its tenacity and specific gravity are diminished by annealing.

White Iron is very brittle and sonorous; it resists file and chisel, and is capable of high polish; surface of its castings is concave; fracture presents a silvery appearance, generally fine grained and compact, sometimes fibrous or lamellar. When melted it is white, throws off a great deal of gas, and its qualities are the reverse of those of gray iron; it is unsuitable for machinery purposes. Its tenacity is increased by annealing.

Best quality of iron has greatest elasticity.

Tests.—It will not blacken if exposed to nitric acid. Long and a fracture denote a soft and strong metal; short black fibres denote refined metal, and a fine grain denotes hardness and condition "cold short." Coarse grain with bright and crystallized fracture colored spots, also denotes "cold short" and brittle metal, working welding well. Cracks upon edges of a bar, etc., indicate "hot short" iron heats readily, is worked easily, and throws off but few sparks.

A high breaking strain may not be conclusive as to quality, but due to a hard, elastic metal, or a low one may be due to great softness.

When iron is fractured suddenly, a crystalline surface is present; when gradually, a fibrous one. Breaking strain of iron is increased by heating it and suddenly cooling it in water. Iron exposed to a weld heat and not reduced by hammering or rolling is weakened.

Specific gravity of iron is a good indication of its quality, as it varies very correctly its relative degree of strength.

LEAD.

Sheet Lead is either *Cast* or *Milled*, the former in sheets 16 inches length and 6 feet in width; the latter is rolled, is thinner than the former, is more uniform in its thickness, and is made into sheets 25 inches length, and from 6 to 7.5 feet in width.

Soft or Rain Water, when aerated, Silt of rivers, Vegetable matter, Mortar, and Vitiating Air will oxidize lead. The waters which have the greatest effect on it are the purest and most highly oxygenated, i.e., nitrates, and chlorides, and those which act with least effect are carbonates, iron carbonate and phosphate of lime.

Coating of Pipes, except with substances insoluble in water, and Sulphide of lead, is objectionable.

Lead-encased Pipes.—An inner pipe of tin is encased in one of lead.

ad Steel, or Steel of Cementation, is prepared by direct combination of carbon. For this purpose, iron in bars is put in layers, alternating with layers of powdered charcoal, in a close furnace, and exposed for 7 or 8 days to a moderate temperature, and then put to cool for a like period. The bars, when taken out, are covered with blisters, have acquired a brittle quality, and are full of fracture, but have a uniform crystalline appearance. The degree of carbonation is varied according to purposes for which the steel is intended; the very best qualities of iron are used for the finest kinds of steel.

Steel is made from blistered steel moderately heated and subjected to a tilt hammer, by which means its tenacity and density are increased.

Steel is made from blistered or natural steel, refined by piling thin layers of iron, which are brought to a welding heat in a reverberatory furnace and hammered or rolled again into bars; this operation is repeated several times to produce finest kinds of shear steel, which are distinguished by the names of *Half shear*, *Single shear*, and *Double shear*, or steel of 1, 2, or 3, etc., according to number of times it has been piled.

Steel is blister steel heated to an orange red color and rolled or drawn.

Crucible Steel is made by breaking blistered steel into small pieces and melting it in close crucibles, from which it is poured into iron molds; when reduced to a bar by hammering or rolling. Cast steel is best for most purposes; it is known by a very fine grain, and a silvery, homogeneous fracture; it is very hard and acquires extreme hardness, but is difficult to weld without use of a blow pipe. Other kinds of steel have a similar appearance to cast steel, but are coarser and less homogeneous; they are softer and less brittle, and weld more readily. A fibrous or lamellar appearance in fracture indicates defective steel. A material of great toughness and elasticity, as well as of great strength, is made by forging together steel and iron, forming the celebrated *Damascus Steel*, which is used for sword-blades, springs, etc.; damask steel is produced by a diluted acid, which gives a black tint to the iron, while the steel remains white.

Cast steel, breaking strength is greater across fibres of rolling than lengthwise.

Process is an improvement on this method, and consists in adding to the steel a small quantity of carburet of manganese.

Process consists in adding nitrate of soda to molten pig-iron, in order to remove carbon and silica.

Process.—Malleable iron is melted in crucibles with oxide of manganese and charcoal.

ad Steel is produced by arresting the puddling in the manufacture of wrought iron before all the carbon has been removed; the small amount of carbon remaining, .3 to 1 per cent., being sufficient to make an excellent steel.

Steel contains from .2 to .5 per cent. of carbon: when more is present it is termed *Hard Steel*.

Steel is made direct from pig-iron. The carbon is first removed, and then pure wrought iron, and to this is added the exact quantity of carbon required for the steel. The pig should be free from sulphur. It is melted in a blast or cupola, and run into a covered iron vessel suspended on hollow trunnions and lined with clay, where it is subjected to an air blast for a period of time, which is sufficient to disperse the carbon, after which from 5 to 10 per cent. of carbon is added.

slag, and the carbon, combining with oxygen, escapes as carbonic oxide, which with the fine iron slag, induces a powerful ebullition.

Modification of this process.—The ore is treated in a separate rotatory furnace with carbonaceous material, and converted into balls of malleable iron, which are then transferred from the rotatory to the bath of the steel-melting furnace.

This process is adapted to the production of steel of a very high quality, in which the sulphur and phosphorus of the ore are separated from the metal in the furnace.

Siemen's-Martin Process.—Scrap-iron or steel is gradually and highly heated condition to a bath of about .25 its weight, of highly malleable pig, and melted. Samples are occasionally taken from the bath, to ascertain the percentage of carbon remaining in the metal, and on the addition of small quantities, in order to reduce the carbon to about .1 per cent.

At this stage of the process, siliceous iron, spiegeleisen, or ferro-manganese is added in such proportions as are necessary to produce steel of the required degree of hardness. The metal is then tapped into a ladle.

Landore-Siemen's Steel is a variety of steel made by the *Modified Siemens Process*. Its great value is due to its extreme ductility, and having nearly like strength in both directions of its plates.

Whitworth's Compressed Steel is molten steel subjected to a pressure of about 6 tons per square inch, by which all its cavities are expelled, and it is compressed to about .875 of its original volume, its density and strength being proportionately increased.

Chrome and Tungsten Steel are made by adding a small percentage of Chromium or Tungsten to crucible steel, the result producing a steel of great hardness and tenacity, suitable for tools, such as drills, etc.

Homogeneous Steel is a variety of cast steel containing .25 per cent carbon.

the crystalline structure takes place entirely in cooling, between a and b ; when temperature sinks below b there is no change of structural forging, therefore, heated ingot, after it is taken out of furnace, is quickly as practicable, so as not to leave any spot untouched by the steel might crystallize quietly, as formation of crystals should the steel should be kept in an amorphous condition until temperature point b .

perature, if piece is cooled in quiet, mass will no longer be disposed to will possess great tenacity and homogeneity of structure.

forged at temperatures lower than b , its crystals or grains, being ch other, change their shapes, becoming elongated in one direction, another; while density and tensile strength are considerably in- able hammer-power is only sufficient for treatment of small steel object of preventing coarse crystalline structure in large forgings and more certainly effected, if, after having given forging desired are be altered to an homogeneous amorphous condition by heating are somewhat higher than b , and the condition be fixed by rapid perature lower than b , the piece should then be allowed to finish r , so as to prevent, as far as practicable, internal strains due to ual contraction.

el with *Silver*, *Platinum*, *Rhodium*, and *Aluminum* have been lew to imitating Damascus steel, Wootz, etc., and improving ome finer kinds of surgical and other instruments.

'*Steel*.—After being tempered it is not easily broken; it welds ot crack or split; bears a very high heat, and preserves the rdening after repeated working.

nd *Tempering*.—Upon these operations the quality of manu- a great measure depends.

ected by heating steel to a cherry-red, or until scales of ed on surface, and plunging it into a cooling liquid; degree pends upon heat and rapidity of cooling. Steel is thus ren- as to resist files, and it becomes at same time extremely e of heat, and temperature and nature of cooling medium, with reference to quality of steel and purpose for which it old water gives a greater hardness than oils or like sub- vet-iron scales, or cinders, but an inferior degree of hardness y acids. Oil, tallow, etc., prevent cracks caused by too rapid r the heat at which steel becomes hard, the better.

-Steel in its hardest state being too brittle for most purposes, rength and elasticity are obtained by tempering—or "*letting* -"—which is performed by heating hardened steel to a certain ing it quickly. Requisite heat is usually ascertained by color of the steel assumes from film of oxide thus formed. Degrees h these several colors correspond are as follows:

| | |
|--------------|---|
| nt yellow... | { Suitable for hard instruments; as hammer-faces, |
| aw color.... | { drills, lancets, razors, etc. |
| low..... | { For instruments requiring hard edges without elastici- |
| color..... | { ty; as shears, scissors, turning tools, penknives, etc. |
| with purple | { For tools for cutting wood and soft metals; such as |
| | { plane-irons, saws, knives, etc. |
| ue..... | { For tools requiring strong e-nt extreme |
| | { hardness; as cold-chisels, a |
| blue, verg. | { For spring-temper, which wil |
| | { as saws, sword-blades, etc. |

ted to a higher temperature than this, e ved.

ig strain may not be conclusive as to c
stic metal, or a low one may be due to

TIN.

Tin is more readily fused than any other metal, and oxidizes very readily. Its purity is tested by its extreme brittleness at high temperatures. *Tin plate* is iron plate coated with tin. *Block Tin* is tin plate with an additional coating of tin.

ZINC.

Zinc, if pure, is malleable at 220° ; at higher temperatures, such as 400° , it becomes brittle. It is readily acted upon by moist air, and when a thin layer of oxide is formed, it protects the surface from further action. Whenever the air is acid, as from the sea or large towns, it is readily oxidized and destroyed.

Iron, Copper, Lead, and Soot are very destructive of it, in consequence of the voltaic action generated, and it should not be in contact with water or acid woods.

The best quality, as that known as "Vielle Montagne," is composed of .995, iron .004, and lead .001. Its expansion and contraction by change of temperature is in excess of that of any other metal.

STRENGTH OF MODELS.

The forces to which Models are subjected are,

1. To draw them asunder by *tensile* stress.
2. To break them by *transverse* stress.
3. To crush them by *compression*.

istance in a model to crushing increases directly as its dimensions; its stress increases as cubes of dimensions, a model is stronger than the true, inversely as the squares of their comparative magnitudes.

ence, greatest magnitude of a structure is ascertained by taking square of quotient, as obtained by preceding rule, instead of quotient itself.

AMPLE.—If greatest weight which a column in a model can sustain is 26 lbs., it is required to bear only 4 lbs.; height of column being 18 ins., what should height of it in structure?

$$\left(\frac{26}{4}\right) = \sqrt{6.5} = 2.55, \text{ and } 2.55 \times 18 = 45.9 \text{ ins., height of column in structure.}$$

when length or height and breadth are retained, and it is required to to the beam, etc., such a thickness or depth that it will not break in consequence of its increased dimensions,

$$\text{then } \sqrt{\left(\frac{26}{4}\right)} = \sqrt{6.5} = 2.55, \text{ which, } \times \text{ square of relative size of model} = \text{thickness required.}$$

• Compute Resistance of a Bridge from a Model.

$$n^2 W - \left[\frac{n^2}{2} (n-1) w \right] = \text{load bridge will bear in its centre.}$$

AMPLE.—If length of the platform of a model between centres of its repose the piers is 12 feet, its weight 30 lbs., and the weight it will just sustain at its ends 350 lbs., the comparative magnitudes of model and bridge as 20, and actual length of bridge 240 feet; what weight will bridge sustain?

$$20^2 \times 350 - \left[\frac{400}{2} \times (20-1) \times 30 \right] = 140000 - 3800 \times 30 = 26000 \text{ lbs.}$$

MOTION OF BODIES IN FLUIDS.

a body move through a fluid at rest, or fluid move against body at resistance of fluid against body is as square of velocity and density of fluid; that is, $R = d v^2$. For resistance is as quantity of matter or mass struck, and velocity with which they are struck. But number of particles struck in any time are as velocity and density of fluid; therefore, resistance of a fluid is as density and square of velocity.

$$= h, \text{ and } \frac{a d v^2}{2 g} = R. \quad h \text{ representing height due to velocity, } d \text{ density of fluid, } R \text{ resistance or motive force.}$$

istance to a plane is as plane is greater or less, and therefore resistance plane is as its area, density of medium, and square of velocity; that is, $a d v^2$.

tion is not perpendicular, but oblique, to plane or to face of body in any direction, sine of which is s to radius r ; then resistance to plane, or force of fluid against plane, in direction of motion, will be diminished in triplicate of radius to sine of angle of inclination, or in ratio of r to s^3 .

$$\text{ie, } \frac{a d v^2 s^3}{2 g} = R, \text{ and } \frac{a d v^2 s^3}{2 g w} = F. \quad w \text{ representing weight of fluid, } F \text{ motive force.}$$

gression of a solid floating body, as a boat in a channel, or rise to a displacement of water surface, which advances in direction of body, and this undulation is termed *undulation*.

Resistance of a fluid to progression of a floating body increases as velocity of body attains velocity of wave of displacement, and it is greatest when the two velocities are equal.

In the motion of elastic fluids, it appears from experiments that oblique action produces nearly same effect as in motion of water, in the passage of curvatures, apertures, etc.

Resistance to an Area of One Sq. Foot moving through Water, or Contrariwise.

| Angle of Surface with Plane of Current. | Pressure per Sq. Foot for following Velocities per Foot per Minute. | | | | Angle of Surface with Plane of Current. | Pressure per Sq. Foot for following Velocities per Foot per Minute. | | | |
|---|---|-------|--------|---------|---|---|--------|--------|---------|
| | 120 | 240 | 480 | 900 | | 120 | 240 | 480 | 900 |
| 0 | Lbs. | Lbs. | Lbs. | Lbs. | 0 | Lbs. | Lbs. | Lbs. | Lbs. |
| 6 | .09 | .359 | 1.435 | 5.046 | 45 | 2.66 | 10.639 | 42.557 | 149.751 |
| 8 | .133 | .53 | 2.122 | 7.459 | 50 | 2.995 | 11.981 | 47.923 | 168.4 |
| 9 | .156 | .624 | 2.496 | 8.775 | 55 | 3.249 | 12.995 | 51.979 | 184.779 |
| 10 | .179 | .718 | 2.87 | 10.091 | 60 | 3.455 | 13.822 | 55.286 | 199.37 |
| 15 | .355 | 1.42 | 5.678 | 19.963 | 65 | 3.607 | 14.43 | 57.72 | 209.96 |
| 20 | .608 | 2.434 | 9.734 | 34.222 | 70 | 3.728 | 14.914 | 59.654 | 219.76 |
| 25 | .94 | 3.76 | 15.038 | 52.869 | 75 | 3.81 | 15.241 | 60.965 | 224.77 |
| 30 | 1.353 | 5.413 | 21.653 | 76.123 | 80 | 3.857 | 15.428 | 61.714 | 228.49 |
| 35 | 1.798 | 7.192 | 28.766 | 101.132 | 85 | 3.892 | 15.569 | 62.275 | 231.07 |
| 40 | 2.258 | 9.032 | 36.13 | 127.018 | 90 | 3.9 | 15.6 | 62.4 | 232.51 |

Resistance to a plane, from a fluid acting in a direction perpendicular to its face, is equal to weight of a column of fluid, base of which is plane and altitude equal to that which is due to velocity of the motion, or through which a heavy body must fall to acquire that velocity.

Resistance to a plane running through a fluid is same as force of fluid in motion with same velocity on plane at rest. But force of fluid in motion is equal to weight or pressure which generates that motion, and this is equal to weight or pressure of a column of fluid, base of which is area of the plane, and its altitude that which is due to velocity.

ILLUSTRATION.—If a plane 1 foot square be moved through water at rate of 32.166 feet per second, then $\frac{32.166^2}{64.333} = 16.083$, space a body would require to fall to acquire a velocity of 32.166 feet per second; therefore 1×62.5 (weight of a cube foot of water) $\times \frac{32.166^2}{64.333} = 1005$ lbs. = resistance of plane.

Resistance of different Figures at different Velocities in Air.

| Velocity per Second. | Cone. | | Sphere. | Cylinder. | Hemisphere. Round. | Velocity per Second. | Cone. | | Sphere. | Cylinder. | Hemisphere. Round. |
|----------------------|---------|-------|---------|-----------|--------------------|----------------------|---------|-------|---------|-----------|--------------------|
| | Vertex. | Base. | | | | | Vertex. | Base. | | | |
| Feet. | Oz. | Oz. | Oz. | Oz. | Oz. | Feet. | Oz. | Oz. | Oz. | Oz. | Oz. |
| 3 | .028 | .064 | .027 | .05 | .02 | 12 | .376 | .85 | .37 | .826 | .36 |
| 4 | .048 | .109 | .047 | .09 | .039 | 14 | .512 | 1.166 | .505 | 1.145 | .47 |
| 5 | .071 | .162 | .068 | .143 | .063 | 15 | .589 | 1.346 | .581 | 1.327 | .53 |
| 8 | .168 | .382 | .162 | .36 | .16 | 16 | .673 | 1.546 | .663 | 1.526 | .61 |
| 9 | .211 | .478 | .205 | .456 | .199 | 18 | .858 | 2.002 | .848 | 1.986 | .81 |
| 10 | .26 | .587 | .255 | .565 | .242 | 20 | 1.069 | 2.54 | 1.057 | 2.528 | 1.01 |

Diameter

A

all the figures was 6.375 ins., and altitude of the cone 6.65 ins. cone and its axis is, consequently, $25^\circ 42'$ nearly.

several practical inferences may be drawn. It is nearly as surface, increasing but a very little on greater surfaces.

2. Resistance to same surface is nearly as square of velocity, but gradually increasing more and more above that proportion as velocity increases.

3. When after parts of bodies are of different forms, resistances are different, though fore parts be alike.

4. The resistance on base* of a cone is to that on vertex nearly as 2.3 to 1. And in same ratio is radius to sine of angle of inclination of side of cone to its path or axis. So that, in this instance, resistance is directly as sine of angle of incidence, transverse section being same, instead of square of sine.

Resistance on base of a hemisphere is to that on convex side nearly as 1 to 1, instead of 2 to 1, as theory assigns the proportion.

Sphere.—Resistance to a sphere moving through a fluid is but half resistance to its great circle, or to end of a cylinder of same diameter, moving in an equal velocity, being half of that of a cylinder of same diameter.

$\sqrt{2g \times \frac{4}{3} d \times \frac{N-n}{n}} = V$. d representing diameter of sphere, and N and n specific gravities of sphere and resisting fluid.

$\times \frac{4}{3} d = S$. S representing space through which a sphere passes while acquiring its maximum velocity, in falling through a resisting fluid.

ILLUSTRATION.—If a ball of lead 1 inch in diameter, specific gravity 11.33, be set in water, specific gravity 1, what is greatest velocity it will attain in descending, and what space will it describe in attaining this velocity?

$$g = 32.166, \quad d = \frac{1}{12} \text{ foot}, \quad N = 11.33, \text{ and } n = 1.$$

$$\text{Then } \sqrt{2 \times 32.166 \times \frac{4}{3} \text{ of } \frac{1}{12} \times \frac{11.33-1}{1}} = \sqrt{7.148 \times 10.33} =$$

$$\text{Hence, } \frac{11.33}{1} \times \frac{4}{3} \text{ of } \frac{1}{12} = 1.259 \text{ feet. } \frac{3nv^2}{8gNd} = f = \text{retardive force} = \frac{v^2}{2gs}.$$

Cylinder. $\frac{na v^2}{2g} = R$, and $\frac{na v^2}{2gw} = f$. a representing area or πr^2 , and height of body.

ILLUSTRATION.—Assume $a = 32$ sq. feet, $v = 10$ feet per second, and $n = .0012$.

$$\text{Then } \frac{.0012 \times 32 \times 10^2}{64 \times 33} = .06 \text{ of a cube foot of water} = .06 \text{ of } 62.5 = 3.75 \text{ lbs.}$$

$$\text{Conical Surface. } \frac{na v^2 s^3}{2g} = R, \text{ also } \frac{np d^2 v^2 s^2}{8g} = R, \text{ and } \frac{np d^2 v^2 s^2}{8gw}$$

s representing sine of inclination, and a convex surface of cone.

$$\text{Curved End as a Sphere or Hemispherical End. } \frac{pn v^2 d^2}{16g}$$

R , and Circle .5 of spherical end.

In general, when n is to water as a standard, result is in cube feet of water, if s is in sq. feet; and in cube ins. of water, if a is in sq. ins., v in ins., and g in ins.

If n is given in lbs. in a cube foot, a is in sq. feet, v and g are in feet, result is in lbs.

Compute Altitude of a Column of Air, Pressure of which shall be equal to Resistance of a Body moving through it, with any Velocity.

$$\frac{1}{2} \times \frac{r}{a} = x = \text{altitude in feet. } a x = \text{volume of column in feet, as}$$

n times a representing area of section of body, similar to an. lar to direction of motion, r resistance to velocity in table, an. column of air, base of which is a , and pressure r .

This is a refutation of the popular assertion that a taper spar can be towed in a is foremost.

When $\alpha = \frac{2}{9}$ of a foot, as in all figures in table, x becomes $\frac{15}{4} r$ when $r = 7$ distance in table to similar body.

ILLUSTRATION.—Assume convex face of hemisphere resistance = .634 oz at a velocity of 16 feet per second.

Then $r = .634$, and $x = \frac{15}{4} r = 2.3775$ feet = altitude of column of air, pressure which = resistance to a spherical surface at a velocity of 16 feet.

To Compute when Pressure of Air in rear of a Projectile is Inferior to Pressure due to its Velocity.

Assume height of barometer = 2.5 feet, and weight of atmosphere = 14.7 lbs.

Weight of cube inch of mercury = $\frac{14.7}{30} = .49$ lbs., and weight of cube inch of air = .00004357 lbs.; hence, $.49 \div .00004357 = 11246$, which $\times 2.5$ feet = 28115 lbs.

Then $\sqrt{16.08} : \sqrt{28115} :: 32.16 : x$, and $x = \frac{32.16 \times \sqrt{28115}}{\sqrt{16}} = 1341.6$ feet.

To Compute Velocity with which a Plane Surface must be projected to generate a Resistance just equal to Pressure of Atmosphere upon it.

By table, resistance on a circle with an area of .222 sq. foot ($2 \div 9$) = .051 oz. at velocity of 3 feet per second. Hence $3^2 : 1^2 :: .051 : .0056$ oz. at a velocity of 1.64, and $1 \times 144 \times 14.7 \times 16 \times 2 \div 9 = 7526.4$ oz. Hence, $\sqrt{.0056} : \sqrt{7526.4} :: 1 : 116.6$

To Compute Velocity lost by a Projectile.

If a body is projected with any velocity in a medium of same density with that in which it describes a space = 3 of its diameters,

$$\text{Then } x = 3d, \text{ and } b = \frac{3n}{8Nd} = \frac{3}{8d}.$$

Hence, $b x = \frac{9}{8}$, and $\frac{c^{bx} - 1}{c^{bx}} = \frac{2.08}{3.08} = \text{velocity lost nearly .66 of projectile velocity}$
 $c =$ base of Nap. system of log.; hence $c^{bx} =$ number corresponding to Nap. log. $b x$. Hence, if $b x \times .4343$, result = com. log. of c^{bx} .

$b x = \frac{9}{8} = 1.125$, which $\times .4343 = .4885875$, and number to this com. log. = 3.0803.

Hence, velocity lost = $\frac{3.0803 - 1}{3.0803} = \frac{2.08}{3.08}$.

ILLUSTRATION.—If an iron ball 2 ins. diam. were projected with a velocity of 1200 feet per second, what would be velocity lost after moving through 500 feet of space?

$$d = \frac{2}{12} = \frac{1}{6}, \quad x = 500, \quad N = 7\frac{1}{3}, \quad \text{and } n = .0012.$$

$$\text{Hence, } b x = \frac{3n x}{8Nd} = \frac{3 \times 12 \times 500 \times 3 \times 6}{8 \times 21 \times 10000} = \frac{81}{440}, \text{ and } v = \frac{1200}{c^{\frac{81}{440}}} = 993 \text{ feet per}$$

second, having lost 202 feet, or nearly $\frac{1}{6}$ of its initial velocity.

$$\frac{12}{10000} = .0012, \quad \frac{3}{22} \text{ and } \frac{6}{3} = \frac{22}{3} \text{ and } \frac{1}{6} \text{ inverted, because } N \text{ and } n \text{ are in denominator.}$$

To Compute Time and Velocity.

$$\frac{1}{b} \left(\frac{1}{v} - \frac{1}{a} \right) = \text{time,} \quad \frac{3n}{8Nd} = b, \text{ and } \frac{a}{c^{bx}} = v.$$

ILLUSTRATION.—If an iron ball 2 ins. in diameter were projected in air with a velocity of 1200 feet per second, in what time would it pass over 1500 feet, and what its velocity at end of that time?

$$b = \frac{3 \times 12 \times 3 \times 6}{8 \times 22 \times 10000} = \frac{1}{2716}, \text{ and } b x = \frac{1500}{2716}; \text{ hence } \frac{1}{b} = \frac{2716}{1}; \quad \frac{1}{a} = \frac{1}{1200}$$

$$\frac{1}{v} = \frac{c^{bx}}{a} = \frac{1.7372}{1200} = \frac{1}{690} \text{ nearly. } \therefore v = 690 \text{ and } t = 2716 \times \left(\frac{1}{690} - \frac{1}{1200} \right) = 1.25$$

NAVAL ARCHITECTURE.

Results of Experiments upon Form of Vessels.

(Wm. Bland.)

Hydrostatic Models. Head Resistance.—Increases directly with area of surface. **Weight Resistance.**—Increases directly as weight.

Hydrostatic Models. Lateral Resistance.—About one twelfth of length of body immersed, varying with speed.

Order of Superiority of Amidship Section.—Rectangle, Semicircular, Square, and Triangle.

Centre of lateral resistance moves forward as model progresses.

Centre of gravity has no influence upon centre of lateral resistance.

Relative Speeds.

Length.—Increased length gives increased speed or less resistance.

Depth of Flotation.—Less depth of immersion of a vessel, less the resistance.

Amidship Section.—Curved sections give higher speed than angled.

Curves.—Slight horizontal curves present less resistance than right lines. Sides with one fourth more beam give equal speeds with straight of less beam. **Keel.**—Length of keel has greater effect than depth. —Parallel-sided after bodies give greater speed than taper-sided.

| Form of Bow. | Order of Speed. |
|---|-----------------|
| Isosceles triangle, sides slightly convex..... | 1 |
| " " right lines..... | 2 |
| " " slightly concave at entrance and running } convex..... | 3 |

Isosceles triangle compared to **Equilateral triangle**, speed is to 12. **Equilateral triangle**, with its isosceles sides bevelled off at an angle of 45° , compared to bow with vertical sides, is as 5 to 4.

Isosceles bow has an angle of 14° with plane of keel, compared with one of 45° , speed is greater.

Bodies Inclined Upwards from Amidship Section.

Model with bow inclined from \boxtimes , has less resistance than model with no inclination.

Model with stern inclined from \boxtimes , has less resistance than model with no inclination.

Model 1 had less resistance than model 2. Model with both bow and stern inclined from \boxtimes , has less resistance than either 1 or 2.

Stability.

Results of Experiments upon Stability of Rectangular Blocks of Wood of Uniform Length and Depth, but of Different Breadths. (Wm. Bland.)

Length 15, Depth 2, and Depression 1 inch.

| Width. | Weight. | Ratio of Stability. | | | |
|--------|---------|---------------------|--------------------|------------------------|----------------------|
| | | As Observed. | With like Weights. | By Squares of Breadth. | By Cubes of Breadth. |
| 24 | 24 | 1 | 1 | 1 | 1 |
| 35 | 35 | 2.5 | 2.4 | 2.25 | 3.375 |
| 45 | 45 | 7 | 3.7 | 4 | 8 |
| 55 | 55 | 11 | 4.8 | 6.25 | 15.625 |

ative Stability of different hulls of vessels is proportionate to the distance for same angles of heeling, or of distance Gz . Oscillations of hull of a vessel be resolved into a rolling about its longitudinal axis, pitching about its se axis, and vertical pitching, consisting in rising and sinking below and position of equilibrium.

verse section of hull of a vessel is such that, when vessel heels, level of f gravity is not altered, then its rolling will be about a permanent longitudinal axis traversing its centre of gravity, and it will not be accompanied by any oscillations or pitchings, and moment of its *inertia* will be constant while

But if, when hull heels, level of its centre of gravity is altered, then axis which it rolls becomes an instantaneous one, and moment of its *inertia* will it rolls; and rolling must then necessarily be accompanied by vertical oscillations.

oscillations tend to strain a vessel and her spars, and it is desirable, therefore, se verse section of hull should be such that centre of its gravity should not it rolls, a condition which is always secured if all water-lines, as wl and ef , ents to a common sphere described about G ; or, in other words, if point of er sections, o , with vertical plane of keel, is always equidistant from centre y of hull.

To Compute Statical Stability.

sin. $M = S$. D representing displacement, M angle of inclination, and S

TRATION I.—Assume a ship weighing 6000 tons is heeled to an angle of 9° , $cM = 3$ feet,

Sin. $9^\circ = .1564$. Then $6000 \times 3 \times .1564 = 2815.2$ foot-tons.

eight of a floating body is 5515 lbs., distance between its centre of gravity z -centre is 11.32 feet, and angle $M = 20^\circ$.

$= .34202$. Hence $5515 \times 11.32 \times .34202 = 21352.24$ foot-lbs.

Statical Surface Stability.

ent of Statical surface stability at any angle is $c z D$. Assuming f gravity of vessel coincided with c ; coefficient of a vessel's stability angle of heel is expressed when the displacement is multiplied by height of the *meta-centre* for given angle of heel above centre of y , or $D c M$.

oximately. RULE.—Divide moment of inertia of plane of flotation ight position, relatively to middle line by volume of displacement; st ient multiplied by sine of angle of heel will give result.

oot of Length of Vessel, $\frac{2}{3} (B^3 \sin. M)$. B representing half breadth.

Dynamical Surface Stability.

ent of Dynamical surface stability is expressed by product of weight d or displacement and depression of centre of buoyancy during the ion , that is, for angle M .

o Compute Dynamical Stability of a Vessel.

oximately. RULE.—Multiply displacement by height of *meta-centre* entre of gravity, and product by versed sine of angle of heel.

ultiply statical stability for given angle by tangent of .5 angle of heel.

ompute Elements of Stability of a Floating Body.

$= s$, $\frac{c}{\sin. M} = r$, $\frac{s}{\sin. M} = g$, and $\sin. M r = c$. A representing area of d section; A' section immersed by careening of body, as fol; s horizontal $c r$, between centres of buoyancy; a horizontal distance between centres of $i z$, of areas immersed and emerged by careening; g distance, $c M$, between f buoyancy or of water displaced and *meta-centre*; r distance, $G M$, between f gravity and *meta-centre*; c horizontal distance, $G z$, between centre of gravity f line of displacement of it when careened; e vertical distance between centres y and buoyancy, all in feet; and M angle of careening.

NOTE.—When centre of gravity, G, is below that of displacement, c, then $e > 0$; when it is above c it is —; and when it coincides with c it is 0; or e is $-\frac{S}{P}$ $\frac{S}{P} < s$; and a body will roll over when $e \sin. M = \text{or} > s$.

Assumed elements of figure illustrated are $A = 86$, $A' = 21.5$, $b = 21.5$, and $c = 5$.

The deduced arc $s = 3.7$, $c = 3.87$, $g = 10.82$, $a = 14.9$, and $r = 11.32$. b representing breadth at water-line or beam in feet, and P weight or displacement in tons.

$$\text{Then } s = \frac{21.5}{86} \times 14.9 = 3.7 \text{ feet, } r = \frac{3.87}{.34202} = 11.32 \text{ feet, } e = r - g, g = \frac{12}{.34202} \\ = 10.82 \text{ feet, } c = .34202 \times 11.32 = 3.87 \text{ feet.}$$

$$\text{Of Hull of a Vessel. } \left(\frac{b^3}{10.7 \text{ to } 13^* A} \pm e \right) P, \sin. M = S; \quad d \cos. .5 M = \frac{b^3}{10.7 \text{ to } 13 (11.93) A} = g, \quad \frac{1}{\sin. M} \left(\frac{S}{P} - s \right) = \pm e; \quad P \left(\frac{b a}{A} + e \sin. M \right) = S; \text{ and}$$

$P (s \pm e \sin. M) = S$. d representing depth of centre of gravity of displacement under water in equilibrium, and d' depth when out of equilibrium, both in feet.

ILLUSTRATION I.—Displacement of a vessel is 10 000 000 lbs.; breadth of beam 50 feet; area of immersed section, 800 sq. feet; vertical distance from centre of gravity of hull up to centre of buoyancy or displacement, 1.9 feet, and horizontal distance a between centres of gravity of areas immersed and emerged, when careened to an angle of $9^\circ 10' = 33.4$ feet, immersed area being 50 sq. feet.

$$\sin. 9^\circ 10' = .1593. \text{ Then } s = \frac{50}{800} \times 33.4 = 2.0875 \text{ feet, } 800 \times 2.0875 = 50 \times 2.4 \\ r = \frac{2.39}{.1593} = 15 \text{ feet. } g = \frac{50^3}{11.93 \times 800} = 13.1 \text{ feet, } S = \left(\frac{50^3}{11.93 \times 800} \right) + 1.9 \\ 10\,000\,000 \times .1593 = 23\,905\,396 \text{ lbs., and } e = \frac{1}{.1593} \left(\frac{23\,905\,396}{10\,000\,000} - 2.0875 \right) = 1.9 \text{ feet.}$$

2.—Assume a ship having a displacement of 5000 tons, and a height of meta-centre of 3.25 feet, to be careened to $6^\circ 12'$. What is her statical stability?

$$\sin. 6^\circ 12' = .1079. \text{ Then } 5000 \times 3.25 \times .1079 = 1753.37 \text{ foot-tons.}$$

3.—Assume a weight, W , of 50 tons to be placed upon her spar deck, having a common centre of gravity of 15 feet above her load-line,

$$\text{Then } 5000 \times 3.25 + 50 \times 15 \times .1079 = 1747.36 \text{ foot-tons.}$$

4.—Assume 100 tons of water ballast to be admitted to her tanks at a common centre of gravity of 15 feet below her load-line,

$$\text{Then } 5000 \times 3.25 + 100 \times 15 \times .1079 = 1915.22 \text{ foot-tons.}$$

5.—Assume her masts, weighing 6 tons, to be cut down 20 feet,

$$\text{Then } \frac{10 \times 20}{5000} = \frac{2}{50} \text{ foot} = \text{fall of centre of gravity, and } 5000 \times \left(3.25 + \frac{2}{50} \right) \times .1079 \\ = 1774.95 \text{ tons.}$$

To Compute Elements of Power, etc., required to Careen a Body or Vessel.

$$\sin. M (h - n \sin. M) + n \sec. M - s = l. \quad \frac{b^3}{10.7 \text{ to } 13^* A} \sqrt[3]{\frac{P}{64.125 l A}} = s$$

$W l r = P c$, and $W l = S$. W representing weight or power exerted at which weight or power acts to careen body, taken from centre of gravity ment perpendicular to careening force, h vertical height from centre of gravity to centre of weight or power to careen body when it is in e , n horizontal distance from centre of vessel to centre of weight or power, l vessel, m meta-centre, and S as in preceding case, all in feet.

* Unit for section of a parallelogram is 10.7; of a semicircle 12, and of a triangle 12.

STRATION.—A weight is placed upon deck of a vessel at a mean height of 3.87 m centre line of hull; height at which it is placed is 11.32, and other elements in first case given.

$20^\circ = .342$. Then $h = 11.32$, $s = 3.87$, and $l = .342 (11.3 - 3.87 \times .342) + 1.0642 - 3.7 = .342 \times 10 + 4.12 - 3.7 = 3.84$ feet.

me $W = 5515$. Then $5515 \times 3.84 = 21187.6$ foot-lbs.

($w \cos. M + h \sin. M$) = S . w representing distance of weight from centre of and h height of w above water-line, both in feet.

STRATION.—If a weight of 30 tons placed at 20 feet from centre of hull or 0 feet above water-line, careens it to an angle of $2^\circ 9'$, what is its stability?

$\cos. 2^\circ 9' = .9993$; $\sin. 2^\circ 9' = .0375$.

$30 (20 \times .9993 + 10 \times .0375) = 30 \times 20.361 = 610.83$ foot-tons.

Bottom and Immersed Surface of Hull of Vessels.

o Compute Bottom and Side Surface of Hull.

om and Side. RULE.—Multiply length of curve of amidship section, from top of tonnage or main deck beams upon one side to same point other (omitting width of keel), by mean of lengths of keel and perpendiculars in feet, multiply product by .85 or .9 (according to the type of vessel), and product will give surface required in sq. feet.

EXAMPLE.—Lengths of a steamer are as follows: keel 201 feet, and between perpendiculars 210 feet, curved surface of amidship section 76 feet; what is surface?

cient. $.87$. $210 + 201 \div 2 = 205.5$, and $76 \times 205.5 \times .87 = 13587$ sq. feet.

—Exact surface as measured was 13650 sq. feet.

om Surface. RULE.—Multiply length of hull at load-line by its 1 , and this product by depth of immersion (omitting the depth of foot; and this product multiplied by from .07 to .08 (according to type of vessel) will give surface required in sq. feet.

EXAMPLE.—Length upon load-line of a vessel is 310 feet, beam 40 feet, depth of foot, and draught of water 20 feet; what is bottom or wet surface?

Coefficient assumed .073. $310 \times 40 \times 20 - 1 \times .073 = 17199$ sq. feet.

o Compute Resistance to Wet Surface of Hull.

$2 = R$. C representing a coefficient of resistance, a area of wet surface in sq. and v velocity of hull in feet per second.

| | | |
|-------------------------------------|--|------------------------------------|
| lues of C , { .007, clean copper. | | .014, iron plate. |
| { .01, smooth paint. | | .019, iron plate, moderately foul. |

r required to propel one sq. foot of immersed amidship section at \boxtimes is .073 smooth wet surface.

To Compute Elements of a Vessel.

Displacement and its Centre of Gravity.

lacement of a vessel is volume of her body below water-line.

re of Gravity, or Centre of Buoyancy of Displacement, is centre of y of water displaced by hull of vessel.

Displacement. RULE.—Divide vessel, on half breadth plan, into a series of equidistant sections, as one, two, or more frames, commencing and running each side of it. Add together lengths of these lines in fore and aft bodies, except first and last, by Simpson's rule for areas (age 344); multiply sum of products by one third distance between 1 s, and product will give area of water-line between fore and aft-sections.

compute areas contained in sections forward and aft of sections taken, in stern and rudder-post, rudder and stem, and add sum to area of body-section already ascertained.*

* To Compute Area of a Water-line, see Mensuration of Surfaces, page 344.

Compute area of remaining water-lines in like manner. Tabulate and multiply them by Simpson's rule in like manner as for a water-line, and consecutive number of water-lines, and sum of products between water product will give volume between load and lower water-line.

Add area of lower water-line to area of upper surface of keel; multiply by distance between them, and product will give volume; then compute tained in sections forward and aft of sections taken as before directed.

If keel is not parallel to lower water-line, take average of distance between

Compute volume of keel, rudder-post and rudder below water-line; add already ascertained; multiply product by two, for full breadth, and give volume required in cube feet, all dimensions being taken in feet.

Fig. 2.



Fig. 3.



ins. apart, and two or more included in a section. Water-lines 2 feet ap

EXAMPLE

a vessel
length by
extreme
load-line
inches i
Figs. 2 and

Distance
sections.
of simple
example,
at 10 feet
frames a

| 1st Water-line. | | | | 2d Water-line. | | | | 3d Water | | | |
|-----------------|-----|-----|--------|----------------|-----|-----|--------|----------|-----|---|-----|
| 4 | 5 | = | 5 | 4 | 2.7 | = | 2.7 | 4 | 1.5 | = | 1.5 |
| 3 | 7.7 | X 4 | = 30.8 | 3 | 6.9 | X 4 | = 27.6 | 3 | 5 | X | |
| 2 | 9.5 | X 4 | = 38 | 2 | 8.7 | X 4 | = 34.8 | 2 | 6.6 | X | |
| 1 | 9.9 | X 4 | = 39.6 | 1 | 9.5 | X 4 | = 38 | 1 | 8.7 | X | |
| 0 | 10 | X 2 | = 20 | 0 | 9.6 | X 2 | = 19.2 | 0 | 8.9 | X | |
| A | 9.6 | X 4 | = 38.4 | A | 9 | X 4 | = 36 | A | 7.6 | X | |
| B | 7.8 | X 2 | = 15.6 | B | 7 | X 2 | = 14 | B | 7 | X | |
| C | 6.8 | X 4 | = 27.2 | C | 5 | X 4 | = 20 | C | 3 | X | |
| D | 4 | = | 4 | D | 2 | = | 2 | D | 1.2 | = | |
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To Compute Centre of Gravity of Displacement.

RULE.—Divide sum of products obtained as above, by consecutive water-lines, by sum of products obtained in column of products by Simpson's multipliers, and quotient, multiplied by distance between water-lines, will give distance of centre below load water-line.

ILLUSTRATION 1. 8096.4, from above, ÷ 5363.8 = 1.5, which × 2 = 3 feet.

$$d = \frac{n}{2 \left(2 - \frac{D}{a n} \right)}$$

a area of immersed surface of hull in sq. feet, and *D* displacement in cube feet.

—Assume draught of water 8 feet, displacement 7152 cube feet, and area of immersed surface of hull 1100 sq. feet.

$$\text{Then } \frac{8}{2 \left(2 - \frac{7152}{1100 \times 8} \right)} = \frac{8}{2 \times 1.187} = 3.37 \text{ feet.}$$

To Compute Displacement Approximately.

Coefficient of Displacement of a vessel is ratio that volume of displacement bears to parallelepipedon circumscribing immersed body.

$$\frac{V}{B D} = C. \quad V \text{ representing volume of displacement in cube feet, } L \text{ length at immersed water-line, } B \text{ extreme breadth, and } D \text{ draught in depth of immersion, both in feet.}$$

Coefficient of Area of Amidship Section in Plane of a Water-line is ratio which their areas bear to that of circumscribing rectangle.

a representing length of water-line, and *D* distance between water-lines, both in feet.

Coefficients. (By S. M. Pook, Constructor U. S. Navy.)

RULE.—Multiply length of vessel at load-line by breadth, and product by *a* (from load-line to under side of garboard-strake) in feet, and this product by coefficient for vessel as follows: divide by 35 for salt water, 36 for fresh water, and quotient will give displacement in tons.

Amidship sections range from .7 to .9 of their circumscribing square, and mean horizontal lines from .55 to .75 of their respective parallelograms. Hence, ranges of vessels of least capacity to greatest are .7 × .55 = .385, and .9 × .75 = .675.

| | | | |
|-----------------------------------|------------|-------------------------------------|------------|
| Merchant ship, very full..... | .6 to .7 | Merchant steamer, medium..... | .52 to .54 |
| “ “ medium..... | .58 to .62 | Clipper..... | 5 to .54 |
| “ “ steamer, stern-wheel..... | .6 to .65 | Schooner, medium..... | .48 to .52 |
| “ “ of the line..... | .5 to .6 | River steamer, tug-boat, sharp..... | .45 to .5 |
| “ “ steamer, first class..... | .5 to .6 | “ “ medium..... | .45 to .5 |
| “ “ “..... | .52 to .58 | “ “ sharp..... | .42 to .45 |
| “ “ steamer, sharp..... | .54 to .58 | Schooner, sharp..... | .46 to .5 |
| “ “ clipper..... | .52 to .56 | Yachts, sharp..... | .4 to .45 |
| “ “ galleons, barks, etc..... | .52 to .56 | “ “ very sharp..... | .3 to .4 |
| “ “ steamer, tug-boat, med'm..... | .52 to .56 | River steamers, very sharp..... | .36 to .42 |

A steam launch *Miranda*, when making 16.2 knots per hour, with a displacement of 58 tons, her coefficient was 3.

To Compute Characteristic

$$\frac{d}{D} \times \frac{L}{m} = d'. \quad D \text{ representing displacement in tons, } L \text{ length in feet, and } m \text{ longitudinal metacentric distance in feet.}$$

ILLUSTRATION.—“*Warrior*,” at draught of 22 feet, *D* = 8625 tons. If, then, a weight of 20 tons

$$\frac{20 \times 100}{8625} \times \frac{380}{475} = .1856,$$

characteristic

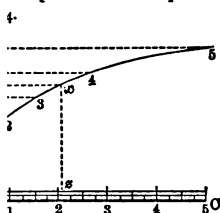
draught in tons, *L* length in feet, *m* = 475 feet, and all 100 feet.

Compute Centre of Gravity or Buoyancy Approximately.

of mean draught of hull, using larger coefficient for full-bodied vessels.

To Delineate Curve of Displacement.

Curve is for purpose of ascertaining volume of water or tons weight, by immersed hull of a vessel at any given or required draught; or required to depress a hull to any given or required draught. From computation for displacement of vessel, proceed as follows, Fig. 4:



On a vertical scale of feet and ins., as A B, set off depths of keel and water-lines, draw ordinates thereto representing displacement of keel, and at each water-line, in tons.

Through points 1, 2, 3, 4, and 5 delineate curve A 5, which will represent displacement at any given or required draught.

Draw a horizontal scale corresponding to weight due to displacement at as A C, and subdivide it into tons and decimals thereof, and a vertical fall from any point, as x , at a given draught, will indicate displacement at depth, on scale A C, and, contrariwise, a line raised point, as z , on A C will give draught at that weight.

Example.—Displacement of hull (page 654) at load-line = 7151.8 cube feet, for salt water = 204.3 tons, hence A C represents tons, and is to be sub-ordinately.

anchoring draught to have been 4 feet, then a vertical let fall from 4 will sight of hull in tons on A C.

Coefficients. (By C. Mackrow, M. I. N. A.)

| TON OF VESSEL. | Length. | Breadth. | Mean Draught. | Displacement. | Coefficient. Amidship Section. | Water-line. |
|------------------|---------|----------|---------------|---------------|--------------------------------|-------------|
| | 225 | 45 | 15 | .715 | .932 | .755 |
| | 325 | 59 | 24.75 | .64 | .81 | .71 |
| | 350 | 35 | 21 | .687 | .85 | .84 |
| ers. | 385 | 42 | 22 | .659 | .88 | .8 |
| | 368.27 | 42.5 | 18.71 | .516 | .812 | .635 |
| small. | 220 | 27 | 8 | .702 | .912 | .742 |
| | 90 | 15 | 4 | .637 | .914 | .704 |
| | 125 | 23 | 8 | .536 | .87 | .616 |
| | 160 | 31.3 | 12 | .466 | .745 | .603 |
| s. | 350 | 49.12 | 23.5 | .47 | .674 | .7 |
| | 340.5 | 46.13 | 15.75 | .4 | .68 | .582 |
| l Steamers. | 337.3 | 50.28 | 22.75 | .483 | .787 | .614 |
| | 270 | 42 | 19 | .497 | .792 | .628 |
| ers, R. N. | 300 | 40.27 | 14 | .414 | .711 | .711 |

Curve of Weight.

Compute Number of Tons required to Depress One Inch at any Draught of Water Plane Water-line.

—Divide area of plane by 12, and again by 35 or 36, as for salt or fresh water.

—Area of load water-line of a vessel is 1422 sq. feet; what is the weight in salt water?

$$1422 \div 12 = 118.5, \text{ which } \div 35 = 3.38 \text{ tons.}$$

To Compute Common Centre of Gravity of Hull, Armament, Engine, Boilers, etc., of a Vessel.

RULE.—Compute moments of the several weights, relatively to assigned horizontal and vertical planes, by multiplying weight of each part by its horizontal and vertical distance from these planes.

Add together these moments, according to their position forward or aft, above or below these planes, and difference between these sums will give position forward or aft, above or below, according to which are greatest.

Divide results thus ascertained by total weight of vessel, and product will give horizontal and vertical distances of centre of gravity from these planes.

It is customary to assume vertical plane at \boxtimes , and horizontal plane at load-line.

NOTE.—In following illustration, in order to simplify computation in table, common centre of gravity of hull, machinery, etc., is taken, instead of centres of individual parts, as engine, boiler, propeller, etc.

ILLUSTRATION.—Assume half-girths as in following table, and distance between sections 10 feet.

| FORWARD. | | | | | | AFT. | | | | | |
|-----------------|--------------|--------------|----------|--------------|----------|----------|--------------|--------------|----------|--------------|----------|
| Section. | Half-Girths. | Multipliers. | Product. | Multipliers. | Moments. | Section. | Half-Girths. | Multipliers. | Product. | Multipliers. | Moments. |
| No. | Feet. | | | | | No. | Feet. | | | | |
| \boxtimes ... | 25 | 1 | 25 | — | — | 1... | 23 | 4 | 92 | 1 | 23 |
| A.... | 23 | 4 | 92 | 1 | 92 | 2... | 20 | 2 | 40 | 2 | 20 |
| B.... | 21 | 2 | 42 | 2 | 84 | 3... | 18 | 4 | 72 | 3 | 54 |
| C.... | 19 | 4 | 76 | 3 | 228 | 4... | 16 | 2 | 32 | 4 | 64 |
| D.... | 17 | 2 | 34 | 4 | 136 | 5... | 14 | 1 | 14 | 5 | 70 |
| E.... | 15 | 1 | 15 | 5 | 75 | | | | | | |
| | | | | | 615 | | | | 534 | | |

Moments forward, 615 — moments abaft, 534 = 29 ÷ sum of product 534 = .54, which \times 10 feet = .54 feet forward of \boxtimes .

Centre of Lateral Resistance.

Centre of Lateral Resistance is centre of resistance of water, and as its position is changed with velocity of vessel, it is variable. It is generally taken at centre of immersed vertical and longitudinal plane of vessel when on an even keel.

If vessel is constructed with a drag to her keel, the centre will be more proportionately abaft of longitudinal centre.

Yacht *America* had a drag to her keel of 2 feet, and centre of lateral resistance of her hull was 8.08 feet abaft of centre of her length on load-line.

Centre of Effort.

Centre of Effort is centre of pressure of wind upon sails of a vessel in vertical and longitudinal plane. Its position varies with area and location of sails that may be spread, and it is usually taken and determined by ordinary standing sails, such as can be carried with propriety in a moderate fresh breeze.

In computing this position, the yards are assumed to be braced down and aft and the sails flat.

NOTE.—Centre of effort of sails, to produce greatest propelling effect, is with capacity of vessel at her load-line, compared with fullness of her body at its extremities. Thus, a vessel with a full load-line and sharp keel below, will sustain a higher centre of effort than one of dissimilar construction.

To Compute Location of Centre of Effort.

RULE.—Multiply area of each sail in square feet by height of its centre of gravity above centre of lateral resistance in feet, divide sum of these products (moments) by total area of sails in square feet, and quotient will give height of centre in feet.

Multiply area of each sail in square feet, centre of which is forward of vertical plane passing through centre of lateral resistance, by direct distance of its centre from that plane in feet, and add products together.

Proceed in like manner for sails that are abaft of this plane, add their products together, and centre of effort will be on that side which has greatest product of sail.

EXAMPLE.—Assume elements of yacht *America* as rigged when in U. S. Service.

| SAIL. | Area. | Height of Cent. of Gravity of Sails. | Vertical Moments. | Distance of Centre of Gravity of Sails. | | Moments. | |
|-------------|---------------|--------------------------------------|-------------------|---|--------|-----------|--------|
| | | | | Foreward. | Abaft. | Foreward. | Abaft. |
| mg Jib..... | Sq. Feet. 656 | Feet. 28 | 18 368 | 52 | — | 34 112 | — |
| | 1087 | 26 | 28 262 | 32 | — | 34 784 | — |
| msail..... | 1455 | 34 | 49 470 | — | 3 | — | 4 365 |
| asail..... | 2185 | 35 | 76 475 | — | 40 | — | 87 400 |
| | 5383 | | 172 575 | | | 68 896 | 91 765 |

Vertical moments 172 575
 Area of sails 5383 = 32.06 = height of centre above centre of lateral resistance.

Moments { $\frac{91\ 765 \sim 68\ 896}{5383} = 4.25 = \text{distance of centre abaft centre of lateral resistance.}$

Relative Positions of Centre of Effort and of Lateral Resistance.

Square Rig. $\frac{L(.75\ d' + d'')}{10\ (d' + d'')} = E.$ **Fore and Aft Rig.** $\frac{L}{10\ (d' + d'')} = E,$

$\frac{4\ A}{5\ d} = E'.$ L representing length of load-line, d distance of centre of buoyancy below it, d' distance of centre of lateral resistance abaft centre of it, d'' distance of centre of buoyancy before centre of it, E distance of centre of effort before centre of lateral resistance, and E' distance of centre of effort above centre of lateral resistance.

Meta-Centre.

Meta-centre of a vessel's hull is determined by location of centre of gravity or buoyancy of immersed bottom of hull, for it is that point in transverse section of hull, where a vertical line raised from its centre of gravity or buoyancy intersects a line passing through centre of gravity of hull, as p. 1, page 650.

To Compute Height of Meta-Centre.

By Moment of Inertia. $\frac{I}{D} = M.$ I representing moment of inertia of area of water-line or plane of flotation, and D volume of displacement in

NOTE.—Moment of Inertia of an area is sum of products of each square by square of its distance from axis, about which moment is computed.

To Ascertain Moment of Inertia approx

Rectangle = CLB^3 ; $C = \frac{1}{12}$ when $L = 4B$; $C = \frac{3}{50}$ when $L =$

when $L = 6B.$ With very fine lines and great proportionate

and B measured at load-line.

ILLUSTRATION.—Assume length of vessel 233 feet, breadth 43, draught 14, and displacement 2700 tons. Length = 5.65 beams; hence C is taken at $\frac{21}{400}$. Volume of displacement = 2700 \times 35 = 92 500 cube feet.

Then $\frac{21 \times 233 \times 43^3}{400 \times 92\,500} = 10.51$. Exact height of moment was 10.44 feet.

By Ordinates. **RULE.**—Divide a half longitudinal section of load water-line by ordinates perpendicular to its length, of such a number that any between any two may be taken as a parallelogram. Multiply sum of cubes of ordinates by respective distances between them, and divide two-thirds of product by volume of immersion, in cube feet.

ILLUSTRATION.—Take dimensions from Figs. 2 and 3, page 654.

| | Length. | Cube. | | Length. | Cube. | | Cube. |
|---|-----------|------------|---|-----------|------------------|--------|----------------|
| 4 | 5 | 125 | A | 9.6 | 885 | | 51 460 |
| 3 | 7.7 | 456 | B | 7.8 | 475 | | 2 |
| 2 | 9.5 | 857 | C | 6.8 | 314 | 3 | 102 920 |
| 1 | 9.9 | 970 | D | 4 | 64 | | |
| 0 | 10 | 1000 | | | | | |
| | | | | | 5146 \times 10 | 7151.8 | 34 306.6 = 411 |

If there are more ordinates, their coefficients must be taken in like manner: 1-4-2-2-4-2-4-1.

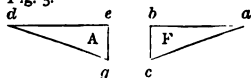
For operation of this method, see Simpson's rule for areas, page 342.

Or, $\frac{2}{3} \int y^3 \frac{dx}{D} = M$. y representing ordinates of half-breadth sections at load line, dx increment of length of load-line section or differential of x , and D displacement of immersed section in cube feet.

$$\frac{2}{3} (a^3 + 4b^3 + 2c^3 + 4d^3 + e^3) \frac{l}{D} + F + A = M. \quad a, b, c, d, e \text{ representing ordinates of 1st or load water-line, } F \text{ area of irregular section between 1st frame and stem, and } A \text{ area of like section between last frame and stern-post, both in sq. feet, } D \text{ displacement, in cube feet, and } l \text{ distance between frames or sections of water-line, as may be taken, in feet.}$$

By Areas. $\frac{2}{3} (a^3 + 4b^3 + 2c^3 + 4d^3 + e^3) \frac{l}{D} + F + A = M$. a, b, c, d, e representing ordinates of 1st or load water-line, F area of irregular section between 1st frame and stem, and A area of like section between last frame and stern-post, both in sq. feet, D displacement, in cube feet, and l distance between frames or sections of water-line, as may be taken, in feet.

Fig. 5.



To Ascertain Areas of F and A —Fig. 5.

$$\frac{2}{3} a b \times b c \div 4 = F, \text{ and } \frac{2}{3} d e \times e g \div 4 = A$$

Elements of Capacity and Speed of Several Types of Steamers of R. N. (W. H. White.)

| CLASSES. | Length. | Length to Breadth. | Displacement. | Speed. | HP to Displacement. | HP to Displacement. |
|----------------|------------|--------------------|---------------|----------------|---------------------|---------------------|
| IRON-CLADS. | Feet. | | Tons. | Knots. | | |
| Recent types. | 300 to 330 | 5.25 to 5.75 | 7500 to 9000 | 14 to 15 | .9 to 1 | 16 to 18 |
| do. twin sc. | 280 to 320 | 4.5 to 5 | 6000 to 9000 | 14 to 15 | .7 to .9 | 15 to 16 |
| UNARMED. | | | | | | |
| Swift cruisers | 270 to 340 | 6.5 to 6.75 | 3000 to 5500 | 15 to 16 | 1.3 to 1.5 | 20 to 25 |
| Corvettes.... | 200 to 220 | 6 | 1800 to 2000 | 12.75 to 13.25 | 1 to 1.2 | 13 to 15 |
| ships..... | 160 | 5 | 850 to 950 | 11 | 1 to 1.2 | 10 to 12 |
| | 125 to 170 | 5.5 to 6.25 | 420 to 800 | 9.5 to 11 | .8 to 1.4 | 7 to 12 |
| | 80 to 90 | 3 to 3.25 | 200 to 250 | 8 to 9 | .8 to 1.1 | 5 to 7 |
| | 0 500 | 9 to 11 | 7000 to 10000 | 14 to 15 | .5 to .6 | 10 to 12 |
| | 0 400 | 8 to 10 | 5000 to 7000 | 13 to 14 | .4 to .5 | 8 to 10 |
| | 0 300 | 7.5 to 10 | 3000 to 6000 | 11 to 15 | .3 to .5 | 5 to 8 |
| | | 7 to 9 | 1500 to 4000 | 9 to 11 | .2 to .3 | 4 to 5 |

Compute Power Required in a Steam Vessel, capacity of another Vessel being given.

vessels of similar models. $\frac{v A}{a} = V$; $\frac{S^3 V}{s^3} = V'$; $\frac{r V'}{r'} = C$; and $\frac{C}{2 r'} = R$;

1' representing product of volumes of given and required cylinders and revolutions in cube feet, *a* and *A* areas of immersed section of given and required in sq. feet at like revolutions and speed of given vessel, *s* and *S* speeds of given required vessel at revolutions of given vessel, both in feet per minute, *r* and *r'* revolutions of given and required vessel per minute, and *C* product of volume of cylinder and revolutions for required vessel.

Illustration.—A steam vessel having an area of amidship section of 675 sq. feet, 5 cylinders of a combined capacity of 533.33 cube feet, and a speed of 10.5 per hour, with 15 revolutions of her engines. Required volume of steamers, with a stroke of 10 feet, for a section of 700 feet and a speed of 13 knots per hour.

$33.33 \times 15 = 8000$ cube feet, $\frac{8000 \times 700}{675} = 8296.3$ cube feet, $\frac{13^3 \times 8296.3}{10.5^3} =$

16288.1 cube feet, $\frac{15 \times 15745.2}{14.5} = 16288.1$ cube feet, and $\frac{16288.1}{2 \times 14.5} = 561.66$ cube

feet $\div 10$ stroke of piston, 12 for ins., and $\times 1728$ ins. in a cube foot = $\times 1728 \div 12 = 8087.9$ sq. ins. area of each cylinder = diameter of 101.5 ins.

Approximate Rules to Compute Speed and IHP of Steam Vessels.

$= C$; $\sqrt[3]{\frac{C \text{ IHP}}{D^{\frac{5}{2}}}} = V$; and $\frac{V^3 D^{\frac{5}{2}}}{C} = \text{IHP}$. Or, $\sqrt[3]{\frac{C \text{ IHP}}{A}} = V$; and $\frac{V^3 A}{C} = \text{IHP}$.

representing coefficient of vessel, A area of immersed amidship section in sq. feet, V velocity of vessel in knots per hour, and D displacement of vessel in tons.

1.—When there exists rig. an unusual surface in free board, deck-houses, etc., element that effects coefficient for class of vessel given, a corresponding addition, or decrease of, following units is to be made:

ge of Coefficients as deduced from observation is as follows:

| SIDE-WHEEL. | | | | | | PROPELLER. | | | | | |
|-------------|---------|-------|----|------------------|---------------------------------|-----------------|---------|-------|----|------------------|---------------------------------|
| VESSEL. | A | | D | C | | VESSEL. | A | | D | C | |
| | Sq. Ft. | Tons. | | V ³ A | V ³ D ^{1/2} | | Sq. Ft. | Tons. | | V ³ A | V ³ D ^{1/2} |
| Steamboat. | — | — | — | — | — | Steamboat. | — | — | — | — | — |
| Lines..... | 43 | 73 | 10 | 470 | 212 | Medium lines.. | 45 | — | 12 | — | 500 |
| "..... | 150 | 465 | 13 | 570 | 219 | "..... | — | — | — | — | — |
| es..... | 136 | 300 | 19 | 540 | 200 | Fine lines..... | 150 | — | 15 | — | 530 |
| Steamer. | — | — | — | — | — | Steamer. | — | — | — | — | — |
| Full lines* | 675 | 3600 | 10 | 650 | 214 | Medium full... | 550 | 2532 | 9 | 194 | 570 |
| "..... | — | — | — | — | — | "..... | — | — | — | — | — |
| et..... | 880 | 5233 | 15 | 650 | 211 | "..... | 390 | 1475 | 10 | 180 | 470 |
| — | — | — | — | — | — | Torpedo boat.. | — | 3600 | 13 | 210 | — |
| | | | | | | | | 27 | 20 | 170 | 500 |

* Full rigged.

† Bark rigged.

efficients as Determined by Several Steamers of H. B. M. Service.

(C. Mackrow, M. I. N. A.)

| h. | Length Beam. | Area of Section at X | Displacement. | IHP | S |
|----|--------------|----------------------|---------------|------|-----|
| | | Sq. Feet. | Tons. | | Kt. |
| | 6.53 | 236 | 775 | 782 | 10 |
| | 5.89 | 277 | 1554 | 1070 | 10 |
| | 7.33 | 814 | 5898 | 2084 | 11 |
| | 6.43 | 632 | 3057 | 2046 | 12 |
| | 6.52 | 1308 | 9487 | 3205 | 12 |
| | 6.73 | 1198 | 9152 | 5971 | 13 |
| | 7.33 | 778 | 5600 | 3945 | 14 |
| | 6.73 | 1185 | 9071 | 6867 | 15 |

3 K

Approximate Rule for Speed of Screw Propell (Molesworth.)

$$\frac{101 V}{P} = N; \quad \frac{P N}{101} = V; \quad \frac{101 V}{N} = P; \quad \frac{88 v}{P} = N; \quad \frac{P N}{88} = v; \quad \text{and } \frac{8}{1}$$

V and *v* representing velocities in knots and miles per hour, *P* pitch of prop foot, and *N* number of revolutions per minute.

This does not include slip, which ranges from 10 to 30 per cent.

Pitch of Screw Propeller.

Pitch ranges with area of circle described by diameter of screw to amidship section.

| | | | | | | | |
|--|---|---|-----|---|-----|---|-----|
| Area of screw circle to amidship section = 1 to..... | 6 | 5 | 4.5 | 4 | 3.5 | 3 | 2.5 |
|--|---|---|-----|---|-----|---|-----|

Two Blades.

| | | | | | | | |
|-----------------------------------|----|------|------|-----|------|------|-----|
| Pitch to diameter of screw = 1 to | .8 | 1.02 | 1.11 | 1.2 | 1.27 | 1.31 | 1.4 |
|-----------------------------------|----|------|------|-----|------|------|-----|

Four Blades.

| | | | | | | | |
|--|------|------|-----|------|------|------|------|
| | 1.08 | 1.38 | 1.5 | 1.62 | 1.71 | 1.77 | 1.89 |
|--|------|------|-----|------|------|------|------|

Length = .166 diameter.

Slip of Side-wheels.

Radial Blades. $\frac{2(A-c)}{A} = S.$ *Feathering.* $\frac{1.5(A-c)}{A} = S.$ *A* repr length of arc of immersed circumference of blades, *c* length of chord of immersion and *S* slip, all in feet.

Area of Blades.

River Service. $\frac{.75 IHP}{D} = A.$ *Sea Service.* $\frac{IHP}{D} = A.$ *D* representing dia of wheel in feet, and *A* area of each blade in square feet.

Length of Blades. .7 in River service and .6 in Sea service.

Distances between Radial Blades. 2.25 in River service and 3 feet in Sea service between Feathering blades, 4 to 6 feet.

Proportion of Power Utilized in a Steam Vess

Side Wheel. $\frac{P-z}{.00000259 d^3 r^2} = C.$ *P* representing gross IHP, *z* effect by slip and oblique action of wheels, *d* diameter of wheels at centre of *r* revolutions per minute, and *C* coefficient for vessel.

ILLUSTRATION.—IHP of engines of a side-wheel steamer is 1120; slip of and loss by oblique action, 33.37 per cent.; diameter of centre of effect of wheel 29.5 feet, and number of revolutions 13.5 per minute; what is coefficient, and power applied to propel vessel?

NOTE.—Slip of wheels from their centre of effect in this case is 15.37 per cent. and loss by oblique action 18 per cent. Hence, representing total power 100—(18+15.37)=66.63 per cent. of power applied to wheels.

As assumed power that operates upon wheels in this case is taken at 86 per cent. of power exerted by engines, 86.12 × 33.37 = 28.74 per cent. for sum by wheels.

$$\frac{1120 - (1120 \times 28.74 \div 100)}{.00000259 \times 29.5^3 \times 13.5^2} = \frac{798.11}{12.16} = 65.63 \text{ coefficient.}$$

Speed of vessel being 10 knots per hour = 17.05 feet per second, power: propel vessel at this speed = 65.63 × 17.05² = 19076.13, and IHP expended = $\frac{13 \times 17.05 \times 60}{13000} = 591.36.$

| | | |
|---|--------|---|
| of engines 1.5 lbs. upon 3848 sq. ins. × 13.5 revolu- | IHP. | F |
| 2.25 × 11200 × 2..... | 94.45 | 0 |
| cent. upon pressure of steam, less 2 lbs. | 60.45 | |
| above..... | 201.6 | |
| | 172.14 | |
| | 591.36 | |
| | 1120 | |

| | HP. | Per cent. of Power. |
|--|--------|------------------------|
| Screw Propeller. Friction of engines..... | 96.06 | 18.83 |
| Friction of load..... | 81.48 | |
| “ of screw surface and resistance of edges of blades.... | 53.44 | 6.83 |
| Slip of propeller..... | 205.55 | 26.27 |
| Absorbed by propulsion of vessel..... | 375.92 | 48.04 |
| | 782.45 | 100 |

NOTE.—From experiments of Mr. Froude, he deduced that, as a rule, only 37 to per cent. of whole power exerted was usefully employed.

With an auxiliary propeller, essential differences are in friction of surfaces and resistance of blades of propeller and slip of propeller, being as 12 to 6.83 in excess in first case, and as 13.7 to 25.27 in second case, or 50 per cent. less.

Resistance of Bottoms of Hulls at a Speed of one Knot per Hour.

| | | | |
|---------------------------------|---------|--------------------------------|----------|
| Smooth wood or painted..... | .01 lb. | Copper..... | .007 lb. |
| Smooth plank..... | .016 “ | Moderately foul..... | .019 “ |
| Shagreened bottom, painted..... | .014 “ | Grass and small barnacles..... | .06 “ |

Sailing.

Ratio of Effective Area of Sails and of Vessel's Speed under Sail to Velocity of Wind.

| COURSE. | Ratio of Effective Area of Sails. | Ratio of Speed of Vessel to Wind. | COURSE. | Ratio of Effective Area of Sails. | Ratio of Speed of Vessel to Wind. |
|---------------------|--|--|-------------------|--|--|
| Points of wind..... | .59 | .33 | Wind abeam..... | .82 | .6 |
| “ abaft beam .. | .91 | .5 | “ astern..... | 1 | .5 |
| “ of wind..... | .68 | .5 | “ on quarter..... | .96 | .66 |

Propulsion and Area of Sails.

Plain sails of a vessel are standing sails, excluding royals and gaff topsails.

Resistance of vessels of similar models but of different dimensions for equal speeds = $D^{\frac{3}{2}}$.

Hence $\frac{a}{a'} = \left(\frac{D}{D'}\right)^{\frac{3}{2}}$. a and a' representing areas of sails of known and given vessels, and D and D' their displacements in tons.

ILLUSTRATION.—Assume D and $D' = 2400$ and 1600 .

Then $\left(\frac{2400}{1600}\right)^{\frac{3}{2}} = \sqrt[3]{1.5^2} = 1.31$, hence area of sails $a' = \frac{1}{1.31} = .763$ per cent.

In Vessels of Dissimilar Models.—Plain sail area should be a multiple of $D^{\frac{3}{2}}$.

Multiples for Different Classes of Vessels, R .

| | Sailing. | Steamer. |
|--------------------|------------|-----------------------|
| Lines of Line..... | 100 to 120 | Ships, iron-clad..... |
| Rigates..... | 120 to 160 | Frigates..... |
| Ships..... | | Sloops..... |
| Brigs..... | | Brigs..... |

English Yachts, designed for high speed, have multiples from 130 to 180. When designed for ordinary speed from 130 to 180.

When Area of Sail to Wet Surface of Hull is taken.—American yacht ratio of 2.7 to 1, and several English yachts nearly the same, while the English was but 2 to 1.

Location of Masts, etc. Load-line = 100.

| VESSEL. | Distance from Stem. | | | Foot of Sail.* | Height of Centre of Mast above Water-line Breadth.* |
|--------------|---------------------|----------|----------|----------------|---|
| | Fore. | Main. | Mizzen. | | |
| Ship..... | 10 to 20 | 53 to 58 | 80 to 90 | 125 to 160 | 1.5 to 2 |
| Bark..... | 12 to 20 | 54 to 60 | 81 to 91 | 130 to 160 | 1.5 to 1.95 |
| Brig..... | 17 to 20 | 64 to 65 | — | 160 to 165 | 1.5 to 1.75 |
| Schooner.... | 16 to 22 | 55 to 61 | — | 160 to 170 | 1.5 to 1.75 |
| Sloop..... | — | 36 to 42 | — | 170 to 190 | 1.25 to 1.75 |

* Measured from Tack of Jib to Clew of Spanker or Mainmast.

Rake of Masts.

Ships.—Foremast 0 to .28 of length from heel, Main and Mizzen 0 to .5.
 Schooners.—Foremast .1 to .25, Mainmast .63 to .77. Sloops.—.08 to .12.

Area of Sails.

| SAILS. | 3 Yards upon each Mast. | 4 Yards upon each Mast. | SAILS. | 3 Yards upon each Mast. | 4 Yards upon each Mast. |
|---------------|-------------------------|-------------------------|----------------|-------------------------|-------------------------|
| Jib..... | .08 | .08 | Mizzenmast.... | .127 | .14 |
| Foremast..... | .295 | .295 | Spanker or } | | |
| Mainmast..... | .417 | .417 | Driver... } | .081 | .08 |

Proportional Area of Sails upon each Mast under above Divisions.

| SAIL. | Fore. | | Main. | | Mizzen. | | Proportion to 1. | |
|-----------------------|-------|------|-------|------|---------|------|------------------|-----|
| Course..... | .115 | .097 | .162 | .138 | — | — | .389 | .31 |
| Topmast..... | .105 | .09 | .149 | .127 | .075 | .063 | .358 | .29 |
| Topgallant sail..... | .075 | .063 | .106 | .089 | .052 | .045 | .253 | .21 |
| Royal..... | — | .045 | — | .063 | — | .032 | — | .19 |
| Spanker or Driver.... | — | — | — | — | .081 | .068 | — | — |
| Jib..... | .08 | .08 | — | — | — | — | — | — |
| | .375 | .375 | .417 | .417 | .208 | .208 | 1 | 1 |

Balance of Sails.—Effect of jib is equal to that of all sails upon mainmast, and sails upon mizzenmast balance those of foremast.

Areas of sails upon masts of a ship should be in following proportion:

Fore..... 1.414 | Main..... 2 | Mizzen..... 1

When, therefore, main yard has a breadth of sail of 100 feet, fore yard should have 70.71 feet, and mizzen 50 feet; topgallant and royal yards and sails being in same proportion.

Angles of Heel for Different Vessels.

Approximately. $\frac{D M a}{H} = S$. D representing displacement of vessel in tons.

M height of meta-centre above centre of gravity in feet, a angle of heel of vessel in circular measure,* and H height of centre of effect above centre of lateral resistance in feet.

Moment of sail should be equal to moment of stability at a defined angle of heel.

| | Angle. | Circular Measure. | | Angle. | Circular Measure. |
|-----------------|--------|-------------------|----------------------|----------|-------------------|
| Ships, etc..... | 4° | .07 | Schooners, etc. | 6° | .105 |
| Yachts..... | 5° | .087 | Yachts..... | 6° to 9° | .105 to .127 |

* displacement 170 tons, height of meta-centre 6.75 feet at 9°; what should be area of sails?

$$2240 = 380800 \text{ lbs. } 9^\circ = .107.$$

$$2 \times 6.75 \times .107 = 7639.8 \text{ sq. feet.}$$

36

Trimming of Sails.

That a vessel's sail may have greatest effect to propel her forward, it should so set between plane of wind and that of her course, that tangent of angle it makes with wind may be twice tangent of angle it makes with her course.
 $r, \tan. a = 2 \tan. b.$ a representing angle of sail with wind, and b angle of sail course of vessel.

Angles of Course and Sails with Wind.

| Wind | Angle of Course. | Tan- gent. | Half Tan- gent. | Angles of Sail with Wind. | Angles of Sail with Course. | Wind Aft. | Angle of Course. | Tan- gent. | Half Tan- gent. | Angles of Sail with Wind. | Angles of Sail with Course. |
|------|------------------|---------------|-----------------------|---------------------------------|-----------------------------------|--------------|---------------------|---------------|-----------------------|---------------------------------|-----------------------------------|
| nd | ad. | | | | | | | | | | |
| its. | | | | | | Points. | | | | | |
| | 45° | .562 | .281 | 29° 18' | 15° 42' | 2 | 112° 30' | 2.166 | 1.082 | 65° 13' | 47° 17' |
| | 56° 15' | .732 | .365 | 36° 12' | 20° 3' | 3 | 123° 45' | 2.737 | 1.368 | 69° 56' | 53° 49' |
| | 67° 30' | .923 | .461 | 42° 43' | 24° 45' | 4 | 135° 0' | 3.562 | 1.781 | 74° 17' | 60° 43' |
| am | 90° | 1.415 | .707 | 54° 45' | 35° 16' | 6 | 157° 30' | 7.511 | 3.754 | 82° 25' | 75° 5' |

Effective Impulse of Wind.

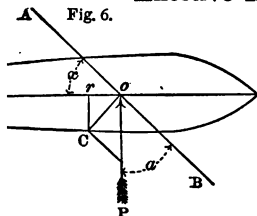


Fig. 6.

Let P , O , Fig. 6, represent direction by compass and force of wind on sail, AB ; from P draw PC parallel to AB , from O draw OC perpendicular to AB ; OC is effective pressure of wind on sail AB , and PC , perpendicular to plane of vessel, is component of OC , which produces lateral motion, as heel and leeway, and PO is component of OC , which propels vessel.

$I \sin. \alpha = P$; $P \cos. \alpha = L$; and $P \sin. \alpha = E$.
 I representing direct impact and P effective pressure of wind on sail, L effective impact producing leeway, and E effective impact which propels vessel.

NOTE.—The law as usually given is $\sin. \alpha$. This is manifestly incorrect, as it gives its less than normal pressure for angles of small incidence. At an angle of incidence of wind of 25° , the law of $\sin.$ is exact. Hence, although it may not be at all angles, it is sufficiently so for practical purposes.

ILLUSTRATION 1.—Assume wind 5 points ahead, and $I = 100$ lbs.

I preceding table angle of course with wind $56^\circ 15'$; hence angle of sail a , with d $36^\circ 12'$, as $\tan. 36^\circ 12' = 2 \tan. 20^\circ 3'$, and angle α $56^\circ 15' - 36^\circ 12' = 20^\circ 3'$.
 Then, $100 \times \sin. 36^\circ 12' = 100 \times .5906 = 59.06$; $59.06 \times \cos. 20^\circ 3' = 59.06 \times .9397 = 55.48$, and $59.06 \times \sin. 20^\circ 3' = 59.06 \times .3426 = 20.23$ lbs.

—Assume wind 4 points abaft, and $I = 100$ lbs.

Then, $100 \times \sin. 74^\circ 17' = 100 \times .9626 = 96.26$; $96.26 \times \cos. 180^\circ - 74^\circ 17' + 45^\circ 0' 43' = 96.26 \times .49 = 47.17$, and $96.26 \times \sin. 60^\circ 43' = 96.26 \times .8722 = 83.82$ lbs.

To Compute Sailing Power of a Vessel.

$$F \sin. w, \sin. s = P.$$

To Compute Careening Power of a Sailing Vessel.

$F \sin. w, \cos. s = P.$ F representing area of sails in sq. feet, w force of wind in per sq. foot, w angle of wind to sails, and s angle of sails to co

To Compute Angle of Steady H

Within a Range of 8° .

$\frac{PE}{M} = \sin. H.$ a representing area of plain sail in sq. feet, P lbs. per sq. foot, E height of centre of effect above mid-draught, in ft of hull, in lbs., and M height of meta-centre in feet.

assumed at 1 lb. per sq. foot, or that due to a brisk wind.

ILLUSTRATION.—Assume $a = 15600$, draught = 20, and $E = 62$; h

$D = 6800000$, and $M = 3$.

$$\text{haz} \frac{15600 \times 1 \times 72}{6800000 \times 3} = \frac{1123200}{20400000} = .05505 = 3^\circ 10'.$$

Course and Apparent Course of Wind.

Apparent course of a wind against sails of a vessel is resultant of *real* course of wind and a course equal and directly opposite to that of vessel.

Fig. 7.

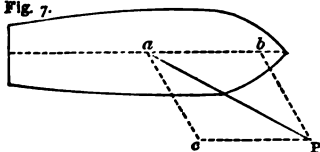


ILLUSTRATION. — If *P*, Fig. 7, represent direction by compass and force of wind, and *a b* direction and velocity of vessel, from *P* draw *P c* parallel and equal to *a b*, join *c a* and it will represent direction and force of apparent wind.

$$\text{Or, } \frac{a}{c} \frac{c}{P} = \text{ratio of velocity of apparent}$$

wind to that of vessel, $\frac{a}{c} \frac{P}{P} = \text{ratio of velocity of wind to that of vessel.}$

Resistance of Air. (Mr. Froude.)

Resistance of wind to a vessel is estimated as equivalent to square of its velocity.

In a calm, resistance of air to a steamer = one thirty-fourth part of resistance of water, and when a steamer's course is head-to, and combined velocity of vessel and wind = 15 knots, resistance is one ninth of that of the water.

Resistance of air to a sq. foot of surface at right angles to course of a vessel is about .33 lb., and when surface is inclined to direction of wind, pressure varies as *sine* of angle of incidence.

Mean of angles of surface of a steamer exposed to wind may be taken as 45°; hence their resistance is about .25 lb. per sq. foot when wind has a velocity of 10 knots per hour.

If sectional area of a steamer's hull above water is 750 sq. feet, resistance to air at a speed of 10 knots in a calm would be 750 × .25 = 187.5 lbs., and resistance to smoke-pipe, spars, and rigging (brig rigged) would be 201 lbs.

Leeway.

Angle of *Leeway* in good sailing vessels, close hauled, varies from 8° to 12°, and in inferior vessels it is much greater.

Ardency is tendency of vessel to fly to the wind, a consequence of the centre of effort being abaft centre of lateral resistance.

Slackness is tendency of vessel to fall off from the wind, a consequence of the centre of effort being forward centre of lateral resistance.

Results of Experiments upon Resistance of Screw-propellers, at High Velocity and Immersed at Varying Depths of Water.

| Immersion of Screw. | Resistance. | Immersion of Screw. | Resistance. | Immersion of Screw. | Resistance. |
|---------------------|-------------|---------------------|-------------|---------------------|-------------|
| Surface. | 1 | 2 feet. | 7 | 4 feet. | 7.8 |
| 1 foot. | 5 | 3 " | 7.5 | 5 " | 8 |

Slip of Propeller, 15 per cent.; of *Side-wheel* (feathering blades), and the axes of blades as the centre of pressure, 23 per cent.

Freeboard.

Deck stringer to surface of water. Depth of Hold from under side of spar deck to top of ceiling.

| Ft. | Hold. | Per Ft. | Hold. | Per Ft. | Hold. | Per Ft. | Hold. | Per Ft. | |
|-----|-------|---------|-------|---------|-------|---------|-------|---------|------|
| 16 | 2.75 | 20 | 3.125 | 24 | 3.375 | 28 | 3.5 | 32 | 3.75 |
| 18 | 3 | 22 | 3.25 | 26 | 3.5 | 30 | 3.75 | 34 | 4 |

Plating Iron Hulls.

T. D representing displacement in tons, L length of hull, b breadth, and r, $.05 f \sqrt{d} = T$. f representing distance between centres of frames, and date below load-line, all in feet, and T thickness of plate in ins.

Masts and Spars. Diameter for Dimensions.

| | | | |
|------------|----------------|------------------------|------------------|
| s..... | at spar deck. | Jib-boom..... | at bowsprit cap. |
| | " stem. | Yards..... | in middle. |
| | " lower cap. | Galls..... | at inner end. |
| masts..... | " topmast cap. | Main and Spanker booms | at taffrail. |

main masts, when of pieces, 1 inch for each 3 to 3.25 feet of whole zzenmast .66 diameter of mainmast. Masts of one piece 1 inch for each feet of whole length.

depth, equal diameter of mainmast; width, diameter equal to foremast.

| | | | | |
|-----------------------|------|-----------------|--------------------------------------|-------------------------|
| ore topmasts..... | 1 | inch for each 3 | to 3.25 | } feet of whole length. |
| nast..... | 1 | " " | 3.25 " 3.33 | |
| masts..... | 1 | " " | 3.25 " 3.33 | |
| s..... | 1 | " " | 3.66 | |
| poles..... | 1 | " " | 2.87 | } feet of whole length. |
| | 1 | " " | 2 ft. of length beyond bowsprit cap. | |
| main yards..... | 1 | " " | 4 | |
| ls..... | .875 | " " | 4 | |
| Topgallant, and | } 1 | " " | 5 | } feet of whole length. |
| ds..... | | | | |
| panker booms..... | 1 | " " | 3.5 | |
| | 1 | " " | 3.5 to 4 | |
| il yards and booms. 1 | " | " " | 4.5 to 4.75 | |

Rudder Head. (Mackrow.)

$.196 C D^3 = M$; $\sqrt[3]{\frac{T}{.196 C}} = D$; and $\frac{A v^2}{2400} = P$. P representing press-

er when hard over, in tons, d distance of geometrical centre of rudder from on, in ins., T stress on head, and M moment of resistance of head, both in immersed area of rudder in sq. feet, v velocity of water passing rudder hour, and C coefficient = 3.5 per sq. inch for Iron, and .125 for Oak.

TON.—Assume area of wooden rudder 24 sq. feet, distance of its geomet. from centre of pintles 2 feet, and velocity of water 10 knots.

$\sqrt[3]{2} = 1 \text{ ton. } 1 \times 2 \times 12 = 24 \text{ inch-tons. } \sqrt[3]{\frac{24}{.196 \times .125}} = 9.93 \text{ ins.}$

Memoranda.

—A man requires in a vessel a displacement of 488 lbs. per month, for res, water, fuel, etc., in addition to his own weight, which is estimated A man and his baggage alone averages 225 lbs.

60 feet in length, 32 beam, and 22.83 in depth, or 664 tons, C. H. (O. M.), 2540 square and 484 round bales of cotton. Total weight of cargo ., equal to 4.57 bales, weighing 1889 lbs., per ton of vessel.

It ship of 1625 tons, N. M., can carry 1800 tons' weight of cargo, or stow f pressed cotton.

on steamboat John Stevens—length 245 feet, beam 3 1d
ght of iron 239 440 lbs. And of one other—length 1
et deep; weight of iron 159 190 lbs.

hull of a vessel with an iron frame and oak planking (t
hull entirely of wood, is as 8 to 15.

all weighs about 45 per cent. less than a wooden hull.

254 feet in length, 42 beam, and 23.5 hold, 1800 tons regis
ons cargo at a draught of 22 feet. Weight of hull in set

ight per Sq. Foot per Month of Metalling of a Vessel's Bottom

61 lb.; Muntz metal .0045 lb.; Zinc .007 lb.; and Iron .00


m between Iron and Steel plated Steamers.—In a vessel

t, hull of steel-plated will weigh 320 tons less = 6.66 per c

[illegible]

1. The first part of the document is a list of names and addresses, which appears to be a directory or a list of contacts. The names are written in a cursive script, and the addresses are listed below them. The list includes names such as "Mr. J. H. Smith", "Mr. W. H. Jones", and "Mr. R. H. Brown".

The following have a direct interest and influence in the work of the Commission and are therefore included in the list of persons to be interviewed:



Wedge. -- A regards compressive dimensions & volume
Linear triangles, that their linear dimensions are directly
with length of curvature.

a ray is diverted from vacuum into any medium, the ratio is greater than unity, and is termed *absolute index* or *index of refraction*.

Mean Indices of Refraction.

| | | | |
|---------------------------|-------|---------------------------|------|
| aqueous humor | 1.339 | Glass, lead, 3 flint..... | 2.03 |
| Stallone lens, under..... | 1.379 | " lead 2, sand 1..... | 1.99 |
| " " central..... | 1.4 | " " 1, flint 1..... | 1.78 |
| Ice..... | 2.6 | Ice..... | 1.31 |
| Quartz..... | 1.57 | Quartz..... | 1.54 |

Indices of other substances, see page 584.

Increases refractive power of fluids and glass.

critical Angle.—Its sine is reciprocal of index of refraction, the incident ray in the less refractive medium.

$$\text{Thus, } \frac{1}{\text{Index}} = \sin. \text{ of angle.}$$

critical Angle is measure of length of image of a straight line on the retina.

critical Reflection is when rays are incident in the more refractive medium, angle greater than the critical angle.

mirage.—An appearance as of water, over a sandy soil when highly heated by sun.

catoptric Curves or Lines are the luminous intersections from curve lines, as on any reflective surface in a circular vessel.

To Compute Index of Refraction.

$$\frac{I}{R} = \text{Index. } I \text{ representing angle of incidence, and } R \text{ that of refraction.}$$

To Compute Refraction.

Concave-Convex and Meniscus.—Effect of a concave-convex in refracting is same as that of a convex lens of same focal distance, and that of a concave is same as a concave lens of same focal distance.

$$\text{Meniscus, with parallel rays } \frac{2 R r}{R - r} = F.$$

magnifying Power.—In *Telescopes* the comparison is the ratio in which the object appears to increase length. In *Microscopes* the comparison is between the object as seen in the instrument and by the eye, at the least distance of vision, which is assumed at 10 ins., and the magnifying power of a microscope is equal to the distance at which an object can be most distinctly examined, divided by the focal length of the lens or sphere.

magnifying power is number of times it is magnified in length, and *Superficial*, number of times it is magnified in surface.

magnifying power of microscopes varies, according to the distance of vision, from 40 to 350 times the linear dimensions of object, and 400 to 1000 times its superficial dimensions.

apparent Area.—As areas of like figures are as the squares of their linear dimensions, the apparent area of an object varies as square of its diameter.

number expressing *Magnification of Apparent Area* is the same as magnifying power as above described.

Illustration.—If diameter of a sphere subtends 1° as seen by the eye, and through a telescope, the telescope is said to have a power of 1.

To Compute Elements of Mirrors and Lenses.

Mirrors. *Spherical Concave.** $\frac{Or}{r-2l} = D$; $\frac{Lr}{r-2l} = L$.

Spherical Convex.† $\frac{Or}{2l+r} = D$; $\frac{Lr}{2l+r} = L$. *Parabolic Concave.* $\frac{d^2}{16h} = L$.

Unequally Convex.‡ $\frac{2Br}{R+r} = F$. *Plano-Convex.§* $2B - .66t = F$.

Hyperbolic Concave.|| *Elliptic Concave.¶* *Sphere.* $\frac{I}{2l-1} = F$.

O representing object = 1, r radius of convexity, l and L length or distance of object from vertex of curve, and from external vertex, D dimension of object, d diameter of base, F focal distance, and h depth of mirror in like dimensions, I index of refraction, and t thickness of lens.

ILLUSTRATION I.—Before a concave mirror of 5 feet radius is set an object at 1.5 feet from vertex of curve; what is ratio of apparent dimension of image, and what is length of and distance of object from external vertex? Object = 1.

$$\frac{1 \times 5}{5 - 2 \times 1.5} = 2.5 \text{ feet, and } \frac{1.5 \times 5}{5 - 2 \times 1.5} = 3.75 \text{ feet.}$$

2.—If object is set at 4.5 feet from vertex of a like mirror, what is length of and distance of inverted object from internal vertex?

$$\frac{1 \times 5}{2 \times 4.5 - 5} = 1.25 \text{ feet, and } \frac{4.5 \times 5}{2 \times 4.5 - 5} = 5.625 \text{ feet.}$$

3.—Before a convex mirror of 3.5 feet radius is set an object at 3 feet from vertex of curve; what is length of and distance of object from external curve?

$$\frac{1 \times 3.5}{2 \times 3 + 3.5} = .368 \text{ foot, and } \frac{3 \times 3.5}{2 \times 3 + 3.5} = 1.105 \text{ feet.}$$

4.—A parabolic reflector has a depth of 1.25 feet and a diameter of 2 feet; what is its focal distance from vertex of internal curve?

$$\frac{2^2}{16 \times 1.25} = .2 \text{ feet or } 2.4 \text{ ins.}$$

Lenses. *Double Convex.* $\frac{Rr}{m-1 \times R+r} = F$. When $R=r$ $\frac{r}{2m-1} = F$.
 $\frac{oF}{F-l} = D$; $\frac{lF}{F-l} = L$; $\frac{S+F}{F} = P$; $\frac{oF}{F-O} = V$; and $\frac{SF}{S+F} = \delta$.

Double Concave. $\frac{Rr}{m-1 \times R+r} = F$; $\frac{oF}{F+L} = D$; $\frac{F o - D}{D} = L$; and $\frac{LF}{L+D} = \delta$.

Optical centres are in centres of lens. *Plano-Convex and Plano-Concave.* $\frac{r}{m-1} = F$. Optical centres are respectively centres of convex and concave

faces. *Convex Concave (Meniscus) and Concavo-Convex.* $\frac{Rr}{m-1 \times R-r} = F$.

Optical Centres. Convex Concave. Delineate lens in half section, draw R from its centre to circumference of lens (intersection of radii), draw r parallel to axis and extending to its circumference, connect R and r at these external points of contact with circumference and external curve, extend line to axis of lens, and point of contact is centre required. *Concavo-Convex.* Proceed in like manner, but in this case r extends to, or delineates, the inner surface of the lens, and point of contact with axis is centre required.

* D or image disappears when $l = .5r$. † When O is beyond F, it will be inverted, and

and $\frac{Lr}{2D+r} = L$. ‡ When equally convex $F = R$. § When convex side is exposed to light and when parallel rays fall upon plane side, $F = 2R$. ¶ Rays of light, bent, or semi-focus of a hyperbola, will diverge from its concave surface, and when from the other will be refracted by surface of the other.

When object is beyond focal distance (F), its image (D) will be inverted, as $\frac{OF}{F-F} = D$, and $\frac{LF}{L-F} = I$, representing magnifying power of lens, 8 limit of normal sight, 10 to 12 ins. for near-sighted eyes and 6 to 8 for near-sighted, ordinarily 10 ins., V limit of distinct vision, O extreme distance of object from optical centre at distinct vision, and m index of refraction.

ILLUSTRATION 1.—If a double convex lens of flint glass has radii of 6 and 6.25 ins., what is its focal distance? Index of refraction = 1.57, see page 584.

$$\frac{6 \times 6.25}{1.57 - 1 \times 6 + 6.25} = 5.37 \text{ ins.}$$

2.—If a double concave lens has a focal distance of 2 ins., and object is 6 ins. from vertex of curve, what is its dimension and what is its distance from vertex of inner curve?

$$\frac{6 \times 2}{2 + 4} = 2 \text{ ins., and } \frac{4 \times 2}{4 + 2} = 1.33 \text{ ins.}$$

3.—If focal distance of a single microscope is 4 ins., what is its limit of distinct vision, and what is its magnifying power? O = 2.857 ins.

$$\frac{2.857 \times 4}{4 - 2.857} = 10 \text{ ins., and } \frac{10 + 4}{4} = 3.5 \text{ times.}$$

Telescopes, Opera-glasses, etc.

$D: o = F: f$; $of \div F = D$, and $\frac{LF}{f} = I$; $\frac{Lf^*}{L-f} \pm \frac{sF}{s+f} = F+f$. f represents length of focal distance from object lens.

ILLUSTRATION.—Principal focal distance of ocular lens of a telescope is .9 in., of objective lens 90 ins.; what is its magnifying power?

$$90 \div .9 = 100 \text{ times the object.}$$

PILE-DRIVING.

Effect of the impact of the ram of a pile-driver is as the square root of its velocity or height of its fall. Thus the theoretical velocity of fall is as $\sqrt{2gh}$ or $8\sqrt{h}$.

The impact or dynamic effect of the blow of a ram on a pile cannot be determined with exactness, so long as it yields under the blow, as the yield-cushions it and reduces its effect.

By my experiments in 1852 to determine the dynamic effect of a falling body, I found it to be far greater than that given by the formula $\sqrt{2gh}$, and upon a late petition of them, under improved conditions of the instrument of registry, I find to be for one pound falling two feet, 52 pounds. One pound falling 2 feet has a velocity of 11.31 feet per second, but its dynamical effect or *vis viva* was 52 pounds, 4.6 times the velocity.

Observation and tests of the sustaining power of piles, at different locations and under different conditions, gave it as 2, 3, and 3.7 to 1 times that deduced by the formula $8\sqrt{h}$, which was but the net effect, or capacity, of ram, less operation.

Wm. J. McAlpine in his operation on the foundations of the dry dock, Brooklyn, estimated the effect of a ram weighing 2240 lbs., fusal, at 224 000 lbs., or 2.28 times that given by the formula $8\sqrt{h}$.

Essayists present a variety of formula, which differ in form. Relatively simple, while others embrace diameter, length, weight, depth driven by last blow in feet or in inches, and Modulus of material of the pile, together with various factors for results.

When the losses of effect in the operation of a pile-driver are due to friction of ram in the guides of the leader, and of the hoisting sheave and over drum (ascertained by experiment with a vertical

* + for telescopes and — for opera-glasses, etc.

be equal to .2 foot of penetration: with a light ram it would be materially so the cushioning of it on head of a pile, however square it may be dressed of want of verticality both of ram in falling and of plane of the pile to the blow consequent lateral vibration of it, the buckling of it in driving, the frequent ting of it on a boulder, and the condition of soil, whether dry, moist, or wet is imbedded or partially exposed to the air, or wholly immersed in wet soil water, and the integrity of the driving—they furnish the elements in determination of a coefficient of safety.

Opposed to these effects is that of the subsidence of the soil around a pile has been disturbed in driving, the effect of which, under favorable conditions, soil, has approached to that of the resistance of the pile at its final blow.

The following formula is constructed on the basis of a pile being driven to a depression of one inch or less, as all estimates based upon a great pressure are not only comparatively valueless, in consequence of the cushioning of the ram, but if piles are not driven to such depression their weight is decreased, and a greater number are rendered necessary to support weight to be imposed upon them, and in it I have omitted an element which is universally given in others, that of the last depression of a pile as a factor, as I not only fail to recognize its connection, but hold its introduction erroneous.

To Compute Safe Load of a Pile Driven to a Depression of 1 Inch or Less.

$4WS\sqrt{h} = L$ *W* representing weight of ram, and *L* load, both in lbs., height of fall in feet.

From which result is to be deducted a factor of safety representing the losses and losses of effect.

Hence, the formula: $\frac{4WS\sqrt{h}}{C} = L$, or safe load in pounds.

For *C*, or coefficient of safety, in consideration of the several losses of effect cited, and especially that of brooming of the heads of a pile, it is assumed at 3 to 6, according to the soil and the integrity of the driving.

Eliminating the numerator 4 and correspondingly reducing the 3 and 6, the formula is, $\frac{WS\sqrt{h}}{.75 \text{ to } 1.5} = L$.

ILLUSTRATION.—Assume an ordinary pile driven in firm soil by a ram of 2000 weight, falling 25 feet, with a final depression of .5 inch, and coefficient of what would be its safe load?

$$\frac{2000 \times .5 \sqrt{25}}{1.25} = \frac{2000 \times .5 \times 5}{1.25} = 64,000 \text{ lbs.}$$

In practice, in the determining the capacity of a range of piles, it is better to reduce the result obtained by the formula, to meet incidental effects of negligence in driving, in the superintendence of it, and the frequent and observed splitting or crushing of a pile on a stone or boulder.

A heavy ram and a low fall is most effective condition of operation in pile-driving, provided height is such that force of blow will not be expended in merely overcoming friction of leader and inertia of pile, and at same time not from such a height as to generate a velocity which will be essentially expended in crushing fibres of head of pile.

When the soil is very soft or wet, concrete should be laid between heads of the piles to a depth of from 1.5 to 3 feet.

When the soil is of fine sand or light gravel piles may be set true from their centres, but if it is saturated with moisture, a greater distance necessary, otherwise small piles are liable to be disturbed by large.

(Continued on page 672.)

Pile-sinking.

Witchell's Screw Piles are constructed of a wrought-iron shaft of suitable diameter, usually from 3 to 8 ins., with 1.5 turns of a cast-iron thread of 1.5 to 3 feet diameter.

Hydraulic Process is effected by the direction of a stream of water under pressure, within a tube or around the base of a pile, by which the sand or mud is removed.

Pneumatic and Plenum Process.—For illustration and details, see Traut-
sch's Engineer's Pocket-book, 647-8. *New Edition.*

Mr. Whewell deduced the following results:

A slight increase in hardness of a pile or in weight of a ram will considerably increase distance a pile may be driven.

Resistance being great, the lighter a pile the faster it may be driven.

Distance driven varies as cube of the weight of ram.

Relative Resistance of Formations to Driving a Pile.

| | | |
|--------------------|-----------------------|---------------------------|
| 100 | Hard clay..... 60 | Light clay and sand... 35 |
| and gravel..... 83 | Clay and sand..... 45 | River silt..... 25 |

PNEUMATICS.—AEROMETRY.

Expansion of gases by operation of gravity is same as that for liquids. The effect of wind increases as square of its velocity.

A volume of air represented by 1, and of 32°, is heated t degrees without changing a different tension, the volume becomes $(1 + .002088 t) = V$; and requires a temperature in excess of t' 32°, it will then assume volume $.002088 t' - 32^\circ$). All aeriform fluids follow this law of dilatation as well as that of compression proportional to weight.

When air passes into a medium of less density, its velocity is determined by difference of its densities. Under like conditions, a conduit will discharge 5 times more air than water.

Compute the Degree of Rarefaction that may be effected in a Vessel.

Let quantity of air in vessel, tube, and pump be represented by 1, and portion of capacity of pump to vessel and tube by .33; consequently, it contains .25 of the air in united apparatus.

Upon the first stroke of piston this .25 will be expelled, and .75 of original quantity will remain; .25 of this will be expelled upon second stroke which is equal to .1875 of original quantity; and consequently there remains .5625 of original quantity. Proceeding in this manner the result is deduced:

| No. of Strokes. | Air Expelled at each Stroke. | Air Remaining |
|-----------------|---|---|
| 1 | .25 = .25 | .75 = .75 |
| 2 | $\frac{3}{16} = \frac{3}{4 \times 4}$ | $\frac{9}{16} = \frac{3 \times 3}{4 \times 4}$ |
| 3 | $\frac{9}{64} = \frac{3 \times 3}{4 \times 4 \times 4}$ | $\frac{27}{64} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4}$ |

so on, multiplying air expelled at preceding stroke by 3, and air remaining after each stroke is ascertained by multiplying remaining after preceding stroke by 3, and dividing it by 4.

Distances at which Different Sounds are Audible.

| | Feet. | Miles. |
|--|---------|--------|
| A full human voice speaking in open air, calm..... | 460 | .87 |
| In an observable breeze, a powerful human voice with the wind can be heard..... | 15 840 | 3 |
| Report of a musket..... | 16 000 | 3.2 |
| Drum..... | 10 560 | 2 |
| Music, strong brass band..... | 15 840 | 3 |
| Cannonading, very heavy..... | 575 000 | 90 |

In Arctic Ocean, conversation has been maintained over water a distance of 6696 feet.

In a conduit in Paris, the human voice has been heard 3300 feet.

For an echo to be distinctly produced, there must be a distance of 55 feet.

Coefficients of Efflux of Discharge of Air. (D'Aubuisson.)

| | | |
|------------------------------|-----|-----|
| Orifice in a thin plate..... | .65 | .75 |
| Cylindrical ajutage..... | .93 | .95 |
| Slight conical ajutage..... | .94 | .97 |

To Compute Volume of Air Discharged through an Opening into a Vacuum, per Second.

$a C \sqrt{2 g h} = V$ in cube feet. a representing area of opening in square feet, C efficient of efflux, and $\sqrt{2 g h} = 1347.4$, as shown at page 428.

ILLUSTRATION.—Area of opening 1 foot square, and $C = .707$.

Then $1 \times .707 \times 1347.4 = 952.61$ cube feet.

Inversely, $V \div a =$ velocity in feet per second.

Velocity and Pressure of Wind.

Pressure varies as square of velocity, or $P \propto V^2$.

$V^2 \times .005 = P$; $\sqrt{200 P} = V$; $v^2 \times .0023 = P$; and $.0023 v^2 \sin. \alpha = P$.

V representing velocity in miles per hour, v in feet per second, P pressure in lbs. per sq. foot, and α angle of incidence of wind with plane of surface.

Table deduced from above Formulas.

| Velocity | | | Character of the Wind. | Velocity | | | Character of the Wind. |
|-----------|-------------|-------------------------|------------------------|-----------|-------------|-------------------------|------------------------|
| per Hour. | per Minute. | Pressure on a Sq. Foot. | | per Hour. | per Minute. | Pressure on a Sq. Foot. | |
| Miles. | Feet. | Lbs. | | Miles. | Feet. | Lbs. | |
| 1 | 88 | .005 | Barely observable. | 25 | 2200 | 3.125 | Very brisk. |
| 2 | 176 | .02 | Just perceptible. | 30 | 2640 | 4.5 | High wind. |
| 3 | 264 | .045 | | 35 | 3080 | 6.125 | |
| 4 | 352 | .08 | Light breeze. | 40 | 3520 | 8 | Very high wind. |
| 5 | 440 | .125 | Gentle, pleasant wind. | 45 | 3960 | 10.125 | Gale. |
| 6 | 528 | .18 | | 50 | 4400 | 12.5 | Storm. |
| 8 | 704 | .32 | | 60 | 5280 | 18 | Great storm. |
| 10 | 880 | .5 | Fresh breeze. | 80 | 7040 | 32 | Hurricane. |
| 15 | 1320 | 1.125 | Brisk blow. | 90 | 7920 | 40.5 | Tornado. |
| 20 | 1760 | 2 | Stiff breeze. | 100 | 8800 | 50 | |

ILLUSTRATION.—What is pressure per sq. foot, when wind has a velocity of 18 miles per hour?

$$18^2 \times .005 = 1.62 \text{ lbs.}$$

Compute Force of Wind upon a Surface.

$F = P \times A$ v representing velocity of wind in feet per second, and α angle of incidence of wind.

It has been observed to have had a velocity of 150 miles

at Greenwich Observatory for a period of 20 years

Force of wind upon a surface, perpendicular to its direction, has been observed as high as 57.75 lbs. per sq. foot; velocity = 159 feet per second.

Dr. Hutton deduced that resistance of air varied as square of velocity nearly, and to an inclined surface as 1.84 power of sine \times cosine.

Figure of a plane makes no appreciable difference in resistance, but convex surface of a hemisphere, with a surface double the base, has only half the resistance.

At high velocities, experiments upon railways show that the resistance becomes nearly a constant quantity.

Course of Wind.

Direction in Northern Hemisphere.



Cyclones.

Wind has its direction nearly at right angles to line between points of highest and lowest pressure of air, or barometer readings, and its course is with the point of lowest pressure at its left, and its velocity is directly as difference of the pressures.

Direction in Southern Hemisphere



In Northern Temperate zone, winds course around an area of low pressure in reverse direction to course of hands of a watch, and they flow away from location of high pressure, and cause an apparent course of the winds in direction of course of the hands.

To Compute Resistance of a Plane Surface to Air.

$.0023 a v^2 = P$ in lbs. a representing area of plane in sq. feet, v velocity in direction of wind in feet per second, + when it moves opposite, and - when with the wind

When Barometer Pressure = 30 Lbs.

(C. F. Martin, U. S. S. S.)

$.004 a V^2 = P$. V representing velocity of wind in miles per hour, and a area of pressure in sq. feet.

To Compute Height of a Column of Mercury to induce an Efflux of Air through a given Nozzle.

Barometer assumed at 2.46 feet = 29.52 ins., and Temperature 52°.

$\frac{P^2}{48.073^2 d^4} = H$, and $48.073 d^2 \sqrt{H} = P$. d representing diameter of nozzle and Height of column of mercury, both in feet, and P volume of air in lbs. per one second.

ILLUSTRATION.—Assume $d = .19$, and $P = .7$ lbs.

$$\frac{.7^2}{48.073^2 \times .19^4} = .1626 \text{ foot. } 48.073 \times .19^2 \sqrt{.1626} = -$$

To Compute Pressure or Weight of Air and Height of Barometer and Temperature, 1 One Second.

$$30.787 d^2 \sqrt{B \frac{b+B}{t}} = \text{pressure in lbs. Or, } 48.073 d^2 \sqrt{B} = lb$$

Height of barometer in external air, B manometer or pressure of a. Mercury, both in feet, and t temperature of air or gas in degrees.

ILLUSTRATION.—Assume $b = 2.5$ feet; $d = .25$ foot; $B = .1$ foot; an.

$$\text{Then } 30.787 \times .0625 \sqrt{.1 \times \frac{2.5 + .1}{1.055}} = 1.924 \times \sqrt{.2465} = .9543 \text{ lbs.}$$

To Compute Temperature for a given Latitude and Elevation.

82.8 $\cos. l - .001981 E - .4 = t$. *E representing elevation in feet.*

ILLUSTRATION.—Assume $l = 45^\circ$; $\cos. = .707$; and $E = 656$ feet.

Then $82.8 \times .707 - .001981 \times 656 - .4 = 58.54 - 1.299 - .4 = 58.54 - .899 = 57.641$.

To Compute Volume of Air or Gas Discharged through an Opening and under a Pressure above that of External Air.

Air. $1347.4 C \frac{d^2}{b} \sqrt{B(b' + B)T} = V$ in cube feet per second.

$T = 1 + .00222 (t - 32^\circ)$, and $b' = 2.5 - .0009$ elevation.

Or, $621.28 d^2 \sqrt{B} = V$.

ILLUSTRATION.—What would be volume of air that would flow through a $\frac{1}{4}$ inch diam. from a reservoir under a pressure of .098 foot of mercury, the air under a barometric pressure of 2.477 feet, temperature of air 55.4° , location of latitude, and at an elevation of 650 feet above level of sea?

$C = .75$; $b' = 2.5 - .0009 \times 650 = 2.4415$ (2.44); and $T = 1.0502$.

Then $1347.4 \times .75 \frac{.246^2}{2.477} \sqrt{.098 (2.44 + .098) \times 1.0502} = 24.689 \times \sqrt{.2617} = 24.689 \times .5116 = 12.63$ cube feet.

When Densities of External Air and that in Reservoir are Equal.

$1347.4 C \frac{d^2}{b} \sqrt{B(b + B)T} = V$. *b' representing height of mercury in reservoir.*

Gas. $\frac{4231}{\sqrt{p}} \sqrt{\frac{B d^5}{L + 42 \times d}} = V$. *p representing specific gravity of gas compared with air, and L, length of pipe or conduit in feet.*

ILLUSTRATION.—If a pipe .05 feet in diameter and 420 feet in length, communicates with a gasometer charged with carburetted hydrogen (illuminating gas), under a water pressure as indicated by a manometer of .1088 foot, what would be the discharge per second?

$d = .05$ foot; $L = 420$ feet; and $B = \frac{.1088}{13.6} = .008$ foot. Specific gravity of gas .5625.

$\frac{4231}{\sqrt{.5625}} \sqrt{\frac{.008 \times .05^5}{420 + 42 \times .05}} = \frac{4231}{.75} \sqrt{\frac{.000000025000}{420 + 2.1}} = .01371$ cube foot.

Resistance of Curves and Angles.—Curves and angles increase resistance to discharge of air or gas very materially. By experiment of D'Aubuisson 7 angles of 45° reduced discharge of gas one fourth.

To Compute Diameter of Discharge-pipe or Nozzle.

When Length and Diameter of Pipe, Volume, and Pressure are given.

$$\sqrt[4]{\frac{42 V^2 d^5}{4230^2 B} - L V^2} = d' \text{ in feet.}$$

ILLUSTRATION.—If a pipe 1000 feet in length, and .4 foot in diameter, leads to a reservoir of air, under a mercurial manometric pressure of .18 foot, what diameter is to be given to a nozzle to discharge 4 cube feet per second?

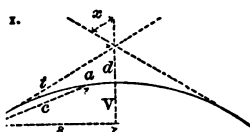
Then $\sqrt[4]{\frac{42 \times 4^2 \times .4^5}{4230^2 \times .18 \times .45 - 1000 \times 4^2}} = \sqrt[4]{\frac{6.88128}{32980.19 - 16000}} = \sqrt[4]{.00040512} = .01703$ ins.

For gases flowing through equal orifices, and under equal pressure, the diameter is proportional to the square roots of their respective densities.

Specific gravity of mercury compared with water.

RAILWAYS.

To Define a Curve.—Fig. 1. (Molenworth.)



$$\frac{1719}{a} c \text{ or } t \tan x = R; \quad R (\cotan. x) = s;$$

$$\frac{1719}{R} c = a; \quad R (\operatorname{cosec}. x - 1) = d;$$

$$R (\cosin. x) = s; \quad R (\operatorname{coversin}. x) = V;$$

$$\frac{5400 - x}{a} = n, \text{ and } (5400 - x) .000582 R = L$$

representing any chord, t length of tangent, d distance of centre of curve from intersection of tangents, s half chord of curve, and l length of curve, all in like dimensions, n gential angle of c in minutes, n number of chords in curve, and x half angle of intersection, but in formulas for number of chords and length of curve to be expressed in minutes.

ILLUSTRATION.—Assume radius 900 and chord 400 feet; angle of intersection = $44' = 764$ minutes, and $x = 56^{\circ} 15' 5''$.

gent of $56^{\circ} 15' 5'' = 1.49673$. Cotangent = .66814.

$$\frac{19 \times 400}{764} = R = 900 \text{ feet}; \quad \frac{1719 \times 400}{900} = 764 \text{ minutes}; \quad 900 \times .66814 = t =$$

$$33 \text{ feet}; \quad 900 \times 1.20269 - 1 = d = 182.42 \text{ feet}; \quad 900 \times .55555 = s = 500 \text{ feet};$$

$$\times .16833 = V = 151.5 \text{ feet}; \quad \frac{5400 - 3379}{764} = 2.645 \text{ times, and } .000582 \times 900 \times$$

$$- 3379 = 1058.6 \text{ feet.}$$

Tangential Angles for Chords of One Chain.

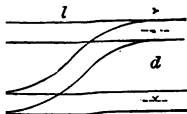
| us of ve. | Tangential Angle. | Radius of Curve. | Tangential Angle. | Radius of Curve. | Tangential Angle. | Radius of Curve. | Tangential Angle. |
|--------------|----------------------|---------------------|----------------------|---------------------|----------------------|---------------------|----------------------|
| ins. | | Chains. | | Chains. | | | |
| 5 | $5^{\circ} 43.8'$ | 15 | $1^{\circ} 54.6'$ | 40 | $42.97'$ | 1 mile | $21.48'$ |
| 8 | $3^{\circ} 34.87'$ | 20 | $1^{\circ} 25.95'$ | 45 | $38.2'$ | 1.25 mil's | $17.19'$ |
| 9 | $3^{\circ} 11'$ | 25 | $8.76'$ | 50 | $34.38'$ | 1.5 miles | $14.33'$ |
| 0 | $2^{\circ} 51.9'$ | 30 | $57.3'$ | 60 | $28.65'$ | 1.75 " | $12.28'$ |
| 2 | $2^{\circ} 23.25'$ | 35 | $49.11'$ | 70 | $24.55'$ | 2 " | $10.74'$ |

NOTE.—Angle for 2 chain chords is double angle for 1 chain chords. Angle for .5 n chords is .5 the angle for 1 chain chords.

Curves of less than 20 chains radius should be set out in .5 chain chords. Curves more than 1 mile radius may be set out in 2 chain chords.

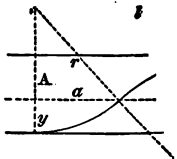
Angles in above Table are in degrees, minutes, and decimals of minutes.

2.



Sidings.

$2 \sqrt{d R - (.5 d)^2} = l$. R representing radius of curve, l length of curve over points, and d distance between tracks, all in feet.



Turn-out of Unequal Radii.

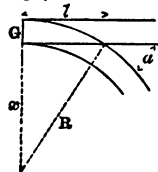
$$x = y; \quad x - y = z; \quad a + b = l; \quad r - y = A;$$

$$(r + A) = a; \quad R - z = B; \quad \sqrt{z(R + B)} = b.$$

and r representing radii of the curves relatively as to length, x distance between outer of tracks and other symbols as shown, all in feet.

Points and Crossings.

Fig. 4.



$\sqrt{(R+x)G} = l$; $\frac{l}{R} = \sin. a$; $\frac{G}{\text{ver. sin. } a} = R$ *R representing radius of curves, G gauge of road, a angle of crossing, and x = R - G, all in feet.*

In horizontal curves, width required for clearance of flange of wheel, and for width of rail at heel of switch render it necessary to make an allowance in length of as ascertained by formula.

For other diagrams and formulas, see Molesworth's Pocket book, pp. 208-18, 21st edition.

To Compute Tangential Angle for Curves. $\frac{1719 C}{R} = a$ *representing chord in feet, and a angle in minutes.*

ILLUSTRATION.—What is angle for a curve with a radius of 900 feet, and a chord of 400 feet?

$$\frac{1719 \times 400}{900} = 764 \text{ minutes.}$$

Curving of Rails.

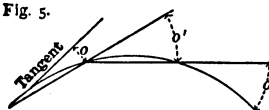
$\frac{1.56 l^2}{R} = v$ *l representing length of rail in feet, v versed sine at centre, and R radius of curve, in ins.*

ILLUSTRATION.—What is curve for a rail 20 feet in length, with a radius of 900 feet?

$$\frac{1.5 \times 20^2}{900} = .666 \text{ ins.}$$

Curves by Offsets in Equal Chords.

Fig. 5.



$$\frac{\text{Chord}^2}{2R} = o \text{ offset.}$$

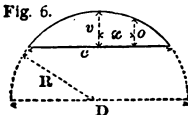
$$\frac{\text{Chord}^2}{R} = 2, o \text{ offset.}$$

ILLUSTRATION.—Assume chords 150, and radius 900 feet.

$$\frac{22500}{2 \times 900} = 12.5 \text{ feet; } \frac{22500}{900} = 25 \text{ feet.}$$

To Compute Versed Sines and Ordinates of Curves.

Fig. 6.



$$R - \sqrt{R^2 - (.5 C)^2} = v; \quad \frac{(.5 C)^2}{v} + v = D;$$

$\sqrt{R^2 - x^2} - (R - v) = 0$. *D representing diameter of circle, and v versed sine of curve.*

ILLUSTRATION.—Assume radius 900, and chord 400 feet.

$$900 - \sqrt{810000 - 40000} = 900 - 877.5 = 22.5 \text{ feet.}$$

Relation of Base of Driving or Rigid Wheels to Curve.

$\frac{R}{G} = B$. *R representing minimum radius of curve, G gauge of road, and B base of wheel, in feet.*

To Compute Elevation of Outer Rail.

For any Radius or Combination of Curve with Straight Line.

$1 - \frac{v}{G} = e$ *v representing velocity of train in feet per second, G gauge of road, and e, both in feet, the versed sine of which = elevation in ins.*

On Curves.

e representing elevation of outer rail in ins.

Radii of Curves set out in Tangential Angles.

| Angle for rd of Feet. | Radius of Curve. | Angle for Chord of 100 Feet. | Radius of Curve. | Angle for Chord of 100 Feet. | Radius of Curve. | Angle for Chord of 100 Feet. | Radius of Curve. |
|-----------------------------|------------------------|------------------------------------|------------------------|------------------------------------|------------------------|------------------------------------|------------------------|
| | Feet. | 0' | Feet. | 0' | Feet. | 0' | Feet. |
| 30 | 5729.6 | 2 30 | 1145.9 | 4 30 | 636.6 | 6 30 | 440.7 |
| | 2864.8 | 3 | 954.9 | 5 | 573 | 7 | 409.3 |
| 30 | 1909.9 | 3 30 | 818.5 | 5 30 | 520.9 | 7 30 | 382 |
| | 1432.4 | 4 | 716.2 | 6 | 447.5 | 8 | 358.1 |

NOTE.—If chords of less length are used, radius will be proportional thereto.

To Ascertain Radius of Curve in Inches for Scale, in Feet per Inch.

Divide radius of curve in feet by scale of feet per inch.

To Compute Required Weight of Rail.

RULE.—Multiply extreme load upon one driving-wheel in lbs. by .005, product will give weight of rail in lbs. per yard.

To Compute Radius of Curve and Wheel Base.

$BG = R \frac{R}{G} = B$. B representing maximum rigid wheel base of cars, and G gauge of way, both in feet.

To Determine Elevation of Outer Rail.

For any Radius or Construction of Curve with Straight.—Fig. 7.

Fig. 7. $V \cdot 5 \sqrt{G} = c$. V representing speed of train in feet per second, G gauge of rails in feet, and c length of chord, versed sine v of which will give at its centre the elevation required.

Thus, determine chord c , align it on inner side of rail, and distance of rail from it at centre of its length will give elevation required, whatever the radius of rail.

For Curves. $\frac{[.782 V^2 (N D W)] - 4 P R}{N D R} = E$; Or, $W \frac{V^2}{1.25 R} = E$. D representing diameter of wheels, W width of gauge, P lateral play between flange and rail, and R radius of curve, all in feet, $N \div D$ ratio of inclination of tire, V velocity of train in ft. per hour, and E elevation of outer rail in ins. (Molesworth.)

$\frac{C(d+l)}{2R}$ = resistance due to curve, and W representing weight of body, both in lbs. C coefficient of friction of wheels upon rails = .1 to .27, according to condition of tire, d distance of rails apart, l length of rigid wheel base, and R radius of curve, in feet. (Morrison.)

ILLUSTRATION.—Assume weight of locomotive 30 tons, radius of curve 1000 feet, distance of rails apart 4 feet 8.75 ins., length of base 10 feet, and rails, dry, $C = 1$.

$$\frac{30 \times 2240 \times 1 \times (4.73 + 10)}{2 \times 1000} = 494.93 \text{ lbs.}$$

Compute Resistance due to Gravity upon Inclination.

$$\frac{2240}{\text{gradient}} = \text{lbs. per ton of train.}$$

Resistance per Mile, and Resistance to Gravity, in Ton.

| Inclination of 1 Inch.. | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 |
|-------------------------|-----|------|------|-----|-----|-----|------|------|----|----|
| In feet..... | 264 | 211 | 176 | 151 | 132 | 117 | 106 | 88 | 75 | 66 |
| Resistance..... | 112 | 89.6 | 74.7 | 64 | 56 | 50 | 44.8 | 37.3 | 32 | 28 |

To Compute Load which a Locomotive will Draw on an Inclination.

$T + r + r' - W = L$. T representing tractive power of locomotive in lbs. distance due to gravity, and r' resistance due to assumed velocity of train in ton, W weight of locomotive and tender, and L load locomotive can draw, in ton, exclusive of its own weight and tender.

Coefficients of Traction of Locomotives.—Railroads in good order, etc., 4 to in ordinary condition, 8 lbs.

In coupled engines adhesion is due to load upon wheels coupled to it

To Compute Traction, Retraction, and Adhesive Power of a Locomotive or Train.

When upon a Level. $a s P \div D = T$. a representing area of one cylinder sq. ins., s stroke of piston and D diameter of driving-wheels, both in feet, P pressure of steam in lbs. per sq. inch, and T traction, in lbs.

When upon an Inclination. $a s P \div D - r w h = T$. r representing resistance per ton, w weight of locomotive upon driving-wheels, in tons, h height of rise in feet per 100 of road, and $R = r w h$ = retraction, in lbs.

$C w b \div 100 = A$. b representing base of inclination in feet per 100 of road

$C w = A$. C = coefficient in lbs. per ton, and A adhesion, in lbs.

When Velocity of a Train is considered.

When upon a Level, $W (C + \sqrt{V}) = R$. **When upon an Inclination,** $W (r h + C + \sqrt{V}) = R$. V representing velocity of train in miles per hour

ILLUSTRATION.—A train weighing 200 tons is to be driven up a grade of 52 per mile, with a velocity of 16 miles per hour; required the retractive power.

$$52.8 \text{ per mile} = 1 \text{ in } 100 \text{ feet} = r = 22.4 \text{ lbs.} \quad C = 5.$$

$$200 (22.4 \times 1 + 5 + \sqrt{16}) = 200 \times 22.4 + 9 = 6280 \text{ lbs.}$$

Velocity of Trains.

| Miles per hour..... | 10 | 15 | 20 | 30 | 40 | 50 | 60 |
|--|------|------|-------|-------|-------|------|------|
| | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| Resistance upon straight line per ton | 8.5 | 9.25 | 10.25 | 13.25 | 17.25 | 22.5 | 29 |
| Do., with sharp curves and strong wind*..... | 13 | 14 | 15.5 | 20 | 26 | 34 | 43.5 |

* Equal to 50 per cent. added to resistance upon a straight line.

Friction of locomotive engines is about 9 per cent., or 2 lbs. per ton of weight. Case-hardening of wheel-tires reduces their friction from 14 to .08 part of 1

To Compute Maximum Load that can be drawn by Engine, up the Maximum Grade that it can Attain, Weight and Grade being given. (Maj. McClellan, U.S.A.)

$$\frac{.2 A}{.4242 G + 8} = L, \text{ and } \frac{.2 A - 8 L}{.4242 L} = G. \quad A \text{ representing adhesive weight of engine in lbs., } G \text{ grade in feet per mile, and } L \text{ load, in tons.}$$

■ L.—When rails are out of order, and slippery, etc., for .2 A , put .143 A .
h an engine of 4 drivers, put .6 as weight resting upon drivers; if entire weight rests upon them.

■ W.—An engine weighing 30 tons has 6 drivers; what are the maximum weight upon a level, and upon a grade of 250 feet, and what is its maximum velocity?

$$\begin{aligned} \text{On a level. } & \frac{.2 \times 2240 \times 30}{.4242 \times 250 + 8} = 1595.4 \text{ tons} \\ \text{On a grade of } & \frac{.2 \times 2240 \times 30 - 8 \times 117.8}{.4242 \times 117.8} = 12.457 \text{ tons} \end{aligned}$$

pared with one of 6 wheels

OPERATION OF LOCOMOTIVES. (*O. Chanute, Am. Soc. C. E.*)

Adhesion.

esion of a locomotive is friction of its driving-wheels upon the rails, g with condition of the surface, and must exceed traction of the engine hem, otherwise the wheels will slip.

rovements heretofore made in the construction of locomotives and have gradually increased the proportion which the adhesion bears to istent weight upon the driving-wheels.

irst accurate experiments were those of Mr. Wood upon the early English lways. He deduced the adhesion to be as follows:

on perfectly dry rails..... .14 of weight on drivers.
 ' damp or muddy rails..... .08 " " "
 ' very greasy rails..... .04 " " "

38, B. H. Latrobe indicated .13 as a safe working adhesion, while modern an practice assumes about .2 of weight as maximum, and .11 as a minimum, perhaps in some mountainous regions, subject to mists. Thus, on the Sem-line, adhesion is generally .16, and between Pontedecimo and Busalla, in ; never exceeds .12 in open cuttings, or .1 in tunnels.

asive experiments made upon French railways, 1862-67, by Messrs. Vuille-ebhard, and Dieudonné gave following coefficients in actual working: *dry*, extreme, .105 to .2; *damp*, .132 to .139; *wet*, .078 to .164; *light rain*, .09; *rain*, .109 to .2; *mean*, .13; *rain and fog*, .115 to .14; *heavy rain*, .16.

rially better results are obtained in United States, partly, perhaps, in con- of greater dryness of the weather, and certainly because of the American of construction and equalizing the weight between the drivers, and of mak- locomotive so flexible as to adapt itself to inequalities in the track.

rn engines in America can safely be relied upon to operate up to an adhesion .222 in summer and .2 in winter, of weight upon the driving wheels.

these data the following tables have been computed:

Coefficients of Adhesion upon Driving Wheels per Ton.

| Con of Rails. | European Practice. | | American Practice. | | Condition of Rails. | European Practice. | | American Practice. | |
|---------------|--------------------|------|--------------------|------|---------------------|--------------------|------|--------------------|------|
| | C. | Lbs. | C. | Lbs. | | C. | Lbs. | C. | Lbs. |
| ry dry..... | .3 | 670 | .33 | 667 | In misty weather . | .015 | 350 | .2 | 400 |
| ry wet..... | .27 | 600 | .25 | 500 | In frost and snow . | .09 | 200 | .16 | 333 |
| y working.. | .2 | 450 | .222 | 444 | | | | | |

Adhesion of Locomotives, in Lbs. (.222 in Summer and .2 in Winter).

| Locomotive. | No. of Drivers. | Weight. | | Adhesion. | |
|-------------|----------------------|-------------|-------------|-----------|---------|
| | | Locomotive. | On Drivers. | Summer. | Winter. |
| | | Lbs. | Lbs. | Lbs. | Lbs. |
| un..... | 4 wheels coupled.... | 64 000 | 42 000 | 9 280 | 8 400 |
| eeled..... | 6 " connected.. | 78 000 | 58 000 | 12 720 | 11 760 |
| | 6 " " .. | 88 000 | 72 000 | 16 160 | 14 720 |
| lation..... | 8 " " .. | 100 000 | 88 000 | 19 360 | 17 600 |
| itching.... | 6 " " .. | 68 000 | 68 000 | 15 040 | 13 920 |
| " | 4 " " .. | 48 000 | 48 000 | 10 640 | 9 600 |

Tractive Power.

tion of a locomotive is the horizontal resultant on the t e of the steam, as applied in the cylinders.

$L \div W = T$. D representing diameter of cylinder, L length of st- r of driving wheels, all in ins., P mean pressure in cylinder, tr. d T tractive force on rails, in lbs.

RATION.—Assume a locomotive, cylinders 18 ins. in diam., 22 in- ins. in diam., and average steam pressure in cylinders 50 lbs. per

Then $18 \times 18 \times 50 \times 22 \div 68 = 5241$ lbs.

Approximation: At 20 miles per hour, the resistance would be:

$$20^2 \div 171 + 8 = 10.3 \text{ lbs. per ton of train.}$$

This formula, however, is empirical. It gives results which are too freight trains at moderate speeds, and too small for passenger trains at high speeds.

Engineers are not agreed as to exact measure and value of each of the of train resistances, but following approximations are sufficient for practical purposes.

Analysis of Train Resistances.

Resistance of trains to traction may be divided into four principal elements: 1st. Grades; 2d. Curves; 3d. Wheel friction; 4th. Atmospheric resistance.

1st. *Grades.*—Gradients generally oppose largest element of resistance to trains. Their influence is entirely independent of speed. The value of this resistance is equal to weight of train multiplied by rate of inclination or per cent. of grade. Thus, a gradient of .5 per 100 feet (or 1 foot per 200 feet) offers a resistance of $\frac{5 \times 2240}{10 \times 100} = 11.2$ lbs. per ton, or 11.2 per 2000 lbs., which is to be multiplied by weight in tons of entire train.

Following table shows resistance, due to gravity alone, for the most usual grades in lbs. per ton of train:

1st. Resistance due to Grades.

| | | | | | | | |
|-----------------------------|------|------|------|------|------|-------|-------|
| Rate per 100 feet..... | .1 | .2 | .3 | .4 | .5 | .6 | .7 |
| Lbs. per ton of 2240 lbs... | 2.24 | 4.48 | 6.72 | 8.96 | 11.2 | 13.44 | 15.68 |
| Rate per mile..... | 5 | 11 | 16 | 21 | 26 | 32 | 37 |
| Lbs. per ton of 2000 lbs... | 2 | 4 | 6 | 8 | 10 | 12 | 14 |

| | | | | | | | |
|-----------------------------|-------|------|-------|-------|-------|-------|------|
| Rate per 100 feet..... | .9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
| Lbs. per ton of 2240 lbs... | 20.16 | 22.4 | 24.64 | 26.88 | 29.12 | 31.36 | 33.6 |
| Rate per mile..... | 47 | 53 | 58 | 63 | 68 | 74 | 79 |
| Lbs. per ton of 2000 lbs... | 18 | 20 | 22 | 24 | 26 | 28 | 30 |

2d. *Curves.*—Recent European formula is that given by Baron von

slightly reduced when curve coincides with that for which wheels are elevated about 3° , and when train runs over it, at precise speed for which elevated, an allowance of .5 lb. per ton per degree is found to give good service.

2d. Resistance on Curves.

From above estimate of curve resistance that, in order to have the same resistance on a curve as on a straight line, the gradient should be diminished by .1 per cent of each degree of curve. Thus a 3° curve requires an easing of the track for 100 feet, a 10° curve an easing of .3 per 100, etc.

It is, however, need only be done upon the limiting gradients, and when sum of curve resistances exceeds resistance which has been assumed as limiting

3d. Resistance due to Wheel Friction.

Engineers are not agreed whether friction of wheels increases simply with the weight they carry, but also in some ratio with the speed. Originally assumed constant at 8 lbs. per ton, improvements in condition of track (steel rails) and in construction and lubrication of rolling-stock have reduced it to 1 lb. per ton for well-oiled trains. Under ordinary circumstances, it is safe to estimate it at 5 lbs. per ton on first-class tracks, and 6 lbs. per ton on second-class tracks. It may run up to 7 or 8 lbs. per ton on bad tracks (iron rails), and all these amounts should be increased from 25 to 50 per cent. in winter, to allow for inferior lubrication.

4th. Resistance due to Atmosphere.

Resistance to trains, complicated as it is by the wind which may be assumed not to be accurately ascertained by experiment. It consists of resistance of first car of train, which is presumably equal to its exposed end area, multiplied by air pressure due to speed.

Resistance of each subsequent car. This varies with distance they are from first car, and so shield each other from end air pressure due to speed.

Resistance of air against sides of each car depending upon the speed. This is small that it may be neglected altogether.

Resistance due to prevailing wind, which modifies above three items of resistance. A head wind retards the train, a rear wind aids it, while a side wind increases resistance by pressing flanges of wheels against one rail, and, in consequence of curves, assumes all of these positions to same wind.

Experiments on Erie Railway seem to indicate that in a dead calm resistance of first car of a freight train may be assumed at an exposed surface of 63 feet multiplied by air pressure due to speed, and that each subsequent car may be assumed to offer a resistance of 20 per cent. of that of first car, while in a passenger train first car may be assumed at an area of 90 sq. feet, multiplied by air pressure due to speed, and that each subsequent car adds an increment equal to 40 per cent. of first car, in consequence of greater distance they are coupled apart.

Resistance is, of course, entirely independent of cars being loaded or empty. It has been found that an allowance of 1.5 to 2 lbs. per ton of weight of train covers atmospheric resistance, except in very high winds.

In consequence of complexity of elements above enumerated, exact formulas cannot be now given for train resistances, but following, if applied with judgment (they are for standard gauge, and in making them, curve resistance has been assumed at .5 lb. per degree, wheel friction at 5 lbs., exposed end area of first car at 63 feet for passenger cars and 90 feet for freight cars, and increment for passenger cars at .4 for passenger trains and .2 for freight trains.

$$\text{Freight Train. } W \left(G + \frac{C^2}{2} + 5 \right) + \left(1 + \frac{n-1}{2.5} \right) 90 P = R.$$

$$\text{Passenger Train. } W \left(G + \frac{C^2}{2} + 5 \right) + \left(1 + \frac{n-1}{5} \right) 63 P = R.$$

W is the total area of car, which generally measures about 71 sq. feet; but part is the area of the wheels, bolts, etc., which offer less resistance than a flat plane. *P* is the pressure of air on the car, which is greater than that of freight cars, but in consequence of the shape of the car, and so opposes greater resistance.

ace, *To Compute Adhesion on a Given Grade and Curve, having Weight in tons.*

L.E.—Multiply tabular number by weight of train in tons (2000 lbs.), product will give adhesion, in lbs.

M.P.L.E.—Assume preceding elements. Then $39.5 \times 405 = 16\,000$ lbs.

E.—A "Consolidation" engine, by its superior adhesion (19 550 lbs.) would pull a like grade and curve 495 tons.

Memoranda on English Railways.

Regulations (Board of Trade).

—iron girders to have a breaking weight = 3 times permanent load, added to its moving load.

—light-iron bridges not to be strained to more than 5 tons per sq. inch.

—minimum distance of standing work from outer edge of rail at level of carriage 3.5 feet in England and 4 feet in Ireland.

—minimum distance between lines of railway, 6 feet.

—**Platforms.**—Minimum width of platform, 6 feet, and 12 at important stations. Minimum distance of columns from edge of platform, 6 feet. Steepest gradient for platforms, 1 in 260. Ends of platforms to be ramped (not stepped). Signals and signalposts in both directions.

—**Carriages.**—Minimum space per passenger 20 cube feet. Minimum area of glass for passenger, 60 sq. ins. Minimum width of seats, 15 ins. Minimum breadth of carriage for passenger, 18 ins. Minimum number of lamps per carriage, 2.

—**Repairs.**—Joints of rails to be fished. Chairs to be secured by iron spikes. Bolts to be used at the joints of flat-bottomed rails.

Construction.

| | Narrow. Feet. Ins. | Broad. Feet. Ins. |
|------------------------------------|-----------------------|----------------------|
| Width, single line..... | 18 | 24 6 |
| “ double line..... | 30 | 38 |
| “ top of ballast, single line..... | 13 6 | 15 6 |
| “ “ double line..... | 24 6 | 29 |

—**Cuttings.**—Minimum width of cutting from centre, 1 in 30. Width of land beyond bottom of slope, 1 foot. Ditch with slopes, 1 foot at bottom, 1 to 1. Quick mound, 18 ins. in height. Post and rail-fence posts, 7 feet 6 ins. \times 6 ins. \times 3.5 ins., 9 feet apart, 3 feet und. Intermediate posts, 5 feet 6 ins. \times 4 ins. \times 1.5 ins., 3 feet apart. Rails \times 1.5 ins.

Parliamentary Regulations for Crossing Roads.

| | Turnpike Road. | Public Road. | Occupation Road. |
|--|-------------------|-----------------|---------------------|
| | Feet. Ins. | Feet. Ins. | Feet. Ins. |
| width of under bridge, or approach.... | 35 — | 25 — | 12 — |
| height of under bridge for a width of 12 ft. | 16 — | — — | — — |
| “ “ “ “ 10 “ | — — | 15 — | — — |
| “ “ “ “ 9 “ | — — | — — | 7.4 — |
| “ “ at springing..... | 12 — | — — | — — |
| bridge, height of parapets..... | 4 — | — — | — — |
| arches, inclination..... | 1 in 30 | — — | — — |
| “ height of fencing..... | 3 — | — — | — — |

—**Limits of Deviation.**—In towns, 10 yards each side of centre line; in country, 5 chains nearly.

—**Clearance.**—In towns, 2 feet. In country, 5 feet.

—**Gradients.**—Gradients flatter than 1 in 100, deviation 10 in 100.

—**Do., steeper,** 3 feet per mile.

—**Curves.**—Curves upwards of .5 a mile radius, may be sharpened.

—**Curves of less than .5 mile radius may not be sharpened.**

ROADS, STREETS, AND PAVEMENTS.

Classification of Roads.

1. Earth. 2. Corduroy. 3. Plank. 4. Gravel. 5. Broken stone (Macadam). 6. Stone sub-pavement with surface of broken stone (Telford). 7. Stone sub-pavement with surface of broken stone and gravel, or gravel alone. 8. Rubble stone bottom with surface of broken stone or gravel, or both. 9. Concrete bottom with surface of broken stone or gravel, or both.

Grade of Roads.

Limit of practicable grade varies with character of road and friction of vehicle. For best carriages on best roads, limit is 1 in 35, or 15 feet in a mile.

Maximum grade of a turnpike road is 1 in 30 feet. An ascent is easier for draught if taken in alternate ascents and levels, than in one continuous rise, although the ascents may be steeper than in a uniform grade.

Ordinary angle of repose is 1 in 40 if roads are bad, and 1 in 30, to 1 in 25.

When roads have a greater grade than 1 in 35, time is lost in descending in order to avoid unsafe speed. Grade of a road should be less than its angle of repose. Minimum grade of a road to secure effective drainage should be 1 in 80. In France it is 1 in 125.

In construction of roads the advantage of a level road over that of an inclined one, in reduction of labor, is superior to cost of an increased length of road in the avoiding of a hill.

Alpine roads over the Simplon Pass average 1 in 17 on Swiss side, 1 in 15 on Italian side, and in one instance 1 in 13.

In deciding upon a grade, the motive power available of ascent and available waste of power in descending are to be first considered.

When traffic is heavier in one direction than the other, the grade is a percent of lighter traffic may be greatest.

When axis of a road is upon side of a hill, and road is made in part by excavation and by embankment, the side surface should be cut into steps in order to afford a secure footing to embankment, and in extreme cases sustaining walls should be erected.

Construction.

Estimate of Labor in Construction of Roads. (M. Ancelet.)

A day's work of 10 hours of an average laborer is estimated as follows:

In Cube Yards.

| WORK. | Ordinary Earth. | Loose Earth. | Mud. | Clay and Earth. | Gravel. | Stone |
|--|-----------------|--------------|---------|-----------------|----------|-------|
| Picking and digging..... | 18 to 23 | 16 | — | 9 | 7 to 11 | 24 |
| Excavation and pitching } 6 to 12 feet..... | 8 to 12 | 8 | 7 to 16 | 4 | — | 21 |
| Loading in barrows..... | 22 | — | 8 | — | 19 | — |
| Wheeling in barrows per } 5 feet..... | 20 to 33 | — | — | — | 24 to 28 | — |
| Wheeling in carts..... | 16 to 48 | — | — | — | 17 to 27 | — |
| Grading and levelling... } | 44 to 88 | — | 25 | — | 30 to 80 | — |

of pitching from a shovel is one third of that of digging.

All ditches should lead to a natural water-course, and their bottom should be 1 in 125.

Elevations in surface of a roadway involve a material reduction in speed. A wheel 4 feet in diameter, in order to be surmounted, and, as a consequence, one seventh of the

An unyielding foundation and surface are indispensable for a perfect roadway.

Earth in embankment occupies an average of one tenth less space than in natural soil, and rock about one third more.

Ruts.—Surface of a roadway should be maintained as intact as practicable, as the rutting of it not only tends to a rapid destruction of it, but involves increased traction.

The general practice of rutting a road displays a degree of ignorance of physical laws and mechanical effects that is as inexplicable as it is injurious and expensive.

On compressible roadways, as earth, sand, etc., resistance of a wheel decreases as width of tire increases.

Depressing of axles at their ends increases friction. Long and pliant springs decrease effect of shock in passing over obstacles in a very great degree.

Transverse Section.—Best profile of section of roadway is held to be one formed by two inclined planes meeting in centre of road and slightly rounded off at point of junction.

Roads having a rough surface or of broken stone should have a rise of 1 in 24, equal to a rise on crown of 6 ins., and on a smooth surface, as a block-stone or wood pavement, the rise may be reduced to 1 in 48.

On roads, when longitudinal inclination is great, the rise of transverse section should be increased, in order that surface water may more readily run off to sides of roadway, instead of down its length, and consequently clogging it.

Stone Breaking. A steam stone-breaking machine will break a cube yard stone into cubes of 1.5 ins. side, at rate of 1 to 1.5 HP per hour.

Macadamized Roads.

In construction of a Macadamized road, the stones (road metal) used should be hard and rough, and cubical in form, the longest diameter of which should exceed 2.5 ins., but when they are very hard this may be reduced to 1.25 in. and 1.5 ins.

The best stones are such as are difficult of fracture, as basaltic and trap, and especially when they are combined with hornblende. Flint and siliceous stone are rendered unfit for use by being too brittle. Light granites are objectionable, in consequence of their being brittle and liable to disintegration; dark granites, possessing hornblende, are less objectionable. Limestones, sandstones, and slate are too weak and friable.

Dimensions of a hammer for breaking the stone should be, head 6 ins. in length, weighing 1 lb., handle 18 ins. in length; and an average laborer can break from 1.5 to 2 cube yards per day.

Stones broken up in this manner have a volume twice as great as in original form. 100 cube feet of rock will make 190 of 1.5 ins. of 2 ins., and 170 of 2.5 ins.

One ton of hard metal has a volume of 1.185 cube yards.

Construction of a Roadway.—Excavate and level to a depth of 12 ins. lay a "bottom" 12 ins. deep of brick or stone spalls or chert concrete, etc., roll down to 9 ins., then add a layer of coarse ballast 5 ins. deep, roll down to 3 ins., and then metal in 2 of 3 ins., laid at an interval, enabling first layer to be fully covered. The second is laid on and rolled to a depth of 4 ins.; a surface of 5 ins. of sharp sand should be laid over last layer of metal with a free supply of water.

depth at sides of road to 4, 5, and 7 inches to centre, and set the broadest edge, free from irregularities in their upper surface, and the terstices filled with stone spalls or chips, firmly wedged in.

Centre portion of road to be metalled first to a depth of 4 ins. after being used for a brief period, 2 ins. more are to be added, surface to be covered, "blinded," with clean gravel 1.5 ins. in depth.

Telford assigned a load not to exceed 1 ton upon each wheel of with a tire 4 ins. in breadth.

Gravel or Earth Roads.

In construction of a gravel or earth road, selection should be made of clean round gravel that will not pack, and sharp gravel intermediate earth or clay, that will bind or compact when submitted to the traffic or a roll.

Surface of an ordinary gravel roadway should be excavated to from 8 to 12 ins. for full width of road, the surface of excavation cut to that of road to be constructed.

The gravel should then be spread in layers, and each layer consolidated by the gradual pressure due to travel over it, or by a roller, the weight increasing with each layer. One of 6 tons will suffice for limit of weight.

If gravel is dry and will not readily pack, it should be wet, and mixed with a binding material, or covered with a thin layer of it, as clay.

In rolling, the sides of road should be first rolled, in order to bind the gravel, when the centre is being rolled, from spreading at the side.

To re-form a mile of gravel or earth road, 30 feet in width between material cast up from sides, there will be required 1640 hours' labour and 20 of a double team.

Corduroy Roads.

A Corduroy road is one in which timber logs are laid transversely to the road.

Asphalt.

asphalt is a bituminous limestone, and is synonymous with bitumen; it consists of from 90 to 94 per cent. of carbonate of lime and 6 to 10 per cent. bitumen.

For forming a pavement the powder is heated to from 212° to 250° , and its particles caused to adhere by pressure, or it is applied as a liquid asphalt or asphaltic tarmac, which is thus manufactured. The powder is heated with from 5 to 8 per cent. of free bitumen for a flux, and the mixture when melted is run into molds. If remelted, additional bitumen must be mixed with it, without which it would become soft.

For paving 60 per cent. of sand or gravel must be mixed with it. No chemical action takes place between the mastic and the sand or gravel, but cohesion is so complete that gravel will fracture with the mastic, and the admixture increases the resistance of the mass to heat of the sun. The roadway should have a convexity of 1/2 of its breadth.

Artificial Asphalt.—Heated limestone and gas tar, when mixed, possess some of the proportions of asphalt mastic, but it is very inferior for the purposes of a pavement.

To repair surface of roadway, dissolve bitumen 1 part in 3 of pitch oil or kerosene oil, apply 10 oz. of mixture over each sq. yard of roadway, sprinkle on 1 lb. of asphalt powder, and then cover surface with sand.

Wood Pavement.

Close-grained and hard woods only are suitable, such as oak, elm, ash, mahogany, and yellow pine, and they should be laid on a foundation of concrete.

Block Stone Pavement.

Paving-blocks, as the Belgian, etc., where crest of street or area of pavement does not exceed 1 inch in 7.5 feet, should taper slightly toward the ends, and the joints be well filled, "blinded," with gravel. The common practice of tapering them downward is erroneous.

The foundation or bottoming of a stone pavement for street travel should consist either of hydraulic concrete or rubble masonry in hydraulic mortar. The practice in this country of setting the stones in sand alone is at variance with endurance and ultimate economy, but when resorted to, there should be a bed of 12 ins. of gravel, rammed in three layers, covered with an inch of sand. Granite or Trap blocks should be $4 \times 9 \times 12$ ins.

Rubble Stone Pavement.

Cobblestones or Beach stone of irregular volumes and forms, set in a bed of concrete, involves great resistance to vehicles and frequent repairs; it is wholly at variance with requirements of heavy traffic or city use.

Concrete Roads.

Concrete roads are constructed of broken stones (road metal) 4 volumes, sharp sand 1.25 to .33 volumes, and hydraulic cement 1 volume. The concrete is laid down in a layer of 3 or 4 ins. in depth, and left to harden during a period of 3 days, when a second and like layer is laid on and then left to harden for a period of from 10 to 20 days, according to temperature and moisture of the weather.

Roads. (Molesworth.)

Ordinary turnpike roads.—30 feet wide, centre 6 ins. higher than surface, .5 inch below centre; 9 feet from centre, 2 ins. lower; 15 feet from centre, 6 ins. below centre.

Foot-paths—6 feet wide, inclined 1 inch towards road, of fine sand and quarry chippings, 3 ins. thick.

Quarry roads—20 feet wide. **Foot-paths**—5 feet.

Drains—3 feet below surface of road.

Bed material—bottom layer gravel, burned clay or chalk, 8 ins. deep; broken granite not larger than 1.5 cube ins., 6 ins. deep.

Metalled Roads should be swept wet.

Rolling.—Steam rolls are most effective and economical. 1000 sq. yards ling will require 24 hours' rolling at 1.5 miles per hour. A roller of 15 ton will roll 1000 sq. yards of Telford or Macadam pavement in from 30 to 40 a speed of 1.5 miles per hour, equal .675 and .9 ton mile per sq. yard.

Sprinkling.—60 cube feet of water with one cart will cover 850 sq. y. cube feet per day will cover 1000 sq. yards; ordinarily two sprinklings are

Granite Pavement.—The wear of granite pavement of London Bridge was per year, and from an average of several streets in London, the wear per 10 per foot of width per day is equal to one sixteenth of an inch per year.

Sweeping and Watering of granite pavement and Macadam road, for ϵ and under alike conditions in every respect, costs as 1 for former to 7 of 1

By men, with cart, horse, and driver, costs 3.25 times more than by a one of which will sweep 16 000 sq. yards of street per period of 6 hours.

Asphalt Pavement.—Average cost per sq. yard in London: foundation, surface, \$3.25; cost of maintenance per sq. yard per year, 40 cents. Wear from .2 to .42 near curb, and .17 to .34 inch on general surface per year.

Washing.—Surface cleaning of stone or asphalt pavement by a jet can b at from 1 to 2 gallons per sq. yard.

Wood Pavement.—Wear of wood pavement in London, per 100 vehicle per foot of width, .083 inch per year.

Macadamized Roads.—Annual cost of maintenance of several such London was 62 cents per sq. yard.

Block Stone Pavement.—Stones should be set with their tapered or least wards, with surface joints of 1 inch.

Fascines, when used, should be in two layers, laid crosswise to each c picketed down.

Bituminous road may be made by breaking up asphalt, laying it 2 is covering with coal tar, and ramming it with a heavy beetle. To repair ous surface, dissolve one part of bitumen (mineral tar) in three of pitch of oil. spread .625 of a lb. of solution over each sq. yard of road. sprinkle 2

SEWERS.

rs are the courses from a series of locations, and are classed as , Sewers, and Culverts.

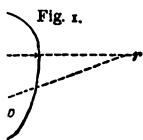
ins are small courses, from one or more points leading to a sewer.

verts are courses that receive the discharge of sewers.

test fall of rain is 2 ins. per hour = 54 308.6 galls. per acre.

nation of sewers should not be less than 1 foot in 240, and for r short lateral service it should be 1 inch in 5 feet.

Fig. 1.



Circular. $55 \sqrt{x^2 f} = v$, and $v a = V$.

Egg. $\frac{D}{3} = w$, $\frac{2D}{3} = w'$, and $D = r$. x representing area of sewer \div wetted perimeter, f inclination of sewer per mile, and v velocity of flow of contents in feet per minute; a area of flow, in sq. feet, V volume of discharge, in cube feet per minute; D height of sewer, w and w' width at bottom and top, and r radius of sides, in feet.

For diameter of sewer exceeding 6 feet. (T. Hawksley.)



$D - \frac{x}{9} = w'$. D diameter of a circular sewer of area required.

Elliptic.—Top and bottom internal should be of equal diameters. Diameter .66 depth of culvert; intersections of top and bottom circles form centres for striking courses connecting top and bottom circles.

Pipes or Small Sewers.—Height of section = 1; diameter = .66; of invert = .33, and radius of sides = 1.

verts less than 6 feet internal depth, brickwork should be 9 ins. thick; ey are above 6 feet and less than 9 feet, it should be 14 ins. thick.

meter of top arch = 1, diameter of inverted arch = .5, and total = sum of the two diameters, or 1.5; then radius of the arcs which are ial to the top, and inverted, will be 1.5.

this any two of the elements can be deduced, one being known.

Drainage of Lands by Pipes.

| SOILS. | Depth of Pipes. | Distance apart. | SOILS. | Depth of Pipes. | Distance apart. |
|-----------------|-----------------|-----------------|----------------------|-----------------|-----------------|
| | Ft. Ins. | Feet. | | Ft. Ins. | Feet. |
| ravel sand | 4 6 | 60 | Loam with gravel ... | 3 3 | 27 |
| nd with gravel | 4 6 | 50 | Sandy loam | 3 9 | 40 |
| um | 3 6 | 33 | Soft clay | 2 9 | 21 |
| th clay | 3 2 | 21 | Stiff clay | 2 6 | 15 |

num Velocity and Grade of Sewers and Drains in Cities. (Wicksteed.)

| Vel. per Minute. | Grade, 1 in | Grade per Mile. | Diam. | Vel. per Minute. | Grade, 1 in | Grade per Mile. | Diam. | Vel. per Minute. |
|------------------|-------------|-----------------|-------|------------------|-------------|-----------------|-------|------------------|
| Feet. | | Feet. | Ins. | Feet. | | Feet. | Ins. | Feet. |
| 240 | 36 | 146.7 | 15 | 180 | 244 | 21.6 | 42 | 18 |
| 220 | 65 | 81.2 | 18 | 180 | 294 | 18 | 48 | 18 |
| 220 | 87 | 60.7 | 24 | 180 | 392 | 13.5 | 54 | 18c |
| 210 | 119 | 44.4 | 30 | 180 | 490 | 10.8 | 60 | 18c |
| 190 | 175 | 30.2 | 36 | 180 | 588 | 9 | | |

of Sewers or Pipes.—An area of 20 acres, miles, etc. times capacity of pipes for one acre, mile, etc., as the d. res, etc., will not flow into the main simultaneously w. etc. Ordinarily in this country an area of sewer or pi. a rainfall of 1 inch per hour (3630 cube feet per acre) is

Sewage.—The excreta per annum of 100 individuals of both sexes and all ages is estimated at 7250 lbs. solid matter and 94 700 fluid, equal to 1000 lbs. *per capita*, and in volume 16 cube feet, to which is to be added the volume of water used for domestic purposes. A velocity of flow of from 2½ to 3 feet per second will discharge a sewer of its sewage matter and prevent deposits. The minimum velocity should not be less than 1.3 feet per second.

Surface from which Circular Sewers with proper Curves will discharge Water equal in Volume to One Inch in Depth per Hour, including City Drainage. (John Roe)

| INCLINATION IN FEET. | DIAMETER OF SEWERS IN FEET. | | | | | | | |
|----------------------|-----------------------------|--------------|------------|------------|------------|-------------|-------------|-------------|
| | 2 | 2.5 | 3 | 4 | 5 | 6 | 7 | 8 |
| None..... | Acres. 38.75 | Acres. 67.25 | Acres. 120 | Acres. 277 | Acres. 570 | Acres. 1020 | Acres. 1725 | Acres. 2500 |
| 1 in 480..... | 48 | 75 | 135 | 308 | 630 | 1117 | 1925 | 3000 |
| 1 in 240..... | 50 | 87 | 155 | 355 | 735 | 1318 | 2225 | 3500 |
| 1 in 160..... | 63 | 113 | 203 | 460 | 950 | 1692 | 2875 | 4500 |
| 1 in 120..... | 78 | 143 | 257 | 590 | 1200 | 2180 | 3700 | 5500 |
| 1 in 80..... | 90 | 165 | 295 | 570 | 1388 | 2486 | 4225 | 6500 |
| 1 in 60..... | 125 | 182 | 318 | 730 | 1500 | 2675 | 4550 | 7000 |

Surface of a Town from which small Circular Drains will discharge Water equal in Volume to Two Inches in Depth per Hour. (John Roe.)

| INCLINATION. Fall of 1 Inch. | DIAMETER OF DRAIN IN INS. | | | | | INCLINATION. Fall of 1 Inch. | DIAMETER OF DRAIN IN INS. | | | |
|---------------------------------|---------------------------|-------|-------|-------|-------|---------------------------------|---------------------------|-------|-------|-------|
| | 3 | 4 | 6 | 7 | 8 | | 9 | 12 | 15 | 18 |
| Acres. | Feet. | Feet. | Feet. | Feet. | Feet. | Acres. | Feet. | Feet. | Feet. | Feet. |
| .125 | 120 | — | — | — | — | 2.1 | 120 | — | — | — |
| .25 | 20 | 120 | — | — | — | 2.5 | 80 | — | — | — |
| .4375 | — | 40 | — | — | — | 2.75 | 60 | — | — | — |
| .5 | — | 30 | 80 | — | — | 4.5 | — | 120 | — | — |
| .6 | — | 20 | 60 | — | — | 5.3 | — | 80 | — | — |
| 1 | — | — | 20 | 60 | — | 5.8 | — | 60 | 240 | — |
| 1.2 | — | — | — | 40 | 20 | 7.8 | — | — | 120 | — |
| 1.5 | — | — | — | 20 | 60 | 9 | — | — | 80 | — |
| 1.8 | — | — | — | — | 80 | 10 | — | — | 60 | 240 |
| 2.1 | — | — | — | — | 60 | 17 | — | — | — | 120 |

Dimensions, Areas, and Volume of Material per Lined Foot of Egg-shaped Sewers of different Dimensions.

| Depth. | INTERNAL DIMENSIONS. | | | VOLUME OF BRICK-WORK. | | |
|--------|----------------------|------------------|-----------|-----------------------|---------------|------------------|
| | Diam. of Top Arch. | Diam. of Invert. | Area. | 4.5 Ins. thick. | 9 Ins. thick. | 13.5 Ins. thick. |
| Feet. | Feet. | Feet. | Sq. Feet. | Cube Feet. | Cube Feet. | Cube Feet. |
| 2.25 | 1.5 | .75 | 2.53 | 2.81 | — | — |
| 3 | 2 | 1 | 4.5 | 3.56 | — | — |
| 3.75 | 2.5 | 1.25 | 7.03 | 4.31 | 9.56 | — |
| 4.5 | 3 | 1.5 | 10.12 | 5.06 | 10.87 | — |
| 5.5 | 3.5 | 1.75 | 13.78 | 5.81 | 12.75 | — |
| 6 | 4 | 2 | 18 | 6.56 | 14.25 | — |
| 6.75 | 4.5 | 2.25 | 22.78 | 7.31 | 15.75 | 24.75 |
| 7.5 | 5 | 2.5 | 28.12 | — | 17.06 | 27 |
| 8.5 | 5.5 | 2.75 | 34.03 | — | 18 | 28.41 |
| 9 | 6 | 3 | 40.5 | — | 19.69 | 30.94 |

product of mean diameter × height.

have a uniform thickness and be uniformly *annally*.

be thicker than those of stone-clay.

STABILITY.

Strength, and Stiffness are necessary to permanence of a structure under all variations or distributions of load or stress to which it is subjected.

1 of a Fixed Body—Is power of remaining in *equilibrium* without deviation of position, notwithstanding load or stress to which it is subjected may have certain directions.

1 of a Floating Body.—A body in a fluid floats, or is balanced, when it displaces a volume of the fluid, weight of which is equal to weight of the body, and when centre of gravity of body and that of volume of fluid displaced are in same vertical plane.

A body in *equilibrium* is free to move, and is caused to deviate in a direction from its position of equilibrium, if it tends to return to its original position, its equilibrium is termed *Stable*; if it does not tend to deviate, or to recover its original position, its equilibrium is termed *Neutral*; and when it tends to deviate further from its original position, its equilibrium is termed *Unstable*.

A body in *equilibrium* may be stable for one direction of stress, and unstable for another.

1 of Stability of a body or structure resting upon a plane is the power of resisting a couple of forces, which must be applied in a plane vertically inclined to the plane of the body, in addition to its weight, in order to remove centre of resistance from the plane, or of the joint, to its extreme position consistent with stability.

The couple generally consists of the thrust of an adjoining structure, or pressure of water, or of a mass of earth against the body, together with the equal and parallel, but not directly opposed, reaction of the plane of foundation or joint of structure to that lateral thrust.

Two forces of equal magnitude applied to same body or structure in parallel and opposite directions, but not in same line of action, constitute a couple.

For Statical and Dynamical Stability, see Naval Architecture, page 649.

Certain Stability of a Body on a Horizontal Plane.

— Fig. 1.

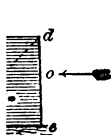


ILLUSTRATION.—Stability of a body, A, Fig. 1, when a thrust is applied as at *o*, to turn it on *a*, is ascertained by multiplying its weight by distance *a s*, from fulcrum *a* to line of centre of gravity, *c s*.

Hence, if cubical block weighed 20 tons and its base is 6 feet, its moment would be $20 \times \frac{6}{2}$.

If upper part, *a b d c*, was removed but 5 tons, but its centre of gravity *s* would be $\frac{2}{3}$ of the height, would be $5 \times 4 = 20$ tons, although it is but half the weight.

Compute Weight of a Given Body to Resist a Given Thrust.

F representing thrust in lbs., *h* height of centre of gravity.

s of fulcrum from centre of gravity = *a s*.

20.—Assume figure to be extended to a height of 20 feet of resisting the extreme pressure of wind.

Pressure estimated at 50 lbs. $F = 6 \times 20 \times 50 = 6000$ lbs. at centre of gravity of surface of body.

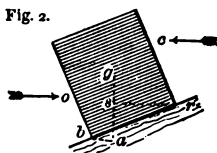
$$\text{Then } \frac{6000 \times 10}{3} = 20000 \text{ lbs.}$$

NOTE 1.—This result is to be increased proportionately with the factor of safety due to character of its material and structure.

2.—If form of body has a cylindrical section, as a round tower, the thrust of the water would be but one half of that of a plane surface.

When the Body is Tapered, as Frustum of Pyramid or Cone.—Ascertain centres of gravity of surface for pressure or thrust, and of body for its stability, and proceed as before.

Fig. 2.



To Ascertain Stability of a Body on an Inclination.—Fig. 2.

ILLUSTRATION.—Stability of body, Fig. 2, when thrust is applied at c , is ascertained by multiplying its weight by distance a b from fulcrum, b , to line of action of gravity, g .

If thrust was applied at o , stability would be maintained by distance r from fulcrum r .

Angles of Equilibrium at which various Substances will Repose, as determined by a Clinometer.

Angle measured from a Horizontal Plane, and falling from a slope.

| Degrees. | Degrees. | Degrees. |
|--------------------|--------------------------|---------------------|
| Lime-dust..... 45 | Sand, less dry..... 39.6 | Common mold... 3 |
| Dry sand..... 40 | Wheat..... 37 | Common gravel.. 3 |
| Moist sand..... 41 | Corn..... 37 | Stones or Coal... 4 |

Weight of a Cube Foot of Materials of Embankment Walls, and Dams.

| | | |
|---------------------------|-----------------|-----------|
| Concrete in cement... 137 | Gravel..... 125 | Clay..... |
| Stone masonry..... 130 | Loam..... 126 | Marl..... |
| Brick "..... 112 | Sand..... 120 | |

Revetment Walls.

When a wall sustains a pressure of earth, sand, or any loose material is termed a Revetment wall, and when erected to arrest the fall or subsidence of a natural bank of earth, it is termed a Face wall.

When earth or banking is level with top of wall, it is termed a Scarped revetment, and when it is above it, or *surcharged*, a Counterscarp revetment.

When face of wall is battered, it is termed Sloping, and when back is battered, Countersloping.

Thrust of earth, etc., upon a wall is caused by a certain portion, in shape of a wedge, tending to break away from the general mass. The pressure thus caused is similar to that of water, but weight of the material must be reduced by a particular ratio dependent upon angle of natural slope, which varies from 45° to 60° (measured from vertical) in earth of mean density.

Or, natural slope of earth or like material lessens the thrust, as the angle of slope.

The line of rupture makes with vertical is .5 of angle which the slope, or angle of repose, makes with same vertical line. At the top, its pressure may be ascertained by considering the cube foot of which is equal to weight of a cube foot of any square of tangent of .5 angle included between

When filling is composed of bowlders and gravel, the thickness of wall must be increased, and contrariwise; when of earth in layers and well rammed, it may be decreased.

Courses of dry wall should be inclined inwards, in order to arrest the flow of water of subsidence in filling from running out upon face of wall.

Less the natural slope, greater the pressure on wall.

Sea walls should have an increased proportion of breadth, as the earth backing is not only subjected to being flooded, but the walls have at times to sustain the weight of heavy merchandise.

Bultruss.—An increased and projecting width of wall on its front, at intervals its length.

Counterfort.—An increased and projecting width of wall at its back and at intervals.

Coefficient of Friction of masonry on masonry .67, of masonry on dry clay 1 and on wet clay .3.

Face of wall should not be battered to exceed 1 to 1.25 ins. in a foot of height; consequence of the facility afforded by a greater inclination to the permanent rain between the joints of the courses.

Footing of a wall, projecting beyond its faces, is not included in its width.

Pressure.—Limit of pressure on masonry 12 500 to 16 500 lbs. per sq. foot wall

Thickness of Walls, in Mortar, Faces vertical. For Railways or Like Structures.

Cut stone or Ranged rubble..... .35 | Brick or Dressed rubble..... .4

When laid dry, add one fourth.

Friction in vegetable earths is .5; pressure in sand .4.

When vegetable earths are well laid in courses, the thrust is reduced .5.

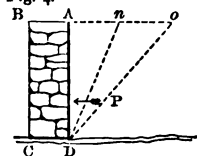
When bank is liable to be saturated with water, thickness of wall should be doubled.

Centre of Pressure of earthwork, etc., coincides with centre of pressure of wall and hence, when surface is a rectangle, it is at .33 of height from base.

The theory of required thickness of a retaining wall, as before stated, is that the lateral thrust of a bank of earth with a horizontal surface is that due to the prism or wedge-shaped volume, included between the vertical inner face of the wall and a line bisecting the angle between the wall and the angle of repose of the material.

To Compute Elements of Revetment Walls.—Fig. 4

Fig. 4.



Let A D o represent angle of repose of material, against a wall, A B C D. $A D n = .5 A D o$. $\tan. A D n = .5 \tan. A D o$.

$$\tan. A D n, \frac{h}{2}, \text{ or } \frac{h^2}{2} \tan. A D n = V;$$

$$\frac{\sin. D \times D A}{\sin. o} = A o; \quad \frac{w h^2}{2} \tan. A D n = P;$$

$$\frac{w h^2}{2} \tan.^2 A D n = P; \quad \frac{w h^2}{2} \tan.^2 A D n = \frac{h}{3} \tan.^2 A D n = E;$$

$$\frac{W h x^2}{2} = m; \quad \frac{w h^3}{6} \tan.^2 A D n = E; \quad \frac{w h^3}{3} \tan.^2 A D n = S; \quad \frac{h^2 \tan.^2 A D n}{2} = v;$$

$$h \tan. A D n \sqrt{\frac{w}{3 W}} = x, \text{ and } h \tan. A D n \sqrt{\frac{2 w}{3 W}} = x'. \quad h \text{ representing height}$$

wall in feet, V volume of section of prism of material A D n one foot in length in feet, W and w weights of a cube foot of wall and of material, P, p, and p' lateral

moments of pressure of prisms of earth A D o and A D n upon wall, M as moments of pressure and weight on and of wall, E and S equilibrium and stability in lbs., and x and x', C D for weights of wall for equilibrium and stability

10X.—A revetment wall, Fig. 4, of 125 lbs. per cube foot and ins a bank of earth having a natural slope of 52° 24', and 1 cube foot; what is pressure or thrust against it, etc.?

$\tan^2 A D n = .242$. Then $.492 \times 40 \times \frac{40}{2} = 393.6$ cube feet.

$$\frac{89.25 \times 40^2}{2} \times .492 = 35128.8 \text{ lbs.} \quad \frac{89.25 \times 40^2}{2} \times .492^2 = 17278.8 \text{ lbs.}$$

$$\frac{89.25 \times 40^2}{2} \times .492^2 \times \frac{40}{3} = 230384 \text{ lbs.} \quad 125 \times 40 \times \frac{9.6^2}{2} = 230400 \text{ lbs.}$$

$$40 \times .492 \sqrt{\frac{89.25}{3 \times 125}} = 9.6 \text{ feet, and } 40 \times .492 \sqrt{\frac{2 \times 89.25}{3 \times 125}} = 13.58 \text{ feet.}$$

For Rubble Walls in Mortar or Dry Rubble, add respectively to base as above retained, .14 and .42 part.

NOTE 1.—When coefficient of friction is known, use it for $\tan^2 A D n$.

2. $\times C$ = base of wall for stability. (Molesworth.)

3.—When either relative weights of equal volumes of wall and bank of earth or their specific gravities are given, S and s may be taken for W and w .

These equations involve simply the operation of a lever, the fulcrum being at the outer edge of wall C . The moment of pressure of bank is product of lateral pressure and perpendicular distance from fulcrum to line of direction of pressure.

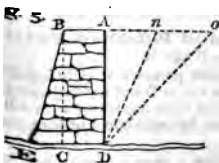
The moment of weight of wall is product of weight of wall and perpendicular distance from fulcrum to vertical line drawn through centre of gravity of wall.

When Weights of Embankment and Wall are equal per Cube Foot.

C for clay = .336, and for sand .267.

When Weights are as 4 to 5. C for clay = .3, and for sand .239.

When Wall has an Exterior Slope or Batter.—Fig. 5.



$$\frac{W h}{2} \left(\frac{C D + E C^2 - E C^2}{3} \right) = M. \quad M \text{ representing moment of weight of wall in lbs.}$$

ILLUSTRATION.—Assume weight of wall 120 lbs. per cube foot, and $C D$ and $E C$ respectively 10 and 2.5 feet, and all other elements as in preceding case.

$$\text{Hence, } \frac{120 \times 40}{2} \times \left(10 + 2.5 - \frac{2.5^2}{3} \right) = 370000 \text{ lbs.}$$

$$\frac{W h}{2} \left(x + n h - \frac{n^2 h^2}{3} \right) = \frac{w h^3}{3} \tan^2 A D n = S.$$

Or, $h \sqrt{\frac{n^2}{3} + \frac{2 w}{3 W} \tan^2 A D n} - n h = x$. x representing $A B$ or $C D$. n ratio of difference of widths of base and top to height. In absence of $\tan^2 A D n$ put C , coefficient of material.

$C = .0424$ for vegetable or clayey earth, mixed with large gravel; .0464 if mixed with small gravel; .1528 for sand, and .166 for semi-fluid earths.

ILLUSTRATION.—Assume elements of preceding case. n = one fortieth, and $\tan^2 A D n = .492$.

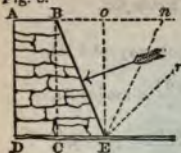
$$40 \sqrt{\frac{1}{3 \times 40^2} + \frac{2 \times 89.25}{3 \times 125} \times .492^2} - 1 = 12.6 \text{ feet.}$$

Hence, thickness of wall at base = 12.6 + 1 (one fortieth of height)

$$\text{Or, } - \text{If } n = \text{one twentieth, } 40 \sqrt{\frac{1}{3 \times 20^2} + \frac{2 \times 89.25}{3 \times 125} \times .492^2} - 1 = 11.63 + 1$$

Hence, wall at base = 11.63 + 1 (one twentieth of height) = 12.63 feet.

Fig. 6.



When Wall has an Interior Slope or Batter, B

Fig. 6.

$$\frac{w h^2}{2} \times \tan^2 \frac{o E r}{2} = P. \quad \frac{w h^3}{6} \times \tan^2 \frac{o E r}{2} =$$

earth for equilibrium; $\frac{w h}{2} \left(D C \times D C + C E - \frac{C E^2}{2} \right)$

M of wall; and $\frac{w h^3}{3} \times \tan^2 \frac{o E n}{3} = M$ of earth for bility.

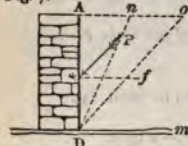
Coefficients for Batter of following Proportions.

Base = Height \times Tab. number.

| BATTER OF WALL. | Weight of Earth to Wall. | | BATTER OF WALL. | Weight of Earth to Wall. | |
|-----------------|--------------------------|------------|-----------------|--------------------------|------------|
| | As 4 to 5. | As 1 to 1. | | As 4 to 5. | As 1 to 1. |
| 1 in 4..... | .083 | .029 | 1 in 8..... | .184 | .125 |
| 1 " 5..... | .122 | .065 | 1 " 12..... | .221 | .16 |
| 1 " 6..... | .149 | .092 | Vertical.... | .3 | .239 |

To Compute Pressure Perpendicular to Back of W
—Fig. 7.

Fig. 7.


 $P = \frac{A D}{3}$ or $\frac{h}{3}$, and f at right angle to back of
whether vertical or inclined.

$$\frac{L \times A n}{h}, \text{ or } L \times \tan. A D n, \text{ or } \frac{w \times h^2 \times \tan^2 A D}{2}$$

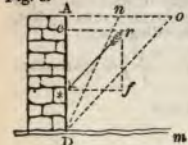
$$\frac{w \times A n^2}{2} = f *. \quad L \text{ representing weight of triangle }$$

bankment, as A D n.

This is pressure independent of friction between surfaces of wall and earth.

To Ascertain and Compute Amount and Effect of F
tion of Wall and Earth.—Fig. 8.

Fig. 8.



Draw $f *$ by scale to computed pressure at right
to back of wall, draw angle $f * r = m D o$ of natural
of earth with horizon, draw $f r$ at right angle to $f *$,
 $r c = f *$, then $c r$ will represent by scale effect of the
against back of wall.

Assume friction to act at point $*$, then $r *$ will give
scale resultant of the two forces of pressure and friction
equal to pressure in force and direction, which
against wall.

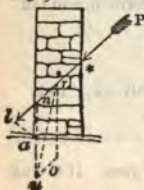
This resultant is also equal to $f * \times \sec. m D o$.

$$\frac{L \times A n \times \sec. m D o}{h} = r *, \text{ or } \frac{w \times h^2 \times \tan^2 m D o}{2} \times \sec. m D o, \text{ or } L \times \tan.$$

$\times \sec. m D o$.

To Ascertain Point of Moment of Pressure of a W
—Fig. 9.

Fig. 9.

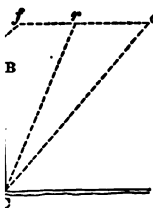


By its resisting lever $l a$, added to its weight.
Weight of wall as computed assumed as concentrated at
centre of gravity.

Draw a vertical line $. o$ through its centre of gravity, and
tinue line of pressure $P *$ to l , take any distance $r o$ by scale
representing weight of wall, and $r n$, by same scale, for amount
pressure or thrust against wall, complete parallelogram $r o n$,
then diagonal $r u$ will give resultant of pressure in amount
direction to overturn wall.

For stability this diagonal should fall inside of base of
not less than one third of its breadth.

Surcharged Revetments.



When the earth stands above a wall, as A B e, Fig. 10, with its natural slope, A f, A B C is termed a *Surcharged Revetment*.

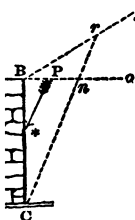
If C r is line of rupture, A f r C is the part of earth that presses upon wall, which part must be taken into the computation, with exception of portion A B e, which rests upon wall; that is, the computation must be for part C e f r, which must be reduced by multiplying weight of a cube foot of it by square of tangent of angle e C r = angle of line of rupture, or half angle e C o, which natural slope makes with vertical, and then proceed as in previous cases for revetments.

$\frac{1}{3} w \tan^2 e C r = \text{breadth or CD.}$ W and w representing weights of wall and ment in lbs. per cube foot, and h' height of embankment, as C e.

ILLUSTRATION.—Height of a surcharged revetment, B C, Fig. 10, is 12 feet, weight per cube foot; what is its width or base to resist pressure of earth of a weight ba. per cube foot, and a height, C e, of 15 feet, angle of repose 45° ?

$$2(45^\circ \div 2) = .1716. \text{ Then } 15 \sqrt{\frac{15 \times 100 \times .1716}{3 \times 12 \times 130}} = 15 \sqrt{.055} = 3.52 \text{ feet.}$$

Ascertain Point of Moment of Pressure of a Surcharged Wall.—Fig. 11.



Draw a line, P *, parallel to slope, C r, through centre of gravity of sustained backing, B C r.

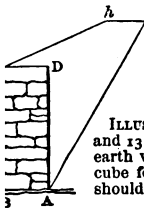
When, as in this case, this section is that of a triangle, point * will be at .33 height of wall.

When natural slope is 1.5 in length to 1 in height, as with gravel or sand, $w \times .64 = \text{pressure P *}$.

In a surcharged revetment, as f B o, at its natural slope, the maximum pressure is attained when the backing reaches to r. When slope of maximum pressure, C n r, intersects face of natural slope, B f, so that if backing is raised to f, or above it, there is theoretically no additional stress exerted at back of or against wall, but practically there is, from effect of impact of vibration of a train, proximity to percussive action, alike to that of a trip-hammer, etc.

1 backing rests on top of wall, as A B e, Fig. 10, small triangle of it is omitted putations. Direction of pressure against wall is same as when wall is not ged.

When Wall is set below Surface of Earth.—Fig. 12.



$$1.4 \tan 45^\circ - \frac{a}{e} \sqrt{\frac{h^2 w \left(\tan 45^\circ - \frac{a}{2} \right)^2}{W}} \sim 2 f V = d$$

a representing angle of repose of earth, w and W weights of earth and wall per cube foot, f friction of wall on base A B, and V weight of wall.

ILLUSTRATION.—If a wall of masonry, Fig. 12, 8 feet in thickness and 13 in height, is to sustain earth level with its upper surface, earth weighing 100 lbs. per cube foot, weight of wall 150 lbs. per cube foot = 15 600 lbs., and angle of repose of earth 30° ; what should be the depth of wall below surface of earth?

$$\tan. 45 - 30 \div 2 = .5774; \text{ and } f = .3.$$

$$1.4 \times .5774 \sqrt{\frac{13^2 \times 100 \times .5774^2 \sim 2 \times .3 \times 15600}{150}} = .8084 \sqrt{\frac{9360 \sim 5634.3}{150}}$$

7 feet.

—Coefficient of stability is assumed by French engineers for walls of fort 1.4 h, and if ground is clayey or wet f = .3.

Fig. 13.



In Computing Stability of a Surcharged Wall, Fig. 13, substitute d for h , as in following illustration. (Molesworth)

d , representing depth at distance l , = h

In slopes of 1 to 1, $d = 1.71 h$; of 1.5 to 1, $d = 1.55$; of 2 to 1, $d = 1.45$; of 3 to 1, $d = 1.31$, and 4 to 1, $d = 1.24$.

To Determine Form of a Pier to Sustain equal Pressure per Unit of Surface at all its Horizontal Sections, or any Height.

A $nd = a$, or $AN = a$. A and a representing areas of sections at summit of pier and at any depth, d , measured from summit, n a number the hyp. log. of which = $1 + \frac{H}{\text{height, } H}$, of a column of the material of which pier is constructed, due to required pressure, and N the number, com. log. of which = $\frac{.4343 d}{H}$.

ILLUSTRATION.—Height of a pier is 20 feet, and area of section of its summit = 1 foot; what should be its areas at 10 feet and base?

$1 \div 20 = .05$, and number = 1.0513 ; $1 \times 1.0513^{20} = 1.649$ feet; and $1 \times 1.0513^{20} = 2.719$ feet.

Counterforts are increased thicknesses of a wall at its back, at intervals of its length.

Embankment Walls and Dams.

Thrust of water upon inner face of an Embankment wall or Dam is horizontal.

When Both Faces are Vertical, Fig. 14.

Assume perpendicular embankment or wall, A B C D, Fig. 14, to sustain pressure of water, B C e f.

Fig. 14.



Let $k i$ be a vertical line passing through o , centre of gravity of wall, c centre of pressure of water, distance $C c$ being $= .33 B C$. Draw $c l$ perpendicular to $B C$; then, since section A C of wall is rectangular, centre of gravity, o , is in its geometrical centre, and therefore $D i = .5 D C$. Now $l D i$ is to be considered as a bent lever, fulcrum of which is D , weight of wall acting in direction of centre of gravity, o , on arm $D i$, and pressure of water on arm $D l$, or a force equal to that pressure thrusting in direction $c l$.

$$\text{Then } P \times D l = P \times \frac{B C}{3} = W \times \frac{D C}{2}, \text{ or } P = \frac{3 D C \cdot W}{2 B C}. \quad P \text{ representing pressure}$$

of water.

NOTE.—When this equation holds, a wall or embankment will just be on the point of overturning; but in order that they may have complete stability, the equation should give a much larger value to P than its actual amount.

The following formulas are for walls or embankments one foot in length, for if they have stability for that length they will be stable for any other length.

$P = \frac{h^2}{2} w$, also $W = h b W$, each value being for 1 foot in length, which, being substituted in the equations, there will result

$\frac{h^2}{2} w = \frac{3 b \times h b W}{2 h}$, or $h^2 w = 3 b^2 W$; $b \sqrt{\frac{3 W}{w}} = h$, and $h \sqrt{\frac{w}{3 W}} = b$, h representing depth of water and wall or embankment, which are here assumed to be equal, b breadth of wall or embankment, and W and w weights of wall and water per cube foot in lbs.

Which gives breadth of a wall or embankment that will just sustain pressure of the water.

To Compute Equilibrium. $h \sqrt{\frac{w}{3W}} = b$.

ILLUSTRATION 1.—Height of a wall, B C, equal to depth of water, is 12 feet, and relative weights of water and wall are 62.5 lbs. and 120 lbs. per cube foot; required width of wall, so that it may have complete stability to sustain the pressure of water.

$$b \sqrt{\frac{62.5}{3 \times 120}} = 12 \times .4166 = 5 \text{ feet, breadth that will just sustain pressure of the water.}$$

Therefore an addition should be made to this to give the wall complete stability, 2 feet; hence $5 + 2 = 7$, required width of wall.

—Width of a wall is 3 feet, and weight of a cube foot of it is 150 lbs.; required height of wall to resist pressure of fresh water to the top.

$$3 \sqrt{\frac{3 \times 150}{62.5}} = 8.049 \text{ feet.}$$

To Compute Stability. $h \sqrt{\frac{2w}{3W}} = b$.

ILLUSTRATION.—Take elements of preceding case.

$$12 \sqrt{\frac{2 \times 62.5}{3 \times 120}} = 12 \times .589 = 7.07 \text{ feet.}$$

Or, Divide 1, 2, or 3, etc., according as the nature of the ground, the material, and the character of the thrust of the water requires, by .05 weight of material of wall, per cube foot, extract the square root of quotient, and multiply result by extreme height of water.

EXAMPLE.—What should be the thickness of a vertical faced wall of masonry, having a weight of 125 lbs. per cube foot, to sustain a head of water of 40 feet, and have stability?

$$\sqrt{(2 \div .05 \times 125)} 40 = \sqrt{.32 \times 40} = 22.63 \text{ feet.}$$

$$\text{Or, } h \sqrt{\frac{2w}{3W}} = 40 \sqrt{.3472} = 23.56 \text{ feet.}$$

When Dam has an Exterior Slope or Batter, as A D.—Fig. 15.



Assume prismoidal wall, A B C D, to sustain pressure of water, B C e f.

Draw A E perpendicular to D C; $h = B C$, the top breadth $A B = E C = b$, and bottom breadth, D E, of sloping part, A E D = S.

Then weights of portions A C and A E D respectively for one foot in length are $h b W$ and $.5 W S h$, these weights acting at points n and i respectively.

To Compute Moment.

$$h b W \times \left(s + \frac{b}{2}\right) = \text{moment for A C, and } \frac{h S W}{2} \times \frac{2S}{3} = \text{moment for A E D.}$$

$$\text{Hence, } \frac{W h}{2} \left(\frac{s^2 + b^2}{3} - \frac{S^2}{3}\right) = \text{moment of dam. } S \text{ representing batter or base E D.}$$

ILLUSTRATION.—Height of a dam, B C, Fig. 15, is 9 feet, base C E 3, and E D 4 feet; find its moment?

$$A C = 9 \times 3 \times 120 \times \left(4 + \frac{3}{2}\right) = 3240 \times 5.5 = 17820 \text{ lbs.}$$

$$A E D = \frac{9 \times 4 \times 120}{2} \times \frac{2 \times 4}{3} = 2160 \times 2\frac{2}{3} = 5760 \text{ lbs.}$$

$$\text{Hence, } 17820 + 5760 = 23580 \text{ lbs. moment. Or, } \frac{120 \times 9}{2} \left(\frac{3^2 + 4^2}{3} - \frac{4^2}{3}\right) = 540 \times 43\frac{1}{3} = 23580 \text{ lbs. moment.}$$

702 STABILITY.—EMBANKMENT WALLS AND DAMS.

To Compute Elements of Walls or Dams with a Exterior Batter.—Fig. 15.

To Compute Width of Top.

When Width of Batter is Given. $\sqrt{\frac{2 h^2 w}{3 W} + \frac{S^2}{3}} - S = b$.

ILLUSTRATION.—Assume height of wall 9 and batter 3 feet, and W and w 120 and 62.5 lbs. per cube foot.

$$\sqrt{\frac{2 \times 9^2 \times 62.5}{3 \times 120} + \frac{3^2}{3}} - 3 = \sqrt{28.125 + 3} - 3 = 2.58 \text{ feet.}$$

To Compute Width of Base.

When Width of Batter is Given. $\sqrt{\frac{2 h^2 w}{3 W} + \frac{S^2}{3}} = B$.

$$\sqrt{\frac{2 \times 9^2 \times 62.5}{3 \times 120} + \frac{3^2}{3}} = 5.58 \text{ feet} = S + b$$

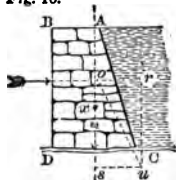
To Compute Width of Batter.

When Width of Top is Given. $\sqrt{\frac{h^2 w}{W} + \frac{3 b^2}{4}} - \frac{3 b}{2} = S$

$$\sqrt{\frac{9^2 \times 62.5}{120} + \frac{3 \times 2.58^2}{4}} - \frac{3 \times 2.58}{2} = \sqrt{42.18 + 4.99} - 3.87 = 3 \text{ feet}$$

When Width of Bottom is Given. $\sqrt{3 B^2 - \frac{2 h^2 w}{W}} = S$.

To Determine Stability of a Retaining Wall or Dam by Protraction.—Fig. 16.



Assume $A B C D$, section of a wall. On horizontal line of centre of thrust or pressure, with a suitable scale, lay off, from vertical line of centre of gravity of wall, line $o r$ = thrust against wall, and on vertical line at centre of gravity of wall, at its intersection with centre of thrust, let fall $o s$ = weight of wall.

Complete parallelogram, and if diagonal $o u$ or its prolongation falls within C , the wall is stable, $W \times$ distance from line $o s$ = moment of wall.

W representing whole weight of wall in lbs.

To Determine Centre of Gravity of a Wall or Dam by Fig. 16.

By Ordinates. $\frac{1}{3} (A B + C D - \frac{A B \times C D}{A B + C D}) = x$, and $\frac{B D}{3} (\frac{2 A B + C D}{A B + C D}) =$

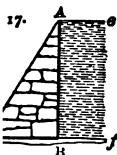
To Compute Base of Dam.

When Height, Rate of Batter, and Weight of Materials are given. R — Multiply square of width of batter by .0166 weight of material per cu foot, add 1, 2, or 3 times square of depth of water, according as resistance due to equilibrium is required, divide result by .05 weight of material per cube foot, and extract square root of quotient.

$$\text{Or, } \sqrt{\frac{x h^2 + b^2 \times .0166 W}{.05 W}} = b. \quad x = \text{number of times of resistance required}$$

Assume a dam 40 feet in height, constructed of masonry weight 150 lbs. per cu foot, to batter 3 ins. per foot, and to have twice the resistance to water; what should be its breadth at its base, $D C$?

$$\text{After, and } \sqrt{\frac{40^2 \times 2 + 10^2 \times .0166 \times 120}{.05 \times 120}} = \sqrt{\frac{3399}{6}} =$$



17. *When Section of Dam is a Triangle, Fig. 17. — Assume dam, A B C, to sustain a head of water, e f.*

RULE.—Proceed as by Rule for Fig. 14; multiply by .033 instead of .05.

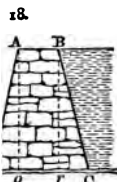
EXAMPLE.—As before.

$$\sqrt{(2 \div .033 \times 125) 40} = \sqrt{.485 \times 40} = 27.84 \text{ feet.}$$

Or, Formula for S (C B), Fig. 15. $\sqrt{\frac{h^2 \times W}{W}} = 28.28 \text{ feet.}$

to Determine Section of a Vertical Wall which shall have Equal Resistance of one having Section of a Triangle. (See J. C. Trautwine, Phila., 1872.)

Compute Thickness of Base of a Wall or Dam.—Fig. 18.



RULE.—Divide 1, 2, or 3 times square of depth of water by .05 weight of material, add quotient to .5 batter on one face, and square root of this sum, added to half batter on other side, will give thickness.

Or, $\sqrt{\frac{h^2 x}{.05 W} + \left(\frac{b}{2}\right)^2} + \frac{b'}{2} = \text{Base.}$ *b and b' representing exterior and interior batters, and x, as before, number of times of resistance or square of depth.*

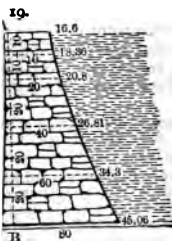
EXAMPLE.—Assume a dam 40 feet in height, to batter 5 feet on each side, constructed of masonry weighing 120 lbs. per cube, and to have twice the resistance due to its equilibrium; what should be width of base, D C?

$$\sqrt{\frac{40^2 \times 2}{.05 \times 120} + \left(\frac{5}{2}\right)^2} + \frac{5}{2} = \sqrt{539.58 + 2.5} = 25.73 \text{ feet.}$$

High Masonry Dams.

ubble Masonry, well laid in strong cement, will bear with safety a load equivalent to weight of a column of it 160 feet in height. Assuming such

masonry as twice weight of water, it is equivalent to a pressure of 20000 lbs. per sq. foot.



Log. B + .434 294 $\times \frac{d}{80} = b.$ *B representing width of wall at top, and d depth at any desired point below top, both in feet.*

Ordinarily, B may be taken at 18 feet, and in cases of extreme and exposed heights of dam at 20 and more, and when b is determined, .9 of it is to be on outer face of wall, as A B, and .1 on inner face.

ILLUSTRATION.—Determine section of a dam, Fig. 19, 80 feet in height, at depths of 10, 20, 40, 60, and 80 feet.

Log. B = 1.2553.

$$\text{og. } 1.2553 + .4343 \times \frac{10}{80} = \text{og. } 1.2553 + .0543 = 20.4, \text{ which } \times .9 = 18.36.$$

$$\text{" } 1.2553 + .4343 \times \frac{20}{80} = \text{og. } 1.2553 + .1086 = 23.11, \text{ which } \times .9 = 20.8.$$

$$\text{" } 1.2553 + .4343 \times \frac{40}{80} = \text{og. } 1.2553 + .2172 = 29.68, \text{ which ;}$$

$$\text{" } 1.2553 + .4343 \times \frac{60}{80} = \text{og. } 1.2553 + .3257 = 38.11, \text{ which}$$

$$1.2553 + .4343 \times \frac{80}{80} = \text{og. } 1.2553 + .4343 = 50.07, \text{ which}$$

STEAM.

STEAM is generated by heating of water until it attains temperature of ebullition or vaporization, and elevation of its temperature is to indications of a thermometer up to point of ebullition; it converted into steam by additional temperature, which cannot be indicated by a thermometer, and is termed latent. (See Heat, p. 6)

Pressure and density of steam, which is generated in free contact with water, rises with the temperature, and reciprocally its temperature rises with pressure and density, and higher the temperature more rapid the pressure. But one and a corresponding pressure and density for each temperature, generated in free contact with water is both at its maximum density and for its temperature, and in this condition it is termed *saturated*, from its capable of vaporizing more water unless its temperature is raised.

Saturated Steam is the normal condition of steam generated in free contact with water, and same pressure always exist in conjunction with its temperature. It therefore is both at its condensing and generating point, it is condensed if its temperature is reduced, and more water is evaporated if its temperature is raised.

If, however, the whole of the water is evaporated, or a volume of saturated steam is isolated from water, in a confined space, and an additional quantity supplied to the steam, its condition of saturation is changed, the steam is termed *superheated*, and both temperature and pressure are increased, while its density is not increased. Steam, when thus surcharged, approaches to condition of dry steam.

With saturated steam, pressure does not rise directly with the temperature.

Steam, at its boiling-point, is equal to pressure of atmosphere, which is 15 lbs. (page 427), at 60° upon a sq. inch.

In all computations concerning steam, it is necessary to have some of the following elements, viz.:

Its *Pressure*, which is termed its tension or elastic force, and is expressed per sq. inch. Its *Temperature*, which is number of its degrees of heat in a thermometer. Its *Density*, which is weight of a unit of its volume with that of water. Its *Relative volume*, which is space occupied by a given volume of it, compared with weight or volume of water that produced it.

Under pressure of the atmosphere alone, temperature of water cannot rise above its boiling-point.

Expansive force of steam of all fluids is same at their boiling-point.

A cube inch of water, evaporated under ordinary atmospheric pressure, expands into 1642* cube ins. of steam, or, in a unit of measure, very nearly 1 and it exerts a mechanical force equal to raising of $14.723307 \times 144 = 2140.1$ lbs. 1 foot high.

A pressure of 1 lb. upon a sq. inch will support a column of mercury of 60°, $1 \div .4907769$ (page 427) = 2.037586 ins. in height; hence raised a mercurial siphon gauge one half of this, or 1.018793 ins.

Velocity of steam, when flowing into a vacuum, is about 1550 feet per second at a pressure equal to the atmosphere; when at 10 atmospheres velocity is 10 but 1780 feet; and when flowing into the air under a similar pressure 650 feet per second, increasing to 1600 feet for a pressure of 20 atmospheres.

Boiling-points of Water, corresponding to different heights of barometer, Heat, page 517.

Volume of a cube foot of water evaporated into steam at 212° is 1642 hence $1 \div 1642 = .000609013$, which represents density or specific gravity of steam at pressure of atmosphere.

Elasticity of vapor of alcohol, at all temperatures, is about 2.125 times the *Specific Gravity*, compared with air, is as weight of a cube foot of it to weight of a cube foot of air. Thus, weight of a cube foot of steam at 212° is 266.124 grains; weight of a like volume of air at 62° is 532.679 grains. Hence $266.124 \div 532.679 = .500$ compared with air at 32°, and with water it is .000

Total Heat of Saturated Steam. (Regnault.)*From Water at 32°.* $1081.4 + .305 T = \text{total heat}$ T representing initial temperature of water.

ILLUSTRATION.—What is total heat of steam at 212°?

$$1081.4 + .305 \times 212 = 1146.06.$$

As Specific heat of water is .9 greater at 212° than at 32°, hence the 212° would be 212.9, and 1146.33 the result.

*Total Heat of Gaseous Steam from Water at 32° = 1074.6 + .475 T.***Absorption of Heat.***In Generation of 1 Lb. of Steam at 212° from Water at 32°.*

| | Thermal Units. | Foot-lbs. |
|---|----------------|--|
| Sensible heat, or heat to raise temperature of water from 32° to 212° | 181.8 | $772 \times 139\,655$ |
| Latent heat to produce steam | 892.9 | |
| “ “ to resist atmospheric pressure 14.7 lbs. per sq. inch | 71.4 | $\frac{964.3 \times 772}{1146.1} = 745\,134$ |
| Total or constituent heat | 1146.1 | 884\,789 |

In Generation of 1 Lb. of Steam at 175 lbs. from Water at 32°.

| | Thermal Units. | Foot-lbs. |
|---|----------------|-----------|
| Sensible heat as in preceding case from 32° to 370.8° | 342.4 | 275\,333 |
| Latent heat to produce steam | 768.2 | 593\,050 |
| “ “ to resist external pressure = 175 lbs. | 83.8 | 64\,694 |
| Total heat from 32° | 1194.4 | 933\,077 |

Mechanical Equivalent of Heat Contained in Steam.

- lb. water heated from 32° to 212° requires as much heat as would raise 180 lbs. 1°. Hence..... 181.8°
- lb. water at 212°, converted into steam at 212° (= 14.7 lbs. pressure), absorbs as much heat as would raise 966.6 lbs. water 1°. Hence..... $\frac{964.3}{1146.1}$

Mechanical Equivalent, or maximum theoretical duty of quantity of heat in one thermal unit or one degree in 1 lb. of water, is 772 foot-lbs., which $\times 1146.1$ units of heat = 884\,789.2 lbs. raised 1 foot high.

To Compute Pressure of Steam*Above Perfect Vacuum.*

When Height of Column of Mercury it will Support is given. RULE.—Divide height of column of mercury in ins. by 2.037 586, and quotient will give pressure per sq. inch in lbs.

EXAMPLE.—Height of a column of mercury is 203.7586 ins.; what pressure per sq. inch will it contain?

$$203.7586 \div 2.037\,586 = 100 \text{ lbs.}$$

To Compute Weight of a Cube Foot of Steam.

RULE.—Multiply its density by 62.425.

EXAMPLE.—Density of a volume of steam is .000609013; what is its weight?

$$.000609013 \times 62.425 = .038\,016\,825 \text{ lbs.}$$

NOTE.—See table, page 708.

- 1 atmosphere or 14.723 307 lbs. per sq. inch = 30 ins. of mercury.

To Compute Temperature of Steam.

RULE.—Multiply 6th root of its force in ins. of mercury by 177.2, subtract 100 from product, and remainder will give temperature in degrees.

EXAMPLE.—When elastic force of steam is equal to a pressure of 64 ins. of mercury, what is its temperature?

NOTE.—To extract 6th root of a number, ascertain cube root of its square root.

$$\sqrt[6]{64} = 8, \text{ and } \sqrt[3]{8} = 2. \text{ Hence, } 2 \times 177.2 - 100 = 254.4^\circ \text{ t.}$$

$$\text{Or, } \frac{2938.16}{6.199\,3544} - \log p - 371.85 = t. \quad p \text{ representing pressure in lbs.}$$

Saturated Steam.

Pressure, Temperature, Volume, and Density.

| Pressure | | Temperature. | Total Heat from Water at 32°. | Volume of 1 Lb. | Density, or Weight of one Cubic Foot. | Pressure | | Temperature. | Total Heat from Water at 32°. |
|---------------|-------------|--------------|-------------------------------|-----------------|---------------------------------------|---------------|-------------|--------------|-------------------------------|
| per Sq. Inch. | in Mercury. | | | | | per Sq. Inch. | in Mercury. | | |
| Lbs. | Ins. | ° | ° | Cub. ft. | Lb. | Lbs. | Ins. | ° | ° |
| 1 | 2.04 | 102.1 | 1112.5 | 330.36 | .003 | 58 | 118.08 | 290.4 | 1170 |
| 2 | 4.07 | 126.3 | 1119.7 | 172.08 | .0058 | 59 | 120.12 | 291.6 | 1170.4 |
| 3 | 6.11 | 141.6 | 1124.6 | 117.52 | .0085 | 60 | 122.16 | 292.7 | 1170.7 |
| 4 | 8.14 | 153.1 | 1128.1 | 89.62 | .0112 | 61 | 124.19 | 293.8 | 1171.1 |
| 5 | 10.18 | 162.3 | 1130.9 | 72.66 | .0138 | 62 | 126.23 | 294.8 | 1171.4 |
| 6 | 12.22 | 170.2 | 1133.3 | 61.21 | .0163 | 63 | 128.26 | 295.9 | 1171.7 |
| 7 | 14.25 | 176.9 | 1135.3 | 52.94 | .0189 | 64 | 130.3 | 296.9 | 1172 |
| 8 | 16.29 | 182.9 | 1137.2 | 46.69 | .0214 | 65 | 132.34 | 298 | 1172.3 |
| 9 | 18.32 | 188.3 | 1138.8 | 41.79 | .0239 | 66 | 134.37 | 299 | 1172.6 |
| 10 | 20.36 | 193.3 | 1140.3 | 37.84 | .0264 | 67 | 136.4 | 300 | 1172.9 |
| 11 | 22.39 | 197.8 | 1141.7 | 34.63 | .0289 | 68 | 138.44 | 300.9 | 1173.2 |
| 12 | 24.43 | 202 | 1143 | 31.88 | .0314 | 69 | 140.48 | 301.9 | 1173.5 |
| 13 | 26.46 | 205.9 | 1144.2 | 29.57 | .0338 | 70 | 142.52 | 302.9 | 1173.8 |
| 14 | 28.51 | 209.6 | 1145.3 | 27.61 | .0362 | 71 | 144.55 | 303.9 | 1174.1 |
| 14.7 | 29.92 | 212 | 1146.1 | 26.36 | .03802 | 72 | 146.59 | 304.8 | 1174.3 |
| 15 | 30.54 | 213.1 | 1146.4 | 25.85 | .0387 | 73 | 148.62 | 305.7 | 1174.6 |
| 16 | 32.57 | 216.3 | 1147.4 | 24.32 | .0411 | 74 | 150.66 | 306.6 | 1174.9 |
| 17 | 34.61 | 219.6 | 1148.3 | 22.96 | .0435 | 75 | 152.69 | 307.5 | 1175.2 |
| 18 | 36.65 | 222.4 | 1149.2 | 21.78 | .0459 | 76 | 154.73 | 308.4 | 1175.4 |
| 19 | 38.68 | 225.3 | 1150.1 | 20.7 | .0483 | 77 | 156.77 | 309.3 | 1175.7 |
| 20 | 40.72 | 228 | 1150.9 | 19.72 | .0507 | 78 | 158.8 | 310.2 | 1176 |
| 21 | 42.75 | 230.6 | 1151.7 | 18.84 | .0531 | 79 | 160.84 | 311.1 | 1176.3 |
| 22 | 44.79 | 233.1 | 1152.5 | 18.03 | .0555 | 80 | 162.87 | 312 | 1176.5 |
| 23 | 46.83 | 235.5 | 1153.2 | 17.26 | .058 | 81 | 164.91 | 312.8 | 1176.8 |
| 24 | 48.86 | 237.8 | 1153.9 | 16.64 | .0601 | 82 | 166.95 | 313.6 | 1177.1 |
| 25 | 50.9 | 240.1 | 1154.6 | 15.99 | .0625 | 83 | 168.98 | 314.5 | 1177.4 |
| 26 | 52.93 | 242.3 | 1155.3 | 15.38 | .065 | 84 | 171.02 | 315.3 | 1177.6 |
| 27 | 54.97 | 244.4 | 1155.8 | 14.86 | .0673 | 85 | 173.05 | 316.1 | 1177.9 |
| 28 | 57.01 | 246.4 | 1156.4 | 14.37 | .0696 | 86 | 175.09 | 316.9 | 1178.1 |
| 29 | 59.04 | 248.4 | 1157.1 | 13.9 | .0719 | 87 | 177.13 | 317.8 | 1178.4 |
| 30 | 61.08 | 250.4 | 1157.8 | 13.46 | .0743 | 88 | 179.16 | 318.6 | 1178.6 |
| 31 | 63.11 | 252.2 | 1158.4 | 13.05 | .0766 | 89 | 181.2 | 319.4 | 1178.9 |
| 32 | 65.15 | 254.1 | 1158.9 | 12.67 | .0789 | 90 | 183.23 | 320.2 | 1179.1 |
| 33 | 67.19 | 255.9 | 1159.5 | 12.31 | .0812 | 91 | 185.27 | 321 | 1179.3 |
| 34 | 69.22 | 257.6 | 1160 | 11.97 | .0835 | 92 | 187.31 | 321.7 | 1179.5 |
| 35 | 71.26 | 259.3 | 1160.5 | 11.65 | .0858 | 93 | 189.34 | 322.5 | 1179.8 |
| 36 | 73.29 | 260.9 | 1161 | 11.34 | .0881 | 94 | 191.38 | 323.3 | 1180 |
| 37 | 75.33 | 262.6 | 1161.5 | 11.04 | .0905 | 95 | 193.41 | 324.1 | 1180.3 |
| 38 | 77.37 | 264.2 | 1162 | 10.76 | .0929 | 96 | 195.45 | 324.8 | 1180.5 |
| 39 | 79.4 | 265.8 | 1162.5 | 10.51 | .0952 | 97 | 197.49 | 325.6 | 1180.8 |
| 40 | 81.43 | 267.3 | 1162.9 | 10.27 | .0974 | 98 | 199.52 | 326.3 | 1181 |
| 41 | 83.47 | 268.7 | 1163.4 | 10.03 | .0996 | 99 | 201.56 | 327.1 | 1181.2 |
| 42 | 85.5 | 270.2 | 1163.8 | 9.81 | .102 | 100 | 203.59 | 327.9 | 1181.4 |
| 43 | 87.54 | 271.6 | 1164.2 | 9.59 | .1042 | 101 | 205.63 | 328.5 | 1181.6 |
| 44 | 89.58 | 273 | 1164.6 | 9.39 | .1065 | 102 | 207.66 | 329.1 | 1181.8 |
| 45 | 91.61 | 274.4 | 1165.1 | 9.18 | .1089 | 103 | 209.7 | 329.9 | 1182 |
| 46 | 93.65 | 275.8 | 1165.5 | 9 | .1111 | 104 | 211.74 | 330.6 | 1182.2 |
| 47 | 95.69 | 277.1 | 1165.9 | 8.82 | .1133 | 105 | 213.77 | 331.3 | 1182.4 |
| 48 | 97.72 | 278.4 | 1166.3 | 8.65 | .1156 | 106 | 215.81 | 331.9 | 1182.6 |
| 49 | 99.76 | 279.7 | 1166.7 | 8.48 | .1179 | 107 | 217.84 | 332.6 | 1182.8 |
| 50 | 101.8 | 281 | 1167.1 | 8.31 | .1202 | 108 | 219.88 | 333.3 | 1183 |
| 51 | 103.83 | 282.3 | 1167.5 | 8.17 | .1224 | 109 | 221.92 | 334 | 1183.3 |
| 52 | 105.87 | 283.5 | 1167.9 | 8.04 | .1246 | 110 | 223.95 | 334.6 | 1183.5 |
| 53 | 107.9 | 284.7 | 1168.3 | 7.88 | .1269 | 111 | 225.99 | 335.3 | 1183.7 |
| 54 | 109.94 | 285.9 | 1168.6 | 7.74 | .1291 | 112 | 228.02 | 336 | 1183.9 |
| 55 | 111.98 | 287.1 | 1169 | 7.61 | .1314 | 113 | 230.06 | 336.7 | 1184.1 |
| 56 | 114.01 | 288.2 | 1169.3 | 7.48 | .1336 | 114 | 232.1 | 337.4 | 1184.3 |
| 57 | 116.05 | 289.3 | 1169.7 | 7.36 | .1364 | 115 | 234.13 | 338 | 1184.5 |

EXAMPLE.—What volume of steam at 212° will raise 100 cube feet of water at 80° 20°?

$$\frac{20 \times 212 - 80}{212 - 80} = 13.68 \text{ cube feet water; or, } (13.68 \times 1642 - 212) = 22250 \text{ of steam.}$$

Compute Volume of Water, at any Given Temperature, that must be Mixed with Steam to Raise or Reduce the Mixture to any Required Temperature.

RULE.—From required temperature subtract temperature of water; then retain how often remainder is contained in required temperature subtracted from sum of sensible and latent heat of the steam, and quotient will be volume required.

Sum of Sensible and Latent Heats for a range of temperatures will be found under pages 508 and 509.

EXAMPLE.—Temperature of condensing water of an engine is 80°, and required temperature 100°; what is proportion of condensing water to that evaporated at a pressure of 34 lbs. per sq. inch?

Sum of sensible and latent heats 1190.4°.

$$100 - 80 = 20. \text{ Then, } 1190.4 \div 20 = 59.52 \text{ to 1.}$$

When Temperature of Steam is given. $\frac{l + T - t}{t - w} = V$. l representing latent heat, t temperature of steam and required temperature, w temperature of condensing water, and V volume of condensing water in cube feet.

ILLUSTRATION.—Temperature of steam in a cylinder is 257.6°, and other elements as in preceding example; required volume of injection water? Latent heat of steam at 230° = 932.8°.

$$\frac{932.8 + 257.6 - 100}{100 - 80} = \frac{1090.4}{20} = 54.52 \text{ volumes.}$$

Compute Temperature of Water in Condenser or Reservoir of a Steam-engine.

$\frac{l + T + V \times w}{V + 1} = t$. **ILLUSTRATION.**—Assume elements as preceding.

$$\frac{932.8 + 257.6 + 54.52 \times 80}{54.52 + 1} = \frac{5552}{55.52} = 100°.$$

To Compute Latent Heat of Saturated Steam.

15.2 — .708 $t = l$. **ILLUSTRATION.**—Assume temperature 257.6° as preceding.

$$1115.2 - .708 \times 257.6 = 932.8°.$$

To Compute Total Heat of Saturated Steam.

15 $t + 1081.4 = H$. **ILLUSTRATION.**—Assume temperature as preceding.

$$.305 \times 257.6 + 1081.4 = 1160.$$

Specific Force and Temperature of Vapors of Alcohol, Ether, Sulphuret of Carbon, Petroleum, and Turpentine.

Force in Ins. of Mercury.

| | Ins. | | Ins. | | Ins. | | Ins. | | Ins. |
|----------|------|----------|------|--------|-------|----------------------|------|------------|------|
| ALCOHOL. | | ALCOHOL. | | ETHER. | | SULPHURET OF CARBON. | | PETROLEUM. | |
| .4 | 140 | 13.9 | 34 | 6.2 | | | | 316 | 30 |
| .86 | 160 | 22.6 | 54 | 15.3 | 53.5 | 7.4 | 345 | 44.1 | |
| 1.23 | 173 | 30 | 74 | 16.2 | 72.5 | 12.55 | 375 | 64 | |
| 1.76 | 180 | 34.73 | 94 | 24.7 | 110 | 30 | | | |
| 2.45 | 200 | 53 | 96 | | 212 | 126 | | | |
| 3.4 | 212 | 67.5 | 104 | 30 | 279.5 | 300 | | | |
| 4.5 | 220 | 78.5 | 120 | 39.47 | 347 | 606 | | | |
| 8.1 | 240 | 111.24 | 150 | 67.6 | | | | | |
| 10.6 | 264 | 166.1 | 212 | 178 | | | | | |

OIL OF TURPENTINE

Saturated Steam.

Pressure, Temperature, Volume, and Density.

| Pressure | | Temperature | Total Heat from Water at 32° | Volume of 1 Lb. | Density, or Weight of one Cubic Foot. | Pressure | | Temperature | Total Heat from Water at 32° | Volume of 1 Lb. | Density, or Weight of one Cubic Foot. |
|---------------|--------------|-------------|------------------------------|-----------------|---------------------------------------|---------------|--------------|-------------|------------------------------|-----------------|---------------------------------------|
| per Sq. Inch. | In. Mercury. | | | | | per Sq. Inch. | In. Mercury. | | | | |
| Lbs. | Ins. | ° | ° | Cub. ft. | Lb. | Lbs. | Ins. | ° | ° | Cub. ft. | Lb. |
| 1 | 2.04 | 102.1 | 1112.5 | 330.36 | .003 | 58 | 118.08 | 290.4 | 1170.4 | 7.24 | .137 |
| 2 | 4.07 | 126.3 | 1119.7 | 172.08 | .0058 | 59 | 120.12 | 291.6 | 1170.4 | 7.12 | .138 |
| 3 | 6.11 | 141.6 | 1124.6 | 117.52 | .0085 | 60 | 122.16 | 292.7 | 1170.7 | 7.01 | .139 |
| 4 | 8.14 | 153.1 | 1128.1 | 89.62 | .0112 | 61 | 124.19 | 293.8 | 1171.1 | 6.9 | .140 |
| 5 | 10.18 | 162.3 | 1130.9 | 72.66 | .0138 | 62 | 126.23 | 294.8 | 1171.4 | 6.81 | .141 |
| 6 | 12.22 | 170.2 | 1133.3 | 61.21 | .0163 | 63 | 128.26 | 295.9 | 1171.7 | 6.7 | .142 |
| 7 | 14.25 | 176.9 | 1135.3 | 52.94 | .0189 | 64 | 130.3 | 296.9 | 1172 | 6.6 | .143 |
| 8 | 16.29 | 182.9 | 1137.2 | 46.69 | .0214 | 65 | 132.34 | 298 | 1172.3 | 6.49 | .144 |
| 9 | 18.32 | 188.3 | 1138.8 | 41.79 | .0239 | 66 | 134.37 | 299 | 1172.6 | 6.4 | .145 |
| 10 | 20.36 | 193.3 | 1140.3 | 37.84 | .0264 | 67 | 136.4 | 300 | 1172.9 | 6.32 | .146 |
| 11 | 22.39 | 197.8 | 1141.7 | 34.63 | .0289 | 68 | 138.44 | 300.9 | 1173.2 | 6.23 | .147 |
| 12 | 24.43 | 202 | 1143 | 31.88 | .0314 | 69 | 140.48 | 301.9 | 1173.5 | 6.15 | .148 |
| 13 | 26.46 | 205.9 | 1144.2 | 29.57 | .0338 | 70 | 142.52 | 302.9 | 1173.8 | 6.07 | .149 |
| 14 | 28.51 | 209.6 | 1145.3 | 27.61 | .0362 | 71 | 144.55 | 303.9 | 1174.1 | 5.99 | .150 |
| 14-7 | 29.92 | 212 | 1146.1 | 26.36 | .03802 | 72 | 146.59 | 304.8 | 1174.3 | 5.91 | .151 |
| 15 | 30.54 | 213.1 | 1146.4 | 25.85 | .0387 | 73 | 148.62 | 305.7 | 1174.6 | 5.83 | .152 |
| 16 | 32.57 | 216.3 | 1147.4 | 24.32 | .0411 | 74 | 150.66 | 306.6 | 1174.9 | 5.76 | .153 |
| 17 | 34.61 | 219.6 | 1148.3 | 22.96 | .0435 | 75 | 152.69 | 307.5 | 1175.2 | 5.68 | .154 |
| 18 | 36.65 | 222.4 | 1149.2 | 21.78 | .0459 | 76 | 154.73 | 308.4 | 1175.4 | 5.61 | .155 |
| 19 | 38.68 | 225.3 | 1150.1 | 20.7 | .0483 | 77 | 156.77 | 309.3 | 1175.7 | 5.54 | .156 |
| 20 | 40.72 | 228 | 1150.9 | 19.72 | .0507 | 78 | 158.8 | 310.2 | 1176 | 5.46 | .157 |
| 21 | 42.75 | 230.6 | 1151.7 | 18.84 | .0531 | 79 | 160.84 | 311.1 | 1176.3 | 5.4 | .158 |
| 22 | 44.79 | 233.1 | 1152.5 | 18.03 | .0555 | 80 | 162.87 | 312 | 1176.5 | 5.35 | .159 |
| 23 | 46.83 | 235.5 | 1153.2 | 17.26 | .058 | 81 | 164.91 | 312.8 | 1176.8 | 5.29 | .160 |
| 24 | 48.86 | 237.8 | 1153.9 | 16.64 | .0604 | 82 | 166.95 | 313.6 | 1177.1 | 5.23 | .161 |
| 25 | 50.9 | 240.1 | 1154.6 | 15.99 | .0625 | 83 | 168.98 | 314.5 | 1177.4 | 5.17 | .162 |
| 26 | 52.93 | 242.3 | 1155.3 | 15.38 | .065 | 84 | 171.02 | 315.3 | 1177.6 | 5.11 | .163 |
| 27 | 54.97 | 244.4 | 1155.8 | 14.86 | .0673 | 85 | 173.05 | 316.1 | 1177.9 | 5.05 | .164 |
| 28 | 57.01 | 246.4 | 1156.4 | 14.37 | .0696 | 86 | 175.09 | 316.9 | 1178.1 | 5 | .165 |
| 29 | 59.04 | 248 | 1157.1 | 13.9 | .0719 | 87 | 177.13 | 317.8 | 1178.4 | 4.94 | .166 |
| 30 | 61.08 | 250 | 1157.8 | 13.46 | .0743 | 88 | 179.16 | 318.6 | 1178.6 | 4.89 | .167 |
| 31 | 63.11 | 251 | 1158.4 | 13.05 | .0766 | 89 | 181.2 | 319.4 | 1178.9 | 4.84 | .168 |
| 32 | 65.15 | 254 | 1158.9 | 12.67 | .0789 | 90 | 183.23 | 320.2 | 1179.1 | 4.79 | .169 |
| 33 | 67.19 | 255.9 | 1159.5 | 12.31 | .0812 | 91 | 185.27 | 321 | 1179.3 | 4.74 | .170 |
| 34 | 69.22 | 257.6 | 1160 | 11.97 | .0835 | 92 | 187.31 | 321.7 | 1179.5 | 4.69 | .171 |
| 35 | 71.26 | 259.3 | 1160.5 | 11.65 | .0858 | 93 | 189.34 | 322.5 | 1179.8 | 4.64 | .172 |
| 36 | 73.29 | 260.9 | 1161 | 11.34 | .0881 | 94 | 191.38 | 323.3 | 1180 | 4.6 | .173 |
| 37 | 75.33 | 262.6 | 1161.5 | 11.04 | .0905 | 95 | 193.41 | 324.1 | 1180.3 | 4.55 | .174 |
| 38 | 77.37 | 264.2 | 1162 | 10.76 | .0929 | 96 | 195.45 | 324.8 | 1180.5 | 4.51 | .175 |
| 39 | 79.4 | 265.8 | 1162.5 | 10.51 | .0952 | 97 | 197.49 | 325.6 | 1180.8 | 4.46 | .176 |
| 40 | 81.43 | 267.3 | 1162.9 | 10.27 | .0974 | 98 | 199.52 | 326.3 | 1181.1 | 4.43 | .177 |
| 41 | 83.47 | 268.7 | 1163.4 | 10.03 | .0996 | 99 | 201.56 | 327.1 | 1181.3 | 4.37 | .178 |
| 42 | 85 | 270.2 | 1163.8 | 9.81 | .102 | 100 | 203.59 | 327.9 | 1181.4 | 4.33 | .179 |
| 43 | 87.54 | 271.6 | 1164.2 | 9.59 | .1042 | 101 | 205.63 | 328.5 | 1181.6 | 4.29 | .180 |
| 44 | 89.58 | 273 | 1164.6 | 9.39 | .1065 | 102 | 207.66 | 329.1 | 1181.8 | 4.25 | .181 |
| 45 | 91.61 | 274.4 | 1165.1 | 9.18 | .1089 | 103 | 209.7 | 329.9 | 1182 | 4.21 | .182 |
| | 93.65 | 275.8 | 1165.5 | 9 | .1111 | 104 | 211.74 | 330.6 | 1182.2 | 4.18 | .183 |
| | 95.69 | 277.1 | 1165.9 | 8.82 | .1133 | 105 | 213.77 | 331.3 | 1182.4 | 4.14 | .184 |
| | | 278.4 | 1166.3 | 8.65 | .1156 | 106 | 215.81 | 331.9 | 1182.6 | 4.11 | .185 |
| | | | 1166.7 | 8.48 | .1179 | 107 | 217.84 | 332.6 | 1182.8 | 4.07 | .186 |
| | | | | 8.31 | .1202 | 108 | 219.88 | 333.3 | 1183 | 4.04 | .187 |
| | | | | 8.17 | .1224 | 109 | 221.92 | 334 | 1183.3 | 4 | .188 |
| | | | | 8.04 | .1246 | 110 | 223.95 | 334.6 | 1183.5 | 3.97 | .189 |
| | | | | 7.88 | .1269 | 111 | 225.99 | 335.3 | 1183.7 | 3.95 | .190 |
| | | | | 7.74 | .1291 | 112 | 228.02 | 336 | 1183.9 | 3.93 | .191 |
| | | | | | .1314 | 113 | 230.05 | 336.7 | 1184.1 | 3.9 | .192 |
| | | | | | .1336 | 114 | 232.1 | 337.4 | 1184.3 | 3.88 | .193 |
| | | | | | .1364 | 115 | 234.13 | 338 | 1184.5 | 3.86 | .194 |

| MEASURE | | | | | PRESSURE | | | | |
|---------------------|--------------|-------------------------------------|--------------------|--|---------------------|---------------------|--------------|-------------------------------------|--------------------|
| in Mer- cury. | Temperature. | Total Heat from Water at 32°. | Volume of 1 Lb. | Density, or Weight of one Cube Foot. | per Sq. Inch. | In Mer- cury. | Temperature. | Total Heat from Water at 32°. | Volume of 1 Lb. |
| Ins. | ° | ° | Cub. ft. | Lb. | Lbs. | Ins. | ° | ° | Cub. ft. |
| 236.17 | 338.6 | 1184.7 | 3.77 | .2649 | 149 | 303.35 | 357.8 | 1190.5 | 2.98 |
| 238.2 | 339.3 | 1184.9 | 3.74 | .2652 | 150 | 305.39 | 358.3 | 1190.7 | 2.96 |
| 240.24 | 339.9 | 1185.1 | 3.71 | .2674 | 155 | 315.57 | 361 | 1191.5 | 2.87 |
| 242.28 | 340.5 | 1185.3 | 3.68 | .2696 | 160 | 325.75 | 363.4 | 1192.2 | 2.79 |
| 244.31 | 341.1 | 1185.4 | 3.65 | .2738 | 165 | 335.93 | 366 | 1192.9 | 2.71 |
| 246.35 | 341.8 | 1185.6 | 3.62 | .2759 | 170 | 346.11 | 368.2 | 1193.7 | 2.63 |
| 248.38 | 342.4 | 1185.8 | 3.59 | .278 | 175 | 356.29 | 370.8 | 1194.4 | 2.56 |
| 250.42 | 343 | 1186 | 3.56 | .2801 | 180 | 366.47 | 372.9 | 1195.1 | 2.49 |
| 252.45 | 343.6 | 1186.2 | 3.54 | .2822 | 185 | 376.65 | 375.3 | 1195.8 | 2.43 |
| 254.49 | 344.2 | 1186.4 | 3.51 | .2845 | 190 | 386.83 | 377.5 | 1196.5 | 2.37 |
| 256.53 | 344.8 | 1186.6 | 3.49 | .2867 | 195 | 397.01 | 379.7 | 1197.2 | 2.31 |
| 258.56 | 345.4 | 1186.8 | 3.46 | .2889 | 200 | 407.19 | 381.7 | 1197.8 | 2.26 |
| 260.6 | 346 | 1186.9 | 3.44 | .2911 | 210 | 427.54 | 386 | 1199.1 | 2.16 |
| 262.64 | 346.6 | 1187.1 | 3.41 | .2933 | 220 | 447.9 | 389.9 | 1200.3 | 2.06 |
| 264.67 | 347.2 | 1187.3 | 3.38 | .2955 | 230 | 468.26 | 393.8 | 1201.5 | 1.98 |
| 266.71 | 347.8 | 1187.5 | 3.35 | .2977 | 240 | 488.62 | 397.5 | 1202.6 | 1.9 |
| 268.74 | 348.3 | 1187.6 | 3.33 | .2999 | 250 | 508.98 | 401.1 | 1203.7 | 1.83 |
| 270.78 | 348.9 | 1187.8 | 3.31 | .302 | 260 | 529.34 | 404.5 | 1204.8 | 1.76 |
| 272.81 | 349.5 | 1188 | 3.29 | .304 | 270 | 549.7 | 407.9 | 1205.8 | 1.7 |
| 274.85 | 350.1 | 1188.2 | 3.27 | .306 | 280 | 570.06 | 411.2 | 1206.8 | 1.64 |
| 276.89 | 350.6 | 1188.3 | 3.25 | .308 | 290 | 590.42 | 414.4 | 1207.8 | 1.59 |
| 278.92 | 351.2 | 1188.5 | 3.22 | .3101 | 300 | 610.78 | 417.5 | 1208.7 | 1.54 |
| 280.96 | 351.8 | 1188.7 | 3.2 | .3121 | 350 | 712.57 | 430.1 | 1212.6 | 1.33 |
| 282.99 | 352.4 | 1188.9 | 3.18 | .3142 | 400 | 814.37 | 444.9 | 1217.1 | 1.18 |
| 285.03 | 352.9 | 1189 | 3.16 | .3162 | 450 | 916.17 | 456.7 | 1220.7 | 1.05 |
| 287.07 | 353.5 | 1189.2 | 3.14 | .3184 | 500 | 1018 | 467.5 | 1224 | .95 |
| 289.1 | 354 | 1189.4 | 3.12 | .3206 | 550 | 1119.8 | 477.5 | 1227 | .87 |
| 291.14 | 354.5 | 1189.6 | 3.1 | .3228 | 600 | 1221.6 | 487 | 1229.9 | .8 |
| 293.17 | 355 | 1189.7 | 3.08 | .325 | 650 | 1323.4 | 495.6 | 1232.5 | .74 |
| 295.21 | 355.6 | 1189.9 | 3.06 | .3273 | 700 | 1425.8 | 504.1 | 1235.1 | .69 |
| 297.25 | 356.1 | 1190 | 3.04 | .3294 | 800 | 1628.7 | 519.5 | 1239.8 | .61 |
| 299.28 | 356.7 | 1190.2 | 3.02 | .3315 | 900 | 1832.3 | 533.6 | 1244.2 | .55 |
| 301.32 | 357.2 | 1190.3 | 3 | .3336 | 1000 | 2035.9 | 546.5 | 1248.1 | .5 |

Saturated Steam from 32° to 212°. (Clausel.)

| PRESSURE. | | | | PRESSURE. | | | |
|---------------|------------------|--------------------------------|-----------------------|------------------------|---------------|------------------|--------------------------------|
| Mer- cury. | Per Sq. Inch. | Weight of 100 Cub. Feet. | Volume of 1 Lb. | Tem- pera- ture. | Mer- cury. | Per Sq. Inch. | Weight of 100 Cub. Feet. |
| Ins. | Lbs. | Lb. | Cub. Feet. | ° | Ins. | Lbs. | Cub. Feet. |
| .181 | .089 | .031 | 3226 | 125 | 3.933 | 1.932 | .554 |
| .204 | .1 | .034 | 2941 | 130 | 4.509 | 2.215 | .63 |
| .248 | .122 | .041 | 2439 | 135 | 5.174 | 2.542 | .714 |
| .299 | .147 | .049 | 2041 | 140 | 5.86 | 2.879 | .806 |
| .362 | .178 | .059 | 1695 | 145 | 6.662 | 3.273 | .909 |
| .426 | .214 | .07 | 1429 | 150 | 7.548 | 3.708 | 1.022 |
| .517 | .254 | .082 | 1220 | 155 | 8.535 | 4.193 | 1.145 |
| .619 | .304 | .097 | 1031 | 160 | 9.63 | 4.731 | 1.333 |
| .733 | .36 | .114 | 877.2 | 165 | 10.843 | 5.327 | 1.432 |
| .869 | .427 | .134 | 746.3 | 170 | 12.183 | 5.985 | 1.602 |
| 1.024 | .503 | .156 | 641 | 175 | 13.654 | 6.708 | 1.774 |
| 1.205 | .592 | .182 | 549.5 | 180 | 15.291 | 7.511 | 1.977 |
| 1.41 | .693 | .212 | 471.7 | 185 | 17.041 | 8.375 | 2.209 |
| 1.647 | .809 | .245 | 408.2 | 190 | 19.001 | 9.335 | 2.475 |
| 1.917 | .942 | .283 | 353.4 | 195 | 21.139 | 10.385 | 2.775 |
| 2.229 | 1.095 | .325 | 307.7 | 200 | 23.461 | 11.526 | 3.115 |
| 2.579 | 1.267 | .373 | 268.1 | 205 | 25.994 | 12.77 | 3.505 |
| 2.976 | 1.462 | .426 | 234.7 | 210 | 28.753 | 14.127 | 3.945 |
| 3.43 | 1.685 | .488 | 204.9 | 212 | 29.922 | 14.7 | 4.245 |

re) as it enters the cylinder, divide product by relative expansion, and quotient will give mean pressure.

NOTE.—Hyp. log. of any number not in table may be found by multiplying a common log. by 2.302585, usually by 2.3.

Proceed by referring to table pp. 331-334.

EXAMPLE.—Assume steam to enter a cylinder at a pressure of 50 lbs. per sq. inch, to be cut off at .25 length of stroke, stroke of piston being 10 feet; what will be the mean pressure?

Expansion assumed at 2 per cent. = .2 feet.

$+.2 = 10.2$ feet, stroke $10 \div 4 + .2 = 2.38$ feet. Then $10.2 \div 2.38 = 4.29$ relative expansion.

P. log. 4.29 (p. 332) = 1.4563, which $+ 1 = 2.4563$, and $\frac{2.4563 \times 50}{4.29} = 28.62$ lbs.

Relative Effect of steam during expansion is obtained from preceding rule.

Mechanical Effect of steam in a cylinder is product of mean pressure in lbs. and distance through which it has passed in feet.

Effects of Expansion. (Essentially from D. K. Clark.)

Back Pressure is force of the uncondensed steam in a cylinder, consequent on the impracticability of obtaining a perfect vacuum, and is opposed to the action of a piston. It varies from 2 to 5 lbs. per sq. inch.

It must be deducted from average pressure. Thus: assume pressure 60 lbs., stroke of piston as in preceding case, and back pressure 2 lbs.

At termination of.... 1st, 2d, 3d, 4th, 5th, and 6th foot of stroke.

Pressure..... 60 30 20 15 12 10 lbs. per inch.

Back pressure..... 2 2 2 2 2 2 " " "

Effective pressure.... 58 28 18 13 10 8 " " "

Total work done by expansion at termination of each foot or assumed distance of stroke of piston is represented by hyp. log. of ratio of expansion, and total work = 1.

Thus, for a stroke of 10 feet and a pressure of 10 lbs.:

End of..... 1st, 2d, 3d, 4th, 5th, 6th, 7th, 8th, 9th, and 10th foot.

Volume expanded }
in vols., hyp. } = .69 1.1 1.39 1.61 1.79 1.95 2.08 2.2 2.3
of which.....

1 duty..... 1 1 1 1 1 1 1 1 1 1

1 duty..... 1 1.69 2.1 2.39 2.61 2.79 2.95 3.08 3.2 3.3

1 duty is represented by 10..... 10 16.9 21 23.9 26.1 27.9 29.5 30.8 32 33

Distance for each }
of stroke..... } = 2 4 6 8 10 12 14 16 18 20

1 effective }
by..... } = 8 12.9 15 15.9 16.1 15.9 15.5 14.8 14 13

by expansion 0 61.25 87.5 98.75 101.25 98.75 93.75 85 75 62.5

Same results would be produced if expansion was applied to a non-condensing engine, exhausting into the atmosphere.

Now, assume total initial pressure in a non-condensing cylinder to be expanded 5 times, or down to 15 lbs., and then exhausted again into the atmosphere and friction of 15 lbs.

At termination of..... 1st, 2d, 3d, 4th, and 5th foot.

Duty..... 1 1.69 2.1 2.39 2.61

" performed... 75 126.75 157.5 179.25 195

Back pressure.... 15 30 45 60 75

Effective duty.... 60 96.75 112.5 119.25 120

by expansion..... 0 61.25 87.5 98.75 101.25

in which it appears that the total duty performed by expanding

the initial volume is full 2.5 times, or as 75 to 195.75.

Relative Effect of Equal Volumes of Steam.

Relative total effect or work of steam is directly as its mean or average pressure (A), and inversely as its final pressure (B), or volume of steam condensed.

If former is divided by latter, quotient will give relative total effect or work of a given volume of steam as admitted and cut off at different points of stroke of piston, with a clearance of 3.125 per cent.

In following computations resistance of back pressure is omitted. If this pressure is uniform with all the ratios of expansion, it is a uniform pressure, to be deducted from the total mean pressure in column (A).

| Cut off at | Pressure. | | (C) Relative Effect. | Cut off at | Pressure. | | (C) Relative Effect. |
|------------|--------------|---------------|----------------------------|------------|--------------|---------------|----------------------------|
| | (A) Mean. | (B) Final. | | | (A) Mean. | (B) Final. | |
| 1 | 1 | 1 | 1 | .375 | .761 | .394 | 1.85 |
| .75 | .969 | .787 | 1.28 | .33 | .702 | .335 | 2.00 |
| .6875 | .946 | .697 | 1.35 | .25 | .628 | .273 | 2.31 |
| .625 | .924 | .636 | 1.45 | .2 | .559 | .224 | 2.48 |
| .5625 | .889 | .576 | 1.54 | .125 | .435 | .15 | 2.90 |
| .5 | .857 | .501 | 1.71 | .1 | .418 | .13 | 3.11 |

To Compute Total Effective Work in One Stroke of Piston, or as Given by an Indicator Diagram.

a P ($P \div \text{hyp. log. } R - c$) = w , and $a b L = w'$. w representing total work on w' back pressure.

NOTE.—Pressure of atmosphere is to be included in computations of expansion; it is therefore to be deducted from result obtained in non-condensing engines. In condensing engines, the deduction due to imperfect vacuum must also be made usually 2.5 lbs. per sq. inch.

ILLUSTRATION.—Assume cylinder of a condensing engine 26.1 ins. in diameter, stroke of 2 feet, pressure of steam 95 lbs. (80.3 + 14.7) per sq. inch, cut off at .5 stroke, with an average back pressure of 2 lbs. per sq. inch, and a clearance of 5 per cent.

Area of piston, deducting half area of rod = 530 sq. ins. $2 \times 5 \div 100 = .1$ clearance, and $2 + .1 \div 1 + .1 = 1.9$ = ratio of expansion, and $1 + \text{hyp. log. } 1.9 = 1.642$.

Then $530 \times 95 \times 1.1 \times 1.642 - .1 = 530 \times 2 \times 2 = 50350 \times 1.706 - 2120 = 81770$ foot-lbs.

ILLUSTRATION.—Assume cylinder of a non-condensing engine having an area of 2000 sq. ins., a stroke of 8 feet, steam at a pressure of 50 lbs. (35.3 + 14.7), cut off at .25 of stroke, and clearance .25 foot.

Ratio of expansion 3.66, back pressure 17 lbs., and $1 + \text{hyp. log. } 3.66 = 2.29$.

$2000 \times 50 (2.25 \times 1 + \text{hyp. log. } 3.66 - .25) = 100000 \times 2.25 \times 1 + 1.297 = 460575$ foot-lbs.

$2000 \times 17 \times 8 = 272000$ foot-lbs. or negative effect, and $460575 - 272000 = 188575$ foot-lbs.

Total Effect of One Lb. of Expanded Steam.

If 1 lb. of water is converted into steam of atmospheric pressure = 14.7 lbs. per sq. inch, or 2116.8 lbs. per sq. foot, it occupies a volume equal to 26.36 cu. ft. and the effect of this volume under one atmosphere = 2116.8 lbs. $\times 26.36$ foot-lbs. = 55799 foot-lbs. Equivalent quantity of heat expended is 1 unit per 772 foot-lbs. = 55799 $\div 772 = 72.3$ units. This is effect of 1 lb. of steam of a pressure of one atmosphere on a piston without expansion.

Gross effect thus attained on a piston by 1 lb. of steam, generated at pressures varying from 15 to 100 lbs. per sq. inch, varies from 56000 to 62000 foot-lbs., equivalent to from 72 to 80 units of heat.

Effect of 1 lb. of steam, without expansion, as thus exemplified, is reduced in clearance according to proportion it bears to volume of cylinder. If clearance per cent. of stroke, then 105 parts of steam are consumed in the work of a stroke which is represented by 100 parts, and effect of a given weight of steam with expansion, admitted for full stroke, is reduced in ratio of 105 to 100. Having ascertained, by this ratio, effect of work by 1 lb. of steam without expansion, we may, by clearance, effect for various ratios of expansion may be deduced from the terms of relative operation of equal weights of steam.

ime of 1 lb. of saturated steam of 100 lbs. per sq. inch is 4.33 cube feet, and re per sq. foot is $144 \times 100 = 14,400$ lbs.; then total initial work = $14,400 \times 4.33$ 52 foot-lbs. This amount is to be reduced for clearance assumed at 7 per cent.

1 $62,352 \times 100 \div 107 = 58,273$ foot-lbs., which, divided by 772 (Joule's equivalent) = 75.5 units of heat.

1 or constituent heat of steam of 100 lbs. pressure per sq. inch, computed from perature of 212° , is 1001.4 units; and from 102° (temperature of condenser a pressure of 1 lb.) the constituent heat is 1111.4 units.

ivalent, then, of net simple effect 75.5 units is 7.5 per cent. of total heat from 1r 6.7 per cent. from 102° .

en steam is cut off at

| | | | | | |
|----------------------|-------|------|------|------|------------------------|
| .75 | .5 | .33 | .25 | .2 | .125 and .1 of stroke, |
| ative effects are as | | | | | |
| 1.26 | 1.616 | 1.92 | 2.14 | 2.27 | 2.51 and 2.6. |

1 effects as given in table, page 718.

ct of 1 lb. of steam, without deduction for back pressure or other effects, varies about 60,000 foot-lbs., without expansion, to about double that, or 120,000 foot-lb. when expanded 3 times, cutting off at about 27 per cent. of stroke; and to 150,000 foot-lbs. when expanded about 6 times, and cut off at about 10 per cent. of stroke.

Effect of Clearance.

arance varies with length of stroke compared with diameter of cylinder, form of valve, as poppet, slide, etc.

th a diameter of cylinder of 48 ins., and a stroke of 10 feet, and poppet s, clearance is but 3 per cent., and with a diameter of 34 ins. and a s of 4.5 feet and slide valves, it is 7 per cent.

ESTRATION OF EFFECT. — Assume steam admitted to a cylinder for .25 of its s, with a clearance of 7 per cent.

n pressure for 1 lb. = .637, and loss by clearance = $7 \div 100 = .07$, which, added 7, = .707, which is effect of a given volume of steam, if there was not any loss arance, or a gain of 11 per cent.

en steam is cut off at 1 .75 .5 .33 .25 .125 and .1 stroke.
s at 7 per cent. clearance. = 7 7.2 8.1 9.6 11 15.3 17 per cent.

Compute Net Volume of Cylinder for Given Weight of Steam, Ratio of Expansion and One Stroke.

LE. — Multiply volume of 1 lb. of steam, by given weight in lbs., by of expansion and by 100, and divide product by 100, added to per cent. arance.

MPLE. — Pressure of steam 95 lbs., cut off at .5, weight .54 lbs., volume of 1 lb. 4.55, and weight = .2198 lbs., stroke of piston 2 feet, and clearance 7 per cent. lo of expansion $2 + .14 \div 1 + .14 = 1.88$.

$$\frac{4.55 \times .54 \times 1.88 \times 100}{100 + 7} = \frac{461.92}{107} = 4.31 \text{ cube feet.}$$

Compute Volume of Cylinder for Given Effect with Given Initial Pressure and Ratio of Expansion.

LE. — Divide given effect or work by total effect of 1 lb. of steam of ressure and ratio of expansion, and quotient will give weight of steam, which compute volume of cylinder by preceding rule.

MPLE. — Assume given work at 50,766 foot-lbs., and pressure and expansion as ding.

al work by 1 lb., 100 lbs. steam, cut off at .5, = by table 94,200 foot-lbs., and by of multipliers for 95 lbs. = .998, which $\times 94,200 = 94,012$ foot-lbs.

$$\text{Then } \frac{50,766}{94,012} = .54 \text{ lbs. weight of steam.}$$

Consumption of Expanded Steam per HP of Effect per Hour.

HP = 33000, which $\times 60 = 1\,980\,000$ foot-lbs. per hour, which steam, the quotient = weight of steam or water required per HP per hour.

ILLUSTRATION.—Effect of 1 lb., 100 lbs. steam, without expansion, with 71 of clearance = 58 273 foot-lbs., and $\frac{1\,980\,000}{58\,273} = 34$ lbs. steam = weight of steam assumed for the effect per HP per hour.

When steam is expanded, the weight of it per HP is less, as effect of 1 lb. is greater, and it may be ascertained by dividing 1 980 000 by the respective or by dividing 34 lbs. by quotient of total mean pressure by final pressure, in table, page 718.

When steam is cut off at } 1 .75 .5 .375 .33 .25 and .20
Volumes consumed per }
HP per hour. } = 34 26.9 21 18.5 17.6 16 14.9

Hence, assuming 10 lbs. steam are generated by combustion of 1 lb. coal of total effect per hour,

The coal consumed per }
HP per hour. } = 3.4 2.69 2.1 1.85 1.76 1.6 1.49

SATURATED STEAM.

To Compute Energy and Efficiency of Saturated Steam.

$$\frac{v}{s} = R; \quad \frac{S}{s} = \frac{1}{r}; \quad p - p' \times a, \text{ or } \frac{X}{R} \text{ or } \frac{X D}{R} = P; \quad \frac{33\,000}{P} = C; \quad \frac{1}{s} \\ \frac{H D}{R} = H'; \quad \frac{D}{R} \text{ or } \frac{1}{R S} = F; \quad J D (t - t') + L = H D; \quad \frac{H D}{D} = H; \quad \frac{1}{s}$$

$$p - p' \times a R S = X; \quad \frac{H D}{1} - X D = H''; \quad \frac{H''}{R} = H'''; \quad 15.5 \text{ I D S}$$

$$h - X = h'; \quad \frac{h}{R S} = P''; \quad \frac{a p - a p'}{P''} \text{ or } \frac{X}{h} = E; \quad \frac{1\,980\,000}{E} \text{ or } 1\,980\,000$$

$$\frac{X}{2 s} = e; \quad n l a p - p' = x, \text{ and } \frac{x}{33\,000} = \text{IHP}; \quad R p - p' a = x; \quad F C X$$

$$\frac{33\,000}{p a - p' a} = \text{cube feet.} \quad \frac{1\,980\,000}{62.5 X} = \text{cube feet water evaporated per hour}$$

V and v representing volumes of mass of steam entering cylinder and termination of stroke of piston; S and s volumes of 1 lb. steam when admitted when at termination of expansion; C volume of cylinder per minute for expansion and effective cut-off; F feed water per cube foot of cylinder per stroke of piston, and f per IHP per hour, all in cube feet; sity or weight of 1 cube foot of steam at temperature of operation, in lbs.; p pressure; p' mean back pressure; I initial pressure; P mean effective pressure per cube foot of volume of cylinder; P' pressure per sq. inch or that equivalent to heat expended, and P'' pressure equivalent to expenditure of available energy, all in lbs. J Joule's equivalent = 772 foot-lbs.; L as per following table; t' absolute temperatures of steam at initial pressure and of feed water in $^{\circ}\text{F}$; H D heat expended per cube foot of steam admitted; H' heat expended per cube foot of volume of cylinder, or pressure equivalent to heat expended per sq. foot; H rejected per cube foot of steam admitted; H'' heat rejected per cube foot of cylinder; A available heat per IHP per hour; e energy per cube foot of steam cylinder to point of cutting off, or of steam admitted; h and h' heat expended, rejected, and X energy exerted, all per lb. of steam and in foot-lbs. E efficiency; ergy exerted per minute and per cube-foot of steam admitted; a area of piston in sq. ins.; l length of stroke of piston in feet, and f feed water per IHP per hour cube feet.

ILLUSTRATIONS.—Assume volume of cylinder and clearance (5 per cent = 1 cube foot, steam (86.3 + 14.7) 100 lbs. per sq. inch, cut off at .5, mean pressure (page 711) 86 lbs., and back pressure 3 lbs.

$$V = 1. \quad S = 4.33. \quad s = 8.31. \quad p = 86. \quad p' = 3. \quad a = 1 \\ t \text{ and } t' = 327.9^{\circ} + 461.2^{\circ} \text{ and } 100^{\circ} + 461.2^{\circ}. \quad l = 2 \text{ feet.} \quad n = 1. \quad 1$$

$$\begin{aligned}
 &= 2 \text{ ratio. } 4.33 \div 8.31 = .521 \text{ effective cut-off. } 86 - 3 \times 144 = 11952 \text{ lbs.} \\
 &\frac{1000}{3 \times 144} = 2.76 \text{ cube feet. } \frac{1}{4.33} = .231 \text{ lbs. } \frac{.231}{2} \text{ or } \frac{1}{2 \times 4.33} = .1154 \text{ cube feet.} \\
 &772 \times .231 (789.1^\circ - 561.2^\circ) + 157748 = 198389 \text{ foot-lbs.} \\
 &\frac{98389}{2} = 99195 \text{ foot-lbs. } \frac{198389}{.231} = 85827 \text{ foot-lbs. } \frac{99195}{144} = 689 \text{ lbs.} \\
 &86 - 3 \times 144 \times 2 \times 4.33 = 103504 \text{ foot-lbs.} \\
 &89 \div .231 - 103504 \times .231 = 174479 \text{ foot-lbs. } 174479 \div 2 = 87239 \text{ foot-lbs.} \\
 &5 \times 100 \times 144 \times 4.33 = 966456 \text{ foot-lbs. } 966456 - 103504 = 862952 \text{ foot-lbs.} \\
 &\frac{.56}{-.33} = 111600 \text{ lbs. } \frac{144 \times 86 - 144 \times 3}{111600} = .107 \text{ E. } \frac{1980000}{.107} = 18504673 \text{ foot-lbs.} \\
 &\text{Or } 1980000 \times \frac{966456}{103504} = 18504673 \text{ foot-lbs. } \frac{23904}{33000} = .725 \text{ HP.} \\
 &1 \times 2 \times 144 \times \frac{86-3}{86-3} = 23904 \text{ foot-lbs. } \frac{1980000}{62.5 \times 103504} = .306 \text{ cube feet.} \\
 &2 \times 86 - 3 \times 144 = 23904 \text{ foot-lbs. } .1154 \times 2.76 \times 60 = 19.11 \text{ cube feet.} \\
 &\frac{103504}{2 \times 4.33} = 11952 \text{ foot-lbs. } \frac{33000}{86 \times 144 - 3 \times 144} = 2.761 \text{ cube feet.}
 \end{aligned}$$

illustration of connection of expenditure of available heat (A) and consumption of coal, assume coal to have a total heat of combustion of 10000000* foot-lbs., corresponding to an equivalent evaporative power under 1 atmosphere at 212° of 13.4 water and efficiency of furnace .5; then available heat of combustion of 1 lb. = 5000000 foot-lbs.

hence, consumption of coal per IHP in an engine of like dimensions and operation with that here given would be $1923000 \div 5000000 = 3.8444 \text{ lbs.}$

Properties of Steam of Maximum Density. (Rankine.) Per Cube Foot.

| T. | Temp. | L. | Temp. | L. | Temp. | L. | Temp. | L. | Temp. | L. |
|-----|-------|------|-------|-------|-------|-------|-------|--------|-------|--------|
| 248 | 95 | 1999 | 158 | 9687 | 221 | 33180 | 284 | 88740 | 347 | 197700 |
| 348 | 104 | 2571 | 167 | 11760 | 230 | 38700 | 293 | 100500 | 356 | 219000 |
| 448 | 113 | 3277 | 176 | 14200 | 239 | 44930 | 302 | 113400 | 365 | 242000 |
| 548 | 122 | 4136 | 185 | 17010 | 248 | 51920 | 311 | 127500 | 374 | 266600 |
| 648 | 131 | 5178 | 194 | 20280 | 257 | 59720 | 320 | 143000 | 383 | 293100 |
| 748 | 140 | 6430 | 203 | 24020 | 266 | 68420 | 329 | 159800 | 392 | 321400 |
| 848 | 149 | 7921 | 212 | 28310 | 275 | 78050 | 338 | 178000 | 401 | 351600 |

* Representing latent heat of evaporation per cube foot of vapor in foot-lbs. of engine. To reduce this to units of heat divide by 772, or Joule's equivalent.

SUPERHEATED STEAM.

The results attained by imparting to steam a temperature moderately in excess of that due to the volume or density of saturated steam are:

An increase of elasticity without a corresponding increase of water evaporated.

Arresting or reducing passage of water, in suspension, to cylinder (foaming), as heat contained in that water is wholly lost without affording any elastic effect.

From these results, by increasing effect of the steam, economize fuel.

Superheated steam should be treated as a gas.

As product of its pressure, p in lbs. per sq. foot, and volume v of 1 lb. of it in cubic feet in the perfectly gaseous condition, is obtained by following formula:

$40 T \div t = p v = 85.44 T$. T temperature of steam + 461.2°, and t 32° + 461.2°.

CONSTANT.—Assume temperature of steam, 327.9°, superheated to 341.1°.

Then $42140 \times 461.2^\circ + 341.1^\circ \div 32 + 461.2^\circ = 68549 \text{ foot-lbs.}$

hence, as pressure of steam at 327.9° = 100 lbs. per sq. inch, and at 341.1° 120

$120 \div 100 = 1.2$ to 1 = a gain of one fifth.

* Coal of average composition, $14133 \times 772 = 10910676$.

Relative Effect of Equal Volumes of Steam.

Relative total effect or work of steam is directly as its mean or average pressure (A), and inversely as its final pressure (B), or volume of steam condensed.

If former is divided by latter, quotient will give relative total effect or work of a given volume of steam as admitted and cut off at different points of stroke of piston, with a clearance of .125 per cent.

In following computations resistance of back pressure is omitted. If this pressure is uniform with all the ratios of expansion, it is a uniform pressure which deducted from the total mean pressure in column (A).

| Pressure. | | | | Pressure. | | | |
|------------|-----------|------------|----------------------|------------|-----------|------------|----------------------|
| Cut off at | (A) Mean. | (B) Final. | (C) Relative Effect. | Cut off at | (A) Mean. | (B) Final. | (C) Relative Effect. |
| 1 | 1 | 1 | 1 | .375 | .761 | .394 | 1.9 |
| .75 | .969 | .737 | 1.28 | .33 | .702 | .335 | 2.1 |
| .675 | .945 | .697 | 1.35 | .25 | .628 | .273 | 2.3 |
| .625 | .924 | .635 | 1.45 | .2 | .559 | .224 | 2.5 |
| .5625 | .899 | .576 | 1.54 | .125 | .435 | .15 | 2.9 |
| .5 | .857 | .501 | 1.71 | .1 | .418 | .13 | 3.1 |

To Compute Total Effective Work in One Stroke of Piston, or as Given by an Indicator Diagram.

$a P (1 + \text{hyp. log. } R - c) = w$, and $a b L = w'$. w representing total work, w' back pressure.

NOTE.—Pressure of atmosphere is to be included in computations of expansion; it is therefore to be deducted from result obtained in non-condensing engine. In condensing engines, the deduction due to imperfect vacuum must also be added, usually 2.5 lbs. per sq. inch.

ILLUSTRATION.—Assume cylinder of a condensing engine 26.1 ins. in diameter, stroke of 2 feet, pressure of steam 95 lbs. (80.3 + 14.7) per sq. inch, cut off at .5 stroke with an average back pressure of 2 lbs. per sq. inch, and a clearance of 5 per cent.

Area of piston, deducting half area of rod = 530 sq. ins. $2 \times 5 \div 100 = .1$ clearance, and $2 + .1 \div 1 + .1 = 1.9 = \text{ratio of expansion}$, and $1 + \text{hyp. log. } 1.9 = 1.46$.

Then $530 \times 95 \times 1.1 \times 1.642 = 1.1 \times 530 \times 2 \times 2 = 50350 \times 1.706 = 2120 = 83777$ 8000 sq. ins., a stroke of 8 feet, steam at a pressure of 50 lbs. (35.3 + 14.7), cut off at .25 of stroke, and clearance .25 foot.

Ratio of expansion 3.66, back pressure 17 lbs., and $1 + \text{hyp. log. } 3.66 = 2.29$.

$2000 \times 50 (2.25 \times 1 + \text{hyp. log. } 3.66 - .25) = 100000 \times 2.25 \times 1 + 1.297 = 460575$ foot-lbs.

$2000 \times 17 \times 8 = 272000$ foot-lbs. or negative effect, and $460575 - 272000 = 188575$ foot-lbs.

Total Effect of One Lb. of Expanded Steam.

If 1 lb. of water is converted into steam of atmospheric pressure = 14.7 lbs. per sq. inch, or 2116.8 lbs. per sq. foot, it occupies a volume equal to 26.36 cubic feet, and the effect of this volume under one atmosphere = 2116.8 lbs. \times 26.36 feet = 55700 foot-lbs. Equivalent quantity of heat expended is 1 unit per 772 foot-lbs. = 77200 foot-lbs. = 72.3 units. This is effect of 1 lb. of steam of a pressure of one atmosphere on a piston without expansion.

Heat effect thus ascertained by 1 lb. of steam, generated at pressure varies from 56000 to 62000 foot-lbs., equal

to volume of cylinder. If clearance is consumed in the work of steam of a given weight of steam without expansion, in ratio of 105 to 100. Having no expansion without expansion, no effect of steam may be deducted from the effect of steam.

EXAMPLE.—If steam is generated at 100 lbs. per sq. inch, and the pressure is reduced to 10 lbs. per sq. inch, the effect of the steam is reduced to 10 per cent of the effect of the steam at 100 lbs. per sq. inch.

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Volume pressure = 62 352

Then (100) = 100

Total effect of steam

under a

Equiv. 212, or

When compared

Total effect

Effect of steam

from above

lbs. which

about 15

cent. of

Clearance with

With 2 valves, clearance

stroke of

ILLUSTRATION.—

stroke, with

Mean pressure

to 57, or

by clearance

When steam

loss at 7

To compare

of

RULE.—

ratio of effect

of clearance

EXAMPLE.—

steam 4.55

Ratio of

To compare

a given

RULE.—

like pressure

from which

EXAMPLE.—

pressure 12.

Effect of

steam 10.

of 1 lb. of saturated steam of 100 lbs. per sq. inch is 4.33 cube feet, and 1 sq. foot is $144 \times 100 = 14,400$ lbs.; then total initial work = $14,400 \times 4.33$ ft.-lbs. This amount is to be reduced for clearance assumed at 7 per cent.

$152 \times \frac{100}{107} = 58,273$ foot-lbs., which, divided by 772 (Joule's equivalent units of heat).

Constituent heat of steam of 100 lbs. pressure per sq. inch, computed from ure of 212° , is 1001.4 units; and from 102° (temperature of condenser ssure of 1 lb.) the constituent heat is 1111.4 units.

nt, then, of net simple effect 75.5 units is 7.5 per cent. of total heat from per cent. from 102° .

am is cut off at

.75 .5 .33 .25 .2 .125 and .1 of stroke,

e effects are as

.26 1.616 1.92 2.14 2.27 2.51 and 2.6.

cts as given in table, page 718.

1 lb. of steam, without deduction for back pressure or other effects, varies 60,000 foot-lbs., without expansion, to about double that, or 120,000 foot-expanded 3 times, cutting off at about 27 per cent. of stroke; and to 60,000 foot-lbs. when expanded about 6 times, and cut off at about 10 per ke.

Effect of Clearance.

pe varies with length of stroke compared with diameter of cylinder, of valve, as poppet, slide, etc.

diameter of cylinder of 48 ins., and a stroke of 10 feet, and poppet arance is but 3 per cent., and with a diameter of 34 ins. and a .5 feet and slide valves, it is 7 per cent.

TION OF EFFECT. — Assume steam admitted to a cylinder for .25 of its 1 a clearance of 7 per cent.

ssure for 1 lb. = .637, and loss by clearance = $7 \div 100 = .07$, which, added 707, which is effect of a given volume of steam, if there was not any loss e, or a gain of 11 per cent.

am is cut off at. 1 .75 .5 .33 .25 .125 and .1 stroke.

per cent. clearance. = 7 7.2 8.1 9.6 11 15.3 17 per cent.

pute Net Volume of Cylinder for Given Weight steam, Ratio of Expansion and One Stroke.

-Multiply volume of 1 lb. of steam, by given weight in lbs., by pansion and by 100, and divide product by 100, added to per cent. e.

—Pressure of steam 95 lbs., cut off at .5, weight .54 lbs., volume of 1 lb. and weight = .2198 lbs., stroke of piston 2 feet, and clearance 7 per cent.

xpansion $2 + .14 \div 1 + .14 = 1.88$.

$$\frac{4.55 \times .54 \times 1.88 \times 100}{100 + 7} = \frac{461.92}{107} = 4.31 \text{ cube feet}$$

pute Volume of Cylinder for Given Effect with ven Initial Pressure and Ratio of Expansion.

- Divide given effect or work by total effect of 1 lb. of steam of ire and ratio of expansion, and quotient will give weight of steam, h compute volume of cylinder by preceding rule.

—Assume given work at 50,766 foot-lbs., and pressure and expansion as

k by 1 lb., 100 lbs. steam, cut off at .5, = by table 94,300 foot-lbs., and by ipliers for 95 lbs. = .998, which $\times 94,300 = 94,012$ foot-lbs.

$$\text{Then } \frac{50,766}{94,012} = .54 \text{ lbs. weight of steam.}$$

ILLUSTRATION.—Assume areas of cylinders 1 and 3 sq. ins., length of stroke of pressure of steam 60 lbs. per sq. inch, cut off at 2 feet, clearance 7 per cent. area of intermediate space, as receiver, one third volume of 1st cylinder.

$$R'' = \text{ratio of expansion in 2d cylinder} \frac{4 - x}{4} \times 3 \times 2.653 = 5.49$$

$$\frac{2.653 \times 2.25 + 1 \times 2.42 - 3 - 1 + 2.653 + 1 \times .42 \times 1 \times 60 = 1.7865 + 1 \times 60}{2 + 2.653 + 1 \times .42 \times 60 = 6.743 - .737 \times 60 = 360.36 \text{ foot-lbs.}}$$

1st Cylinder.

| | |
|--|----------|
| Effect on piston 60 lbs. \times 1 inch \times 2 feet..... | = 120 |
| " of clearance 60 lbs. \times .42 foot..... | = 25.2 |
| Total initial effect = 60 \times 2 \times .42..... | = 145.2 |
| Then 145.2 \times 1 + hyp. log. 2.653 or 1.976..... | = 285.2 |
| Less effect of clearance..... | = 25.2 |
| Net effect on piston above vacuum line..... | = 260.0 |
| Less effect of back pressure 60 + 2.653 = 22.62, which, \times 3 sq. ins. and 2 feet stroke..... | = 133.06 |
| Net effect on piston..... | = 126.94 |

2d Cylinder.

$$145.2 \times 1 + \text{hyp. log. } 2.25 \text{ or } 1.81 \dots \dots \dots = 262.81 \text{ foot-lbs.}$$

$$\text{Effect of clearance } 22.62 \times 3 \times .42 \dots \dots \dots = 28.49$$

$$360.36$$

Intermediate reduction of pressure, as given at page 721, = .25 \times 22.62 = 5.65 per sq. inch, which, \times 3 sq. ins. and by 2 per foot of stroke, = 33.9 foot-lbs.

Hence 360.36 + 33.9 = 394.26 foot-lbs.

Or, by sum of the three results, viz :

| | |
|-----------------------------|--------|
| 1st cylinder..... | 126.94 |
| Intermediate expansion..... | 33.9 |
| 2d cylinder..... | 234.32 |
| | 394.26 |

WOOLF ENGINE. D. K. Clark.

Ratio of Expansion.—In 1st cylinder as per formula, page 710. In 2d cylinder $r \frac{l}{l'} + z \div 1 + z = \text{ratio}$. r representing ratio of area of 1st cylinder to that of 2d, and l' lengths of stroke and of stroke added to clearance, in ins. or feet, and s value of intermediate volume.

ILLUSTRATION.—Assume $l = 6$ feet, $l' = 7$ per cent. = .42, $r = 3$, and $s = .333$

$$3 \times \frac{6}{6.42} + 333$$

$$\text{Then } \frac{3 \times \frac{6}{6.42} + 333}{1 + 333} = 2.353, \text{ ratio of expansion in 2d cylinder.}$$

Total Actual Ratio of Expansion. $R' \left(r \frac{l}{l'} + z \right) = \text{ratio}$.

PROOF.—Assume preceding elements, $R = 2.653$.

$$3 \times \frac{6}{6.42} + .333 = 2.653 \times 3.137 = 8.322, \text{ total actual ratio}$$

Ratio of Expansion. $R' \left(r \frac{l}{l'} + z \right) \div 1 + s = \text{ratio}$

Assume preceding elements.

$$8.322 \div 1 + .333 = \frac{8.322}{1.333} = 6.242, \text{ combined ratio}$$

Attain Combined Ratio of Expansion and Final Pressure in 2d Cylinder.

uming four cases as taken for Receiver Engine with a clearance of Cent. The relations would be as follows:

| | | | | | |
|--|-------|-------|-------|-------|---|
| Intermediate spaces are | 0 | .333 | .5 | 1 | { part of volume of 1st cylinder plus its clearance, or, |
| of 1st cylinder..... | 0 | .357 | .535 | 1.07 | |
| These 1.07, the volume of 1st cylinder plus its clearance, and.... | 1.07 | 1.427 | 1.605 | 2.14 | { total initial volumes for expansion in 2d cylinder or times volume of 1st cyl'r. |
| 2 values of intermediate space = the volume of 2d cylinder, the sums are the final volumes expansion in 2d cylinder..... | 3 | 3.357 | 3.535 | 4.07 | { times volume of 1st cylinder. |
| Expansion in 2d cyl'r are equal of final by initial volumes.. | 2.804 | 2.352 | 2.202 | 1.902 | ratios of expansion. |
| Intermediate falls of pressure are, in of final pressure in 1st cylinder } | 0 | .25 | .333 | .5 | { of final pressure; or, assuming initial pressure at 63 lbs., and final pressure at 23.75 lbs., they are |
| | 0 | 5.94 | 7.92 | 11.87 | lbs. per sq. inch. |
| Initial pressures for expansion in cylinder are | 1 | .75 | .66 | .5 | { of final pressure in 1st cylinder, or |
| | 23.75 | 17.81 | 15.83 | 11.87 | lbs. per sq. inch. |
| Final pressures in 2d cyl'r are.. | 8.47 | 7.57 | 7.19 | 6.24 | lbs. per sq. inch. |

Combined Ratios in these Four Cases.

| 1st. | 1st ratio of expansion.... | 1 to 2.653 | Combined Ratio. |
|------|----------------------------|------------------------------------|-----------------|
| | 2d do. do. | 1 to 2.804 = 2.653 × 2.804 = 7.44. | |
| 2d. | 1st do. do. | 1 to 2.653 | |
| | 2d do. do. | 1 to 2.352 = 2.653 × 2.352 = 6.24. | |
| 3d. | 1st do. do. | 1 to 2.653 | |
| | 2d do. do. | 1 to 2.202 = 2.653 × 2.202 = 5.84. | |
| 4th. | 1st do. do. | 1 to 2.653 | |
| | 2d do. do. | 1 to 1.905 = 2.653 × 1.905 = 5.05. | |

Initial effect of steam at 63 lbs. pressure, admitted to 1st cylinder, for 2 feet, or one of stroke of piston, and with a clearance of 7 per cent. or .42 feet, is as follows:

on piston..... 63 × 2 feet = 126 foot-lbs. { Total initial
in clearance.. 63 × .42 foot = 26.46 = 63 × 2.42 = 152.46 foot-lbs. } effect.

Sum is initial effect, on which effect by expansion is computed, while it is foot-lbs. in excess of the initial effect on the piston.

total effect, then, is computed as follows:

| 1st case. | 152.46 × (1 + hyp. log. 7.44) or 3.0069 = 458.27 | Net Effect. |
|-----------|--|------------------|
| | Less effect of clearance..... 26.46 | 431.81 foot-lbs. |
| 2d case. | 152.46 × (1 + hyp. log. 6.24) or 2.831 = 431.47 | |
| | Less effect of clearance..... 26.46 | 405.01 " |
| 3d case. | 152.46 × (1 + hyp. log. 5.84) or 2.7647 = 421.35 | |
| | Less effect of clearance..... 26.46 | 394.89 " |
| 4th case. | 152.46 × (1 + hyp. log. 5.05) or 2.6294 = 399.29 | |
| | Less effect of clearance..... 26.46 | 372.83 " |

Reductions of net effect in 2d, 3d, and 4th cases are 6.2, 8.6, and 13.7 per cent. in 1st case.

Compute Effect for One Stroke and a Given Combined Actual Ratio of Expansion.

1. To hyp. log. of combined actual ratio of expansion (bel)
2. add 1; multiply sum by period of admission of steam to
3. added to clearance, and from product subtract clearance.

4. multiply area of 1st cylinder in sq. ins. by initial pressure of steam
5. inch and by above remainder. Product is net effect in foot-lbs.

EXAMPLE.—Assume elements of 1st illustration page 723.

Hyp. log. $6.24 + 1 = 2.831$, which, $\times 2.42 = 6.85$, and $6.85 - .42$ and $\times 60 = 385.8$ foot-lbs.

$$\text{Or, } l'(1 + \text{hyp. log. } R') - C \times a P = E.$$

Comparative Effect of Steam in Receiver and Engines.

The effect of steam in a compound engine, without clearance and without intermediate reduction of pressure, is the same whether operated in a Woolf engine.

When, however, there is an intermediate space between the two cylinders, there is an intermediate reduction of the pressure of the steam upon the increase of its volume in the receiver; the reduction of pressure, therefore, being less rapid than with the Woolf engine, the effect is greater.

In illustration, the following comparative elements of the effect of steam are furnished.

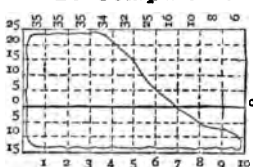
| RECEIVER. (7 per cent. clearance.) | | | WOOLF. | | |
|------------------------------------|---------------------|-------------|---------------|---------------------|-------------|
| | Ratio of Expansion. | Net Effect. | | Ratio of Expansion. | Net Effect. |
| 1st case..... | 7.96..... | 422.3 | 1st case..... | 7.64..... | 431.7 |
| 2d "..... | 5.97..... | 421.55 | 2d "..... | 6.24..... | 405.1 |
| 3d "..... | 5.31..... | 417.96 | 3d "..... | 5.84..... | 394.5 |
| 4th "..... | 3.98..... | 402.78 | 4th "..... | 5.05..... | 372.5 |

From which it appears, that although the effect of a receiver engine is less, its ratio of expansion is less than with the Woolf engine.

Also, that by the addition of clearance to the pistons of each engine, ratios of expansion are sensibly reduced, as compared with the ratio of expansion.

INDICATOR.

To Compute Mean Pressure by an Indicator.



sq. inch upon piston.

EXAMPLE.—Pressures, as above given, are:

$$35 + 35 + 35 + 34 + 32 + 25 + 16 + 10 + 8 + 6 = 236, \text{ which, } \div 10, = 23.6$$

NOTE.—If it were practicable to run an engine without any load, and times is, the mean pressure, as exhibited by an indicator, would be an exact measure of the friction of the engine.

Conclusions on Actual Efficiency of Steam

For development of highest efficiencies of steam, as used in an engine, protecting it from cooling and condensing in the cylinder must be employed. Heating of it prior to its introduction into a cylinder is probably most effective that may be employed for this purpose. Application to cylinders is next best means; and next is the steam-jacket.

As of locomotive and portable engines, consumption of steam per horse power is less than for that of single-cylinder condensing engines for like rate of speed, which is due to effect of temperature of non-condensing cylinder.

Results that the compound engine is more efficient than the two kinds of compound engines, the Woolf.

Steam coal per IHP per hour, for compound engines, ranged from 1.7 to 2.3 lb.

Compute Volume of Water Evaporated per Lb. of Coal.

$\frac{v W}{d}$ = volume of water, in lbs. V and v representing volume of steam and of water, in cube feet, W weight of cube foot of water, and F weight of unsaturated, both in lbs., and d density of water, in degrees of saturation.

STRACTION.—Take case at foot of page. $V = 449\ 887$ cube feet, $v = 838$ cube feet, $E = 1$, and $F = 4061$ lbs.

$$\frac{449\ 887 \div 838 \times 64.3}{4061 \times 1} = 3.5 \text{ lbs. per hour.}$$

in Fuel, and Initial Pressure of Steam required, when Acting Expansively, compared with Non-Expansion or Full Stroke.

| of off. | Gain in Fuel. | Cutting off. | Point of Cutting off. | Gain in Fuel. | Cutting off. | Point of Cutting off. | Gain in Fuel. | Cutting off. |
|---------|---------------|--------------|-----------------------|---------------|--------------|-----------------------|---------------|--------------|
| 3. | Per Cent. | Lbs. | Stroke. | Per Cent. | Lbs. | Stroke. | Per Cent. | Lbs. |
| | 22.4 | 1.03 | .5 | 41 | 1.18 | .25 | 58.2 | 1.67 |
| 5 | 32 | 1.09 | .375 | 49.6 | 1.32 | .125 | 67.6 | 2.6 |

STRACTION.—What must be initial pressure of steam cut off at .5, to be equivalent to 100 lbs. per sq. inch at full stroke?

$$100 \text{ at full stroke} = 100, \text{ and } 100 \times 1.18 = 118 \text{ lbs.}$$

To Compute Gain in Fuel.

EX.—Divide relative effect of steam by number of times the steam is expanded, and divide 1 by quotient; result is the initial pressure of steam required to be expanded to produce a like effect to steam at full stroke.

Find this pressure by number of times the steam is expanded, and subtract quotient from 1, remainder will give gain per cent. in fuel.

EXAMPLE.—When steam is cut off at .5, what is gain in fuel, and what mechanical

effect, including clearance of 5 per cent., = 1.64; number of times of expansion = 2.

$$1.64 \div 2 = .82, \text{ and } 1 \div .82 = 1.22 \text{ initial pressure.}$$

$$1.22 \div 2 = .61, \text{ and } 1 - .61 = .39 \text{ per cent.}$$

Mechanical effects of steam at full and half strokes are 2 — 1.64 = .36 difference.

EX., 1.64 : .36 :: 50 (half volume of steam used) : 10.97 per cent. more fuel to produce same effect at half stroke, compared with steam at full stroke.

Compute Consumption of Fuel in a Furnace.

EX.—Dimensions of Cylinder, Pressure of Steam, Point of Cut-off, Revolution, and Evaporation per Lb. of Fuel per Minute are given.

EX.—Compute volume of cylinder to point of cutting off steam, including clearance. Multiply result by number of cylinders, by twice number of revolutions of piston, and by 60 (minutes); divide product by density of steam pressure in cylinder, and quotient will give number of cube feet of steam expended in steam.

Multiply number of cube feet by 64.3 for salt water (62.425 for fresh), product by evaporation per lb. of fuel consumed, and quotient will give consumption in lbs. per hour.

EXAMPLE.—Cylinder of a marine engine is 95 ins. in diameter by 10 feet stroke; pressure of steam in steam-chest is 15.3 lbs. per sq. inch, cut off at .5; number of revolutions 14.5, and evaporation estimated at 8.5 lbs. of salt water per lb. of coal; what is consumption of coal per hour, when density of water is 62.425? (See Saturation, page 726.)

EX.—10 feet of steam at above pressure, compared with water (15.3 + 15 ins. + 2.5 per cent. for clearance $\div 144 = 50.45$ cube feet.

50.45 feet + 2.5 per cent. = 51.66 feet 1.5 ins., and 50.45 \times 5 feet 1.5 ins. = 252.25 cube feet steam per hour.

Steam and Exhaust Valves.—(Poppel), $\frac{a s n}{24000} = \text{area for steam}$, $\frac{615}{2000}$ exhaust; (Slide), $\frac{a s n}{30000}$ for steam, and $\frac{a s n}{22750}$ for exhaust. a represents area of steam cylinder in sq. ins., s stroke of piston in ins., and n number of revolutions per minute.

Injection Pipes.—One each Bottom and Side to each condenser; each equal to supply 70 times volume of water evaporated when engine working at a maximum; and in Marine and River engines the addition of a Bilge, which is properly a branch of bottom pipe.

NOTE 1.—Proportions given will admit of a sufficient volume of water when engine is in operation in the Gulf Stream, where the water at times is at temperature of 84° , and volume required to give water of condensation a temperature of 70 times that of volume evaporated.

2. Velocity of flow of water through cock or valve 20 feet per second in the shallow draught, and 30 feet in sea or deep draught service.

*Feed Pump.**—(Single acting, Marine), Volume, .006 to .01 steam cylinder (River and Land), or when fresh water alone or a surface condenser is used, .004 to .007.

NON-CONDENSING.

For a Range of Pressures of from 50 to 150 lbs. (Manometer Gauge) per Sq. Inch, Cut Off at Half Stroke.

Piston-rod.—(Wrought Iron), Diam., .125 to .2 steam cylinder. .8 diam.

Steam and Exhaust Valves.—Area is determined by rules given for a condensing engine, using for divisors 30000 and 22750.

A decrease in volume of cylinder is not attended with a proportionate decrease of their area, it being greater with less volume.

*Feed-pump.**—(Single acting, Marine), Volume, .008 to .016 steam cylinder. (River and Land), or where fresh water alone is used, .005 to .007.

General Rules.

Engines.

Cylinder. Thickness.—(Vertical), $\frac{D p}{2400} = t$; (Horizontal), $\frac{D p}{2000} = t$ (inclined), divide by 2000 in a ratio inversely as sine of angle of inclination. D representing diameter of cylinder, p extreme pressure in lbs. per sq. inch may be subjected to, and t thickness in ins.

Shafts, Gudgeons, Journals, etc. To resist Torsion. See rules, pp. 75

Coupling Bolts. $\frac{D}{2} \sqrt{\frac{D}{n d}} = d$. n representing number of bolts, D diameter of shaft, d distance of centre of bolts from centre of shaft, and d diameter of bolts.

Cross-head, Wrought-iron. (Cylinder), $\frac{a p l}{700} = S$, and $\sqrt{\frac{S}{b}} = d$, or $d = \sqrt{\frac{S}{b}}$. a representing area of cylinder in sq. ins., l length of cross-head between centre of cylinder in feet, and S product of square of depth d , and breadth b , of section.

(Air-pump), $\frac{a l}{18} = S$, and as above for d and b .

m is cylindrical, for S put $\sqrt[3]{S \times 1.7}$.

* end journals same as that of piston.

2e.—Its area should exceed that of steam-valve, proportionate to its exposure to the air.

g-rod.—Length, 2.25 times stroke of piston when it is at all to afford the space; when, however, it is imperative to reduce weight, it may be twice the stroke.

Friction of long and short connecting-rods is, for length of stroke of 1 cent. additional; twice, 3 per cent.; and for thrice, 1.33.

Diam. 1 to 1.1 that of piston-rod. Centre of body (*Horizontal*), *Vertical*), .06 inch per foot of length of rod.

Connecting-rods or links, area of necks .65 to .75 area of attached rod. Second set of rods is used, as in some air-pump connections, area of ratio, inversely as their lengths to that of first set.

* *Connecting-rods, Links, etc.*—(*Strap*), area at its least section attached rod; (*Gib and Key*), .3 diam. of neck, width, 1.25 times, times (*Draft*) of keys .6 to .8 inch per foot. Distance of *Slot* rod .5 diam. of pin.

Links, Beams, etc.).
$$\sqrt[3]{\frac{Pl}{C} \cdot 355} = d. \quad P \text{ representing pressure or thrust}$$

 $m, l \text{ length of journal in ins., and } C, \text{ for Wrought iron} = 640, \text{ Cast, } 560, 600, \text{ and Cast, } 1200.$

.3 to 1.5 times their diam., and pressure should not exceed 750 inch for propeller engine, and 1000 for side-wheel.

Wrought-iron).—*Hub*, compared with neck of shaft, 1.75 diam., 1. Eye, compared with pin, 2 diam., and 1.5 depth. *Web*, at periph. width, .7 width, and in depth .5 depth of hub; and at periph. width, .8 width, and in depth, .6 depth of eye.

1.) Diameters of *Hub* and *Eye* respectively, twice diam. of neck and 2.25 times that of crank pin.

Fillet of sides of web .5 width of web at end for which fillet is designed; back of web, .5 that at sides of their respective ends.

pen or Trussed.—Length from centres 1.8 to 2 stroke of piston, 5 length. If strapped, *Strap* at its least dimensions .9 area of its depth equal to .5 its breadth. *End centre journals* each 1, and *journals* 2.5 times area of piston or driving-rod.

Proportion for strap is when depth of beam is .5 length, as above; consequently its depth is less, area of strap must be increased; and when depth of beam is less than .5 width, its area is determined by product of its $b d^2$, as if its depth was .5 its width.

m). *Area of Section of Centre*.
$$\frac{p \times l \div 2}{500 d} = A. \quad p \text{ representing pressure upon piston in lbs., } d \text{ depth in ins., and } l \text{ length in feet.}$$

Centre .5 to .75 diam. of cylinder, and, when of uniform thickness of not less than .1 of depth.

1 of *End Centres*.— $l \div 2 - \sqrt{(l \div 2)^2 - (s \div 2)^2}$ = vibration at each *senting stroke of piston, in feet.*

Blocks (Shaft).—Binder $d \sqrt{\frac{l}{b}} = \text{depth.}$ d representing diam. 1 two to binder, l distance between bolts, b breadth of binder, all in ins., wrought iron 1, steel .85, and cast iron .2.

Down Bolts. $P \div 3 C$ = area at base of thread of each bolt. C for mild and large bolts 6000 and 7000, for wrought iron 4500 and 6000, if by

ass).
$$\frac{d}{3} \sqrt{\frac{l}{b}} = \text{depth.}$$

Cocks.—Angles of sides of plug from 7° to 8° from plane of it.

Pumps.—Velocity of water in pump openings should not exceed 300 ft. per minute.

Fly-wheels and Governors.—See Rules, pages 451 and 452.

Water-wheels.

Water-wheels (Arms).—Number from .75 to .8 diam. of wheel is in (Blades) Wood.—For a distance of from 5 to 5.5 feet between arms, thickness from .09 to .1 inch for each foot of diam. of wheel.

Area of blades, compared with area of immersed amidship section of vessel, depends upon dip of wheels, their distance apart, model and rig of vessel.

In *River service*, area of a single line of blade surface varies from .3 to that of immersed section; in *Bay or Sound service*, it varies from .15 to .2 and in *Sea service*, it varies from .07 to .1.

NOTE.—A wrought-iron blade .625 inch thick bent at a stress withstood by oak blade 3.5 ins. thick.

Radial and Feathering.

Radial.—Loss of effect is sum of loss by oblique action of wheel blades upon the water, their slip, and thrust and drag of arms and blades as they enter and leave the water.

Loss by oblique action is computed by taking mean of square of sine of angles of blades when fully immersed in the water.

Loss by oblique action of blades of wheel of steamer *Arctic*, when her wheels were immersed 7 feet 9 ins. and 5 feet 9 ins., was 25.5 and 18.5 per cent. which was the loss of useful effect of the portion of total power developed by expansion which was applied to wheels.

Feathering.—Loss of effect is confined to thrust and drag of arms and blades as they enter and leave the water.

Comparative Effects.—In two wheels of a like diameter (26 feet, and 6 feet immersion), like number and depth of blades, etc., the losses are as follows:

Radial 26.6 per cent. | Feathering 15.4 per cent.

Loss of effect by thrust and drag in a feathering wheel, having these elements and included in the above given loss, is computed at 2 per cent.

Relative loss of effect of the two wheels is, approximately, for ordinary immersions, 20 and 15 per cent. from circumference of wheel.

Centre of Pressure, $\frac{2}{3} \frac{d^3 - d'^3}{d^2 - d'^2} - d = c$. d and d' representing depths of blades below surface of water, and c centre of pressure, all in like dimensions, from leading edge.

In the cases here given, centres of pressure are as follows:

Radial 6.4 ins. | Feathering 8.5 ins.

Propellers.

Propellers (Screw).—Pitch should vary with area of circle described by tip to area of midship section of vessel.

AREA, TWO-BLADED.

| r to mid- | 6 | 5 | 4.5 | 4 | 3.5 | 3 | 2.5 |
|--------------------|----|------|------|-----|------|-----|-----|
| er of | .8 | 1.02 | 1.11 | 1.2 | 1.27 | 1.3 | 1.4 |
| | | | | | | | |

multiply ratio of pitch to diam. as given in table.

7.—Slip of a screw propeller is directly as its pitch, and economical of a screw is inversely as its pitch, greater the pitch less the effect. expanding pitch has less slip than a uniform pitch, and, consequently, is effective.

To Compute Thrust of a Propeller.

$$\frac{217}{S} = T. \quad S \text{ representing speed of vessel in knots per hour.}$$

SLIDE VALVES.

All Dimensions in Inches.

Compute Lap required on Steam End, to Cut-off at any given Part of Stroke of Piston.

LE.—From length of stroke subtract length of stroke that is to be made steam is cut off; divide remainder by stroke, and extract square root of quotient.

Multiply this root by half throw of valve, from product subtract half lead, remainder will give lap required.

EXAMPLE.—Having stroke of piston 60 ins., stroke of valve 16 ins., lap upon exhaust side .5 in. = one thirty-second of valve stroke, lap upon steam side 3.25 ins., ins., steam to be cut off at five sixths stroke; what is the lap?

$$-\frac{5}{6} \text{ of } 60 = 10. \quad \sqrt{\frac{10}{60}} = .408. \quad .408 \times \frac{16}{2} = 3.264, \text{ and } 3.264 - \frac{2}{2} = 2.264 \text{ ins.}$$

Ascertain Lap required on Steam End, to Cut-off at various Portions of Stroke.

| Valve out Lead. | Distance of piston from end of its stroke when steam is cut off, in parts of length of its stroke. | | | | | | | | | |
|----------------------|---|---------------|---------------|---------------|---------------|---------------|---------------|-----------------|---------------|---------------|
| | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | $\frac{15}{16}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
| parts of ke. | .354 | .323 | .286 | .27 | .25 | .228 | .204 | .177 | .144 | .102 |

NOTATION.—Take elements of preceding case.

Let $\frac{1}{8}$ is .204, and $.204 \times 16 = 3.264$ ins. lap.

Then Valve is to have Lead.—Subtract half proposed lead from lap assumed by table, and remainder will give proper lap to give to valve.

Then, as last case, valve was to have 2 ins. lead, then $3.264 - 2 \div 2 = 2.264$ ins.

Compute at what Part of Stroke any given Lap on Steam Side will Cut off.

LE.—To lap on steam side, as determined above, add lead; divide sum by length of throw of valve. From a table of natural sines (pages 390—find the arc, sine of which is equal to quotient; to this arc add 90° , from their sum subtract arc, cosine of which is equal to lap on steam divided by half throw of valve. Find cosine of remaining arc, add 1 and multiply sum by half stroke, and product will give length of that stroke that will be made by piston before steam is cut off.

EXAMPLE.—Take elements of preceding case.

$$\left(\sin. \frac{2.264 + 2}{16 \div 2} + 90^\circ - \cos. \frac{2.264}{16 \div 2} \right) + 1 \times \frac{60}{2} = \cos. (32^\circ 13' + 90^\circ - 73^\circ 34')$$

$$39^\circ, \text{ and } \cos. 48^\circ 39' + 1 \times \frac{60}{2} = 1.66 \times 30 = 49.8 \text{ ins.}$$

To Ascertain Breadth of Ports.

Throw of valve should be at least equal to lap on steam side, If this breadth does not give required area of port, throw until required area is attained.

Portion of Stroke at which Exhausting Port is Closed and Opened.

| Lap on Exhaust Side of Valve in Parts of its Throw. | Portion of Stroke at which Steam is cut off. | | | | | | Lap on Exhaust Side of Valve in Parts of its Throw. | Portion of Stroke at which Steam is cut off. | | | | | |
|---|--|---------------|---------------|---------------|---------------|---------------|---|--|---------------|---------------|---------------|---------------|---------------|
| | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{1}{2}$ |
| A | | | | | | | B | | | | | | |
| .125 | .178 | .161 | .143 | .126 | .109 | .093 | .074 | .125 | .033 | .006 | .019 | .012 | .008 |
| .0625 | .113 | .118 | .1 | .085 | .071 | .056 | .043 | .0625 | .00 | .052 | .04 | .03 | .022 |
| .03125 | .113 | .101 | .085 | .069 | .053 | .043 | .033 | .03125 | .073 | .066 | .051 | .042 | .033 |
| 0 | .092 | .082 | .067 | .055 | .041 | .033 | .022 | 0 | .092 | .082 | .067 | .055 | .041 |

Units in columns of table A express distance of piston, in parts of its stroke, from end of stroke when exhaust port in advance of it is closed; and those in columns of table B express distance of piston, in parts of its stroke, from end of its stroke when exhaust port behind it is opened.

ILLUSTRATION.—A slide valve is to be cut off at one sixth from end of stroke on exhaust side is one thirty-second of stroke of valve (16 ins.), and stroke of piston is 60 ins. At what point of stroke of piston will exhaust port in advance of it be closed and the one behind it open.

Under one sixth in table A, opposite to one thirty-second, is .053, which \times length of stroke = 3.18 ins.; and under one sixth in table B, opposite to one thirty-second, is .033, which \times 60 = 1.98 ins.

If lap on exhaust side of this valve was increased, effect would be to cause port in advance of valve to be closed sooner and port behind it opened later. And if lap on exhaust side was removed entirely, the port in advance of piston would be closed and the one behind it open, at same time.

Lap on steam side should always be greater than that on exhaust side, and difference greater the higher the velocity of piston.

In fast-running engines, alike to locomotives, it is necessary to open exhaust port before end of stroke of piston, in order to give more time for escape of the steam.

To Compute Stroke of a Slide Valve.

RULE.—To twice lap add twice width of a steam port in ins., and sum will give stroke required.

Expansion by lap, with a slide valve operated by an eccentric alone, cannot be extended beyond one third of stroke of a piston without interfering with efficient operation of valve; with a link motion, however, this distortion of the valve is somewhat compensated. When lap is increased, throw of eccentric should also be increased.

When low expansion is required, a cut-off valve should be resorted to in addition to main valve.

To Compute Distance of a Piston from End of its Stroke, when Lead produces its Effect.

RULE.—Divide lead by width of steam port, both in ins., and term the quotient sine; multiply its corresponding versed sine by half stroke of piston; product will give distance of piston from end of its stroke, when steam is admitted for return stroke and exhaustion ceases.

EXAMPLE.—Stroke of piston is 48 ins., width of port 2.5 ins., and lead .5 ins. What will be distance of piston from end of stroke when exhaustion commences?

$$.5 \div 2.5 = .2 = \text{sine, ver. sin. of } .2 = .0202, \text{ and } .0202 \times \frac{48}{2} = .4848 \text{ ins.}$$

To Compute Lead, when Distance of a Piston from the End of Stroke is given.

RULE.—Divide distance in ins. by half stroke in ins., and term quotient sine; multiply corresponding sine by width of steam port, and product will give lead.

elements of preceding case.

$$\text{ver. sin. and sine of ver. sin. } .0202 = .2, \text{ and } .2 \times 2.5 = .5 \text{ ins.}$$

Compute Distance of a Piston from End of its Stroke, When Steam is admitted for its Return Stroke.

RULE.—Divide width of steam port, and also that width, less the lead, by stroke of slide, and term quotients *versed sines first and second*. Ascertain their corresponding arcs, and multiply *versed sine* of difference between *first and second* by .5 stroke, and product will give distance.

EXAMPLE.—Assume elements of preceding case. lap = .5 inch, and stroke of 6 ins.

$$\frac{5}{6} \text{ and } \frac{2.5 - .5}{6 \div 2} = .8333, \text{ and } .667 \text{ and ver. sin. } 80^{\circ} 24' \sim 70^{\circ} 33' \times \frac{48}{2} = .3528 \text{ inch.}$$

To Compute Lap and Lead of Locomotive Valves.

Cut off at .33, .25, and .125 of stroke of piston, lap = 289, .25, and .177 *t*, outside = .07 *t*, and inside lead = .3 *t*. *t* representing stroke of valve, all in ins.

HORSE-POWER.

Horse-power is designated as *Nominal*, *Indicated*, and *Actual*.

Nominal, is adopted and referred to by Manufacturers of steam-engines, in order to express capacity of an engine, elements thereof being confined to dimensions of steam cylinder, and a conventional pressure of steam and lead of piston.

Indicated, designates full capacity in the cylinder, as developed in operation, and without any deductions for friction.

Actual, refers to its actual power as developed by its operation, involving elements of mean pressure upon piston, its velocity, and a just deduction for friction of operation of the engine.

To Compute Horse-power of an Engine.

Nominal.—*Non-condensing*, $\frac{D^2 v}{1000}$, and *Condensing*, $\frac{D^2 v}{1400} = \text{HP}$. *D* representing diameter of cylinder in ins., and *v* velocity of piston in feet per minute.

Non-condensing is based upon uniform steam-pressure of 60 lbs. per sq. inch (steam-gauge), cut off at .5 stroke, deducting one sixth for friction and losses, with a mean velocity of piston, ranging from 250 to 450 feet per minute.

Condensing is based upon uniform steam-pressure of 30 lbs. per sq. inch (steam-gauge), cut off at .5 stroke, deducting one fifth for friction and losses, with a mean velocity of piston of 300 feet per minute for an engine of short stroke, and of 400 feet for one of long stroke.

Actual.—*Non-condensing*, $\frac{A P^* - (f + 14.7) 2 s r}{33000} = \text{HP}$. *A* representing area of cylinder in sq. ins., *P* mean effective pressure upon cylinder piston, inclusive of atmosphere, *f* friction of engine in all its parts, added to friction of load, both in lbs. per sq. inch, *s* stroke of piston in feet, and *r* number of revolutions per minute. The sum of these resistances is from 12.5 to 20 per cent., according to pressure of steam, being least with highest pressure.

h is value is best obtained by an *Indicator*; when one is not used, refer to rule above, pp. 710-12. In estimating value of *P*, add 14.7 lbs., for atmospheric pressure, that indicated by steam gauge or safety-valve. Clearance of piston at each end of cylinder is included in this estimate.

h is value may be safely estimated in engines of magnitude, for friction of engine in all its parts, and friction of load, 10 per cent. of remaining pressure. Sum of these resistances is from 10 to 20 per cent., according to pressure of steam, and is to deliver water of condensation at level of discharge. For pressure representing friction for different designs, as estimated by English authority, see pp. 473-5 and 66.

ILLUSTRATION.—Diameter of cylinder of a non-condensing engine is 10 ins., stroke of piston 4 feet, revolutions 45 per minute, and mean pressure of steam (per gauge) 60 lbs. per sq. inch.

$$A = 78.54 \text{ sq. ins. } P = 60 + 14.7 = 74.7 \text{ lbs. } f = 2.5 + \frac{(60 + 14.7 - 2.5) \times .075}{1.5} = 7.92$$

$$\text{Then } \frac{78.54 \times (60 + 14.7 - 7.92 + 14.7) \times 2 \times 4 \times 45}{33000} = 44.6 \text{ HP.}$$

NOTE 1.—Power of a non-condensing engine is sensibly affected by character of exhaust, as to whether it is into a heater, or through a contracted pipe, to admit blast to combustion.

2.—If an indicator is not used to determine pressure of steam in a cylinder, a safe estimate of it, when acting expansively, is .9 of full pressure, and when at full stroke from .75 to .8.

$$\text{Condensing. } \frac{A P \bar{s} - f \bar{s} r}{33000} = \text{HP.}$$

Power required to work the air-pump of an engine varies from .7 to .9 lbs. per sq. inch upon cylinder piston.

ILLUSTRATION.—Diameter of cylinder of a marine steam-engine is 60 ins., stroke of piston 10 feet, revolutions 15 per minute, pressure of steam 50 lbs. per sq. inch, cut off at .25 stroke, and clearance 2 per cent.

$$A = 2827.4 \text{ sq. ins. } P \text{ (per Ex., page 713)} = 28.62 \text{ lbs. } f = 1.5 + \frac{28.62 - 1.5}{1.5} = 2.467 \text{ lbs.}$$

$$\text{Then } \frac{2827.4 \times 28.62 - 2.467 \times 2 \times 10 \times 15}{33000} = 662.23 \text{ HP.}$$

From which is to be deducted in marine engines power necessary to discharge water of condensation at level of load-line, which is determined by pressure and elevation of water, area of air-pump piston, and velocity of its discharge in feet per second.

$$\text{Indicated. } \frac{A P \bar{s} r}{33000} = \text{HP, and } \frac{33000 \text{ HP}}{P \bar{s} r} = A.$$

$$\text{British Admiralty Rule.—Nominal. } \frac{7 A v}{33000} \text{ or } \frac{D^2 v}{6000} = \text{HP.}$$

$$\text{French.—(Force de Cheval.) } 1.695 D^2 L r = \text{HP. } D \text{ and } L \text{ in meters.}$$

ILLUSTRATION.—Assume a diameter of cylinder of .254 meters, with a stroke of piston of .3 meters and 250 revolutions per minute.

$$1.695 \times .254^2 \times .3 \times 250 = 8.18 \text{ HP.}$$

$$A \text{ Force de Cheval} = 4500 \text{ kilograms per minute} = 32549 \text{ foot-lbs.} = .98757 \text{ HP.}$$

$$\text{One HP} = 1.0139 \text{ Force de Cheval.}$$

$$\text{Compound Indicated. } A L r \left(\frac{P}{R''} \cdot 1 \text{ hyp. log. } R'' - b \right) \cdot .00053 = \text{HP.}$$

L, representing length of stroke in feet, *R''* combined ratio of both cylinders, and *b* back pressure.

ILLUSTRATION.—Assume area of cylinder 3 sq. ins., stroke 6 feet, one stroke of piston, and steam 60 lbs. per sq. inch, cut off at .25.

$$A = 3 \text{ sq. ins., } L = 6 \text{ feet, } n = 1 \text{ stroke, } P = 60 \text{ lbs., } R'' = 5.969, b = .25 \text{ per sq. inch, and } r = .5, \text{ and } 1 + \text{hyp. log. } R'' = 1 + 1.7865.$$

$$\text{Then } 3 \times 6 \times .5 \times \left(\frac{60}{5.969} \times 1 + 1.7865 - .25 \right) \times .00053 = 9 \times \frac{10.052 \times 2.7865}{10000} = .0132 \text{ HP, which, } \times 2 \text{ for 1 revolution,} = .0264 \text{ HP per revolution.}$$

Compute Volume of Water required to be Evaporated in an Engine.

—Multiply volume of steam expended in cylinder and steam-chamber by number of revolutions, and multiply product by density of steam.

PLE.—What volume of water will an engine require to be evaporated per on, diam. of cylinder being 70 ins., stroke of piston 10 feet, and pressure of 4 lbs. per sq. inch, including atmosphere, cut off at .5 of stroke?

of cylinder = 3848.5 ins. $10 \times 12 \div 2 = 60$ ins., $60 \times 3848.5 = 230910$ cube ins. for clearance at one end, volume of nozzle, steam-chest, etc., 17 317 cube ins. $230910 + 17317 = 1728 \times 2 = 287.3$ cube feet, which, $\times .001336$, density of at 34 lbs. pressure (see Note 2), = .3838 cube feet.

1.—This refers to expenditure of steam alone; in practice, however, a large y of water "foaming," differing in different cases, is carried into cylinder in ation with the steam; to which is to be added loss by leaks, gauges, etc.

Volume of steam is readily computed by aid of table, pp. 708-9. Thus, den- weight of one cube foot of steam at above pressure = .0835 lbs. Hence, as : 1 cube foot :: .0835 lbs. : .001336 cube foot.

Compute Volume of Circulating Water required by an Engine.

$\frac{t + .3T - t'}{t' - t} = V$. T representing temperature of steam upon entering the con-
t, t' , and t'' temperatures of feed water, of water of condensation discharged, circulating water, all in degrees.

STRATION.—Assume exhaust steam at 8 lbs. per sq. inch, temperatures of dis- 100°, feed water 120°, and sea-water 75°.

perature at 8 lbs. pressure = 183°. $\frac{1114 + .3 \times 183 - 120}{100 - 75} = 41.95$ times.

Compute Volume of Flow through an Injection Pipe.

E.—Multiply square root of product, of 64.33 and depth of centre of g into condenser, from surface of external water, added to height of a a of water due to vacuum in condenser, all in feet, by area of opening ins.; and .6 product, divided by 2.4 (144 ÷ 60) will give volume in eet per minute.

PLE.—Diameter of an injection pipe is 5.375 ins., height of external water condenser 6.13 feet, and vacuum 24.45 ins.; what is volume of flow per min.?

of 5.375 ins. = 22.69 ins., $c = .6$. Vacuum $\frac{24.45 \text{ ins.}}{2.04} = 12$ lbs.; 12×2.24 (a-water) = 26.88 feet, and $26.88 + 6.13 = 33.1$ feet.

Then $\frac{\sqrt{64.33 \times 33.1} \times 22.69 \times .6}{2.4} = \frac{628.15}{2.4} = 261.73$ cube feet.

To Compute Area of an Injection Pipe.

LE.—Ascertain volume of water required by rule, page 706, in cube ins. econd, multiply it by number of volumes of water required for con- tion, by rule, page 707, divide it by velocity due to flow in feet per 1, and again by 12, and quotient will give area in sq. ins.

MPLE.—An engine having a cylinder 70 ins. diam., stroke of piston 10 feet, tions per minute 15, and steam 19.3 lbs., mercurial gauge cut off at .5; what l be area of its injection pipe at its maximum operation?

ime of cylinder 267.25 cube feet, cut off at .5 = 133.625 ins. sity of steam at 34 lbs. (19.3 + 14.7) = .001336. Velocity of flow of injected (computed from vacuum and elevation of condensing water) 33 feet per second.

1 $133.625 \times 15 \times 2 \times 1728 \div 60 = 115452$ cube ins. second, and
2 $\times .001336 = 154.24$ cube ins. water per second.

imum volume of water required to condense steam 1 evaporated, which only occurs in the Gulf of Mexk
t 40 times.

1 + 11.59 (= 7.5 per cent. for leakage of valves, etc.

2 = 11608.1 cube ins., and $11608.1 \div 33 \times 12 = 29.31$

Coefficient of velocity for flow under like conditions = .6; hence, $2.3 \div 48.85$ sq. ins.

NOTE.—This is required capacity for one pipe. It is proper and customary there should be two pipes, to meet contingency of operation of one being out.

To Compute Area of a Feed Pump. (Sea-water.)

RULE.—Divide volume of water required in cube ins. by number of strokes of piston, both per minute, and divide quotient by stroke of pump in ins.; multiply this quotient by 6 (for waste, leaks, "running up," etc.) product will give area of pump in sq. ins.

EXAMPLE.—Assume volume to be 5 cube feet and revolutions of engine: minute, with a stroke of pump of 3.5 feet.

$$\frac{5 \times 1728}{15} = 576, \text{ which } \div 3.5 \times 12 = 13.72, \text{ and } 13.72 \times 6 = 82.32 \text{ sq. ins.}$$

NOTE.—In fresh water, this proportion may be reduced one half.

STEAM-INJECTOR. William Sellers & Co.

Self-adjusting.

Volume of Water Discharged per Hour.

| No. | Pressure of Steam in Lbs. | | | | No. | Pressure of Steam in Lbs. | | | |
|-----|---------------------------|------------|------------|------------|-----|---------------------------|------------|------------|------------|
| | 60 | 80 | 100 | 120 | | 60 | 80 | 100 | 120 |
| | Cub. feet. | Cub. feet. | Cub. feet. | Cub. feet. | | Cub. feet. | Cub. feet. | Cub. feet. | Cub. feet. |
| 3 | 28.12 | 31.66 | 35.2 | 38.75 | 7 | 162.65 | 182.1 | 201.55 | 221 |
| 4 | 52.16 | 58.44 | 64.72 | 71 | 8 | 213.2 | 238.8 | 264.4 | 290 |
| 5 | 82.18 | 92.02 | 101.86 | 111.7 | 9 | 269.97 | 302.28 | 334.59 | 366.9 |
| 6 | 119.09 | 133.33 | 147.57 | 161.82 | 10 | 333.64 | 373.57 | 413.49 | 453.4 |

Highest temperature admissible of feed water 135°.

To Compute Size of Injector required.

One IP per hour will require from 15 to 40 lbs. of water per hour, according to character of engine.

When the lbs. of coal burned per hour can be ascertained, divide the 7.5, and quotient will give the volume of water in cube feet per hour.

When the area of grate-surface is known, multiply it by 1.6 for IP.

In case of plain cylindrical boilers, divide the number of sq. feet of ing-surface by 10 for the IP. In case of flue boilers, divide by 12, and multi-tubular boilers, by 15, for the nominal IP.

Minimum capacity of Injectors, about 50 per cent. of Tabular capacity.

To Compute Volume of Injection Water required IHP per Hour.

OPERATION.—Assume temperature of water 80°, and of condensation 100°. Volume of cylinder per IHP as per formula, page 716, and illustration, page 717: feet per minute.

Then, as per rule page 707, $\frac{1146.1 - 100}{100 - 80} = 52.3$ cube ins. per cube foot of steam.

$$\frac{2.76 \times 52.3 \times 62.5}{1728} = 5.22 \text{ lbs., which, } \times 60, = 313.2 \text{ lbs.}$$

To Compute Net Volume of Feed Water required IHP per Hour.

elements of formula, page 716, and illustration, page 717.

Then $1154 \times 2.76 \times 60 = 19.11$ lbs.

= diameter for small, and $\frac{d}{32} \sqrt{v}$, for large pump
ins., and v its velocity in feet per minute.

Results of Operations of Steam-engines. (D. K. Clark.)

| CONDENSING ENGINE. | Actual Ratio of Expansion. | Steam per HP as cut-off. | Coal per HP. | Initial Pressure at cut-off. | Steam per HP per hour. |
|------------------------------|----------------------------|--------------------------|--------------|------------------------------|------------------------|
| SINGLE. | | | | | |
| iss, Saltaire..... | 5.2 | 14.51 | 2.5 | 34.5 | 17.4 |
| ping, Crossness..... | 6.07 | 14.27 | 2.2 | 46 | 18.7 |
| East London..... | 3.62 | 12.92 | — | 23.25 | 20.72 |
| er, Corliss valves..... | 10 | — | 3.3 | 50 | 19.6 |
| erheated, Hira..... | 4.132 | — | — | 60 | 18.62 |
| COMPOUND. | | | | | |
| lder & Co..... | { Receiver..... 1.85 | 14.45 | 1.61 | 56 | — |
| | { Marine, jacketed 1.852 | 14.85 | | | |
| E. Wood..... | { Receiver..... 4.01 | 10.94 | 2.14 | 85.5 | — |
| | { stationary..... 1.857 | 13.34 | | | |
| kin..... | { Woolf, stationary 2.486 | 13.18 | — | 50.5 | — |
| | { jacketed..... 3.221 | 13.87 | — | — | 22.51 |
| erican, Woolf..... | { 1st cylinder..... 2.31 | actual | — | 90 | 15.37 |
| | { both..... 5.63 | 23.21 | | | |
| " " jacketed | { 1st cylinder..... 3.77 | 20.71 | — | 90 | 14.1 |
| | { both..... 9.19 | | | | |
| NON-CONDENSING. | | | | | |
| shall, Sons, & Co..... | 4.8 | 16.87 | — | 76 | 25.9 |
| ey, Paxman, & Co..... | 5 | 14.93 | — | 73 | 29.6 |
| omotive "Great Britain"..... | 1.45 | 31.36 | — | 102 | 31.36 |
| " " "..... | 2.94 | 21.24 | — | 87 | 21.24 |

Practical Efficiency of Steam-engines. Initial Volume = 1.

| CYLINDERS. | Most Efficient Ratio of Expansion. | Steam * per HP per hour. | CYLINDERS. | Most Efficient Ratio of Expansion. | Steam * per HP per hour. |
|--------------------------|------------------------------------|--------------------------|-------------------------------|------------------------------------|--------------------------|
| CONDENSING. | | | | | |
| le cylinder, jacketed... | 6 | 19.5 | Compound, jacketed, Woolf | 10 | 1.62 |
| le cylinder..... | 4 | 24 | Compound, Woolf..... | 7 | 20.5 |
| " superheated | 4 | 18.5 | NON-CONDENSING. | | |
| pound, jacketed, Re-} | 6 | 19 | Single cylinder, † jacketed.. | 4 | 24 |
| ver..... | | | Single cylinder, ‡..... | 3 | 21 |

* From boiler.

† 70 lbs. pressure.

‡ 90 lbs. pressure.

Standard Operation of a Portable Engine.

| | | | | |
|----------------------|------|-----------|-------------------------|----------|
| sq. feet..... | 5.5 | sq. feet. | Water evaporated from } | |
| ing surface..... | 220 | " " | and at 212° per hour. } | 450 lbs. |
| per HP per hour.... | 6.25 | lbs. | " " per HP per hour | 62.5 " |
| " sq. foot of grate. | 9 | " | " " " sq. foot of } | 81.8 " |
| " hour..... | 50 | " | grate..... | |

Ratio of heating surface of grate..... 40 to 1.

MIXTURE OF AIR AND STEAM.

Water contains a portion of air or other uncondensable gaseous converted into steam, this air is mixed with it, and when left in a gaseous state. If means were not taken to remove from condenser of a steam-engine, it would fill it and destroy operation; but, notwithstanding the ordinary means of condensation, a certain quantity of it always remains in condenser.

volumes of water absorb 1 volume of air.

3 Q*

BOILER.

efficiency is determined by proportional quantity of heat of combustion of fuel used, which it applies to the conversion of water into steam, or it may be determined by weight of water evaporated per lb. of fuel.

Following results and computations, water is held to be evaporated from standard temperature of 212°.

Portion of surplus air, in operation of a furnace, in excess of that which is actually required for combustion of the fuel, is diminished as rate of combustion is eased; and this diminution is one of the causes why the temperature in a furnace is increased with rapidity of combustion.

When combustion is rapid, some air should be introduced in a furnace through the grates, in order the better to consume the gases evolved.

Natural Draught.

Grate (Coal) should have a surface area of 1 sq. foot for a combustion of 12 lbs. of coal per hour, length not to exceed 1.5 times width of furnace, and an inclination toward bridge-wall of 1 to 1.5 ins. in every foot of length. When, however, rate of combustion is not high, in consequence of low velocity of draught of furnace, or fuel being insufficient, this proportion of area be increased to one sq. foot for every 12 lbs. of fuel.

Thickness of bars the least practicable, spaces between them being from .5 to 1 inch, according to fuel used. Anthracite requiring less space than bituminous. Short grates are most economical in combustion, but generate less rapidly than long.

Height of grate under a plain cylindrical boiler gives best effect with a fire deep, when grate is but 7.5 ins. from lowest point.

Width, Cast-iron, .6 square root of length in ins.

Area, their area should be 1.25 to 1.4 that for coal.

Automatic (Vicar's).—Its operation effects increased rapidity in firing and more effective evaporation.

Pit.—Transverse area of it, for a combustion of 15 lbs. of coal per hour, to .25 area of grate surface for bituminous coal, and .25 to .3 for anthracite. Or 15 to 20 ins. in depth for a width of furnace of 42 ins.

Space or Combustion Chamber.—(Coal) Volume of it from 2.75 to 3 cube feet per sq. foot of grate surface. (Wood) 4.6 to 5 cube feet.

Higher the rate of combustion the greater the volume, bituminous coal requiring more than anthracite. Velocity of current of air entering grate-pit may be estimated at 12 feet per second.

Volume of air and smoke for each cube foot of water converted into steam is, for coal, 1780 to 1950 cube feet, and for wood, 3900.

Rate of Combustion.—In lbs. of coal per sq. foot of grate per hour. *Marine* Boilers, slowest, 4; ordinary, 10. *Stationary*, 12 to 16. *Marine*, 14. Quickest: complete combustion of dry coal, 20 to 23; of caking coal, 4 to 27; *Blast or Fan and Locomotive*, 40 to 120.

Bridge-wall (Calorimeter).—Cross-section of an area of 1.2 sq. ins. for 1 lb. of bituminous coal consumed per hour, or from 1.5 to 2 sq. ins. for 1 sq. foot of grate, for a combustion of 15 lbs. of coal per hour.

Temperature of a furnace is assumed to range from 1500 to 2000°. Volume of air required for combustion of 1 lb. of bituminous coal, is 154.81 cube feet, which, at furnace temperatures, makes volume of heated air at bridge-wall 1.2 feet for each lb. of coal consumed upon grate.

Hence, at a velocity of draught of about 12 feet per second, area of bridge-wall, required to admit of this volume being passed off in an hour, is 25 sq. ins., and proportionately for increased velocity, but in practice it might be 1.2 to 1.6 ins.

When 20 lbs. of coal per hour are consumed upon a sq. foot of grate, at a draught of 1.6 to 2.4 or 32 sq. ins. are required, and in a like proportion for other quantities.

Or, When area of flues is determined upon, and area over bridge-wall required, it should be taken at from .7 to .8 area of lower flues for a draught, and from .5 to .6 for a blast.

When one half of tubes were closed in a fire-tubular marine boiler, the evaporative power per lb. of coal was reduced but 1.5 per cent.

Firing.—Coal of a depth up to 12 ins. is more effective than at a greater depth. Admission of air above the grate increases evaporative effect, but diminishes the rapidity of it.

Air admitted at bridge-wall effects a better result than when admitted above, and when in small volumes, and in streams or currents, it arrests or prevents smoke. It may be admitted by an area of 4 sq. ins. per sq. foot of grate.

Combustion is the most complete with firings or charges at intervals of 15 to 20 minutes.

With a fuel economizer (Green's) an increased evaporative effect of 9 per cent. has been obtained.

When external flues of a Lancashire boiler were closed, evaporative power was slightly increased, but evaporative efficiency was decreased; and when 25 per cent. of like surface in setting of a plain cylindrical boiler was cut off, evaporation was reduced but 1.5 per cent. When temperature at base of chimney was 650°, with a fire 12 ins. in depth, it was decreased to 550° with one 9 ins. in depth, and to 500° with one 6 ins.

High wind increases evaporative effect of a furnace.

Stationary or Land.—Set at an inclination downward of .5 inch in 10 lbs.

Smoke Preventing.—A test of C. Wye Williams's design of preventing smoke at Newcastle, 1857, as reported by Messrs. Longridge, Armstrong, and Richardson, gave an increased evaporative effect with the "practical prevention of smoke." Hence it was concluded, "That by an easy method of firing, combined with the admission of air in front of furnace, and a proper arrangement of grate, emission of smoke may be effectually prevented in ordinary marine multi-tubular boilers with suitable coals." 2d. That prevention of smoke increases economic value of boiler and evaporative power of boiler. 3d. That coals from the Hartley district have an evaporative power fully equal to that of the best Welsh steam-coals."

Heating Surfaces.

Marine (Sea-water).—Grate and heating surfaces should be increased about .07 over that for fresh water.

Relative Value of Heating Surfaces.

Horizontal surface above the flame = 1 | Horizontal beneath the flame..... = .1
Vertical..... = .5 | Tubes and flues..... = .5

Minimum Volumes of Fuel Consumed per Sq. Foot of Grate per Hour, for given Surface-ratios. (D. K. Clark)

| CRIPION OF BOILER. | Surface-ratios of Heating Surface to Grate. | | | | | | | | | |
|-----------------------|---|------|------|------|------|------|------|------|------|------|
| | 10 | 15 | 20 | 30 | 40 | 50 | 60 | 75 | 90 | 100 |
| | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| WY... | 1.7 | 3 | 6.8 | 12.1 | 18.9 | 26 | — | — | — | — |
| | .6 | 2.8 | 6.3 | 11.2 | 17.5 | 24 | — | — | — | — |
| | 4 | .8 | 1.8 | 3.2 | 5 | — | — | — | — | — |
| | — | 1.3 | 2.9 | 5.2 | 8.1 | 11.7 | 18.3 | 26.3 | 34 | 40 |
| | — | 1.8 | 4 | 7 | 11 | 16 | 25 | 36 | 46 | 56 |

of fuel (120 lbs.) coke will withstand disturbing draught

le of sediment one sixteenth of an inch thick will effect a loss of 14.7 per fuel.

sq. foot of fire surface is held to be as effective as three of heating.

Relation of Grate, Heating Surface, and Fuel.

When Grate and Heating Surface are constant, greater the weight of fuel used per hour, greater the volume of water evaporated; but the volume decreased proportion to fuel consumed.

On the subject of relations of grate, surface, and fuel, D. K. Clark, in his valuable work, submits, that in 1852 he investigated the question of evaporative performance of locomotive-boilers, using coke; and he deduced from them, that, assuming constant efficiency of fuel, or proportion of water evaporated to fuel, evaporative power or volume of water which a boiler evaporates per hour, decreases directly as area of grate is increased; that is to say, larger the grate, less the evaporation of water, and the rate of efficiency of fuel, even with same heating surface.

That evaporative effect increases directly as square of heating surface, with same area of grate and efficiency of fuel.

Necessary heating surface increases directly as square root of effect—viz., for same effect, with same efficiency, twice heating surface only is required.

Necessary heating surface increases directly as square root of grate, with same effect; that is, for instance, if grate is enlarged to four times its first area, twice the surface would be required, and would be sufficient, to evaporate same volume of water per hour with same efficiency of fuel.

Result of 40 experiments with a stationary boiler (fresh water), with an evaporation of 9 lbs. water per lb. of fuel consumed, the coefficient .00222 is deduced.

Equation, $\left(\frac{h}{g}\right)^2 \cdot .00222 = W$. W representing volume of water in cube feet, and g areas of grate and heating surfaces in sq. feet.

EXAMPLE.—Assume a heating surface of 90 feet, and a grate of 3; what will be evaporation?

$$\text{Then } 90 \div 3 \times .00222 = 1.998 \text{ cube feet.}$$

E.—A Galloway stationary boiler, with a ratio of grate area of 34.3 and a consumption of 21.8 lbs. coal per hour, evaporated 2.9 cube feet of water per sq. foot of grate. Hence the coefficient in this case would be .002466.

Compute Areas of Grate and Heating Surfaces, Volume of Water, and Weight of Fuel.

For a Temperature of 281°, or Pressure of 50 lbs. per Sq. Inch.

To Compute Weight of Fuel.

Water per Sq. Foot of Grate per Hour and Surface Ratio are Given.

$$\frac{W - x R^2}{C} = F, \text{ and } x R^2 = (E - C) F.$$

EXAMPLE.—Assume elements as preceding.

$$\frac{200 - .02 \times 50^2}{10} = 15, \text{ and } .02 \times 50^2 = \left(\frac{200}{15} - 10\right) \times 15 = 50.$$

Compute Ratio of Heating Surface to Area of Grate, and to Effect a Given Evaporation.

When Water and Fuel per Sq. Foot of Grate are Given. $\sqrt{\frac{W - C F}{R}} = R$.
 W representing water evaporated per sq. foot of grate, and F fuel consumed, both per hour. C and x specific constants for each type of boiler, and R $(h + g)$ heating surface to grate.

EXAMPLE.—Assume $W = 200$, $C = 10$, $F = 15$, and

$$\frac{200 - 10 \times 15}{.02} = 50; \quad \frac{200 - .02 \times 50^2}{10} = 15; \text{ and } .$$

When Efficiency of Fuel and Fuel consumed per Sq. Foot of Grate per Hour are given. $\frac{W}{F} = E$ or efficiency of fuel or weight of water evaporated per lb. of fuel. $\sqrt{\frac{(E-C)F}{x}} = R$.

To Compute Fuel that may be consumed per Sq. Foot of Grate per Hour, corresponding to a Given Efficiency.

When Efficiency of Fuel, that is, Weight of Water evaporated per lb. of Fuel, and the Surface Ratio, are given.

$$\frac{xR^2 + CF}{F}, \quad C + \frac{xR^2}{F} = E, \quad \text{and} \quad \frac{xR^2}{E-C} = F.$$

ILLUSTRATION.—Assume elements as preceding.

$$\frac{.02 \times 50^2 + 10 \times 15}{15} = 13.33; \quad 10 + \frac{.02 \times 50^2}{15} = 13.33, \quad \text{and} \quad \frac{.02 \times 50^2}{13.33 - 10} = 15 \text{ lbs.}$$

Combustion of Coal per sq. foot of grate.—Natural Draught, from 30 to 25 lbs. to be consumed per hour.—Steam-jet, 30 lbs., and Exhaust-blast 65 to 80 lbs.

From Results of Experiments upon Marine Boilers, see Manual of D. K. Clark, page 808; he deduced the following formula, as applicable to all surface ratios of such boilers.

$$\text{Newcastle } .02156 R^2 + 9.71 F, \quad \text{and for Wigan } .01 R^2 + 10.75 F = W \text{ in lbs.}$$

And the general formulas he deduced from all the various experiments are as follows.

From and at 212°.

$$\begin{array}{ll} \text{Portable} \dots .008 R^2 + 8.6 F = W & \text{Marine} \dots .016 R^2 + 10.25 F = W \\ \text{Stationary} \dots .0222 R^2 + 9.56 F = W & \text{Locomotive, coal, } .009 R^2 + 9.7 F = W \\ \text{Locomotive, coke} \dots .0178 R^2 + 7.94 F = W. \end{array}$$

As the maximum evaporative power of fuel is a fixed quantity, the preceding formulas are not fully applicable in minimum rates of its consumption and evaporative quality.

With coal and coke the limits of evaporative efficiency may be taken respectively at 12.5 and 12 lbs. water from and at 212°.

ILLUSTRATION 1.—Assume a marine fire-tubular boiler with a surface ratio of heating surface to grate of 30 and a consumption of coal of 15 lbs. per sq. foot of grate per hour, what will be its evaporation per sq. foot of grate?

$$.016 \times 30^2 + 10.25 \times 15 = 168.15 \text{ lbs.}$$

2.—Assume a like boiler, using fresh water, to have a ratio of heating surface to grate of 30 and an evaporation of 165 lbs. water per sq. foot of grate per hour, what would be consumption of coal per sq. foot of grate per hour?

$$\frac{165 - .016 \times 30^2}{10.25} = 14.69 \text{ lbs.}$$

Tube Surface (Iron) per lb. of coal 1.58, per sq. foot of grate 32, and per 100 sq. feet.

Locomotive Boiler has from 60 to 90 sq. feet per foot of grate, and consumes 15 lbs. coal per sq. foot per hour.

Evaporative Capacity of Tubes of Varying Length

Total Length of Tubes 12 Feet 3 ins. (M. Paul Hever, 1874.)

| SURFACE AND WATER. | Furnace and 3 ins. in Length of Tubes. | TUBES. | | | |
|--|--|---------------|---------------|---------------|---------------|
| | | 3.02 Feet. | 3.02 Feet. | 3.02 Feet. | 3.02 Feet. |
| Surface in sq. feet..... | 76.43 | 179 | 179 | 179 | 179 |
| Water evaporated per sq. } foot per hour in lbs. } | 24.5 | 8.72 | 4.42 | 2.9 | 2.9 |

Results of Operation of Boilers under Varying Proportions of Grate, Area, and Length of Heating Surface, Draught of Furnace, and Rate of Combustion.

| DESCRIPTION. | Area of Grate. | Heating Surface. | Grate to Heating Surface. | Coal per Sq. Foot of Grate per Hour. | Evaporation of Water from 212° per sq. ft. of grate. | per lb. of Coal. | FUEL. |
|-----------------------|----------------|------------------|---------------------------|--------------------------------------|--|------------------|--------|
| Fire-tubular. | Sq. Feet. | Sq. Feet. | Ratio. | Lbs. | Lbs. | Lbs. | |
| Cultural and Hoisting | 4.7 | 158 | 34 | 13 | 119 | 9.33 | Welsh. |
| " " " | 3.2 | 220 | 69 | 12.8 | 151 | 11.83 | " |
| Motive..... | 26.25 | 963.5 | 36.7 | 30.86 | 327 | 10.6 | " |
| ash..... | 16 | 818 | 51 | 38 | 375 | 10.47 | " |
| " " " " " " " | 10.5 | 788 | 75 | 45 | 419 | 11.04 | " |
| " " " " " " " | 10.6 | 1056 | 100 | 157 | 1401 | 10.41 | " |
| 2e ¹ | 22 | 748 | 34 | 24.3 | 265 | 10.7 | " |
| 1..... | 18 | 749 | 41.6 | 23.6 | 264 | 11.2 | " |
| 2..... | 10.3 | 915 | 50 | 41.25 | 468 | 11.36 | " |
| 2*..... | 10.3 | 508 | 49.3 | 27.63 | 309.8 | 11.54 | Lanc'r |
| 3..... | 10.8 | 151.2 | 14 | 27.76 | 205 | 7.39 | Anth'o |
| Binary 4..... | 31.5 | 945 | 30 | 28.87 | 293.7 | 10.17 | Welsh. |
| " 2..... | 31.5 | 767 | 24.4 | 14 | 141.4 | 10.1 | " |

1 New Castle.

2 and 4 Wigan.

3 Experimented at New York.

Test of reducing the tube-surfaces was tried by stopping one half the number of tubes in alternate rows, so that the tube surface was reduced 206.5 sq. feet. The results with fires 12 ins. were as follows:

| | Tubes open. | Tubes half closed. |
|---|--------------|--------------------|
| Coal per sq. foot of grate per hour | 25 lbs. | 24 lbs. |
| Water from 212° per lb. of coal..... | 12.41 " | 12.23 " |
| Smoke per hour, very light..... | 2.8 minutes. | 8 minutes. |

Comparative Effects of Boilers for Different Rates of Combustion, and Surface Ratios. (D. K. Clark.)

Water from and at 212° per Hour.

Surface Ratio 30.

| per sq. foot rate sur. | STATIONARY. | | MARINE. | | PORTABLE. | | LOCOMOTIVE. | | Coke. | |
|---------------------------------|------------------|---------------------|------------------|---------------------|------------------|---------------------|------------------|---------------------|------------------|---------------------|
| | Water | | Water | | Water | | Coal. | | Water | |
| | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. |
| 1 | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 2 | 116 | 11.6 | 117 | 11.7 | 93 | 9.3 | 105 | 10.5 | 95 | 9.5 |
| 3 | 163 | 10.9 | 168 | 11.2 | 136 | 9 | 154 | 10.3 | 135 | 9 |
| 4 | 211 | 10.6 | 219 | 10.9 | 179 | 9 | 202 | 10.1 | 175 | 8.7 |
| 5 | 307 | 10.2 | 322 | 10.7 | 265 | 8.8 | 299 | 10 | 254 | 8.5 |

Surface Ratio 50.

| | | | | | | | | | | |
|---|-----|------|-------|------|-----|-----|-----|------|-----|------|
| 1 | 187 | 12.5 | 187.5 | 12.5 | 149 | 9.9 | 168 | 11.2 | 164 | 10.9 |
| 2 | 247 | 12.3 | 248 | 12.5 | 192 | 9.6 | 217 | 10.9 | 203 | 10.2 |
| 3 | 342 | 11.4 | 348 | 11.6 | 278 | 9.3 | 314 | 10.4 | 283 | 9.4 |
| 4 | 438 | 10.9 | 450 | 11.3 | 364 | 9.1 | 411 | 10.3 | 362 | 9 |
| 5 | 534 | 10.7 | 552 | 11 | 450 | 9 | 508 | 10.1 | 442 | 8.8 |

Surface Ratio 75.

| | Water. | | Fuel per Sq. Foot of Grate per Hour in Lbs. | | | | | | |
|----------------|---------------|------|---|------|------|------|------|------|------|
| | 30 | 40 | 50 | 60 | 75 | 90 | 100 | | |
| MOTIVE, coal.. | Per sq. foot. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| " " " " " " " | 342 | 439 | 536 | 633 | 778 | 927 | 1000 | | |
| " " " " " " " | 11.4 | 11 | 10.7 | 10.7 | 10.4 | 10.3 | 10.2 | | |
| " " " " " " " | 338 | 418 | 497 | 576 | 695 | 815 | 894 | | |
| " " " " " " " | 11.3 | 10.4 | 9.9 | 9.6 | 9.3 | 9 | 8.9 | | |

When a heater is used, and temperature of feed-water is raised above that of in a condensing engine, the proportions of surfaces may be correspondingly reduced.

ILLUSTRATION.—Assume $W = .16$, $v = 150$, $t = 1000^\circ$, and $a = 5$.

$$\frac{.16 \times 150 \times 1000}{5 \times 493.2^\circ} = \frac{24,000}{2466} = 9.73 \text{ feet.}$$

.084 to .087 = D. D representing weight of a cube foot of gas discharged by

meq, in lbs. ILLUSTRATION. $\frac{493.2^\circ}{1000} \times .086 = .0424 \text{ lb.}$

$\frac{2}{3} \left(1 + G + \frac{f l}{m} \right) = H$. G representing a coefficient of resistance and friction of
through grate and fuel,* f coefficient of friction of gas through flues and over
surfaces,† l length of flues and chimney, m hydraulic mean depth,‡ and H height
chimney, all in feet.

ILLUSTRATION I.—Assume $C = 9.73$, $l = 60$, and $m = .72$, all in feet.

$$\frac{9.73^2}{64.33} \left(1 + 12 + \frac{.012 \times 60}{.72} \right) = \frac{94.67}{64.33} \times 14 = 20.6 \text{ feet.} \quad \frac{C a v'}{v t} = W.$$

—Assume preceding elements. $\frac{9.73 \times 5 \times 493.2^\circ}{150 \times 1000} = .16 \text{ lb.}$

When H is given. $\sqrt{\left(H 2 g \div 1 + G + \frac{f l}{m} \right)} = C$

ILLUSTRATION.—Assume preceding elements. $\sqrt{20.6 \times 64.33 \div 14} = 9.73 \text{ feet.}$

22 x pressure in lbs. per sq. foot = head in ins. of water.

Temperature at base of smoke-pipe or chimney, or termination of flues or
is estimated at 500° ; and base of chimney, or its calorimeter, should
be an area of 1.3 to 1.6 sq. ins. for every lb. of coal consumed per hour.
If tubes of small diameter, compared to their length, this proportion may
be reduced to 1 and 1.2 ins.

Admission of air behind a bridge-wall increases temperature of the gases,
it must be at a point where their temperature is not below 800° .

Loss of Pressure by Flow of Air in Pipes.

Length 3280 Feet, or 1000 Meters.

| Velocity at Entrance of Pipe. | | Diameter of Pipe in Ins. | | | | | |
|--|-------------------|--------------------------|-------|-------|-------|-------|------|
| Feet per Second. | Meter per Second. | 4 | 6 | 8 | 10 | 12 | 14 |
| Loss of Pressure in Lbs. per Sq. Inch. | | | | | | | |
| 28 | 1 | .114 | .076 | .057 | .057 | .038 | .038 |
| 56 | 2 | .5 | .343 | .25 | .21 | .172 | .153 |
| 84 | 3 | 1.183 | .8 | .592 | .477 | .394 | .343 |
| 112 | 4 | 2.06 | 1.374 | 1.03 | .84 | .687 | .6 |
| 140 | 5 | 3.2 | 2.16 | 1.61 | 1.29 | 1.1 | .923 |
| 168 | 6 | 4.446 | 2.964 | 2.223 | 1.778 | 1.482 | 1.28 |

Mount Cenis Tunnel, the loss of pressure from 84 lbs. per sq. inch, in a pipe
5 ins. in diameter and 1 mile 15 yards in length, was but 3.5 per cent.

Artificial Draught.

production of draught in an ordinary marine boiler, from 20 to 33 per
cent of total heat of combustion of fuel is expended.

Fact.—By experiments of D. K. Clark and others it was deduced that the vacuum
back connection is about .7 of blast pressure, and in the furnace from .33 to .5
that in back connection; that rate of evaporation varies nearly as square root of
vacuum in back connection; that best proportions of chimney and passages thereto
those which enable a given draught to be produced with greatest diameter of
pipe; for the manifest reason, that the greater that diameter, the less the back-
pressure due to resistance of orifice, and that these proportions are best at all rates
expansion and speeds.

Which, in furnaces consuming from 20 to 24 lbs. coal per sq. foot of grate per hour, is assigned
at 22. † Estimated by same authority at .012
or a square of circular flue is .25 its diameter.

Thermal Efficiency of Fuel and Fuel consumed per Sq. Foot of Grate
 When the given $\frac{W}{F} = 1$ or efficiency of fuel or weight of water evaporated
 of fuel $\frac{W - 1}{1} = 1$

Thermal Efficiency of Fuel and Fuel consumed per Sq. Foot of Grate per Hour corresponding to a Given $\frac{W}{F}$

When Efficiency of Fuel and $\frac{W}{F}$ are given or $\frac{W}{F}$ and $\frac{W}{F}$ are given

$$\frac{W - 1}{1} = \frac{W}{F} \quad \frac{W}{F} = \frac{1}{1} = 1 \quad \text{and} \quad \frac{W}{F} = F$$

Illustration.—Taking elements as preceding

$$\frac{W}{F} = \frac{100 - 100}{100} = 100 \quad \frac{W}{F} = \frac{100 - 100}{100} = 100 \quad \text{and} \quad \frac{W}{F} = \frac{100 - 100}{100} = 100$$

Combustion of Coal per sq. foot of grate.—Normal Evaporation from 100 to 100 lbs. of water per hour.—Normal per sq. foot of grate

From Table of Experiments of Marine Boilers see Table of D. & C. page 101. It contains the data of thermal efficiency as applied to all surface and fire-tube boilers

Normal weight $W = 100$ lbs. and $F = 100$ lbs. $W = 100$ lbs. $F = 100$ lbs.

And the general formulae be deduced from all the various experiments of boilers

From end of 100

Portable..... $W = 100$ lbs. $F = 100$ lbs. Marine..... $W = 100$ lbs. $F = 100$ lbs.
 Stationary.... $W = 100$ lbs. $F = 100$ lbs. Locomotive, coal, $W = 100$ lbs. $F = 100$ lbs.

Locomotive coke..... $W = 100$ lbs. $F = 100$ lbs.

As the maximum evaporative power of fuel is a fixed quantity, the preceding formulae are not fully applicable in maximum rates of its consumption and evaporative quality

With coal and coke the limits of evaporative efficiency may be taken respectively at 12.5 and 12.5 lbs. water from and at 12.5

Illustration 1.—Assume a marine fire-tubular boiler with a surface ratio of 12.5 sq. feet to grate of 30 and a consumption of coal of 15 lbs. per sq. foot of grate per hour. What will be its evaporation per sq. foot of grate?

$$100 \times 30 + 10.25 \times 15 = 163.15 \text{ lbs.}$$

2.—Assume a like boiler, using fresh water, to have a ratio of heating surface to grate of 30 and an evaporation of 165 lbs. water per sq. foot of grate per hour. What would be consumption of coal per sq. foot of grate per hour?

$$\frac{165 - 10.25 \times 30}{10.25} = 14.69 \text{ lbs.}$$

Surface (Iron) per lb. of coal 1.58, per sq. foot of grate 32, and per HP

Boiler has from 60 to 90 sq. feet per foot of grate, and consumption of coal per hour.

Capacity of Tubes of Varying Length
 Length of Tubes 12 Feet 3 ins. (M. Paul Hever, 1874)

| Furnace and sq. in. Length of Tubes. | TUBES | | | |
|--|---------------|---------------|---------------|---------------|
| | 3.02 Feet. | 3.02 Feet. | 3.02 Feet. | 3.02 Feet. |
| 76.43 | 179 | 179 | 179 | 179 |
| 44.5 | 8.72 | 4.42 | 2.9 | 2.9 |

Its of Operation of Boilers under Varying Proportions of Grate, Area, and Length of Heating Surface, Height of Furnace, and Rate of Combustion.

| DESCRIPTION. | Area of Grate. | Heating Surface. | Grate to Heating Surface. | Coal per Sq. Foot of Grate per Hour. | Evaporation of Water from 212° per sq. ft. of grate. | per lb. of Coal. | FUEL. |
|------------------------|----------------|------------------|---------------------------|--------------------------------------|--|------------------|--------|
| Fire-tubular. | Sq. Feet. | Sq. Feet. | Ratio. | Lbs. | Lbs. | Lbs. | |
| tubular and Hoisting | 4.7 | 158 | 34 | 13 | 119 | 9.33 | Welsh. |
| " | 3.2 | 220 | 69 | 12.8 | 151 | 11.83 | " |
| ative..... | 26.25 | 963.5 | 36.7 | 30.86 | 327 | 10.6 | " |
| " | 16 | 818 | 51 | 38 | 375 | 10.47 | " |
| " | 10.5 | 788 | 75 | 45 | 419 | 11.04 | " |
| " | 10.6 | 1056 | 100 | 157 | 1401 | 10.41 | " |
| " | 22 | 748 | 34 | 24.3 | 265 | 10.7 | " |
| " | 18 | 749 | 41.6 | 23.6 | 264 | 11.2 | " |
| " | 10.3 | 915 | 50 | 41.25 | 468 | 11.36 | " |
| "* | 10.3 | 508 | 49.3 | 27.63 | 309.8 | 11.54 | Lanc'r |
| " | 10.8 | 151.2 | 14 | 27.76 | 205 | 7.39 | Anth'e |
| ary ⁴ | 31.5 | 945 | 30 | 28.87 | 293.7 | 10.17 | Welsh. |
| " ² | 31.5 | 767 | 24.4 | 14 | 141.4 | 10.1 | " |

¹ New Castle.

² and 4 Wigan.

³ Experimented at New York.

t of reducing the tube-surfaces was tried by stopping one half the number of tubes in alternate rows, so that the tube surface was reduced 206.5 sq. feet. The results with fires 12 ins. as follows:

Tubes open. Tubes half closed.

Coal per sq. foot of grate per hour 25 lbs. 24 lbs.

Water from 212° per lb. of coal..... 12.41 " 12.23 "

Smoke per hour, very light..... 2.8 minutes. 8 minutes.

Comparative Effects of Boilers for Different Rates of Combustion, and Surface Ratios. (D. K. Clark.)

Water from and at 212° per Hour.

Surface Ratio 30.

| STATIONARY. | | MARINE. | | PORTABLE. | | LOCOMOTIVE. | | Coke. | |
|---------------|------------------|---------------|------------------|---------------|------------------|---------------|------------------|---------------|------------------|
| Water | | Water | | Water | | Coal. | | Water | |
| per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. | per Sq. foot. | per lb. of Coal. |
| Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 116 | 11.6 | 117 | 11.7 | 93 | 9.3 | 105 | 10.5 | 95 | 9.5 |
| 163 | 10.9 | 168 | 11.2 | 136 | 9 | 154 | 10.3 | 135 | 9 |
| 211 | 10.6 | 219 | 10.9 | 179 | 9 | 202 | 10.1 | 175 | 8.7 |
| 307 | 10.2 | 322 | 10.7 | 265 | 8.8 | 299 | 10 | 254 | 8.5 |

Surface Ratio 50.

| | | | | | | | | | |
|-----|------|-------|------|-----|-----|-----|------|-----|------|
| 187 | 12.5 | 187.5 | 12.5 | 149 | 9.9 | 168 | 11.2 | 164 | 10.8 |
| 247 | 12.3 | 248 | 12.5 | 192 | 9.6 | 217 | 10.9 | 203 | 10.7 |
| 342 | 11.4 | 348 | 11.6 | 278 | 9.3 | 314 | 10.4 | 283 | 10.3 |
| 438 | 10.9 | 450 | 11.3 | 364 | 9.1 | 411 | 10.3 | 362 | 10.3 |
| 534 | 10.7 | 552 | 11 | 450 | 9 | 508 | 10.1 | 442 | 10.1 |

Surface Ratio 75.

| | Water. | Fuel per Sq. Foot of Grate per Hour in Lb | | | | | |
|------------------|---------------|---|----------|----------|----------|----------|----------|
| | | 30 | 40 | 50 | 60 | 75 | 90 |
| PORTABLE, coal.. | Per sq. foot. | Lbs. 342 | Lbs. 439 | Lbs. 536 | Lbs. 633 | Lbs. 778 | Lbs. 927 |
| " " " " | " lb. coal. | 11.4 | 11 | 10.7 | 10.7 | 10.4 | 10.3 |
| coke. | " sq. foot. | 338 | 418 | 497 | 576 | 695 | 815 |
| " " " " | " lb. coal. | 11.3 | 10.4 | 9.9 | 9.6 | 9.3 | 9 |

in a heater is used, and temperature of feed-water is raised above that in a condensing engine, the proportions of surfaces may be correspondingly altered.

Results of Operation of various Designs of Boiler, under varying Proportions of Grate, Calorimeter, Area and Length of Heating Surface, Draught, Firing, Rate of Combustion.

| STATIONARY. | Area of Grate. | Heat- ing Surface. | Grate to Heating Surface. | Circuit of Heating Surface. | Temperature of Chimney. | Fuel per Sq. Foot per Hour. | Water Expended from 212° per lb. of Coal. |
|--|-------------------------------|-----------------------------------|--|--|--|--|--|
| | Sq. Ft. | Sq. Ft. | Ratio. | Feet. | ° | Lbs. | Lbs. |
| Lancashire double internal and external flued¹..... | 20.5 | 612 | 29.8 | 79 | 511 | 15.35 | 8.32 |
| " " "²..... | 21 | 767 | 36.5 | 80 | 505 | 21.5 | 10.88 |
| Galloway vertical water-tubular²..... | 21 | 719 | 22.8 | 79 | 505 | 22.7 | 10.77 |
| " " "²..... | 31.5 | 719 | 34.3 | 80 | 630 | 18.3 | 10.17 |
| Fairbairn¹..... | 20.5 | 1017 | 49.5 | — | 387 | 15.27 | 8.67 |
| French¹..... | 20.1 | 607 | 30.3 | — | 510 | 16.42 | 8.12 |
| Cylindrical flued³.... | 14.2 | 377.5 | 26.8 | 56 | 292 | 7.43 | 9.08 |

MARINE.

At Pressure of Atmosphere.

| Horizontal fire-tub. | 2 | 10.3 | 508 | 49.3 | 27.5 | 11.92 |
|----------------------|---|------|------|------|-------|-------|
| " | " | 10.3 | 508 | 49.3 | 41.25 | 11.36 |
| " | " | 2 | 10.3 | 302 | 30 | 12.23 |
| " | " | | 19.3 | 749 | 39 | 10 |
| " | " | | 28.5 | 749 | 26.3 | 8.94 |
| " | " | | 28.5 | 749 | 26.3 | 11.13 |
| (C. Wye Williams) | | 15.5 | 749 | 48.3 | 600 | 10.63 |
| " | | 22 | 749 | 34 | 600 | 11.7 |
| " | | 42 | 749 | 17.6 | 16 | 9.65 |
| " | 3 | 10.8 | 150 | 13.9 | 8. | 8.95 |
| " | 3 | 4.32 | 147 | 34 | 8. | 7.24 |

¹ Trial in France. ² At Wigan, 1866-68, height of chimneys 100 feet. ³ yard, Washington, U. S., chimney 61 feet. ⁴ At pressure of atmosphere, fires deep, at 40 lbs. pressure, evaporation was reduced 12 per cent. ⁵ Bituminous. ⁶ Anthracite, at pressure of 6.5 lbs. above atmosphere. ⁷ Fires 14 ins. deep. ⁸ Mitted through furnace-doors. ⁸ Ditto do., jet blast. ⁹ Half tubes cool. ¹⁰ Air through grate only. ¹¹ Air through grate and door, no smoke. ¹² One ing in door, temp. 625°, with two 633°, with four 638°, and with six 600°. ¹³ grates, air spaces fully open, no smoke. ¹⁴ One furnace, anthracite coal, 5 ins.

Draught.

Draught of Furnace.—Volume of gas varies directly as its absolute temperature, and draught is best when absolute temperature of gas in chimney is to that of external air as 25 to 12.

$$\frac{T + 461.2^{\circ}}{+ 461.2^{\circ}} = \frac{V}{V'} = V'' \quad V, V', \text{ and } V'' \text{ representing absolute temperatures}$$

ex.—Assume temperature of furnace or $T = 1500^{\circ}$, and 12 lbs.

$2^0 = 2.08$, and as 150 cube feet is volume of gas per lb. of fuel
 $= 507$ cube feet.

weight of fuel consumed in furnace per second

lb. of fuel in cube feet, t absolute temperature
in, a area of chimney in sq. feet, and C velocity

ILLUSTRATION.—Assume $W = .16$, $v = 150$, $t = 1000^\circ$, and $a = 5$.

$$\frac{.16 \times 150 \times 1000}{5 \times 493.2^\circ} = \frac{24000}{2466} = 9.73 \text{ feet.}$$

34 to .087 = D. D representing weight of a cube foot of gas discharged by

y, in lbs. ILLUSTRATION. $\frac{493.2^\circ}{1000} \times .086 = .0424 \text{ lb.}$

$1 + G + \frac{f l}{m} = H$. G representing a coefficient of resistance and friction of
ough grate and fuel,* f coefficient of friction of gas through flues and over
urfaces,† l length of flues and chimney, m hydraulic mean depth,‡ and H height
ney, all in feet.

ILLUSTRATION 1.—Assume $G = 9.73$, $l = 60$, and $m = .72$, all in feet.

$$\frac{9.73^2}{64.33} \left(1 + 12 + \frac{.012 \times 60}{.72} \right) = \frac{94.67}{64.33} \times 14 = 20.6 \text{ feet.} \quad \frac{C a V'}{v t} = W.$$

Assume preceding elements. $\frac{9.73 \times 5 \times 493.2^\circ}{150 \times 1000} = .16 \text{ lb.}$

When H is given. $\sqrt{\left(H^2 g \div 1 + G + \frac{f l}{m} \right)} = C$

ILLUSTRATION.—Assume preceding elements. $\sqrt{20.6 \times 64.33 \div 14} = 9.73 \text{ feet.}$

× pressure in lbs. per sq. foot = head in ins. of water.

Temperature at base of smoke-pipe or chimney, or termination of flues or
is estimated at 500° ; and base of chimney, or its calorimeter, should
in area of 1.3 to 1.6 sq. ins. for every lb. of coal consumed per hour.
tubes of small diameter, compared to their length, this proportion may
used to 1 and 1.2 ins.

Passage of air behind a bridge-wall increases temperature of the gases,
must be at a point where their temperature is not below 800° .

Loss of Pressure by Flow of Air in Pipes.

Length 3280 Feet, or 1000 Meters.

| Diameter of Pipe in Ins. | Diameter of Pipe in Ins. | | | | | |
|--|--------------------------|-------|-------|-------|-------|------|
| | 4 | 6 | 8 | 10 | 12 | 14 |
| Loss of Pressure in Lbs. per Sq. Inch. | | | | | | |
| at Entrance of Pipe. | | | | | | |
| t ond. | Meter per Second. | | | | | |
| 8 | 1 | .114 | .076 | .057 | .057 | .038 |
| 6 | 2 | .5 | .343 | .25 | .21 | .153 |
| 4 | 3 | 1.183 | .8 | .592 | .477 | .343 |
| 2 | 4 | 2.06 | 1.374 | 1.03 | .84 | .6 |
| | 5 | 3.2 | 2.16 | 1.61 | 1.29 | .923 |
| 8 | 6 | 4.446 | 2.964 | 2.223 | 1.778 | 1.28 |

Mount Cenis Tunnel, the loss of pressure from 84 lbs. per sq. inch, in a pipe
ns. in diameter and 1 mile 15 yards in length, was but 3.5 per cent.













Artificial Draught.

Production of draught in an ordinary marine boiler, from 20 to 33 per
of total heat of combustion of fuel is expended.

2.—By experiments of D. K. Clark and others it was deduced that the vacuum
k connection is about .7 of blast pressure, and in the furnace from .33 to .5
in back connection; that rate of evaporation varies nearly as square root of
n in back connection; that best proportions of chimney and passages thereto
use which enable a given draught to be produced with least diameter of
pipe; for the manifest reason, that the greater that d
re due to resistance of orifice, and that these propo
ansion and speeds.

inch, in furnaces consuming from 20 to 24 lbs. coal per sq. foot of
† Estimate
a square of circular flue is .25 its diameter.

Proportions of Eggs of *Staph. Eridani*, etc.

| Drillpress | Drill | Drill | No. 4 |
|--------------------------------|---|---|---|
| | Drill | Drill | Drill |
| No. 1. 1/2" x 1/2" x 1/2" 1/2" |  |  |  |
| No. 2. 1/2" x 1/2" x 1/2" 1/2" |  |  |  |
| No. 3. 1/2" x 1/2" x 1/2" 1/2" |  |  |  |
| No. 4. 1/2" x 1/2" x 1/2" 1/2" |  |  |  |

When drilled from upset bar, dimensions same as for N.O.D. (See end of neck of rod)

Special Notes.—Iron are not to be subjected to a greater stress than 100,000 lbs. per sq. inch of section; Steel 150,000 lbs., both cases on the weakest part of rod, and when of steel they are not to be welded.

To Compute Diameter and Pitch of Stay-bolts, and Resistance they will Sustain.

Revised. $\frac{\partial \sqrt{V}}{\partial \sigma} = D_1$, $\frac{\partial \sqrt{V}}{\partial \mu} = D_2$ and $\left(\frac{\partial \sqrt{V}}{\partial \mu}\right)^2 = V$. Since $\frac{\partial \sqrt{V}}{\partial \mu} = \frac{1}{2}$,
 $\frac{\partial \sigma}{\partial \mu} = \bar{\sigma}$, and $\left(\frac{\partial \sigma}{\partial \mu}\right)^2 = V$. $\bar{\sigma}$ representing diameter in ins.

EXERCISE 69.—Assume pitch of stay bolts 6 ins. and working pressure 80 lbs. per sq. inch; what should be diameters of bolts, both screw and socket?

$$\frac{6 \times \sqrt{100}}{95} = 0.57 \text{ inch Screwed, and } \frac{6 \times \sqrt{100}}{95} = 0.51 \text{ inch Solder}$$

Pharmac. (Eloin²2)

length of gears, d its depth, t its thickness at centre or mean of its thickness, L distance apart from centre to centre, and p pitch of staves, will be $\frac{L}{C} \cos \alpha$, and C as usual as before.

Find stay for each girder, $C = 6000$. If two or three = 9000. If four = 10,000.
 11. Parameters = Assume triple stayed girder, 24 ins. in length, 3 ins. in depth,
 1 inch thick, and stayed at intervals of 6 ins.; what working pressure will it stand?

$$t = 9 \text{ sec. Then } \frac{9000 \times 6^3 \times 1}{(34 - 6) \times 6 \times 24} = \frac{324,000}{2502} = 125 \text{ Me.}$$

Flues, Arched or Circular Furnaces. U. S. Ld.

stress high for each 16 ins. of diameter. English iron, being harder than American, is better constructed to resist compression, and consequently less thickness of metal is required for like stress.

Elonch's.

$$\frac{80,600 \text{ lb}}{100} = R \quad \sqrt{\frac{P L}{80,600}} = t \quad \frac{80,600 \text{ lb}^2}{P L} = D_1 \text{ and } \frac{80,600 \text{ lb}^2}{P D} = L \quad D \text{ represents}$$

Actual size. Actual diameter of the 16 ins., length 5 feet, and weight 100 lbs. The 16 ins. dia. up to 100 lbs.

[Faint handwritten notes at the bottom of the page]

Velocities of Current of Heated Air in a Chimney 100 Feet in Height.

In Feet per Second.

| Air at Base of Chimney. | | | | | External Air. | | | | |
|-------------------------|-------|-------|-------|------|---------------|-------|-------|-------|-------|
| | 150° | 250° | 350° | 450° | | 150° | 250° | 350° | 450° |
| Feet. | Feet. | Feet. | Feet. | | | Feet. | Feet. | Feet. | Feet. |
| 24 | 30 | 33 | 35 | | 60° | 19 | 26 | 29 | 33 |
| 22 | 28 | 31 | 34 | | 70° | 18 | 25 | 29 | 32 |
| 20 | 27 | 30 | 33 | | 80° | 17 | 24 | 28 | 32 |

Height of Chimney is less than 100 feet.—Multiply velocity as observed temperature by .1 square root of height of chimney in feet.

It is consequent upon a steam-jet in a smoke-pipe or chimney is equal to that of a moderate blast.

Most effective draught is when absolute temperature of heated air or that of external air as 25 to 12, or nearly equal to temperature of lead.

Chimneys of gas retorts, ovens, and like furnaces, the draught is more or a like height of chimney than in ordinary furnaces, in consequence of the great mass of brick masonry, which, becoming heated, adds to the draught.

Chimneys. Lawrence Manufacturing Co., Mass. Octagonal.

above ground 211 feet. Diameters 15, and 10 feet 1.5 ins. Wall at base at top 11.5 ins. Shell at base 15 ins., at top 3.75 ins. Foundation 22 feet deep.

d.—Square.—Height.....190 feet. Diameter at base.....20 feet.

Round. "300 " " "29 "

Round. "312 " " "30 "

Round. "300 " " "20 "

at base usually 1 of height above ground.

at base of chimney ranges from .375 to .43 ins. of water.

Circulating Pumps.

single-acting. — .6 volume of single-acting air-pump and .32 of double-

double-acting. — .53 volume of double-acting air-pump.

Volume of Pump compared to Steam Cylinder or Cylinders.

| Engine. | Pump. | Volume. |
|------------------------------------|---------------------|---------------|
| single-acting, 1.5 to 5 times..... | Single-acting | .08 to .045. |
| double-acting, 1.5 to 5 times..... | do. | .045 to .035. |
| single-acting, 1.5 to 5 times..... | Double-acting..... | .045 to .025. |
| double-acting, 1.5 to 5 times..... | do. | .025 to .02. |

1.—Area such as to restrict the mean velocity of the flow to 450 feet etc.

PLATES AND BOLTS.

light-iron.—Tensile strength ranges from 45 500 to 70 000 lbs. inch for plates, and 60 000 to 65 000 lbs. for bolts, being increased subjected to a moderate temperature.

th plates range from 45 000 to 56 000 lbs., and 45 000 to 56 000 lbs.

dark gives best quality of Yorkshire 56 000 lbs., of

of Plates. (U. S.)—All plates to be stamped 17 ins. from edge, and also in or near to their cent.

his location, and tensile stress they will bear. subjected to a tensile stress under 45 000 lbs. per sq. section 12 per cent, 45 000 and under 50 000, 15, at rupture.

ILLUSTRATION.—Assume $T = 50,000$ lbs. tensile strength of plate, $B = 75$ per cent. $D = 120$ ins., $C = 5$, and $t = .5$. What pressure will shell sustain, and what should be thickness of plates for such pressure and diameter?

$$\frac{50,000 \times .75 \times .5 \times 2}{120 \times 5} = 62.5 \text{ lbs., and } \frac{120 \times 62.5 \times 5}{50,000 \times .75 \times 2} = .5 \text{ inch.}$$

For all practicable deficiencies in drilling, punching, and riveting in transverse courses, if existing, this coefficient is increased up to 6.75, and in longitudinal courses to 8.75, and when courses are not properly broken, in addition is made to above of .4.

Diameter of rivets should not be less than thickness of plates.

Molesworth.

$\frac{P d}{2 t} = C$, $\frac{2 t C}{d} = P$, and $\frac{P d}{2 C} = t$. d representing diameter and t thickness of metal, both in ins., P working pressure in lbs. per sq. inch, and C as follows:

| | | Single riveted. | Double riveted. |
|----------------------------|--------------------------------|-----------------|------------------------|
| Best Yorkshire plates..... | one ninth of tensile strength. | $C = 6200$ | and $\frac{7}{8}$ inch |
| Staffordshire plates.... | | $C = 5000$ | " $\frac{7}{8}$ inch |
| Ordinary plates..... | | $C = 3000$ | " $\frac{3}{4}$ inch |

Working stress not to exceed .2 tensile strength of joint or riveted plate. Then for a pressure of 110 lbs., and a diameter of 42 ins., as given for a standard U. S. boiler.

Taking C as above for best single-riveted plate at 6200, $\frac{110 \times 42}{2 \times 6200} = .3717$ in thickness, or .122 inch in excess of U. S. Law for a plain cylindrical boiler, riveted.

Lloyd's.

Thickness of shells to be computed from strength of longitudinal joint.

$\frac{t J C}{D} = P$, $\frac{P D}{C J} = t$, $\frac{t J C}{P} = D$, $\frac{p - d}{p} = z$, and $\frac{n a}{p t} = z$. t representing thickness of plate, D diameter of shell, p pitch and d diameter of rivets, all in ins.; z per cent. of strength of joint or rivets, the least to be taken; C a constant as per table; P working pressure in lbs. per sq. inch; n number and a area of rivet; z per cent. of strength of plate at joint compared with solid plate, and z per cent. of strength of rivets compared with solid plate.

When plates are drilled, take .9 of z , and when rivets are in double strap put 1.75 a for a .

Constants.

| JOINT. | IRON PLATES. | | | STEEL PLATES. | | |
|----------------------------|--------------------|---------------------|-----------------|----------------------|-----------------------|---------------------|
| | .5 inch and under. | .75 inch and under. | Above .75 inch. | .375 inch and under. | .5625 inch and under. | .75 inch and under. |
| Lap { punched holes..... | 155 | 165 | 170 | 200 | 215 | 230 |
| do. { drilled do..... | 170 | 180 | 190 | — | — | — |
| Double but { punched holes | 170 | 180 | 190 | 215 | 230 | 250 |
| strap { drilled do. | 180 | 190 | 200 | — | — | — |

When plates, as in steam-chimneys, superheaters, etc., are exposed to direct action of the flame, these constants are to be reduced .33.

ILLUSTRATIONS.—Assume pitch 4 ins., diam. of rivet 1.375 ins., and thickness of plate 1 inch, both single and double riveted. Area 1.375 = 1.43 sq. ins.

$$\frac{4 - 1.375}{4} = .656 \text{ per cent. strength of joint compared to solid plate. } \frac{1 \times 1.43}{4 \times 1} = .3575$$

per cent. strength of rivet to solid plate when single riveted, and $\frac{1.75 \times 1.43}{4 \times 1} = .3106$

per cent. when double riveted. Rivets at Joint. $\frac{n a}{p t} \times 100$ with punched holes by 90 with drilled.

Plates.

Compute Thickness of Plates for a Given Pressure Pitch, and Pressure and Pitch for Given Thick-

$\sqrt{\frac{t^2 C}{P}} = p$, and $\sqrt{\frac{P p^2}{C}} = t$. *t* representing thickness of metal in inches, *p* pitch of stays or distance apart at centres in ins., *P* working pressure in lbs. per sq. inch, and *C* a constant, as follows:

for a Tensile Strength of Metal of 50 000 Lbs. per Sq. Inch.

Stay-bolts with Riveted Heads.—Plates up to .4375 inch in thickness *C* = 90, that 100.

Stay-bolts with Nuts.—Plates up to .4375 inch in thickness *C* = 110, and 120.

Stay-bolts with Double Nuts and Washers.—Up to .4375 ins. in thickness and above that 160.

Stay-bolts are not exposed to corrosion, these constants may be reduced .2.

Force of a flat surface decreases in a higher ratio than space between fence, *C* must be decreased in proportion to increase of pitch above ordinary boiler-plates.

EXAMPLE 1.—Assume pressure 110 lbs. per sq. inch, and pitch of stays 5 ins.; find thickness of plate for screw-bolts and riveted heads?

$$C = 95. \text{ Then } \sqrt{\frac{110 \times 5^2}{95}} = \sqrt{\frac{2750}{95}} = 5.38 \text{—sixteenth.}$$

Assume thickness of metal 5 sixteenths inch thick, stay-bolts screwed and cover its threads, and working pressure of steam 80 lbs. per sq. inch.

$$C = 95. \text{ Then } \sqrt{\frac{5^2 \times 95}{80}} = 5.45 \text{ ins. pitch.}$$

Abut Straps.

Abuts should be at least .625 thickness of plate covered. *Single*, thicker than plate covered, and *Double*, .625.

Stays.

—Tensile stress should not exceed 5000 lbs. per sq. inch for Iron, and for Steel.

Normal or Oblique.—Ascertain area of direct stay required to sustain pressure; multiply it by length of diagonal stay, and divide product by a line drawn at a right angle to surface stayed, to end of diagonal; quotient will give area of stay increased to that which is required.

Area upon an oblique stay is also equal to stress which a perpendicular supporting a like surface would sustain, divided by sine of angle formed with perpendicular to surface to be stayed.

EXAMPLE.—Assume pressure 110 lbs. per sq. inch, and angle of stay 45°; what would be pressure?

$$\text{Cosine } 45^\circ = .70711. \text{ Then } 110 \times 36 \div .70711$$

$$\text{Efficiency of boiler. } 1.833 \left(\frac{1600}{1600 \times 2 + 800} \right) = .733$$

The evaporative power of different fuels, from and at 212° F. is for each lb. to 16.8 lbs., the average of Newcastle being 15.3, for patent fuels 15.65, L. 17.1, Coke 13.3, Peat 10.3, and Woods, when dry, 8.1. See *A. E. Seaton, London*.

Notes on Horse-power.

A Lancashire boiler with a heating surface of 610 sq. feet and a grate-area will evaporate in ordinary operation 50 cube feet of water per hour; 3.12 sq. horizontal section per cube foot of water, and .5 sq. foot of grate-area per cube foot of water.

Nominal. Flue Boilers.—Usually computed at 5.5 to 6 sq. feet of horizontal section, 15 sq. feet of heating surface, and 1 sq. foot of grate-area.

The IHP of such boilers will range from 3 to 4 times that of the nominal.

Multitubular Boilers.—75 sq. foot of grate-area and 2.5 of heating surface.

Weights of Steam-engines.

Side-wheels.—American Marine (Condensing).

| ENGINE. | Frame. | Water-wheels. | Cylinders. | | Weight per | |
|--------------------|--------|---------------|------------|------------|------------|-----|
| | | | No. | Volume. | Cube Foot. | Sq. |
| | | | | Cube Feet. | Lbs. | |
| Vertical beam..... | Wood.* | Wood. | 1 | 63 | 1100 | 15 |
| "..... | Wood.* | Wood. | 2 | 216 | 1040† | 30 |
| "..... | Wood.* | Wood. | 1 | 430 | 1225 | 30 |
| "..... | Wood.* | Wood. | 2 | 253 | 1480† | 30 |
| "..... | Wood.* | Iron. | 1 | 725 | 1089† | 30 |
| Oscillating..... | Iron. | Iron. | 2 | 540 | 850 | 30 |
| "..... | Iron. | Iron. | 2 | 1502 | 550† | 30 |
| Inclined..... | Iron. | Iron. | 2 | 535 | 1100 | 30 |

* Without frame.

† With frame 1109.

‡ Including boilers.

§ Single list

Screw Propellers.—American Marine (Condensing).

| ENGINE. | No. | Cylinders. | | Weights. | | Per C. Ft. Cylinder. |
|----------------------------------|-----|------------|-----------|----------|----------------------|----------------------|
| | | Volume. | Engine. | Boilers. | Per C. Ft. Cylinder. | |
| | | Cube Feet. | Lbs. | Lbs. | Lbs. | |
| Vertical direct, Jet Condens'g.. | 1 | 4 | 22 040 | 12 100 | 8 535 | |
| " " Surface Cond'g. | 1 | 12.5 | 59 000 | 32 000 | 7 280 | |
| " " Jet " " | 1 | 12.5 | 48 130 | 35 000 | 6 650 | |
| " " " " " | 1 | 33 | 120 450 | 98 000 | 6 620 | |
| " " " " " | 4 | 506 | 1 523 060 | 985 600 | 4 938 | |
| Horizontal back-action..... | 2 | 68 | 289 680 | 200 800 | 7 212 | |
| " direct..... | 2 | 67 | 201 000 | 200 593 | 6 009 | |
| Vertical compound..... | 2 | 4.8 | 24 705 | 26 372 | 10 641 | |
| " " " " " Surface Condens'g. | 2 | 24.3 | 94 196 | 88 050 | 7 500 | |
| " " " " " " " | 2 | 425 | 1 022 400 | 840 000 | 4 380 | |
| " " direct..... | 1 | 3.6 | 30 534 | 27 301 | 10 066 | |
| " " " " " " " | 1 | 35 | 172 028 | 100 065 | 7 774 | |
| " " Non-Condensing. | 1 | 1.86 | 14 410 | 22 481 | 19 534 | |
| " " " " " " " | 1 | 2.77 | 14 759 | 22 417 | 13 421 | |

English Marine (Condensing).

| DESCRIPTION. | Cylinders. | | Weights. | | | | |
|----------------------|------------|----------|----------|-------------------------|--------------------|--------|---------|
| | No. | Volume. | Engines. | Propeller and Shafting. | Boilers and Water. | Total. | Per IHP |
| | | Cube Ft. | Tons. | Tons. | Tons. | Tons. | Lbs. |
| Vertical direct..... | 2 | 230 | 121 | 47 | 257 | 425 | 465 |
| " " " " " " " | 2 | 382 | 223 | 85 | 303 | 611 | 336 |
| " " " " " " " | 2 | 393 | 165 | 48 | 144 | 357 | 781 |
| " " " " " " " | 2 | 440 | 117 | 43 | 135 | 295 | 560 |
| " " " " " " " | 2 | 24 | 4.25 | .75 | 7.25 | 12.25 | 60 |
| " " " " " " " | 6 | 707 | 497 | 167 | 656 | 1320 | 358 |
| " " " " " " " | 2 | 52 | 55 | 15 | 110 | 180 | 351 |
| " " " " " " " | 2 | 143 | 130 | 87 | 162 | 319 | 791 |

Land-engines.—(Non-condensing.)

| ENGINE. | Volume of Cyl'r. | Engine. | Spar-wheel and Connections. | Sugar-Mill Complete. | Boilers, Grates, etc. | Engine per Cube Foot of Cylinder. |
|---------------------|------------------|---------|-----------------------------|----------------------|-----------------------|-----------------------------------|
| | | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| 1) 18 ins. X 4 feet | 7 | 67 200 | 37 800 | 80 600 | 26 880 | 9600 |
| 1) 30 ins. X 5 feet | 24.5 | 105 000 | 137 179 | 265 879 | 75 000 | 4290 |
| 2) 14 ins. X 2 feet | 2.2 | 10 914 | — | — | 8 200 | 5100 |
| 2) 22 ins. X 4 feet | 10.6 | 56 000 | — | — | 30 140 | 5600 |

compute Weight of a Vertical Beam and Side-wheel Jet Condensing Engine. (T. F. Rowland, A.S.C.E.)

ing all Metals, Boiler and Attachments, Smoke-pipe, Grates, Iron Floors, and Iron in Wooden Water-wheels, omitting Coal-bunkers.

For a Pressure per Mercurial Gauge of 40 lbs. per Sq. Inch.

For surface condenser add 10 to 15 per cent.

E.—Multiply volume of cylinder in cube feet by Coefficient in following corresponding to length of stroke, and product will give rough in lbs. For finished weight deduct 6 per cent.

| Coefficient. | Stroke. | Coefficient. | Stroke. | Coefficient. | Stroke. | Coefficient. |
|--------------|---------|--------------|---------|--------------|---------|--------------|
| | Feet. | | Feet. | | Feet. | |
| 2467 | 7 | 2213 | 9 | 1865 | 11 | 1619 |
| 2340 | 8 | 2000 | 10 | 1730 | 12 | 1546 |

PLATE I.—What are the rough and finished weights of a vertical beam engine, 80 ins. in diameter and 12 feet stroke of piston?

of 80 ins. = 5026.56, which X 12 feet = 419 cube feet, and X 1546 for 12 feet = 647 774 lbs. rough weight.

647 774 X .06 = 38 866, and 647 774 — 38 866 = 608 908 lbs. finished weight.

WEIGHTS OF BOILERS.

ts of Iron Boilers (including Doors and Plates, and exclusive of Smoke-pipes and Grates) per Sq. Foot of Heating Surface.

are Measured from Grates to Base of Smoke-pipe or Top of Steam Chimney.

| BOILER. For a Working Pressure of 40 Lbs. | Weight. |
|--|--------------|
| | Lbs. |
| return, Flue*.....water bottom.... | 25.6 to 32.9 |
| " " Multi-flue*.....water bottom.... | 24 to 30 |
| " " ".....water bottom.... | 27 to 45 |
| " " ".....water bottom.... | 25 to 43 |
| total return, Tubular†.....water bottom.... | 22.5 to 35 |
| " " ".....water bottom.... | 21 to 33 |
| " " ".....water bottom.... | 17.7 to 20.7 |
| total direct, Tubular*.....water bottom.... | 18.5 to 26.5 |
| " " ".....water bottom.... | 19.8 to 23.8 |
| " " ".....water bottom.... | 17 to 21 |
| ical, external furnace,† 36 ins. in diam., .25 inch thick..... | 23.5 to 24 |
| Flue "† 36 to 42 " .25 "..... | 18.1 to 18.6 |
| total direct, Tubular.....Locomotive..... | 16.3 to 17.3 |
| total Cylinder direct, Tubular..... | 24 to 25 |

ht of Cylindrical Furnace and Shell Boilers, all complete for Sea Service and pressure of 60 lbs. steam, 200 lbs. per I.P.

ton of furnace square. Shell cylindrical. † Section of furnace and shell square. eight-iron heads, .375 inch thick, flues, .25 inch, and surface computed to half diameter of shell.

ts.—1. The range in the units of weight arises from peculiarities of construction subsequent upon proportionate number of flues.
"shell compared with heating surface,†

pressure is increased the above units

Boiler-power.

The power of a boiler is the volume or weight of steam alone (independent of any water that it may hold in suspension) that it will generate at a given operating pressure in a unit of time.

Marine boilers of the ordinary type and proportions, with natural draught, burning anthracite coal, produce 3.5 to 5.5 HP per sq. foot of grate per hour; with free burning or a semi-bituminous coal, 5 to 7.5 HP; and with a forced draught with 25 to 30 lbs. best coal per sq. foot of grate per hour, 8 to 10 HP.

Marine engines, operating with a steam-pressure of 35 lbs. (m. g.), and with moderate expansion, consume 30 lbs. steam per HP per hour, and with a high rate of expansion, under a pressure of 70 lbs., 20 lbs. steam.

With a blast draught and consuming 30 to 40 lbs. of a fair quality of coal per sq. foot of grate per hour, 7 to 10 HP per hour can be attained.

In locomotive boilers, having from 50 to 90 sq. feet of heating surface per sq. foot of grate, and at a rate of combustion of from 45 to 125 lbs. of coke, an average evaporation of 9 lbs. of water per lb. of coke has been attained at ordinary temperatures and pressure.

To Compute Volume of Air and Gas in a Furnace.

When Volume at a Given Temperature is known. RULE.—Multiply given volume by its absolute temperature, and divide product by the given absolute temperature.

NOTE.—Absolute temperature is obtained by adding 461° to given or required temperature.

EXAMPLE.—Assume volume of air entering a furnace at 1 cube foot, its temperature 60° , and temperature of furnace 1623° ; what would be the increase of volume?

$$\frac{1 \times 1623^{\circ} + 461^{\circ}}{60^{\circ} + 461^{\circ}} = \frac{2084}{521} = 4 \text{ times.}$$

Volume of Furnace Gas per Lb. of Coal. (Rankine.)

| Temperature. | Air Supplied. | | | Temperature. | Air Supplied. | | |
|--------------|---------------|---------|---------|---------------|---------------|---------|---------|
| | 12 Lbs. | 18 Lbs. | 24 Lbs. | | 12 Lbs. | 18 Lbs. | 24 Lbs. |
| 32° | 150 | 225 | 300 | 752° | 369 | 553 | 737 |
| 68 | 161 | 241 | 322 | 1112 | 479 | 718 | 957 |
| 104 | 172 | 258 | 344 | 1472 | 588 | 882 | 1176 |
| 212 | 205 | 307 | 409 | 1832 | 697 | 1045 | 1393 |
| 572 | 314 | 471 | 628 | 2500 | 906 | 1357 | 1810 |

Temperature of ordinary boiler furnaces ranges from 1500° to 2500° .

The opening of a furnace door to clean the fire involves a loss of from 4 to 15 per cent. of fuel.

For other illustrations, see *ante*, page 744-6.

Rate of Combustion.

The rate of combustion in a furnace is computed by the lbs. of fuel consumed per sq. foot of grate per hour.

In general practice the rate for a natural draught is, for anthracite coal, from 16 lbs., for bituminous, from 10 to 25 lbs., and with artificial or forced draught, with a blower, exhaust-blast, or steam-jet, the rate may be increased from 30 to 125 lbs.

The dimensions or size of coal must be reduced and the depth of the fire increased directly, as the intensity of the draught is increased.

Temperature of gases at base of chimney or pipe should be 600° , and resistance of surface of chimney is as square of velocity of current of gas.

Ordinarily from 20 to 32 per cent. of total heat of combustion is expended in the production of the chimney draught in a marine boiler, to which is to be added losses by incomplete combustion of the gaseous portion of the fuel, and of the gases by an excess of air, making a total of fully 60 per cent.

Wm. H. Shock, U. S. N., 1881.)

STRENGTH OF MATERIALS.

Length of a material is measured by its resistance to alteration of form when subjected to stress and to rupture, which is designated as Strength. Detrusive, Tensile, Torsion, and Transverse, although transverse is a combination of tensile and crushing, and detrusive is a form of rupture at short lengths of application.

ELASTICITY AND STRENGTH.

Length of a material is resistance which a body opposes to a permanent separation of its parts, and is measured by its resistance to rupture of form, or to stress.

Adhesion is force with which component parts of a rigid body adhere to each other.

Elasticity is resistance which a body opposes to a change of form.

Elasticity and Strength, according to manner in which a force is exerted on a body, are distinguished as *Crushing Strength*, or Resistance to Compression; *Detrusive Strength*, or Resistance to Shearing; *Tensile Strength*, or Resistance to Tension; *Torsional Strength*, or Resistance to Torsion; and *Transverse Strength*, or Resistance to Flexure.

Limit of Stiffness is flexure, and limit of Resistance is fracture.

Neutral Axis, or Line of Equilibrium, is the line at which extension terminates and compression begins.

Stiffness, or toughness of bodies, is strength and flexibility combined; any material or body which bears greatest load, and bends most at fracture, is toughest.

Stiffest bar or beam that can be cut out of a cylinder is that of which the breadth is square root of 3 to 1; *strongest*, as square root of 2 to 1 most resilient, that which has breadth and depth equal.

Stiffness expresses condition of a material when it is loaded, or extended in length to its elastic limit.

General law regarding deflection is, that it increases, *ceteris paribus*, directly as cube of length of beam, bar, etc., and inversely as breadth and cube of depth.

Resistance of Flexure of a body at its cross-section is very nearly .9 of its resistance to rupture.

Coefficient of Elasticity.

Elasticity of any material subjected to a tensile or compressive force, within its limits, is measured by a fraction of the length, per unit of force at a sectional area, termed a *constant*, and coefficient of elasticity is defined as the weight which would stretch a perfectly elastic bar of unit section to double its length.

Force and area is usually taken at one lb. per sq. inch. *E* represents the denominator of fraction.

EXAMPLE.—If a bar of iron is extended one 12 000 000th part of its length per lb. per sq. inch of section,

$$\frac{1}{12\,000\,000} = \frac{1}{E}.$$

It would, therefore, be stretched 12 000 000 lbs. per sq. inch, if the material

by a force of

The same method of expressing coefficient of elasticity is applied to resistance to compression. That is, coefficient, in weight, is expressed by the nominator of fraction of its length by which a bar is compressed per unit weight per sq. inch of section.

Ultimate extension of cast iron is 500th part of its length.

Extension of Cast-iron Bars, when suspended Vertically
1 Inch Square and 10 Feet in Length. Weight applied at one End.

| Weight. | Extension. | Set. | Weight. | Extension. | Set. | Weight. | Extension. | Set. |
|---------|------------|---------|---------|------------|---------|---------|------------|-------|
| Lbs. | Ins. | Ins. | Lbs. | Ins. | Ins. | Lbs. | Ins. | Ins. |
| 529 | .0044 | — | 2117 | .0190 | .000059 | 8468 | .0871 | .0035 |
| 1058 | .0092 | .000015 | 4234 | .0397 | .00265 | 14820 | .1829 | .0035 |

Woods.—MM. Chevaudier and Wertheim deduced that there was a limit of elasticity in woods, there being a permanent set for every extension. They, however, adopted a set of .00005 of length as limit of elasticity. This is empirical.

MODULUS OF ELASTICITY.

Modulus or Coefficient of Elasticity of any material is measure of its elastic reaction or force, and is height of a column of the material pressing on its base, which is to the weight causing a certain degree of compression as length of material is to the diminution of its length.

It is computed by this analogy: As extension or diminution of length of any given material is to its length in inches, so is the force that produced that extension or diminution to the modulus of its elasticity.

Or, $x : P :: l : w$; $w = \frac{Pl}{x}$. x representing length of a substance 1 inch square and 1 foot in length would be extended or diminished by force P , and w weight of modulus in lbs.

To Compute Weight of Modulus of Elasticity.

RULE.—As extension or compression of length of any material in inches, is to its length, so is the weight that produced that extension or compression, to modulus of elasticity in lbs.

EXAMPLE.—If a bar of cast iron, 1 inch square and 10 feet in length, is extended .008 inch, with a weight of 1000 lbs., what is the weight of its modulus of elasticity?
 $.008 : 120 (10 \times 12) :: 1000 : 15000000 \text{ lbs.}$

To Compute Modulus of Elasticity.

When a Bar or Beam is Supported at Both Ends and Loaded in Center.
RULE.—Multiply weight or stress per sq. inch in lbs. by length of material in ins., and divide product by modulus of weight.

$$\text{Or, } \frac{lW}{M} = E; \quad \frac{lW}{E} = M; \quad \frac{EM}{l} = W. \quad l \text{ representing length in ins., } M \text{ modulus}$$

W weight in lbs. per sq. inch, and E compression or extension.

EXAMPLE 1.—If a wrought-iron rod, 60 feet in length and .2 inch in diameter, subjected to a stress of 150 lbs., what will it be extended?

Modulus of elasticity of iron wire is 28 230 500 lbs. (see following table), and modulus = .314 16.

$$= 477.46 \text{ lbs. per sq. inch, and } 60 \times 12 = 720 \text{ ins.}$$

$$= \frac{343771.2}{.31416} = .01218 \text{ inch.}$$

— under rule for weight of modulus

$$\frac{.008 \times 15000000}{120} = 1000 \text{ lbs.}$$

Modulus of Elasticity and Weight of Various Materials.

| SUBSTANCES. | Height. | Weight. | SUBSTANCES. | Height. | Weight. |
|---------------------|-----------|------------|-----------------------|-----------|------------|
| | Feet. | Lbs. | | Feet. | Lbs. |
| Aluminum..... | 4 970 000 | 1 656 570 | Larch..... | 4 415 000 | 1 074 000 |
| Brass..... | 4 600 000 | 1 345 000 | Lead, cast..... | 146 000 | 720 000 |
| Copper, yellow..... | 2 460 000 | 8 464 000 | Lignum-vitæ..... | 1 850 000 | 1 080 400 |
| Wire..... | 4 112 000 | 14 632 720 | Limestone..... | 2 400 000 | 3 300 000 |
| Per, cast..... | 4 800 000 | 18 240 000 | Mahogany..... | 6 570 000 | 2 071 000 |
| Steel..... | 5 680 000 | 1 499 500 | Marble, white..... | 2 150 000 | 2 508 000 |
| Red..... | 8 330 000 | 2 016 000 | Oak..... | 4 750 000 | 1 710 000 |
| SS..... | 4 440 000 | 5 550 000 | Pine, pitch..... | 8 700 000 | 2 430 000 |
| Metal..... | 2 790 000 | 8 844 300 | " white..... | 8 970 000 | 1 830 000 |
| Open fibres..... | 5 000 000 | 170 000 | Steel, cast..... | 8 530 000 | 26 650 000 |
| Cast..... | 6 000 000 | 2 370 000 | " wire..... | 9 000 000 | 28 680 000 |
| Wrought..... | 5 750 000 | 17 068 500 | Stone, Portland... .. | 1 672 000 | 1 718 800 |
| Wire..... | 8 377 000 | 25 820 000 | Tin, cast..... | 1 053 000 | 3 510 000 |
| | | 28 230 500 | Zinc..... | 4 480 000 | 13 440 000 |

Weight a Material will bear per Sq. Inch, without Permanent Alteration of its Length.

| MATERIAL. | Lbs. | MATERIAL. | Lbs. | MATERIAL. | Lbs. |
|-----------------|--------|---------------------|------|---------------|------|
| <i>Metals.</i> | | <i>Stones, etc.</i> | | <i>Woods.</i> | |
| SS..... | 6 700 | Marble..... | 4900 | Beech..... | 2360 |
| Cast metal..... | 10 000 | Limestone*..... | 2000 | Elm..... | 3240 |
| Cast..... | 15 000 | Portland..... | 1500 | Fir, red..... | 4290 |
| Wrought..... | 17 800 | | | Larch..... | 2060 |
| D..... | 1 500 | <i>Woods.</i> | | Mahogany..... | 3000 |
| Al..... | 45 000 | Ash..... | 3540 | Oak..... | 3960 |

* Tensile strength 2800.

Comparative Resilience of Woods.

| | | | | | | | |
|--------|-----|---------------|-----|-----------------|-----|-------------------|-----|
| | 1 | Chestnut..... | .73 | Larch..... | .84 | Spruce..... | .64 |
| h..... | .86 | Elm..... | .54 | Oak..... | .63 | Teak..... | .59 |
| r..... | .66 | Fir..... | .4 | Pitch Pine..... | .57 | Yellow Pine... .. | .64 |

MODULUS OF COHESION.

Compute Length of a Prism of a Material which would be Severed by its own Weight when Suspended.

RULE.—Divide tensile resistance of material per sq. inch by weight of a foot in length, and quotient will give length in feet.

ILLUSTRATION.—Assume tensile resistance of a wrought-iron rod to be 60 000 lbs. Weight of 1 foot = 3.4 lbs.

Then $60\,000 \div 3.4 = 17\,647.06$ feet.

Length in Feet required to Tear Asunder the following Substances:

Aluminum..... 15 375 feet. | Hemp twine... 75 000 feet. | Catgut..... 25 000 feet.

Elasticity of Ivory as compared with Glass is as .95 to 1.

When Height is given. **RULE.**—Multiply weight of 1 foot in length and height of material by height of its modulus in feet, and product will give weight.

To Compute Height of Modulus of Elasticity.

RULE.—Divide weight of modulus of elasticity of material by weight of it, and quotient will give height in feet.

SAMPLE.—Take elements of preceding case (page 762), w = 15 000 000; what is height of its modulus of elasticity?

$15\,000\,000 \div 3 = 5\,000\,000$ feet.

CRUSHING STRENGTH.

Crushing Strength of any body is in proportion to its area and inversely as its height.

In tapered columns, it is determined by the least diameter.

When height of a column is not 5 times its side or diameter, strength is at its maximum.

Cast Iron.—Experiments upon bars give a mean crushing strength of 100 000 lbs. per sq. inch of section, and 5000 lbs. per sq. in. to overcome elasticity of metal; and when height exceeds the iron yields by flexure. When it is 10 times, it is reduced when it is 15 times, as 1 to 2; when it is 20 times, as 1 to 3; when it is 30 times, as 1 to 4; and when it is 40 times, as 1 to 6.

Experiments of Mr. Hodgkinson have determined that the strength of about one eighth of destructive weight is obtained at the diameter of a column in its middle.

In columns of same thickness, strength is inversely as the square power of length nearly.

A hollow column, having a greater diameter at one end than the other, has not any additional strength over that of an uniform column.

Wrought Iron.—Experiments give a mean crushing strength of 27 000 lbs. per sq. inch, and it will yield to any extent with 27 000 lbs. while cast iron will bear 80 000 lbs. to produce same effect.

Effects.—A wrought bar will bear a compression of $\frac{1}{8}$ of its length out its utility being destroyed.

With cast iron, a pressure beyond 27 000 lbs. per sq. inch will destroy it.

ing Strength of various Materials, deduced from
periments of Maj. Wade, Hodgkinson, Capt. Meigs,
S. A., Stevens Institute, and by G. L. Vose.

Reduced to a Uniform Measure of One Sq. Inch.

CAST IRON.

| FIGURES AND MATERIAL. | Crushing Weight. | FIGURES AND MATERIAL. | Crushing Weight. |
|-------------------------|------------------|-----------------------------------|------------------|
| | Lbs. | | Lbs. |
| etal, American..... | 174 803 | Clyde, average, English..... | 82 000 |
| " { | 85 000 | Stirling, mean of all, English .. | 122 395 |
| mean..... | 125 000 | " extreme, English..... | 134 400 |
| or, No. 1, English..... | 100 000 | Extrem, English..... | 53 760 |
| No. 2, " { | 62 450 | Average (Hodgkinson), English | 153 200 |
| No. 3, " { | 92 330 | Blaenavon No. 2..... | 84 240 |
| | 106 039 | | 109 700 |

WROUGHT IRON.

| | | | |
|--------------------|---------|---------------------|--------|
| an, extreme..... { | 127 720 | English..... { | 65 200 |
| mean..... { | 83 500 | " average..... { | 40 000 |
| | 47 040 | | 37 850 |

VARIOUS METALS.

| | | | |
|------------------------|---------|----------------------|---------|
| ium bronze, 95 cop.... | 129 920 | Steel, Bessemer..... | 50 000 |
| ass..... | 164 800 | " soft..... | 66 200 |
| pper..... | 117 000 | " tempered..... | 335 000 |
| ast..... { | 105 000 | " Siemens..... | |
| " { | 250 000 | Tin, cast..... | 15 500 |
| agersta | 154 500 | Lead..... | 7 730 |

tic Crushing Strength of Wrought Iron and Crucible Steel is equal to its ten-
Bessemer Steel, 50 per cent. of its transverse strength.

WOODS.

| | | | |
|------------------|--------|----------------------------|--------|
| | 6 663 | Maple..... { | 8 100 |
| | 6 963 | Oak, American white..... | 10 000 |
| { | 3 300 | " Canadian white..... | 7 500 |
| | 7 900 | " live..... | 5 982 |
| | 10 513 | " English..... { | 6 850 |
| red..... | 5 968 | " Dantzic, dry..... | 9 500 |
| seasoned..... | 6 500 | " Pine, pitch..... | 6 484 |
| ut..... | 5 350 | " white..... | 7 700 |
| | 6 831 | " yellow..... | 8 947 |
| soned..... | 10 000 | " Deal, Christiana..... | 5 775 |
| glish..... | 10 300 | Spruce, white..... | 8 200 |
| y, white..... | 8 925 | Teak..... | 5 850 |
| { | 3 200 | Walnut..... | 5 950 |
| | 5 500 | Willow, seasoned..... | 12 100 |
| | 9 113 | | 6 645 |
| ny, Spanish..... | 8 198 | | 6 000 |

Crosswise of Fibre.

..... 2300 | Larch..... 1300 | Pines..... 550

se in Strength of Cubes of Sandstone, per Sq. Inch (under Blocks
Wood), as Area of Surface is increased. (Gen'l Gillmore, U. S. A.)

| STONE. | INCHES. | | | | | | | |
|--------------------|--------------|--------------|--------------|--------------|--------------|--------------|---------------|---------------|
| | .5 | 1 | 1.5 | 2 | 2.25 | 2.75 | 3 | 4 |
| Berea sandstone .. | Lbs. 6080 | Lbs. 6990 | Lbs. 8226 | Lbs. 8955 | Lbs. 9130 | Lbs. 9838 | Lbs. 10125 | Lbs. 11120 |
| " " " | — | 9500 | 10730 | 12000 | 12500 | 13200 | — | — |

Stones, Cements, etc. (Per Sq. Inch.)

| FIGURES AND MATERIAL. | | Crushing Weight. | FIGURES AND MATERIAL. | |
|----------------------------------|--|------------------|------------------------------|--|
| | | Lbs. | | |
| Basalt, Scotch..... | | 8 300 | Granite, Patapsco, Md..... | |
| " Welsh..... | | 16 800 | " Portland, Eng..... | |
| Beton, N. Y. S. Concreting Co. { | | 800 | " Quincy, Mass..... | |
| Brick, pressed..... | | 1 400 | Greenstone, Irish..... | |
| " Gloucester, Mass..... | | 6 222 | Limestone..... | |
| " hard burned..... | | 10 219 | " compact, Eng..... | |
| " common..... | | 14 216* | " Magnesian, " | |
| " yellow-faced burned, Eng. | | 3 630 | " Anglesea " | |
| " Stourbridge fire-clay, " | | 800 | " Irish " | |
| " Staffordshire blue, " | | 4 000 | Marble, Baltimore, Md..... | |
| " stock, English..... | | 1 440 | " East Chester, N. Y.† | |
| " Fareham, English..... | | 1 650 | " Hastings, N. Y..... | |
| " red, English..... | | 7 200 | " Irish..... | |
| " Sydney, N. S..... | | 2 250 | " Italian..... | |
| Cuen, France..... | | 5 000 | " " white..... | |
| Cement, Hydraulic, pure, Eng. { | | 17 000 | " Lee, Mass..... | |
| " Portland, sand 1..... | | 32 000 | " Montgomery Co., Pa | |
| " " sand 3..... | | 1 280 | " Statuary..... | |
| " " 3 mos..... | | 600 | " Stockbridge, Mass.† | |
| " " 1 sand, 3 mos..... | | 3 800 | " Symington, large... | |
| " " 9 mos..... | | 2 464 | " " fine crystal... | |
| " " 1 sand, 9 mos..... | | 5 980 | " " strata horizon | |
| " " 12 inch cubes, } | | 2 330 | Masonry, brick, common... | |
| 12 mos. } | | | " " in cement.. | |
| " 1 sand and gravel } | | 2 650 | Mortar, good..... | |
| 3 " " " } | | 1 800 | " lime and sand..... | |
| " Roman..... | | 342 | " " beaten | |
| " " pure, Eng..... | | 750 | " common..... | |
| " Rosendale..... | | 3 270 | Oolite, Portland..... | |
| " Sheppey, Eng..... | | 1 280 | Pottery-pipe, Chelsea..... | |
| Concrete, lime 1, gravel 3.... | | 460 | Sandstone, Aquia Creek §.... | |
| Freestone, Belleville, N. J..... | | 775 | " Arbroath, Eng..... | |
| " Connecticut..... | | 3 522 | " Connecticut..... | |
| " Dorchester, Mass.... | | 3 319 | " Craigleth, Eng.... | |
| " Little Falls, N. Y.... | | 3 069 | " Derby grit " | |
| Glass, crown..... | | 2 991 | " Holyh'd quartz, Et | |
| Gneiss..... | | 31 000 | " Seneca §..... | |
| Granite, Aberdeen, Eng..... | | 19 600 | " Yorkshire, Eng.... | |
| " Cornish, " | | 10 760 | Slate, Irish..... | |
| " Dublin, " | | 6 339 | Terra Cotta..... | |
| " Newry, " | | 10 450 | Whinstone, Scotch..... | |
| | | 12 850 | | |

* Tested by author at Stevens' Institute, N. J. † Post-office, Wash. ‡ City Hall, § Capitol, Treasury Department, and Patent Office, Washington, D. C. § Smithsonian

Safe Load of Hollow, Cylindrical, and Solid Columns, Arches, Chords, etc., of Cast Iron.

Hollow Columns. Per Sq. Inch. (F. W. Shields, M. I. C. E.)

| Length. | Thick- ness. | Load. | Length. | Thick- ness. | Load. | Length. | Thick- ness. | Load. | Length. | Thick- ness. | Load. |
|---------|-----------------|-------|----------|-----------------|-------|----------|-----------------|-------|----------|-----------------|-------|
| Inch. | Inch. | Lbs. | Inch. | Inch. | Lbs. | Inch. | Inch. | Lbs. | Inch. | Inch. | Lbs. |
| 375 | .375 | 2800 | 20 to 24 | .625 | 3920 | 25 to 30 | .375 | 2240 | 25 to 30 | .375 | 2240 |
| | | 3360 | diam's. | .75 | 4480 | diam's. | .5 | 2800 | diam's. | | |

Columns, etc.—3360 lbs. per sq. inch. (Brund.)

—2800 lbs. per sq. inch.

and Posts.—1 inch diameter and not more than 15 diameters in breaking weight of metal.

meter and not more than 25 diameters in length .5 of breaking weight when more than 25 diameters in length from .1 to .025 of breaking L. (Baltimore Bridge Co.)

Cast-iron Cylinders and Rectangular Tubes.

| | External Diameter. | Internal Diameter. | Thickness. | Area. | Crushing Weight per Sq. Inch. |
|----------|--------------------|--------------------|------------|----------|-------------------------------|
| S. | Ins. | Ins. | Ins. | Sq. Ins. | Lbs. |
| | 1.495 | 1.292 | .1 | .444 | 14 661 |
| | 2.49 | 2.275 | .107 | .804 | 29 779 |
| | 6.366 | 6.106 | .13 | 2.547 | 35 886 |
| TUBES. | | | | | |
| | 4.1 | 4.1 | .03 | .504 | 10 980 |
| | 4.1 | 4.1 | .03 | .504 | 11 514 |
| | 4.1 | 4.1 | .06 | 1.02 | 19 261 |
| | 4.25 | 4.25 | .134 | 2.395 | 21 585 |
| | 4.25 | 4.25 | .134 | 2.395 | 23 202 |
| | 8.4 | 4.25 | .126 | 6.89 | 29 981 |
| | 8.1 | 8.1 | .06 | 2.07 | 13 276 |
| | 8.1 | 8.1 | .06 | 2.07 | 13 300 |
| Internal | 8.1 | 8.1 | .0637 | 3.551 | 19 732 |
| phrag's | 8.1 | 8.1 | .0637 | 3.551 | 23 208 |

per Sq. Inch of 2-Inch Cubes under Blocks of Wood. (Gen'l Gillmore, U. S. A.)

Surfaces Worked to a Clear Bed.

| | Lbs. | LIMESTONE. | Lbs. |
|---------------------|--------|------------------------------|--------|
| blue | 22 250 | Bardstown, Ky., dark | 16 250 |
| | 15 000 | Cooper Co., Mo., dark drab | 6 650 |
| | 17 750 | Erie Co., N. Y., blue | 12 250 |
| | 14 750 | Caen, France | 3 650 |
| Co., N. Y. | 18 250 | MARBLE. | |
| ut, Conn. | 16 187 | East Chester, N. Y. | 13 504 |
| Conn. | 12 500 | Italian, common | 13 062 |
| a. | 21 250 | Dorset, Vt. | 7 612 |
| ' gray | 14 100 | Mill Creek, Ill., drab | 9 687 |
| ass. | 12 423 | North Bay, Wis., drab | 20 025 |
| | 19 500 | SANDSTONE. | |
| I., gray | 14 937 | Little Falls, N. Y., brown | 9 850 |
| ass., gray | 15 937 | Bellefonte, N. J., gray | 11 700 |
| udson River, gray | 13 370 | Middletown, Conn., brown | 6 950 |
| .. dark | 17 750 | Haverstraw, N. Y., red | 4 350 |
| .. bluish gray | 12 875 | Medina, N. Y., pink | 17 725 |
| al Park, N. Y., red | 17 500 | Berea, O., drab | 7 250 |
| N. J., soap | 20 750 | | 10 250 |
| " gray | 24 040 | Vermillion, O., drab | 8 850 |
| | | Fond du Lac, Wis., purple | 6 250 |
| N. Y. | 11 475 | Marquette, Mich., " | 150 |
| lain, N. Y. | 25 000 | Seneca, O., red brown | |
| N. Y. | 20 700 | Cleveland, O., olive green | |
| " | 13 900 | Albion, N. Y., brown | |
| " | 18 500 | Kasota, Minn., pink | |
| O., white | 12 600 | Fontenac, Minn., light bui | |
| hite | 16 900 | Craigleth, Edinburgh | |
| Mich., drab | 18 000 | Dorchester, N. B., freestone | |
| | 25 000 | Massillon, O., yellow drab | |
| ; Wis., bluish drab | 21 500 | Warrensburg, Mo., bluish | |

STRENGTH OF MATERIALS.—CRUSHING.

Safe Crushing Weight of Columns.

B. Gordon from Results of Experiments of various Authors.

Cast Iron. (Hodgkinson.)

Round Solid or Hollow. $\frac{36 a}{1 + \frac{r^2}{400}} = W.$ For rectangular put 500.

Rectangular Solid or Hollow. $\frac{36 a}{1 + \frac{r^2}{500}} = W.$ For L, T, U, etc., put $\frac{19 a}{1 + \frac{r^2}{900}}.$

Wrought Iron. (Stoney.)

Round Solid. $\frac{16 a}{1 + \frac{r^2}{2400}} = W.$ *Rectangular Solid.* $\frac{16 a}{1 + \frac{r^2}{3000}} = W.$

Steel. (Er.)

Round Solid.—Strong steel, $\frac{51 a}{1 + \frac{r^2}{900}} = W.$ mild steel, $\frac{30 a}{1 + \frac{r^2}{1400}} = W.$

Rectangular Solid.—Strong steel, $\frac{51 a}{1 + \frac{r^2}{1600}} = W;$ mild steel, $\frac{30 a}{1 + \frac{r^2}{2480}} = W.$

a representing area of metal in sq. ins., *r* ratio of length to least external diameter or side, and *W* crushing weight in tons.

ILLUSTRATION.—What is the crushing weight of a hollow cylindrical column of cast iron 10 ins. in diameter, 20 feet in length, and 1 inch in thickness?

a = area of 10 ins. — area of 10 — 1 × 2 = 28.28 ins. $r = \frac{20 \times 12}{10} = 24$, and $\frac{1}{1 + \frac{24^2}{400}} = 576.$ Then, $\frac{36 \times 28.28}{1 + \frac{576}{400}} = \frac{1018.08}{1 + 1.44} = 417.25 \text{ tons} = 934640 \text{ lbs.}$

NOTE.—This is for hard English iron.

Weight borne with Safety by Solid Cast-iron Columns. In 1000 Lbs.—(New Jersey Steel and Iron Co.)

| Length. Feet. | DIAMETER. | | | | | | | | | | | | | | |
|------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------|------------|------------|------------|------------|------------|--|
| | 2 Ins. | 3 Ins. | 4 Ins. | 5 Ins. | 6 Ins. | 7 Ins. | 8 Ins. | 9 Ins. | 10 Ins. | 11 Ins. | 12 Ins. | 13 Ins. | 14 Ins. | 15 Ins. | |
| 5 | 12.4 | 44 | 102 | 184 | 288 | 414 | 560 | 728 | 916 | 1126 | 1354 | — | — | — | |
| 6 | 9.4 | 36 | 88 | 164 | 264 | 386 | 532 | 698 | 884 | 1082 | 1320 | 1570 | — | — | |
| 7 | 7.2 | 30 | 76 | 146 | 242 | 360 | 502 | 660 | 850 | 1056 | 1282 | 1530 | 1798 | 2086 | |
| 8 | — | 24 | 66 | 130 | 218 | 332 | 470 | 630 | 812 | 1016 | 1240 | 1486 | 1754 | 2040 | |
| 9 | — | 20 | 56 | 114 | 198 | 306 | 440 | 596 | 774 | 974 | 1196 | 1440 | 1706 | 1990 | |
| 10 | — | 18 | 48 | 102 | 180 | 282 | 410 | 560 | 739 | 932 | 1152 | 1392 | 1656 | 1940 | |
| 12 | — | — | 38 | 80 | 136 | 238 | 354 | 494 | 658 | 846 | 1056 | 1292 | 1550 | 1836 | |
| 14 | — | — | — | 28 | 64 | 122 | 200 | 304 | 432 | 586 | 774 | 966 | 1192 | 1440 | |
| 16 | — | — | — | — | 52 | 100 | 170 | 262 | 378 | 520 | 686 | 878 | 1094 | 1332 | |
| 18 | — | — | — | — | 44 | 84 | 144 | 226 | 332 | 462 | 616 | 796 | 1000 | 1228 | |
| 20 | — | — | — | — | — | 72 | 124 | 196 | 292 | 410 | 552 | 720 | 912 | 1130 | |

For Tubes or Hollow Columns.

Subtract weight that may be borne by a column of diameter of internal diameter of tube from external diameter, and remainder will give weight that may be borne. Thickness of metal should not be less than one twelfth diameter of column.

ILLUSTRATION.—Required the safe load of a solid cast-iron column 6 and 20 feet in length.

or 6 and in a line with 20 is 72, which × 1000 = 72 000 lbs.

—This is about one sixth of destructive weight.

Safe Loads as determined by Preceding Formulas.

st Iron, one fifth to one sixth. *Wrought Iron*, one fourth to one fifth.
Is, one seventh to one tenth.

WOODS.

To Compute Destructive Weight of Column.

inder. $\frac{d^4 \cdot 6}{l^2} C = W.$ *Rectangle.* $\frac{s^4}{l^2} C = W.$ *Short Columns, or less*

30 diameters in length. $\frac{W' \bar{a} S}{W' + .75 \bar{a} S} = W.$ *d representing diameter and s*
n ins., a area of section in sq. ins., l length in feet, S crushing strength of ma-
C coefficient of material, and W' destructive weight, as ascertained by compu-
for a long column of like dimensions in lbs.

Coefficients.

| | | |
|----------------------|--------------------------|-------------------------|
| 22 000 | Elm, rock..... 26 000 | Red Pine..... 17 500 |
| Canadian..... 17 000 | Fir, Dantzic..... 22 000 | Yellow pine..... 12 000 |
| 17 500 | Oak, white..... 20 000 | White "..... 9 000 |
| 14 000 | " Eng. 23 000 | Spruce..... 14 000 |
| 17 500 | Pitch Pine..... 20 000 | Walnut..... 12 500 |

ESTIMATION.—What is destructive weight of a column of yellow pine 10 ins.
 8 and 12 feet in length or height?

$$\frac{10^4}{12^2} \times 12\,000 = \frac{10\,000}{144} \times 12\,000 = 833\,333 \text{ lbs.}$$

long square columns of the following: Hodgkinson put $C =$ Dantzic oak, dry,
 ; red deal, dry, 17 472; and French oak, dry, 15 456.

To Compute Safe Weight in Tons.*

angular Oak Columns. Secured at Both Ends. RULE.—Divide length
 column by thickness or least dimension, multiply unit in column C, of
 wing table, corresponding to this quotient, by width of column, all di-
 mensions in inches; and product will give weight.

| C | $\frac{L}{T}$ | C | $\frac{L}{T}$ | C | $\frac{L}{T}$ | C | $\frac{L}{T}$ | C | $\frac{L}{T}$ | C |
|-----|---------------|-----|---------------|-----|---------------|-----|---------------|------|---------------|------|
| .43 | 7 | .36 | 13 | .26 | 19 | .18 | 25 | .12 | 31 | .093 |
| .43 | 8 | .35 | 14 | .24 | 20 | .17 | 26 | .12 | 32 | .089 |
| .42 | 9 | .33 | 15 | .23 | 21 | .17 | 27 | .11 | 33 | .084 |
| .4 | 10 | .31 | 16 | .21 | 22 | .15 | 28 | .1 | 34 | .08 |
| .39 | 11 | .29 | 17 | .2 | 23 | .14 | 29 | .098 | 35 | .077 |
| .38 | 12 | .27 | 18 | .19 | 24 | .13 | 30 | .097 | 36 | .073 |

ESTIMATION.—Assume a white-oak column, secured at both ends, 12 by 8 ins.,
 5 feet in length.

$\frac{12}{8} = 1.5$. C for which = .097. Hence, $12 \times 8 \times .097 = 9.312 \text{ tons.}$

other woods take the values in following table. Thus, if an oak column, as
 will sustain 9.312 tons, the strength of one of yellow pine is thus obtained:
 3 : 9.312 :: 3 : 4.816 tons.

Relative Value of various Woods, their Crushing
Strength and Stiffness being Combined.

| | | | |
|----------------|-------------------|-------------------|-------------------|
| 9.4 | Elm..... 5 | Mahogany..... 3.7 | Yellow pine... 3 |
| sh oak ... 5.8 | Beech..... 4.4 | Spruce..... 3.6 | Sycamore..... 2.6 |
| 5.1 | Quebec oak... 4.1 | Walnut..... 3.4 | Cedar..... 1 |

Comparative Value of Long Solid Columns of various
Materials. (Hodgkinson.)

Iron.... 1000 | Cast Steel.... 2518 | Oak..... 108.8 | Pine..... 78.5

ice, To compute destructive weight of an Oak or Pine column, take weight
 of Cast iron of like dimensions, and if for Oak divide by 9, and for Pine by

* All tons, except when otherwise designated, are 2240 lbs.

DEFLECTION.

Deflection of Bars, Beams, Girders, etc.

Experiments of Barlow upon deflection of wood battens determined, that deflection of a beam from a transverse strain, varied directly as cube of length and inversely as breadth and cube of depth, and that with like beams and within limits of elasticity it was directly as the weight.

In bars, beams, etc., of an elastic material, and having great length compared to their depth, deductions of Barlow will apply with sufficient accuracy for all practical purposes; but in consequence of varied proportions of depth to length, of varied character of materials, of irregular resistance of beams constructed with scarphs, trusses, or riveted plates, and of unequal deflection at initial and ultimate strains, it is impracticable to deduce any exact laws regarding degrees of deflection of different and dissimilar figures and proportions.

From an experiment of Mr. Tredgold it was shown that deflection of cast iron is exactly proportionate to load until stress reaches a certain magnitude, when it becomes irregular.

In experiments of Hodgkinson, it was further shown that sets from deflections were very nearly as squares of deflections.

In a rectangular bar, beam, etc., position of neutral axis is in its centre, and it is not sensibly altered by variations in amount of strain applied. In bars, beams, etc., of cast and wrought iron, position of neutral axis varies in same beam, and is only fixed while elasticity of beam is perfect. When a bar, beam, etc., is bent so as to injure its elasticity, neutral line changes, and continues to change during loading of beam, until its elasticity is destroyed.

When bars, beams, etc., are of same length, deflection of one, weight being suspended from one end, compared with that of a beam *Uniformly Loaded*, is as 8 to 3; and when bars, etc., are supported at both ends, deflection in like case is as 5 to 8. Whence, if a bar, etc., is in first case supported in middle, and ends permitted to deflect, and in second, ends supported, and middle permitted to descend, deflection in the two cases is as 3 to 5.

Of three equal and similar bars or beams, one inclined upward, one downward, at same angle, and the other horizontal, that which has its angle upward is weakest, the one which declines is strongest, and the one horizontal is a mean between the two.

When a bar, beam, etc., is *Uniformly Loaded*, deflection is as weight, and approximately as cube of length or as square of length; and element of deflection and strain upon beam, weight being the same, will be but half of that when weight is suspended from one end.

Deflection of a bar, beam, etc., *Fixed at one End, and Loaded at other*, compared to that of a beam of twice length, *Supported at both Ends, and Loaded in Middle*, strain being same, is as 2 to 1; and when length and loads are same, deflection will be as 16 to 1, for strain will be four times greater on beam fixed at one end than on one supported at both ends; therefore, all other things being same, element of deflection will be four times greater; also, as deflection is as element of deflection into square of length, then, as lengths at which weights are borne in their cases are as 1 to 2, deflection is as $1:2^2 \times 4 = 1$ to 16.

Deflection of a bar, beam, etc., having section of a triangle, and supported at ends, is .33 greater when edge of angle is up than when it is down.

To counteract deflection of a beam, etc., under stress of its load, horizontal surface is required, it should be *cambered* on its upper surface equal to computed deflection.

Safe Deflection.—One fortieth of an inch for each foot of span, with a factor of safety for load of .33 of destructive weight = $\frac{1}{1440}$, but for ordinary loads and purposes,

Cast Iron, $\frac{1}{1200}$ to $\frac{1}{3000}$; and **Wrought Iron,** $\frac{1}{1800}$ to $\frac{1}{2400}$ or $\frac{1}{1200}$, after beam, etc., has become set.

When Length is uniform, with same weight, deflection is inversely as breadth and square of depth into element of deflection, which is inversely as depth. Hence, other things being equal, deflection will vary inversely as breadth and cube of depth.

ILLUSTRATION.—Deflections of two pine battens, of uniform breadth and depth, and equally loaded, but of lengths of 3 and 6 feet, were as 1 to 7.8.

Deflection of different bars, beams, etc., arising from their own weight, having their several dimensions proportional, will be as square of either of their like dimensions.

NOTE.—In construction of models on a scale intended to be executed in full dimensions, this result should be kept in view.

When a continuous girder, uniformly loaded, is supported at three points by two equal spans, middle portion is deflected downwards over middle bearing, and it sustains by suspension the extreme portions, which also have a bearing on outer bearings. Middle portion is, by deflection, convex upwards, and outer portions are concave upwards; and there is a point of "contrary flexure," where curvature is reversed, being at junction of convex and concave curves, at each side of middle bearing. This point is distant from middle bearing, on each side, one fourth of span. Of remaining three fourths of each span, a half is borne by suspension by middle portion, and a half is supported by abutment. Hence, distribution of load on bearings is easily computed, as given above. Deflection of each span is to that of an independent beam of same length of span as 2 to 5.

In a beam of three equal spans, deflection at middle of either of side spans is to that of an independent beam as 13 to 25.

In a long continuous beam, supported at regular intervals, deflection of each span is to that of an independent beam of one span as 1 to 5.

Cylinder.—If a bar or beam is cylindrical, deflection is 1.7 times that of a square beam, other things being equal.

Formulas for Deflection of Beams of Rectangular Section, etc.

| | | | |
|-------------------------------|---|---------------------|---|
| Fixed at One End. | { | Loaded at One End. | $\frac{l^3 W}{b d^3 C} = D$; and $\frac{b d^3 C D}{l^3} = W$. |
| | | " Uniformly. | $\frac{3 l^3 W}{8 b d^3 C} = D$; and $\frac{8 b d^3 C D}{3 l^3} = W$. |
| Fixed at Both Ends. | { | Loaded in Middle. | $\frac{l^3 W}{24 b d^3 C} = D$; and $\frac{24 b d^3 C D}{l^3} = W$. |
| | | " Uniformly. | $\frac{5 l^3 W}{8 \times 24 b d^3 C} = D$; and $\frac{8 \times 24 b d^3 C D}{5 l^3} = W$. |
| Supported at Both Ends. | { | Loaded in Middle. | $\frac{l^3 W}{16 b d^3 C} = D$; and $\frac{16 b d^3 C D}{l^3} = W$. |
| | | " Uniformly. | $\frac{5 l^3 W}{8 \times 16 b d^3 C} = D$; and $\frac{8 \times 16 b d^3 C D}{5 l^3} = W$. |
| | | " at any one Point. | $\frac{m^2 n^2 W}{l b d^3 C} = D$; and $\frac{l b d^3 C D}{m^2 n^2} = W$. |

Supported in Middle.

$$\text{Ends Uniformly loaded. } \frac{3 l^3 W}{5 \times 16 b d^3 C} = D; \text{ and } \frac{5 \times 16 b d^3 C}{3 l^3} = W.$$

l representing length in feet, *b* breadth, and *d* depth, both in ins., *W* in lbs., *m* and *n* distances of weight between supports, *C* a constant, *D* in ins. — *etcres*

STRENGTH OF MATERIALS.—

Deflection of Beams of Rectangular Section.

Supported at Both Ends.

CAST IRON.

Rectangular Beams. *Loaded in the Middle*, $\frac{l^3 W}{36000 b d^3} = D$.

Round Beams. For 36000 put 24000.

EXAMPLE.—Assume a rectangular bar of cast iron, 1 inch square and 12 feet between its supports.

$$\text{Then } \frac{224 \times 4.5^3}{36000 \times 1 \times 1^3} = \frac{20412}{36000} = .567 \text{ ins.}$$

Actual experiment of Mr. Hodgkinson the deflection was .561 ins.

WROUGHT IRON.

Rectangular Beams. *Loaded in the Middle*, $\frac{l^3 W}{60000 b d^3} = D$.

Round Beams. For 60000 put 42000.

WOODS.

Mean of Laslett's, Barlow, etc. (D. K. Clark.)

Supported at Both Ends. Loaded in Middle.

$\frac{l^3 W}{5 d^3 C} = D$. *l representing length in ins. and W weight in lbs.*

| | C | | C | | C |
|--------------------|------|-------------------------|------|------------------|------|
| ash, Canadian..... | 1476 | Iron-wood..... | 4228 | Oak, French..... | 2100 |
| " Eng..... | 2722 | Larch..... | 2100 | " white..... | 2100 |
| Beech..... | 2418 | Mahogany, Honduras..... | 2118 | Pitch pine..... | 2100 |
| Blue Gum..... | 2559 | " Mexican..... | 3608 | Red "..... | 2100 |
| elm..... | 1227 | " Spanish..... | 3360 | Rock-elm..... | 2100 |
| Flr, Dantzic..... | 2490 | Norway spar..... | 2465 | Spruce..... | 2100 |
| " Memel..... | 3630 | Oak, Baltimore..... | 2761 | " Amer..... | 2100 |
| " Riga..... | 2920 | " Canadian..... | 3445 | " Scotch..... | 2100 |
| Greenheart..... | 1888 | " Dantzic..... | 2080 | Teak..... | 2100 |
| Iron Bark..... | 4378 | " Eng..... | 1848 | Yellow pine..... | 2100 |

Application of Table: To Compute Deflection of a Rectangular Beam of W

ILLUSTRATION.—What is the deflection of a floor beam of yellow pine, 3 by 12 feet between its supports, under a uniformly distributed load of 3000 lbs.

$$C = 2084. \quad \frac{5 \times 12^3 \times 3000}{8 \times 3 \times 12^3 \times 2084} = \frac{15000}{50016} = .299 \text{ inch.}$$

Hence, To compute weight that may be borne by a given deflection of such a

$$\frac{8 \times 3 \times 12^3 \times 2084 \times .299}{5 \times 12^3} = \frac{14955}{5} = 2991 \text{ lbs.}$$

Deflection of Continuous Girders or Beams of Uniform Dimensions, Supported at Three or More Bearings.

(D. K. Clark.)

| | |
|---|--|
| 1. Two Equal Spans or 3 Bearings. | 2. Three Equal Spans or 4 Bearings. |
| Weight on 1st and 3d bearing = .375 W l | Weight on 1st and 4th bearing = .375 W l |
| " " 2d bearing = 1.25 W l | " " 2d " 3d " = .375 W l |

3. Four Equal Spans or 5 Bearings.

Weight on 2d and 4th bearing = .39 W l | Weight on 2d and 4th bearing = .39 W l
Weight on 3d bearing = .93 W l

Cylindrical Beam. $\frac{l^3 W}{d^4 C} = D$; and $\frac{d^4 C D}{l^3} = W$.

Compute Maximum Load that may be borne by a Rectangular Beam.

Deflection not to exceed Assigned Limit of one hundred and twentieth of an Inch for each Foot of Span.

Supported at Both Ends. Loaded in Middle.

= W . b and d representing breadth and depth in ins., l length in feet, C constant W weight or load in lbs.

Constants.

| | | | | | |
|----------------|-------|------------------|------|----------------------|------|
| Iron..... | .0003 | Oak, white..... | .027 | Oak, red..... | .039 |
| Cast Iron..... | .0021 | Ash, white..... | .03 | Hemlock..... | .039 |
| Steel..... | .018 | Pine, pitch..... | .033 | Pine, white..... | .039 |
| | .024 | " yellow..... | .036 | Chestnut, horse..... | .051 |





DEFLECTION.—What is maximum load that may be borne by a beam of white pine by 12 ins., 20 feet between its supports, and loaded in its middle?

$$C = .039. \quad \text{Then } \frac{3 \times 12^3}{20^2 \times .039} = \frac{5184}{15.6} = 332.3 \text{ lbs.}$$

WROUGHT IRON.

Deflection of Wrought-iron Bars.

Supported at Both Ends. Weight applied in Middle.

| Form. | Length of Beam- ing. | Breadth. | Depth. | Weight and Deflection | | | | | | | | Constant at Reduced Weight and Deflection. $\frac{W}{l^3} = C$. |
|--|-------------------------|----------|--------|---------------------------|------|---|------|--|------|------|--|---|
| | | | | by Actual Observation. | | at one sixth of Destructive Weight. | | at $\frac{1}{125}$ th of an inch for each Foot of Span. | | C | | |
| | | | | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | | | |
| American.  | 1.83 | 1 | 1 | 600 | .06 | 266 | .027 | 148 | .015 | 1 | | |
| British...  | 2.75 | 2 | 2 | 4480 | .08 | 1310 | .022 | 1310 | .022 | 1.29 | | |
| "  | 2.75 | 1.5 | 2.5 | 8960 | .104 | 2128 | .025 | 1873 | .022 | 1.25 | | |
| "  | 2.75 | 1.5 | 3 | 8960 | .088 | 3800 | .037 | 2250 | .022 | .88 | | |

Compute Deflection of, and Weight that may be borne by, a Rectangular Bar or Beam of Wrought Iron.

$$\frac{W l^3}{60000 b d^3 D} = C. \quad \frac{W l^3}{60000 b d^3 C} = D. \quad \frac{60000 b d^3 C D}{l^3} = W.$$

DEFLECTION.—What weight will a beam 2 ins. in breadth, 5 ins. in depth, and between its supports, bear with safe deflection of $\frac{1}{125}$ of an inch for each space, or $\frac{1}{1500}$ of its length?

From table = .88. $D = \frac{1}{125}$ of 15 = .12 inch.

$$\frac{60000 \times 2 \times 5^3 \times .88 \times .12}{15^3} = \frac{1584000}{3375} = 469.33 \text{ lbs.}$$

Clark gives for Elastic deflection, 47000 for Rectangular bars, and 32000 for circular.


2.—Deflection of $\frac{1}{400}$ to $\frac{1}{600}$ of the length may be allowed under special circumstances; but under ordinary loads the deflection should not exceed one fourth of $\frac{1}{1600}$ to $\frac{1}{2400}$.

3.—In U. S. is to allow $\frac{1}{1800}$ after girder has taken its permanent set. In all bridges there is a slight increase in deflection from high speeds, about .144 of the normal deflection, with the same load moving at slow speeds. For girders there is no perceptible difference between the deflection at high speeds.

Deflection of Wrought-iron Rolled Beams.

Supported at Both Ends. Weight applied in Middle.

$$\frac{W l^3}{70000 d^2 (4a + 1.155 a')} = C \text{ at Reduced Weight and Deflection.}$$

| No. | FORM. | Length. Feet. | Flanges. | | Web. Inch. | Depth. Ins. | Weight and Deflection by Actual Observation. | | at one sixth of Destructive Weight. | | C |
|-----|---|------------------|----------------|-------------------------------|---------------|----------------|---|------|--|-------|------|
| | | | Width. Ins. | Mean Thick- ness. Inch. | | | Lbs. | Ins. | Lbs. | Inch. | |
| 1. |  | 10 | 3 | .485 | .5 | 7 | 12000 | .4 | 3800 | .127 | 1.05 |
| 2. | " | 20 | 4.6 | .8 | .5 | 9.85 | 16000 | 1.15 | 6300 | .453 | .72 |
| 3. | " | 20 | 5.7 | .643 | .6 | 11.75 | 20000 | .85 | 8000 | .34 | .58 |

To Compute Deflection of, and Weight that may be borne by, a Wrought-iron Rolled Beam of Uniform and Symmetrical Section.

Supported at Both Ends. Weight applied in Middle. (D. K. Clark.)

$$\frac{W l^3}{70000 d^2 (4a + 1.155 a')} = D. \quad \frac{70000 d^2 (4a + 1.155 a') D}{l^3} = W.$$

l representing span in feet, *d* reputed depth, or depth less thickness of lower flange in ins., *a* area of section of lower flange, *a'* area of section of web for reputed depth of beam, both in sq. ins., and *W* weight or stress in lbs.

ILLUSTRATION.—What is deflection of a wrought-iron rolled beam of New Jersey Steel and Iron Co., 10.5 ins. in depth, flanges 5 by .5 ins., and width of web .47 inch, when loaded in its middle with 8000 lbs., and supported over a span of 20 feet?

d = 10.5 — .5 = 10 ins., *a* = 5 × .5 = 2.5 sq. ins., and *a'* = 10 × .47 = 4.7 sq. ins.

$$\text{Then } \frac{8000 \times 20^3}{70000 \times 10^2 \times (4 \times 2.5 + 1.155 \times 4.7)} = \frac{64000000}{107999500} = .59 \text{ inch.}$$

If weight is uniformly distributed, divide by 112 500 instead of 70 000.



A like beam 6 ins. in depth, loaded with 2608 lbs., and supported over a span of 12 feet, gave by actual test a deflection of .3 inch, and by above formula it is also .3 inch.

NOTE.—Deflection for such a beam, for a statical weight or stress of 17 100 lbs., uniformly distributed, by rules of N. J. Steel and Iron Co., would be .54 inch, which, with difference in weights, will make deflections alike.

Deflection of Wrought-iron Riveted Beams.

Supported at Both Ends. Weight applied in Middle.

$$\frac{W l^3}{168000 \left(\frac{a + a'}{2} + \frac{a''}{4} \right) d^2 D} = C \text{ at Reduced Weight and Deflection.}$$

| No. | FORM. | Length. | Flanges. | Angles. | Web. | Depth. | Weight and Deflection by Actual Observation. | | at one sixth of Destructive Weight. | | C |
|-----|---|---------|---|--|--------------------|--------------|---|-------------|--|--------------|--------------|
| | | | | | | | Lbs. | Inch. | Lbs. | Inch. | |
| 1. |  | 7 | — | 2.125 × 2 × .28 2.125 × 2 × .29 | .25 | | 4216 | .1 | 4062 | .096 | .63 |
| 2. |  | 11.66 | 4.5 × 4.5 × 3.75 4.5 × 4.5 × 7 × .5 | 2 × 2 × .3125 × .3125 2 × 2 × .375 3 × 3 × .4375 | .25 .25 .375 | 12.5 16.5 | 77280 115584 | .46 .875 | 12880 19265 | .075 .148 | 1.96 3.86 |

To Compute Deflection of, and Weight that may be borne by, a Riveted Beam of Wrought Iron.

$$\frac{W l^3}{168000 \left(\frac{a+a'}{2} + \frac{a''}{4} \right) d^3 C} = D. \quad 168000 \left(\frac{a+a'}{2} + \frac{a''}{4} \right) d^3 C D = W.$$

a , a' , and a'' representing areas of upper and lower flanges with their angle pieces, and of web for its entire depth, all in sq. ins.

NOTE.—If there are not any flanges, as in No. 1, angle pieces alone are to be computed for flange area.

ILLUSTRATION.—What weight will a riveted and flanged beam of following dimensions sustain, at a distance between its supports of 25 feet, and at a safe deflection of .2 inch or $\frac{1}{1500}$ of its length?

Top flange $6 \times .5$ ins. | Web5 ins.
Bottom flange $6 \times .5$ " | Depth 17 "
Angles $2.25 \times 2.25 \times .5$ ins.



a and a' each = $6 \times .5 = 3 + 2.25 + 2.25 - .5 \times .5 \times 2 = 7$ sq. ins.
 $a'' = .5 \times 17 = 8.5$ sq. ins. C , as per No. 2, = .43, but inasmuch as flanges in this case are much heavier, assume .5.

$$\text{Then } \frac{168000 \left(\frac{7+7}{2} + \frac{8.5}{4} \right) 17^2 \times .2 \times .5}{25^3} = \frac{44393700}{15625} = 2835.4 \text{ lbs.}$$

Strength of a Riveted beam compared to a Solid beam is as 1 to 1.5, while for equal weights its deflection is 1.5 to 1.

Tubular Girders. Wrought Iron.

Supported at Both Ends. Weight applied in Middle.

| SECTION. | Length of Bearing. | Breadth. | Depth. | | Weight. | Deflection. | Deflection at .003 inch for each foot of span. | $\frac{13 W}{16 b d^3 D} = C$ |
|--|--------------------|----------|-----------|-----------|---------|-------------|--|-------------------------------|
| | | | Internal. | External. | | | | |
| | Feet. | Ins. | Ins. | Ins. | Lbs. | Ins. | Inch. | C |
|  Thickness .03 inch | 3.75 | 1.9 | 2.94 | 3 | 448 | .1 | .03 | 288 |
| " " .525 " | 30 | 15.5 | 22.95 | 24 | 33685 | .56 | .24 | 473 |
| " top .372 " | 30 | 16 | 23.28 | 24 | 32538 | 1.11 | .24 | 224 |
| " bottom .244 " | | | | | | | | |
| " sides .125 " | 45 | 24 | 34.25 | 35.75 | 128850 | 1.85 | .36 | 362 |
| " Thickness .75 " | | | | | | | | |
|  Thickness .0375 " | 17 | 12 | 11.925 | 12 | 2755 | .65 | .136 | 62.8 |
| " " .0416 " | 17 | 9.25 | 13.535 | 13.62 | 2262 | .62* | .136 | 47.9 |
| " " .143 " | 17 | 9.25 | 14.714 | 15 | 16800 | 1.39* | .136 | 119 |

* Destructive weight.

To Compute Deflection of, and Weight that may be borne by, a Wrought-iron Tubular Girder.

$$\frac{16 b d^3 C D}{l^3} = W. \quad \frac{W l^3}{16 b d^3 C} = D.$$

ILLUSTRATION.—What weight may be safely borne by a wrought-iron tube, alike No 3 in preceding table, for a length of 30 feet, and a deflection of .32 inch?

$$\frac{16 \times 16 \times 24^3 \times 224 \times .24}{30^3} = \frac{190253629}{27000} = 7046 \text{ lbs.}$$

Flanged Rails.

Deflection of Iron and Steel Flanged Rails within their elastic strength, is as 17 to 20, and with double-headed

MATERIALS.

tie p
tim
to
n of
position o
compare

of Unsymmetrical Section.—Elastic strength is ultimate strength, according to ordinary ratio defined experimentally. Elastic strength and deflection of beam of any section is same, whether in its not side down.

Strength and Deflection of Cast-iron Flanged Beams.

| Description of Beam | Comp. Strength. | Description of Beam. | | | | Comp. Strength. |
|---------------------|-----------------|--|---|---|---|-----------------|
| | | Beam with flanges as 1 to 4-5-1 to 5-5-1 to 6-1 to 6-73- | | | | |
| | .58 | " | " | " | " | .78 |
| ange. | .72 | " | " | " | " | .82 |
| 2.... | .63 | " | " | " | " | .92 |
| 4.... | .73 | " | " | " | " | |

SHAFTS.

To Compute Deflection and Distributed Weight for Limit of Deflection. Wrought Iron.

| Supported at Ends. | | Fixed at Ends. | | Supported at Ends. | | Fixed at Ends. | |
|--------------------|---------------------------|----------------|---------------------------------|-----------------------|-----|-----------------------------|--|
| Round. | $\frac{W l^3}{66400 d^4}$ | and | $\frac{W l^3}{133000 d^4} = D.$ | $\frac{664 d^4}{l^2}$ | and | $\frac{1330 d^4}{l^2} = W.$ | |
| Square. | $\frac{W l^3}{97500 s^4}$ | and | $\frac{W l^3}{195000 s^4} = D.$ | $\frac{975 s^4}{l^2}$ | and | $\frac{1950 s^4}{l^2} = W.$ | |
| Cast Iron. | | | | | | | |
| Round. | $\frac{W l^3}{39400 d^4}$ | and | $\frac{W l^3}{79000 d^4} = D.$ | $\frac{394 d^4}{l^2}$ | and | $\frac{790 d^4}{l^2} = W.$ | |
| Square. | $\frac{W l^3}{58000 s^4}$ | and | $\frac{W l^3}{116000 s^4} = D.$ | $\frac{580 s^4}{l^2}$ | and | $\frac{1160 s^4}{l^2} = W.$ | |

| | | | | | |
|---------|----------------------------|--------|---------------------------------|------------------------|----------------------------|
| | | Steel. | | | |
| Round. | $\frac{W l^3}{78800 d^4}$ | and | $\frac{W l^3}{158000 d^4} = D.$ | $\frac{788 d^4}{l^2}$ | and $\frac{1576 d^4}{l^2}$ |
| Square. | $\frac{W l^3}{116000 s^4}$ | and | $\frac{W l^3}{232000 s^4} = D.$ | $\frac{1160 s^4}{l^2}$ | and $\frac{2320 s^4}{l^2}$ |

d representing diameter and s side of shaft, in ins., l length between centres in feet, and W weight in lbs.

Deflection of a Cylindrical Shaft from its Weight when Supported at Both Ends.

$\Delta = .007318 \frac{l^4}{d^2 C} = D.$ l representing length in feet, d diameter in ins., a constant, ranging from 475 to 550.

The greatest admissible deflection for any diameter is $.00167 \frac{l}{d} = D.$

Admissible Distances between Bearings.

| Diam. of Shaft. | Distance. | | Diam. of Shaft. | Distance. | | Diam. of Shaft. | Wron Iron |
|-----------------|---------------|--------|-----------------|---------------|--------|-----------------|-----------|
| | Wrought Iron. | Steel. | | Wrought Iron. | Steel. | | |
| Ins. | Feet. | Feet. | Ins. | Feet. | Feet. | Ins. | Feet. |
| 1 | 12.27 | 12.61 | 5 | 20.99 | 21.57 | 9 | 25 |
| 2 | 15.46 | 15.84 | 6 | 22.3 | 22.92 | 10 | 26 |
| | 17.7 | 18.19 | 7 | 23.48 | 24.13 | 11 | 27 |
| | | 20.02 | 8 | 24.55 | 25.23 | 12 | 28 |

Ends of Shaft are rigidly connected at Ends.
Values obtained by above formula; but Δ = .25 of above.

Deflection of Mill and Factory Shafts.

$\frac{28 W}{\pi d^4 C} = D$. l representing length between supports in ins., W weight at middle in lbs., d diameter of shaft in ins., and C as follows:

Bessemer steel..... 3 800 000 | Wrought iron..... 3 500 000

To Compute Deflection of a Cylindrical Shaft.

RULE.—Divide square of three times length in feet by product of following constants and square of diameter in ins., and quotient will give deflection.

| | | | |
|-----------------------------|------|-------------------------------|------|
| Cast iron, cylindrical..... | 1500 | Wrought iron, cylindrical.... | 1980 |
| “ square..... | 2560 | “ “ square..... | 3360 |

EXAMPLE.—Length of a cast-iron cylindrical shaft is 30 feet, and its diameter in ins. is 15 ins.; what is its deflection?

$$\frac{30 \times 3^2}{1500 \times 15^2} = \frac{8100}{337500} = .024 \text{ ins.}$$

SPRINGS.

Flexure of a spring is proportional to its load and to cube of its length.

Deflection of a Carriage Spring.

A railway-carriage spring, consisting of 10 plates .3125 inch thick, and 2 .375 inch, length 2 feet 8 ins., width 3 ins., and *camber* or spring 6 ins., loaded as follows, without any permanent set:

| | | | | | |
|----------|----------|---------------|----------|-------------|--------|
| ton..... | .5 inch. | 1.5 tons..... | 1.5 ins. | 3 tons..... | 3 ins. |
| “ “ “ “ | “ “ | 2 “ “ “ “ | 2 “ “ | 4 “ “ “ “ | 4 “ “ |

Compression of an India-rubber Buffer of 3 Ins. Stroke.

| | | | | | |
|-----------|----------|-------------|---------|-------------|-----------|
| ton..... | 1.3 ins. | 2 tons..... | 2 ins. | 5 tons..... | 2.75 ins. |
| tons..... | 1.75 “ | 3 “ “ “ “ | 2.375 “ | 10 “ “ “ “ | 3 “ “ |

General Deductions.

Deflection depends essentially upon form of Girder, Beam, etc.

A continuous weight, equal to that a beam, etc., is suited to sustain, will cause deflection of it to increase unless it is subjected to considerable changes of temperature.

Heaviest load on a railway girder should not exceed .16 of that of deflective weight of girder when laid on at rest.

Semi-girders or Beams.—Deflection of a beam, etc., fixed at one end and loaded at other, is 32 times that of same beam supported at both ends and loaded in middle.

Deflection consequent upon Velocity of Load.—Deflection is very much increased by instantaneous loading; by some authorities it is estimated to be doubled.

Momentum of a railway train in deflecting girders, etc., is greater than that from dead weight of it, and deflection increases with velocity.

When motion is given to load on a beam, etc., point of greatest deflection does not remain in centre of beam, etc., as beams broken by a travelling load are always fractured at points beyond their centres, and often into several pieces.

Heaviest running weight that a bridge is subjected to is five and tender, which is equal to 2 tons per lineal foot.

Girders should not, under any circumstances, be deflected more than 1 inch to a foot in length.

Deflection of Solid rolled beams compared to Riveted beams is as 1 to 1.5. Wrought-iron Girders of ordinary construction are not safe when subjected to violent impacts or disturbances, with a load equal to .33 of their struative weight.

Wood.—In consequence of wood not being subjected to weakening by the fect of impact, a factor of safety of 5 for single pieces is held to be sufficient, but for structures, in consequence of loss of strength in its connections, factor of from 8 to 10 becomes necessary.

Working Strength or Factors of Safety.*

Elastic strength of materials is, in general terms, half of its ultimate destructive or breaking strength. If a working load of .5 elastic strength, or .25 of ultimate strength, be accepted, equal range for fluctuation within lastic limit is provided. But, as bodies of same material are not all uniform in strength, it is necessary to observe a lower limit than .25 where material is exposed to great or to sudden variations of load or stress.

Cast Iron.—Mr. Stoney recommends .25 of ultimate tensile strength, for lead weights; .16 for bridge girders; and .125 for crane posts and machinery. In compression, free from flexure, cast iron will bear 8 tons (17 920 ba.) per sq. inch; for arches, 3 tons (6720 lbs.) per sq. inch; for pillars, upporting dead loads, .16 of ultimate strength; for pillars subject to ibration from machinery, .125; and for pillars subject to shocks from eavy-loaded wagons and like, .1, or even less, where strength is exerted in assistance to flexure.

Wrought Iron.—For bars and plates, 5 tons (11 200 lbs.) per sq. inch of et section is taken as safe working tensile stress; for bar iron of extra uality, 6 tons (13 440 lbs.). In compression, where flexure is prevented, tons (8960 lbs.) is safe limit; in small sizes, 3 tons (6720 lbs.). For col- rns subject to shocks, Mr. Stoney allows .16 of calculated breaking weight; ith quiescent loads, .25. For machinery, .125 to .1 is usually practised; nd for steam-boilers, .25 to .125.

Mr. Roebling claims that long experience has proved, beyond shadow of oublet, that good iron, exposed to a tensile strain not above .2 of its ultimate strength, and not subject to strong vibration or torsion, may be de- ended upon for a thousand years.

Steel.—A committee of British Association recommended a maximum working tensile stress of 9 tons (20 160 lbs.) per sq. inch. Mr. Stoney recommends, for mild steel, .25 of ultimate strength, or 8 tons (17 920 lbs.) per sq. inch. Limit for compression must be regulated very much by nature of steel, and whether it be annealed or unannealed. Probably a limit of 9 tons (20 160 lbs.) per sq. inch, same as limit for tension, would be safe maximum for general purposes. In absence of experience, Mr. Stoney further ecommends that, for steel pillars, an addition not exceeding 50 per cent. be made to safe load for wrought-iron pillars of same dimensions.

Wood.—One tenth of ultimate stress is an accepted limit. Piles have, in some situations, borne permanently .2 of their ultimate compressive strength.

Foundations.—According to Professor Rankine, maximum pressure on foundations in firm earth is from 17 to 23 lbs. per sq. inch; and, on rock, it should not exceed .125 of its crushing load.

Masonry.—Mr. Stoney asserts that working load on rut- rick-work, or concrete rarely exceeds .16 of crushing weight mass; and that this seems to be a safe limit. In an arch, calcu- should not exceed .05 of crushing pressure of stone.

* Essentially from Manual of D. K. Clark, London, 1877.

Ropes.—For round, working load should not exceed .14 and for flat .11.

Perfect material and workmanship....

| | | |
|--------------------|--------------------------|----------------|
| Dr. Rankine gives | { Good ordinary material | { Metal..... |
| following factors: | | |
| | | { Masonry..... |

A *Dead Load* is one that is laid on very gradually and

A *Live Load* is one that is laid on suddenly, as a load passing swiftly over a bridge.

DETRUSIVE OR SHEARING STRESS

Detrusive or Shearing Strength of any body is direct as force, or thickness, or area of shearing surface.

Results of Experiments upon Detrusive Metals with a Punch.

| METALS. | Diameter of Punch. | Thickness of Metal. | Power exerted. |
|-------------------|--------------------|---------------------|----------------------|
| | Ins. | Ins. | Lbs. |
| Brass..... | 1 | .045 | 5 448 |
| Cast iron..... | — | — | — |
| Copper..... | .5 | .08 | 3 983 |
| Steel..... | 1 | .3 | 21 250 |
| | .5 | .25 | 34 720 |
| " Bessemer..... | .875 | .75 | { 103 600 184 800 |
| Wrought iron..... | .5 | .17 | 11 950 |
| | 1 | .615 | 82 870 |
| | 2 | 1.06 | 207 400 |

To Compute Power to Punch Iron. Br

RULE.—Multiply product of diameter of punch and thickness of plate by 150 000 if for wrought iron, by 128 000 if for brass, and by 100 000 if for cast iron or copper, and product will give power required in horse power.

EXAMPLE.—What power is required to punch a hole .5 inch of brass .25 inch thick?

$$.5 \times .25 \times 128\,000 = 16\,000 \text{ lbs.}$$

Comparison between Detrusive and Strengths.

Assuming compression and abrasion of metal in application one inch in diameter to extend to .125 of an inch beyond comparative resistance of wrought iron to *destructive* and latter estimated at 600 lbs. per sq. inch, for a bar 1 foot in

WOODS.

Detrusive Strength of Woods. *Pe*

| | Lbs. | | Lbs. | | Lbs. |
|------------------|------|----------------|------|---------------|------|
| Spruce..... | 470 | Pine, pitch... | 510 | Ash..... | 65 |
| Pine, white..... | 490 | Hemlock..... | 540 | Chestnut..... | 60 |

To Compute Length of Surface of Resistor to Horizontal Thrust.

RULE.—Divide 4 times horizontal thrust in lbs. by pressure in wood in ins., and deduct resistive resistance per sq. inch in lbs. and quotient will give length required.

EXAMPLE.—Thrust of a rafter is 5600 lbs., breadth of tie beam is 6 ins.; what should be length of beyond score for raft

th 510 as above. Then $\frac{4 \times 5600}{6 \times 510} = \frac{22400}{3060} =$

Shearing.

Wrought Iron.

Resistance to shearing of American is about 75 per cent., and of English 70 per cent., of its tensile strength.

Resistance to shearing of plates and bolts is not in a direct ratio. It approximates to that of square of depth of former, and to square of diameter of latter.

Results of Experiments upon Shearing Strength of Various Metals by Parallel Cutters.

Wrought Iron.—Thickness from .5 to 1 inch, 50,000 lbs. per sq. inch.

Made by Inclined Cutters, angle = 7°.

| PLATES. | Thickness. | Power. | Bolts. | Diam. | Power. |
|-------------------|------------|--------|-------------------|-------|--------|
| | Inch. | Lbs. | | Inch. | Lbs. |
| Steel..... | .05 | 540 | Brass..... | 1.11 | 29 700 |
| Per..... | .297 | 11 196 | Copper..... | .775 | 11 310 |
| Al..... | .24 | 14 930 | Steel..... | .775 | 28 720 |
| Wrought iron..... | .51 | 39 150 | Wrought iron..... | .32 | 3 093 |
| | 1 | 44 800 | | 1.142 | 35 410 |

Result of Experiments in Shearing, made at U. S. Navy Yard, Washington, on Wrought-iron Bolts.

| Diam. | Stress. | | Per Sq. Inch. | Diam. | Stress. | | Per Sq. Inch. |
|-------|----------|----------|---------------|-------|----------|----------|---------------|
| | Minimum. | Maximum. | | | Minimum. | Maximum. | |
| Ch. | Lbs. | Lbs. | Lbs. | Inch. | Lbs. | Lbs. | Lbs. |
| 5 | 8 500 | 9 400 | 44 149 | .875 | 25 500 | 27 600 | 41 503 |
| 75 | 18 400 | 19 650 | 39 553 | 1 | 32 900 | 35 800 | 40 708 |

Mean 41 033 lbs.

Result of Experiments on .875 Inch Wrought-iron Bolts. (E. Clark.)

| | Lbs. | Tons. | | Lbs. | Tons. |
|-------------------|--------|-------|--|--------|-------|
| | | | | | |
| Single shear..... | 54 096 | 24.15 | Double shear of two .625-inch plates riveted together (one section) | 45 696 | 20.4 |
| Double " | 46 904 | 22.1 | | | |

Tensile strength..... 50 176 lbs.

Riveted Joints.

Experiments on strength of riveted joints showed that while the plates were destroyed with a stress of 43 546 lbs., the rivets were strained by a stress of 39 088 lbs.

Cast Iron.

Resistance to shearing is very nearly equal to its tensile strength. An average of English being 24 000 lbs. per sq. inch.

Steel.

Shearing strength of steel of all kinds (including Fagersta) is about 72 per cent. of its tensile strength.

Treennails.

Oak treennails, 1 to 1.75 ins. in diameter, have an average shearing strength 1.8 tons per sq. inch, and in order to fully develop their strength, the planks in which they are driven should be 3 times their diameter.

Woods.

When a beam or any piece of wood is let in (not mortised) at right angles to another piece, so that thrust will bear in direction of fibres, if it is cut, depth of cut at right angles to fibres should not be more than length of piece, fibres of which, by their cohesion, resist thrust.

TENSILE STRENGTH.

Tensile Strength is resistance of the fibres or particles of a body to separation. It is therefore proportional to their number, or to area of its transverse section, and in metals it varies with their temperature, generally decreasing as temperature is increased. In silver, tenacity decreases more rapidly than temperature; and in copper, gold, and platinum less rapidly.

Cast Iron.

Experiments on Cast-iron bars give a tensile strength of from 4000 to 5000 lbs. per sq. inch of its section, as just sufficient to balance elasticity of the metal; and as a bar of it is extended the 12 300th part of its length for every 1000 lbs. of direct strain, or one sixteenth of an inch in 64.06 feet per sq. inch of its section, it is deduced that its elasticity is fully excited when it is extended less than the 2400th part of its length, and extension of it at its limit of elasticity, which is about .5 of its destructive weight, is estimated at 1500th part of its length.

Average ultimate extension is 500th part of its length.

A bar will contract or expand .000006 173 inch, or the 162 000th of its length, for each degree of heat; and assuming extreme of moderate range of temperature in this country 140° ($-20^{\circ} + 120^{\circ}$), it will contract or expand with this change .000 864 2 inch, or the 1157th part of its length.

It follows, then, that as 1000 lbs. will extend a bar the 12 300th part of its length, contraction or extension for 1157th part will be equivalent to a force of 10 648 lbs. (4.75 tons) per sq. inch of section. It shrinks in cooling from one eighty-fifth to one ninety-eighth of its length.

Mean tensile strength of American, as determined by Maj. Wade for U. S. Ordnance Corps, is 31 829 lbs. (14.21 tons) per sq. inch of section; mean of English, as determined by Mr. E. Hodgkinson for Commission on Application of Iron to Railway Structures, 1849, is 19 484 lbs. (8.7 tons); and by Col. Wilmot, at Woolwich, in 1858, for gun-metal, is 23 257 lbs. (10.35 tons), varying from 12 320 lbs. (5.5 tons) to 25 520 lbs. (10.5 tons).

Mean ultimate extension of four descriptions of English, as determined for Commission above referred to, was, for lengths of 10 feet, .1997 inch, being 600th part of its length; and this weight would compress a bar the 775th part of its length.

Tensile strength of strongest piece ever tested—45 970 lbs. (20.52 tons). This was a mixture of grades 1, 2, and 3 from furnace of Robert P. Parrott at Greenwood, N. Y., and at 3d fusion.

At 2.5 tons per sq. inch it will extend same as wrought iron at 5.6 tons.

From experiments of Maj. Wade he deduced the following mean results:

| Density. | Tensile. | Transverse. | Torsion. | Crushing. | Hardness. |
|----------|----------|-------------|----------|-----------|-----------|
| 7.225 | 31 829 | 8182 | 8614 | 144 916 | 22.34 |

Tensile per sq. inch of section; *Transverse* per sq. inch, one end fixed, pulled at other end at a distance of 1 foot; and *Torsion* per sq. inch, pulled at end of a lever 1 foot in length.

are 6 per cent. stronger than dry, and 30 per cent. but when castings are chilled and annealed, a gain of 1 over those made in green sand.

—Stress is for American as 4.55 to 1, and strength increasing with density.

elling.—Strength, as well as density, are increased by repeated re-
s. The increase is the result of the gradual abstraction of the con-
t carbon of the iron, and the consequent approximation of the metal
ught iron.

ult of the 4th melting of pig iron, as determined by Major Wade, was
ease its strength from 12 880 lbs. (5.75 tons) to 27 888 lbs. (12.45
and its specific gravity from 6.9 to 7.4.

se successive meltings of Greenwood iron, N. Y., gave tensile strength
00, 30 100, and 35 700 lbs.

ult of 5th melting by Mr. Bramwell was to increase strength of Acadian
om 16 800 lbs. (7.5 tons) to 41 440 lbs. (18.5 tons).

melting increases its resistance to a crushing stress from 70 to 80 tons
r cent.) per sq. inch of section.

Hot and Cold Blast.

Hodgkinson deduced from experiments that relative strength of 1.2
ins. square was as 100, 80, and 77, and that hot blast had less tensile
th than cold blast, but greater resistance to a crushing stress.

tain James ascertained that tensile strength of .75 inch bars, cut out
id 3 inch bars, had only half strength of a bar cast 1 inch square.

Robert Stephenson concluded, from experiments of recent date, that
ge strength of hot blast was not much less than that of cold blast; but
ld blast, or mixtures of cold blast, were more regular, and that mixt-
f cold blast and hot blast were better than either separate.

Stirling's Mixed or Toughened Iron.

mixture of a portion of malleable iron with cast iron, carefully fused
rucible, a tensile strain of 25 764 lbs. has been attained. This mixt-
hen judiciously managed and duly proportioned, increases resistance
t iron about one third; greatest effect being obtained with a propor-
f about 30 per cent. of malleable iron.

Malleable Cast Iron.

nsile strength of annealed malleable is guaranteed by some Manufact-
of it at 56 000 lbs.; it is capable of sustaining 22 400 lbs. without per-
t set.

Wrought Iron.

periments on English bars gave a tensile strength of from 22 000 lbs.
400 lbs. per sq. inch of its section, as just sufficient to balance elasticity
metal; and as a bar of it is extended the 28 000th part of its length
ery 1000 lbs. of direct strain, or one sixteenth of an inch in 116.66 feet
1. inch of its section, it is deduced that its elasticity is fully excited
it is extended the 1000th part of its length, and extension of it at its
of elasticity, which is from .45 to .5 of its destructive weight, is esti-
l at 1520th part of its length.

ar will expand or contract .000 006 614 inch, or 151 200 part of its length
ach degree of heat; and assuming, as before stated for cast iron, that
me range of temperature in air in this country is 140°, it will contract
pand with this change .000 926, or 1080th of its length, which is eq
o a force of 20 740 lbs. (9.25 tons) per sq. inch of section.

an tensile strength of American bars and plates (45 000 to 76
0 lbs. (27 tons) per sq. inch of section; as determined by Prof. Jo
36, is 55 900 lbs.; and mean of English, as determined by Capt. E
W, Brunel, and Fairbairn, is 53 900 lbs.; and by Mr. Kirkaldy, bar
(47 040 to 55 910) 51 475 lbs. (22.97 tons).

Greatest strength observed 73 449 lbs. (32.79 tons).

Ultimate strength, as given by Mr. D. K. Clark, 59 732 lbs. (26.66 tons).

Average ultimate extension is 600th part of its length.

Strength of plates, as determined by Sir William Fairbairn, is fully 9 per cent. greater with fibre than across it.

Resistance of wrought iron to crushing and tensile strains is, as a mean, as 1.5 to 1 for American; and for English 1.2 to 1.

Reheating.—Experiments to determine results from repeated heating and laminating, furnished following:

From 1 to 6 reheatings and rollings, tensile stress increased from 43 904 lbs. to 61 824 lbs., and from 6 to 12 it was reduced to 43 904 again.

Effect of Temperature.—Tensile strength at different temperatures is as follows: 60°, 1; 114°, 1.14; 212°, 1.2; 250°, 1.32; 270°, 1.35; 325°, 1.41; 435°, 1.4.

Experiments of Franklin Institute gave at

| | | | | | |
|-----------|-------------|------------|-------------|------------|-------------|
| 80°..... | 56 000 lbs. | 720°..... | 55 000 lbs. | 1240°..... | 22 000 lbs. |
| 570°..... | 66 500 " | 1050°..... | 32 000 " | 1317°..... | 9 000 " |

Annealing.—Tensile strength is reduced fully 1 ton per sq. inch by annealing.

Cold Rolling.—Bars are materially stronger than when hot rolled, strength being increased from one fifth to one half, and elongation reduced from 21 to 8 per cent.

Hammering increases strength in some cases to one fifth.

Welding.—Strength is reduced from a range of 3 to 44 per cent. 20 per cent., or one fifth, is held to be a fair mean.

Temperature.—From 0° to 400° strength is not essentially affected, but at high temperature it is reduced. When heated to redness its strength is reduced fully 25 per cent.

Tensile strength at 23° was found to be .024 per cent. less than at 64°.

Cutting Screw Threads reduces strength from 11 to 33 per cent.

Hardening in water, oil, etc., reduces elongation, but does not essentially increase the strength.

Case Hardening reduces strength fully 10 per cent.

Galvanizing does not affect strength of plates.

Angled Bars, etc.—Their strength is fully 10 per cent. less than for bolts and plates.

Elements connected with Tensile Resistance of various Substances.

| SUBSTANCES. | Stress per Sq. Inch for limit of Elasticity. | Ratio of Stress to that causing Rupture. | SUBSTANCES. | Stress per Sq. Inch for limit of Elasticity. | Ratio of Stress to that causing Rupture. |
|----------------------------|--|--|----------------------------|--|--|
| | Lbs. | | | Lbs. | |
| Beech | 3 355 | .3 | Wrought iron, ordinary.... | 17 600 | .3 |
| Cast iron, English..... | 4 000 | .22 | " " Swedish.... | 24 400 | .34 |
| " " American..... | 5 000 | .2 | " " English..... | 18 850 | .35 |
| Oak | 2 856 | .23 | " " "..... | 22 400 | .35 |
| Steel plates, .5 inch..... | 52 000 | .62 | " " American.. | 15 000 | .36 |
| " wire..... | 75 700 | .5 | " " No. 9, unannealed | 47 532 | .46 |
| " "..... | 3 332 | .23 | " " annealed.. | 36 300 | .45 |

oving outer surface does not reduce the strength of bolts

TIE-RODS.

Results of Experiments on Tensile Strength of Wrought-iron Tie-rods.

Common English Iron, 1.1875 Ins. in Diameter.

| DESCRIPTION OF CONNECTION. | Breaking Weight. |
|--|------------------|
| | Lbs. |
| micircular hook fitted to a circular and welded eye..... | 14 000 |
| 70 semicircular hooks hooked together..... | 16 220 |
| ght-angled hook or goose-neck fitted into a cylindrical eye..... | 29 120 |
| 70 links or welded eyes connected together..... | 48 160 |
| raight rod without any connective articulation..... | 56 000 |

Ratio of Ductility and Malleability of Metals.

| In order of Wire-drawing Ductility. | In order of Laminable Ductility. | In order of Wire-drawing Ductility. | In order of Laminable Ductility. | In order of Wire-drawing Ductility. | In order of Laminable Ductility. |
|-------------------------------------|----------------------------------|-------------------------------------|----------------------------------|-------------------------------------|----------------------------------|
| Gold. | Iron. | Tin. | Gold. | Tin. | Zinc. |
| Silver. | Copper. | Lead. | Silver. | Platinum. | Iron. |
| Platinum. | Zinc. | Nickel. | Copper. | Lead. | Nickel. |

Relative resistance of Wrought Iron and Copper to tension and compression is as 100 to 54.5.

Steel.

Experiments of Mr. Kirkaldy, 1858-61, give an average tensile strength of bars of 134 400 lbs. (60 tons) per sq. inch for tool-steel, and 62 720 lbs. (28 tons) for puddled. Greatest observed strength being 148 288 lbs. (66.2 tons). Plates, mean, 86 800 lbs. (32 to 45.5 tons) with fibre, and 81 760 lbs. (36.5 tons) across it.

Its resistance to crushing compared to tension is as 2.1 to 1.

Hardening.—Its strength is very materially increased by being cooled in oil, ranging from 12 to 55 per cent.

Crucible.—Experiments by the Steel Committee of Society of Civil Engineers, England, 1868-70, give a tensile strength of 91 571 lbs. per sq. inch (40.88 tons), with an elongation of .163 per cent., or 1 part in 613, and 1 elastic extension of .000034 7th part for every 1000 lbs. per sq. inch, or 1 part in 28 818.

Bessemer.—Experiments by same Committee give a tensile strength of 76 653 lbs. per sq. inch (34.22 tons) with an elongation of .144 per cent., or 1 part in 695, and an elastic extension of .000034 82d part for every 1000 lbs. per sq. inch, or 1 part in 28 719.

Result of Experiments by Committee of Society of Civil Engineers of England, 1868-70, and Mr. Daniel Kirkaldy, 1875.

Per Sq. Inch.

| STEEL. | Elastic Strength. | | Elastic Extension | | Ratio of Elastic to Ultimate Strength | Destructive Weight. |
|-----------------------|-------------------|-------|---------------------|---------------|---------------------------------------|---------------------|
| | | | In Parts of Length. | per 1000 Lbs. | | |
| | Lbs. | Tons. | Per Cent. | In Length. | Per Cent. | Tons. |
| Crucible..... | 49 840 | 22.25 | .225 | .000 5 | 58. | |
| Bessemer..... | 44 800 | 20 | .204 | .000 45 | 59. | |
| Crucible, unannealed. | 48 608 | 21.7 | — | — | 59. | |
| “ annealed.... | 39 200 | 17.5 | — | — | 51. | |
| Bessemer, unannealed. | 32 080 | 14.56 | — | — | 46. | |
| “ annealed.... | 28 784 | 12.85 | — | — | 44. | |

Average Tensile Elasticity of Steel Bars and Plates.
(Com. of Civil Engineers, 1870.)

| DESCRIPTION. | Elasticity per Sq. Inch. | Elastic Extension in Parts of Length. | Ratio of Elastic to Descriptive Strength. |
|------------------------------------|--------------------------|---------------------------------------|---|
| Bars. | | | |
| Crucible, hammered and rolled..... | Lbs. 50 557 | Parts. 1 in 485 | Per Cent. 58.2 |
| Bessemer, " " "..... | 43 814 | 1 in 675 | 55 |
| Fagersta, rolled..... | 56 560 | — | 64.8 |
| " unannealed..... | 34 048 | — | 55.6 |
| " hammered and rolled..... | 55 574 | — | 64.7 |
| " " annealed..... | 40 858 | — | 54 |
| " plates, unannealed..... | 30 710 | 1 in 980 | 59.2 |
| " " annealed..... | 26 940 | 1 in 1020 | 56.5 |
| Siemens, " unannealed..... | 32 500 | — | 46.4 |
| " " annealed..... | 28 780 | — | 44.4 |
| " tires..... | 40 174 | — | 58.8 |
| Krupp's shaft..... | 42 112 | 1 in 185 | — |

Tensile strength of steel increases by reheating and rolling up to second operation, but decreases after that.

Tensile Strength of Various Materials, deduced from Experiments of U. S. Ordnance Department, Fairbairn, Hodgkinson, Kirkaldy, and by the Author.

Power or Weight required to tear asunder One Sq. Inch, in Lbs.

| METALS. | Lbs. | METALS. | Lbs. |
|----------------------------|---------|--|---------|
| Antimony, cast..... | 1 053 | Steel, Pittsburgh, mean..... | 94 450 |
| Bismuth, cast..... | 3 248 | " Bessemer, rolled..... | 76 650 |
| Cast Iron, Greenwood..... | 45 970 | " " hammered..... | 125 000 |
| " mean, Major Wade..... | 31 829 | " Eng., cast..... | 134 000 |
| " gun-metal, mean..... | 37 232 | " " plates, mean..... | 93 500 |
| " malleable, annealed..... | 56 000 | " " plates..... | 86 800 |
| " Eng., strong..... | 29 000 | " " puddled plates..... | 62 750 |
| " " weak..... | 13 400 | " " crucible..... | 91 570 |
| " " averages..... | 15 600 | " " homogeneous..... | 96 280 |
| " " "..... | 21 280 | " " blistered, bars..... | 104 000 |
| " " gun-metal..... | 23 257 | " " Fagersta bars..... | 89 600 |
| " " mean*..... | 19 484 | " " plates..... | 98 560 |
| " " Low Moor, No. 2..... | 14 076 | " " Whitworth's..... | 89 600 |
| " " Clyde, No. 1..... | 16 125 | " " Siemens's plates..... | 64 000 |
| " " " No. 3..... | 23 468 | " " Krupp's shaft..... | 69 880 |
| " " Stirling, mean..... | 25 764 | " " "..... | 92 243 |
| Copper, wrought..... | 34 000 | " " "..... | 5 000 |
| " rolled..... | 36 000 | " Banca..... | 2 100 |
| " cast..... | 24 250 | Wire rope, per lb. w't per fathom..... | 4 480 |
| " bolt..... | 36 800 | " " galvanized steel, "..... | 6 750 |
| " wire..... | 61 200 | " " "..... | 45 500 |
| Gold..... | 20 384 | Wrought Iron, boiler plates..... | 62 000 |
| Lead, cast..... | 1 800 | " rivets..... | 65 000 |
| " pipe..... | 2 240 | " bolts, mean..... | 60 500 |
| " " encased..... | 3 759 | " " inferior..... | 30 000 |
| " rolled sheet..... | 3 320 | " hammered..... | 54 000 |
| Platinum wire..... | 53 000 | " shaft..... | 44 750 |
| Silver, cast..... | 40 000 | " wire..... | 73 600 |
| " l, cast, maximum..... | 142 000 | " " No. 9..... | 100 000 |
| " " mean..... | 88 560 | " " No. 20..... | 120 000 |
| " puddled, maximum..... | 173 817 | " " diam. .0069 inch..... | 301 168 |
| " W. Tool Co..... | 179 980 | " " galvanized .058 "..... | 64 060 |
| " " "..... | 210 000 | " " Eng., heavy forging..... | 33 000 |
| " " "..... | 300 000 | " " plates, lengthwise..... | 53 800 |
| " " "..... | 96 300 | " " " crosswise..... | 48 800 |
| " " "..... | 93 700 | | |
| " " "..... | 180 000 | | |

application of force

| METALS. | | Lbs. | WOODS. | | Lbs. |
|---|--|--------|------------------------------|--|--------|
| Frought Iron, Eng., mean. | | 51 000 | Larch | | 4 200 |
| “ Eng., Low Moor. | | 57 600 | Lignum vitæ | | 9 500 |
| “ “ Lancashire. | | 48 800 | Locust. | | 11 800 |
| “ “ Thames. | | 65 920 | | | 16 000 |
| “ armor-plates | | 40 000 | Mahogany, Honduras. | | 20 500 |
| “ “ bar. | | 31 300 | | | 21 000 |
| “ “ charcoal. | | 56 000 | “ Spanish. | | 8 000 |
| “ “ rivet, scrap. | | 63 000 | Oak, Pa., seasoned. | | 12 000 |
| “ Russian, bar, best. | | 51 760 | “ Va., “ | | 20 333 |
| “ “ “ | | 59 500 | “ white | | 25 220 |
| “ “ “ | | 49 000 | “ live, Ala. | | 16 500 |
| “ Swedish, “ best. | | 72 000 | “ red. | | 16 380 |
| “ “ “ | | 48 900 | “ African. | | 10 250 |
| inc. | | 3 500 | “ English. | | 9 500 |
| “ sheet. | | 7 000 | “ Dantzic. | | 4 500 |
| | | 16 000 | Pear. | | 7 571 |
| ALLOYS OR COMPOSITIONS. | | | | | 4 200 |
| Alloy, Cop. 60, Iron 2, Zinc 35, Tin 2. | | 85 120 | Pine, Va. | | 9 860 |
| “ Tin 10, Antimony 1. | | 11 000 | “ Riga | | 10 200 |
| Aluminium, Cop. 90. | | 71 600 | “ yellow. | | 14 000 |
| “ maximum. | | 96 320 | “ white | | 13 000 |
| bell-metal. | | 3 070 | “ red. | | 11 800 |
| brass, cast. | | 18 000 | Poon. | | 13 000 |
| “ wire. | | 49 000 | Poplar. | | 13 300 |
| bronze, Phosphor., extreme. | | 50 915 | Redwood, Cal. | | 7 000 |
| “ mean. | | 34 464 | | | 10 833 |
| “ ordinary. | | 23 500 | Spruce, white. | | 10 290 |
| “ Cop. 10, Tin 1. | | 33 000 | | | 12 400 |
| “ “ 9, “ 1. | | 38 080 | Sycamore. | | 9 600 |
| “ “ 8, “ 1. | | 36 000 | | | 13 000 |
| “ “ 2, Zinc 1. | | 29 000 | Teak, India. | | 15 000 |
| gun-metal, ordinary. | | 18 000 | “ African. | | 21 000 |
| “ mean. | | 33 600 | Walnut, Eng. | | 7 800 |
| “ bars. | | 42 040 | “ black. | | 16 633 |
| peculum metal. | | 7 000 | “ Mich. | | 17 580 |
| yellow metal. | | 48 700 | Willow | | 13 000 |
| | | | Yew. | | 8 000 |
| WOODS. | | | Across Fibre. | | |
| ash, white. | | 14 000 | Oak | | 2 300 |
| “ American. | | 9 500 | Pine. | | 550 |
| “ English. | | 16 000 | MISCELLANEOUS. | | |
| bamboo. | | 6 300 | Basalt, Scotch. | | 1 469 |
| bay. | | 14 000 | Beton, N. Y. Stone Con'g Co. | | 300 |
| beech, English. | | 11 500 | | | 500 |
| birch. | | 15 000 | Blue stone. | | 77 |
| “ Amer., black. | | 7 000 | Brick, extreme. | | 750 |
| box, African. | | 23 000 | “ inferior. | | 100 |
| bullet. | | 19 000 | | | 290 |
| cedar, Lebanon | | 11 400 | Cement, Portland, 7 days. | | 400 |
| “ West Indian. | | 7 500 | | | 860 |
| “ American. | | 11 600 | “ pure, 1 mo. | | 393 |
| chestnut. | | 12 500 | “ sand 2, 320 days. | | 713 |
| “ horse. | | 10 000 | “ “ 1 | | 948 |
| ypress. | | 6 000 | “ pure, “ | | 1 152 |
| oak, Christiana. | | 12 400 | “ sand 1, in water | | 201 |
| ony. | | 27 000 | | | 1 mo. |
| “ | | 6 000 | “ “ “ 1 | | “ |
| “ | | 13 000 | “ “ “ 3, 1 ye | | “ |
| “ blue. | | 18 000 | “ “ “ 5, 1 | | “ |
| “ Alabama. | | 15 860 | “ “ “ 7, 1 | | “ |
| “ oak, mack. | | 12 000 | Hydraulic. | | “ |
| “ oak, kory. | | 11 000 | Rosedale, Ust. Co. | | “ |
| “ oak. | | 16 000 | “ sand 1, | | “ |
| “ oak. | | 17 350 | “ 9 mos. | | “ |
| “ oak. | | 23 000 | | | |

| MISCELLANEOUS. | Lbs. | MISCELLANEOUS. |
|---------------------------------|--------|---------------------|
| Cement, Roman, in water 7 days. | 90 | Mortar, 1 year..... |
| " " " 1 mo... | 115 | " hydraulic |
| " " " 1 year. | 286 | " ordinary |
| " " sand 1, 42 days.. | 284 | Oxhide |
| " " " 2, " .. | 199 | Rope, Manila..... |
| " " " 3, " .. | 160 | " tarred hemp..... |
| Flax..... | 25 000 | Sandstone..... |
| Glass, crown | 2 546 | " fine green..... |
| Glue..... | 4 000 | " Arbroath..... |
| Granite..... | 578 | " Caithness..... |
| Gutta Percha..... | 3 500 | " Portland..... |
| Hemp rope..... | 12 000 | " Craigleith..... |
| Ivory..... | 16 000 | Silk fibre..... |
| Leather belting..... | 1 000 | Slate |
| Limestone..... | 330 | Whalebone..... |
| Marble, statuary..... | 670 | |
| " Italian..... | 2 800 | |
| Marble, white..... | 3 200 | |
| " Irish..... | 5 200 | |
| | 9 000 | |
| | 17 600 | |

TORSIONAL STRENGTH.

SHAFTS AND GUDGEONS,

Shafts are divided into *Shafts* and *Spindles*, according to nitide, and are subjected to Torsion and Lateral Stress com Lateral Stress alone.

A *Gudgeon* is the metal journal or *Arbor* upon which a w revolves.

Lateral Stiffness and Strength.—Shafts of equal length have *ness* as their breadth and cube of their depth, and have *latera* their breadth and square of their depths.

Shafts of different lengths have *lateral stiffness* directly as t and cube of their depth, and inversely as cube of their lengt *lateral strength* directly as their breadth and as square of thei inversely as their length.

Hollow Shafts having equal lengths and equal quantities of n *lateral stiffness* as square of their diameter, and have *lateral stre* diameters. Hence, in hollow shafts, one having twice the dia other will have four times the stiffness, and but double the st when having equal lengths, by an increase in diameter they inci ness in a greater proportion than in strength.

When a solid shaft is subjected to torsional stress, its centre *axis*, about which both intensity and leverage of resistance incre or side; and the two in combination, or moment of resistance increase as square of radius or side.

and Shaft.—Radius of ring of resistance is radius of gyr sing alike to that of a circular plate revolving on its axi The ultimate moment of resistance then is expresse nal area of shaft, by ultimate shearing resistance per by radius, and by .7071.

$$R \cdot W = \frac{1}{2} \pi d^2 r S \times .7071 = .278 d^3 S = R W. \quad (D. K. Clark)$$

meter of shaft and r radius, S ultimate shearing s
1, R radius through which stress is expressed, in
active stress, in lbs.

$$R W = S; \text{ and } \sqrt[3]{\frac{R W}{S}} \times 1.534 = d$$

d Shaft.—Strength, compared to a square of equal sectional area, as 1 to .85. Diameter of a round section, compared to side of section of equal resistance, is as 1 to .96.

re Shaft.—Moment of torsional resistance of a square shaft exceeds a round of same sectional area, in consequence of projection of corner square; but inasmuch as material is less disposed to resist torsional resistance of a square shaft, compared to a round one of like area on, is as 1 to 1.18, and of like side and diameter, as 1.08 to 1.

$$\frac{.278 \times 1.08 s^2 S}{R} = W. \quad \text{Hollow Round Shafts.} \quad \frac{.278 (d^4 - d'^4) S}{R d} = W.$$

Section is comparatively Thin. $\frac{1.57 d^3 t S}{R} = W.$ *s representing side, external and internal diameters, and t thickness of metal in ins.*

onal Angle of a bar, etc., under equal stress, will vary as its length. torsional strength of bars of like diameters is inversely as their

upon a shaft from a weight upon it is proportional to product of the parts multiplied into each other. Thus, if a shaft is 10 feet in length, and a weight centre of gravity of the stress is at a point 2 feet from one end, the parts 2 multiplied together, are equal to 16; but if weight or stress were applied in of the shaft, parts 5 and 5, multiplied together, would produce 25.

load upon a shaft is uniformly distributed over any part of it, it is considered in middle of that part; and if load is not uniformly distributed, it is considered as united at its centre of gravity.

tion of a shaft produced by a load which is uniformly distributed over its same as when .625 of load is applied at middle of its length.

tance of body of a shaft to lateral stress is as its breadth and square length; hence diameter will be *as product of length of it, and length one side of a given point, less square of that length.*

FRACTION.—Length of a shaft between centres of its journals is 10 feet; what be relative cubes of its diameters when load is applied at 1, 2, and 5 feet e end? and what when load is uniformly distributed over length of it?

$l \times l^3 - l'^2 = d^3$; and when uniformly distributed, $d^3 \div 2 = d'^3$.

$10 - 1^2 = 9 = \text{cube of diameter at 1 foot}; 10 \times 2 = 20 - 2^2 = 16 = \text{cube of diameter at 2 feet}; 10 \times 5 = 50 - 5^2 = 25 = \text{cube of diameter at 5 feet}.$

if a load is uniformly distributed, stress is greatest at middle of length, and to half of it; $25 \div 2 = 12.5 = \text{cube of diameter at 5 feet}.$

ional Strength of any square bar or beam is as cube of its side, and slender as cube of its diameter. Hollow cylinders or shafts have great-torsional strength than solid ones containing same volume of material.

Compute Diameter of a Solid Shaft of Cast or Wrought Iron to Resist Lateral Stress alone.

Stress is in or near Middle. RULE.—Multiply weight by length of shaft in feet; divide product by 500 for cast iron and 560 for wrought iron; the root of quotient will give diameter in ins.

EXAMPLE.—Weight of a water-wheel upon a cast-iron shaft is 50 000 lbs., and centre of stress of wheel 7 feet from one end; what should be diameter?

$$\sqrt[3]{\left(\frac{50000 \times 30}{500}\right)} = 14.42 \text{ ins., if weight was in middle of its length}$$

the diameter at 7 feet from one end will be, as by preceding Rule, $\sqrt[3]{12.5} = \text{relative cube of diameter at 7 feet}; 30 \times 15 - 15^2 = 225 = \text{relative cube at 15 feet, or at middle of its length}.$

Ans $\sqrt[3]{225} = 14.42 :: \sqrt[3]{12.5} = 12.89 \text{ ins., diameter of shaft at 7 feet from one}$

For Bronze, 420; Cast steel, 1000 to 1500; and Puddled steel, 50

When Stress is uniformly laid along Length of Shaft. RULE.—Cube root of product of weight and length by 9.3 for Cast iron; for Wrought iron, and quotient will give diameter in ins.

EXAMPLE.—Apply rule to preceding case. $\sqrt[3]{\frac{50\,000 \times 30}{9.3}} = 12.31 \text{ ins.}$

For Bronze, 8.5; Cast steel, 18.6 to 27.9; and Puddled steel, 9.3.

When Diameter for Stress applied in Middle is given. RULE.—T root of .625 of cube of diameter, and this root will give diameter re

EXAMPLE.—Diameter of a shaft when stress is uniformly applied along its 14.43 ins.; what should be its diameter, stress being applied in middle?

$$\sqrt[3]{.625 \times 14.43^3} = \sqrt[3]{.625 \times 3000} = 12.33 \text{ ins.}$$

To Compute Diameter of a Solid Shaft of Cast I Resist its Weight alone.

RULE.—Multiply cube of its length by .007, and square root of will give diameter in ins.

EXAMPLE.—Length of a shaft is 30 feet; what should be its diameter in

$$\sqrt{(30^3 \times .007)} = \sqrt{189} = 13.75 \text{ ins}$$

HOLLOW SHAFTS.

To Compute Diameter of a Hollow Shaft of Cas to Sustain its Load in Addition to its Weig

When Stress is in or near Middle. RULE.—Divide continued p .012 times cube of length, and number of times weight of shaft in square of internal diameter added to 1, and twice square root of added to internal diameter will give whole diameter in ins.

EXAMPLE.—Weight of a water-wheel upon a hollow shaft 30 feet in len times its own weight, and internal diameter is 9 ins.; what should be wh eter of shaft?

$$2\sqrt{\left(\frac{.012 \times 30^3 \times 2.5}{1 + 9^2}\right)} + 9 = 2\sqrt{\frac{810}{82}} = 6.28 \text{ ins.}, \text{ and } 6.28 + 9 = 15.2$$

To Compute Diameter of a Round or Square S Resist Combined Stress of Torsion and Weig

RULE.—Multiply extreme of pressure upon crank-pin, or at pit pinion, or at centre of effect upon the blades of a water-wheel, et shaft may at any time be subjected to; by length of crank or wheel, etc., in feet; divide the product by Coefficient in following T cube root of quotient will give diameter of shaft or its journal in in

$$\text{Or, } \sqrt[3]{\frac{P R}{C}} = d.$$

EXAMPLE.—What should be diameter for journal of a wrought-iron w shaft, extreme pressure upon crank-pin being 59 400 lbs., and crank 5 feet

$$C = 120. \quad \sqrt[3]{\frac{59\,400 \times 5}{120}} = \sqrt[3]{2475} = 13.53 \text{ ins.}$$

When Two Shafts are used, as in Steam-vessels, etc., with On

RULE.—Divide three times cube of diameter for one shaft by cube root of quotient will give diameter of shaft in ins.

$$\text{Or, } \sqrt[3]{\frac{3 d^3}{4}} = d.$$

—Area of journal of a shaft is 113 ins.; what should be dia

$$113 = 12. \quad \text{Then } \frac{3 \times 12^3}{4} = 1296, \text{ and } \sqrt[3]{1296} =$$

Torsional Strength of Various Metals.

Wm. Wade, U. S. Ordnance Corps, 1851, Steel Committee [England, 1868], and Stevens Institute, N. J., 1878.)

*Reduced to a Uniform Measure of One Inch in Diameter or Side.
Stress applied at One Foot from Axis of Body and at Face of Axis.*

| S AND METALS. | Tensile Strength. | Destructive Stress | | Torsional Strength Wt. $\frac{C d^3}{d^2} = T$. | Coefficient $\frac{C d^3}{R} = W$. | | | | | |
|----------------------|-------------------|--------------------|---------------------------|---|-------------------------------------|-----------|------------|------------|------------|--|
| | | at 25 Ins. | Computed at 12 Ins. | | 3 Ins. | 5 Ins. | 10 Ins. | 15 Ins. | 20 Ins. | |
| CAST IRON. | Lbs. | Lbs. | Lbs. | | | | | | | |
| Diam. { 1.3 ins. } | 45 000 | 520 | 1082 | 492 | 100 | 95 | 90 | 85 | 80 | |
| Area 1 sq. inch } | | | | | | | | | | |
| Dim. { 3.25 ins. } | " | 3800 | 7904 | 230 | 45 | 40 | 35 | 30 | 25 | |
| Area 2.97 sq. ins. } | | | | | | | | | | |
| Diam. } Least ... | 9 000 | 1550 | 3664 | 530 | 130 | 125 | 120 | 115 | 110 | |
| = 1.9 } Mean ... | 31 829 | 2145 | 4462 | 650 | | | | | | |
| ins. } Greatest. | 45 000 | 2840 | 5907 | 850 | | | | | | |
| Side 1 inch | " | 350 | 728 | 728 | 125 | 120 | 115 | 110 | 105 | |
| Area 1 sq. inch } | | | | | | | | | | |
| WROUGHT IRON. | | | | | | | | | | |
| Diam. } Least ... | 38 027 | 1250 | 2600 | 376 | 120 | 115 | 110 | 105 | 100 | |
| = 1.9 } Mean ... | 56 300 | 1375 | 2860 | 416 | | | | | | |
| ins. } Greatest. | 74 592 | 1500 | 3120 | 452 | | | | | | |
| Area 2.83 sq. ins. | | | | | | | | | | |
| BRONZE. | | | | | | | | | | |
| Dim. } Least ... | 17 698 | 500 | 1040 | 152 | 30 | 28 | 26 | — | — | |
| = 1.9 } Mean ... | 56 786 | 650 | 1352 | 197 | | | | | | |
| ins. } Greatest. | | | | | | | | | | |
| Area 2.83 sq. ins. | | | | | | | | | | |
| CAST STEEL. | | | | | | | | | | |
| Dim. } Least ... | 42 000 | 2600 | 5408 | 788 | 160 | 155 | 150 | — | — | |
| = 1.9 } Mean ... | 128 000 | 7760 | 16140 | 2353 | | | | | | |
| ins. } Greatest. | | | | | | | | | | |
| Area 2.83 sq. ins. | | | | | | | | | | |
| SEMI-STEEL. | | | | | | | | | | |
| Dim. } Least ... | 36 960 | 1568 | 3261 | 1236 | 245 | 240 | 235 | 230 | 225 | |
| = 1.382 ins. } | | | | | | | | | | |
| Area 1.5 sq. ins. } | | | | | | | | | | |

Compute Diameter of Shafts of Oak and Pine.
Multiply diameter ascertained for Cast Iron as follows: Oak by 1.83,
Pine by 1.716.

Metals and Woods.

mate Torsional Strength.—Of Cast Iron may be taken as equal to its tensile strength for American and .9 for English, or as .26 of its tensile strength for American and .23 for English. Of Wrought Iron, as .7 to .8 of its tensile strength for American and .7 to 1 for English, and of Steel as its tensile strength.

astic Torsional Strength.—Of Cast Iron may be taken as equal to its tensile strength, of Wrought Iron 40 per cent. of its ultimate torsional strength, of Steel 44 per cent. of its tensile strength, and 45 per cent. of its ultimate torsional strength.

semi-Steel.—Has a torsional strength of 6670 lbs. per sq. inch at a distance of one foot, being somewhat less than that of Cast Iron, Fagersta has an ultimate transverse strength, and Siemens 44.5 per cent. of its ultimate tensile.

NOTE.—Examples here given are deduced from instances of successful practice, where diameter has been less, fracture has almost universally taken place, strain being increased beyond ordinary limit.

2.—When shafts of less diameter than 12 ins. are required, *Coefficients* here given may be slightly reduced or increased, according to quality of the metal and diameter of shaft; but when they exceed this diameter, *Coefficients* may not be increased, as strength of a shaft decreases very materially as its diameter increases.

Order of shafts, with reference to degree of torsional stress to which they may be subjected, is as follows:

1. Fly-wheel | 2. Water-wheel | 3. Secondary shaft. | 4. Tertiary, etc.
Hence, diameters of their journals may be reduced in this order.

To Compute Diameter of a Wrought-iron Centre Shaft for connecting Two Engines at a Right Angle.

Conditions of such a shaft are as follows:

Greatest stress that it is subjected to is when leading engine is at .75 of its stroke, and following engine .25 of its stroke; hence, position of each crank is as $\sin. 22^{\circ} 30' \times 2 = .7071$ of length of crank or radius of power.

Consequently,
$$\sqrt[3]{\frac{2 P \cdot 707 R}{125}} = d. \quad P \text{ representing extreme pressure on piston.}$$

NOTE.—In computing *P* it is necessary to take very extreme pressure that piston may be subjected to, however short the period of time. Average pressure does not meet requirement of case.

ILLUSTRATION.—Extreme pressure upon each piston of two engines connected at a right angle was 111 592 lbs., and stroke of pistons 10 feet; what should have been diameter of centre shaft? and what of each wheel or driving shaft?

$$\sqrt[3]{\left(\frac{111\,592 \times 2 \times .707 \frac{10}{2}}{125}\right)} = \sqrt[3]{\frac{788\,955}{125}} = 18.42 \text{ ins. centre shaft.}$$

For ordinary mill purposes, driving shafts should be as cube root of .75 cube of centre shaft.

$$\text{Thus } \sqrt[3]{\frac{18.48^3 \times 3}{4}} = 16.79 \text{ ins.}$$

To Compute Torsional Strength of Hollow Shafts and Cylinders.

RULE.—From fourth power of exterior diameter subtract fourth power of interior diameter, and multiply remainder by *Coefficient* of material; divide this product by product of exterior diameter and length or distance from axis at which stress is applied in feet, and quotient will give resistance in lbs.

$$\text{Or, } \frac{(d^4 - d'^4) C}{d l} = R.$$

EXAMPLE.—What torsional stress may be borne by a hollow cast-iron shaft, having diameters of 3 and 2 ins., power being applied at one foot from its axis?

$$C = 130. \quad 3^4 - 2^4 \times 130 = 8450, \text{ which } \div 3 \times 1 = \frac{8450}{3} = 2816.6 \text{ lbs.}$$

To Compute Torsional Strength of Round and Square Shafts.

RULE.—Multiply *Coefficient* in preceding Table by cube of side or of diameter of shaft, etc., and divide product by distance from axis at which stress is applied in feet; quotient will give resistance in lbs.

ILLUSTRATION.—What torsional stress may be borne by a cast-iron shaft of 2 ins. in diameter, power applied at 2 feet from its axis.

$$\text{Table} = 130. \quad \frac{130 \times 2^3}{2} = \frac{1040}{2} = 520 \text{ lbs.}$$

Feeling of vessel or roughness of sea the stress may be equal to diameter of journal of its shaft should be equal

GUDGEONS.

Compute Diameter of a Single Gudgeon of Cast Iron, to Support a given Weight or Stress.

LE.—Divide square root of weight in lbs. by 25 for Cast iron, and 26 for wrought iron, and quotient will give diameter in ins.

EXPL.—Weight upon a gudgeon of a cast-iron water-wheel shaft is 62 500 lbs.; should be its diameter?

$$\frac{\sqrt{62\,500}}{25} = \frac{250}{25} = 10 \text{ ins.}$$

Compute Diameter of Two Gudgeons of Cast Iron, to Support a given Stress or Weight.

LE.—Proceed as for two shafts, page 792.

Compute Ultimate Torsional Strength of Round and Square Shafts. (D. K. Clark.)

Cast Iron. Round. $\frac{.278 d^3 S}{R} = W$; $1.534 \sqrt[3]{\frac{WR}{S}} = d$; and $\frac{WR}{.278 d^3} = S$.

Wrought Iron. $\frac{.4 s^3 S}{R} = W$, and $1.36 \sqrt[3]{\frac{WR}{S}} = s$. Hollow. $\frac{.278 (d^4 - d'^4) S}{R d} = W$.

representing ultimate shearing strength, and W moment of load, both in lbs., s side of shaft, and R radius of stress, both in ins.

ILLUSTRATION.—What is ultimate torsional strength of a round cast-iron shaft in diameter, stress applied at 5 feet from its axis?

Let $S = 20\,000$ lbs. Then $\frac{.278 \times 4^3 \times 20\,000}{5 \times 12} = 5930$ lbs.

Experiments of Major Wade, ordinary foundry iron has a torsional strength of 15 lbs., or 644 lbs. per sq. inch at radius of one foot.

Now, take preceding illustration. Then $\frac{7725 \times 4^3}{5 \times 12} = 8240$ lbs.

Wrought Iron. Round. $\frac{.2224 d^3 S}{R} = W$. Square. $\frac{.32 s^3 S}{R} = W$.

When Torsional Strength per sq. inch for radius of 1 inch is ascertained, substitute C for .278, .4, .2224, or .32.

Stress which will give a bar a permanent set of $.5^\circ$ is about .7 of that which will break it, and this proportion is quite uniform, even when strength of material may vary essentially.

Wrought Iron, compared with Cast Iron, has equal strength under a stress which does not produce a permanent set, but this set commences under a less strain in wrought iron than cast, and progresses more rapidly thereafter. The strongest bar of wrought iron acquired a permanent set under a less strain than a cast-iron bar of lowest grade.

Longest bars give longest fractures.

Steel. Round. $\frac{.2 d^3 S}{R} = W$. When S is not known, substitute for S $72 s = 72$ per cent. of tensile strength.

Torsional Strength of Cast Steel is from 2 to 3 times that of Cast Iron.

Following rules are purposed to apply in all instances to diameters of shafts, or to diameter or side of bearings of beams, etc., height of journal or distance upon which strain bears does not govern diameter of journal or side of beam, etc.; hence, when length or stress is increased, diameter or side must be correspondingly increased.

Coefficients for torsional breaking stress of Iron, Bronze, and Steel.

Given by Major Wade, are: Wrought Iron, 640; Cast Iron, 560; and

Cast Steel, 1120 to 1680. Puddled Steel does not differ essentially from cast iron.

Formulas for Minimum and Maximum Diam. of Wrought-iron Shafts.

(A. E. Seaton, London, 1883, and Board of Trade, Eng.)

Compound Engines. $\sqrt[3]{\frac{D^2 p d^2 15}{C}} S = \text{diameter.}$ D and d representing diameter of low and high pressure cylinders, and S half stroke, all in ins., p pressure of steam in boiler, in lbs. per sq. inch, and C a coefficient, as follows:

| Angle of Crank. | Shafts. | | Angle of Crank. | Shafts. | | Angle of Crank. | Shafts. | | Angle of Crank. | Shafts. | |
|-----------------|----------------|----------------|-----------------|----------------|----------------|-----------------|----------------|----------------|-----------------|----------------|----------------|
| | Crank. | Propeller. | | Crank. | Propeller. | | Crank. | Propeller. | | Crank. | Propeller. |
| 90° | { 2468 4000 | { 2880 5400 | 100° | { 2279 4000 | { 2659 5400 | 110° | { 2131 4000 | { 2487 5400 | 120° | { 2016 4000 | { 2331 5400 |

$\sqrt[3]{\frac{HP}{r}} C = \text{diameter.}$ A. E. Seaton, London, 1883.

Side-wheel Engines, Sea Service.—One cylinder crank journal, C=80; outboard 100; Two cylinder crank journal 50; outboard 65; and centre shaft 58.

Propeller Engines.—One cylinder crank journal 150; Tunnel 130; Two cylinder compound crank 130; Tunnel 110; Two cranks, crank 100; Tunnel 85; Three cranks, crank 90; and Tunnel 78.

River Service.—C may be reduced one fifth.

ILLUSTRATION.—With a compound propeller engine, steam cylinders 20 and 4 ins. in diameter, by 40 ins. stroke, operating under a pressure of 80 lbs. steam (mercurial gauge), what should be the diameter of the shafts of wrought iron?

$$\sqrt[3]{\frac{20^2 \times 80 + 4^2 \times 15}{4000}} \times 40 = \sqrt[3]{\frac{56000}{4000}} \times 40 = 8.24 \text{ ins. crank shaft;}$$

$$\text{and } \sqrt[3]{\frac{56000}{5400}} \times 40 = 7.46 \text{ ins. propeller shaft.}$$

Journals of Shafts, etc.

Journals or bearings of shafts should be proportioned with reference to pressure or load to be sustained by the journal. Simplest measure of bearing capacity of a journal is product of its length by its diameter, in sq. ins.; and axial area or section thus obtained, multiplied by a *coefficient of pressure* per sq. inch, will give bearing capacity.

Sir William Fairbairn and Mr. Box give instances of weights on bearings of shafts, etc., from which following deductions are made, showing pressure per sq. inch of axial section of journal:

Crank pins, 687 to 1150 lbs. per sq. inch.

Link bearings, 456 to 690 lbs. per sq. inch.

Pressure on bearings, as a general rule, should not exceed 750 lbs. per sq. inch of axial area.

Length of Journals should be 1.12 to 1.5 times diameter.

Journals of Locomotives or Like Axles are usually made twice diameter, and to sustain a pressure of 300 lbs. per sq. inch of axial area, or 10 sq. ins. per ton of load.

Solid Cylindrical Couplings or Sleeves.

$d + \sqrt{5.5} d = D$; $3 d = L$; $.8 d = l$; $.25 d + .12 = k$. d representing diameter, and L length of sleeve, l length of lap or scarf of shaft, k breadth of key, its depth being half its breadth, and D diameter of coupling or sleeve, all in ins.

Flanged Couplings.

$\sqrt{3.5} d = D$; $3 d = F$; $.3 d + .4 = l$; $d + 1 = L$; $l \div 4 = s$. D representing diameter of body of coupling, F diameter of flanges, l thickness of both flanges, of each coupling, s projection of end of one shaft and retrocession of other of coupling, and d diameter of shaft, all in ins.

Supports for Shafts. (Molesworth.)

representing distance of supports apart, in feet.

Resist Lateral Stress. $\sqrt[3]{\frac{L W}{C}} = D$. W representing weight or pressure
re of length in lbs., and D diameter or side, if square, in ins.

of C.—Wrought Iron, 560; Cast Iron, 500; Cast Steel, 1000 to 1500; Bronze,
and Wood, 40. When Weight is distributed put 2 C.

of C for Shafting of Various Metals, as observed by different
authorities, and deduced from Formulas of Navier. $\frac{16 W r}{\pi d^3} = C$.

Ultimate Resistance.

| METAL. | C | METAL. | C | METAL. | C |
|-----------------|--------|-------------------|--------|--------------------|---------|
| FOUGHT IRON. | | CAST IRON. | | STEEL. | |
| Can, Pemb., Me. | 61 673 | | 36 846 | American, Conn.. | 82 926 |
| Ulster..... | 61 815 | American, mean { | 38 300 | " Spindle | 102 131 |
| mean..... | 66 436 | " { | 42 821 | " Nash. I. Co. | 95 213 |
| h, refined..... | 49 148 | " 18 trials { | 44 957 | English, Shear.... | 111 191 |
| "..... | 54 585 | English, mean.. { | 22 132 | Bessemer..... | 73 060 |
| th..... | 61 909 | | 38 217 | | 79 662 |

Mill and Factory Shafts. (J. B. Francis.)

Cylindrical.

$$\frac{V R}{d^3} = T. \quad \frac{\pi d^3 T}{16 R} = W.$$

Square.

$$\frac{3 \sqrt[3]{2} W R}{s^3} = T. \quad \left(\frac{s^3 T}{R} \div 3 \right) \div \sqrt[3]{2} = W.$$

Mean value of T.

| | | | | | |
|---------------|--------|--------------------|--------|--------------|---------|
| ron..... { | 22 000 | Wrought Iron.... { | 49 000 | Steel..... { | 76 000 |
| | 65 000 | " { | 94 000 | " { | 111 000 |
| mean..... | 35 000 | " mean..... | 50 000 | " mean..... | 86 000 |
| " Eng. 30 000 | | " " Eng. 45 000 | | " " Bessemer | 78 000 |

STRATION.—What is the ultimate or destructive weights that may be borne
round Cast-iron shaft 2 ins. in diameter, and by a Square shaft 1.75 ins. side,
applied at 25 ins. from axis? Assume $T = 36\,000$.

Round.

$$\frac{16 \times 2^3 \times 36\,000}{16 \times 25} = 2261.95 \text{ lbs.}$$

Square.

$$\left(\frac{1.75^3 \times 36\,000}{25} \div 3 \right) \div \sqrt[3]{2} = 1819.2 \text{ lbs.}$$

ir lengths should be reduced, and diameter increased, in following cases :

At high velocities, to admit of increased diameter of journals, thereby
ring them less liable to heating. 2d. As they approach extremity of a
f shafting. 3d. Attachment of intermediate pulleys or gearing.

Prime Movers of Power.

Transmitters of Power.

$$\text{ght } \sqrt[3]{\frac{100 \text{ HP}}{n}} = d, \text{ and } .01 n d^3 = \text{HP.} \quad \sqrt[3]{\frac{50 \text{ HP}}{n}} = d, \text{ and } .02 n d^3 = \text{HP.}$$

$$\sqrt[3]{\frac{62.5 \text{ HP}}{n}} = d, \text{ and } .016 n d^3 = \text{HP.} \quad \sqrt[3]{\frac{31.25 \text{ HP}}{n}} = d, \text{ and } .032 n d^3 = \text{HP.}$$

$$\sqrt[3]{\frac{167 \text{ HP}}{n}} = d, \text{ and } .006 n d^3 = \text{HP.} \quad \sqrt[3]{\frac{83.5 \text{ HP}}{n}} = d, \text{ and } .012 n d^3 = \text{HP.}$$

representing horse-power transmitted, n number of revolutions, and d diameter
ft in ins.

STRATION 1.—What should be diameter of a wrought-iron s
nit 128 HP at 100 revolutions per minute?

$$\sqrt[3]{\frac{50 \times 128}{100}} = \sqrt[3]{\frac{6400}{100}} = 4 \text{ ins.}$$

What HP will a steel shaft of 4 ins. diameter transmit at 100
e?

$$.032 \times 100 \times 4^3 = 204.8 \text{ horses.}$$

$$3 \times 4^3$$

TRANSVERSE STRENGTH.

Transverse or Lateral Strength of any Bar, Beam, Rod, etc., is in proportion to product of its breadth and square of its depth; in like-sided bars, beams, etc., it is as cube of side, and in cylinders as cube of diameter of section.

When One End is Fixed and the Other Projecting, strength is inversely as distance of weight from section acted upon; and stress upon any section is directly as distance of weight from that section.

When Both Ends are Supported only, strength is 4 times greater for an equal length, when weight is applied in middle between supports, than if one end only is fixed.

When Both Ends are Fixed, strength is 6 times greater for an equal length, when weight is applied in middle, than if one end only is fixed.

When Ends Rest merely upon Two Supports, compared to one When Ends are Fixed, strength of any bar, beam, etc., to support a weight in centre of it, is as 2 to 3.

When Weight or Stress is Uniformly Distributed, weight or stress that can be supported, compared with that when weight or stress is applied at one end or in middle between supports, is as 2 to 1.

Metals.

In Metals, less dimension of side of a beam, etc., or diameter of a cylinder, greater its proportionate transverse strength, in consequence of their having a greater proportion of chilled or hammered surface, compared to their elements of strength, resulting from dimensions alone.

Strength of a *Cylinder*, compared to a *Square* of like diameter or side, is as 5.5 to 8. Strength of a *Hollow Cylinder* to that of a *Solid Cylinder*, of same area of section, is about as 1.65 to 1, depending essentially upon the proportionate thickness of metal compared to diameter.

Strength of an *Equilateral Triangular Beam, Fixed at One End and Loaded at the Other*, having an *edge up*, compared to a *Square* of the same area, is as 22 to 27; and strength of one, having an *edge down*, compared to one with an *edge up*, is as 10 to 7.

NOTE.—In Barlow and other authors the comparison in this case is made when the beam, etc., rested upon *supports*. Hence the stress is contrariwise.

Strongest rectangular bar or beam that can be cut out of a cylinder is one of which the squares of breadth and depth of it, and diameter of the cylinder, are as 1, 2, and 3 respectively.

Cast Iron.

Mean transverse strength of American, as determined by Major Wade, is 681 lbs. per sq. inch, suspended from a bar fixed at one end and loaded at the other; and mean of English, as determined by Fairbairn, Barlow, and others, is 500 lbs.

Experiments upon bars of cast iron, 1, 2, and 3 ins. square, give a result of transverse strength of 447, 348, and 338 lbs. respectively; being in the ratio of 1, .78, and .756.

Woods.

Beams of wood, when laid with their annular layers vertical, are stronger than when they are laid horizontal, in the proportion of 8 to 7.

Relative Stiffness of Materials to Resist a Transverse Stress.

| | | | |
|----------|-----------------|-------------------|------------------|
|089 | Cast Iron.... 1 | Oak..... .095 | Wrought iron 1.5 |
| .. .073 | Elm..... .073 | White pine.... .1 | Yellow pine. .08 |

ngth of a Rectangular Beam in an *Inclined position*, to resist a vertical is to its strength in a horizontal position, as square of radius to square ine of elevation; that is, as square of length of beam to square of dis- between its points of support, measured upon a horizontal plane.

Transverse Strength of Various Materials.

Ordinance Department, Hodgkinson, Fairbairn, Kirkaldy, and by the Author.)

reduced to uniform Measure of One Inch Square, and One Foot in Length; Weight suspended from one End.

| METALS. | WOODS. |
|-----------------------------------|------------------------------|
| on, mean of 4 grades..... 260 | Hickory..... { 170 |
| “ (Maj. Wade)..... 660 | Iron wood, Burmah..... { 250 |
| ordinary..... 631 | Larch, Russian..... 240 |
| extreme, West P't F'dry..... 575 | Lignumvitæ..... 118 |
| gun-metal, * “ “..... 980 | Locust..... 162 |
| Eng., Low Moor, cold blast.. 740 | Mahogany..... 295 |
| “ Ponkey, “ “..... 472 | Mangrove..... 112 |
| “ Ystalyfera “ “..... 581 | Maple..... 162 |
| “ mean, 65 kinds..... 770 | Oak, white..... 202 |
| “ “ 15 kinds, cold blast..... 500 | Oak, live..... 150 |
| “ planed bar..... 641 | “ red, black..... 160 |
| “ rough bar..... 518 | “ African..... 135 |
| hammered, mean..... 534 | “ English..... { 207 |
| cast, soft..... 244 | “ French..... { 105 |
| “ hard..... 1500 | “ Dantzic..... 157 |
| hematite, hammered..... 160 | “ Canada..... 88 |
| Krupp's shaft..... 1620 | “ Sardinia..... 146 |
| Fagersta, hammered..... 2096 | “ Spanish..... 142 |
| ht Iron, mean..... 1200 | “ Pine, white..... 105 |
| “ English..... 600 | “ pitch..... 125 |
| “ Swedish†..... 475 | “ yellow..... 137 |
| | “ Georgia..... 130 |
| WOODS. | “ Poon..... 200 |
| English..... 220 | Poplar..... 184 |
| nada..... 160 | Spruce, Canada..... 112 |
| “ black..... 120 | Spruce, Canada..... 125 |
| a, Canada..... 87 | “ black..... 87 |
| white..... 130 | Sycamore..... 125 |
| white..... { 112 | Tamarack..... 100 |
| “ { 160 | Teak..... 165 |
| “ { 115 | Walnut..... 112 |
| white..... 160 | Willow..... 87 |
| Cuba..... { 63 | Whitewood..... 116 |
| ut..... { 105 | |
| ut..... 160 | STONES, BRICKS, ETC. |
| anada, red..... 125 | Brick, common, mean..... 20 |
| altic, mean..... 170 | “ pressed, “..... 40 |
| anada, yellow..... { 153 | “ English, stock..... 11.8 |
| “ red..... { 58 | “ “ fine..... 14 |
| “ { 117 | Brick arch..... 15 |
| “ { 120 | Cement, mean..... 15 |
| orway..... 123 | “ “ Portland..... { 10.2 |
| antzie..... 163 | “ “ “..... { 37.5 |
| iga..... 112 | “ “ “..... 5 |
| emel..... 161 | “ “ “..... 5 |
| “ red..... 75 | “ “ “..... 2 |
| heart, Guiana..... 160 | “ Puz..... 4.5 |
| blue..... 136 | “ Port..... 8 |
| natack..... 102 | “ Rom..... 2.5 |
| ck..... 100 | Concrete, Eng..... |

* This was with a tensile strength of 27,000 lbs.

† With 840 lbs. the deflection was 1 inch, and the elasticity of

800 STRENGTH OF MATERIALS.—TRANSVERSE

STONES, BRICKS, ETC.

| | |
|---|-------|
| Concrete, Eng. fire-brick, sand 3, } lime 1..... | .7 |
| " Eng. clay and chalk.... | 5.4 |
| Flagging, blue, New York..... | 31.25 |
| Freestone, Conn..... | 13 |
| " Dorchester..... | 10.8 |
| " New Jersey, mean..... | 19 |
| " New York..... | 24 |
| " Eng., Craigleth..... | 10.7 |
| " Darby, Victoria..... | 1.3 |
| " Park Spring..... | 4.3 |
| Glass, flooring..... | 42.5 |
| Granite, blue, coarse..... | 18 |
| " Quincy..... | 26 |
| " mean..... | 25 |
| " Eng., Cornish..... | 22 |
| Limestone..... | |
| " English..... | 11 |
| Marble, Vermont, mean..... | 92 |

STONES, BRICKS, ETC.

| | |
|---------------------------------|------|
| Marble, Adelaide..... | 45 |
| " Italian, white..... | 16 |
| Mortar, lime, 60 days..... | 24 |
| " 1 lime, 1 sand..... | 1 |
| " 1 " 2 "..... | 1.75 |
| " 1 " 4 "..... | 1.5 |
| Oolite, English, Portland..... | 22.5 |
| Paving, Scotch, Caithness..... | 68 |
| " Ireland, Valentia..... | 61.5 |
| " Welsh..... | 157 |
| " English, Yorkshire, blue..... | 10.4 |
| " Arbroath..... | 17 |
| Slate..... | 81 |
| " Bangor..... | 90 |
| " English, Llangollen..... | 43 |
| Stones, English, Bath..... | 5.2 |
| " Kentish, Rag..... | 35.5 |
| " Yorkshire, landing..... | 22.5 |
| " Caen..... | 12.5 |

Elastic Transverse Strength of Woods, compared with their Breaking Weight, is as follows:

| Per Cent. | Per Cent. | Per Cent. |
|---------------|----------------------|---------------------|
| Ash..... 29 | Norway Spruce.... 30 | Red Pine..... 29 |
| Beech..... 25 | Oak, Dantzic..... 36 | Riga Fir..... 30 |
| Elm..... 32 | " English..... 33 | Teak..... 32 |
| Larch..... 38 | Pitch Pine..... 24 | Yellow Pine..... 30 |








Increase in Strength of several Woods by Seasoning.
Per Cent.

Ash.....44.7 | Beech.....61.9 | Elm.....12.3 | Oak.....26.1 | White pine.....9

Concretes, Cements, etc.

| MATERIALS. | Breaking Weight. | MATERIALS. | Breaking Weight. |
|---------------------------------|------------------|-------------------------------|------------------|
| CONCRETES (English). | Lbs. | BRICKS (English). | Lbs. |
| Fire-brick beam, Portl'd cement | 3.1 | Best stock..... | 11.8 |
| " sand 3 parts, lime 1 part | .7 | Fire-brick..... | 14 |
| CEMENTS (English). | | New brick..... | 10.7 |
| Blue clay and chalk..... | 5.4 | Old brick..... | 9.1 |
| Portland..... | 37.5 | Stock-brick, well burned..... | 5.8 |
| Sheppey..... | 10.2 | " inferior, burned... | 2.5 |
| | 5 | | |

Transverse Strength of Various Figures of Cast Iron.
Reduced to Uniform Measure of Sectional Area of One Inch Square and One Foot in Length. Fixed at one End; Weight suspended from the other.

| Form of Bar or Beam. | Breaking Weight. | Form of Bar or Beam. | Breaking Weight. |
|---|------------------|---|------------------|
| | Lbs. | | Lbs. |
|  Square..... | 673 |  Rectangular prism. | |
| | | 2 X .5 ins. in depth... | 1496 |
| | | " 3 X .33 " in depth... | 2390 |
| | | " 4 X .25 " in depth... | 2652 |
|  Square, diagonal vertical.... | 568 |  Equilateral triangle, an edge up..... | 560 |
| | |  Equilateral triangle, an edge down..... | 958 |
| Cylinder..... | 573 |  2 ins. in depth X 2 X .268 inch in width... | 2068 |
| w cylinder; greater diameter twice that of sq..... | 794 |  2 ins. in depth X 2 X .268 inch in width... | 558 |

Solid and Hollow Cylinders of various Materials.*One Foot in Length. Fixed at one End; Weight suspended from the other.*

| Specimens. | External Diam. | Internal Diam. | Breaking Weight. | MATERIALS. | External Diam. | Internal Diam. | Breaking Weight. |
|------------|----------------|----------------|------------------|------------------|----------------|----------------|------------------|
| 1. | Ins. | Inch. | Lbs. | METAL. | Ins. | Ins. | Lbs. |
| | 2 | — | 685 | Cast iron, cold | 3 | — | 12000 |
| | 2 | 1 | 604 | blast | | | |
| | 2 | — | 772 | STONE-WARE. | | | |
| pine.. | 1 | — | 75 | Rolled pipe of } | 2.87 | 1.928 | 190 |
| " .. | 2 | — | 610 | fine clay..... } | | | |

* An inch-square batten, from same plank as this specimen, broke at 139 lbs.

Formulas for Transverse Stress of Rectangular Bars, Beams, Cylinders, etc.*Fixed at One End. Loaded at the Other.*

$$\text{1, Beams, etc. } \frac{l W}{b d^2} = S; \quad \frac{S b d^2}{l} = W; \quad \frac{S b d^2}{W} = l; \quad \frac{l W}{S d^2} = b; \quad \sqrt{\frac{l W}{S b}} = d;$$

$$\text{and Cylinder } \sqrt[3]{\frac{l W}{S}} = b \text{ and } d.$$

Fixed at Both Ends. Loaded in Middle.

$$\text{1, Beams, etc. } \frac{l W}{6 b d^2} = S; \quad \frac{6 S b d^2}{l} = W; \quad \frac{6 S b d^2}{W} = l; \quad \frac{l W}{6 S d^2} = b;$$

$$\sqrt{\frac{l W}{6 S b}} = d; \text{ and Cylinder } \sqrt[3]{\frac{l W}{6 S}} = b \text{ and } d.$$

Fixed at Both Ends. Loaded at any Other Point than in Middle.

$$\text{1, Beams, etc. } \frac{2 m n W}{3 l b d^2} = S; \quad \frac{3 l b d^2 S}{2 m n} = W; \quad \frac{2 m n W}{3 S b d^2} = l; \quad \frac{2 m n W}{3 S l d^2} = b;$$

$$\sqrt{\frac{2 m n W}{3 S l b}} = d; \text{ and Cylinder } \sqrt[3]{\frac{2 m n W}{3 S l}} = b \text{ and } d.$$

Supported at Both Ends. Loaded in Middle.

$$\text{1, Beams, etc. } \frac{l W}{4 b d^2} = S; \quad \frac{4 S b d^2}{l} = W; \quad \frac{4 S b d^2}{W} = l; \quad \frac{l W}{4 S d^2} = b;$$

$$\sqrt{\frac{l W}{4 S b}} = d; \text{ and Cylinder } \sqrt[3]{\frac{l W}{4 S}} = b \text{ and } d.$$

Supported at Both Ends. Loaded at any Other Point than in Middle.

$$\text{1, Beams, etc. } \frac{m n W}{l b d^2} = S; \quad \frac{S l b d^2}{m n} = W; \quad \frac{m n W}{S b d^2} = l; \quad \frac{m n W}{S l d^2} = b;$$

$$\sqrt{\frac{m n W}{S l b}} = d; \text{ and Cylinder } \sqrt[3]{\frac{m n W}{S l}} = b \text{ and } d.$$

Square Beams, etc., for b and d put $\sqrt[3]{\frac{l W}{S}} = \sqrt{\frac{l W}{S b}} = d$. In Cylinders, for b and d as above.

When weight is uniformly distributed, same formulas will apply. W represents only half required or given weight.

Representing stress in a Bar, Beam, or Cylinder, one foot in length, S in lbs. per sq. in. side, or in diameter; and W weight, in lbs.; b breadth, or diameter; l , m distance of weight from one end, and n from the other.

Brick-work.

A brick arch, having a rise of 2 feet, and a span of 12 feet, with a depth at its crown of 4 ins., bore 358 lbs. pressure.

Coefficient or Factor of Safety.

Coefficient or factor of safety of different materials must be taken in view of importance of structure, or instrument, probable or required period of duration of it, and if it is to bear a quiescent, vibratory, gradual, or percussive stress, and to meet these varied conditions, it will range from .125 to .3 of the maximum or ultimate strength here given or ascertained.

To Compute Transverse Strength of a Rectangular Bar or Beam.

When a Bar or Beam is Fixed at One End, and Loaded at the Other. RULE.—Multiply *Coefficient* of material in preceding Tables, or, as may be ascertained, by breadth and square of depth in ins., and divide product by length in feet.

NOTE.—When a beam, etc., is loaded uniformly throughout its length, result must be doubled.

EXAMPLE.—What weight will a cast-iron bar, 2 ins. square and projecting 30 ins. in length, bear without permanent injury?

Assume strength of material at 660, and its elasticity at one fifth or .2 of its strength.

$$\text{Then } \frac{660 \times .2 \times 2 \times 2^2}{2.5} = \frac{1056}{2.5} = 422.4 \text{ lbs.}$$

If Dimensions of a Beam or Bar are Required to Support a Given Weight at its End. RULE.—Divide product of weight and length in feet by *Coefficient* of material, and quotient will give product of breadth and square of depth.

EXAMPLE.—What is the depth of a wrought-iron beam, 2 ins. broad, necessary to support 576 lbs. suspended at 30 ins. from fixed end?

Assume strength of iron at 150.

$$\text{Then } \frac{2.5 \times 576}{150} = 9.6, \text{ and } \sqrt{\frac{9.6}{2}} = 2.19 \text{ ins. depth.}$$

When a Beam or Bar is Fixed at Both Ends, and Loaded in the Middle. RULE.—Multiply *Coefficient* of material by 6 times breadth and square of depth in ins., and divide product by length in feet.

NOTE.—When beam is loaded uniformly throughout its length, result must be doubled.

EXAMPLE.—What weight will a bar of cast iron, 2 ins. square and 5 feet in length, support in middle, without permanent injury?

Assume strength of material as in a previous case at .2 of 660.

$$\text{Then } \frac{660 \times .2 \times 2 \times 6 \times 2^2}{5} = \frac{6336}{5} = 1267.2 \text{ lbs.}$$

If Dimensions of a Beam or Bar are Required to Support a Given Weight in Middle, between Fixed Ends. RULE.—Divide product of weight and length in feet by 6 times *Coefficient* of material, and quotient will give product of breadth and square of depth.

EXAMPLE.—What dimensions will a square cast-iron bar, 5 feet in length, require to support without permanent injury a stress of 2160 lbs.?

Assume strength of material at .2 of 660 or 132, as preceding.

$$\frac{2160 \times 5}{2 \times 6} = \frac{10800}{792} = 13.64, \text{ which, divided by 2 for assumed breadth } = 6.82, \\ = 2.61 \text{ ins. depth.}$$

Breadth or Depth is Required. RULE.—Divide product obtained by square of depth, and quotient is breadth; or by root of quotient is depth.

product, and depth is 8; then $128 \div 8^2 = 2$, breadth
 $= 8$, depth.

Weight is not in Middle between Ends. RULE.—Multiply *Coefficient* of material by 3 times length in feet, and breadth and square of depth in inches, and divide product by twice product of distances of weight, or stress from either end.

EXAMPLE.—What weight will a cast-iron bar, fixed at both ends, 2 ins. square and 10 feet long, bear without permanent injury, 2 feet from one end?

SOLUTION.—The strength of material at 2 of 660 or 132, as preceding.

$$\text{Then } \frac{132 \times 3 \times 5 \times 2 \times 2^2}{2 \times (2 \times 3)} = \frac{15840}{12} = 1320 \text{ lbs.}$$

Weight of a Beam or Bar is Supported at Both Ends, and Loaded in Middle. RULE.—Multiply *Coefficient* of material by 4 times breadth and square of depth in inches, and divide product by length in feet.

—When beam is loaded uniformly throughout its length, result must be

EXAMPLE.—What weight will a cast-iron bar, 5 feet between the supports, and 2 ins. square, bear in middle, without permanent injury?

SOLUTION.—The strength of iron at 132, as preceding.

$$\text{Then } 132 \times 2 \times 4 \times 2^2 = 4224 \div 5 = 844.8 \text{ lbs.}$$

Dimensions are Required to Support a Given Weight. RULE.—Divide weight by *Coefficient* of material, and length in feet by 4 times *Coefficient* of material, and square of depth. It will give product of breadth, and square of depth.

Weight is not in Middle between Supports. RULE.—Multiply *Coefficient* of material by length in feet, and breadth and square of depth in inches, and divide product by product of distances of weight, or stress from either end.

EXAMPLE.—What weight will a cast-iron bar, 2 ins. square and 5 feet in length, without permanent injury, at a distance of 2 feet from one end, or support?

SOLUTION.—The strength of iron at 132, as preceding.

$$\text{Then } \frac{132 \times 5 \times 2 \times 2^2}{2 \times (5 - 2)} = \frac{5280}{6} = 880 \text{ lbs.}$$

How to Compute Pressure upon Ends or upon Supports.

RULE 1.—Divide product of weight and its distance from nearest end or support, by whole length, and quotient will give pressure upon end or support nearest from weight.

Divide product of weight and its distance from farthest end, or support, by whole length, and quotient will give pressure upon end or support farthest from weight.

EXAMPLE.—What is pressure upon supports in case of preceding example?

SOLUTION.— $2 = 352 \text{ lbs. upon support farthest from the weight; } \frac{880 \times 3}{5} = 528 \text{ lbs. upon support nearest to weight.}$

Weight of a Bar or Beam, Fixed or Supported at Both Ends, and Loaded at Unequal Distances from Ends.

$$\frac{W}{L} + \frac{l'w}{L} = \text{pressure at } w \text{ end, and } \frac{n'w}{L} + \frac{l'W}{L} = \text{pressure at } w' \text{ end,}$$

where *n* representing distances of greatest and least weights from nearest end, *w* greatest and least weights, *L* whole length, *l* distance from nearest end, and *l'* distance of greatest weight from farthest end.

EXAMPLE.—A beam 10 feet in length, having both ends fixed, and loaded with weights—viz., one of 1000 lbs., at 4 feet from one of its ends, and another of 4 feet from the other end; what is pressure upon each end?

$$\frac{1000}{10} + \frac{6 \times 1000}{10} = 1400 \text{ lbs. at } w; \quad \frac{4 \times 1000}{10} + \frac{6 \times 2000}{10} = 1600 \text{ lbs. at } w'$$

When Plane of Bar or Beam Projects Obliquely Upward or Downward.

When Fixed at One End and Loaded at the Other. RULE.—Multiply *efficient* of material by breadth and square of depth in ins., and divide product by product of length in feet and *cosine* of angle of elevation or depression.

NOTE.—When beam is loaded uniformly along its length, result must be doubled.

EXAMPLE.—What is weight an ash beam, 5 feet in length, 3 ins. square, and projecting upward at an angle of $7^{\circ} 15'$, will bear without permanent injury?

Assume breaking weight of ash at 160, and its elasticity at .25 of its strength at *cosine* of $7^{\circ} 15' = .992$.

$$\text{Then } \frac{160 \times .25 \times 3 \times 3^2}{5 \times .992} = \frac{1080}{4.96} = 217.74 \text{ lbs.}$$

To Compute Transverse Strength of an Equilateral Triangle or T Beam.

RULE.—Proceed as for a rectangular beam, taking following properties of *Coefficient* of material:

| | | | |
|-----------------------------------|---|--------------------------------------|---------------------------|
| <i>Fixed at One or Both Ends.</i> | { | Equilateral triangle, edge up..... | $b \times d^2 \times .12$ |
| | | Equilateral triangle, edge down..... | $b \times d^2 \times .34$ |
| <i>Supported at Both Ends.</i> | { | T beam, flange up..... | $b \times d^2 \times .42$ |
| | | Equilateral triangle, edge up..... | $b \times d^2 \times .34$ |
| | { | Equilateral triangle, edge down..... | $b \times d^2 \times .12$ |
| | | T beam, flange up..... | $b \times d^2 \times .42$ |

To Compute Transverse Strength of a Solid Cylinder.

RULE.—Proceed as for a rectangular beam, and take .6 of *Coefficient* of product.

A mean of 18 results with cold blast gun-metal, gave a coefficient for 740 lbs.

When Fixed at One End, and Loaded at the Other. RULE.—Multiply weight to be supported in lbs. by length of cylinder in feet, divide product by .6 of *Coefficient* of material, and cube root of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly throughout its length, cube root of half quotient will give diameter.

EXAMPLE.—What should be diameter of a cast-iron cylindrical beam of gun-metal 8 ins. in length, to break at 15000 lbs.?

$$\sqrt[3]{\frac{15000 \times 8 \div 12}{.6 \times 740}} = \sqrt[3]{\frac{10000}{444}} = 2.82 \text{ ins.}$$

When Fixed at Both Ends, and Loaded in Middle. RULE.—Multiply weight to be supported in lbs. by length of cylinder between supports in feet; divide product by .6 of *Coefficient* of material, and cube root of sixth of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly along its length, cube root of half quotient will give diameter.

EXAMPLE.—What is the diameter of a cast-iron cylinder of gun-metal 2 feet between supports, that will break at 35964 lbs.?

$$\frac{35964 \times 2}{.6 \times 740} = 162, \text{ and } \sqrt[3]{\frac{162}{6}} = 3 \text{ ins.}$$

Mean results of cylinder and square bars gave 444 and 740 lbs. Hence, strength of a cylinder compared to a square is as 444 to 740 or .6 to 1.

$$\text{Then } \frac{4 \times 3^3 \times 444}{1} = 47952 \text{ lbs.}$$

To Compute Diameter of a Solid Cylinder to Support a given Weight.

When Supported at Both Ends, and Loaded in Middle. RULE.—Multiply weight to be supported in lbs. by length of cylinder between supports in feet; divide product by .6 of *Coefficient* of material, and cube root of fourth of quotient will give diameter.

NOTE.—When cylinder is loaded uniformly along its length, cube root of half the weight will give diameter.

SAMPLE.—What is diameter of a cast-iron gun-metal cylinder, 1 foot between its ports, that will break at 48 000 lbs.?

$$\frac{48\,000 \times 1}{.6 \times 740} = 108, \text{ and } \sqrt[3]{\frac{108}{4}} = 3 \text{ ins.}$$

Rectangular. (D. K. Clark.)

(1) Loaded at Middle. $\frac{8 b d^2}{l} = W.$ (2) Loaded at One End. $\frac{2 b d^2}{l} = W.$

Cylindrical.

(1) Loaded at Middle. $\frac{5.5 b d^2}{l} = W.$ (4) Loaded at One End. $\frac{1.375 b d^2}{l} = W.$

W representing ultimate stress in tons.

Above Coefficients are for iron of a tensile strength of 7 tons per sq. inch.

| | (1) | (2) | (3) | (4) | | (1) | (2) | (3) | (4) |
|-----------------------|------|-----|-----|-----|-----------------|------|-----|------|-----|
| Hence, for 8 tons put | 9.2 | 2.3 | 6.3 | 1.6 | For 12 tons put | 13.8 | 3.4 | 9.4 | 2.4 |
| 9 " | 10.4 | 2.6 | 7.1 | 1.8 | 13 " | 14.5 | 3.6 | 10.2 | 2.6 |
| 10 " | 11.5 | 2.9 | 7.9 | 2 | 14 " | 16 | 4 | 11 | 2.8 |
| 11 " | 12.7 | 3.2 | 8.6 | 2.2 | 15 " | 17.2 | 4.3 | 11.8 | 3 |

3 Compute Destructive Weight, or Loads that may be borne by Wrought-iron Rolled Beams and Girders, or Riveted Tubes of various Figures and Sections.

Supported at Both Ends. Load applied in Middle.

When Section of Beam or Girder is that of any of the Figures in following Table. **RULE.**—Divide product of area of section, depth, and Coefficient girder, etc., from following Table, by length between supports in feet, quotient will give destructive weight in lbs.

NOTE.—The Coefficients given are based upon experiments with English iron.

Solid Beams.

ILLUSTRATION.—What load will destroy a wrought-iron grooved beam of following dimensions, 10 feet in length between supports, and loaded in its middle?

Flanges, 5.7 × .6 inch; Web, .6 inch; Depth, 11.75 ins.; Area, 13.34 sq. ins.

Assume Coefficient 4638 as for like case (11) in following table, page 806.

$$\frac{13.34 \times 11.75 \times 4638}{10} = \frac{726\,983}{10} = 72\,698.3 \text{ lbs.}$$

Ultimate stress for such a beam by experiment was estimated at 97 997 lbs.

Formulas of Various Authors give following Results:

1. **K. CLARK.** $\frac{d(4a + 1.1555a')}{.6l} = W.$ a representing area of section of lower flange, a' area of section of web, less one flange, d depth of beam, less average depth of one flange, all in ins., l length in feet, and W ultimate destructive weight in tons. This formula is based upon the assumption that the beam has no notch.

$$\frac{1.75 - .6(4 \times 5.7 \times .6 + 1.155 \times 11.75 - .6 \times .6)}{.6 \times 10} = \frac{238.69}{6} = 39.78 \text{ tons.}$$

39.78 tons.

2. **MOLESWORTH.** $\frac{4 C b d^2}{l} = W.$ $C = 7616 \text{ lbs.}$, and for $b d^2$ p

and d representing exterior and b' and d' interior dimensions, as

$$5.7 \times 11.75^2 - [5.7 - .6 \times 11.75 - (.64 \times 2^2)] = 786.6 \rightarrow 558.$$

$$\text{Then } \frac{4 \times 7616 \times 227.7}{10 \times 12} = \frac{6936652}{120} = 57805.4 \text{ lbs.}$$



























Fairbairn's formula would give a result less than half of the first, & is alike to that of Molesworth.

WROUGHT IRON.

Transverse Strength of Wrought-iron Rolled Beams and Girders. (Barlow, Fairbairn, Hughes, Kirkaldy, etc.)

Reduced to Uniform Measure of One Foot in Length.

Supported at Both Ends; Stress or Weight applied in Middle.

| Section. | Flanges. | Web. | Depth (d). | Distance. | Area (A). | Destructive Weight For Length of One Foot (l). | W A d |
|---|---------------|-------|------------|-------------|-----------|--|------------|
| | Inch. | Inch. | Inch. | Feet. Inch. | Sq. Inch. | Lbs. | Lbs. (W) |
|  | — | 1 | 1 | 1 | 1 | 2 500 | 2 500 |
|  | — | 2 | 2 | 2 9 | 4 | 6 600 | 18 150 |
|  | 1.5 | 3 | 2 9 | 4.5 | 10 080 | 27 720 | 205 |
|  | 1 | 3 | 3 | 3 | 7 050 | 21 150 | 139 |
|  | 1 | 1 | 5 | .78 | 474 | 2 370 | 138 |
|  | 3.5 X .6 | .8 | 3.5 | 2 7 | 5.65 | 20 160 | 52 480 |
|  | 2.5 X 1 | .325 | 7 | 2 9 | 5.9 | 44 000 | 121 000 |
|  | 4 X .38 | .85 | 5 | 4 6 | 7.44 | 19 000 | 85 500 |
|  | 2.6 X 1.25 | .5 | 7.07 | 10 | 5.87 | 24 200 | 242 000 |
|  | 3 X .49 | .5 | 9.85 | 20 | 11.5 | 38 080 | 761 600 |
|  | 4.6 X .8 | .6 | 11.75 | 10 | 13.34 | 72 688 | 726 880 |
|  | 5.7 X .64 | .31 | 2.5 | 4 | 1.75 | 3 150 | 12 600 |
|  | 2.85 X .38 | .38 | 16.5 | 22 6 | 18.9 | 49 280 | 1108 800 |
|  | 7 X .5 | .38 | 14.25 | 16 5 | 10.5 | 47 000 | 775 500 |
|  | 4 X .5 | .25 | 7 | 7 | 6.35 | 24 380 | 170 660 |
|  | 4.5 X .375 | .13 | 6 | 7 6 | 2.62 | 9 976 | 74 820 |
|  | 2 X 2 X .3125 | .53 | 24 | 30 | 41.4 | 128 885 | 3 866 550 |
|  | 4.5 X 28 | .75 | 35.75 | 45 | 87.38 | 257 080 | 11 568 600 |
|  | 4.5 X 3 | .131 | 12.4 | 10 | 5.05 | 17 885 | 178 850 |
|  | 3.9 | .75 | 12.138 | 10 | 5.56 | 26 250 | 262 500 |
|  | 3.9 | .143 | 15 | 10 | 7.72 | 102 480 | 512 400 |
|  | 15.5 | .75 | 5.2 | 5 | | | |
|  | 15.5 | | | | | | |
|  | 15.5 | | | | | | |
|  | 24 | | | | | | |
|  | 24 | | | | | | |

STEEL.

I

Results are very conclusive of the correctness of above formula, as in cases given, and they are deduced from beams and girders to 45 feet in length; hence, when length of a beam or girder sections given is less, relative breaking weight may be influence of increased stability of beam or girder.

As on Tubes and Tubular Girders, etc., see *Rep. of Comm.* London, 1849.

Iron assumed at 45 000 lbs. per sq. inch.

ed Wrought-iron Beams and Channel Bars.

With Safe Load Uniformly Distributed for Length of One Foot.

New Jersey Steel and Iron Co., Trenton, N. J.

(Beams Supported Sidewise.)

| I. | Designation. | Width. | | Area. | | Weight per Foot. | Load. |
|-----|--------------------|--------|---------|----------|----------|------------------------|-------|
| | | Web. | Flange. | Web. | Flange. | Total. | |
| | | Inch. | Ins. | Sq. Ins. | Sq. Ins. | Sq. Ins. | Lbs. |
| | I BEAMS. | | | | | | Lbs. |
| ins | Extra Light | .1875 | 2 | .75 | 1.02 | 1.77 | 6 |
| " | Light | .25 | 2.75 | 1 | 1.91 | 2.91 | 10 |
| " | Heavy | .3125 | 3 | 1.25 | 2.41 | 3.66 | 12.3 |
| " | Light | .25 | 2.75 | 1.2 | 1.79 | 2.99 | 10 |
| " | Heavy | .3125 | 3 | 1.56 | 2.34 | 3.9 | 13.3 |
| " | Light | .25 | 3 | 1.5 | 2.51 | 4.01 | 13.3 |
| " | Heavy | .3 | 3.5 | 1.8 | 3.11 | 4.91 | 16.7 |
| " | 90 lbs. | .5 | 5 | 3 | 5.7 | 8.7 | 30 |
| " | 120 " | .625 | 5.25 | 3.75 | 8.09 | 11.84 | 40 |
| " | 55 " | .3 | 3.75 | 2.1 | 3.4 | 5.5 | 18.3 |
| " | Light | .3 | 4 | 2.4 | 3.97 | 6.37 | 21.7 |
| " | Heavy | .375 | 4.5 | 2.96 | 5.07 | 8.03 | 26.7 |
| " | Light | .3 | 4 | 2.7 | 4.3 | 7 | 23.3 |
| " | Heavy | .375 | 4.5 | 3.38 | 5.12 | 8.5 | 28.3 |
| " | Extra Heavy | .57 | 4.5 | 5.13 | 7.2 | 12.33 | 41.7 |
| " | Extra Light | .3125 | 4.5 | 3.28 | 5.62 | 8.9 | 30 |
| " | Light | .375 | 4.5 | 3.94 | 6.5 | 10.44 | 35 |
| " | Heavy | .47 | 5 | 4.93 | 8.43 | 13.36 | 45 |
| " | Extra Light | .32 | 5.25 | 3.84 | 5.62 | 9.46 | 32 |
| " | Light | .39 | 5.5 | 4.68 | 7.05 | 11.73 | 40 |
| " | Light | .47 | 4.8 | 5.75 | 6.58 | 12.33 | 41.7 |
| " | Heavy | .6 | 5.5 | 7.39 | 9.38 | 16.77 | 56.66 |
| " | Extra Light | .42 | 5 | 6.35 | 6.01 | 12.36 | 41.7 |
| " | Light | .5 | 5 | 7.59 | 7.45 | 15.04 | 50 |
| " | Heavy | .6 | 5.75 | 9.07 | 10.95 | 20.02 | 66.66 |
| " | Light | .5 | 6 | 10.5 | 9.97 | 19.97 | 66.66 |
| " | Heavy | .6875 | 6.75 | 13.75 | 13.45 | 27.2 | 90.66 |
| | DECK BEAMS. | | | | | | |
| " | — | .3125 | 4.5 | 2.17 | 3.18 | 5.35 | 18.3 |
| " | — | .375 | 4.5 | 3.04 | 3.25 | 6.29 | 21.7 |
| | CHANNELS. | | | | | | |
| " | Extra Light | .2 | 1.5 | .6 | .85 | 1.45 | 5 |
| " | " " | .2 | 1.5 | .8 | .85 | 1.65 | 5.5 |
| " | " " | .2 | 1.625 | 1 | .92 | 1.92 | 6.3 |
| " | " " | .18 | 1.875 | 1.03 | 1.17 | 2.25 | 7.5 |
| " | Light | .28 | 2.25 | 1.68 | 1.52 | 3.2 | 11 |
| " | Heavy | .4 | 2.5 | 2.4 | 1.92 | 4.32 | 15 |
| " | Extra Light | .2 | 2 | 1.4 | 1.14 | 2.54 | 8.5 |
| " | Light | .25 | 2.5 | 1.75 | 1.85 | 3.6 | 12 |
| " | Extra Light | .2 | 2.2 | 1.6 | 1.7 | 3.3 | 11 |
| " | Light | .26 | 2.5 | 2.08 | 2.4 | 4.48 | 15 |
| " | " | .33 | 2.5 | 2.97 | 2.11 | 5.08 | 16.7 |
| " | Heavy | .43 | 3.125 | 3.87 | 3.15 | 7.02 | 23.3 |
| " | Extra Light | .3125 | 2.5 | — | — | 4.8 | 16 |
| " | Light | .375 | 2.75 | 3.94 | 2.06 | 6 | 20 |
| " | " | .33 | 3 | 4.04 | 2.96 | 7 | 23.3 |
| " | Heavy | .68 | 4 | 8.33 | 5.77 | 14.1 | 46.7 |
| " | Light | .5 | 4 | 7.5 | 4.5 | 12 | 40 |
| " | Heavy | .75 | 4.75 | 11.25 | 7.6 | 18.1 | 56.66 |

s given in table are such as will effect a maximum str
o lbs. per sq. inch. For permanent stress, absolutely fr
strain would be allowable, and, contrariwise, if the st
e load, the loads here given should be reduced.

A difference of 25 per cent. in either direction should be made, according to character of load to be supported or stress to be borne.

Elastic Transverse Strength of Wrought-iron Bars is about 45 per cent. of their transverse strength, of Solid rolled beams, 50 per cent.; and of double-headed rails, 46 per cent. of their transverse strength; of Fagersta Steel, 56 per cent. of its transverse strength; of double-headed Steel rails, 47 per cent.; of Bessemer Steel, 37.5 to 48 per cent.; and of Steel flanged, 68 per cent.

Transverse strength of Solid Cast-iron Beams or Girders is about 50 per cent. of ultimate strength.

NOTE.—Actual breaking weight of a 10.5 ins. beam of New Jersey Steel and Iron Co., weight 35 lbs. per foot, for a length of span of 20 feet, is 60000 lbs.

Rolled Steel Beams.

With Safe Load Uniformly Distributed for Length of One Foot.

The New Jersey Steel and Iron Co., Trenton, N. J.
(Beams Supported Sidewise.)

| Depth. | Designation. | Width. | | Area. | | Total Area. | | Weight per foot. | |
|--------|--------------|--------|---------|-----------|-----------|-------------|-----------|------------------|-------|
| | | Web. | Flange. | Web. | Flange. | Min. | Max. | Min. | Max. |
| Inch. | I BEAMS. | Inch. | Inch. | Sq. inch. | Sq. inch. | Sq. inch. | Sq. inch. | Lbs. | Lbs. |
| 1.25 | — | .125 | 1.5 | — | — | .515 | .515 | 1.75 | 1.75 |
| 4 | Light | .2 | 2.62 | — | — | 2.21 | 3.15 | 7.5 | 10.7 |
| 4 | Heavy | .24 | 2.75 | — | — | 2.94 | 3.91 | 10 | 13.3 |
| 5 | Light | .22 | 3 | — | — | 2.94 | 4.12 | 10 | 14 |
| 5 | Heavy | .26 | 3.13 | — | — | 3.81 | 5.09 | 13 | 17.3 |
| 6 | Light | .25 | 3 | — | — | 3.91 | 5.39 | 13.3 | 18.3 |
| 6 | Heavy | .3 | 3.5 | — | — | 4.89 | 6.35 | 16.6 | 21.6 |
| 7 | Light | .23 | 4 | — | — | 4.56 | 6.35 | 15.5 | 21.6 |
| 7 | Heavy | .27 | 4.25 | — | — | 5.88 | 7.82 | 20 | 26.6 |
| 8 | Light | .25 | 4.25 | — | — | 5.29 | 7.35 | 18 | 25 |
| 8 | Heavy | .27 | 4.5 | — | — | 6.47 | 8.33 | 22 | 28.3 |
| 9 | Light | .27 | 4.5 | — | — | 6.17 | 8.33 | 21 | 28.3 |
| 9 | Heavy | .31 | 4.75 | — | — | 7.94 | 10.29 | 27 | 35 |
| 10 | Light | .32 | 4.75 | — | — | 7.15 | 9.89 | 25.3 | 33.3 |
| 10 | Heavy | .37 | 5 | — | — | 9.71 | 12.26 | 33 | 41.7 |
| 10 | Extra Heavy | .45 | 5.25 | — | — | 13.24 | 15.69 | 45 | 53.33 |
| 12 | Light | .32 | 5.25 | — | — | 9.41 | 12.26 | 32 | 41.7 |
| 12 | Heavy | .39 | 5.5 | — | — | 11.74 | 14.7 | 40 | 50 |
| 15 | Light | .4 | 5.5 | — | — | 12.59 | 15.69 | 41 | 53.33 |
| 15 | Heavy | .45 | 5.75 | — | — | 14.71 | 18.63 | 50 | 63.33 |

Loads given in table are such as will effect a maximum strain upon the metal of 16000 lbs. per sq. inch, and are for the Minimum area.

Steel beams have greater estimated strength than iron, but their stiffness is not materially greater.

Operation of Tables.

To Compute Depth of a Beam to Support a Uniformly Distributed Load.

RULE.—Multiply load in lbs. by length of span in feet, and take from table the beam, load of which is nearest and in excess of product obtained.

EXAMPLE.—What should be depth of an iron beam to sustain with safety a uniformly distributed load of 30000 lbs., over a span of 15 feet?

$30000 \times 15 = 450000$, which is load for a heavy beam 12.25 ins. in depth.

Weight of beam should be added to load.

Inversely.—If the load is required, divide load in table by span of beam in feet, subtract weight of beam.

Compute Deflection of Like Beams.

Take square of span in feet by 70 times depth of beam in ins. same beam as preceding.

$$\frac{15^2}{70 \times 12.25} = \frac{225}{857.5} = .262 \text{ ins.}$$

Comparative Strength and Deflection of Cast-iron Flanged Beams.

| DESCRIPTION OF BEAM. | Comp. Strength. | DESCRIPTION OF BEAM. | Comp. Strength. |
|----------------------------|-----------------|----------------------------------|-----------------|
| of equal flanges..... | .58 | Beam with flanges as 1 to 4.5... | .78 |
| with only bottom flange.. | .72 | “ with flanges as 1 to 5.5... | .82 |
| with flanges as 1 to 2.... | .63 | “ with flanges as 1 to 6.... | 1 |
| with flanges as 1 to 4.... | .73 | “ with flanges as 1 to 6.73... | .92 |

Dimensions and Proportions of Wrought-iron Flanged Beams. (D. K. Clark.)

| th. | Breadth of Flanges. | Thickness. | | Weight per Lineal Foot. | Ultimate Strength. Loaded in Middle. | Safe Stress Uniformly Distributed. |
|-----|---------------------|------------|----------|-------------------------|--------------------------------------|------------------------------------|
| | | Web. | Flanges. | | | |
| 1. | Inch. | Inch. | Inch. | Lbs. | Lbs. | Lbs. |
| | 2 | .1875 | .2187 | 5.5 | 2 800 | 910 |
| | 3 | .25 | .3125 | 10 | 5 600 | 1 860 |
| 25 | 1.625 | .1875 | .2187 | 5.5 | 2 490 | 830 |
| | 2 | .25 | .3125 | 8 | 5 490 | 1 830 |
| | 3 | .25 | .375 | 12 | 8 510 | 2 830 |
| 5 | 2 | .25 | .3125 | 8 | 6 940 | 2 310 |
| | 3 | .3125 | .4375 | 13 | 13 440 | 4 480 |
| | 4.5 | .375 | .5 | 23 | 19 270 | 6 420 |
| 1 | 2 | .375 | .4375 | 10 | 11 880 | 3 060 |
| | 5 | .4375 | .5625 | 30 | 23 830 | 7 940 |
| | 2 | .3125 | .4375 | 11 | 13 440 | 4 440 |
| 15 | 2.25 | .3125 | .375 | 18 | 13 000 | 4 330 |
| 15 | 3.25 | .3125 | .4062 | 12.5 | 17 470 | 5 820 |
| 5 | 2.25 | .281 | .375 | 14 | 14 790 | 4 930 |
| | 2.25 | .3125 | .4375 | 14 | 17 020 | 5 070 |
| | 3.625 | .3125 | .4375 | 19 | 23 300 | 7 760 |
| 5 | 3.625 | .3125 | .5 | 19 | 25 980 | 8 660 |
| | 2.375 | .3125 | .4375 | 15 | 20 830 | 6 940 |
| | 2.5 | .375 | .375 | 15 | 21 280 | 7 090 |
| 5 | 4 | .375 | .5 | 21 | 4 500 | 11 500 |
| | 5 | .375 | .5625 | 29 | 44 800 | 14 930 |
| | 5.125 | .4375 | .5625 | 29 | 47 040 | 15 680 |
| 5 | 3.75 | .4375 | .5 | 24 | 41 560 | 13 850 |
| | 4.5 | .375 | .6875 | 30 | 59 360 | 19 750 |
| | 4.5 | .4375 | .5625 | 32 | 56 000 | 18 660 |
| 5 | 4.75 | .4375 | .5625 | 32 | 58 240 | 19 410 |
| | 4.75 | .75 | .625 | 36 | 76 160 | 25 390 |
| | 5 | .5625 | .8175 | 42 | 100 800 | 33 600 |
| 5 | 5 | .5625 | .9375 | 56 | 136 640 | 45 530 |
| | 5.5 | .5625 | .875 | 60 | 150 020 | 50 000 |
| | 6 | .5625 | .8175 | 60 | 152 260 | 50 750 |
| 5 | 5.625 | .75 | .8175 | 62 | 188 160 | 62 720 |

Wrought-iron Rectangular Girders or Tubes. (Riv'd.) Supported at Both Ends. Loaded in Middle.

$C = W$. A representing area of section in sq. ins., d depth in ins., l length between supports in feet, and W destructive weight in lbs.

ILLUSTRATION.—What is the destructive weight of a rectangular girder, 35.75 ins. thick by 24 in breadth, metal .75 inch thick, and length between supports 47 ft. Use C or coefficient = 37 00, as per case (18) in preceding table, $p = 87.375$ ins.

$$\text{Then } \frac{87.375 \times 35.75 \times 3700}{45} = \frac{11\,557\,528}{45} = 256\,833.9 \text{ lbs.}$$

experiment it was 257 080 lbs. By Inversion $\frac{W l}{C d} = A$, and $\frac{l W}{A C} = d$.

IRKINSON'S formula would give a result of 259 373 lbs., and MOLESWORTH 27 lbs.

Unequally Loaded Beams, etc.

FORM.—Representing length between supports, and a and b distance from each support, also the denomination, and W weight destructive and safe weight, with the denomination.

or Composite Destructive Weight and Area of Bottom Plate.

$$\frac{W_d}{L} = \frac{W_s}{L} = \frac{W}{L} \quad \text{and} \quad \frac{W_d}{L} = \frac{W_s}{L} = \frac{W}{L} \quad \text{representing area of plate in sq. in.}$$

a and b length and length a and b distance of load at other points from left and right, and W weight in lbs.

NOTE.—Sufficient metal should be provided in order to resist transverse crushing stress, and a upper flange to resist crushing.

EXAMPLE.—What area of wrought iron is necessary in bottom plate of a 10 ft. span, 12 in. deep, supported at both ends, and loaded in middle with 20,000 lbs.?

SOLUTION.—By experiment for destructive stress, see case 11a, and area 7.19 sq. in.

$$\frac{20,000 \times 12}{10 \times 12} = 20,000 \text{ lbs.}$$

Wrought-iron Cylindrical Beams or Tubes.

FORM.— $\frac{A d^2}{L} = W$. **EXPLANATION.**—What is destructive weight of a cylindrical beam, 12 in. in diameter, 12 ft. in thickness, and 10 ft. between its supports? Area of metal = 3.14 sq. in., and $C = 25,000$, as in the 15th case of table, page 104.

$$\text{Then } \frac{3.14 \times 12 \times 25,000}{10} = 23,568 \text{ lbs.}$$

D. E. CLARK. $\frac{3.14 d^2 t S}{L} = W$. d representing diameter, t thickness of metal, all in in., S tensile strength of metal per sq. inch, and W weight, both in lbs.

$$S = 45,000 \text{ lbs. } \frac{3.14 \times 12^2 \times 45,000}{10 \times 12} = 23,568 \text{ lbs.}$$

MINNEMOTA'S FORMULA gives a result of 23,568 lbs.

Wrought-iron Elliptical Beams or Tubes.

FORM.— $\frac{A d^2}{L} = W$. **EXPLANATION.**—Assume diameter of tube 9.75 and 15 in., and 12 ft. in thickness, and distance between supports 10 ft.

$A = 3.14 \text{ sq. in. } C = 25,000$, as per case (15) in preceding table, page 104.

$$\text{Then } \frac{3.14 \times 15 \times 25,000}{10} = 23,568 \text{ lbs.}$$

D. E. CLARK. $\frac{3.14 (b^2 + d^2) t S}{L} = W$. b and d representing conjugate extreme diameter, L length between supports, t thickness of metal, all in in., S tensile strength of metal per sq. inch, and W destructive weight, both in lbs.

$$S = 44,000 \text{ lbs. } \frac{3.14 (9.75^2 + 15^2) \times 25,000}{10 \times 12} = 23,568 \text{ lbs.}$$

NOTE.—B. Baker, in his work on Strength of Beams, etc., London, 1870, page 104, shows that ordinary method of computing transverse strength of a hollow cast-iron difference of diameter alone is erroneous, in consequence of loss of resistance to flexure in a hollow beam.

Girders and Beams of Unsymmetrical Section.

$\frac{4 S d}{L} = W$. S representing tensile resistance of metal, and W destructive weight both in lbs., d distance between centres of compression and extension, or crushing and tensile resistances, in in., and L length between supports, in feet.

NOTE.—To ascertain d , see Rule, page 819.

ILLUSTRATION.—Dimensions of a rolled wrought-iron girder, 11 feet in length between its supports, is as follows :

| | | | |
|-----------------|---------------|--------------------|---------------|
| Top flange..... | 2.5 × 1 inch. | Bottom flange..... | 4 × .38 inch. |
| Web..... | .325 " | Depth..... | 7 ins. |

What is its destructive weight?

$$W = 5.22 \text{ ins. } S \text{ assumed at } 45000 \text{ lbs. Then } \frac{4 \times 45000 \times 5.22}{11 \times 12} = 7118.18 \text{ lbs.}$$

Strength of Riveted Beams or Girders, compared with Solid, is less, and deflection is greater

Wrought-iron Inclined Beams, etc.

$\frac{L W}{l} = w$. L and l representing lengths or inclination, and horizontal line, in like nominations, and W and w destructive and safe weights on horizontal line and inclination, also in like denominations.

Plate Girders.

$\frac{A d C}{l} = W$. A representing section in sq. ins., d depth in ins., and l length between supports in feet.

ILLUSTRATION—What load will destroy a wrought-iron plate girder or beam of following dimensions, 10 feet in length between its supports?

| | | | |
|--------------------|------------------|--------------------|------------|
| Top flange..... | 4.5 × .375 inch. | Width of web..... | .375 inch. |
| Bottom flange..... | 4.5 × .375 " | Depth of web..... | 13.5 ins. |
| Angle pieces..... | 2 × .3125 " | Depth of beam..... | 14.25 " |

Area of Section = 13 sq. ins.

Assume coefficient of 5180 as per case (14) in preceding Table, page 806.

$$\text{Then } \frac{13 \times 14.25 \times 5180}{10} = \frac{959595}{10} = 95959.5 \text{ lbs.}$$

MOLESWORTH. $\frac{L l}{8 d} = S$. L representing load equally distributed, and S stress on centre, both in tons, and d effective depth of girder in feet.

By actual experiment $L = 48$ tons for 16.5 feet between supports; hence, 16.5 : 48 :: 79.2 tons = 39.6 when supported in middle, and 14.25 ins. = 1.1875 feet.

$$\text{Then } \frac{39.6 \times 10}{8 \times 1.1875} = \frac{396}{9.5} = 41.68, \text{ which } \times 2240 = 93363.2 \text{ lbs.}$$

D. K. CLARK. $\frac{d (4 a + 1.155 a')}{.6 l} = W$. d representing depth of girder or beam

depth of lower flange in ins., a and a' areas of sections of bottom flange and of web, at its reputed depth, both in sq. ins., and l length between supports in feet.

$$d = 14.25 - .375 = 13.875 \text{ ins. } a = 3, \text{ and } a' = 5 \text{ sq. ins.}$$

$$\text{Then } \frac{13.875 (4 \times 3 + 1.155 \times 5)}{.6 \times 10} = \frac{246.63}{6} = 41.105, \text{ which } \times 2240 = 92075.2 \text{ lbs.}$$

Mr Clark assumes, however, that for girders of like construction the destructive stress should be taken at two thirds of that deduced by the formula.

Girders or Beams without Upper and Lower Flanges.

ILLUSTRATION.—Assume angles 2.125 × .28 above, 2.125 × .3 below, web .25, depth 7 ins., and length between supports 7 feet.

Area of section = 6.35 sq. ins., and $C = 3840$, as per case (15) in preceding Table, page 806.

$$\text{Then } \frac{6.35 \times 7 \times 3840}{7} = \frac{170688}{7} = 24384 \text{ lbs.}$$

Approximate. $\frac{\frac{a}{2} + .25 a' \times 5 d}{l} = W$. a representing area of sections of upper and lower angles, a' area of section of web for total depth, both in sq. ins., d depth in ins., and W load or stress in lbs.

$a = 4.6$ sq. ins., and $a' = 7 \times .25 = 1.75$ sq. ins.

$$\text{Then } \frac{\frac{4.6}{2} + \frac{1.75}{4} \times 5 \times 7}{7} = \frac{95.81}{7} = 13.687, \text{ which } \times 2240 = 30658.8 \text{ lbs.}$$

IRON AND STEEL RAILS.

Symmetrical Section.

To Compute Transverse Strength. (D. K. Clark)

$$S \left(4 a \frac{d'^2}{d} + 1.155 t d^2 \right) = W, \text{ and } \frac{W l}{\left(4 a \frac{d'^2}{d} + 1.155 t d^2 \right)} = S. \text{ S representing}$$

the strength in lbs. or tons per sq. inch, a area of one head or flange exclusive of central portion composing web, in sq. ins., d' depth or distance between centres of heads, d depth of rail, t thickness of web, l distance between supports, all in ins., and W weight in lbs. or tons, alike to S .

ILLUSTRATION 1.—What is destructive weight of a wrought-iron double-headed rail, 5.4 ins. deep, having a web of .8 ins., an area of head of 1.9 sq. ins., distance between centres of its heads 4.2 ins., and between its supports 5 feet?

S assumed at 50 000 lbs.

$$\text{Then } \frac{50\,000 \left(4 \times 1.9 \times \frac{4.2^2}{5.4} + 1.155 \times .8 \times 5.4^2 \right)}{5 \times 12} = \frac{50\,000 \times (24.82 + 26.91)}{60} = 43\,125 \text{ lbs.}$$

2.—What is destructive weight of a Bessemer steel double-headed rail, 5.4 ins. deep, having a web of .75 inch, an area of head of 2 sq. ins., and distance between heads 4.2 ins.?

S assumed at 80 000 lbs.

$$\text{Then } \frac{80\,000 \left(4 \times 2 \times \frac{4.2^2}{5.4} + 1.155 \times .75 \times 5.4^2 \right)}{5 \times 12} = \frac{80\,000 \times 51.39}{60} = 68\,520 \text{ lbs.}$$

NOTE.—Transverse strength of Bessemer Rails increases very generally, in direct proportion to the proportion of Carbon in it.

Unsymmetrical Section.

$\frac{6.92 S d'' A}{l h} = W.$ d'' representing vertical distance between centres of tension and compression, h height of neutral axis above base of section, and l length between supports, all in ins., and A sum of products, obtained by multiplying areas of fibres of reduced section under tensile stress, by their mean distances, respectively, from the distances of their centres of gravity, from the neutral axis, in ins.

Bowstring Girder.

To Compute Diameter of a Wrought-iron Tie-rod of an Arched or Bowstring Girder of Cast Iron.

$\sqrt{\frac{W l}{4500 \times h}} = d.$ W representing weight distributed over beam in lbs., l length between piers or supports in feet, and h height between centre of area of section of girder and centre of rod in ins.

ILLUSTRATION.—Required diameter of tie-rod for an arched girder, 25 feet between its piers, and 30 ins. between centres of its area and of rod, to safely support a uniformly distributed load of 25 000 lbs.?

$$\sqrt{\frac{25\,000 \times 25}{4500 \times 30}} = \sqrt{\frac{625\,000}{135\,000}} = \sqrt{4.62} = 2.15 \text{ ins.}$$

If two rods are used Then $\sqrt{\frac{4.62}{2}} = 1.52 \text{ ins.} = \text{diameter of each rod.}$

CAST IRON.

Transverse Strength of Girders and Beams.

iced from Experiments of Barlow, Hodgkinson, Hughes, Bramah, Cubitt, Tredgold, and others.)

Reduced to a Uniform Measure of One Foot in Length.

Supported at Both Ends. Stress or Weight applied in Middle.

| Flanges. | Web. | Depth. | Distance. | Area. | Destructive Weight. | | $\frac{1}{2} \frac{W}{Ad} = C.$ |
|-------------|--|--------|------------|----------|---------------------|---------------------|---------------------------------|
| | | | | | For Distance. | Length of One Foot. | |
| Ins. | Ins. | Ins. | Feet. Ins. | Sq. Ins. | Lbs. | Lbs. | |
| — | 1 | 1 | 1 | 1 | 2 240 | 2 240 | 2240 |
| — | 1 | 1 | 4 6 | 1 | 500 | 2 250 | 2250 |
| — | 3 | 3 | 13 6 | 9 | 5080 | 63 580 | 2540 |
| — | 1 | 3 | 4 6 | 3 | 5 100 | 22 950 | 2550 |
| — | 1 | 4 | 4 6 | 4 | 10 300 | 46 350 | 2896 |
| 4 X 2 | 2 | 4 | 5 | 12 | 6 720 | 33 600 | 700 |
| 1.52 X .78 | 1.56 | 4.07 | 4 6 | 2.35 | 6 666 | 30 000 | 3136 |
| 1.5 X .5 | .5 | 3 | 3 1 | 2 | 5 208 | 16 145 | 2676 |
| 1.5 X .5 | .5 | 3 | 3 1 | 2 | 4 536 | 14 062 | 2331 |
| 1.5 X .5 | .5 | 4 | 3 1 | 1 | 7 104 | 22 420 | 5475 |
| 1.5 X .5 | .5 | 4 | 3 1 | 1 | 3 312 | 10 267 | 2553 |
| 1.53 X 1 | .5 | 2.04 | 4 | 2.6 | 4 004 | 16 016 | 3019 |
| 2 X .51 | 1 | 2.02 | 4 | 2.59 | 2 569 | 10 276 | 1963 |
| — | — | 2.52 | 5 | 4.98 | 4 143 | 20 715 | 1650 |
| — | — | 2.83 | 5 | 4 | 2 988 | 14 940 | 1320 |
| 2.28 X .53 | $\left\{ \begin{array}{l} .3 \\ .425 \end{array} \right\}$ | 5.13 | 4 6 | 2.28 | 9 503 | 42 763 | 3656 |
| 23.9 X 3.12 | 3.29 | 36.1 | 20 | 183.5 | 403 312 | 8 066 240 | 1220 |
| 1.76 X .4 | .29 | 5.13 | 4 6 | 2.82 | 6 678 | 30 512 | 2077 |
| 1.74 X .26 | .3 | 5.13 | 4 6 | 2.87 | 7 368 | 33 200 | 2250 |
| 1.78 X .55 | .32 | 5.13 | 4 6 | 3.02 | 8 270 | 37 215 | 2402 |
| 1.07 X .3 | .34 | 5.13 | 4 6 | 5.41 | 21 009 | 94 540 | 3406 |
| 2.1 X .57 | 1.25 | 8.18 | 11 | 15 | 35 620* | 391 853 | 3193 |
| 1.54 X .32 | | | | | | | |
| 6.5 X .51 | | | | | | | |
| 2.5 X 1.5 | | | | | | | |
| 3.75 X 1.4 | | | | | | | |

* Stirling Iron.

on, $\frac{AdC}{l} = W.$ A representing area of section, d depth in ins., l length in feet, destructive weight in lbs.

—When lengths are less than those instanced, breaking weight will be 1 in consequence of increased stability of girder.

To Compute Transverse Strength or Destructive Stress of Cast-iron Beams or Girders, of various Figures.

Supported at Both Ends. Weight applied in Middle.

When Section of Beam or Girder is alike to any of Examples given in preceding Table. RULE 1.*—Divide product of area of section and depth in ins., and Coefficient for girder, etc., from preceding Table, by length between supports in feet, and quotient will give breaking weight in lbs.

EXAMPLE.—Dimensions of a beam, having top and bottom flanges in proportion of 1 to 6, give an area of section of 25.6 sq. ins., a depth of 15.5 ins., and a length between its supports of 18 feet; what is its destructive weight?

NOTE.—In consequence of increased area of metal over case No. 21 in Table, Coefficient of 3402 is reduced to 3300.

Dimensions.—Top flange, $3 \times .75$ ins.; bottom, $18 \times .75$ a = 13.5 sq. ins.; web, $15.5 \times .7$ a' = 10.8 sq. ins., and d' = 15.5 — .75 = 14.75 ins.

$$\text{Then } \frac{25.6 \times 15.5 \times 3300}{18} = \frac{1309440}{18} = 72746.6 \text{ lbs.}$$

D. K. CLARK. $\frac{d' (6.5 \uparrow a + 2 a')}{3 \downarrow} = W$. a representing area of bottom flange, d of web at depth d' of beam, less depth of bottom flange in sq. ins., l length between supports in feet, and W destructive weight in tons.

$$\text{Then } \frac{14.75 (7 \times 13.5 + 2 \times 10.8)}{3 \times 18} = \frac{1712.4}{54} = 31.71, \text{ which } \times 2240 = 71030.4 \text{ lbs.}$$

HODGKINSON'S formula would give a result of 53491.2 lbs., and MOLESWORTH'S 54248.3 lbs.

RULE 2.—From product of breadth and square of depth in ins. of rectangular solid, the dimensions of which are the depth and greatest breadth of beam in its centre, subtract product of breadths and square of depths of that part of the beam which is required to make it a rectangular solid, and then determine its resistance by rule for the particular case as to its being supported or fixed, etc.

This rule is applicable only in case referred to, viz., when area of section is great compared with area of extreme dimensions.

Mr. Baker, in case of a hollow cylindrical shaft, where thickness of metal is but one eighth of extreme diameter, computes result at but .4 of that of a solid beam. This is in consequence of resistance to flexure in hollow beam being more than proportionally greater than in solid.

EXAMPLE.—Take 7th case from preceding Table, page 813, for length of one foot. Coefficient for cold-blast iron = 500.

$$\text{Then } 1.52 \times 4.07^2 - 1.52 \times 2.51^2 \times 4 \times 500 = (25.17 - 9.58) \times 2000 = 31180 \text{ lbs.}$$

Result as by experiment, 30000 lbs.

NOTE 1.—These rules are applicable to all cases where flange of beam is as shown in Table, and beam rests upon two supports, or contrariwise, as to position of flange, when beam is fixed at one end only.

2.—When case under consideration is alike in its general character to one in Table, but differs in some one or more points, an increase or decrease of metal is obtained by an increase or reduction of the Coefficient, according as the differences may affect resistance of beam.

3.—The Coefficients here given are based altogether upon experiments with English iron.

* Utility of these rules in preference to those of Hodgkinson, Fairbairn, Tredgold, Hughes, and Barlow is manifest, as in one case the Coefficient of the metal is considered, and in the other cases the metal is assumed to be of a uniform value or strength.

variable element not embraced in this rule is that consequent upon any peculiarity of form of section, for instance, in that of a Hodgkinson, or like beam, where area of one flange greatly exceeds that of the other, and this flange is other than below, when beam rests upon two supports or is fixed at one end, and this flange is other than below, when beam rests upon two supports or is fixed at one end, when beam is fixed at one or both ends.

to some extent by the three cases in table, where proportion of flanges are 1 to 6.

— Coefficient same as tensile strength of metal in tons per sq. inch.

Flanged Hollow or Annular Beams of Symmetrical Sections. (*D. K. Clark.*)

When Depth is Great Compared with Thickness of Flanges.—Figs. 1, 2, and 3.

1. 2. 3.
$$\frac{d \times S (4a + 1.155 a')}{l} = W.$$
 a representing area of one flange, *a'* area of web or ribs, both in sq. ins., *d* depth of beam, less depth of one flange, and *l* distance between supports, both in ins., *S* tensile strength of metal, and *W* weight between supports, both in lbs.

Then Depth of Flanges is Great Compared with Depth of Beam.—Figs. 4 and 5.

4. 5.
$$\frac{S (4a \frac{d'^2}{d} + 1.155 t d^2)}{l} = W.$$
 a representing area of one flange less thickness of web, in sq. ins., *t* thickness of web, *d'* reputed depth or distance between centres of flanges, and *d* depth of beam, all in ins.

Then Section of Circular or Elliptic Beam is Small Compared with Diameter.—Figs. 6, 7, and 8.

6. 7. 8.
$$\frac{3.14 d^2 t S}{l} = W. \quad \frac{1.57 (b^2 + d^2) t S}{l} = W.$$

and *d* representing mean breadth and depth.

ILLUSTRATION 1.—Assume Figs. 1, 2, and 3. 20 ins. in depth, width of flanges on top and bottom ribs 5 ins., thickness of flanges and webs 1 inch, and of sides of g. 3.5 inch; length between supports 10 feet, and *S* 20 000 lbs.; what would be the weight of each?

Then
$$\frac{20 - 1 \times 20000 (4 \times 5 + 1.155 \times 18)}{10 \times 12} = \frac{380000 (20 + 20.79)}{120} = 129\ 168.4 \text{ lbs.}$$

2.—Assume Figs. 4 and 5. 6 ins. in depth, area of flanges 3 ins., widths of webs 1 in., and length and *S* as in preceding case.

Then
$$\frac{20000 \left(4 \times 3 \times \frac{6-1}{6} + 1.155 \times 1 \times 6^2 \right)}{10 \times 12} = \frac{20000 \times 91.58}{120} = 15\ 263.3 \text{ lbs.}$$

3.—Assume Fig. 6 10 ins. in diameter, Fig. 7. 7.5 ins. in depth and 12 ins. in width, Fig. 8. 12 ins. in depth and 7.5 ins. in width, and thickness of all metal 1 inch.

Then, Fig. 6
$$\frac{3.14 \times 10^2 \times 1 \times 20000}{10 \times 12} = \frac{6280000}{120} = 52\ 333.3^* \text{ lbs., which is .4 of wt of solid cylinder.}$$

Figs. 7 and 8
$$\frac{1.57 \times (12^2 + 7.5^2) \times 1 \times 20000}{10 \times 12} = \frac{6287850}{120} = 52\ 398.75 \text{ lbs.}$$

NOTE.—For all ordinary purposes, operation of computing their strength, by first computing that of their circumscribing figure, and then deducting from it strength due to difference between it and section of beam under computation, will be sufficiently accurate. See Illustration, page 814.

If greater accuracy is required, see page 810, or *D. K. Clark's Manual*, pp. 513

NOTE.—To compute location of neutral axis of beams of unsymmetrical section, also *D. K. Clark*, pp. 514-15.

This result agrees with deduction of Mr. Baker, as given by him in his work on Strength of B. pp. 25-7, for hollow or annular beams of small area of section compared with that of diameter up to 1 thickness of metal of one eighth of diameter. He assigns their strength so low as that of solid cylinder, in consequence of loss of resistance to flexure.

General Formulas for Destruction of Beams of Symmetrical

Supported at Both Ends. Weight

Line of Neutral Axis runs through centre of representing area and depth of section, r radius of length of beam between its supports in ins., W and S tensile strength of material in like tons or lbs.

ILLUSTRATION.—Assume dimensions of cast-iron follows, viz.: 1 and 2, 5×5 ins.; 3, 2.5×10 ; 4, or equal areas; distance between supports 60 ins. 20000 lbs.



3.

Areas of each 25 sq. ins. Radius of gyration, .5; and 5, 1.43.

$$1. \frac{2 \times 25 \times 10 \times .5775 \times 26000}{60}$$

$$2. \frac{2 \times 25 \times 7.07 \times .4083 \times 26000}{60}$$

$$3. \frac{2 \times 5^3 \times .5775 \times 26000}{60}$$

4. For formula for square beams

$$\text{Then } 4. \frac{25 \times 5.64 \times 26000}{60} = 61100 \text{ lbs.};$$

$$.7854 \times 4.39 \times 7.25^2 \times 26000$$

These formulas give a result equal to a transverse tensile strength of 26000 lbs., and of Wrought of 50000 lbs. (as per table, page 788).



$\frac{C b d^2}{l} = W$. C representing coefficient of and depth in ins., l length in feet, and W

$\frac{4 - r^4}{R} 4.7 = b d^2$. R and r representing

$\frac{b^3 - b' d'^3}{4} = W$. b' and d' representing

$\frac{b d^2}{4} = W$. d representing

and d representing

width and depth of

to strength of Cast

Wrought Iron of 50

at of Inertia of a Solid Beam.—Fig. 2.

2.  $\frac{b d^3}{12} = M.$

ents of preceding case.

$\frac{20^3}{12} = \frac{64000}{12} = 5333.33 \text{ moment.}$

enting breadth of vertical divisions, n number of horizontal neutral axis, b breadth, and d depth of beam.

nts of preceding case.


$t=2$, $n=5$, and $b=8$.


20×2 for lower half = 4800 = moment.

Beams of Various Figures.—Figs. 3, 4, 5.


3. $\frac{b d^3 - b' d'^3}{12}$, 4 and 5. $\frac{b d^3 - 2 b' d'^3}{12} = M.$

b' and d' representing respectively breadth less thickness of web, and depth less thickness of flanges.

 $.7854 c t^3 = M.$

 $\frac{b d^3}{36} = M.$

 $\frac{s^4}{12} = M.$

 $.11 r^4 = M.$

verse and c conjugate diameters, and s side.

on Centre of Gravity and Vertical Centres of Crushing and Tensile or Beam.

of section of each part or figure composing the from centre of one of the two extreme parts of their products by sum of surfaces of secance of common centre of gravity from centres re.

ure.

$1 \times 0 = 2.5 \times 0 = .0$

$\left(\frac{5.62}{2} + \frac{1}{2}\right) = .325 \times 3.31 = 1.076$

$4 \times \left(\frac{.38}{2} + 5.62 + \frac{1}{2}\right) = 1.52 \times 6.31 = 9.591$

55 = distance of common centre from centre of upper

$1.52 \times 0 = .0$

$\left(\frac{5.62}{2} + \frac{.38}{2}\right) = 1.826 \times 3 = 5.478$

$\left(\frac{1}{2} + 5.62 + \frac{.38}{2}\right) = 2.5 \times 6.31 = 15.775$

31 = distance of common centre from centre of low

stance of common centre from bottom, and $3.631 +$

centre of gravity

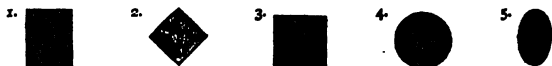
General Formulas for Destructive Weight of Solid Beams of Symmetrical Section.

Supported at Both Ends. Weight applied in Middle.

Line of Neutral Axis runs through centre of gravity of section.

$\frac{a d r S}{l} = W$, and $\frac{l W}{a d r} = S$. In square beams for a & d put d^2 . a and d representing area and depth of section, r radius of gyration (half depth of beam = $\frac{1}{2} l$ length of beam between its supports in ins., W destructive weight in tons or lbs., and S tensile strength of material in like tons or lbs. per sq. inch.

ILLUSTRATION.—Assume dimensions of cast-iron beams, Figs. 1, 2, 3, 4, and 5 as follows, viz.: 1 and 2, 5×5 ins.; 3, 2.5×10 ; 4, 5.64 diameter; and 5, 7.25×4.39 or equal areas; distance between supports 60 ins., and tensile strength of iron = 20 000 lbs.



Areas of each 25 sq. ins. Radius of gyration, No. 1, .5775; 2, .4083; 3, .5775; 4, .5; and 5, 1.43.

$$1. \frac{2 \times 25 \times 10 \times .5775 \times 20\,000}{60} = 125\,125 \text{ lbs.}$$

$$2. \frac{2 \times 25 \times 7.07 \times .4083 \times 20\,000}{60} = 62\,545 \text{ lbs.}$$

$$3. \frac{2 \times 5^2 \times .5775 \times 20\,000}{60} = 62\,562 \text{ lbs.}$$

$$4. \text{ For formula for square beams substitute } \frac{a d S}{l} = W$$

$$\text{Then } 4. \frac{25 \times 5.64 \times 20\,000}{60} = 61\,100 \text{ lbs.; and for } 5. \frac{.7854 d^2 S}{l} = W.$$

$$\frac{.7854 \times 4.39 \times 7.25^2 \times 20\,000}{60} = 78\,532 \text{ lbs.}$$

These formulas give a result equal to a transverse strength for Cast iron of 50 000 lbs. a tensile strength of 26 000 lbs., and of Wrought iron of 60 000 lbs. for a like strength of 50 000 lbs. (as per table, page 788).



$\frac{4 C b d^2}{l} = W$. C representing coefficient of strength of metal in lbs., b and d breadth and depth in ins., l length in feet, and W destructive weight in tons.

$$6. \frac{R^4 - r^4}{R} 4.7 = b d^2. \text{ } R \text{ and } r \text{ representing external and internal radius}$$

$$\frac{b d^3 - b' d'^3}{d} = b d^2. \text{ } b' \text{ and } d' \text{ representing interior breadth and depth}$$

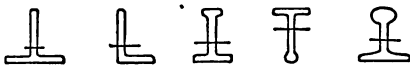
$$38 R^3 = b d^2. \quad 9. \frac{b d^2}{4} = W. \text{ } d \text{ representing depth or height.}$$

$d'^2 = W$. b and d representing breadth and depth of centre and d' breadth and depth of horizontal rib, external to central rib
 r a tensile strength of Cast Iron of 26 000 lbs. per sq. inch, and
 g th of Wrought Iron of 50 000 lbs., and *pro rata*.

mal of square.

† In square beams $d = a \times d$.

Beams of Unsymmetrical Section. (D. K. Clark.)



S representing total tensile strength of section in lbs. per sq. inch, d distance between centres of tension and compression in ins., l length in ins., w weight in lbs.

EX.—If the sectional area of a beam of cast iron is 5.9 sq. ins., the distance between centres of tension and compression 5.6 ins., distance between supports 5.5 feet, and tensile strength of metal 30000 lbs. per sq. inch.

$$\text{Then } \frac{4 \times 5.9 \times 30000 \times 5.6}{5.5 \times 12} = \frac{3964800}{66} = 60072.7 \text{ lbs.}$$

STEEL.

Compute Transverse Strength of Steel Bars.

Supported at Both Ends. Weight applied in Middle.

$= W.$ S representing tensile strength in lbs., l length between supports in feet, w weight in lbs.

EX.—What is ultimate destructive stress of a bar of Crucible steel, 2 feet between supports? $S = 90000$ lbs.

$$\text{Then } \frac{1.155 \times 90000 \times 2^3}{2 \times 12} = \frac{831600}{24} = 34650 \text{ lbs.}$$

Compute Section of Lower Flange of a Girder or Circular Shaft of Cast Iron to Sustain a Safe Load in Middle. (Baker.)

l representing distance between supports in feet, d depth of girder, etc., W weight in tons, C coefficient, and M moment of weight around support.

EX.—What should be section of a girder, 12 ins. deep, to sustain a safe load in its middle, between supports 16 feet apart?

$$\text{Load } 2 \text{ tons per sq. inch, and Factor of safety } 4. \quad \frac{16 \times 12 \times 10}{4} = 480 = M.$$

$= a.$ S representing stress assumed in tons, and a area of section of ins.

$$\text{Then } \frac{480}{12 \times 2} = 20 \text{ sq. ins.}$$

Angular, Diagonal, or Circular Beam or Shaft.

$$\frac{d^2 b}{6} = M.$$



$$\frac{d^3}{10.2} = M.$$



$$\frac{d^3}{8.4} = M.$$

Formulas for Computation of Destructive Stress of a Beam or Girder of any form of Cross Section and of any Material. (B. Baker.)




Load applied at Middle.

$= W.$ S representing tensile strength of material per sq. inch in tons, d resistance of section = product of effective depth of girder or beam, and of flange portion of section, in sq. ins., Q resistance of supports in feet, and $Q' = Q \times$ thickness of web in ins.

Average Values of S for Various Materials

| Tons. | | Tons. | |
|-------|----------|----------|----|
| 7 | Steel | 40 to 50 | 0 |
| 21 | " plates | 35 | P. |

Substituting Values of S and Q in a General Equation.

| Section. | Cast Iron. | Wrought Iron. | Steel. | Oak. | Pine. |
|---|----------------------------|--------------------------|--|---|--|
|  | $W = .875 \frac{d^2 b}{l}$ | $= 1.75 \frac{d^2 b}{l}$ | $= 3 \text{ to } 5 \frac{d^2 b}{l}$ | $= .14 \text{ to } .25 \frac{d^2 b}{l}$ | $= .11 \text{ to } .1 \frac{d^2 b}{l}$ |
|  | $W = .75 \frac{d^3}{l}$ | $= 1.5 \frac{d^3}{l}$ | $= 2.625 \text{ to } 4.25 \frac{d^3}{l}$ | $= .1 \text{ to } .16 \frac{d^3}{l}$ | $= .08 \text{ to } .14 \frac{d^3}{l}$ |
|  | $W = .5625 \frac{d^3}{l}$ | $= 1.125 \frac{d^3}{l}$ | $= 2 \text{ to } 3.25 \frac{d^3}{l}$ | $= .08 \text{ to } .14 \frac{d^3}{l}$ | $= .06 \text{ to } .11 \frac{d^3}{l}$ |

d representing depth of a rectangular bar, side of a square, or diameter of a round bar, all in ins., and l distance between supports in feet.

Moment of Resistance.

Moment of Resistance of a cross section is the static force resisting an external force of tension or compression, and it is equal to moment of inertia divided by distance of centre of effect of the area of fibres which are respectively the most extended or compressed from the neutral axis of the section.

To Compute Moment of Resistance.

$\frac{I}{d} = M$. I representing moment of inertia, and d distance of centre of effect of area of fibres of extension or compression.

Work of Resistance.

Under a Quiescent Load.—Intensity of Elastic resistance increases uniformly with total space through which action of stress operates; hence, S may be defined by a triangular section.

Consequently, $.5 S L = R$. S representing space passed through, L load, and R resistance.

To Compute Moment of Resistance.

$\frac{6 C I}{h}$ and $\frac{M I}{h} = R$. C a coefficient = one sixth of destructive weight, I moment of inertia, h height of neutral axis from base of section, R moment of resistance, and M modulus of rupture.

NOTE.—Neutral axis, for all practical purposes, is at centre of gravity of any section.

For Radius of Gyration, see Centre of Gyration, page 609.

For other rule for computation of Moment of Resistance, see Strength of Beam, B. Baker, London, 1870.

Moment of Inertia.

Moment of Inertia is resistance of a beam to bending, and moment of any transverse section is equal to sum of products of each particle of its area into square of their distance from neutral axis of section.

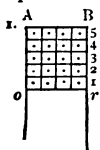


ILLUSTRATION.—If transverse section of a beam, $A B C D$, Fig. 1, is 8×20 ins., its neutral axis will be at middle of its depth, O ; divide $A B$, $O r$, into any number of equal spaces, as shown, then each space will be $2 \times 2 = 4$ sq. ins., and the distances of the centre of each square from neutral axis will be as follows:

1, 1. $2 \times 2 \times 4 \times 1^2 = 16$ | 4, 4. $2 \times 2 \times 4 \times 7^2 = 784$
 2, 2. $2 \times 2 \times 4 \times 3^2 = 144$ | 5, 5. $2 \times 2 \times 4 \times 9^2 = 1296$
 3, 3. $2 \times 2 \times 4 \times 5^2 = 400$ | 2640×2 for lever half = $5280 = \text{moment}$.

If the area of the figure in illustration had been more minutely divided, it would have approximated more nearly to the above result.

of Inertia of a Revolving Body, see Centre of Gyration, page 609.

compute Moment of Inertia of a Solid Beam.—Fig. 3.

$$2. \quad \frac{b d^3}{12} = M.$$

TRATION.—Take elements of preceding case.

$$\text{Then } \frac{8 \times 20^3}{12} = \frac{64\,000}{12} = 5333.33 \text{ moment.}$$

$t^3 n^3 b = M.$ t representing breadth of vertical divisions, n number of horizontal divisions from plane of neutral axis, b breadth, and d depth of beam.

TRATION.—Take elements of preceding case.

$$t = 2, n = 5, \text{ and } b = 8.$$

$$.3 \times 2^3 \times 5^3 \times 8 = 2400 \times 2 \text{ for lower half} = 4800 = \text{moment.}$$

Beams of Various Figures.—Figs. 3, 4, 5.
 $3. \quad \frac{b d^3 - b' d'^3}{12}, 4 \text{ and } 5. \quad \frac{b d^3 - 2 b' d'^3}{12} = M.$
 b' and d' representing respectively breadth less thickness of web, and depth less thickness of flanges.

$$\begin{array}{lll} \text{Fig. 3: } .7854 r^4 = M. & \text{Fig. 4: } .7854 c t^3 = M. & \text{Fig. 5: } \frac{b d^3}{36} = M. \\ .7854 (r^4 - r'^4) = M. & \text{Fig. 6: } \frac{s^4}{12} = M. & \text{Fig. 7: } .11 r^2 = M. \end{array}$$

representing radius, t transverse and c conjugate diameters, and s side.

compute Common Centre of Gravity and Vertical distance between Centres of Crushing and Tensile stress of a Girder or Beam.

E.—Multiply surface of section of each part or figure composing by distance of its centre from centre of one of the two extreme parts res, as .; divide sum of their products by sum of surfaces of second result will give distance of common centre of gravity from centres of extreme part or figure.

PLE.—Take annexed figure.

$$\begin{array}{l} \text{Above} \left\{ \begin{array}{l} 2.5 \times 1 \times 0 = 2.5 \times 0 = .0 \\ .325 \times \left(\frac{5.62}{2} + \frac{1}{2} \right) = .325 \times 3.31 = 1.076 \\ .38 \times 4 \times \left(\frac{.38}{2} + 5.62 + \frac{1}{2} \right) = 1.52 \times 6.31 = 9.591 \end{array} \right. \\ \hline 4.345 \qquad 10.667 \end{array}$$

ing 10.667 by 4.345 = 2.455 = distance of common centre from centre of upper


$$\begin{array}{l} \text{Below} \left\{ \begin{array}{l} 1.52 \times 0 = 1.52 \times 0 = .0 \\ .325 \times 5.62 \times \left(\frac{5.62}{2} + \frac{.38}{2} \right) = 1.826 \times 3 = 5.478 \\ 2.5 \times \left(\frac{1}{2} + 5.62 + \frac{.38}{2} \right) = 2.5 \times 6.31 = 15.775 \end{array} \right. \\ \hline 5.846 \qquad 21.253 \end{array}$$

ing 21.225 by 5.846 = 3.631 = distance of common centre from centre of

$3, 3.631 + \frac{.38}{2} = 3.821$ = distance of common centre from bottom, and
 1.283 = distance between centres of gravity.

Girders, Beams, Lintels, etc.

Transverse or Lateral Strength of any Girder, Beam, Breast-summer, Lintel, etc., is in proportion to product of its breadth and square of its depth, and area of its cross-section.

Best form of section for Cast-iron girders or beams, etc., is deduced from experiments of Mr. E. Hodgkinson, and such as have this form of section  are known as Hodgkinson's.

Rule deduced from his experiments directs, that *area of bottom flange should be 6 times that of top flange*—flanges connected by a thin vertical web, sufficiently rigid, however, to give the requisite lateral stiffness, tapering both upward and downward from the neutral axis; and in order to set aside risk of an imperfect casting, by any great disproportion between web and flanges, it should be tapered so as to connect with them, with a thickness corresponding to that of flange.

As both Cast and Wrought iron resist compression or crushing with a greater force than extension, it follows that the flange of a girder or beam of either of these metals, which is subjected to a crushing strain, according as the girder or beam is *supported at both ends, or fixed at one end*, should be of less area than the other flange, which is subjected to extension or a tensile stress.

When girders are subjected to impulses, and sustain vibrating loads, as in bridges, etc., best proportion between top and bottom flange is as 1 to 4; as a general rule, they should be as narrow and deep as practicable, and should never be deflected to more than .002 of their length.

In Public Halls, Churches, and Buildings where weight of people alone are to be provided for, an estimate of 175 lbs. per sq. foot of floor surface is sufficient to provide for weight of flooring and load upon it. In computing other weight to be provided for it should be that which may at any time bear upon any portion of their floors; usual allowance, however, is for a weight of 280 lbs. per sq. foot of floor surface for stores and factories.

In all uses, such as in buildings and bridges, where the structure is exposed to sudden impulses, the load or stress to be sustained should not exceed from .2 to .16 of breaking weight of material employed; but when load is uniform or stress quiescent, it may be increased to .3 and .25 of breaking weight.

An open-web girder or beam, etc., is to be estimated in its resistance on the same principle as if it had a solid web. In cast metals, allowance is to be made for loss of strength due to unequal contraction in cooling of web and flanges.

In Cast Iron, the mean resistances to Crushing and Extension are, for American as 4.55 to 1, and for English as 5.6 to 7 to 1; and in Wrought Iron are, for American as 1.5 to 1, and for English as 1.2 to 1; hence the mass of metal below neutral axis will be greatest in these proportions when stress is intermediate between ends or supports of girders, etc.

Wooden Girders or Beams, when sawed in two or more pieces, and slips are set between them, and whole bolted together, are made stiffer by the union, and are rendered less liable to decay.

Beams cast with a face up are stronger than when cast on a side, in the ratio of 1 to .96, and they are strongest also when cast with bottom

optimal construction of a Girder or Beam, with reference to strength with least material, is as follows: The outline of

tom, and sides should be a curve of various forms, according as or depth throughout is equal, and as girder or beam is loaded only end, or in middle, or uniformly throughout.

King Weights of Similar Beams are to each other as Squares of their near Dimensions.

Board of Trade regulations in England, iron may be strained to 5 tons inch in tension and compression, and by regulation of the Ponts et Chênes, France, 3.81 tons.

ts .75 and 1 inch in diameter, and set 3 ins. from centre in top of and 4 ins. at bottom.

acter of fracture, as to whether it is crystalline or fibrous, depends character of blows; thus, sharp blows will render it crystalline, and ill not disturb its fibrous structure.

spans exceeding 40 feet, wrought iron is held to be preferable to n.

ting, when well executed, is not liable to be affected by impact or 7 of load.

upled Girder or Beam is one composed of two, fastened together, and over the other.

Trussed Beams or Girders.

ght and Cast Iron possess different powers of resistance to tension and com- 1; and when a beam is so constructed that these two materials act in uni- h each other at *stress due to load required to be borne*, their combination will n essential economy of material. In consequence of the difficulty of adjust- nsion-rod to the stress required to be borne, it is held to be impracticable to ct a perfect truss beam.

airn declares that it is better for tension of truss-rod to be low than high, osition is fully supported by following elements of the two metals.

ght Iron has great tensile strength, and, having great ductility, it undergoes elongation when acted upon by a tensile force. On the contrary, *Cast Iron* at crushing strength, and, having but little ductility, it undergoes but little ion when acted upon by a tensile stress; and, when these metals are re- from the action of a high tensile stress, the *set* of one differs widely from the other, that of wrought iron being the greatest.

r same increase of temperature, expansion of wrought is considerably great- that of cast iron; 1.87* tons per sq. inch is required to produce in wrought ne extension as in cast iron by 1 ton.

airn, in his experiments upon English metals, deduced that within limits ss of 13,440 lbs. per sq. inch for cast iron, and 30,240 lbs. per sq. inch for t iron, tensile force applied to wrought iron must be 2.25 times tensile force to cast iron, to produce equal elongations.

ive tensile strengths of cast and wrought iron being as 1 to 1.35, and their ice to extension as 1 to 2.25, therefore, where no initial tension is applied to rod, cast iron must be ruptured before wrought iron is sensibly extended.

tance of cast iron in a trussed beam or girder is not wholly that of tensile h, but it is a combination of both tensile and crushing strengths, or a trans- strength; hence, in estimating resistance of a trussed beam or girder, trans- length of it is to be used in connection with tensile strength of truss.

: transverse strength of a cast-iron bar, one inch square and or supported at both ends, stress applied in the middle, *without se* ; and as mean tensile strength of wrought iron, also *without se* bs. per sq. inch, ratio between sections of beams and of truss sb 'transverse strength per sq. inch of beam and of tensile strength c rs under consideration are those alone in which truss is attachc rwer flange, in which case it presents following conditions:

Elongation of cast and wrought iron being 5500 and 10 000, hence $10\ 000 \div 5500 = 1.8$

1. When truss runs parallel to lower flange. 2. When truss runs at an inclination to lower flange, being depressed below its centre. 3. When beam is arched upward, and truss runs as a chord to curve.

Consequently, in all these cases section of beam is that of an open one with a cast-iron upper flange and web, and a wrought-iron lower flange, increased in its resistance over a wholly cast-iron beam in proportion to the increased tensile strength of wrought iron over cast iron for equal sections of metals.

From various experiments made upon trussed beams, it is shown :

1. That their rigidity far exceeds that of simple beams; in some cases it was from 7 to 8 times greater. 2. That when truss resists rupture, upper flange of beam being broken by compression, there is a great gain in strength. 3. That their strength is greatly increased by upper flange being made larger than lower one. 4. That their strength is greater than that of a wrought-iron tubular beam containing same area of metal.

Comparative Value of Wrought-iron Bars, Hollow Girders, or Tubes of Various Figures (English).

| | | | |
|----------------------------------|-----|--------------------------------------|----|
| Circular tubes, riveted | 1 | Circular, uniform thickness | 17 |
| Flanged beams | 1.2 | Plate beams | 17 |
| Elliptic tubes, riveted | 1.3 | Elliptic, uniform thickness | 17 |
| Rectangular tubes, riveted | 1.5 | Rectangular, uniform thickness | 17 |

General Deductions from Experiments of Stephenson, Fairbairn, Cubitt, Hughes, etc.

Fairbairn shows in his experiments that with a stress of about 12 320 lbs. per sq. inch on cast iron, and 28 000 lbs. on wrought iron, the sets and elongations are nearly equal to each other.

A cast-iron beam may be bent to .3 of its breaking weight if load is laid on gradually; and .16 of it, if laid on at once, will produce same effect, if weight of beam is small compared with weight laid on. Hence, beams of cast iron should be made capable of bearing more than 6 times greatest weight which will be laid upon them.

In beams of cast or wrought iron, if fixed or supported at both ends, flanges should be in proportion to relative resistances of material to crushing or extension.

Breaking weights in similar beams are to each other as squares of their like linear dimensions; that is, breaking weights of beams are computed by multiplying together area of their section, depth, and a *Constant*, determined from experiments on beams of the particular form under investigation, and dividing product by distance between supports.

Cast and wrought-iron beams, having similar resistances, have weights nearly as 2.44 to 1.

A box beam or girder, constructed of plates of wrought-iron, compared to a single rib and flanged beam **I**, of equal weights, has a resistance as 100 to 93.

Resistance of beams or girders, where depth is greater than their breadth, when supported at top, is much increased. In some cases the difference is fully one third.

When a beam is of equal thickness throughout its length, its curve of equilibrium, to enable it to support a uniform stress with equal resistance in every part, should be an *Ellipse*, and if beam is an open one, its curve of equilibrium, for a uniform load, should be that of a *Parabola*. Hence, when middle portion is not wholly removed, its curve should be a compound of an ellipse and a parabola, approaching nearer to the latter as the middle part is decreased.

Girders of cast iron, up to a span of 40 feet, involve a less cost than of wrought iron.

Cast-iron beams and girders should not be loaded to exceed .2, or subjected to a stress than .166 of their destructive weight; and when the stress is attended with concussion and vibration, this proportion must be increased.

le cast-iron girders may be made 50 feet in length, and best form is that of a parabola when subjected to a fixed load, flanges should be as 1 to 6, and when as 1 to 4.

aces exceeding limit of those of simple cast iron are varied are those of straight or arched cast-iron girders in d together—Trussed, Bowstring, and wrought-iron Box

Straight or Arched Girder, formed of separate castings, is entirely dependent on bolts of connection for its strength.

Trussed or Bowstring Girder is made of one or more castings to a single piece, its strength depends, other than upon the depth or area of it, upon the proper adjustment of the tension, or the initial strain, upon the wrought-iron truss.

Box or Tubular Girder is made of wrought iron, and is best constructed with stiff-iron tops, in order to resist compression: this form of girder is best adapted to ordinary lateral stiffness.

When a girder has four or more supports, its condition as regards a stress on its middle is essentially that of a beam fixed at both ends.

The following results of the resistances of materials will show how they could be distributed in order to obtain *maximum* of strength with *minimum* dimensions:

| | To Tension. | To Crushing. | | To Tension. | To Crushing. |
|----------------|-------------|--------------|-------------------|-------------|--------------|
| Cast iron..... | { 21 000 | 90 300 | Oak, white, mean. | 11 000 | 7 500 |
| | { 32 000 | 140 500 | “ English “ | 6 500 | 3 100 |
| “ English.. | { 13 000 | 58 000 | Wrought iron.... | { 45 000 | 47 000 |
| | { 23 000 | 116 000 | | { 59 000 | 83 000 |
| Granite..... | { 578 | 15 000 | “ English | { 31 000 | 40 000 |
| | { 670 | 4 000 | | { 53 000 | 65 000 |
| Limestone..... | { 2 800 | 9 000 | Yellow pine..... | 16 000 | 4 000 |

The best iron has greatest tensile strength, and least compressive or crushing.

Conditions of Forms and Dimensions of a Symmetrical Beam or Girder.

When Fixed at One End, and Loaded at the Other.

1. *When Depth is uniform throughout entire Length*, section at every point must be in proportion to product of length, breadth, and square of depth, and square of depth is in every point the same, breadth must vary directly as length; consequently, each side of beam must be a vertical plane, tapering gradually to end.

2. *When Breadth is uniform throughout entire Length*, depth must vary as square root of length; hence upper or lower sides, or both, must be determined by a parabolic curve.

3. *When Section at every point is similar, that is, a Circle, an Ellipse, a Square, or a Rectangle, Sides of which bear a fixed Proportion to each other*, the section at every point being a regular figure, for a circle, the diameter at every point must be as cube root of length; and for an ellipse or a rectangle, breadth and depth must vary as cube root of length.

ILLUSTRATION.—A rectangular beam as above, 6 ins. wide and 1 foot in depth at its extreme end, and 4 feet in length, is capable of bearing 6480 lbs.; what should its dimension at 3 feet?

$$\sqrt[3]{4} = 1.587, \text{ and } \sqrt[3]{3} = 1.442.$$

Then $1.587 : 1.442 :: 1 : .9086$, and $6 \text{ and } 12 \times .9086 = 5.452 \text{ and } 10.9$.

$$\text{Hence } \frac{5.452 \times 10.9^2}{3} = 216, \text{ and } \frac{6 \times 12^2}{4} = 216.$$

When Fixed at One End, and Loaded uniformly throughout its Length.

1. *When Depth is uniform throughout its entire Length*, breadth must increase as the square of length.

2. *When Breadth is uniform throughout its entire Length*, depth varies directly as length.

3. *When Section at every point is similar, as a Circle, Ellipse, Square, or Rectangle*, section at every point being a regular figure, cube of depth is in ratio of square of length.

ILLUSTRATION.—Take preceding case.

Then $4^2 : 3^2 :: 12^2 : 972$, and $\sqrt{972} = 9.9$ in depth.

When Supported at Both Ends.

1. *When Loaded in the Middle, Coefficient or Factor of Safety* of the beam, or product of breadth and square of depth, must be in proportion to distance from nearest support; consequently, whether the lines forming the beam are straight or curved, they meet in the centre, and of course the two halves are alike.

2. *When Depth is Uniform throughout*, breadth must be in ratio of length

3. *When Breadth is Uniform throughout*, depth will vary as square root of length.

4. *When Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle*, section at every point being a regular figure, cube of depth will be as square of distance from supported end.

When Supported at Both Ends, and Loaded uniformly throughout its Length.

1. *When Depth is Uniform*, breadth will be as product of length of beam and length of it on one side of given point, less square of length on one side of given point.

2. *When Breadth is Uniform*, depth will be as square root of product of length of beam and length of it on one side of given point, less square of length on one side of given point.

3. *When Section at every point is similar, as a Circle, Ellipse, Square, and Rectangle*, section at every point being a regular figure, cube of depth will be as product of length of beam and length of it on one side of given point, less square of length on one side of given point.

Elliptical-sided Beams.

To Determine Side or Curve of an Elliptical-sided Beam.

$\sqrt{\frac{Ll}{2Cb}} = d$. L representing load in lbs., l length in feet, C coefficient, and b breadth in ins.

ILLUSTRATION.—What should be depth in centre of a beam of white pine, 10 feet in length between its supports, and 5 ins. in breadth, to support a load of 10 000 lbs.?

Assume $C = 100$. Then $\sqrt{\frac{10\,000 \times 10}{2 \times 100 \times 5}} = \sqrt{\frac{100\,000}{1000}} = 10$ ins.

Hence, outline of beam is that of a semi-ellipse, having 10 feet for its transverse diameter, and 9 ins. for its semi-conjugate.

NOTE.—Weight of Girder, Beam, etc., should in all cases be added to stress or load

Miscellaneous Illustrations.

—What should be side of a rectangular white oak beam, 2 ins. in width, and 6 between its supports, to sustain a load of 360 lbs.?

stress at .2 of breaking weight of 150 lbs. = 30.

$$\sqrt{\frac{6 \times 360}{4 \times 2 \times 30}} = \sqrt{\frac{2160}{240}} = 3 \text{ ins.}$$

be breadth and depth of such a beam if square?

$$\sqrt[3]{\frac{6 \times 360}{4 \times 30}} = \sqrt[3]{\frac{2160}{120}} = 2.62 \text{ ins.}$$

or of a cylinder?

$$\sqrt[3]{\frac{120}{4}} = 3.1 \text{ ins.}$$

STEEL.

To Compute Transverse Strength of Steel Bars.

Supported at Both Ends. Weight applied in Middle.

$\frac{1.155 S b d^2}{l} = W$. S representing tensile strength in lbs., l length between supports in ins., and W weight in lbs.

ILLUSTRATION.—What is ultimate destructive stress of a bar of Crucible steel 2 ins. square, and 2 feet between supports? $S = 90\ 000$ lbs.

$$\text{Then } \frac{1.155 \times 90\ 000 \times 2^3}{2 \times 12} = \frac{831\ 600}{24} = 34\ 650 \text{ lbs.}$$

Elastic Transverse Strength is 50 per cent. of its ultimate strength.

Hardening in oil increases its strength from 12 to 56 per cent. Thus, Soft steel, 121 520 lbs.; soft steel, cooled in water, 90 160 lbs.; soft steel cooled in oil, 215 120 lbs.

Krupp's is about .45 of its tensile breaking weight, .24 of its compressive or crushing strength, .38 of its transverse, and .39 of its torsional.

Friction of a steel shaft compared to one of wrought iron is as .625 to 1.

Capacity of steel to resist a transverse stress is much less than to resist torsion.

Relative diameters of steel and wrought-iron shafts, to resist equal transverse stress, are as .98 to 1, and weight of such a proportion of steel shaft compared with one of wrought iron will be about 4 per cent. less, and friction of bearing will be 6 per cent. less.

CYLINDERS, FLUES, AND TUBES.

Hollow Cylinders. Cast Iron.

To Compute Elements of Hollow Cylinders within Limits of Elastic Strength. (D. K. Clark.)

$S \times \text{hyp. log. } R = P$. $\frac{P}{\text{hyp. log. } R} = S$. $\frac{P}{S} = \text{hyp. log. } R$. S representing elastic tensile strength of metal in lbs. per sq. inch, R ratio of external diameter to internal, $\frac{d'}{d} = \frac{r'}{r}$, and P internal pressure in lbs. per sq. inch. d and d' representing internal and external diameter, and r and r' internal and external radii, all in ins.

NOTE.—Hyperbolic Logarithm of a number is equal to product of its common logarithm and 2.3026

ILLUSTRATION 1.—Diameters of a hydrostatic cylinder 5.3 by 13.125 ins.; what pressure within its elastic strength will it sustain per sq. inch?

$$\text{Assume } S = 10\ 000 \text{ lbs. } \text{Hyp. log. } R = \frac{13.125}{5.3} \times 2.3026 = \log. 2.5 \times 2.3026 = .92$$

$$\text{Then } 10\ 000 \times .92 = 9200 \text{ lbs. per sq. inch.}$$

NOTE.—For Bursting Strength take maximum strength of metal.

2.—A water-pipe .75 inch thick has an internal diameter of 10 ins., what is its bursting pressure?

$$S = 30\ 000 \text{ lbs. } \text{Hyp. log. } \frac{10 + .75 \times 2}{10} = .1398$$

$$\text{Then } 30\ 000 \times .1398 = 4194 \text{ lbs.}$$

3.—If it were required of a hydrostatic press to sustain a pressure of r upon a ram of 5 ins. in diameter, what would be pressure on ram, and v be thickness of metal, assuming it equal to an elastic tensile stress of S per sq. inch?

$$\text{Area of 5 ins.} = 19.635. \frac{589\ 050}{19.635} = 30\ 000 = \text{pressure per sq. inch on}$$

$$\text{Then } \frac{30\ 000}{15\ 000} = 2, \text{ which } = \text{hyp. log. } R = 7.39, \text{ and } 7.39 \times 5 = 36.95 = \text{ex}$$

$$\text{meter. } 36.95 - 5 = 31.95, \text{ which } \div 2 = 15.975 \text{ ins. thickness of metal.}$$

Wrought Iron and Steel.

$$\frac{R + \text{hyp. log. } \frac{d'}{d} - 1}{2} S = P. \quad \frac{2P}{R + \text{hyp. log. } \frac{d'}{d} - 1} = S. \quad \frac{2P}{S} + 1 = (P + \text{hyp. log. } \frac{d'}{d}) S$$

ILLUSTRATION 1.—If diameters of a wrought-iron cylinder are 5 and 15 ins., and ultimate or destructive strength of metal is 40 000 lbs. per sq. inch, what is its bursting pressure?

$$\frac{15}{5} = 3. \quad \text{Hyp. log. } 3 = .47712 \times 2 \times 3026 = 1.0986.$$

Then $\frac{3 + 1.0986 - 1}{2} \times 40\,000 = 61\,972 \text{ lbs. per sq. inch} = 61\,972 \times 5 \div 15 - 3 = 30\,986.2 \text{ lbs. per sq. inch of section of metal.}$

2.—A steam-boiler 6 feet in internal diameter, of wrought-iron plates .375 inch thick and double riveted longitudinally, burst at a joint by a pressure of 300 lbs. per sq. inch; what was resistance of joint per sq. inch of its section?

$$\frac{72 + \frac{.375 \times 2}{72}}{72} = 1.0104. \quad \text{Hyp. log. } 1.0104 = .010345.$$

$$\text{Then } \frac{2 \times 300}{1.0104 + .010345 - 1} = \frac{600}{.020745} = 29\,405 \text{ lbs. per sq. inch of section of joint.}$$

SHIP AND BOILER PLATES.

(See pages 751-757 for Boiler Riveting.)

Ultimate Tensile Strength of Riveted and Welded Joints of Wrought-iron Plates. (D. K. Clark.)

Entire Plate = 100.

| JOINTS. | Plate. | | | Average. | JOINTS. | Plate. | | | Average. |
|--|----------|----------|----------|----------|--|--------|------|-------|----------|
| | .5 | .375 | .4375 | | | .5 | .375 | .4375 | |
| Scarf-welded..... | — | 102 | 106 | 104 | Double riv'd, snap- headed..... | 59 | 72 | 70 | 67 |
| Lap-welded..... | 50 | 66 | 69 | 62 | “ “ counter- sunk and snap- headed..... | 53 | 69 | 72 | 68 |
| Single hand riveted. “ “ snap- headed..... | 40 50 | 60 56 | 50 52 | 50 53 | “ “ with single welt, counters'k and snap-headed | 52 | 65 | 60 | 58 |
| “ “ by machine | 40 | 52 | 54 | 49 | | | | | |
| “ “ counter- sunk head... | 44 | 52 | 50 | 49 | | | | | |

Strength of Riveted Joints per Sq. Inch of Single Plate. (Wm. Fairbairn.)

Single Lapped.—Machine riveted. Pitch 3 times, 25 000 lbs.

Hand riveted. Pitch 3 times, 24 000 lbs.

Rivets “staggered,” and equidistant from centres, 30 500 lbs.

Abut Joints.—Hand riveted. Rivets not “staggered,” and equidistant from centres, single cover or strap, 30 000 lbs.

Rivets “square,” single cover or strap, 42 000 lbs.; double covers or straps, 55 000 lbs.

Comparative Strength of Riveted Joints.

Entire Plate .375 ins. thick = 100.

| | | | |
|---|----|---|----|
| Double riveted, double strap, or fish- plated joint..... | 80 | Double riveted, single strap, or fish- plated joint..... | 80 |
| Double riveted lap joint..... | 72 | Single riveted lap joint..... | 72 |

For all joints of plates over .5 inch, other than double welded, these proportions are too high.

A closer pitch of rivets should be adopted in single than in double riveted joints, etc.

Dimensions of Rivets, Pitch, Lap, etc.

| No. 1088. | Diam. of Rivet. | Length from Head. | Pitch. | L a p. | | |
|--------------|--------------------|----------------------|--------|---------|---------|------------|
| | | | | Single. | Double. | Staggered. |
| 1. | Ina. | Ina. | Ina. | Ina. | Ina. | Ina. |
| | .5 | 1.125 | 1.5 | 1.5625 | 2.75 | 2.4375 |
| 25 | .625 | 1.375 | 1.625 | 2 | 3.4375 | 3 |
| 5 | .75 | 1.625 | 1.75 | 2.4375 | 4.125 | 3.625 |
| | .8125 | 2.25 | 2.125 | 2.625 | 4.4375 | 3.9375 |
| 25 | .9375 | 2.75 | 2.375 | 3 | 5.1875 | 4.5625 |
| 5 | 1 | 3 | 2.625 | 3.25 | 5.5 | 4.8125 |
| | 1.125 | 3.25 | 3 | 3.625 | 6.1875 | 5.4375 |
| 5 | 1.25 | 4 | 3.375 | 4 | 6.875 | 6.0625 |
| | 1.5 | 4.5 | 4.375 | 4.875 | 8.25 | 7.25 |

ps. — Single, .125 thicker than the plate; Double, each .625 of thickness of

To Compute Diameter of Rivet.

linarily, $T 1.25 + .1875 = d$. T representing thickness of plate, and d diameter k .

Pitch of Rivets. (Nelson Foley.)

| No. | Metal between the Holes. | Diam. of Rivets. | Plates. | Metal between the Holes. | Diam. of Rivets. |
|------|-----------------------------|---------------------|-----------|-----------------------------|---------------------|
| | | | | | |
| | 52 to 62 per cent. | 1.4 to 2.3 | Square.. | 70 to 78 per cent. | .99 to 1.7 |
| red. | 68 to 75 " " | 1.4 to 2.1 | Triple... | 76 to 80 " " | .77 to 1 |

Proportions of Single Rivet Wrought-iron Joints.

(French.)

| No. | Diameter of Rivets. | | Pitch of Rivets. | | Width of Lap. | | Thickness of Plate. | | Diameter of Rivets. | | Pitch of Rivets. | | Width of Lap. | |
|-----|------------------------|-------|---------------------|------|------------------|------|------------------------|-------|------------------------|-------|---------------------|------|------------------|------|
| | Mil's. | Inch. | Mil's. | Ina. | Mil's. | Ina. | Mil's. | Inch. | Mil's. | Ina. | Mil's. | Ina. | Mil's. | Ina. |
| 18 | 8 | .315 | 27 | 1.06 | 30 | 1.18 | 10 | .394 | 20 | .787 | 56 | 2.2 | 58 | 2.28 |
| 58 | 10 | .394 | 32 | 1.26 | 34 | 1.34 | 11 | .433 | 21 | .827 | 57 | 2.24 | 60 | 2.36 |
| 77 | 12 | .472 | 37 | 1.46 | 40 | 1.58 | 12 | .472 | 22 | .866 | 58 | 2.28 | 60 | 2.36 |
| 36 | 14 | .551 | 43 | 1.69 | 44 | 1.73 | 13 | .512 | 23 | .906 | 60 | 2.36 | 62 | 2.44 |
| 76 | 16 | .63 | 48 | 1.89 | 50 | 1.97 | 14 | .551 | 24 | .945 | 62 | 2.44 | 64 | 2.52 |
| 15 | 17 | .669 | 51 | 2.01 | 54 | 2.13 | 15 | .591 | 25 | .984 | 63 | 2.48 | 66 | 2.6 |
| 54 | 19 | .748 | 54 | 2.13 | 56 | 2.2 | 16 | .63 | 26 | 1.024 | 65 | 2.56 | 68 | 2.68 |

Alt of Experiments on Double Riveted and Double
Strapped Plate Joints. (Mr. Brunel.)

20 ins. in width, 5 inch thick, Abut jointed, with a Strap or Fish-plate on each side, 10 ins. in width. Holes Punched.

.6875 inch rivets, 4 ins. pitch, set "square," tensile strength 77 per cent.

75 " " " " "staggered," " " 78.6 "

75 " " " 5 " " "square," " " 84 "

Boiler Riveting see pp. 755-57.

Hulls of Vessels.

Diameter of Rivets.

| Plate. | U. S. and British Lloyds. | Liverpool Reg'y. | Admiralty, Eng. | Millwall, Eng. | Pitch of Rivets. | Length of Rivet Counter- sunk. | Length of Rivet Square- headed. |
|--------|---------------------------------|---------------------|--------------------|-------------------|---------------------|--------------------------------------|---------------------------------------|
| Inch. | Inch. | Inch. | Inch. | Inch. | Inch. | Inch. | Inch. |
| .3125 | .625 | .5 | .5 | .625 | 1.75 | 1.125 | 1.5 |
| .375 | .625 | .625 | .625 | .625 | 2 | 1.25 | 1.625 |
| .4375 | .625 | .625 | .75 | .625 | 2.125 | 1.375 | 1.75 |
| .5 | .75 | .75 | .75 | .75 | 2.25 | 1.5 | 2 |
| .5625 | .75 | .75 | .875 | .75 | 2.437 | 1.6875 | 2.1875 |
| .625 | .75 | .8125 | .875 | .875 | 2.50 | 1.9375 | 2.375 |
| .6875 | .875 | .875 | .875 | .875 | 2.812 | 2.1875 | 2.625 |
| .75 | .875 | .875 | 1 | .875 | 3.125 | 2.375 | 2.75 |
| .8125 | .875 | .9375 | 1 | .875 | 3.375 | 2.5 | 2.875 |
| .875 | 1 | 1 | 1.125 | 1 | 3.625 | 2.625 | 3 |
| .9375 | 1 | 1.0625 | 1.125 | 1 | 3.875 | 2.75 | 3.125 |
| 1 | 1 | 1.125 | 1.125 | 1 | 4.125 | 2.875 | 3.25 |

Lap of Joint or Course should be .5 pitch of rivets added to .3 diam. of rivet.

NOTE.—Lloyd's requires a spacing of 4.5 diameter. Liverpool Registry, 4 diam. in edges and abuts of bottom and bulkhead plates, and 5 to 6 in other water-tight work. Bureau Veritas, 4 diameters for single riveting, and 4.5 for double.

STEEL PLATES.

Steel Plates, according to M. Barba, .354 inch thick are equal to wrought iron .472 inch thick, or as 3 to 4; consequently, when iron rivets are used their diameter should be in proportion to an iron plate.


It is ascertained also that they are best united by iron rivets.


A steel plate .3125 inch thick requires an iron rivet .5625 inch in diameter, and 1.375 ins. apart.

Bridge Plates and Rivets.

Plates .25 to .5 inch thick. Rivets .75 to 1 inch diameter, and 3 ins. apart from centres in upper flange or girder, and 4 ins. in lower

Rivet Heads.

1.  Ellipsoidal, Fig. 1.—D diameter, R radius of head = D, r radius of flange = .4 D, c depth at centre = .5 D.

2.  Segmental, Fig. 2.—D diameter, c depth at centre = .625 D, R radius of head = .75 D, o depth below head = .125 D.

Countersunk.—Head 1.52 D, angle 60°. Countersink .45 diam. of plate. Cheesehead or heads, section of which is a parallelogram. Head .45 D, diameter 1.5 D.

Rivets.

Shearing strength of a Lowmoor rivet = $40\ 320\ d^2$ or $18\ d^2$ in tons. d representing diameter of rivet in ins.

Memoranda.

Punching holes for riveting weakens plates, varying from 10 to 20 per cent. according to their temper, hardest losing most.

Countersunk riveting does not impair strength of joint, as compared with external head.

Diagonal abut joints are stronger than square.

Shearing strength of rivets should not exceed that of plates.

Maximum strength of joint is attained at 90 to 100 per cent. of net section of plates.

Shearing strength of English wrought iron is taken at 80 per cent. of its tensile strength.

LEAD PIPE.

Resistance of Lead Pipe to Internal Pressure.
(Kirkaldy, Jardine, and Fairbairn.)

| Thick- ness. | Weight per Foot. | Bursting Pressure. | Diam. | Thick- ness. | Weight per Foot. | Bursting Pressure. | Diam. | Thick- ness. | Weight per Foot. | Bursting Pressure. |
|-----------------|------------------------|-----------------------|-------|-----------------|------------------------|-----------------------|-------|-----------------|------------------------|-----------------------|
| Inch. | Lbs. | Lbs. | Inch. | Inch. | Lbs. | Lbs. | Inch. | Inch. | Lbs. | Lbs. |
| .2 | 2.3 | 1579 | 1.25 | .21 | 5.3 | 683 | 2 | .21 | 9.2 | 498 |
| .2 | 2.6 | 1349 | 1.5 | .24 | 7.1 | 734 | 2 | .2 | — | 448 |
| .22 | 3.8 | 1191 | 1.5 | .2 | — | 528 | 3 | .25 | — | 364 |
| .2 | 4.1 | 911 | 1.5 | .2 | — | 626 | 3 | .25 | — | 374 |

Strength of metal = 2240 lbs. per sq. inch.

Compute Thickness of a Lead Pipe when Diameter and Pressure in Lbs. per Sq. Inch is given.

E.—Multiply pressure in lbs. per sq. inch by internal diameter of pipe and divide product by twice tensile resistance of metal in lbs. per sq.

EXAMPLE.—Diameter of a lead pipe is 3 ins., and pressure to which it is to be fitted is 370 lbs. per sq. inch; what should be thickness of metal?

$$\frac{370 \times 3}{2240 \times 2} = \frac{1110}{4480} = .248 \text{ ins.}$$

Difference in Weight between Pipes of "Common," "Middling," and "Strong" is 10 per cent.

To Compute Weight of Lead Pipe.

$\overline{d^2} \times 3.86 = W$. D and d representing external and internal diameters in ins., weight of a lineal foot in lbs.

Compute Maximum or Bursting Pressure that may be borne by a Lead Pipe.

E.—Multiply tensile resistance of metal in lbs. per sq. inch by twice thickness of pipe, and divide product by internal diameter, both in ins.

EXAMPLE.—What is bursting pressure of a lead pipe 3 ins. in diameter and .24 inch thick?

$$\frac{2240 \times .5 \times 2}{3} = \frac{2240}{3} = 746.6 \text{ lbs.}$$

How much water will a column of water 34 feet in height to weigh 15 lbs. per sq. inch; what water would such a pipe sustain at point of rupture?

$$15 : 34 :: 746.6 : 1692.3 \text{ feet.}$$

Resistance of Glass Globes and Cylinders to Internal Pressure and Collapse. (Flint Glass.)

Bursting Pressure.

| GLOBES. | | | CYLINDER. | | | |
|-----------------|---------------|--|---------------------------|---------|-----------------|---------------|
| Thick- ness. | Per Sq. Inch. | | Diameter. | Length. | Thick- ness. | Per Sq. Inch. |
| Inch. | Lbs. | | Inch. | | Inch. | Lbs. |
| .024 | 84 | | 4 | 7 | .079 | 282 |
| .022 | 90 | | Elliptical (Crown Glass). | | | |
| .059 | 152 | | 4.1 | 7 | .019 | 109 |

Collapsing Pressure.

| | | | | | |
|------|-------|---|----|------|-----|
| .014 | 292 | 3 | 14 | .014 | 85 |
| .025 | 1000* | 4 | 7 | .034 | 202 |
| .059 | 900* | 4 | 14 | .064 | 297 |

* Unbroken.

Manganese Bronze.

Manganese Bronze, No. 2, has a Tensile strength of 72 000 to 78 600 lbs. per sq. inch, its elastic limit is from 35 000 to 50 000 lbs., its ultimate elongation 12 to 22 per cent., and its hardness alike to that of *mild steel*.

Transverse Strength.—Destructive stress of a bar 1 inch square, supported at both ends at a distance of 1 foot = 4200 lbs., bending to a right angle before breaking, and requiring 1700 lbs. to give it a permanent set.

MEMORANDA.**Cast Iron.**

Beams cast horizontally are stronger than when cast vertically.

Relative strength of columns of like material and of equal weights is: Cylindrical, 100; Square, 93; Cruciform, 98; Triangular, 110. (*Hodgkinson*.)

If strength of a cylindrical column is 100, one of a square, a side of which is equal to diameter of the cylinder, is as 150.

Repetition of Stress.—A piece submitted to transverse stress broke at 1956th strain, with a stress .75 of that of its original ultimate resistance.

Resistance to Bursting of Thick Cylinders.—Mean resistance to bursting of chambers of cast-iron guns is as follows (*Major Rodman, U.S.A.*):

Thickness of metal = 1 calibre, length = 3 calibres, 52 217 lbs. per sq. inch.

Thickness of metal = .5 calibre, length = 3 calibres, 49 100 lbs. per sq. inch.

The tensile strength of the iron being 18 820 lbs.

Diam. of cylinder 2 ins., length 12 ins., metal 2 ins., 80 229 lbs. per sq. inch.

Diam. of cylinder 3 ins., length 12 ins., metal 3 ins., 93 702 lbs. per sq. inch.

Tensile strength of iron being 26 866 lbs.

Sudden Applications of Stress.—Loss of strength by sudden application of load was, by experiment, 18.6 per cent. in excess of load applied gradually, and its elongation 20 per cent. greater.

Low Temperature.—Tensile strength at 23° under sudden application of load, was reduced 3.6 per cent., and elongation 18 per cent.

Wrought Iron.

Increased Hammering gives 20 per cent. greater strength with decreased elongation.

Hardening.—Water increases strength more than oil or tar. A bar $\frac{1}{2}$ inch in diameter, forged and hardened in water, attained a tensile strength of 73 448 lbs. (*Mr. Kirkaldy*.)

Case Hardening.—Loss of tensile strength 4950 lbs. per sq. inch.

Cold Rolling added 18.5 per cent. to tensile strength, and when plates were reduced .33 in thickness, strength was nearly doubled, with but .1 per cent. elongation. Specific gravity was reduced.

Fibre.—Plates are about 12 per cent. stronger with fibre than across it.

Angles, Tees, etc., have from 2200 to 4500 lbs. less tensile strength than rectangular bars.

Welding does not perceptibly affect strength.

Welding.—Strength as affected by welding varies by experiment from 26 per cent. less, average being 19.4.

is about .45 of its tensile breaking weight, .15 of its compressive strength, and .5 of its transverse strength.

Welding.—1 inch bolts lose by dies 6.11 per cent., and by

Steel.

ter at a temperature of 310°.

WOODS.

pute Transverse Strength of Large Timber.

Destructive Stress.

ne End, and Loaded at the Other. $\frac{.3 S b d^2}{l} = W.$

oth Ends, and Loaded in Middle. $\frac{1.8 S b d^2}{l} = W.$

l at Both Ends, and Loaded in Middle. $\frac{1.2 S b d^2}{l} = W.$

oth Ends, and Loaded at any other point than } $\frac{.45 S b d^2}{l} = W.$

at Both Ends, and Loaded at any other point } $\frac{.3 S b d^2 l}{m n} = W.$

* Hence, $\frac{W l}{1.2 b d^2} = S$, and $\frac{W l}{b d^2} = 1.2 S.$

representing breadth, depth, and length to or between supports, all in
f tensile and crushing strengths of material at two thirds of its Value,
by experiments, W ultimate weight or stress in lbs., and m and n dis-
from nearest supports in ins.

n is uniformly loaded, the stress is twice that if applied in its middle

Values of 1.2 S.

ther coefficients, as .3, 1.8, etc., the values will be proportional.

| WOODS. | 1.2 S | WOODS. | 1.2 S |
|-------------|-------|-------------------------|-------|
| | 2.38 | Locust..... | 3.7 |
| | 2.4 | Mahogany, Honduras..... | 2.3 |
| | 2.46 | Oak, Pa..... | 2 |
| | 2.55 | " Va..... | 2.3 |
| | 2.5 | " white..... | 2.5 |
| | 1.6 | " English..... | 1.7 |
| | 1.6 | " Dantzic..... | 1.35 |
| | 1.53 | " French..... | 2.44 |
| | .85 | Pine, Va..... | 3 |
| | 1.12 | " pitch..... | 2.2 |
| Canada..... | 2.63 | " white..... | 2.71 |
| | 2.5 | " yellow..... | 3.87 |
| | 3.81 | " Canada..... | 1.8 |
| | 2 | Redwood, Cal..... | 1.1 |
| | 1.36 | Spruce..... | 1.2 |
| | 3.64 | Teak..... | 3.17 |
| | 1.77 | Walnut, black..... | 1.25 |

EX 1.—What is destructive stress of a beam of English oak, 2 ins.
feet between its supports?

ble = 1.7, and S = .66 of 5700 (mean of tensile and crushing strength)

$$\frac{1.7 \times 2 \times 2^2 \times 3762}{6 \times 12} = \frac{51163}{72} = 710.6 \text{ lbs.}$$

ent of Mr. Laslett it was 688 lbs.

destructive stress of a beam of yellow pine, 3 ins. by 12, and 14
upports?

le = 3.87, and S = .66 of 10200 (mean of tensile and crushing strei

$$\frac{3.87 \times 3 \times 12^2 \times 6732}{14 \times 12} = \frac{11254827}{168} = 66993 \text{ lbs.}$$

was fixed at both ends then 3.87 would be 5.8.

Or, as 1.2 : 1.8 :: 3.87 : 5.8.

Loads for Rectangular Beams of Various Materials, One Inch in Breadth and One Foot in Length.

Supported at Both Ends and Loaded in Middle.

African, B Baltic, B K Black, C Canadian, D Danish, E English, G Georgia, M Mowee, P Pitch, R Riga, W White, Y Yellow.—Figures at Head of Columns denote Destructive Weight of Material in Lbs.

| Depth. | B K Ash. Red Cedar. | C Fir. Spruce. Larch. | Birch. Hack- matack. Hemlock. | W Beech. Walnut. R Fir. | Spruce. Syramore. W Elm. | Beech. Y Pine. Oak. | P Pine. Oak. Spruce. | W Oak. B Fir. | M Fir. Chestnut. Ash. | Ash. D Fir. Rock Elm. | Hickory. Maple. Alder. G Pine. | Locust. |
|--------|------------------------|-----------------------------|--|-------------------------------|--------------------------------|---------------------------|----------------------------|------------------|-----------------------------|-----------------------------|---|---------|
| 1 | 300 | 320 | 400 | 450 | 500 | 525 | 550 | 600 | 640 | 680 | 800 | 1180 |
| 2 | 60 | 64 | 80 | 90 | 100 | 105 | 110 | 120 | 128 | 136 | 160 | 236 |
| 3 | 240 | 256 | 320 | 360 | 400 | 420 | 440 | 480 | 512 | 544 | 640 | 944 |
| 4 | 540 | 576 | 720 | 810 | 900 | 945 | 990 | 1080 | 1152 | 1224 | 1440 | 2124 |
| 5 | 900 | 1024 | 1280 | 1440 | 1600 | 1680 | 1760 | 1920 | 2048 | 2176 | 2560 | 3776 |
| 6 | 1500 | 1600 | 2000 | 2250 | 2500 | 2625 | 2750 | 3000 | 3200 | 3400 | 4000 | 5900 |
| 7 | 2160 | 2304 | 2880 | 3240 | 3600 | 3780 | 3960 | 4320 | 4608 | 4896 | 5760 | 8496 |
| 8 | 2940 | 3136 | 3920 | 4410 | 4900 | 5145 | 5390 | 5880 | 6272 | 6664 | 7840 | 11564 |
| 9 | 3840 | 4096 | 5120 | 5760 | 6400 | 6720 | 7040 | 7680 | 8102 | 8704 | 10240 | 15104 |
| 10 | 4860 | 5184 | 6480 | 7290 | 8100 | 8505 | 8910 | 9720 | 10368 | 11016 | 12960 | 19116 |
| 11 | 6000 | 6400 | 8000 | 9000 | 10000 | 10500 | 11000 | 12000 | 12800 | 13600 | 16000 | 23600 |
| 12 | 7200 | 7744 | 9680 | 10890 | 12100 | 12605 | 13110 | 14320 | 15488 | 16456 | 19360 | 28556 |
| 13 | 8640 | 9216 | 11520 | 12960 | 14400 | 15120 | 15840 | 17280 | 18432 | 19584 | 23040 | 33984 |
| 14 | 10140 | 10816 | 13520 | 15210 | 16900 | 17745 | 18590 | 20280 | 21632 | 22984 | 27040 | 39884 |
| 15 | 11760 | 12544 | 15680 | 17640 | 19600 | 20580 | 21560 | 23520 | 25088 | 26656 | 31360 | 46256 |
| 16 | 13500 | 14400 | 18000 | 20250 | 22500 | 23625 | 24750 | 27000 | 28800 | 31600 | 36000 | 53100 |
| 17 | 15300 | 16384 | 20480 | 23040 | 25600 | 26880 | 28160 | 30720 | 32708 | 34816 | 40960 | 60416 |

Illustrations of Table.

1.—What is safe statical load for a white-pine beam, 4 ins. by 12, and 15 feet between its supports, loaded in middle.

A like beam, 1 inch in width, 12 ins. in depth, and 1 foot between its supports, will bear as per table 14 400 lbs.

2.—What should be depth of a like beam, 3 ins. in width, and 10 feet between its supports, to bear a statical weight of 1900 lbs.?

Coefficient or Safe load for the material = $500 \div 3 = 166$.

$$\sqrt{\frac{10 \times 1900 \times 1}{4 \times 166 \times 1}} = \sqrt{\frac{19000}{664}} = \sqrt{28.61} = 5.35 \text{ ins.}$$

To Compute Depth of a Header Beam.

LE.—See rule for depth of a floor beam, page 835, with the exception a header is assumed to be always uniformly loaded.

$$\text{Or, } \sqrt{\frac{l \cdot 5 W}{b C}} = d$$

To Compute Breadth of a Trimmer Beam.

With One Header and One Set of Tail Beams. RULE.—Proceed as for computation of dimension of a beam loaded at any other point than middle.

$$\frac{n n W}{l d^2 C} = b. \quad m \text{ and } n \text{ representing distances of the weight or load from each end feet.}$$

ILLUSTRATION.—What should be breadth of a trimmer or carriage beam of Georgia pine 23 feet in length, 15 ins. in depth, sustaining a header 10 feet in length, with beams 19 feet, and designed for a load of 540 lbs. per sq. foot of floor?

Assume $C = 100$; $d = 15 - 1 = 14$; m and $n = 19$ and 4 feet.

$$.5 \times \frac{19 \times 4 \times 19 \times 10 \div 2 \times 540}{23 \times 100 \times 14^2} = .5 \times \frac{3898800}{450800} = 4.32 \text{ ins.}$$

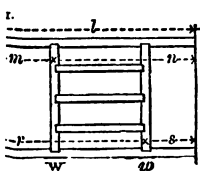
RE 1.—Depth of trimmer beams is usually determined by depth of floor beams; not, proceed to determine it as for a header.

—When a trimmer beam is mortised to receive headers, it is proper to deduct $\frac{1}{2}$ from its depth, as in preceding illustrations. When bridle or stirrup irons are used to suspend headers, a deduction of the thickness of the iron only is necessary, usually $\frac{1}{2}$ inch.

With Two Headers and One Set of Tail Beams.—Fig. 1.

OPERATION.—Proceed for each weight or load as for a beam, when weights sustained or stress borne at other point than the middle.

$$\frac{L}{4} = W \text{ and } w. \quad a \text{ representing area of floor in sq. feet, } L \text{ load per sq. foot, } W \text{ and } w \text{ weights or loads at points of rest on trimmers.}$$



NOTE.—Hatfield and some other authors give complex and extended formulas, to deduce the dimensions of a Girder or Beam, under a like stress.

Upon consideration, however, it will readily be recognized that a beam loaded at more than one point is simply two or more beams, as the case may be, loaded at different points, and connected together.

ILLUSTRATION.—What should be breadth of a trimmer beam of Yellow or Georgia pine, 25 feet in length, 12 ins. in depth, sustaining two headers 15 feet in length, set at 15 feet from one wall and 5 feet from the other, to support safely 300 lbs. per sq. foot of floor?

$= 25$, $m = 15$, $n = 10$, $s = 5$, $r = 20$, $C = 100$, and $d = 12 - 1 = 11$ ft by mortising.

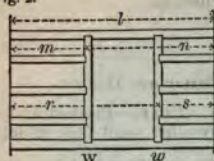
$$\frac{5 \times 5 \times 300}{4} = \frac{18000}{4} = 4500 \text{ lbs. at } W, \text{ and } \frac{12 \times 5 \times 300}{4} = \frac{18000}{4} = 4500 \text{ lbs. at } w$$

$$\text{then } \frac{15 \times 10 \times 4500}{25 \times 11^2 \times 100} = \frac{675000}{302500} = 2.23 \text{ ins. breadth for load on header at 15 ft}$$

$$\frac{5 \times 20 \times 4500}{25 \times 11^2 \times 100} = \frac{450000}{302500} = 1.48 \text{ ins. breadth for load on header at 5 feet, and } 2.23 + 1.48 = 3.71 \text{ ins. combined breadth.}$$

With Two Headers and Two Sets of Tail Beams.—Fig. 2.

Fig. 2.



OPERATION.—Proceed as directed for Fig. 1.

ILLUSTRATION.—What should be breadth of trimmer beam of yellow pine 25 feet in length, 4 ins. in depth, sustaining two headers 12 feet in length, set at 15 feet from one wall and 5 feet from the other, to support with safety 300 lbs. per sq. foot of floor?

$l = 25$, $m = 15$, $n = 10$, $s = 5$, $r = 20$, $C = 100$
and $d = 15 - 1 = 14$ for loss by mortising.

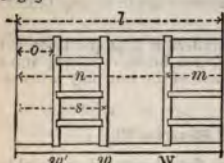
$$\frac{12 \times 15 \times 300}{4} = \frac{54000}{4} = 13500 \text{ lbs. at } W, \text{ and}$$

$$\frac{12 \times 5 \times 300}{4} = \frac{18000}{4} = 4500 \text{ lbs. at } w.$$

Then $\frac{15 \times 10 \times 13500}{25 \times 14^2 \times 100} = \frac{2025000}{490000} = 4.14 \text{ ins.}$, and $\frac{5 \times 20 \times 4500}{25 \times 14^2 \times 100} = \frac{450000}{490000} = .92 \text{ ins.}$, and $4.14 + .92 = 5.06 \text{ ins. combined breadth.}$

With Three Headers and Two Sets of Tail Beams.—Fig. 3.

Fig. 3.



OPERATION.—Proceed as directed for Fig. 1.

ILLUSTRATION.—What should be breadth of trimmer beam of yellow pine, 20 feet in length, 4 ins. in depth, sustaining 3 headers 15 feet in length, set at 3, 7, and 13 feet from one wall, to sustain load of 200 lbs. per sq. foot of floor?

$l = 20$, $m = 7$, $n = 13$, $s = 7$, $o = 3$, $d = 13 - 1 = 12$ ins., and $C = 100$.

$$\frac{15 \times 7 \times 200}{4} = \frac{21000}{4} = 5250 \text{ lbs. at } W;$$

$$\frac{15 \times 7 - 3 \times 200}{4} = \frac{12000}{4} = 3000 \text{ lbs. at } w; \text{ and } \frac{15 \times 7 - 3 \times 200}{4} = 3000 \text{ lbs. at } w.$$

Then $\frac{7 \times 13 \times 5250}{20 \times 12^2 \times 100} = \frac{477750}{288000} = 1.66 \text{ ins.}$; $\frac{7 \times 13 \times 3000}{20 \times 12^2 \times 100} = \frac{273000}{288000} = .95 \text{ ins.}$
and $\frac{3 \times 17 \times 3000}{20 \times 12^2 \times 100} = \frac{153000}{288000} = .53 \text{ ins.}$ Hence, $1.66 + .95 + .53 = 3.14 \text{ ins. combined breadth.}$

Stirrups or Bridles.

Stirrups are resorted to in flooring designed for heavy loads, in order to avoid the weakening of the trimmers by mortising.

Average wrought iron will sustain from 40000 to 50000 lbs. per sq. inch.

Hence 45000 lbs. as a mean, which $\div 5$ for a factor of safety, = 9000 lbs.

A stirrup supports one half weight of header, and being doubled (looped) the stress on it is but $.5 \div 2 = .25$ of load on header.

To Compute Dimensions of Stirrups or Bridles.

$$\frac{W \div 2}{2 \times 9000} = \text{area. Hence } \frac{\text{area}}{\text{thickness}} = \text{width.}$$

ILLUSTRATION.—What should be area and width of .75 inch wrought-iron stirrups for a weight on a header beam of 240000 lbs.?

$$\frac{240000 \div 2}{2 \times 9000} = \frac{120000}{18000} = 6.66 \text{ sq. ins.}, \text{ and } \frac{6.66}{.75} = 8.8 \text{ ins.} = \text{width.}$$

Girder.

Condition of stress borne by a Girder is that of a beam fixed or supported both ends, as the case may be, supporting weight borne by all beams resting thereon, at the points at which they rest.

To Compute Dimensions of a Girder.

RULE.—Multiply length in feet by weight to be borne in lbs., divide product by twice* the *Coefficient*, and quotient will give product of breadth d square of depth in ins.

$$\text{Or, } \frac{l W}{2 C} = b \text{ and } d^2, \text{ and } \sqrt{\frac{l W}{2 C}} = d.$$

EXAMPLE.—It is required to determine dimensions of a yellow-pine girder, 15 feet between its supports, to sustain ends of two lengths of beams, each resting upon it adjoining walls, 15 feet in length, having a superincumbent weight, including 12 of beams, of 200 lbs. per sq. foot.

Condition of stress upon such a girder is that of a number of beams, 30 feet in length (15 × 2), supported at their ends, and sustaining a uniform stress along their length, of 200 lbs. upon every superficial foot of their area.

Coefficient .2 of 500 = 100.

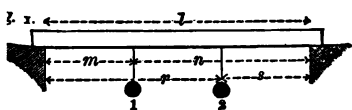
30 × 15 × 200 ÷ 2, for half support on their walls = 45 000 lbs.

Then $\frac{15 \times 45\,000}{2 \times 100} = 3375 = b \text{ and } d^2$. Assuming $b = 12 \text{ ins.}$, then $\sqrt{\frac{3375}{12}} = 16.77$

2. Or, if 15 ins., then $\sqrt{\frac{3375}{15}} = 15 \text{ ins.}$

To Compute Greatest Load upon a Girder, and Dimensions thereof.—Fig. 1.

When a Beam is Loaded at Two Points.



$\frac{m n}{l} = \text{effect of weight at 1,}$

$\frac{r s}{l} = \text{effect of weight at 2,}$

$\frac{m}{l} (W \times n + w s) = \text{the two effects}$

1, and $\frac{s}{l} (w r + W m) = \text{two effects at 2.}$

Then, for weight and dimensions, same formulas will apply.

ILLUSTRATION.—Assume weight of 8000 lbs. at 3 feet from one end of a white-pine beam 12 feet in length between its bearings, and another weight of 3000 lbs. at 5 feet from other end. C. 2 of 500 = 100.

$3000 \times 3 \times \frac{12-3}{12} = 216\,000$ effect of weight at location 1, and $3000 \times 5 \times \frac{12-5}{12} = 105\,000$ effect of weight at location 2. Hence 1, being greatest, = W, and 2 = w.

Then, $\frac{3 \times 9}{12} \times 8000 = 18\,000$ at W, and $\frac{5 \times 7}{12} \times 3000 = 8750$ at w; and

$(18000 \times 9 + 3000 \times 5) = 217\,500 = \text{total effect at W, and } \frac{5}{12} (3000 \times 7 + 8000 \times 3)$

$18\,750 = \text{total effect at w.}$

Hence, to ascertain dimensions at greatest stress,

$\frac{217\,500 \times 3 \times 9}{12 \times 300} = 163.12$, and assume $d = 10$, then $\frac{163.12}{10^2} = 1.63 \text{ ins. breadth;}$

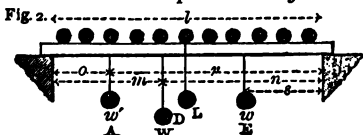
$\frac{163.12}{1.63} = 10 \text{ ins. depth.}$

* For being uniformly loaded.

Verification.—Assume a beam as above loaded with 21 750 lbs. at 3 feet from end

Then, by formula for 801, $\frac{3 \times 9 \times 21\,750}{12 \times 10^2 \times 100} = \frac{587\,250}{120\,000} = 4.89 \text{ ins.}$

Equivalent Weight at Middle.—Fig. 2.



$$\frac{w'o}{l \div 2} = A; \quad \frac{Wn}{l \div 2} = B;$$

$$\frac{ws}{l \div 2} = E; \quad \text{and} \quad \frac{Li}{l} \div 2 = D =$$

equivalent load at middle.

uniformly loaded with 4000 lbs., and sustaining 3 headers of breadth of a beam of Georgia pine, 20 feet in length, 15 ins. in depth, 6000 lbs., at respective distances of 4 and 9 feet from one end and 7000 lbs. at 6 feet from other end?

$$o = 4, \quad r = 16, \quad m = 9, \quad n = 11, \quad s = 6, \quad d = 15 - 1 = 14, \quad L = 4000, \text{ and}$$

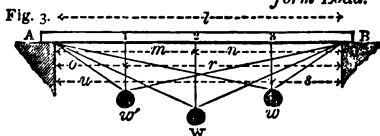
$$C = 850 \times .2 = 170. \quad \frac{6000 \times 4}{20 \div 2} = 2400; \quad \frac{6000 \times 9}{20 \div 2} = 5400; \quad \frac{7000 \times 6}{20 \div 2} = 4200;$$

$$\text{and} \quad \frac{4000 \times 20}{20} \div 2 = 2000. \quad 2400 + 5400 + 4200 + 2000 = 14\,000 \text{ lbs.}$$

$$\text{Then} \quad \frac{14\,000 \times 10 \times 10}{20} = 70\,000 \text{ lbs., effect at middle.}$$

$$\text{Hence,} \quad \frac{20 \times 70\,000}{4 \times 14^2 \times 170} = \frac{1\,400\,000}{133\,280} = 10.5 \text{ ins.}$$

Operation deduced by Graphic Delineation of Greatest Stress without uniform Load.



Moments of weights =

$$\frac{w'o'r}{l}; \quad \frac{Wm'n}{l}; \quad \text{and} \quad \frac{ws}{l} =$$

19 200, 29 700, and 29 400, and let fall perpendiculars 1, 2, and 3 proportionate thereto.

Connect w' , W , and w'' with A , B , and sum of distances of in-

tersections of these lines upon perpendiculars, from 1, 2, and 3, respectively, will give stress upon AB at these points.

Whence, greatest stress at greatest load will be ascertained to be 61 800 lbs.

When Loaded at Three Points, $\frac{m}{l} (Wn + ws) + w' \frac{n'o}{l} = \text{Greatest Stress as in Fig. 2.}$

ILLUSTRATION.—Take elements of above case, omitting uniformly distributed load

$$\frac{9}{20} (6000 \times 11 \times 7000 \times 6) + 6000 \frac{11 \times 4}{20} = \frac{9}{20} \times 108\,000 + 13\,200 = 61\,800 \text{ lbs.}$$

Deflection of Girders and Beams.

$$D = \frac{C b d^3}{l^3} = W; \quad \sqrt[3]{\frac{W l^3}{C b d}} = d; \quad \text{and} \quad \sqrt[3]{\frac{C b d^3 D}{W}} = l. \quad l \text{ represent}$$

feet, b and d breadth and depth, and D deflection in ins.

Values of C for Various Woods. (Hatfield.)

| | | | | |
|----------|-------------------|------|--------------------|------|
| ... 4000 | Larch..... | 2093 | Pine, Georgia..... | 9000 |
| .. 2550 | Oak, white..... | 3100 | " pitch..... | 2696 |
| .. 2800 | " English, mean.. | 2686 | " white..... | 2900 |
| 3850 | Spruce..... | 3500 | " red..... | 4239 |

would be deflection of a floor beam of white pine, 10 feet and 8 in depth, with 4000 lbs. loaded in its middle?

$$\frac{4000 \times 10^3}{939\,200} = .674 \text{ inch.}$$

When Weight is Uniformly Distributed.

$$\frac{625 W l^3}{C b d^3} = D; \quad \frac{C b d^3}{.625 l^3} = W; \quad \sqrt[3]{\frac{C b d^3 D}{.625 W}} = l; \text{ and } \sqrt[3]{\frac{W .625 l^3}{C b}} = d.$$

Hence, Deflection in preceding illustration would be $.674 \times .625 = .421$ ins.

ILLUSTRATION.—What should be length of a white-pine beam 3 by 10 ins., to support 6000 lbs. uniformly distributed, with a deflection of 2 ins. ? $C = 2900$.

$$\sqrt[3]{\frac{2900 \times 3 \times 10^3 \times 2}{.625 \times 6000}} = \sqrt[3]{\frac{17400000}{3750}} = 16.68 \text{ feet.}$$

A fair allowance for deflection of floor beams, etc., is .03 inch per foot of length; 1 inch may be safely resorted to.

Weights of Floors and of Loads.

Dwellings.—Weight of ordinary floor plank of white pine or spruce, 3 lbs. per sq. foot, and of Georgia pine, 4.5 lbs.

Plastering, Lathing, and Furring will average 9 lbs. per sq. foot.

Clay Blocks (Flat Arch) 5.25 \times 7.25 ins. in depth and 1 foot in length, lbs. = 80 lbs. per cube foot of volume.

Floors of dwellings will average 5 lbs. per sq. foot for white pine or spruce, 1 on iron girders will average from 17 to 20 lbs. per sq. foot.

Weight of men, women, and children over 5 years of age, 105.5 lbs., and a third of each will occupy an average area of 12 \times 16 ins. = 192 sq. ins. 78.5 lbs. per sq. foot.

Of men alone 15 \times 20 ins. = 300 sq. ins. = 48 in 100 sq. feet.

Bridges, etc.—Weight of a body of men, as of infantry closely packed, = 3 lbs. each, and they will occupy an area of 20 \times 15 ins. = 300 sq. ins. = 24 lbs. per sq. foot of floor of bridge, and as a live or walking load, 80 lbs. per sq. foot.

Weight of a dense and stationary crowd of men, 120 lbs. per sq. foot.

Bridging of Floor Beams increases their resistance to deflection in a very partial degree, depending upon the rigidity and frequency of the bridges.

Weight on Floors, etc., in addition to Weight of Structure, per Sq. Foot.

| | | | |
|--------------------------|--------------|--------------------------|---------------|
| 11 rooms..... | 85 lbs. | Roofs, wind and snow.... | 30 to 35 lbs. |
| Back or stone walls..... | 115 to 150 " | Slate roofs..... | 45 " |
| Arches and Theatres.... | 80 " | Snow, per inch..... | .5 lb. |
| Cellings..... | 40 " | Street bridges..... | 80 lbs. |
| Stories..... | 200 to 400 " | Warehouses..... | 250 to 500 " |
| Attics..... | 100 " | Wind..... | 50 " |

Scarfs.

Relative resistance of scarfs in Oak and Pine, 2 ins. square, and 4 feet in length, by experiments of Col. Beaufoy.

Scarf 12 ins. in Length and 13 ins. from End, or 1 inch from Fulcrum.

Vertical.—110 lbs. gave away in scarf.

Horizontal, large end uppermost and towards fulcrum.—101 lbs. fastenings gave through small end of scarf; small end uppermost, etc., 87 lbs. gave away in thick part of scarf.

Factors of Safety.

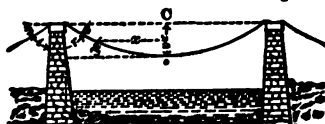
Statical or Dead Load at .2 of destructive stress, but for ordinary purposes it may be increased to .25, and in some cases with good materials to .3.

Live Load at .1 to .125 of destructive stress.

See also page 802.

SUSPENSION BRIDGE.

To Compute Elements.



$$\frac{CL}{8v} \text{ or } \frac{Qa^2}{2v} = S; \quad \frac{va^2}{(5C)^2} = i;$$

$$S \sqrt{\left(\frac{2h}{s}\right)^2 + 1} = s = S \sin i;$$

$$\frac{4h}{2s} = \tan. \text{ angle } o; \quad \frac{L}{s-1} = f;$$

$$2\sqrt{(5C)^2 + \frac{4}{3}v^2} = l; \quad \frac{4v}{C} = \cot. r;$$

$$\frac{CL}{8S} = v; \quad \frac{8vt}{C} \sqrt{\left(\frac{4v}{C}\right)^2 + 1} = i;$$

$$S + \sqrt{\left(\frac{4v}{C}\right)^2 + 1} = t; \text{ or } \frac{L+s}{s} = t; \quad \frac{2v}{\sqrt{(2v)^2 + (C+s)^2}} = \sin i \text{ and } \frac{L+2X \sin i}{\sin i} = t;$$

= stress at a . C representing chord or span, a half chord, and v vertical distance of chord or curve of deflection, in feet, L distributed load inclusive of suspended structure, Q load per lineal foot, and S stress at centre, all in tons, a distance of any point from centre of curve, and h height of chain at a above centre of it, both in feet, stress on chain at any point, as s , from centre of span, s stress on any tension, and t stress at abutments, all in tons, n number of tension-rods, o angle of length of chain with horizon at any point, as a , r angle of chain with vertical at which, i length of chain, in feet, and s angle of direction of chain.

Assume $C = 300$ feet, $L = 1000$ tons, $v = 25$ feet, $a = 100$ feet, $n = 30$, $r = 71^\circ 34'$, and $o = 12^\circ 32'$.

$$\text{Then, } \frac{300 \times 1000}{8 \times 25} = 1500 \text{ tons} = S;$$

$$\frac{25 \times 100^2}{(5 \times 300)^2} = 11.11 \text{ feet} = h;$$

$$1500 \sqrt{\left(\frac{2 \times 11.11}{100}\right)^2 + 1} = 1536.56 \text{ tons} = s;$$

$$\frac{4 \times 11.11}{2 \times 100} = .2222 = \tan o = 12^\circ 32' = o;$$

$$\tan. \angle o; \quad 2 \sqrt{(5 \times 300)^2 + \frac{4}{3} 25^2} = 305.5 \text{ feet} = l; \quad \frac{300 \times 1000}{8 \times 1500} = 25 = v;$$

$$\frac{4 \times 25}{300} = .3333 = 71^\circ 34' = \cot. \text{ angle } r; \quad \frac{1000}{30 - 1} = 34.48 \text{ tons} = t;$$

$$\sqrt{\left(\frac{4 \times 25}{300}\right)^2 + 1} = 1.23.1 \text{ tons} = t; \text{ and } \frac{2 \times 25}{\sqrt{(2 \times 25)^2 + (300 + 2)^2}} = .3162 = \sin i.$$

For a deflection of .125 of span, horizontal stress is equal to total load.

To Construct curve, see Geometry, page 230.

To Compute Ratio which Stress on Chains or Cables at either Point of Suspension Bears to whole Suspended Weight of Structure and Load.

$$\frac{1}{2 \times \sin. i} = R. \text{ R representing ratio.}$$

ILLUSTRATION.—Assume elements of preceding case.

$$\frac{1}{2 \times .3162} = 1.58 \text{ ratio. By a preceding formula it would be } 1.536.$$

Stress on Back Stays.—The cables being led over rollers, having free motion, tension upon them is same, whether angle i is same as that of r or not.

Stress on Piers.—When angles r and i are alike, stress on piers will be vertical, but when angle of i is greater or less than r , stress will be oblique.

Compute Horizontal Stress and Vertical Pressure on Piers.

$$S \cos. n = S o, \quad S \sin. s = P i, \text{ and } S \sin. n = P o. \quad S i \text{ and } S o \text{ stress, and } P i \text{ and } P o \text{ pressure, inward and outward.}$$

of New York and Brooklyn Bridge 1595.5 feet, deflection 12 feet, n at piers from horizontal $15^\circ 10'$.

TRACTION.

Results of Experiments on Traction of Roads and Pavements. (*M. Morin.*)

- 1st. Traction is directly proportional to load, and inversely proportional diameter of wheel.
- 2d. Upon a paved or Macadamized road resistance is independent of dth of tire, when it exceeds from 3 to 4 ins.
- 3d. At a walking pace traction is same, under same circumstances, for triages with or without springs.
- 4th. Upon hard Macadamized, and upon paved roads, traction increases with velocity: increments of traction being directly proportional to increments of velocity above velocity of 3.28 feet per second, or about 2.25 miles per hour. The equal increment of traction thus due to each equal increment of velocity is less as road is more smooth, and carriage less rigid or better adapted.
- 5th. Upon soft roads of earth, sand, or turf, or roads thickly gravelled, traction is independent of velocity.
- 6th. Upon a well-made and compact pavement of dressed stones, traction at a walking pace is not more than .75 of that upon best Macadamized roads under similar circumstances; at a trotting pace it is equal to it.
- 7th. Destruction of a road is in all cases greater as diameters of wheels are less, and it is greater in carriages without springs than with them.
- Experiments made with the carriage of a *siege train* on a solid gravel road and on a good sand road gave following deductions:
1. That at a walk traction on a good sand road is less than that on a good gravel road.
 2. That at high speeds traction on a good sand road increases very rapidly with velocity.
- Thus, a vehicle without springs, on a good sand road, gave a traction 2.64† times greater than with a similar vehicle on same road with springs.

Results with a Dynamometer.

Wagon and Load 2240 lbs.*

| ROADWAY. | Relat'e number of horses for like effect. | ROADWAY. | Relat'e number of horses for like effect. |
|--------------------------------|---|-----------------------------------|---|
| 1 railway, 8 lbs..... | 1 | Telford road, 46 lbs..... | 5.75 |
| 1 best stone tracks, 12.5 lbs. | 1.56 | Broken stone or concrete, 46 lbs. | 5.75 |
| 1 plank road, 32 to 50 lbs. | 4 to 6.25 | Gravel or earth, 140-147 lbs. { | 17.5 |
| 1 one block pavement, 32.5 " | 4.06 | Common earth road, 200 lbs.. | 18.37 |
| 1 macadamized road, 65 lbs.... | 8.12 | | 25 |

NOTE.—By recent experiments of M. Dupuit, he deduced that traction is inversely proportional to square root of diameter of wheel.

Relation of force or draught to weight of vehicle and load over 6 different constructions of road, gave for different speeds as follows:

| | Walk. | Trot. | | Walk. | Trot. |
|-----------------------|-------|-------|------------------------------------|-------|-------|
| Stage coach, 5 tons.. | 1.3 | 1 | Carriage, seats only, on springs.. | 1.29 | 1 |

Resistance to Traction on Common Roads.

On Macadamized or Uniform Surfaces. (*M.*)

1. Resistance is directly proportional to pressure.
2. It is independent of width of tire.
3. It is inversely as square root of diameter of wheel.
4. It is independent of speed.

* See *Traction on Roads, Streets, and Pavements*, by Brev. Maj.-Gen'l G.
† Telford estimated it at 3.5.

On Paved and Rough Roads.

Resistance increases with speed, and is diminished by an enlargement of tire up to a moderate limit.

Traction on Various Roads.—Traction of a wheeled vehicle is to its weight upon various roads as follows:

| | Per Ton. | Per 100 lbs. | | Per Ton. | Per 100 lbs. |
|-------------------|------------|--------------|------------------|------------|--------------|
| Stone track, best | 12.5 to 15 | .55 to .58 | Telford road.... | 46 to 78 | 2.1 to 3.5 |
| " " " " " " " " | 28 to 39 | 1.25 to 1.3 | Macadamized... | 46 to 90 | 2 to 4 |
| " " pavement. | 14 to 36 | .5 to 1.5 | " loose | 67 to 112 | 3 to 5 |
| Asphalted..... | 22 to 28 | 1 to 1.25 | Gravel..... | 134 to 180 | 6 to 8 |
| Plank..... | 22 to 45 | .98 to 2 | Sandy..... | 140 to 313 | 6.3 to 14 |
| Block stone | 32 to 35 | 1.4 to 1.6 | Earth..... | 200 to 290 | 9 to 13 |
| pavement.... | | | | | |

Hence, a horse that can draw 140 lbs. at a walk, can draw upon a gravel road

$$140 \div \frac{6+8}{2} \times 100 = 2000 \text{ lbs.}$$

Resistance on Common Roads or Fields.

(Bedford Experiments, 1874. *)

| GRAVELLED ROAD. (Hard and dry, rising 1 in 430.) | Maximum Draft. | Average Draft. | Average Speed per Hour. | H ² de- veloped per Minute. | Draft per Ton on Level. | Work per H. per Hour. |
|---|-------------------|-------------------|----------------------------------|---|----------------------------|--------------------------------|
| | Lbs. | Lbs. | Miles. | HP. | Lbs. | H. |
| 2 horse wagon without springs. | 320 | 159 | 2.5 | 1.06 | 43.5 or .092 | .53 |
| 4 " " " " " " | 400 | 251 | 2.6 | 1.74 | 44.5 " .02 | .87 |
| 2 " " " with " " " | 300 | 133 | 2.47 | .88 | 34.7 " .015 | .44 |
| 1 " cart without " " " | 180 | 49.4 | 2.65 | .35 | 28 " .0125 | .35 |
| ARABLE FIELD. (Hard and dry, rising 1 in 1000.) | | | | | | |
| 2 horse wagon without springs. | 1000 | 700 | 2.35 | 4.36 | 210 or .099 | 2.18 |
| 4 " " " " " " | 1200 | 997 | 2.52 | 6.7 | 194 " .083 | 3.35 |
| 2 " " " with " " " | 1000 | 710 | 2.35 | 4.45 | 210 " .099 | 1.22 |
| 1 " cart without " " " | 400 | 212 | 2.61 | 1.48 | 140 " .0625 | 1.48 |

Fore wheels of wagons were 39 ins., and hind 57 ins. in diam.; tires varying from 2.25 to 4 ins.; and wheels of cart were 54 ins. in diam., and tires 3.5 and 4 ins.

Springs reduced resistance on road 20 per cent., but did not lessen it in the field.

From these data it appears, that on a hard road, resistance is only from .25 to .16 of resistance in field. Lowest resistance is that of cart on road = 28 lbs. per ton; due, no doubt, to absence of small wheels alike to those of the wagons.

Assuming average power without springs to be .6 HP on road, as average for a day's work, it represents .6 × 33000 = 19800 foot-lbs. per minute for power of a horse on such a road.

Resistance of a smooth and well-laid granite track (tramway), alike to those in London and on Commercial Road, is from 12.5 to 13 lbs. per ton.

Omnibus.† (Weight 5758 lbs.)

| | Average Speed per Hour. | Per Ton. | Total. |
|---|-------------------------|------------|------------|
| Granite pavement (courses 3 to 4 ins.)..... | 2.87 miles. | 17.41 lbs. | 44.75 lbs. |
| Asphalt roadway..... | 3.56 " | 27.14 " | 69.75 " |
| Wood pavement..... | 3.34 " | 41.6 " | 106.88 " |
| Macadam road gravelly..... | 3.45 " | 44.48 " | 114.32 " |
| " " " " " " " " " " " " | 3.51 " | 101.09 " | 259.8 " |

It is noted for an asphalt roadway is apparently inconsistent with pavement, for when it is properly constructed it is least resistance.

Wagon. (Sir John Macneil.)

Weight 2342 lbs. Speed 2.5 Miles per Hour.

| | Resistance. | |
|---|-------------|---------|
| | Per Ton. | Total. |
| made stone pavement..... | 31.2 lbs. | 33 lbs. |
| made with 6 ins. of broken hard stone, on a foundation of stones in pavement, or upon a bottom of concrete..... | 44 " | 46 " |
| on road, or a road made with a thick coating of broken stones, on earth..... | 62 " | 65 " |
| made with a thick coating of gravel, on earth..... | 140 " | 147 " |

Stage Coach. (Sir John Macneil.)

Weight 3192 lbs. Gradients 1 to 20 to 600.

| Speed. | Metalled Road. |
|-----------------------|------------------|
| 6 miles per hour..... | 62 lbs. per ton. |
| 8 " " | 73 " " |
| 10 " " | 79 " " |

—It was found that, from some unexplained cause, the net frictional resistance at equal speeds was considerably, according to gradient, resistances being a maximum for steepest gradient, and a minimum for gradients of 1 in 30 to 1 in 40; for these they are less than 1 in 600. Mode of action of wheels on the carriage may have been an influential element. (D. K. Clark.)

Compute Resistance to Traction on Various Roads.

(Sir John Macneil.)

ON A LEVEL.

EXAMPLE.—Divide weight of vehicle and load in lbs. by its unit in following table and to quotient add .025 of load; add sum to product of velocity of vehicle in feet per second, and Coefficient in following table for the particular road and result will give power required in lbs.

$$\frac{V+w}{\text{unit}} + w \cdot .025 + Cv = T. \quad W \text{ and } w \text{ representing weights of vehicle and load.}$$

Coefficients for Traction of Various Vehicles.

| | | | |
|----------------------------|-----|------------------------------------|----|
| Stage coach..... | 100 | 2 horse wagon without springs..... | 54 |
| Wagon..... | 93 | 2 " " with " | 42 |
| Wagon without springs..... | 55 | 1 " " cart without " | 36 |

Coefficients for Roads of Various Construction.

| | | | |
|---------------------------|------|-----------------------|-----|
| Metalled..... | 2 | Macadamized road..... | 4.3 |
| Stone, dry and clean..... | 5 | Gravel, clean..... | 13 |
| " covered with dust..... | 8 | " muddy..... | 32 |
| " muddy..... | 10 | Stone tramway..... | 1.2 |
| Sand and Gravel..... | 12.1 | | |

EXAMPLE.—What is the traction or resistance of a stage coach weighing 2200 lbs. with a load of 1600 lbs., when driven at a velocity of 9 feet per second over a clean broken stone road?

$$\frac{2200 + 1600}{100} + 1600 \times .025 + 5 \times 9 = 123 \text{ lbs.}$$

Compute Power necessary to Sustain a Vehicle upon an Inclined Road, and also its Pressure thereon, omitting Effect of Friction.

AT AN INCLINATION.

$$W : AC :: o : BC, \text{ and } \therefore p : AB.$$

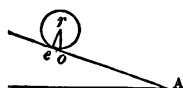
$$\text{Or, } re : eo :: AB : B$$

$$\therefore h : \text{whence,}$$

$$W \frac{h}{l} = eo.$$

Assume AB of such a

rise,



1 foot; then,

$$= \frac{W}{\sqrt{AB^2 + 1}} = W \sin. A = o, \text{ and } \frac{W AB}{AC} = \frac{W A}{\sqrt{AB^2}}$$

To Compute Frictional Resistance to Traction of Stage Coach on a Metalled Road in Good Condition

$30 + 4v + \sqrt{10v} = R$ v representing speed in miles per hour, and R frictional resistance to traction per ton.

NOTE.—Formula is applicable to wagons at low speeds.

Canal, Slackwater, and River.

On a canal and water, resistance to traction varies as square of velocity from that of 2 feet per second to that of 11.5 feet.

When velocity is less than .33 miles per hour, resistance varies in a 1 degree.

In towing, velocity is ordinarily 1 to 2.5 miles per hour.

Resistance of a boat in a canal depends very much upon the comparative areas of transverse sections of it and boat, it being reduced as difference increases.

In a mixed navigation of canal and slack-water, 3 horses or strong men will tow a full-built, rough-bottomed canal boat, with an immersed section area of 94.5 sq. feet, and a displacement of 240 tons, 1.75 to 2 miles per hour for periods of 12 hours.

With a section of but 24.5 sq. feet, or a displacement of 65 tons, an average speed of 2.5 miles is attained for a like period.

By the observations of Mr. J. F. Smith, Engineer of the Schuylkill Navigation Co., a canal boat, with an immersed section alike to that above given, can be towed for 10 hours per day as follows:

Per Hour.

| By 1 horse or mule. | By 2 horses or mules. | By 3 horses or mules. | By 4 horses or mules. | By 5 horses or mules. |
|---------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1 mile. | 1.5 miles. | 1.75 miles. | 1.875 miles. | 2.5 miles. |

Assuming then, the tractive power of a horse as given in table, page 437, the above elements determine results as follows:

| Horses. | Miles. | Tractive Power divided by Load. | Traction | |
|-----------------|--------|---------------------------------|------------------|---|
| | | | In Lbs. per Ton. | In Lbs. per Sq. Foot of Immersed Section. |
| 1 | 1 | $250 \div 240$ | 1.04 | 2.65 |
| 2 | 1.5 | $165 \times 2 \div 240$ | 1.38 | 3.49 |
| 3 | 1.75 | $140 \times 3 \div 240$ | 1.75 | 4.44 |
| 3 | 1.875 | $132 \times 3 \div 240$ | 1.65 | 4.19 |
| 3 | 2 | $125 \times 3 \div 240$ | 1.56 | 3.98 |
| 3 (light) | 2.5 | $100 \times 3 \div 65$ | 4.61 | 12.24 |

Upon a canal of less section and depth, a displacement of 105 tons, with an immersed section of 43 sq. feet, a speed of 2 miles with 2 horses was readily obtained which would give a traction of 2.38 lbs. per ton, and of 5.71 lbs. per sq. foot of immersed section.

Maximum Power of a Horse on a Canal. (Molesworth.)

| | | | | | | | | | | |
|-------------------------|------|-----|-----|-----|-----|----|-----|-------|----|-----|
| per hour | 2.5 | 3 | 3.5 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| tion of work in } | 11.5 | 8 | 5.9 | 4.5 | 2.9 | 2 | 1.5 | 1.125 | .9 | .7 |
| rawn in tons .. | 520 | 243 | 153 | 102 | 52 | 30 | 19 | 13 | 9 | 6.5 |

Street Railroads or Tramways. (Gen'l Gillmore.)

and at a speed of 5 miles per hour, the power required to draw from $\frac{1}{100}$ to $\frac{1}{10}$ of total weight, varying with condition of moisture of their surface.

To Compute Resistance of a Car.

$f; \frac{T \times v}{3} = c; \frac{v^2 \times a}{400} = r; \text{ and } f + c + r = R.$ *T representing weight in lbs., v speed in miles per hour, a area of front or section of car in sq. ins., r resistance of atmosphere, and R total resistance, all in lbs.*
 ILLUSTRATION.—Assume a car and load of 8960 lbs., with an area of section of 56 sq. ins. at a speed of 5 miles per hour.

$$\frac{60}{40} = 1.5 \text{ tons}; \quad 4 \times 6 = 24 \text{ lbs. friction}; \quad \frac{4 \times 5}{3} = 6.66 \text{ lbs. concussion};$$

3.5 lbs. resistance of air; and $24 + 6.66 + 3.5 = 34.16 \text{ lbs.}$

Under the best condition of a road, the resistance of a car may be taken at $\frac{1}{10}$, which, in the worst case, would be 74.66 lbs. On a descending grade, therefore, of 1 in 100, the application of a brake would not be required.

WATER.

WATER. Constitution of it by weight and measure is

| By Weight. | By Measure. | By Weight. | By Measure. |
|-------------------|-------------|-------------------|-------------|
| Hydrogen ... 88.9 | 1 | Hydrogen ... 11.1 | 2 |

A cubic foot of distilled water at its maximum density of 39.1°, barometer 30 ins., weighs 252.879 grains, and it is 772.708 times heavier than atmospheric air.

A cubic foot of water at (at 39.1°) weighs 998.8 ounces, or 62.425 lbs.

—For facility of computation, weight of a cube foot of water is taken at 1000 ounces and 62.5 lbs.

At a temperature of 32° it weighs 62.418 lbs., at 62° (standard temperature) 62.355 lbs., and at 212° 59.64 lbs. Below 39.1° its density decreases, at first very slowly, but progressing rapidly to point of congelation, a cubic foot of ice being but 57.5 lbs.

Sea-water is lighter as compared with sea-water is nearly as 39 to 40.

At 32° it expands .08553 its volume in freezing. From 40° to 12° it expands .0236 its volume, and from 40° to 212° it expands .0467— .0002715 for each degree, giving an increase in volume of 1 in 21.41 feet.

Volumes of Pure Water.

| | |
|--------------------------------|-------------------------------------|
| 7.684 cube ins. = 1 cube foot. | At 62° 1 Ton = 35.923 cube feet. |
| 7.68 " " = 1 " " | " " 1 Lb. = 27.71 " ins. |
| 7.712 " " = 1 " " | " 39.1° 1 Tonneau = 35.3156 " feet. |
| 8.078 " " = 1 " " | " " 1 Kilogr. = .0353 " " |

Height of a Column of Water at 62° or 62.355 lbs.

1 sq. inch = 2.3093 feet, and at pressure of atmosphere = 33.947 feet = 10.33 fathoms.

Ice and Snow

A cubic foot of Ice at 32° weighs 57.5 lbs. A cubic foot of water at 32°, compared with ice, weighs 8.553 per cent.

A cubic foot of new fallen snow weighs 5.2 lbs. A cubic foot of snow weighs 5.2 lbs.

te, containing Carbonate of iron—as, Hampstead, Tunbridge, Cheltenham, England; Spa, Belgium; Ballston and Saratoga, N. Y.; and

containing salts—as, Epsom, Cheltenham, and Bath, England; Baden-utzer, Germany; Kissingen, Bavaria; Plombières, France; Seidlitz, Ucca, Italy; Yellow Springs, Ohio; Warm Springs, N. C.; Congress; and Grenville, Ky.

Chief Rules for Qualitative Analysis of Mineral Waters.

to be determined, in examination of a mineral water, is to which of does water in question belong.

reddens blue litmus paper before boiling, but not afterwards, and blue red paper is restored upon warming, it is Carbonated.

esses a nauseous odor, and gives a black precipitate, with acetate of shurous.

ddition of a few drops of hydrochloric acid, it gives a blue precipitate, r red prussiate of potash, water is a Chalybeate.

res blue color to litmus paper after boiling, it is Alkaline.

esses neither of above properties in a marked degree, and leaves a upon evaporation, it is a Saline water.

: canal water contains .05 } of its volume of gaseous matter.
r well water " .07 }

Re-agents.

er is pure it will not become turbid, or produce a precipitate following *Re-agents*:

ter, if a precipitate or opaqueness appear, Carbonic Acid is present.

terium indicates Sulphates, Nitrate of Silver, Chlorides, and Oxalate Lime salts. Sulphide of Hydrogen, slightly acid, Antimony, Arsenic, Iodine, Platinum, Mercury, Silver, Lead, Bismuth, and Cadmium; Sulmonium, solution alkaloid by ammonia, Nickel, Cobalt, Manganese, Imina, and Chromium. Chloride of Mercury or Gold and Sulphate of matter.

Filter Beds.

feet 6 ins.; Coarse sand, 6 ins.; Clean shells, 6 ins., and Clean gravel er 700 gallons water in 24 hours by gravitation.

ER. Composition of it per volume:

| | | | |
|----------------------|------|------------------------|------|
| ium (common salt) .. | 2.51 | Carbonate of Lime | |
| Magnesium | .53 | " of Magnesia } | .02 |
| " | .33 | Sulphate of Lime | .01 |
| | | Water | 96.6 |

is of Dr. Murray, at specific gravity of 1.029, it contains

| | | | |
|---------------|--------|---------------------------|--------|
| f Soda | 220.01 | Muriate of Magnesia | 42.08 |
| of Soda | 33.16 | Muriate of Lime | 7.84 |
| | | | 303.09 |

contains .030 309 parts of salt = $\frac{1}{33}$ part of its weight.

me of solid matter in solution is 3.4 per cent., .75 of which is

Points at Different Degrees of Saturation.

| Bolling Point. | Salt, by Weight, in 100 Parts. | Bolling Point. | Bolling lat. |
|----------------|--------------------------------|----------------|--------------|
| 213.2° | 15.15 = $\frac{5}{8}$ | 217.9° | |
| 214.4° | 18.18 = $\frac{6}{8}$ | 219° | |
| 215.5° | 21.22 = $\frac{7}{8}$ | 220.2° | |
| 216.7° | 24.25 = $\frac{8}{8}$ | 221.4° | |

* Saturated.

To Compute Number of Teeth.

RULE.—Divide circumference by pitch.

To Compute Number of Teeth in a Pinion or F.
to have a given Velocity.

RULE.—Multiply velocity of driver by its number of teeth, a product by velocity of driven.

EXAMPLE 1.—Velocity of a driver is 16 revolutions, number of its teeth velocity of pinion is 48; what is number of its teeth?

$$16 \times 54 \div 48 = 18 \text{ teeth.}$$

2.—A wheel having 75 teeth is making 16 revolutions per minute; what number of teeth required in pinion to make 24 revolutions in same time?

$$16 \times 75 \div 24 = 50 \text{ teeth.}$$

To Compute Proportional Radius of a Wheel or P

RULE.—Multiply length of line of centres by number of teeth for wheel, and in pinion, for pinion, and divide by number of teeth wheel and pinion.

EXAMPLE.—Line of centres of a wheel and pinion is 36 ins., and number in wheel is 60, and in pinion 18; what are their radii?

$$\frac{36 \times 60}{60 + 18} = 27.69 \text{ ins. wheel.} \quad \frac{36 \times 18}{60 + 18} = 8.3 \text{ ins. pinion.}$$

To Compute Diameter of a Pinion.

When Diameter of Wheel and Number of Teeth in Wheel and P given. **RULE.**—Multiply diameter of wheel by number of teeth in and divide product by number of teeth in wheel.

EXAMPLE.—Diameter of a wheel is 25 ins., number of its teeth 210, and of teeth in pinion 30; what is diameter of pinion?

$$25 \times 30 \div 210 = 3.57 \text{ ins.}$$

To Compute Number of Teeth required in a Train of Wheels to produce a given Velocity.

RULE.—Multiply number of teeth in driver by its number of revolutions, and divide product by number of revolutions of each pinion, for each and pinion.

EXAMPLE.—If a driver in a train of three wheels has 90 teeth, and makes 10 revolutions, and velocities required are 2, 10, and 18, what are number of teeth of other two?

$$10 : 90 :: 2 : 18 = \text{teeth in 2d wheel.} \quad 18 : 90 :: 2 : 10 = \text{teeth in 3d wheel.}$$

To Compute Velocity of a Pinion.

RULE.—Divide diameter, circumference, or number of teeth in driver by diameter, etc., of pinion.

there are a Series or Train of Wheels and Pinions. **RULE.**—

1 product of diameter, circumference, or number of teeth in driver, and

2 product of diameter, etc., of pinions.

3 If a wheel of 32 teeth drives a pinion of 10, upon axis of which is driving a pinion of 8, what are revolutions of last?

$$\frac{32}{10} \times \frac{30}{8} = \frac{960}{80} = 12 \text{ revolutions.}$$

of wheels are 6, 9, 9, 10, and 12 ins.; of pinions, 10, 8, 10, 12, and 10 ins.; revolutions of driving shaft or prime mover is 10?

$$\frac{10 \times 10}{6} = \frac{583200}{7776} = 75 \text{ revolutions.}$$

Compute Proportion that Velocities of Wheels in a Train should bear to one another.

ULE.—Subtract less velocity from greater, and divide remainder by one man number of wheels in train quotient is number, rising in arithmetico-geometric progression from less to greater velocity.

MPLE.—What should be velocities of 3 wheels to produce 18 revolutions, the making 3?

$-3 = \frac{15}{2} = 7.5 = \text{number to be added to velocity of driver} = 7.5 + 3 = 10.5, \text{ and}$
 $-1 = 2$
 $7.5 = 18 \text{ revolutions. Hence } 3, 10.5, \text{ and } 18 \text{ are velocities of three wheels.}$

Pitch of Wheels.

Compute Diameter of a Wheel for a given Pitch, or Pitch for a given Diameter.

From 8 to 192 Teeth.

| Diameter. | No. of Teeth. | Diameter. | No. of Teeth. | Diameter. | No. of Teeth. | Diameter. | No. of Teeth. | Diameter. |
|-----------|---------------|-----------|---------------|-----------|---------------|-----------|---------------|-----------|
| 2.61 | 45 | 14.33 | 82 | 26.11 | 119 | 37.88 | 156 | 49.66 |
| 2.93 | 46 | 14.65 | 83 | 26.43 | 120 | 38.2 | 157 | 49.98 |
| 3.24 | 47 | 14.97 | 84 | 26.74 | 121 | 38.52 | 158 | 50.3 |
| 3.55 | 48 | 15.29 | 85 | 27.06 | 122 | 38.84 | 159 | 50.61 |
| 3.86 | 49 | 15.61 | 86 | 27.38 | 123 | 39.16 | 160 | 50.93 |
| 4.18 | 50 | 15.93 | 87 | 27.7 | 124 | 39.47 | 161 | 51.25 |
| 4.49 | 51 | 16.24 | 88 | 28.02 | 125 | 39.79 | 162 | 51.57 |
| 4.81 | 52 | 16.56 | 89 | 28.33 | 126 | 40.11 | 163 | 51.89 |
| 5.12 | 53 | 16.88 | 90 | 28.65 | 127 | 40.43 | 164 | 52.21 |
| 5.44 | 54 | 17.2 | 91 | 28.97 | 128 | 40.75 | 165 | 52.52 |
| 5.76 | 55 | 17.52 | 92 | 29.29 | 129 | 41.07 | 166 | 52.84 |
| 6.07 | 56 | 17.8 | 93 | 29.61 | 130 | 41.38 | 167 | 53.16 |
| 6.39 | 57 | 18.15 | 94 | 29.93 | 131 | 41.7 | 168 | 53.48 |
| 6.71 | 58 | 18.47 | 95 | 30.24 | 132 | 42.02 | 169 | 53.8 |
| 7.03 | 59 | 18.79 | 96 | 30.56 | 133 | 42.34 | 170 | 54.12 |
| 7.34 | 60 | 19.11 | 97 | 30.88 | 134 | 42.66 | 171 | 54.43 |
| 7.66 | 61 | 19.42 | 98 | 31.2 | 135 | 42.98 | 172 | 54.75 |
| 7.98 | 62 | 19.74 | 99 | 31.52 | 136 | 43.29 | 173 | 55.07 |
| 8.3 | 63 | 20.06 | 100 | 31.84 | 137 | 43.61 | 174 | 55.39 |
| 8.61 | 64 | 20.38 | 101 | 32.15 | 138 | 43.93 | 175 | 55.71 |
| 8.93 | 65 | 20.7 | 102 | 32.47 | 139 | 44.25 | 176 | 56.02 |
| 9.25 | 66 | 21.02 | 103 | 32.79 | 140 | 44.57 | 177 | 56.34 |
| 9.57 | 67 | 21.33 | 104 | 33.11 | 141 | 44.88 | 178 | 56.66 |
| 9.88 | 68 | 21.65 | 105 | 33.43 | 142 | 45.2 | 179 | 56.98 |
| 10.2 | 69 | 21.97 | 106 | 33.74 | 143 | 45.52 | 180 | 57.23 |
| 10.52 | 70 | 22.29 | 107 | 34.06 | 144 | 45.84 | 181 | 57.62 |
| 10.84 | 71 | 22.61 | 108 | 34.38 | 145 | 46.16 | 182 | 57.93 |
| 11.16 | 72 | 22.92 | 109 | 34.7 | 146 | 46.48 | 183 | 58.25 |
| 11.47 | 73 | 23.24 | 110 | 35.02 | 147 | 46.79 | 184 | 58.57 |
| 11.79 | 74 | 23.56 | 111 | 35.34 | 148 | 47.11 | 185 | 58.89 |
| 12.11 | 75 | 23.88 | 112 | 35.65 | 149 | 47.43 | 186 | |
| 12.43 | 76 | 24.2 | 113 | 35.97 | 150 | 47.75 | 187 | |
| 12.74 | 77 | 24.52 | 114 | 36.29 | 151 | 48.07 | 188 | |
| 13.06 | 78 | 24.83 | 115 | 36.61 | 152 | 48.39 | 189 | |
| 13.38 | 79 | 25.15 | 116 | 36.93 | 153 | 48.7 | 190 | |
| 13.7 | 80 | 25.47 | 117 | 37.25 | 154 | 49.02 | 191 | |
| 14.02 | 81 | 25.79 | 118 | 37.56 | 155 | 49.34 | 192 | |

ch in this table is true pitch, as before described.

To Compute Circumference of a Wheel.

ULE.—Multiply number of teeth by their pitch.

To Compute Revolutions of a Wheel or Pinion.

RULE.—Multiply diameter or circumference of wheel or number of its teeth in ins., as case may be, by number of its revolutions, and divide product by diameter, circumference, or number of teeth in pinion.

EXAMPLE.—A pinion 10 ins. in diameter is driven by a wheel 2 feet in diameter making 46 revolutions per minute; what is number of revolutions of pinion?

$$2 \times 12 \times 46 \div 10 = 110.4 \text{ revolutions.}$$

To Compute Number of Teeth of a Wheel for a given Diameter and Pitch.

RULE.—Divide diameter by pitch, and opposite to quotient in preceding table is given number of teeth.

EXAMPLE.—Diam. of wheel is 40 ins., and pitch 1.675; what is number of its teeth?
 $40 \div 1.675 = 23.88$, and opposite thereto in table is 75 = number of teeth.

To Compute Diameter of a Wheel for a given Pitch and Number of Teeth.

RULE.—Multiply diameter in preceding table for number of teeth by pitch, and product will give diameter at pitch circle.

EXAMPLE.—What is diameter of a wheel to contain 48 teeth of 2.5 ins. pitch?

$$15.29 \times 2.5 = 38.225 \text{ ins.}$$

To Compute Pitch of a Wheel for a given Diameter and Number of Teeth.

RULE.—Divide diameter of wheel by diameter in table for number of teeth, and quotient will give pitch.

EXAMPLE.—What is pitch of a wheel when diameter of it is 50.94 ins., and number of its teeth 80?

$$50.94 \div 25.47 = 2 \text{ ins.}$$

General Illustrations.

1.—A wheel 96 ins. in diameter, making 42 revolutions per minute, is to drive a shaft 75 revolutions per minute; what should be diameter of pinion?

$$96 \times 42 \div 75 = 53.76 \text{ ins.}$$

2.—If a pinion is to make 20 revolutions per minute, required diameter of wheel to make 58 revolutions in same time.

$58 \div 20 = 2.9 = \text{ratio of their diameters.}$ Hence, if one to make 20 revolutions given a diameter of 30 ins., other will be $30 \div 2.9 = 10.345 \text{ ins.}$

3.—Required diameter of a pinion to make 12.5 revolutions in same time as wheel of 32 ins. diameter making 26.

$$32 \times 26 \div 12.5 = 66.56 \text{ ins.}$$

4.—A shaft, having 22 revolutions per minute, is to drive another shaft at the distance of 15, distance between two shafts upon line of centres is 45 ins.; what should be diameter of wheels?

Then, 1st. $22 + 15 : 22 :: 45 : 26.75 = \text{ins. in radius of pinion.}$

2d. $22 + 15 : 15 :: 45 : 18.24 = \text{ins. in radius of spur.}$

5.—A driving shaft, having 16 revolutions per minute, is to drive a shaft 81 revolutions per minute, motion to be communicated by two geared wheels and two pulleys, with an intermediate shaft; driving wheel is to contain 54 teeth, and driven pulley upon driven shaft is to be 25 ins. in diameter; required number of teeth in driven wheel, and diameter of driven pulley.

Let driven wheel have a velocity of $\sqrt{16 \times 81} = 36$, a mean proportional between extreme velocities 16 and 81.

Then, 1st. $36 : 16 :: 54 : 24 = \text{teeth in driven wheel.}$

2d. $81 : 36 :: 25 : 11.11 = \text{ins. diameter of driven pulley.}$

6.—If, as in preceding case, whole number of revolutions of driving shaft and number of teeth in its wheel, and diameters of pulleys are given, what are revolutions of shafts?

Then, 1st. $18 : 16 :: 54 : 48 = \text{revolutions of intermediate shaft.}$

2d. $15 : 48 :: 25 : 80 = \text{revolutions of driven shaft.}$

Teeth of Wheels.

epicycloidal.—In order that teeth of wheels and pinions should work truly and without unnecessary rubbing friction, the face (*from pitch line to* ρ) of the outline should be determined by an epicycloidal curve (see 228), and that of the flank (*from pitch line to base*) by an hypocycloidal also page 228).

When generating circle is equal to half diameter of pitch circle, hypocycloidal described by it is a straight diametrical line, and consequently outline of a flank is a right line, and radial to centre of wheel.

A like generating circle is used to describe face of a tooth of other wheel in proportion respectively, the wheel and pinion will operate evenly.

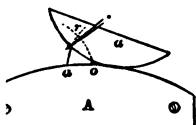
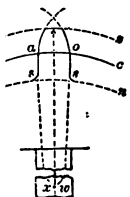


ILLUSTRATION.—Determine all elements of wheel—viz., Pitch circle, Number of teeth, Pitch, Length, Face, and Flank.

Cut a template A to pitch circle cc of wheel, and secure it temporarily to a board.

Having determined depth of tooth, set it off on pitch line, as ao , Fig. 1, and above it apply a second template, a ; radius of wheel is equal to half of pinion; insert into, or attach exactly at its edge, a tracer t , roll template a , and tracer will describe an epicycloidal curve, ar , and by inverting a the or , and faces of a tooth are delineated.



To describe flanks, define pitch line cc , Fig. 2, and arc nn , drawn at base of teeth or board A (as in Fig. 1), secure a strip of wood, w , equal in length to radius of wheel, and locate centre of it, x , draw radii xa and xo , and they will define flanks, which should be filleted, as shown at ss . Define arc sz , and length of tooth is determined.

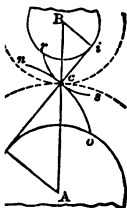
Proceed in like manner conversely for teeth of pinion, and wheel and pinion thus constructed will operate truly.

In construction of the teeth of a wheel or pinion in the pattern-shop, it is customary to construct the wheel or pinion complete, out to face of wheel at base of teeth, and then to insert the teeth in rough, approximately shaped blocks, by a dovetail at their base, fitting into face of wheel, and then outline of a tooth is described thereon; the block is then removed, finished as a tooth, replaced, fastened, and filleted.

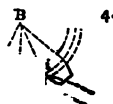
Involute.

Teeth of two wheels will work truly together when their face is that of an involute (see page 229), and that two such wheels should work truly, the arcs from which the involute lines for each wheel are generated must be concentric with the wheels, with diameters in same ratio as those of the wheels.

Assume Ac , Bc , Fig. 3, pitch radii of two wheels designed to work together, through c , draw a right line, eti , and with perpendiculars ec , ic , describe arcs no , rs , and involutes nc and rc define a face of each of the teeth.



To describe teeth of a pair of wheels of which Ac , Bc , Fig. 3, are pitch radii, draw eti , ec , ic , perpendicular to radii Bc and Ac and they are to be taken as radii of the involute arcs i which the faces of the teeth to be defined; then fillet flanks to base, as before described, Fig. 2.



Involute teeth will work with truth, even at varying distances apart of the centres of the wheels, and any wheels, however varied their diameters.

To Compute Depth of a Tooth.

When Stress is given. RULE.—Extract square root of stress, and multiply by .02 for cast iron, and .027 for hard wood.

When HP is given. RULE.—Extract square root of quotient of HP divided by velocity in feet per second, and multiply it by .466 for cast iron, .637 for hard wood.

EXAMPLE.—HP to be transmitted by a tooth of cast iron is 60, and velocity of its pitch-line is 6.66 feet per second; what should be depth of tooth?

$$\sqrt{\frac{60}{6.66}} \times .466 = 1.398 \text{ ins.}$$

To Compute HP of a Tooth.

RULE.—Multiply pressure at pitch-line by its velocity in feet per minute, divide product by 33 000.

EXAMPLE.—What is HP of a tooth of dimensions and at velocity given in preceding example.

$$4886 \times 6.66 \times 60'' \div 33\,000 = 59.16 \text{ horses.}$$

To Compute Stress that may be borne by a Tooth.

RULE.—Multiply Coefficient of material of tooth to resist a transverse stress, as estimated for this character of stress, by breadth and square of its length, and divide product by extreme length of it in decimal of a foot.

EXAMPLE.—Dimensions of a cast-iron tooth in a wheel are 1.38 ins. in depth, 2.1 in length, and 7.5 ins. in breadth; what is the stress it will bear?

$$\text{Coefficient assumed at 60.} \quad \frac{60 \times 7.5 \times 1.38^2}{2.1 \div 12} = 4897 \text{ lbs.}$$

Following deductions by the rules of different authors for like elements are substituted for a cast-iron tooth:

..... 3 ins. | Depth.... 1.38 ins. | Breadth... 7.5 ins. | Length.... 2.1 ins.

| AL POWER IN STRESS EXERTED at a velocity of 400 feet per min., 4886 lbs. | Depth of Tooth. | ACTUAL POWER IN STRESS EXERTED at a velocity of 400 feet per min., 4886 lbs. | Depth of Tooth. |
|---|--------------------|---|--------------------|
| By Rule $\sqrt{\frac{H}{v}} \times .446$ | Ins. 1.398* | By Rankine $\sqrt{\frac{W}{1500}}$ | Ins. 1.8 |
| Fairbairn .025 \sqrt{W} | 1.75 | " Tredgold $\frac{3}{4} \sqrt{\frac{H}{v}}$ | 2.25 |
| Imperial Journal $\sqrt{\frac{W}{1576}}$.. | 1.76 | " Buchanan $\sqrt{\frac{.556 H}{v}}$ | 2.24 |

representing horse-power (60), W stress in lbs., and v velocity in feet per second.

Depth, Pitch, and Breadth. (M. Morin.)

Cast iron..... .028 $\sqrt{W} = d$, .057 $\sqrt{W} = P$.
Hard wood..... .038 $\sqrt{W} = d$, .079 $\sqrt{W} = P$.
representing weight or stress upon tooth in lbs., d depth of tooth, and P pitch in feet.

When velocity of pitch-circle does not exceed 5 feet per second, $d = 4$, and if wheels are exposed to the weather, $d = 5$, representing breadth.

ILLUSTRATION.—Assume pressure at pitch-line of a cast iron wheel is 6000 lbs., and velocity 5 feet per second.

Then .028 $\sqrt{6000} = 2.17$ ins. Depth, and .057 $\sqrt{6000} = 4.5$ feet Pitch.

NOTE.—For further Illustrations of Formation of Teeth, Trundles, etc., see Mosely's Engineering, Shelton's Machinery of Construction, etc.

its depth, with a breadth of 7.5 ins., is .1 of ultimate strength.

EXAMPLE.—What is diameter of roll in preceding example?

Area of $94.59 = 7027.2 + (\text{area of } \frac{7200 \times 1}{2}) = 7200 = 14227.2$, and $\sqrt{14227.2 + 154} = 151.85$ ins.

Or, Radius of drum is increased number of revolutions multiplied by thickness of rope; as $\frac{94.59}{2} + 20 \times 1 = 67.295$ ins.

To Compute Number of Revolutions.

RULE.—To area of drum add area of edge surface of rope; from diameter the circle having that area subtract diameter of drum, and divide remainder by twice thickness of rope; quotient will give number of revolutions.

EXAMPLE.—Length of a rope is 2600 ins., its thickness 1 inch, and diameter of drum 20 ins.; what is number of revolutions?

Area of $20 + \text{area of rope} = 314.16 + 2600 = 2914.16$, diameter of which is 60.91, d $\frac{60.91 - 20}{1 \times 2} = 20.45$ revolutions.

Or, subtract diameter of drum from diameter of roll, and divide remainder by twice thickness of rope; as $60.91 - 20 = 40.91$, and $40.91 \div 1 \times 2 = 20.45$ revolutions.

o Compute Point of Meeting of Ascending and Descending Buckets when two or more are used.

To Compute Point of Meeting of Buckets. **RULE.**—Divide sum of length turns of rope by 2, and to quotient add length of last turn; divide sum by 2, multiply quotient by half number of revolutions, and product will be distance from centre of drum at which buckets will meet.

NOTE 1.—Meetings will always be below half depth of pit.

2.—At half number of revolutions buckets will meet.

EXAMPLE.—Diameter of a drum is 9 feet, thickness of rope 1 inch, and revolutions 20; what is depth of pit, and at what distance from top will buckets meet?

$$\frac{28.54 + 38.48}{2} + 38.48 \div 2 \times \frac{20}{2} = \frac{71.99 \times 10}{2} = 35.995 \times 10 = 359.95 \text{ feet.}$$

To Compute this Depth. **RULE.**—To diameter of drum add thickness of rope in feet, and ascertain its circumference; to diameter of drum add quotient of product of twice thickness of rope and number of revolutions less 1, divided by 12 for a diameter, and circumference of this diameter is length last turn, also in feet; add these two lengths together, multiply their sum by half number of revolutions, and product will give depth of pit.

+ thickness of rope = $9 + \frac{1}{12}$ of 1 = 9.083, which $\times 3.1416 = 28.54$ feet = length

first turn. $9.083 + \frac{1 \times 2 \times 20 - 1}{12} \times 3.1416 = 38.48$ feet = length of last turn.

Then $28.54 + 38.48 \times \frac{20}{2} = 67.02 \times 10 = 670.2$ feet, depth of pit.

WINDMILLS.

Driving Shaft of a vertical windmill should be set at an elevating angle to horizon when set upon low ground, and at a depressing angle when set on elevated ground. Range of these angles is from 3° to 15° . A velocity wind of 10 feet per second is not generally sufficient to drive a loaded mill, and if velocity exceeds 35 feet per second the force is generally too great for ordinary structures.

Angle of Sails should be from 18° to 30° at their least radius, and from 17° at their greatest radius, mean angle being from 15° to 17° to allow motion of sails. Length of a whip (arm) is divided into 7 parts, dividing over 6 parts.

Whip in parts of its length: Breadth .033, at top .016; Depth .025, .0125; Width of sail .33, at axis .2. Distance of sail from axis x length of whip, and cross-bars 16 to 18 ins. from centres.

To Compute Angles of Sails.

$23^\circ - \frac{18 d^2}{r^2} = \text{angle of sail with plane of its motion at any part of it. } d$ representing distance of part of sail from its axis, and r extreme radius of sail, both:

ILLUSTRATION.—Assume $r = 14$, and length of sail 12 feet, $d = .5$ of 12 or sixths of sail $= .5 \times 12 + (14 - 12) = 8$ feet.

$$\text{Then } 23^\circ - \frac{18 \times 8^2}{14^2} = 23 - 5.88^\circ = 17.12^\circ.$$

Hence, angle of sail with axis $= 90^\circ - 17.12^\circ = 72.88^\circ$.

If radius of sails is divided into 6 equal parts, angles at each of these parts be as follows:

| | Distance from Axis. | | | | |
|-------------------------------|---------------------|-----|-------|-----|-------|
| Angle of sail with axis..... | 67.5° | 69° | 71.5° | 75° | 79.5° |
| “ “ with plane of motion..... | 22.5° | 21° | 18.5° | 15° | 10.5° |

To Compute Elements of Windmills.

$$\frac{3.16 v}{r \sin. \alpha} = n; \quad \frac{11.5 v}{r} = n; \quad .1047 n = \alpha v; \quad \frac{A v^3}{1080000} =$$

$$\frac{IP \times 1080000}{v^3} = A; \quad \sqrt{\frac{R^2 + r^2}{2}} = r'. \quad v \text{ representing velocity of wind per}$$

ond, r' radius of centre of percussion of sails, and R and r outer and inner radii of sails, all in feet, α mean angle of sail to plane of motion, n number of revolution arms per minute, αv angular velocity, A area of sails in sq. feet, and IP horse-

ILLUSTRATION.—If a windmill has 4 arms of 28 feet, with a mean angle (α) of with an area of sail of 150 sq. feet each, having an inner radius of 4 feet, and is erated by wind at a velocity of 40 feet per second; what are its elements?

$$\text{Then } \frac{11.5 \times 40}{20} = n = 23; \quad \sqrt{\frac{28^2 + 4^2}{2}} = r' = 20 \text{ feet}; \quad \frac{3.16 \times 40}{20 \times .27564} = n = 23$$

$$\frac{4 \times 150 \times 64000}{1080000} = IP = 35.55; \quad \frac{35.55 \times 1080000}{64000} = A = 599.9 \text{ sq. feet.}$$

Deductions from Velocities varying from 4 to 9 Feet per Second. (Mr. Smeaton.)

1. Velocity of windmill sails, so as to produce a maximum effect, is nearly as velocity of wind, their shape and position being same.

2. Load at maximum is nearly, but somewhat less than, as square of velocity of wind, shape and position of sails being same.

3. Effects of same sails, at a maximum, are nearly, but somewhat less than, as cubes of velocity of wind.

4. Load of same sails, at maximum, is nearly as squares, and their effects as cubes of their number of turns in a given time.

5. In sails where figure and position are similar, and velocity of wind same, number of revolutions in a given time will be reciprocally as radius of sail.

at a maximum, which sails of a similar figure and position at a given distance from centre of motion, will be as cube of radius of sails of similar figure and position are as square of radius of sails of Dutch sails, as well as of enlarged sails when unloaded, or even loaded to a maximum that of wind.

Results of Experiments on Effect of Windmill Sails.—When a vertical windmill is employed to grind corn, the millstone usually makes 5 revolutions to 1 of the sail.

When velocity of wind is 19 feet per second, sails make from 11 to 12 revolutions in a minute, and a mill will grind from 880 to 990 lbs. in an hour or about 22 440 lbs. in 24 hours.

When velocity of wind is 30 feet per second, a mill will carry all sail, make 22 revolutions in a minute, grinding 1984 lbs. of flour in an hour, 616 lbs. in 24 hours.

Results of Operation of Windmills. (A. R. Woolf, M. E.)

Velocity of Wind 15 to 20 Miles per Hour.

Revolutions of Wheel and Gallons of Water raised per Minute.

| Revolutions of Wheel. | Water raised to an Elevation of | | | | Power developed. | Cost per Hour. | |
|-----------------------|---------------------------------|----------|-----------|-----------|------------------|----------------|---------|
| | 25 Feet. | 50 Feet. | 100 Feet. | 200 Feet. | | Actual.* | Per HP. |
| No. | Gallons. | Gallons. | Gallons. | Gallons. | HP | Cents. | Cents. |
| 70 to 75 | 6.16 | 3.02 | — | — | .04 | .60 | 15 |
| 60 to 65 | 19.18 | 9.56 | 4.75 | — | .12 | .70 | 5.8 |
| 50 to 55 | 45.14 | 22.57 | 11.25 | 5 | .28 | 1.63 | 5.8 |
| 40 to 45 | 97.68 | 52.16 | 24.42 | 12.21 | .61 | 2.83 | 4.6 |
| 35 to 40 | 124.95 | 63.75 | 31.25 | 15.94 | .78 | 3.56 | 4.5 |
| 30 to 35 | 212.38 | 106.96 | 49.73 | 26.74 | 1.34 | 4.26 | 3.2 |

* Including interest at 5 per cent. per annum.

WOOD AND TIMBER.

Selection of Standing Trees.—Wood grown in a moist soil is lighter, decays sooner, than that grown in dry, sandy soil.

Best Timber is that grown in a dark soil, intermixed with gravel. Fir, Cypress, Willow, and all others which grow best in a wet soil, are exceptions.

Hardest and densest woods, and least subject to decay, grow in warm states; but they are more liable to split and warp in seasoning.

Trees grown upon plains or in centre of forests are less dense than those from edge of a forest, from side of a hill, or from open ground.

Trees (in U. S.) should be selected in latter part of July or first part of August; for at this season leaves of sound, healthy trees are fresh green, while those of unsound are beginning to turn yellow. A good, healthy tree is recognized by its top branches being well leaved, even and of a uniform color. A rounded top, few leaves, some of them turned yellow, a rougher bark than common, covered with parasitic lichens, and with streaks or spots upon it, indicate a tree upon the decline.

Decay of branches, and separation of bark from the wood, are reliable indications that the wood is impaired.

Green timber contains 37 to 48 per cent. of liquids.

After seasoning one year, it loses from 17 to 25 per cent.

When dried it retains from 10 to 15 per cent.

According to M. Leplay, green wood contains about 40 per cent. of moisture. In Central Europe, wood cut in winter, during summer, fully 40 per cent. of water, and when dried retains from 15 to 20 per cent. of water.

Best Time for Felling Timber.—Most suitable time for felling timber is in summer. Recent experiments indicate latter season as best.

A tree should be allowed to attain full maturity before being felled. Oak matures at 75 to 100 years and upwards, according to circumstances; Ash, Larch, and Elm at 75; and Spruce and Fir at 80. Age and rate of growth of a tree are indicated by number and width of the rings of annual increase which are exhibited in a cross-section of its body.

A tree should be cut as near to the ground as practicable, as the lower part furnishes best timber.

Dressing Timber.—As soon as a tree is felled, it should be stripped of its bark, raised from the ground, reduced to its required dimensions, and its sap-wood removed.

Inspection of Timber.—Quality of wood is in some degree indicated by its color, which should be nearly uniform, and a little deeper towards its centre, and free from sudden transitions of color. White spots indicate decay. Sap-wood is known by its white color; it is next to the bark, and soon rots.

Defects of Timber.

Wind-shakes are serious defects, being circular cracks separating the concentric layers of wood from each other.

Splits, Checks, and Cracks, extending toward centre, if deep and strongly marked, render timber unfit for use, unless purpose for which it is intended will admit of its being split through them.

Brash is when wood is porous, of a reddish color, and breaks short, without splinters. It is generally consequent upon decline of tree from age.

Belled is that which has been killed before being felled, or which has died from other causes. It is objectionable.

Knotty is that containing many knots, though sound; usually of stunted growth.

Twisted is when grain of it winds spirally; it is unfit for long pieces.

Dry-rot is indicated by yellow stains. Elm and Beech are soon affected, if left with the bark on.

Large or decayed knots injuriously affect strength of timber.

Heart-shake.—Split or cleft in centre of tree, dividing it into segments.

Star-shake.—Several splits radiating from centre of timber.

Cup-shake.—Curved splits separating the rings wholly or in part.

Rind-gall.—Curved swelling, usually caused by growth of layers over spot where a branch has been removed.

Upset.—Fibres injured by crushing.

Foriness.—Yellow or red tinge, indicating incipient decay.

Doutiness.—A speckled stain.

Seasoning and Preserving Timber.

Seasoning is extraction or dissipation of the vegetable juices and moisture and solidification of the albumen. When wood is exposed to currents of air and high temperature, the moisture evaporates too rapidly, and it cracks; if temperature is high and sap remains, it ferments, and dry-rot

quires time in which to season, very much in proportion to density

is total immersion of timber in water, for purpose of when thus seasoned it is less liable to warp and crack.

For purpose of seasoning, it should be piled under shelter and kept dry; it should have a free circulation of air, without being exposed to strong currents. Bottom pieces should be placed upon skids, which should be free from decay, raised not less than 2 feet from ground; a space of an inch should intervene between pieces of same horizontal layers, and slats or pilings should be placed between each layer, one near each end of pile, and others at equal distances, in order to keep the timber from winding. These strips should be one over the other, and in large piles should not be less than 1 inch apart. Light timber may be piled in upper portion of shelter, heavy timber on ground floor. Each pile should contain but one description of timber, and they should be at least 2.5 feet apart.

Timber should be repiled at intervals, and all pieces indicating decay should be removed, to prevent their affecting those which are still sound.

Timber requires from 2 to 8 years to be seasoned thoroughly, according to its species, and it should be worked as soon as it is thoroughly dry, for it deteriorates after that time.

Natural seasoning is most favorable to strength and durability of timber. Various methods have been proposed for hastening the process, as *Steaming*, which has been applied with success; and results of experiments of various processes of saturating it with a solution of *Corrosive sublimate* and *Antiseptics*; fluids are very satisfactory. Such process hardens and seasons wood, at the same time that it secures it from dry-rot and from attacks of worms. Woods are densest and strongest at the roots and at their centres. Their strength decreasing with the decrease of their density.

Soft timber loses *one fifth of its weight* in seasoning, and about *one third* coming perfectly dry.

Soft pine, from the presence of pitch, requires time in excess of that due to the density of its fibre.

Softwood should be seasoned slowly, Pine quickly. Whitewood should be dried artificially, as the effect of heat is to twist it.

Water renders wood harder, heavier, and more durable than fresh.

Condition of timber, as to its soundness or decay, is readily recognized by striking with a quick blow.

Timber that has been for a long time immersed in water, when brought into the air and dried, becomes brashy and useless.

When trees are barked in the spring, they should not be felled until the sap is dead.

Timber cannot be seasoned by either smoking or charring; but when it is exposed to worms or to the production of *fungi*, it is proper to smoke or steam it, and it may be partially seasoned by being boiled or steamed.

Timber houses are best provided with blinds which can be turned to admit air in fine weather, but which can be kept entirely free from any pieces of decayed wood.

Un-drying is suited only for boards and pieces of small timber, to cause cracks and to impair the strength, unless they are dried.

Painting, or covering the surface is highly injurious to wood, as it effectually prevents drying of the interior, in consequence of which fermentation and decay soon follow.

Timber is subject to *Common or Dry-rot*, former occasioned by exposure to moisture and dryness, and as progress of it is from the surface, if seasoned, with paint, tar, etc., is a preventive.

Common-rot is the consequence of its being piled in badly-ventilated sheds. Obvious indications are yellow spots upon ends of pieces, and a yellow tint in the checks and cracks, particularly where the pieces rest upon plating-strips.

Dry or Sap-rot is inherent in timber, and it is the putrefaction of the vegetable albumen. Sap wood contains a large proportion of fermentable elements.

Insects attack wood for the sugar or gum contained in it, and fungi subsist upon the albumen of wood; hence, to arrest dry-rot, the albumen must be either extracted or solidified.

Most effective method of preserving timber is that of expelling or exhausting its fluids, solidifying its albumen, and introducing an antiseptic liquid.

Strength of impregnated timber is not reduced, and its *resilience* is improved.

In desiccating timber by expelling its fluids by heat and air, its strength is increased fully 15 per cent.

The saturation of wood with creosote, tar, antiseptics, etc., preserves it from the attack of worms. Jarroo wood, from Australia, is not subject to their attack.

In a perfectly dry atmosphere durability of woods is almost unlimited. Rafters of roofs are known to have existed 1000 years, and piles submerged in fresh water have been found perfectly sound 800 years from period of their being driven.

Resistance of woods to extension is greater than that of compression.

Impregnation of Wood.

Several of the successful processes are as follows:

Kyan, 1832.—Saturated with corrosive sublimate. Solution 1 lb. of chloride of mercury to 4 gallons of water.

Burnett (Sir Wm.), 1838.—Impregnation with chloride of zinc by submitting the wood endwise to a pressure of 150 lbs. per sq. inch. Solution, 1 lb. of the chloride to 4 gallons of water.

Bouchéri.—Impregnation by submitting the wood endwise to a pressure of about 15 lbs. per sq. inch. Solution, 1 lb. of sulphate of copper to 200 gallons of water.

Bethel.—Impregnation by submitting the wood endwise to a pressure of 150 to 200 lbs. per sq. inch, with oil of creosote mixed with bituminous matter.

Robbins, 1865.—Aqueous vapor dissipated by the wood being heated in a chamber, the albumen solidified, then submitted to vapor of coal tar, or bituminous oils, which, being at a temperature not less than 325°, readily takes the place of the vapor expelled by a temperature of 212°.

Hayford, 187.—Aqueous vapor dissipated by the wood being heated in a chamber to a temperature of from 250° to 270°, the albumen solidified, the air introduced to assist the splitting of the outer surfaces. When vapor is dissipated, dead oils are introduced under a pressure of 75 lbs. per sq. inch.

Planks, Deals, and Battens.—When cut from Northern pine (*Pinus strobus*) are termed *yellow or red deal*, and when cut from spruce (*Abies*, etc.) they are termed *white deal*.

Desiccated wood, when exposed to air under ordinary circumstances absorbs 5 per cent. of water in the first three days; and will continue to absorb it until it reaches from 14 to 16 per cent., the amount varying with the condition of the atmosphere.

WOOD AND TIMBER.

Durability of Various Woods.

Pieces 2 feet in Length, 1.5 ins. Square, driven 28.5 ins. into the Earth.

| Wood. | Condition | |
|------------------|---------------------------|---|
| | After 2.5 Years. | After 5 Years. |
| Acacia..... | Good | { Externally decayed, rest fectly sound. |
| Ash, Amer..... | Much decayed..... | Decayed. |
| Cedar, Va..... | Very good..... | Sound as when driven. |
| " Lebanon..... | Good | Tolerable. |
| Elm, Eng..... | Much decayed..... | Entirely decayed. |
| " Can..... | " | Decayed. |
| Fir..... | " attacked..... | Much decayed. |
| Larch..... | Surface only attacked.... | { Attacked in part only, re condition. |
| Oak, Can..... | Very much decayed..... | Very rotten. |
| " Memel..... | " | " |
| " Dantzic..... | " | " |
| " Chestnut..... | Very good | { Some moderately, most much, decayed. |
| Pine, pitch..... | Surface only attacked.... | { Attacked in part only, re condition. |
| " yellow..... | Attacked | Much decayed. |
| " white..... | Very much decayed..... | Very rotten. |
| Teak | Very good..... | Somewhat soft, but good. |

Effect of Creosoting.

Results of Experiments with Various Woods (E. R. Andrews).

| Wood. | Water absorbed. | Per cent. | Wood. | Water absorbed. | Per cent. |
|--------------|--------------------|-----------|----------------|--------------------|-----------|
| | | | | | |
| Spruce | dried..... | .2543 | Hard pine.... | dried..... | .11 |
| | creosoted... | .0261 | | creosoted.. | .0 |
| Oak | dried..... | .2 | Gum, black.. | dried..... | .1 |
| | creosoted... | .0 | | creosoted.. | .1 |
| Cotton-wood | dried..... | .714 | Birch, white.. | dried..... | .4 |
| | creosoted... | .347 | | creosoted.. | .1 |

Sequoia Gigantea of California, dried, .4722; creosoted, .0.

Fluids will pass with the grain of wood with great facility, but will enter it except to a very limited extent when applied externally.

Absorption of Preserving Solution by different Woods for a Period of 7 Days. Average Lbs. per Cube Foot.

| | | | | | |
|----------------|-----|--------------|-----|----------------|--|
| Black Oak..... | 3.6 | Hemlock..... | 2.6 | Rock Oak..... | |
| Chestnut..... | 3 | Red Oak..... | 3.9 | White Oak..... | |

Proportion of Water in various Woods.

| | | | |
|--|------|--|--|
| Alder (<i>Betula alnus</i>)..... | 41.6 | Pine (<i>Pinus Sylvestris L.</i>)..... | |
| Ash (<i>Fraxinus excelsior</i>)..... | 28.7 | Red Beech (<i>Fagus sylvatica</i>)..... | |
| Beech (<i>Fagus sylvatica</i>)..... | 33 | Red Pine (<i>Pinus picea dur</i>)..... | |
| Birch (<i>Betula alba</i>)..... | 30.8 | Spruce (<i>Abies, alba, nigra rubra,</i> <i>excelsa</i>)..... | |
| Elm (<i>Ulmus campestris</i>)..... | 44.5 | Sycamore (<i>Acer p</i> <i>white</i>)..... | |
| Horse-chestnut (<i>Æsculus hippocast.</i>) | 38.2 | White Oak (<i>Querc</i> <i>alba</i>)..... | |
| Larch (<i>Pinus larix</i>)..... | 48.6 | White Pine (<i>Pinus</i> <i>banksiana</i>)..... | |
| Mountain Ash (<i>Sorbus aucuparia</i>).. | 28.3 | White Poplar (<i>Pop</i> <i>ulus alba</i>)..... | |
| Oak (<i>Quercus robur</i>)..... | 34.7 | | |
| Willow (<i>Salix caprea</i>)..... | | | |

Decrease in Dimensions of Timber by

| Woods. | Ins. | Ins. | Woods. |
|---------------------|-------|--------------|----------------------|
| Cedar, Canada..... | 14 | to 13.25 | Pitch Pine, South.. |
| Elm..... | 11 | to 10.75 | Spruce..... |
| Oak, English..... | 12 | to 11.625 | White Pine, American |
| Pitch Pine, North.. | 10X10 | to 9.75X9.75 | Yellow Pine, North.. |

Weight of a beam of English oak, when wet, was reduced from 67.25 to 63.05 lbs.

Weight of a Cube Foot of Oak and Yellow Pine.

| Age. | White Oak, Va. | | Yellow Pine, Va. | | Live Oak. |
|--------------|----------------|---------|------------------|---------|-----------|
| | Round. | Square. | Round. | Square. | |
| Green..... | 64.7 | 67.7 | 47.8 | 39.2 | 78.7 |
| 1 Year..... | 53.6 | 53.5 | 39.8 | 34.2 | — |
| 2 Years..... | 46 | 49.9 | 34.3 | 33.5 | 66.7 |

In England, Timber sawed into boards is classed as follows:

6.5 to 7 ins. in width, *Battens*; 8.5 to 10 ins., *Deals*; and 11 to 12 ins. *Planks*. (See also page 62.)

Distillation.—From a single cord of pitch pine distilled by chemical apparatus, following substances and in quantities stated have been obtained:

| | | | |
|-----------------------------|----------------------|----------------------------|-------------|
| Charcoal..... | 50 bushels. | Pyroligneous Acid..... | 100 gallons |
| Illuminating Gas..... | about 1000 cu. feet. | Spirits of Turpentine..... | 20 " |
| Illuminating Oil and Tar... | 50 gallons. | Tar..... | 1 barrel |
| Pitch or Resin..... | 1.5 barrels. | Wood Spirit..... | 5 gallons |

Strength of Timber.

Results of experiments have satisfactorily proved: That deflection was sensibly proportional to load; That extension and compression were nearly the same, though former being the greater; That, to produce equal deflection, load, when placed in the centre, was to a load uniformly distributed, as 5 to 1; That deflection under equal loads is inversely as breadths and cubes of the depths, and directly as cubes of the spans. (*M. Morin*.)

It has also been shown, that density of wood varies very little with its age. That coefficient of elasticity diminishes after a certain age, and that it depends also on the dryness and the exposure of the ground where the wood is grown. Woods from a northerly exposure, on dry ground, have a high coefficient, while those from swamps or low moist ground have a low one. That tensile strength is influenced by age and exposure. The coefficient of elasticity of a tree cut down in full vigor, or before it arrives at this condition, does not present any sensible difference. That there is no limit of elasticity in wood, there being a permanent set for every extension.

Average Result of Experiments on Tensile Strength of Wood in Various Positions per Sq. Inch. (*MM. Chevandier and Wertheim*.)

With the fibre, 6900 lbs. Radially, 683 lbs., and Tangentially, 723 lbs.

To Compute Volume of an Irregular Body.

By "Simpson's Rule."

OPERATION.—Take a right line in the figure for a base line, as A B, divide the figure into any number of equal parts, and compute the areas of their plane sections as 1, 2, 3, etc., at the points of division, by rules applicable to area of a plane. Then, erate these areas as if they were the ordinates of a plane curve or figure of same with as the figure, and result will give volume required.

STRATION.—Assume a figure having areas as follows, and A B = 24 feet.

| Sections, 1 | Areas, 3 feet | Multiplier, 1 | Products, 3 |
|-------------|---------------|---------------|-------------|
| 1 | 2 | 4 | 20 |
| 2 | 5 | 2 | 14 |
| 3 | 7 | 4 | 56 |
| 4 | 9 | 2 | 11 |
| 5 | 11 | 1 | 84 |

$$4 \times 24 \div 4 \div 3 = 168 \text{ cube feet.}$$

MISCELLANEOUS MIXTURES.

Cements.

ends upon manner in which a cement is applied as upon the
f, as best cement will prove worthless if improperly applied.
iles must be rigorously adhered to to attain success:

ment into intimate contact with surfaces to be united. This is best
ment pieces to be joined in cases where cement is melted by heat, as
shellac, marine glue, etc. Where solutions are used, cement must be
nto surfaces, either with a brush (as in case of porcelain or glass),
; the two surfaces together (as in making a glue joint between pieces

cement as practicable should be allowed to remain between the united
secure this, cement should be as liquid as practicable (thoroughly
d with heat), and surfaces should be pressed closely into contact until
ardened.

ould be allowed for cement to dry or harden, and this is particularly
il cements, such as copal varnish, boiled oil, white lead, etc. When
each .5 inch across, are joined by means of a layer of white lead
en them, 6 months may elapse before cement in middle of joint be-
At the end of a month the joint will be weak and easily separated; at
years it may be so firm that the material will part anywhere else than
nce, when article is to be used immediately, the only safe cements
ch are liquefied by heat and which become hard when cold. A joint
arine glue is firm an hour after it has been made. Next to cements
died by heat are those which consist of substances dissolved in water
A glue joint sets firmly in 24 hours; a joint made with shellac varnish
in 2 or 3 days. Oil cements, which do not dry by evaporation, but
idation (boiled oil, white lead, red lead, etc.) are slowest of all.

sin, Yellow Wax, and Venetian Red, each 1 oz.; melt and mix.

Aquarium.

ne white dry Sand, and Plaster of Paris, each 1 gill; finely pulverized
l.

ly and make into a paste with boiled linseed oil to which drier has been added. Beat
nd 4 or 5 hours before using it. After it has stood for 15 hours, however, it loses its
cemented into a frame with this cement will resist percolation for either salt or fresh

Adhesive for Fractures of all Kinds.

l ground with Linseed-oil Varnish, and kept from contact with the air.
v weeks to harden.

Stone or Iron.

equal parts of Sulphur and Pitch.

Brass to Glass.

—Resin, 5 ozs.; Beeswax, 1 oz.; Red Ochre or Venetian Red, in pow-
dry earth thoroughly on a stove at above 212°. Melt Wax and Resin
stir in powder by degrees. Stir until cold, lest earthy matter settle

ming brass-work to glass tubes, flasks, etc.

Chinese Waterproof.

—To 3 parts of Fresh Beaten Blood add 4 parts of
a thin, pasty mass is produced, which can be used

ich are to be made specially waterproof are painted twice, &
buildings of China are painted with *schio-liao*, which gives t
s, but adds to their durability. Pasteboard treated with it r
ad.

China.

lk, dried and powdered, 10 ozs.; Quicklime, 1 oz.; Cal
p air-tight. When used, a portion is to be mixed with a little w

Cisterns and Water-casks.

ie, 8 parts; Linseed oil, boiled into a varnish with Lith.
hardens in about 48 hours, and renders the joints of wooden cisterns

Cloth or Leather.

Shellac, 1 part; Pitch, 2 parts; India Rubber, 4 parts; and Gutta Percha, 2 parts; cut small; Linseed oil, 2 parts; melted together and mixed.

Earthen and Glass Ware.

Heat article to be mended a little above 212° , then apply a thin coating of Shellac upon both surfaces of broken vessel.

Or, dissolve gum Shellac in alcohol, apply solution, and bind the parts firmly together until cement is dry.

Or, dilute white of egg with its bulk of water and beat up thoroughly. Mix consistency of thin paste with powdered Quicklime.

Use immediately.

Entomologists'.

Thick Mastic Varnish and Isinglass size, equal parts.

Gutta Percha.

Melt together, in an iron pan, 2 parts Common Pitch and 1 part Gutta Percha.

Stir well together until thoroughly incorporated, and then pour liquid into cold water. When it is black, solid, and elastic; but it softens with heat, and at 100° is a thin fluid. It may be used as soft paste, or in liquid state, and answers an excellent purpose in cementing metal, glass, porcelain, ivory, etc. It may be used instead of putty for glazing.

Glass.

Sorel's.—Mix commercial Zinc White with half its bulk of fine Sand, add a quantity of Chloride of Zinc of 1.26 spec. grav., and mix thoroughly in a mortar.

Apply immediately, as it hardens very quickly.

Holes in Castings.

Sulphur in powder, 1 part; Sal-ammoniac, 2 parts; powdered Iron turnings, 1 part. Make into a thick paste.

Make only as required for immediate use.

Hydraulic Paint.

Hydraulic cement mixed with oil forms an incombustible and waterproof paint for roofs of buildings, outhouses, walls, etc.

Iron Ware.

Sulphur, 2 parts; fine Black-lead, 1 part. Heat sulphur in an iron pan until it melts, then add the lead; stir well, and remove. When cool, break into pieces as required. Place upon opening of the ware to be mended, and solder with iron.

Kerosene Lamps, etc.

Resin, 3 parts; Caustic Soda, 1; Water, 5, mixed with half its weight of Phosphorus of Paris.

It sets firmly in about three quarters of an hour. Is of great adhesive power, not permeable to kerosene, a low conductor of heat, and but superficially attacked by hot water.

Leather to Iron, Steel, or Glass.

1.—Glue, 1 quart, dissolved in Cider Vinegar; Venice Turpentine, 1 oz.; boil gently or simmer for 12 hours.

Or, Glue and Isinglass equal parts, soak in water 10 hours, boil and add tannin until mixture becomes "ropy;" apply warm.

Remove surface of leather where it is to be applied.

2.—Steep leather in an infusion of Nutgall, spread a layer of hot Glue on surface of metal, and apply flesh side of leather under pressure.

Leather Belting.

Common Glue and Isinglass, equal parts, soaked for 10 hours in enough water to cover them. Bring gradually to a boiling heat and add pure Tannin until whole becomes ropy or appears alike to white of eggs.

Clean and rub surfaces to be joined, apply warm, and clamp firmly.

Molding and Temporary Adhesion.

Soft.—Melt Yellow Beeswax with its weight of Turpentine, and color with finely powdered Venetian red.

When cold it has the hardness of soap, but is easily softened and molded with the finger.

Maltha, or Greek Mastic.

ne and Sand mixed in manner of mortar, and made into a proper consistency milk or size without water.

Marble.

ster of Paris, in a saturated solution of Alum, baked in an oven, and reduced wder. Mixed with water, and color if required.

Metal to Glass.

pal Varnish, 15 parts; Drying Oil, 5; Turpentine, 3. Melt in a water bath and o of Slaked Lime.

Mending Shells, etc.

m Arabic, 5 parts; Rock Candy, 2; and White Lead, enough to color.

Large Objects.

ollaston's White.—Beeswax, 1 oz.; Resin, 4 ozs.; powdered Plaster of Paris, 5 Melt together.

rm the edges of the object and apply warm.

means of this cement a piece of wood may be fastened to a chuck, which will hold when cool; and work is finished it may be removed by a smart stroke with tool. Any traces of cement may be red by Benzine.

Marble Workers and Coppersmiths.

bite of egg, mixed with finely-sifted Quicklime, will unite objects which are submitted to moisture.

Porcelain.

ld Plaster of Paris to a strong solution of Alum till mixture is of consistency eam.

ets readily, and is suited for cases in which large rather than small surfaces are to be united.

Rust Joint.

uick Setting.)—Sal-ammoniac in powder, 1 lb.; Flour of Sulphur, 2 lbs.; Iron ogs, 80 lbs. Made to a paste with water.

low Setting.)—Sal-ammoniac, 2 lbs.; Sulphur, 1 lb.; Iron borings, 200 lbs.

latter cement is best if joint is not required for immediate use.

Steam Boilers, Steam-pipes, etc.

nely powdered Litharge, 2 parts; very fine Sand, 1; and Quicklime slaked by sure to air, 1.

a mixture may be kept for any length of time without injuring. In using it, a portion is mixed wate with linseed oil, boiled or crude. Apply quickly, as it soon becomes hard.

ft.—Red or White Lead in oil, 4 parts; Iron borings, 2 to 3 parts.

ard.—Iron borings and salt water, and a small quantity of Sal-ammoniac with a water.

Transparent—Glass.

dia-rubber, 1 part in 64 of chloroform; gum Mastic in powder, 16 to 24 parts. st for two days, with frequent shaking.

pulverized Glass, 10 parts; powdered Fluor-spar, 20; soluble Silicate of Soda,

Both glass and fluor-spar must be in finest practicable condition, which is best y shaking each in fine powder, with water, allowing coarser particles to de-, and then by pouring off remainder, which holds finest particles in suspension.

a mixture must be made very rapidly, by quick stirring, and applied immediately.

Uniting Leather and Metal.

nsh metal with hot Gelatine; steep leather in an infusion of Nutgalls, hot, bring the two together.

Waterproof Mastic.

ed Lead, 1 part; ground Lime, 4 parts; sharp Sand and boiled Oil, 5 parts.

; Red Lead, 1 part; Whiting, 5; and sharp Sand and boiled Oil, 10.

Wood to Iron.

harge and Glycerine.—Finely powdered Oxide of Lead (litharge) and Concen-d Glycerine.

a composition is insoluble in most acids, is unaffected by action of moderate heat, sets rapidly, equires an extraordinary hardness.

erner's.—Melt 1 lb. of Resin, and add .25 lb. of Pitch.

hile boiling add Brick dust to give required consistency. In winter it may e sary to add a little Tallow.

GLUES.

Marine.

Dissolve India Rubber, 4 parts, in 34 parts of Coal-tar Naphtha; add powdered Shellac, 64 parts.

While mixture is hot pour it upon metal plates in sheets. When required in use, heat it, and apply with a brush.

Or, India Rubber, 1 part; Coal Tar, 12 parts; heat gently, mix, and add powdered Shellac, 20 parts. Cool. When used, heat to about 250°.

Or, Glue, 12 parts; Water, sufficient to dissolve; add Yellow Resin, 3 parts; and, when melted, add Turpentine, 4 parts.

Strong Glue.—Add Powdered Chalk to common Glue.

Mix thoroughly.

Mucilage.

Curd of Skim Milk (carefully freed from Cream or Oil), washed thoroughly, and dissolved to saturation in a cold concentrated solution of Borax.

This mucilage keeps well, and, as regards adhesive power, far surpasses gum Arabic.

Or, Oxide of Lead, 4 lbs.; Lamp-black, 2 lbs.; Sulphur, 5 ozs.; and India Rubber dissolved in Turpentine, 10 lbs.

Boil together until they are thoroughly combined.

Preservation of Mucilage.—A small quantity of Oil of Cloves poured into a bottle containing Gum Mucilage prevents it from becoming sour.

To Resist Moisture.

Glue, 5 parts; Resin, 4 parts; Red Ochre, 2 parts; mixed with least practicable quantity of water.

Or, Glue, 4 parts; Boiled Oil, 1 part, by weight; Oxide of Iron, 1 part.

Or, Glue, 1 lb., melted in 2 quarts of skimmed Milk.

Parchment.

Parchment Shavings, 1 lb.; Water, 6 quarts.

Boil until dissolved, then strain and evaporate slowly to proper consistence.

Rice, or Japanese.

Rice Flour; Water, sufficient quantity.

Mix together cold, then boil, stirring it during the time.

Liquid.

Glue, Water, and Vinegar, each 2 parts. Dissolve in a water-bath, then add Alcohol, 1 part.

Or, Cologne or strong Glue, 2.2 lbs.; Water, 1 quart; dissolve over a gentle heat; add Nitric Acid 36°, 7 ozs., in small quantities.

Remove from over fire, and cool.

Or, White Glue, 16 ozs.; White Lead, dry, 4 ozs.; Rain Water, 2 pints. Add Alcohol, 4 ozs., and continue heat for a few minutes.

Elastic and Sweet.—Stamps or Rolls.

Elastic.—Dissolve good Glue in water by a water-bath. Evaporate to a thick consistence, and add equal weight of Glycerine to Glue; submit to heat until all water is evaporated, and pour into molds or on plates.

Sweet.—Substitute Sugar for the Glycerine.

For Engravings or Lithographs upon Wood.
parts; Mastic in tears, 64 parts; Resin, 125 parts; Venice Turpentine, and Alcohol, 1000 parts by measure.

Gilding, or Bronzing, Liquid.

Sweet Spirit of Nitre, 1 oz.; Water, 1 pint.

use.

MISCELLANEOUS MIXTURES.

Gun Barrels.

Infuse of Muriate of Iron, 1 oz.; Nitric Ether, 1 oz.; Sulphate of Iron, 1 pint. If the process is to be hurried, add 2 oz. of Muriate of Mercury.

When barrel is finished, let it remain a short time in lime-water, to neutralize any acid penetrated; then rub it well with an iron wire scratch-brush.

After Browning.—Shellac, 1 oz.; Dragon's-blood, .25 oz.; rectified Spirit, 1 pint. Dissolve and filter.

Or, Nitric Acid, spec. grav. 1.2; Nitric Ether, Alcohol, and Muriate of Iron, 1 pint. Mix, then add Sulphate of Copper 2 parts, dissolved in Water 10 parts.

LACQUERS.

Small Arms, or Waterproof Paper.

Beeswax, 13 lbs.; Spirits Turpentine, 13 gallons; Boiled Linseed Oil, 1 gallon. All ingredients should be pure and of best quality. Heat them together in a copper over a gentle fire, in a water-bath, until they are well mixed.

Bright Iron Work.

Linseed Oil, boiled, 80.5 parts; Litharge, 5.5 parts; White Lead, in oil, 1 part; Resin, pulverized, 2.75 parts.

Add litharge to oil; simmer over a slow fire 3 hours; strain, and add resin and white lead, gradually warmed, and stir until resin is dissolved.

Or, Amber, 6 parts; Turpentine, 6 parts; Resin, 1 part; Asphaltum, 1 part; Rye Oil, 3 parts; heat and mix well.

Or, Shellac, 1 lb.; Asphaltum, 6 lbs.; and Turpentine, 1 gallon.

Iron and Steel.

Clear Mastic, 10 parts; Camphor, 5 parts; Sandarac, 15 parts; and Ether, 10 parts. Dissolve in Alcohol, filter, and apply cold.

Brass.

Shellac, 8 ozs.; Sandarac, 2 ozs.; Annatto, 2 ozs.; and Dragon's-blood, 1 part; and Alcohol, 1 gallon.

Or, Shellac, 8 ozs.; and Alcohol, 1 gallon. Heat article slightly, and apply with a soft brush.

Wood, Iron, or Walls, and rendering Cloth, Paper, or Canvas, Waterproof.

Heat 120 lbs. Oil Varnish in one vessel, 33 lbs. Quicklime in 22 lbs. water. Soon as lime effervesces, add 55 lbs. melted India Rubber. Still pour into vessel of hot Varnish. Stir, strain, and cool.

When used, thin with Varnish and apply, preferably hot.

To Clean Soiled Engravings.

Ozone Bleach, 1 part; Water, 10; well mixed.

INKS.

Indelible, for Marking Linen, etc.

1.—Juice of Sloes, 1 pint; Gum, .5 oz.

This requires no "preparation" or mordant, and is very durable.

2.—Nitrate of Silver, 1 part; Water, 6 parts; Gum, 1 part; Dissolve.

3.—Lunar Caustic, 2 parts; Sap Green and Gum Arabic, each 1 part; distilled water.

"Preparation."—Soda, 1 oz.; Water, 1 pint; Sap Green, .5 drs. Dry wet article to be marked, then dry and apply the ink.

Perpetual, for Tomb-stones, Marble, etc.—Pitch, 11 parts; Linseed Oil, 1 part; Turpentine sufficient. Warm and mix.

Copying Ink.—Add 1 oz. Sugar to a pint of ordinary Ink.

SOLDERING.

Base for Soldering.

Strips of Zinc in diluted Muriatic, Nitric, or Sulphuric Acid, until exposed as acid will effect. Add Mercury, let it stand for a day, then pour off the acid, and bottle the Mercury.

When required, rub surface to be soldered with a cloth dipped in the

VARNISHES.

Waterproof.

Flour of Sulphur, 1 lb.; Linseed Oil, 1 gall.; boil them until they are thoroughly combined.

Good for waterproof textile fabrics.

Harness.

India Rubber, .5 lb.; Spirits of Turpentine, 1 gall.; dissolve into a jelly; then mix hot Linseed Oil, equal parts with the mass, and incorporate them well over a slow fire.

Fastening Leather on Top Rollers.

Gum Arabic, 2.75 ozs., and a like volume of Isinglass, dissolved in Water.

To Preserve Glass from the Sun.

Reduce a quantity of Gum Tragacanth to fine powder, and dissolve it for 24 hours in white of egg well beat up.

Water-color Drawings.

Canada Balsam, 1 part; Oil of Turpentine, 2 parts.

Mix and size drawing before applying.

Objects of Natural History, Shells, Fish, etc.

Mucilage of Gum Tragacanth and of Gum Arabic, each 1 oz.

Mix, and add spirit with Corrosive Sublimate, to precipitate the more stringy portion of the Gum.

Iron and Steel.

Mercury, 120 parts; Tin, 10 parts; Green Vitriol, 20 parts; Hydrochloric Acid of 1.2 sp. gr., 15 parts, and pure Water, 120 parts.

Blackboards.

Shellac Varnish, 5 gallons; Lamp-black, 5 ozs.; fine Emery, 3 ozs.; thin with Alcohol, and lay in 3 coats.

Black.

Heat, to boiling, Linseed Oil Varnish, 10 parts, with Burnt Umber, 2 parts, and powdered Asphaltum, 1 part.

When cooled, dilute with Spirits of Turpentine as may be required.

Balloon.

Melt India Rubber in small pieces with its weight of boiled Linseed Oil. Thin with Oil of Turpentine.

Transfer.

Alcohol, 5 ozs.; pure Venice Turpentine, 4 ozs.; Mastic, 1 oz.

To render Canvas Waterproof and Pliable.

Yellow Soap, 1 lb., boiled in 6 pints of Water, add, while hot, to 112 lbs. of oil Paint.

Waterproof Bags.

Pitch, 8 parts, Wax and Tallow, each 1 part.

To Clean Varnish.

Mix a lye of Potash or Soda, with a little powdered Chalk.

STAINING.

Wood and Ivory.

Yellow

1 Acid will produce it on wood.

1 azil Wood in Stale Urine, in the proportion of 1 lb. to 1 on when boiling hot, also Alum water before it dries food in Spirits of Wine.

2 Nitric Acid

Verdigris in Nitric Acid, which will turn th boiling hot.

amoniac into four times its weight of

dissolved in water and put on hot

MISCELLANEOUS.

Blacking for Harness.

.5 lb.; Ivory Black, 2 ozs.; Spirits of Turpentine, 1 oz.; Prussian Blue oil, 1 oz.; Copal Varnish, .25 oz.
and stir it into other ingredients before mixture is quite cold; make it Rub a little upon a brush, and apply it upon harness, then polish lightly

To Clean Brass Ornaments.

naments that have not been gilt or lackered may be cleaned, and a very for given to them, by washing them in Alum boiled in strong Lye, in the of an ounce to a pint, and afterwards rubbing them with strong Tripoli.

To Harden Drills, Chisels, etc.

them in Mercury.

To Clean Coral.

ith equal parts Spirits of Salts and cold water.
n a hot solution of Potash or Chloride of Lime. If much discolored, let n solution for a few hours.

Blacking, without Polishing.

4 ozs.; Lamp-black, .5 oz.; Yeast, a table-spoonful; Eggs, 2; Olive Oil, ful; Turpentine, a teaspoonful. Mix well.
ied with a sponge, without brushing.

Dubbing.

lbs.; Tallow, 1 lb.; Train-oil, 1 gallon.

Anti-friction Grease.

oo lbs.; Palm-oil, 70 lbs. Boiled together, and when cooled to 80°, strain sieve, and mix with 28 lbs. of Soda, and 1.5 gallons of Water.
ter, take 25 lbs. more oil in place of the Tallow.
c Lead, 1 part; Lard, 4 parts.

To Attach Hair Felt to Boilers.

d, 1 lb.; White Lead, 3 lbs.; and Whiting, 8 lbs. Mixed with boiled Lin-consistency of paint.

Pastils for Fumigating.

ibic, 2 ozs.; Charcoal Powder, 5 ozs.; Cascarilla Bark, powdered, .75 oz.; 25 drachm. Mix together with water, and make into shape.

Writing upon Zinc Labels.—Horticultural.

100 grains of Chloride of Platinum in a pint of water; add a little Mu-Lamp-black.
ummoniac, 1 dr.; Verdigris, 1 dr.; Lamp-black, .5 dr.; Water, 10 drs. Mix.

To Remove old Ironmold.

en part stained with ink, remove this by use of Muriatic Acid diluted by s its weight of water, when old and new stain will be removed.

To Cut India Rubber.

ide of knife wet with water or a strong solution of Po

Adhesive for Rubber Belts.

ving surface with Boiled Oil or Cold Tallow, and the.

Liard.

of finest Rape-oil, and 1 part of Caoutchouc, cut small.
' all dissolved.

To Preserve Leather Belting or Ho-
arm Castor Oil. For hose, force it through it.

To Oil Leather Belting.

solution of India Rubber and Linseed Oil.

Dressing for Leather Belts.

1.—Beef Tallow, 1 part, and Castor Oil, 2 parts. Apply warm.

2.—Beef Tallow, 3 lbs.; Beeswax, 1 lb. Heated and applied warm to both sides of the Belts.

Lay dull files in diluted Sulphuric Acid until they are bitten deep enough.

To Remove Oil from Leather.

Apply Aqua-ammonia.

To Clean Paint.

Wash with a solution of Pearlash in water. If greasy, use Quicklime.
Or, Extract of Luthetium diluted with from 200 to 300 parts of water.

To Remove Paint.

Mix Soft Soap, 2 ozs., and Potash, 4 ozs., in boiling Water, with Quicklime; apply hot, and let remain for 1 day.

Or, Extract of Luthetium, thinly brushed over the surface 2 or 3 times.

To Clean Marble.

Chalk, powdered, and Pumice-stone, each 1 part; Soda, 2 parts. Mix with water. Wash the spots, then clean and wash off with Soap and Water.

Paste for Cleaning Metals.

Oxalic Acid, 1 part; Rottenstone, 6 parts. Mix with equal parts of Train Oil and Spirits of Turpentine.

Watchmaker's Oil, which never Corrodes or Thickens.

Place coils of thin Sheet Lead in a bottle with Olive Oil. Expose it to the sun for a few weeks, and pour off the clear oil.

Durable Paste.

Make common Flour paste rather thick (by mixing some Flour with a little oil and water until it is of uniform consistency, and then stir it well while boiling water is being added to it); add a little Brown Sugar and Corrosive Sublimate, which will prevent fermentation, and a few drops of Oil of Lavender, which will prevent it becoming moldy. When dried, dissolve in water.

It will keep for two or three years in a covered vessel.

To Extract Grease from Stone or Marble.

Soft Soap, 1 part; Fuller's Earth, 2 parts; Potash, 1 part. Mix with boiling water. Lay it upon the spots, and let it stand for a few hours.

Stains.

To Remove.—Stains of Iodine are removed by rectified Spirit; Ink stains by Oxalic or Superoxalate of Potash; Ironmolds by same; but if obstinate, moisten them with Ink, then remove them in the usual way.

Red spots upon black cloth, from acids, are removed by Spirits of Hartshorn, or other solutions of Ammonia.

Stains of Marking-ink, or Nitrate of Silver.—Wet stain with fresh solution of Lime, and, after 10 or 15 minutes, if marks have become white, dip the solution of Ammonia or of Hyposulphite of Soda. In a few minutes wash with water.

Then lay the stained linen over a basin of hot water, and wet mark with Time.

Paste for Objects of Natural History.

b.; Powdered Hellebore, 2 lbs.

Bottoms of Iron Steam-boilers.

Red, 17 parts; Whiting, 6.5 parts; and Litharge.

Preserve Sails.

Use lime-water, and mix it with 120 gallons

Whitewash.

Inside exposure, slack Lime, .5 bushel, in a barrel; add common Salt, 1 lb.; of Zinc, .5 lb.; and Sweet Milk, 1 gallon.

To Preserve Woodwork.

Oil and finely powdered Charcoal, each 1 part; mix to the consistence of Apply 2 or 3 coats.

position is well adapted for caaks, water-spouts, etc.

To Polish Wood.

Surface with Pumice Stone and water until the rising of the grain is removed. With powdered Tripoli and boiled Linseed Oil, polish to a bright surface.

Paint for Window Glass.

16 Green, .25 oz.; Sugar of Lead, 1 lb.; ground fine, in sufficient Linseed Oil to mix to the consistency of cream, and apply with a soft brush.

It should be well cleansed before the paint is applied. The above quantity is sufficient for feet of glass.

To Make Drain Tiles Porous.

Dust with the clay before burning.

CELLANEUS OPERATIONS AND ILLUSTRATIONS.

It is required to lay out a tract of land in form of a square, to be enclosed with a post and rail fence, 5 rails high, and each rod of fence to contain 10 rails. What must be side of this square to contain just as many rods as there are rails in fence?

SOLUTION. 1 mile = 320 rods. Then $320 \times 320 \div 160$, sq. rods in an acre = 640 and 320×4 sides and $\times 10$ rails = 12 800 rails per mile.

As 640 acres : 12 800 rails :: 12 800 acres : 256 000 rails, which will enclose 12 800 acres, and $\sqrt{256\,000} = 506.1$ = number of yards in side of a sq. acre, and yards in a mile = 20 miles.

How many fifteens can be counted with four fives?

$$\text{OPERATION. } \frac{4 \times 3 \times 2 \times 1}{1 \times 2 \times 3} = \frac{24}{6} = 4.$$

What are the chances in favor of throwing one point with three dice?

SOLUTION.—Assume a bet to be upon the ace. Then there will be $6 \times 6 \times 6 = 216$ ways which the dice may present themselves, that is, with and without an ace.

If the ace side of the die is excluded, there will be 5 sides left, and $5 \times 5 \times 5 = 125$ ways without the ace.

Therefore, there will remain only $216 - 125 = 91$ ways in which there could be an ace chance, then, in favor of the ace is as 91 to 125; that is, out of 216 throws, probability is that it will come up 91 times, and lose 125 times.

The hour and minute hand of a clock are exactly together at 12; when will they next together?

SOLUTION.—As the minute hand runs 11 times faster than the hour hand, then, 11 : 1 :: 1 min. 27 $\frac{1}{11}$ sec. = time past 1 o'clock.

Assume a cube inch of glass to weigh 1.49 ounces, and of brandy .53. A gallon of this liquor weighs 3.84 lbs., is thrown into sea-water. It is asked, will it sink, and, if so, how much force will just be required to support it?

SOLUTION. $3.84 \times 12 \div 1.49 = 30.92$ cube ins. of glass in bottle. In a gallon $\times .53 = 122.43$ ounces of brandy.

A bottle and brandy weigh $3.84 \times 12 + 122.43 = 168.51$ ounces, which $\times .59 = 154.53$ ounces, weight of an equal volume of water. $168.51 - 154.53 = 13.98$ ounces, weight necessary to support it.

6.—A fountain has 4 supply cocks, A, B, C, and D, and under it is a cistern, which can be filled by the cock A in 6 hours, by B in 8 hours, by C in 10 hours, and by D in 12 hours; now, the cistern has 4 holes, designated E, F, G, and H, and it can be emptied through E in 6 hours, F in 5 hours, G in 4 hours, and H in 3 hours. Suppose the cistern to be full of water, and that all the cocks and holes were opened together, in what time would the cistern be emptied?

OPERATION.—Assume the cistern to hold 120 gallons.

| hrs. | gall. | hrs. | gall. |
|-------------------------------|--------|------|----------|
| If 6 : | 120 :: | 1 : | 20 at A. |
| 8 : | 120 :: | 1 : | 15 at B. |
| 10 : | 120 :: | 1 : | 12 at C. |
| 12 : | 120 :: | 1 : | 10 at D. |
| Run in in 1 hour, 57 gallons. | | | |

| hrs. | gall. | hrs. | gall. |
|---------------------------------|--------|------|----------|
| If 6 : | 120 :: | 1 : | 20 at E. |
| 5 : | 120 :: | 1 : | 24 at F. |
| 4 : | 120 :: | 1 : | 30 at G. |
| 3 : | 120 :: | 1 : | 40 at H. |
| Run out in 1 hour, 114 gallons. | | | |

Run out in 1 hour more than run in, 57 gallons.

Then, as 57 gallons : 1 hour :: 120 gallons : 2.1054 hours.

7.—A cistern, containing 60 gallons of water, has 3 cocks for discharging it; one will empty it in 1 hour, a second in 2 hours, and a third in 3 hours; in what time will it be emptied if they are all opened together?

OPERATION.—1st, .5 would run out in 1 hour by the 2d cock, and .333 by the 3d; consequently, by the 3 would the reservoir be emptied in 1 hour. $.5 + .333 + .166 = 1$, being reduced to a common denominator, the sum of these $3 = \frac{1}{1}$; whence the proportion, 11 : 60 :: 6 : 32 $\frac{8}{11}$ minutes.

8.—A reservoir has 2 cocks, through which it is supplied; by one of them it will fill in 40 minutes, and by the other in 50 minutes; it has also a discharging cock, by which, when full, it may be emptied in 25 minutes. If the 3 cocks are left open, in what time would the cistern be filled, assuming the velocity of the water to be uniform?

OPERATION.—The least common multiple of 40, 50, and 25, is 200.

Then, the 1st cock will fill it 5 times in 200 minutes, and the 2d, 4 times in 200 minutes, or both, 9 times in 200 minutes; and, as the discharge cock will empty it 8 times in 200 minutes, hence $9 - 8 = 1$, or once in 200 minutes = 3.2 hours.

9.—The time of the day is between 4 and 5, and the hour and minute hands are exactly together; what is the time?

OPERATION.—Difference of speed of the hands is as 1 to 12 = 11.

4 hours $\times 60 = 240$, which $\div 11 = 21$ min. 49.09 sec., which is to be added to 4 hours.

10.—Out of a pipe of wine containing 84 gallons, 10 were drawn off, and the vessel refilled with water, after which 10 gallons of the mixture were drawn off, and then 10 more of water were poured in, and so on for a third and fourth time. It is required to compute how much pure wine remained in the vessel, supposing the two fluids to have been thoroughly mixed.

OPERATION. $84 - 10 = 74$, quantity after the 1st draught.

Then, $84 : 10 :: 74 : 8.8095$, and $74 - 8.8095 = 65.1905$, quantity after 2d draught.

$84 : 10 :: 65.1905 : 7.7608$, and $65.1905 - 7.7608 = 57.4297$, quantity after 3d draught.

$84 : 10 :: 57.4297 : 6.8367$, and $57.4297 - 6.8367 = 50.593$, quantity after 4th draught, = result required.

11.—A reservoir having a capacity of 10000 cube feet, has an influx of 250 and a discharge of 1000 cube feet per day. In what time will it be emptied?

OPERATION. $\frac{10000}{1000 - 250} = 40$ days.

∴ The discharge being 1000 and the influx 250 cube feet per hour

$\frac{10000}{1000 - 250} = 40$ hours = 1 day 16 hours.

—A son asked his father how old he was. His father answered him : If you take away 5 from my years, and divide the remainder by 8, the quotient will be one third of your age; but if you add 2 to your age, and multiply the whole by 3, and then subtract 7 from the product, you will have the number of years of my age. What were the ages of father and son?

SOLUTION.—Assume father's age 37.

In $37 - 5 = 32$, and $32 \div 8 = 4$, and $4 \times 3 = 12$, son's age. Again: $12 + 2 = 14$, $4 \times 3 = 12$, and $42 - 7 = 35$. Therefore $37 - 35 = 2$, error too little.

Ans: Assume father's age 45; then $45 - 5 = 40$, and $40 \div 8 = 5$. Therefore $5 \times 3 = 15$, son's age. Again: $15 + 2 = 17$, and $17 \times 3 = 51$, and $51 - 7 = 44$. Therefore $45 - 44 = 1$, error too little.

Since (45 sup. $\times 2$ error) — (37 sup. $\times 1$ error) = $90 - 37 = 53$, and $2 - 1 = 1$.

Consequently, 53 is father's age. Then $53 - 5 = 48$, and $48 \div 8 = 6$. .333 of son's age and $6 \times 3 = 18$ years, son's age.

—Two companions have a parcel of guineas. Said A to B, if you will give me one of your guineas I shall have as many as you have left. B replied, if you will give me one of your guineas I shall have twice as many as will have left. How many guineas had each of them?

SOLUTION.—Assume B had 6.

Ans A would have had 4, for $6 - 1 = 4 + 1 = 5$. Again: 4 (A's parcel) — 1 = 3, $3 + 1 = 4$, and $3 \times 2 = 6$. Therefore $7 - 6 = 1$, error too little.

Ans: Assume B had 8.

Ans A would have had 6, for $8 - 1 = 6 + 1 = 7$. Again: 6 (A's parcel) — 1 = 5, and $5 \times 2 = 10$. Therefore $10 - 9 = 1$, error too great.

Since $8 \times 1 = 8$, and $6 \times 1 = 6$. Then $8 + 6 = 14$, and $1 + 1 = 2$. Whence, dividing products by sum of errors, $14 \div 2 = 7 =$ B's parcel, and $7 - 1 = 5 + 1 = 6$ when he had received 1 of B; also $5 - 1 \times 2 = 7 + 1 = 8 =$ B's parcel when he received 1 of A.

—If a traveller leaves New York at 8 o'clock in the morning, and walks towards New London at the rate of 3 miles per hour, without intermission; another traveller starts from New London at 4 o'clock in the evening, walks towards New York at the rate of 4 miles per hour continuously; what distance between the two cities to be 130 miles, whereabouts upon the road will they meet?

SOLUTION.—From 8 to 4 o'clock is 8 hours; therefore, $8 \times 3 = 24$ miles, per-
formed by A before B set out from New London; and, consequently, $130 - 24 = 106$
he miles to be travelled between them after that.

Since, as $(3 + 4) 7 : 3 :: 106 : 21\frac{2}{3} = 45\frac{2}{3}$ more miles travelled by A at the meeting; consequently, $24 + 45\frac{2}{3} = 69\frac{2}{3}$ miles from New York is place of their meeting.

—If from a cask of wine a tenth part is drawn out and then it is filled with water; after which a tenth part of the mixture is drawn out; again filled, and again a tenth part of the mixture is drawn out: now, assume fluids to mix uniformly at each time the cask is replenished, what fractional part of wine will remain after the process of drawing out and replenishing has been repeated four times?

SOLUTION.—Since .1 of the wine is drawn out at first drawing, there must remain .9 of wine. After cask is filled with water, .1 of whole being drawn out, there will remain .81 of wine; but .9 of this mixture is wine; therefore, after second drawing, there remain .9 of .9 of wine, or $\frac{9^2}{10^2}$; and after third drawing, there will remain

of .9 of wine, or $\frac{9^3}{10^3}$.

Since, the part of wine remaining is expressed by the ratio, the ratio of which is number of times cask has been drawn from.

Therefore, fractional part of wine is $\frac{9^4}{10^4} = .6561$.

16.—There is a fish, the head of which is 9 ins. long, the tail as long as the head and half the body, and the body as long as both the head and tail. Required the length of the fish.

OPERATION.—Assume body to be 24 ins. in length. Then $24 \div 2 + 9 = 21$, length of tail.

Hence $21 + 9 = 30$, length of body, which is 6 ins. too great.

Again: assume the body to be 26 ins. in length. Then $26 \div 2 + 9 = 22$, length of tail. Hence $22 + 9 = 31$, length of body, which is 5 ins. too great.

Therefore, by *Double Position*, divide difference of products (see rule, page by difference of errors (the errors being alike), $26 \times 6 - 24 \times 5 = 36 =$ difference of products, and $6 - 5 = 1 =$ difference of errors.

Consequently, $36 \div 1 = 36$, length of body, and $36 \div 2 + 9 = 27$, length of tail. $36 + 27 + 9 = 72$ ins., length required.

17.—A hare, 50 leaps before a greyhound, takes 4 leaps to the greyhound's 3, but 2 leaps of the hound are equal to 3 of the hare's. How many leaps must the greyhound take before he can catch the hare?

OPERATION.—As 2 leaps of the greyhound equal 3 of the hare, it follows that 4 the greyhound equal 9 of the hare.

While the greyhound takes 6 leaps, the hare takes 8; therefore, while the hare takes 8, the greyhound gains upon her 1.

Hence, to gain 50 leaps, she must take $50 \times 8 = 400$ leaps; but, while hare takes 400 leaps, greyhound takes 300, since number of leaps taken by them are as 4 to 3.

18.—If a basket and 1000 eggs were laid in a right line 6 feet apart, and 10 men (designated from A to J) were to start from basket and to run alternately, collect the eggs singly, and place them in basket as collected, and each man to collect but 10 eggs in his turn, how many yards would each man run over, and what would be entire distance run over?

OPERATION.—A's course would be 6×2 feet (first term) $+ 10 \times 6 \times 2$ feet (last term) $= 132 =$ sum of first and last terms of progression.

Then $132 \div 2 \times 10 = 660$ feet $=$ number of times \times half sum of extremes $=$ sum of all the terms, or the distance run by A in his first turn.

B's course would be $11 \times 6 \times 2 = 132$ feet (first term) $+ 20 \times 6 \times 2 = 240$ feet (last term) $= 372 =$ sum of first and last terms.

Then $372 \div 2 \times 10 = 1860 =$ sum of all the times, or B's first turn.

A's last course would be $901 \times 6 \times 2 = 10812$ feet for the first term, and $900 \times 11 \times 6 \times 2 = 11920$ feet for the last term of his last turn.

Then $10812 + 11920 \div 2 \times 10 = 108660 =$ sum of the terms, or distance run.

B's last course would be $911 \times 6 \times 2 = 10932$ feet for the first term, and $900 \times 20 \times 6 \times 2 = 11040$ feet for the last term of his last turn.

Then $10932 + 11040 \div 2 \times 10 = 109860 =$ sum of the terms or distance run.

Therefore, if A's first and last runs $= 660$ and 108660 feet, and the number of terms 10, then, by *Progression*, the sum of all the terms $= 546600$ feet.

And if B's first and last runs $= 1860$ and 109860 feet, and the number of terms 10, then the sum of all the terms $= 558600$ feet.

Consequently, $558600 - 546600 = 12000 =$ common difference of runs, which being added to each man's run $=$ sum of all runs, or entire distance run over.

A's run, 546 600 $=$ 182 200 yds. F's run, 606 600 $=$ 202 200 yds.

B's " 558 600 $=$ 186 200 " G's " 618 600 $=$ 206 200 "

C's " 570 600 $=$ 190 200 " H's " 630 600 $=$ 210 200 "

D's " 582 600 $=$ 194 200 " I's " 642 600 $=$ 214 200 "

E's " 594 600 $=$ 198 200 " J's " 654 600 $=$ 218 200 "

6006 000 feet, which $\div 5280 = 11375$ yds.

19.—If, in a pair of scales, a body weighs 90 lbs. in one scale, and 40 lbs. in the other, what is the true weight?

$$\sqrt{(40 \times 90)} = 60 \text{ lbs.}$$

—If a steamboat, running uniformly at the rate of 15 miles per hour through the water, were to run for 1 hour with a current of 5 miles per hour, to return against that current, what length of time would she require to reach the place from whence she started?

RATION. $15 + 5 = 20$ miles, the distance run during the hour.

At $15 - 5 = 10$ miles is her effective velocity per hour when returning, and $10 = 2$ hours, the time of returning, and $2 + 1 = 3$ hours, or the whole time occupied.

Let d represent distance in one direction, t and t' greater and less times of running in hours, and c current or tide.

$$d \frac{t+t'}{t \times t'} = \text{velocity of boat through the water, and } \frac{v \times t' - d}{t'} = c.$$

—Flood-tide wave in a given river runs 20 miles per hour, current of 3 miles per hour. Assume the air to be quiescent, and a floating body free at commencement of flow of the tide; how long will it drift in one direction, the tide flowing for 6 hours from each point of river?

RATION.—Let x be the time required; $20x =$ distance the tide has run up, together with the distance which the floating body has moved; $3x =$ whole distance the body has floated.

At $20x - 3x = 6 \times 20$, or the length in miles of a tide.

$$x = \frac{20}{20-3} \times 6 = 7 \text{ hours, 3 minutes, 31.765 seconds.}$$

—A steamboat, running at the rate of 10 miles per hour through the water, descends a river, the velocity of which is 4 miles per hour, and returns in 10 hours; how far did she proceed?

RATION.—Let $x =$ distance required, $\frac{x}{10+4} =$ time of going, $\frac{x}{10-4} =$ time of returning.

Then, $\frac{x}{14} + \frac{x}{6} = 10$; $6x + 14x = 840$; $20x = 840$; $840 \div 20 = 42$ miles.

—From Caldwell's to Newburgh (Hudson River) is 18 miles; the current of the river is such as to accelerate a boat descending, or retard one ascending, 1.5 miles per hour. Suppose two boats, running uniformly at the rate of 15 miles per hour through the water, were to start one from each end at the same time, where will they meet?

RATION.—Let $x =$ the distance from N. to the place of meeting; its distance from C., then, will be $18 - x$.

Speed of descending boat, $15 + 1.5 = 16.5$ miles per hour; of ascending boat, $15 - 1.5 = 13.5$ miles per hour. $\frac{x}{16.5} =$ time of boat descending to point of meeting. $\frac{18-x}{13.5} =$ time of boat ascending to point of meeting.

These times are of course equal; therefore, $\frac{x}{16.5} = \frac{18-x}{13.5}$. Then, $13.5x = 297 - 16.5x$ and $13.5x + 16.5x = 297$, or $30x = 297$.

At $x = \frac{297}{30} = 9.9$ miles, the distance from Newburgh.

—There is an island 73 miles in circumference; 3 men start together to walk around it and in the same direction: A walks 5 miles per hour, B 3 miles per hour; when will they all come aside of each other again?

RATION.—It is evident that A and C will be together every round it remains to ascertain when A and B will be in conjunction at a point. As miles are gained every day by B. Therefore, as $3 : 1 :: 73 : 24 \frac{1}{3}$, in conjunction is a fractional number, it is necessary to ascertain what multiplier will make the division a whole number.

At $24 \div 3 = 8$, the number of days required in which A will go 1 round C 10 times.

25.—Assume a cow, at age of 2 years, to bring forth a cow-calf, to continue yearly to do the same, and every one of her produce forth a cow-calf at age of 2 years, and yearly afterward in like manner how many would spring from the cow and her produce in 40 years?

OPERATION.—The increase in 1st year would be 2, in 2d year 1, in 3d 1, in 5th 3, in 6th 5, and so on to 40 years or terms, each term being = sum of preceding ones. The last term, then, will be 165 580 141, from which is subtracted 1 for the parent cow, and the remainder, 165 580 140, will represent required.

26.—The interior dimensions of a box are required to be in the ratios of 2, 3, and 5, and to contain a volume of 1000 cube ins.; what be the dimensions?

OPERATION.— $\sqrt[3]{\frac{1000 \times 2^3}{2 \times 3 \times 5}} = 6.43$; $\sqrt[3]{\frac{1000 \times 3^3}{2 \times 3 \times 5}} = 9.65$; and $\sqrt[3]{\frac{1000 \times 5^3}{2 \times 3 \times 5}} = 15$.

And what for a box of one half the volume, or 500 cube ins., and same proportionate dimensions?

OPERATION.— $2 \times 3 \times 5 = 30$, and $\frac{30}{2} = 15$.

Then, $\sqrt[3]{\frac{15 \times 6.43^3}{30}} = 5.1$; $\sqrt[3]{\frac{15 \times 9.65^3}{30}} = 7.66$; and $\sqrt[3]{\frac{15 \times 15^3}{30}} = 12$.

27.—The chances of events or games being equal, what are the odds for or against the following results?

| Five Events. | | | Four Events. | | |
|--------------|---|------------|--------------|--|-----------|
| Odds. | Against. | In favor. | Odds. | Against. | In favor. |
| 31 to 1 | All the 5 | 1 out of 5 | 15 to 1 | All the 4 | 1 out |
| 4.33 to 1 | 4 out of 5 | 2 out of 5 | 2.2 to 1 | 3 out of 4 | 2 out |
| 5 to 3 | in favor of the 5 events resulting 3 and 2. | | 5 to 3 | against 2 events only, 0 the 4 events do not result 2 and 2. | |

| Three Events. | | | Two Events. | | |
|---------------|---|---------------------|--------------------------------------|-------------------|-------------------|
| Odds. | Against. | In favor. | Odds. | Against. | In favor. |
| 7 to 1 | All the 3 | 1 out of 3 | 3 to 1 | Both events | 1 out |
| Even | { 2 or all out of 3 | { 2 or all out of 3 | Even | { 1 only out of 2 | { 1 only out of 2 |
| 3 to 1 | in favor of the 3 events resulting 2 and 1. | | Even that the events result 1 and 2. | | |

28.—Required the chances or probabilities in events or games, where chances or probabilities of the results, or the players, are equal.

| Events or Games. | That a named event occurs a majority or more of times. | Against a named event occurring an exact majority of times. | Against each event occurring an equal number of times. | Events or Games. | That a named event occurs a majority or more of times. | Against a named event occurring an exact majority of times. | Against each event occurring an equal number of times. |
|------------------|--|---|--|------------------|--|---|--|
| 21 | Even | 5 to 1 | — | 11 | Even | 3.4 to 1 | — |
| 20 | 1.33 to 1 | — | 4.66 to 1 | 10 | 1.7 to 1 | — | 3.0 |
| 19 | Even | 4.5 to 1 | — | 9 | Even | 3 to 1 | — |
| 18 | 1.55 to 1 | — | 4.4 to 1 | 8 | 1.75 to 1 | — | 2.6 |
| 17 | Even | 4.4 to 1 | — | 7 | Even | 2.7 to 1 | — |
| 16 | 1.5 to 1 | — | 4.1 to 1 | 6 | 2 to 1 | — | 2.2 |
| 15 | Even | 4 to 1 | — | 5 | Even | 2.2 to 1 | — |
| 14 | 1.5 to 1 | — | 3.8 to 1 | 4 | 2.2 to 1 | — | 1.6 |
| 13 | Even | 3.7 to 1 | — | 3 | Even | 1.66 to 1 | — |
| 12 | 1.6 to 1 | — | 3.44 to 1 | 2 | 3 to 1 | — | 1.0 |

29.—The chances of consecutive events or results are as follows:

to 1. | 10.—1023 to 1. | 9.—511 to 1. | 8.—255 to 1. | 7.—127 to 1. | 6.—63 to 1. | 5.—31 to 1. | 4.—15 to 1. | 3.—7 to 1. | 2.—3 to 1. | 1.—1 to 1.

of 11 consecutive events compared with 10,

30.—Required the chances or probabilities of events or results in a given number of times.

The *numerator* of a fraction expresses the chance or probability either for the result or event to occur or fail, and the *denominator* all the chances or probabilities for it to occur or fail.

Thus, in a given number of events or games, if the chances are even, the probability of any particular result is as $\frac{1}{1+1} = \frac{1}{2}$; $\frac{2}{2+2}$; $\frac{3}{3+3}$, etc., being 1 out of 2 out of 4, etc., or even.

If the number of events or games are 3, then the probability of any particular result, as 2 and 1, or 1 and 2, is determined as follows:

Number of permutations of 3 events are $1 \times 2 \times 3 = 6$, which represents number of times that number of events can occur, 2 and 1, or 1 and 2, to which is to be added the 2 times or chances they can occur all in one way or the reverse thereto.

Hence, $\frac{6}{2+6} = \frac{3}{4} = \frac{3}{4-1} = \frac{3}{3}$, or 3 to 1 in favor of result; and probability of 1 party naming or winning two precise events or results, as winning 2 out of 3, determined as follows: Number of permutations and chances, as before shown,

is 8. Hence, number of his chances being 3, $\frac{3}{3+5} = \frac{3}{8} = \frac{3}{8-3} = \frac{3}{5}$, or 3 to 5 in favor of result; and probability of one party naming or winning all, or 3 events or results, is determined as follows: Number of permutations and chances being 8, as before shown, 8. Hence, as there is but one chance of such a result, $\frac{1}{1+7} = \frac{1}{8} = \frac{1}{8-1} = \frac{1}{7}$, or 1 to 7 in favor of result.

If number of events, etc., are 4, then probability of any particular result, 2 and 2, or of winning 2 or more of them, is determined as follows:

Number of permutations and chances of 4 events are 16. Hence, as number of chances of such a result are 11, $\frac{11}{5+11} = \frac{11}{16} = \frac{11}{16-11} = \frac{11}{5}$, or as 11 to 5 in favor of the result, and that the results do not occur precisely 2 and 2. The number of chances of such a result being 10, $\frac{10}{6+10} = \frac{5}{8} = \frac{5}{8-5} = \frac{5}{3}$, or 5 to 3 against it.

If number of events, etc., are 5, then probability of any particular result, 3 and 2, is determined as follows:

Number of permutations and chances being 32, and number of chances of such result being 20, $\frac{20}{12+20} = \frac{10}{16} = \frac{10}{16-10} = \frac{10}{6} = \frac{5}{3}$, or as 5 to 3 in favor of the result; and that it may occur precisely 3 out of 5, the number of chances are $\frac{10}{10+22} = \frac{10}{32} = \frac{5}{16} = \frac{5}{16-5} = \frac{5}{11}$, or 11 to 5 against it.

31.—What is the dilatation of the iron in a railway track per mile, between the temperatures of -20° and $+130^{\circ}$?

OPERATION.— $-20^{\circ} + 130^{\circ} = 150^{\circ}$. The dilatation of wrought iron (as per table, page 519) is, from 32° to $212^{\circ} = 180^{\circ} = .0012575$ times its length.

Hence, as $180 : 150 :: .0012575 : .0010479 = \frac{.0010479}{1}$ of 5280 (feet in a mile) = 53 feet per mile.

32.—A steamer having an immersed amidship section of 125 sq. feet, has a speed of 15 miles per hour with 300 HP. What power would be required of one of like model, having a section of 150 sq. feet for a speed of

As power required for like models is as cube of speeds.

Then $\frac{150}{125} = 1.2$ relative sections, and $\frac{20^3 = 8000}{15^3 = 3375} = 2.37$ relative p

Hence, $1 : 1.2 :: 2.37 : 2.844$ times HP.

ELEMENTS AND CAPACITIES OF NAVAL MARINE STEAMERS.

Cruisers, Iron-clad and Protected.

Compound, Triple, and Quadruple Expansion.

Dimensions and Hull, in feet and ins.; Engines and Propeller, in feet and ins.; Revolutions, per minute; Surfaces, in sq. feet; Pressure, in pounds, and Weights and Displacements, in tons of 2240 lbs.

Speed in Knots per Hour.

| ENGINES AND CAPACITIES. | Cushing.* | American. | | | | | | | | | | English. | | | | Rattle- snake and Class. |
|-----------------------------|-------------------|----------------|--------------------|----------------|--------------------|--------------------|------------------|--------------------|------------------|--------------------|------------------------|-------------------------|-----------------------------------|------------------|------------------|-----------------------------------|
| | | Maime. | Charles- ton. | York- town. | Chi- cago. | Balti- more. | Yan- kee. | Phila- delphia. | New- ark. | San Francisco. | Medea and China. | Bombay and China. | Tran- sylvania and Nile. | Arch- er. | Unarm. Steel. | |
| Construction..... | Unarm'd Steel. | Iron- clad. | Prot'd Cruiser. | Gun- boat. | Prot'd Cruiser. | Prot'd Cruiser. | Unarm. Steel. | Prot'd Steel. | Prot'd Steel. | Prot'd Cruiser. | Prot'd Cruiser. | Prot'd Cruiser. | Iron- clad. | Unarm. Steel. | Unarm. Steel. | 200 |
| Length..... | 137.5 | 310 | 300 | 228 | 315 | 315 | 246.25 | 315 | 310 | 310 | 265 | 325 | 345 | 225 | 225 | |
| Beam..... | 14.1 | 57 | 46.2 | 36 | 48.3 | 48.6 | 26.6 | 48.7 | 48.1 | 49.2 | 41 | 68 | 73 | 36 | 36 | 23 |
| Hold..... | — | — | — | 18.9 | 34.9 | — | 14.1 | — | 31.8 | 28.8 | — | — | — | — | — | — |
| Draught, medium..... | 4.5 | 21.6 | 17.10 | 14 | 19.1 | 19.2 | 9.3 | 19.2 | 18.9 | 18.9 | 16.6 | 26.4 | 27.6 | 13.6 | 13.6 | 8.33 |
| Displacement at do. do..... | 91.34 | 6648 | 3557 | 1703 | 4526 | 4500 | 805 | 4325 | 4083 | 7400 | 2800 | 9500 | 11940 | 1630 | 1630 | 475 |
| Immersed Sec'n at do..... | 42.36 | 1080 | 712 | 435 | — | 833 | — | 845 | 807 | 770 | — | — | — | — | — | — |
| Cylinders, HP..... | 11.25 | 35.5 | 44.125 | 22 | 45 | 42 | 21.5 | 38 | 34 | 42 | 33.5 | 52 | 43 | 27 | 27 | 18.5 |
| " Int..... | 22.5 | 57 | 88 | 31 | — | 60 | 31 | 86 | 76 | 59 | 47 | — | 62 | 50 | 42 | 27 |
| stroke of Piston..... | 3 of 22.5 | 15 | 36 | 30 | 78 | 94 | 2 of 34 | 86 | 40 | 92 | 74 | 2 of 74 | 96 | 50 | 42 | 27 |
| steam, Pressure of..... | 245 | 135 | 91.5 | 157 | 85 | 133 | 157 | 153 | 160 | 160 | 36 | 39 | 45 | 51 | 33 | 18 |
| Revolutions..... | 370 | 133 | 160.8 | 70 | 116.25 | 670 | 269 | 119.6 | 127 | 129 | 135 | 89.2 | 134 | 127 | 136 | 136 |
| Grate Surface..... | 76.5 | 553 | 436.2 | 220 | 672 | 670 | 195 | 624 | 561.4 | 1268 | 456 | 756 | 604 | 157 | 309.5 | 309.5 |
| Heating..... | 4750 | 18300 | 15377 | 8032 | 21306 | 17175 | 8941 | 20458 | 17295 | 43272 | 14070 | 20394 | 19390 | 5900 | 5000 | 5000 |
| Condensing "..... | 1052 | 14020 | 13795 | 4760 | 9174 | 12369 | 4518 | 13510 | 12510 | 18948 | 14518 | 17850 | 15000 | 5640 | 4000 | 4000 |
| Propeller, diam..... | 2 of 2 | 14 | 14 | 10.5 | 15.6 | 14.5 | 7.9 | 14.6 | 14.6 | 3 of 3 | 2 of 2 | 2 of 2 | 2 of 2 | 2 of 2 | 2 of 2 | 2 of 2 |
| " pitch..... | 8.5 | 17.5 | 17.6 | 12.5 | 24.54 | 18.6 | 9.4 | 20.5 | 19 | — | 13.6 | 18.6 | 16.6 | 16.6 | 16.6 | 6.6 |
| Coal, weight..... | 25 | 400 | 328 | 300 | 940 | 400 | 9.4 | 20.5 | 19 | 750 | 400 | 1200 | 940 | 250 | 700 | 700 |
| Speed..... | 22.5 | 17 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 | 16.6 |
| HP..... | 22.5 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 | 270 |
| Combustion..... | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |

ELEMENTS AND CAPACITIES OF NAVAL MARINE STEAMERS.

Compound and Triple Expansion.

Length, between perpendiculars and Hull, in feet and ins.; Engines and Propeller, in feet and ins.; Revolutions, per minute; Surfaces, in sq. feet; Pressure, in pounds, and Weights and Displacements, in tons of 2240 lbs.

Speed in Knots per Hour.

| | English. | | French. | | Italian. | | Spanish. | | Austrian. | | Brazilian. | | Chilian. | | Jap. | |
|-------------------------|----------------|-----------------|-----------------|-----------------|-----------------|------------|----------------|----------------|----------------|-----------------|------------|------------|-----------------|-------------------|-----------------|-----------------|
| | Sunder-land.* | Galleon. | Cécille. | Forbin. | Plemonite. | Le-panto. | Nib-ble.* | Tripoli. | De-struct-or. | Reina Regie. | Ardie. | Torpedo. | Prot'd Cruiser. | Almirante Condal. | Emen-ralda. | Prot'd Cruiser. |
| Construction..... | Unarm'd Steel. | Belted Cruiser. | Prot'd Cruiser. | Prot'd Cruiser. | Prot'd Cruiser. | Iron-clad. | Unarm'd Steel. | Unarm'd Steel. | Unarm'd Steel. | Prot'd Cruiser. | Torpedo. | Iron-clad. | Prot'd Cruiser. | Torpedo. | Prot'd Cruiser. | Prot'd Cruiser. |
| Length..... | 137 | 300 | 378.9 | 311.7 | 300 | 400.5 | 151.8 | 229.6 | 192.5 | 320 | 147.5 | 395 | 270 | 240 | 300 | 300 |
| Beam..... | 13-9 | 56 | 49.3 | 30.6 | 38 | 72.9 | 17 | 25.7 | 25 | 50.6 | 14.5 | 52 | 45 | 27.6 | 46.16 | 46.16 |
| Hold..... | — | — | — | — | — | — | — | 18.31 | 6.6 | 20 | 3.33 | 19.6 | 18.6 | 9 | 18.6 | 18.6 |
| Draught, medium..... | 4 | 21 | 19.8 | 13.11 | 15 | 30.4 | — | — | 385 | 4800 | 97 | 52 | 3000 | 710 | 3730 | 3730 |
| Displacement at do. do. | 84 | 5040 | 5766 | 1848 | 2500 | 14860 | 145 | — | — | — | — | — | — | — | — | — |
| Immersed Sec'n at do. | 40 | 8375 | 8375 | 3499 | 3499 | 1999 | — | — | — | — | — | — | — | — | — | — |
| Cylinder, HP..... | 15 | 36 | 39 | 36.8 | 36 | — | 17 | 19.5 | 18.5 | 40 | 14.5 | 52 | 43 | 22 | 44 | 44 |
| " " Int..... | 22 | 51 | 55 | — | 55 | — | 26 | 27 | 60 | — | — | 2 of 74 | 82 | 33 | 85 | 85 |
| Stroke of Piston..... | 33.5 | 77 | 72 | 73.6 | 2 of 60 | 54 | 37 | 35 | 42 | 92 | 24.5 | 18 | 36 | 21 | 36 | 36 |
| Steam, Pressure of..... | 160 | 138 | — | 100 | — | 49 | 195 | 130 | 145 | 140 | 139 | 143 | 90 | 143 | 88.5 | 88.5 |
| Revolutions..... | 352 | 113.5 | 85.8 | 140 | — | 93.5 | 325 | 207 | 202 | 116 | 375 | 355 | 81.5 | 116.5 | 270 | 221.4 |
| Grate Surface..... | — | 500 | 852 | 291 | — | 1153.5 | 3400 | 6696 | 5920 | 658 | 75.4 | 44 | 585 | 450 | 190 | 392 |
| Heating "..... | 2174 | 15900 | 23919 | 14100 | — | 42080 | — | — | — | 22500 | 4240 | 2000 | 15356 | 6000 | 15114 | 15114 |
| Condensing Surface..... | 1745 | 12000 | 15494 | — | — | 31360 | 2 of 2 | 3 of 2 | 5000 | 15000 | 2 of 2 | — | — | — | 13500 | 13500 |
| Propellers, diam..... | 6 | — | 2 of 2 | 13.2 | 2 of 2 | 20.6 | 5.11 | 7.1 | 2 of 2 | 2 of 2 | — | — | 2 of 2 | 2 of 2 | — | — |
| " " pitch..... | 8.3 | 23.3 | 650 | 200 | 200 | 18.8 | 14 | 100 | 45 | 500 | — | — | 18 | 9 | — | — |
| Coal, weight..... | Oil | 440 | — | 20.6 | 22 | 18.38 | 26.8 | 19.8 | 22.08 | 20.6 | 24.9 | 15.3 | 18.28 | 20.3 | 18.9 | 18.9 |
| " " "..... | 1204 | 9203 | 6348 | 5000 | 12700 | 16150 | 2200 | 3016 | 3829 | 12000 | 1500 | 4537 | 6750 | 4350 | 7650 | 7650 |
| Blast..... | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast | Blast |

* Torpedo-boats.

Freight.

Expansion.

*ins.; Engines, in ins.; Pressure, in lbs.; Revolutions, per minute;
lbs. per Hour; P Passengers and F Freight.*

Speed in Miles per Hour.

| Puritan. | Tuscaro- ra. | City of Racine. | John F. Smith. | New York. | Ata- lanta. | Susque- hanna. | Robert E. Lee. | Mary- land. |
|--------------------------|--------------------|-------------------------|-------------------------|-------------------|-----------------------|-----------------------|-------------------------|--------------------------|
| 10. Steel. P and F | 11. Steel. F | 12. Iron. F and P | 13. Iron. P and F | 14. Iron. P | 15. Iron. Yacht | 16. Iron. Yacht | 17. Wood. P and F | 18. Steel. F and F |
| — | 306.7 | 220 | 130 | 315 | 240 | 166.5 | 315 | 332 |
| 402 | 296.7 | — | 122 | — | 228.5 | 150 | 306 | — |
| 403.5 | 289.3 | 203.5 | 122 | 301 | 222.9 | 164 | 315.8 | 316.4 |
| 52 | 40 | 35 | 42 | 40.2 | 26.33 | 22 | 48.5 | 42 |
| 18.1 | 23 | — | 9 | 11 | 15.2 | 13 | 9.2 | 20.4 |
| 1 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| 3075 | 1937 | 802 | 142.39 | 1092 | 284 | 117 | — | 1892 |
| 4593 | 2669 | 1041 | 135.60 | 1553 | 568 | 233 | 1479 | 2419 |
| 13 | 16 | — | 5.3 | 6.33 | 12 | 9.3 | — | 16 |
| 4775 | 3570 | — | 155 | 1000 | 1042 | 310 | — | 4690 |
| 643 | 624 | 218 | 83 | 235 | 246 | 128 | — | 650 |
| 7 | 9.7 | — | 3 | 6 | 4.75 | 4.5 | — | 8 |
| 75 | 24 | 28 | 14 | — | 30 | 17 | of 40.5 | 22 |
| — | 38 | — | — | — | — | 28 | — | 35 |
| 110 | 61 | 50 | 26 | 75 | 60 | 42 | — | 56 |
| and 14 | 42 | 36 | 18 | 12 | 30 | 22 | 10 | 44 |
| 110 | 160 | 110 | 125 | 50 | 115 | 160 | 148 | 160 |
| 24 | 90 | 100 | — | 30 | 128 | 164 | 21 | 85 |
| 8 | 3 | 2 | 1 | 3 | 2 | 1 | 9 | 2 |
| 850 | 162 | 90 | — | 230 | 146 | 65 | 118 | 152 |
| 26 000 | 5574 | 4000 | 1258 | 5360 | 4534 | 2180 | 3360 | 4656 |
| 15 000 | Jet | — | 624 | 5700 | 2226 | 1470 | — | — |
| — | 14 | 10.5 | 6.4 | — | — | 8 | — | 13.2 |
| — | 17.5 | 16.5 | 9 | — | — | — | — | 16 |
| 35 | — | — | — | 30.16 | — | — | 39 | — |
| 14 | — | — | — | 12.5 | — | — | 17 | — |
| 200 | 230 | 110 | 15 | 50 | 170 | 50 | — | 200 |
| 9 p. HP | 2340 | 1400 | 4 | 2.5 p. HP | — | — | — | — |
| Natural | Natural | Natural | Natural | Blast | Blast | Blast | Natural | Natural |
| 900 | 2140 | 310 | — | — | — | — | 1425 | 3100 |
| 1200 | — | — | 150 | 2100 | 18 | — | 300 | 8 |
| 200 | 29 | — | 8 | 50 | 45 | — | 150 | 19 |
| 7500 | 1800 | 750 | 300 | 3700 | 1950 | 925 | — | 1200 |
| 21 | 16 | 14.5 | 13 | 23 | 17.45 | 18 | 21 | 12.5 |
| — | Sch'r | Sch'r | — | — | 3-m Sch'r | Sch'r | — | — |

nd W. and A. Fletcher Co., Hoboken, N. J. ——— No. 11.

Cleveland, Ohio; Hull 1240 tons, Engines 200, and Boilers 70 t.

Chas. F. Elmes, Chicago, Ill., and Burger & Burger, Wis.;

1 deck; Hull 350 tons, Engines 40, Boilers 36, and Water 23.—

ey & Jones Co., Wilmington, Del. ——— No. 14. W. & A. Fle

1, N. J.; Water-wheel blades, 13 of 45 ins. ——— No. 15. Sam

——— No. 16. The Harlan & Hollingworth Co., Wilmington, 1

2, Engines 20, and Boilers 25. ——— No. 17. Jas. Howard &

lle and American Foundry, New Albany, Ind.; Water-wheel t

a ——— No. 18. Detroit Dry Dock Co., Detroit, Mich.; Hull 1.

140, and Boilers 70.

Ferry Passenger and Team, and Tow-boat
Single, Compound, and Triple Expansion.

Length and fuel, in feet and tenths; Draught, Propeller, and Sigs Wink
and in.; Engines, in in.; Pressure, in lb.; Revolutions per minute; Speed
in sq. feet; Weights and Displacements, in Tons of 2240 lb.; Fuel, in lb.
Hour; P Passengers and T Tonnage.

Speed in Miles per Hour.

| Dimensions and Capacity | Maxim and Washburn Prop. | John G. Mott and Washburn Prop. | Barnes Prop. | De-Long Prop. | Mattison Prop. | Low and Washburn Prop. | East Prop. |
|-------------------------|--------------------------|---------------------------------|--------------|---------------|----------------|------------------------|------------|
| | 1. In. | 2. In. | 3. In. | 4. In. | 5. In. | 6. In. | 7. In. |
| Service | Full T | T and P | Full T | Twining | T and P | Twining | Twining |
| Length on deck | 209 | 215 | 209 | 210 | 212.5 | 210 | 210 |
| " " bet. perp. st. | 195 | 198.5 | 200 | 210 | 215 | 210 | 210 |
| " " beam | 195 | 198.5 | 200 | 210 | 215 | 210 | 210 |
| Beam do. | 37.4 | 45 | 37 | 38.5 | 41.5 | 42 | 42.5 |
| " " over guards | 65 | 62 | — | 62.5 | — | — | — |
| Hold, tonnage | 14.2 | 14.5 | 16.6 | 22.5 | 13.3 | 16.3 | 3.5 |
| Decks | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Tons | 839 | 1008 | 734 | 1088.0 | 545.7 | 400 | 55.6 |
| " " draught, load | 9 | 11 | 9.5 | 9.5 | 7.2 | 12 | 8 |
| Displacement do. | 880 | 1340 | 560 | 303 | 678.3 | 530 | 150 |
| Immersed Sec'n at do. | 215 | 450 | 225 | 164 | 206 | 260 | — |
| Freeboard | 7.5 | 7.75 | 6.9 | 4 | 5 & 6.5 | 5.5 | — |
| Cylinders, 1P | — | 22 | 18.5 | 22 | 46 | 16 | 16 |
| " " Int. | — | 50 | 27 | — | — | 24 | — |
| " " L.P. | 50 | — | 42 | 40 | — | 41 | 32 |
| Stroke of Piston | 10 | 36 | 24 | 26 | 120 | 30 | 28 |
| Steam Pressure | 50 | 100 | 160 | 160 | 22 | 160 | 100 |
| Revolutions | 32 | 120 | 162 | 90 | 24 | — | 100 |
| Boilers | 1 | 2 | 2 | 1 | 1 | 2 | 1 |
| Grate surface | 168 | 140 | 81 | 71.5 | 76 | 80 | 45.5 |
| Heating do. | 1380 | — | 3462 | 2503 | 2259 | 2400 | 1710 |
| Condensing do. | Jet | — | — | 1105 | Jet | 1100 | 553 |
| Propeller, diam. | — | 2 of | 2 of | 9.5 | — | 9.5 | 8 |
| " " Pitch | — | — | 8 | — | — | — | — |
| Blades | — | — | 8.91 | 14 & 16 | — | — | 14 |
| Side-wheel diam. | 20.5 | — | 4 | — | 20.5 | — | 4 |
| " " width | 8.66 | — | — | — | 8.6 | — | — |
| Coal, weight | 5 | — | 15 | 60 | 40 | 270 | 10 |
| Consumption | — | — | 1580 | — | — | — | 200 |
| Combustion | Natural | Natural | Natural | Natural | Natural | Natural | Natural |
| HP | 1010 | — | 1007 | — | 650 | 800 | 250 |
| Team space | 3500 | 4530 | 3448 | — | 3420 | — | — |
| Passenger do. | 4130 | 5200 | 4330 | 450 | 2896 | — | — |
| Weight, Hull | 470 | — | 321 | — | — | — | 50 |
| Engine | 104 | — | 177 | 13 | 12 | — | 21 |
| Boilers | 51 | — | 48.5 | 20.5 | 39.75 | — | 21 |
| Water | 29.5 | — | 25 | 17.6 | 29.6 | — | — |
| Speed | 12 | 12 | 14.6 | 13.5 | 12 | 15 | 12 |

Remarks.—No. 1. Side-wheel, T. S. Marvel & Co., Newburgh, and
Iron Works, N. Y.; Double ends. No. 2. Neale and Levy, Penn
Phila., Pa.; Propeller at each end. No. 3. Hull same as 1, and
Iron Works, N. Y.; Propeller at each end; Weights: of Hull as launched; Iron
including steering and ventilating; donkey pumps, piping and chimney;
Surfaces, 7500 sq. feet. Nos. 4, and 5. The Boston and Fall River
Wilmington, Del.; Propeller and side-wheel. No. 6. Neale and Le
one Wrecking pump, 16 and 20 x 8 ins., three 3-inch sections on each v
500 tons water per hour; one fire-pump, eight 2½-inch streams; En
lights, 6000 candle power, several of 2000 candle wicks. Lights.
Penny & Jones Co., Wilmington, Del. No. 7. Hull same as No
Vigors W. & A. Fletcher Co., Hoboken, N. J.; Propeller at each
three * designed by Col. E. A. Stevens, Hoboken, N. J.

Wood Propellers.

RESHOFF, R. N., VERTICAL DIRECT ENGINE (Compound).—Length on deck, 46 over all, 48 feet; beam, 9 feet; hold, 5 feet.

Displacement at load-line, 7.44 tons. Area of section at load-line, 217.8 sq. feet of wetted surface, 365.5 sq. feet. Coefficient of fineness, .396.

Inders.—8 and 14 ins. in diam. by 9 ins. stroke of piston.

Condenser, External.—Surface.

Propeller.—4 blades, 3 feet in diam. by 4 feet 1 inch pitch.

Rev., 42 ins. in diam.

Heating surface, 174 sq. feet. Grates, 12.5 sq. feet.

Pressure of Steam, 53 lbs. per sq. inch. Revolutions, 333 per minute. IHP, 68.4.

10.18 knots per hour. With 129 lbs. and 466 revolutions, 14.26 knots. IHP, 100. Weight of Engines, Boiler, and Water, 5300 lbs.

RESHOFF, VERTICAL DIRECT ENGINE (Compound).—Length over all, 86 feet; 11 feet. Displacement, 27 tons.

Inders.—13 and 22 ins. in diam. by 12 ins. stroke of piston.

Condensing.

Pressure, 130 lbs. per sq. inch.

Revolutions, 460 per minute. Speed, 20 knots per hour. IHP, 425.

Propeller, 3 blades. Pitch, 5 feet.

RESHOFF, R. I. N.—VERTICAL DIRECT ENGINE (Compound).—Length over all, 77 feet; beam, 7 feet; hold, 5.5 feet. Displacement at load-draught of 32 ins., 7 tons (bs.).

Inders.—8 and 14 ins. in diam. by 9 ins. stroke of piston. Surface condenser.

Pressure of Steam.—140 lbs. per sq. inch, cut off at .5.

Revolutions, 600 per minute. Speed, 19.875 knots per hour.

Cable or Rope Towing.

FITRA. 11.—HORIZONTAL DIRECT ENGINES (Condensing).—Length of boat, 138 feet; 24.5 feet; hold, 7.5 feet.

Reversed section, 74.4 sq. feet. Displacement, 200 tons at load-line of 3.75 feet.

Reversed section, 263.7 sq. feet. Displacement, 949 tons. Tow.—3 barges.

Inders.—2 of 14.18 ins. in diam. by 23.625 ins. stroke of piston.

Net effective, 100. Speed, 7.73 miles per hour.

Propellers.—Twin, 4 feet 2 ins. in diam.

Wires.—Cable, 7485 lbs. Per ton of displacement, 6.5 lbs.; per sq. foot of immersed section, 22 lbs.

Weight.—Per mile and ton of displacement (1149), .078 lbs.

Towing. Wood Side Wheels.

M. H. WEBB. 11.—HARBOR AND COAST.—VERTICAL BEAM ENGINES (Condensing). 7th upon deck, 185.5 feet; beam, 30.25 feet; hold, 10.8 feet.

Reversed Section at load-line, 194 sq. feet. Displacement 498.25 tons, at load-line of 7.25 feet.

Inders.—2, of 44 ins. in diam. by 10 feet stroke of piston; volume, 211 cube feet.

Condensers.—Jet, 2, volume 105 cube feet. Air-pumps.—2, volume 45 cube feet.

Water-wheels.—Diam., 30 feet. Blades (divided), 21; breadth of do., 4.6 of do., 2.33 feet. Dip at load-line, 3.75 feet.

Return pipes.—2 (return flue). Heating surface, 3280 sq. feet. Grates, 147.5 sq. ft.

Smoke-pipe.—Area, 11.6 sq. feet, and 35 feet in height above the grate level.

Pressure of Steam.—35 lbs. per sq. inch, cut off at .5 stroke. Revolutions, 2

a. IHP, 1500.

Fuel.—Anthracite or Bituminous. Consumption, 1680 lbs. per hour.

Speed.—20 miles per hour.

Weights.—Engines, Wheels, Frame, and Boilers, 310 579 lbs.

Wood Side Wheels.

Passenger.

"MARY POWELL," HUDSON RIVER.—VERTICAL BEAM ENGINE (Condensing).—Length on water-line, 286 feet; over all, 294 feet; beam, 34 feet 3 ins.; over all, 64 feet; hold, 9 feet. Deck to promenade deck, 10 feet.

Immersed section at load-line of 6 feet, 200 sq. feet. Displacement, 800 tons at mean load-draught of 6 feet.

Area of transverse head surface of hull above water, 2000 sq. feet.

Cylinder.—72 ins. in diam. by 12 feet stroke of piston; volume, 338 cube feet. Clearance at each end, 12.5 cube feet.

Steam and Exhaust Valves, 14.75 ins. in diam. Air-pump, 40 ins. in diam. by 5 feet 2 ins. stroke of piston. Condenser.—Jet, 128 cube feet. Crank-pin, 8.75 ins. in diam. \times 10.75 ins.

Beam, 22.5 feet in length; centre, 9.75 in diam.

Water-wheels.—Diam. 31 feet; blades (divided), 26; breadth of do., 10 feet 6 ins.; width, 1 foot 6 ins.; immersion, 3 feet 6 ins. Shafts.—Journal, 15.625 ins. by 17 ins.

Boilers.—2 (flue and return tubular), of steel, 11 feet front by 26 feet in length; shell, 10 feet in diam. and 16 feet 1 inch in length. Furnaces, 2 in each, of 4 feet 10 ins. by 8 feet in length. Heating Surface, 2660 sq. feet; and Superheating, 30 sq. feet in each. Grates, 152 sq. feet. Flues, 10 in each, transverse area, 11 feet 7 ins. Tubes, 80 in each, 4.5 ins. in diam., 6 feet 6 ins. in length, and 8 feet 7 ins. in transverse area.

Steam Chimneys, 8 feet in diam. \times 12 feet in height. Smoke-pipe, 4 feet 6 ins. in diam. and 68 feet in height from grates.

Combustion, Blast. Blowers, 4 feet in diam. and 3 feet in width. Revolutions, 78 per minute. Fuel (anthracite), 6280 lbs. per hour, or 40 lbs. per sq. foot of grate per hour. Per sq. foot of heating surface, 2.25 lbs.

Speed, 23.65 miles per hour.

Pressure of Steam, 28 lbs. per sq. inch, cut off at .47 stroke; terminal pressure, 16.4 lbs.; throttle, .625 open. Vacuum, 25 ins. Revolutions, 22.75 per minute.

Temperatures.—Reservoir, 120°. Feed water, 120°. Chimney, 740°. IP.—Total, 1900. IHP, 1560. Net, 1450.

Evaporation.—Water per lb. of coal, from 120°, 7 lbs.; per lb. of combustible, from 120°, 8.2 lbs. Steam per total HP per hour, 21.1 lbs. Coal per do. do., 3.14 lbs.

Weights. Engine.—Frame, keelson, out-board wheel-frames donkey engine, and boiler, blower engines and blowers, all complete, 360 000 lbs. Boilers.—Iron return flue, 120 000 lbs. Steel return tubular, 116 000 lbs. Water, 128 000 lbs.

Capacity.—2000 passengers and their baggage.

Memoranda.—This vessel was originally but 266 feet in length, and when lengthened the cylinder of 62 ins. in diam. was removed and replaced with one of 72 ins. Engine designed throughout for original cylinder and a pressure of from 50 to 55 lbs., cutting off at .625 of stroke, with throttle wide open.

Engines and Boilers built by Fletcher, Harrison, & Co., New York, 1861 and 1875.

Iron Stern Wheels.

Passenger and Freight.

HORIZONTAL ENGINES (Non-condensing).—Length upon deck, 110 feet; beam, 14 feet (deck projecting over, 4 feet); hold, 3.5 feet.

Immersed section at load-line, 10.25 sq. feet. Displacement at load-draught of 11 tons.

—Two, of 10 ins. in diam. by 3 feet stroke of piston; volume of piston be feet.

Diam. 13 feet. Blades, 13; breadth of do., 8.5 feet; depth of do., 8 ins. , 33 per minute. Boiler.—One (horizontal tubular). Tubes, 100 of 1

Consumption, 4480 lbs. in 24 hours.

; bilges, No. 4; bottom, No. 5; sides, Nos. 6 and 7, 1 ins. apart from centres.

Wood Stern Wheels.

Passenger and Deck Freight.

ONTANA."—HORIZONTAL ENGINES (*Non-condensing*).—*Length upon deck* (over 48 feet; at water-line, 245 feet; beam, 48 feet 8 ins. (over all, 50 feet 4 ins.); draught of water at load-line, 5.5 feet.

versed section at load-line, 244 sq. feet. Displacement at mean light draught ns., 594 tons (2000 lbs.)

nders.—Two, 18 ins. in diam. by 7 feet stroke of piston.

ves, 4.5 and 5 ins. in diam. Piston-rod, 4 ins. Steam-pipe, 4.5 ins. Connect- d, 30 feet in length.

ter-wheel, 19 feet in diam. by 35 feet face; blades, 3 feet in depth. Shaft, ins. in diam.

lers.—Four (horizontal tubular), 42 ins. in diam. by 26 feet in length. Two in each, 15 ins. in diam. Heating surface, effective, 1023, total 1431 sq. feet. ice, 6.5 X 17 feet. Grates, 4.16 X 17 feet; surface, 70.8 sq. feet. Smoke-pipes, 3 feet in diam. by 55 feet 3 ins. in height. Exhaust or Blower draught.

rimeter.—Of Bridge, 15.27; of Flues, 9.82; and of Chimneys, 14.14 sq. feet. of grate, compared to calorimeter of flues, 7.2; to ditto. of chimneys, 5; and dge, 4.6 sq. feet.

um-room, 562; and water space, 294 cube feet.

ll.—Frames, 4 X 6 ins. and 15 ins. apart at centres. Intermediate do., 4 X 6 and running for 7.5 feet each side of keelson. Planking.—Bottom, oak, 4 ins.; o., 2.5 to 4 ins. Deck beams, pine, 3 X 6 ins. Deck plank, 2.5 ins. Keelson, side do., eight each side, one each 7, 8.75, and 9 ins., and five 6.75 ins. Wales, each side, 9 and 7 ins. by 3, and one 10 X 2.5 ins. Deck posts, 3.5 X 3 ins. and 4 part. Deck beams, 5.5 X 3 ins. Knuckles, oak, 6 X 12 ins. Bulkheads, one udinal and one athwartship at shear of stern. Sheathing of wrought iron, to .125 inch from just below light to load-line.

Posts.—White pine, 8.5 and 11 ins. square. Chains, 1.5 ins. in diam.

ights.—Boilers, 29 264; water, 18 351; and boilers, chimneys, grates, and water, lbs. Hull, oak, 520 560; Pine, 91 437; Bolts, spikes, etc., 8000, and Deck and 8, 76 000 lbs.; Hull alone, 310 tons.

ight of hull compared to one of iron as 8 to 5, effecting a difference of about ns.

ITTSBURGH."—HORIZONTAL ENGINES (*Non-condensing*).—*Length on deck*, 252 beam, 39 feet; hold, 6 feet; draught of water at load-line, 2 feet.

versed section at load-line, 75 sq. feet. Displacement at load-draught of 2 feet, ns (2000 lbs.).

nders.—Two, 21 ins. in diam. by 7 feet stroke of piston.

ter-wheel.—21 feet in diam. by 28 feet face.

lers.—2 (horizontal tubular), 47 ins. in diam. by 28 feet in length. Two fires ch.

Oil Engine Launch.

Elements of Engine and Dimensions of Launch.

Consumption .9 pint ordinary Mineral Oil per HP per

| HP* | Launch. | | Weight.† | Type. | HP* | L Length |
|-----|---------|----------|----------|-------|-----|----------|
| | Length. | Breadth. | | | | |
| No. | Feet. | Feet. | Lbs. | No. | No. | Feet. |
| 1 | 16 | 4 | 896 | 3 | 5 | 30 |
| 2 | 21 | 5 | 1332 | 2 | 10 | 40 |
| 3 | 27 | 6 | 1568 | 1 | 15 | 45 |

* Developed by Brake.

† Of engine without of

Passenger and Deck Freight.

"**PERMEEUX**."—HORIZONTAL ENGINES (Non-condensing).—Length on deck 100 feet; beam, 32 feet; hold, 5 feet; draught of water at load-line, 2 feet. Immersed section at load-line, 75 sq. feet. Displacement at load draught 170 tons (2000 lbs.).

Cylinders.—Two, 21 ins. in diam. by 7 feet stroke of piston.

Water-wheel.—20 feet in diam. by 20 feet diam.

Boilers.—2 (horizontal tubular), 47 ins. in diam. by 20 feet in length. Two in each.

Iron Stern Wheels.

HORIZONTAL ENGINES (Non-condensing).—Length upon deck, 120 feet; beam, 32 feet (deck projecting over, 4 feet); hold, 3.5 feet.

Immersed section at load-line, 102.75 sq. feet. Displacement at load draught 170 tons.

Cylinders.—Two, of 20 ins. in diam. by 3 feet stroke of piston; volume of piston space, 1.6 cubic feet.

Wheel.—Diam. 13 feet. Blades, 15; breadth of do., 8.5 feet; depth of do., 10 inches. Revolutions, 33 per minute. Boiler.—One (horizontal tubular). Tubes, 20 inches in diam.

Fuel.—Stearious coal. Consumption, 4450 lbs. in 24 hours.

Hull.—Plater, keel, No. 3; bilges, No. 4; bottoms, No. 5; sides, Nos. 6 and 7; frames, 2.5 X .5 ins., and 20 ins. apart from centres.

Steel.

"**CHATTAGOOCHER**."—INCLINED ENGINES (Non-condensing).—Length on deck 100 feet; beam, 32.5 feet; hold, 5 feet.

Immersed section at load-line, 153 sq. feet. Freight capacity, 400 tons (2000 lbs.).

Cylinders.—Two, 15 ins. in diam. by 5 feet stroke; volume of piston space, 1.6 cubic feet.

Wheel.—One, 18 feet in diam.; blades, 2 feet in depth.

Boilers.—Three (cylindrical flued). Diam. 42 ins.; length, 22 feet; 2 in each. Heating surface, 690 sq. feet. Grates, 48 sq. feet.

Pressure of Steam, 160 lbs. per sq. inch, cut off at .375. Revolutions, 22 per minute.

Consumption of Fuel, 12 tons (2000 lbs.) in 24 hours. Plating of Hull, 1/16 inch. Light draught, 21 ins.

Iron Propellers.

VERTICAL DIRECT ENGINES (Non-condensing).—Length on deck, 70 feet; beam, 32 feet; draught, 12 ins.

Propellers, 2.—2 blades, 16 ins. in diam., set 11 ins. below water-line.

Boiler (tubular coil). Revolutions, 480 per minute.

Speed, 10.49 miles per hour.

Water led to propellers through tunnels in bottom at sides.

"**LOUISE**."—VERTICAL TANDEM ENGINES (Compound).—Length, 60 feet; beam, 32 feet; hold, 4.25 feet.

Displacement at load-draught of 2.5 feet, 8 tons.

Cylinders, 5 and 10 ins. in diam. by 8 ins. stroke of piston.

Surface Condenser.—Boiler (vertical tubular), 4 feet in diam. by 8.5 in length.

Iron Sailing Vessels.

Passenger and Freight.

ENGLISH.—SHIP.—Length upon deck, 178 feet; do. at mean load-line of 19.15 feet; keel, 171 feet; beam, 32.88 feet; depth of hold, 21.75 feet; keel (mean), 171 feet.

Immersed section at load-line, 387 sq. feet. Displacement at load-draught, 1383 tons; at deep load-draught of 20 feet, 1495 tons; and, in proportion to circumscribing parallelepipedon, 1524.

Load-line.—Area at load-draught, 4557 sq. feet. Angle of entrance, 57° 10'. Area in proportion to its circumscribing parallelogram, 74.

e of Gravity, 6.416 feet below mean load-line. *Centre of Displacement* (grav-
6.25 feet below load-line; and 4.33 feet before middle of length of load-line.
rsed Surface.—*Bottom*, 7370 sq. feet. *Keel*, 1130 sq. feet. *Sails*, 13282 sq. feet.
centre, 6.66 feet above centre of gravity of displacement. *Centre of Effort*
entre of displacement, 3.5 feet; height of do. above mean load-line, 55.5 feet.

Launch. Wood.

LAUNCH "HERRESHOFF."—*VERTICAL ENGINE (Compound)*.—*Length*, 33 feet
beam, 8.75 feet.
acement at mean load-draught of (to rabbet of keel) 19 ins., 8929 lbs.
hts.—*Hull and Machinery*, 6555 lbs. Coal, 1120 lbs.

Yachts. Wood.

ERICA," SCHOONER.—*Length over all*, 98 feet; *upon deck*, 94 feet; *at load-line*,
t; *beam*, 22.5 feet; *at load-line*, 22 feet; *depth of hold*, 9.25 feet. *Height at*
m under side of garboard strake, 11 feet. *Sheer, forward*, 3 feet; *aft*, 1.5 feet,
rsed section at load-line, 121.8 sq. feet. *Displacement at load-draught of* 8.5
m under side of garboard strake and of 11 feet *aft*, 191 tons; and, *in pro-*
to Volume of circumscribing parallelepipedon, .375.

acement at 4 feet (from garboard strake), 43 tons; *at* 5 feet, 66 tons; *at* 6
tons; *at* 7 feet, 127 tons; and *at* 8 feet, 167 tons.

e of Gravity.—*Longitudinally*, 1.75 feet aft of centre of length upon load-
sectional, 2.58 feet below load-line. Of Fore body, 14.25 feet forward; and
body, 19 feet aft. *Meta-centre*, 6.72 feet above centre of gravity.

e of Effort, 31 17 feet from load-line. *Centre of Lateral Resistance*, 6.33 feet
f centre of gravity. *Area of Load-line*, 1280 sq. feet. *Mean girths of im-*
section to load-line, 25 feet.

-draught.—*Forward*, 4.91 feet; *aft*, 11.5 feet. *Rake of Stem*, 17 feet

1.—*Mainmast*, 81 feet in length by 22 ins. in diam. *Foremast*, 79.5 feet in
y 24 ins. in diam. *Main boom*, 58 feet in length. *Main gaff*, 28 feet. *Fore*
feet. *Rake*, 2.7 ins. per foot. *Drag of Keel*, 3 feet. *Tons*, 170.56.

IA," SLOOP.—*Length for tonnage*, 72.25 feet; *on water-line*, 70 feet 7 ins.;
3 feet 8 ins.; *hold*, 6 feet 8 ins. *Tons*, U. M. 83.4; N. M. 43.98.

-draught, 6.25 feet.

—*Mainsail*, hoist, 49.75 feet, foot 54.25, and gaff 27.66; *Jib*, hoist, 49.75 feet,
5, and stay 63.5. *Gaff topsail*, hoist, 24.5 feet.

1.—*Mainsail*, 2322 sq. feet. *Jib*, 986, and *Topsail*, 454.

Cutters.

A" (English) SLOOP.—*Length on load-line*, 66 feet; *beam*, 11.5 feet.

rsed section at load-line, 11.5 sq. feet. *Displacement*, 75 tons.

1.—*Mast*, deck to hounds, 42 feet. *Boom*, 58 feet. *Gaff*, 39 feet. *Bowsprit*
of stem, 30 feet. *Mast to stem*, 26 feet. *Topmast*, foot to hounds, 25 feet.
topsail yard, 46 feet. *Canvas, area*, 3450 sq. feet. *Tons*, C. H., 90.

st.—*At Keel*, 38.5 tons. *Hull*, 1.5 tons.

CHIEF" (English), SLOOP.—*Length on load-line*, 61 feet; *beam*, 19.9 feet.
rsed section at load-line, 60 sq. feet. *Displacement*, 55 tons.

Pilot Boat.

L H. ASPINWALL," SCHOONER.—*Length of keel*, 74 feet; *upon deck*
9 feet; *hold*, 7.6 feet. *Draught of water*, 6 feet forward; *aft*, 9.5 J
22 ins. in depth. *False keel*, 12 ins. in depth at centre.

1.—*Mainmast*, 77 feet in length. *Foremast*, 76 feet. *Main bo-*
ff, 21 feet. *Fore gaff*, 20 feet.

—*N. M.*, 46.32.

Robinson Street to Kingston Light, 90.375 miles, in 4 hours, making 5 landings.

1850, *Buckeye State*, of Pittsburgh, Penn. (non-condensing), Cincinnati, 500 miles (200 passengers), 53 landings, in 1 day 19 hours, adverse current. Speed = 15.63 miles and 1.23 landings per hour. of water in channel 7 feet.

1852, *Reindeer*, of N. Y., New York to Hudson, 116.5 miles, in making 5 landings. Flood tide.

1853, *Shotwell*, of Louisville, Ky. (non-condensing), New Orleans 1450 miles, 8 landings, in 4 days 9 hours; 4.5 to 5.5 miles per hour. Speed = 18.81 miles per hour.

NOTE.—In 1817-18 the average duration of a passage from New Orleans to Louisville, 12 hours; the shortest, 25 days.

1855, *New Princess*, of New Orleans (non-condensing), New Orleans to Mississippi, 310 miles, in 17 hours 30 min.; 3.5 to 4 miles per hour. Speed = 20.98 miles per hour.

1864, *Daniel Drew*, of N. Y., Jay Street, N. Y., to Albany, 148 miles, 9 landings. Flood tide. Speed of boat = 22.6 miles per hour.

1867, *Mary Powell*, of N. Y., Desbrosses Street, N. Y., to Newburgh, 2 hours 50 min., 3 landings; from Poughkeepsie to Rondout Light, 39 min., flood tide. 1873, Milton to Poughkeepsie, light draught, 16 miles, in 9 min.; and 1874, Desbrosses Street to Piermont, 24 miles, Caldwell's, 43.25 miles, in 1 hour 50 min. Speed = 22.77 to 23 miles per hour.

Runs from New York to Albany, 146 miles, by different boats.

| | | |
|-------------------------------|------------------|---------------------------------|
| 1826, <i>Sun</i> | 12 hours 16 min. | 1852, <i>Fr. Skiddy</i> §..... |
| 1826, <i>North America</i> *. | 10 " 20 " | 1860, <i>Armenia</i> |
| 1841, <i>Troy</i> †..... | 8 " 10 " | 1864, <i>Daniel Drew</i> ‡.... |
| 1841, <i>South America</i> ‡. | 7 " 28 " | 1864, <i>Ch'ncey Vibbard</i> ‡. |

* 7 landings. † 4 landings. ‡ 9 landings. § 6 landings.

Timing Distance.—From 14th St., Hudson River, N. Y., to College at Mount St.

NOTE.—Where landings have been made, and the river crossed, the distance given is correspondingly increased.

1870, *R. E. Lee*, of St. Louis (non-condensing), New Orleans to St. Louis, 1400 miles (without passengers or freight), 4 to 5 miles per hour adverse

PASSAGES OF STEAMERS AND SAILING VESSELS.

Distances in Geographical Miles or Knots.

Steamers. Side-wheels.

- 807, *Phoenix*, of Hoboken, N. J. (John Stevens), New York, N. Y., to Philadelphia, in. First passage of a steam vessel at sea.
- 814, *Morning Star*, of Eng., River Clyde to London, Eng. First passage of an English steamer at sea.
- 817, *Caledonia*, of Eng., Margate, Eng., to Cassel, Germ., 180 miles, in 24 hours.
- 819, *Savannah*, of N. Y., about 340 tons O. M., Tybee Light, Savannah River, Ga., Rock Light, Liverpool, Eng., 3640 miles, in 25 days 14 hours; 6 days 21 hours of which were under steam.
- 825, *Enterprise*, of Eng., 500 tons, Falmouth, Eng., to Table Bay, Africa, in 57 days; and to Calcutta, India, in 113 days. First passage of a steamer to India.
- 830, *Hugh Lindsay*, 411 tons, 80 HP, Bombay, India, to Suez, Egypt, 3103 miles, 11 days running time.
- 837, *Atlanta*, of Eng., 650 tons, Falmouth, Eng., to Calcutta, in 91 days.
- 839, *Great Western*, of Eng., Liverpool to New York, N. Y., 3017 miles, in 12 days 18 hours.
- 870, *Scotia*, of Eng., Queenstown, Ireland, to Sandy Hook, N. J., 2780 miles, in 7 days 7 hours 31 min. 1866, New York to Queenstown, 2798 miles, in 8 days 2 hours 48 min.; thence to Liverpool, Eng., 270 miles, in 14 hours 59 min.; total, 8 days 17 hours 47 min.

Screw.

- 874, *India Government Boat*, Steel, length 87 feet, beam 12 feet, draught of water 5 feet, mean speed for one mile 20.77 miles per hour, and maintained a speed of 32 miles in 1 hour.
- 877, *Lusitania*, of Eng., London to Melbourne, Australia, via Cape, 11 445 miles, 38 days 23 hours 40 min.

Sailing Vessels.

- 851, *Chrysolite* (clipper ship), of Eng., Liverpool, Eng., to Anjer, Java, 13 000 miles, in 88 days. The *Oriental*, of N. Y., ran the same course in 89 days.
- 853, *Trade Wind* (clipper ship), of N. Y., San Francisco, Cal., to New York, N. Y., 510 miles, in 75 days.
- 854, *Lightning* (clipper ship), of Boston, Mass., Melbourne, Australia, to Liverpool, Eng., 12 190 miles, in 64 days.
- 854, *Comet* (clipper ship), of N. Y., Liverpool, Eng., to Hong Kong, China, 13 040 miles, in 84 days.
- 854, *Sierra Nevada* (schooner), of N. H., Hong Kong, China, to San Francisco, 6000 miles, in 34 days.
- 854, *Red Jacket* (clipper ship), of N. Y., Sandy Hook, N. J., to Melbourne, Australia, 12 720 miles, in 69 days 11 hours 1 min.
- 855, *Euterpe* (half-clipper ship) of Rockland, Me., New York to Calcutta, India, 100 miles, in 78 days.
- 860, *Andrew Jackson* (clipper ship), of Boston, New York, N. Y., to San Francisco, Cal., 13 610 miles, in 80 days 4 hours.
- 865, *Dreadnought* (clipper ship), of Boston, Honolulu, Sandwich Islands, to New Bedford, Mass., 13 470 miles, in 82 days; and 1859, Sandy Hook, N. J., to Rock Light, Liverpool, Eng., 3000 miles, in 13 days 8 hours.
- 865, *Sovereign of the Seas* (medium ship), of Boston, Mass., in 22 days sailed 11 miles = 245 miles per day. For 4 days sailed 341.78 miles per day, and for 1 day 375 miles.
- 866, *Henrietta* (schooner yacht), of N. Y., Sandy Hook, N. J., to Gloucester, Mass., in 13 days 21 hours 55 min. 16 sec.
- 866, *Ariel* and *Serica* (clipper ships), of England, Foo-chou-foo, China, to Downs, Eng., 13 500 miles, in 98 days.
- 869, *Sappho* (schooner yacht), of N. Y., Light-ship off Sandy Hook, Queenstown, Ireland, 2857 miles, in 12 days 9 hours 34 min.

2.17 lbs. per sq. inch. *Revolutions*, 22 per minute.

Pipes, 3 feet in diam. = 168 area of cylinder.

Tuyeres.—Each Furnace, 2 of 3 ins. in diam.; 1 of 3.25 ins.; and Each Flinery, 6 of 1.33 ins.; and 1, 4 of 1.125 ins.

Temperature of Blast, 600°. *Ore*, 40 to 45 per cent. of iron.

Furnaces.—Eight, diam. 16 to 18 feet. *Dowlais Iron Works*
1300 Tons Forge Iron per Week; discharging 44 000 Cube Feet
Minute.

Engine (non-condensing).—Cylinder, 55 ins. in diam. by 13 feet stroke.
Pressure of Steam.—60 lbs. per sq. inch, cut off at .33 the stroke of piston.
120 ins. in area.

Boilers.—Eight (cylindrical flued, internal furnace), 7 feet in diam.
length; one flue 4 feet in diam. *Grates*, 288 sq. feet.

Fly Wheel.—Diam., 22 feet; weight, 25 tons.

Blowing Cylinder, 144 ins. in diam. by 12 feet stroke of piston.

Revolutions, 20 per minute. *Blast*, 3.25 lbs. per sq. inch. *Discharge*
5 feet, and 420 feet in length. *Valves*.—Exhaust, 56 sq. feet; Delivery,

Furnaces.— *Lackenby (England)*.

800 Tons Iron per Week.

Engine (horizontal, compound condensing).—32 and 60 ins. in diam.
stroke of piston.

Blowing Cylinders.—Two, 80 ins. in diam. by 4.5 feet stroke of piston.
4.5 lbs. per sq. inch. *Revolutions*, 24 per minute.

Pipe, 30 ins. in diam.; *volume*, 12.25 times that of blowing cylinder

HP.—Engine, 290 lbs.; Blowing cylinders, 258; efficiency, 89 per cent.

Valves.—Area of admission, .16 of area of piston; of exit, .125.

Volume.—190 000 cube feet of air are supplied per ton of air.

Blower and Exhausting Fan.

COTTON FACTORIES. (*English.*)

Driving 22060 Hand-mule Spindles, with Preparation, and 260 Looms, with common Sizing.

Engine (condensing).—Cylinder, 37 ins. in diam. by 7 feet stroke of piston; volume of piston space, 53.6 cube feet.

Pressure of Steam.—(Indicated average) 16.73 lbs. per sq. inch. *Revolutions, 17* minute.

Friction of Engine and Shafting.—(Indicated) 4.75 lbs. per sq. inch of piston.

125. Total power = 1. Available, deducting friction = .717.

ES.—Each IHP will drive $\left\{ \begin{array}{l} 305 \text{ hand-mule spindles, with preparation,} \\ \text{or } 230 \text{ self-acting} \quad \quad \quad \text{"} \\ \text{or } 104 \text{ throstle} \quad \quad \quad \text{"} \\ \text{or } 10.5 \text{ looms, with common sizing.} \end{array} \right.$

Working preparation:

1 throstle spindle = 3 hand-mule, or 2.25 self-acting spindles.

1 self-acting spindle = 1.2 hand-mule spindles.

DREDGING MACHINES.

*Working 20 Feet from Water-line, or 180 Tons of Mud or Silt per Hour
11 Feet from Water-line.*

Length upon deck, 123 feet; beam, 26 feet. Breadth over all, 41 feet.

*Reversed section at load-line, 60 sq. feet. Displacement, 141 tons, at load-draught
3 feet.*

*Engine (non-condensing).—Cylinders, two, 12.125 ins. in diam. by 4 feet stroke
on.*

*Boilers.—Two (cylindrical flue), diam. 40.5 ins., and length, 20 feet 3 ins.; two
14.625 ins. in diam. Heating surface, 617 sq. feet. Grates, 37 sq. feet.*

*Pressure of Steam, 25 lbs. per sq. inch; throttle .25 open, cut off at .5 the stroke
on. Revolutions, 42 per minute.*

*Buckets.—Two sets of 12, 2.5 feet in length by 15 ins. at top and 2 feet deep; vol-
ume .25 cube feet. Chain Links, 8 ins. in length by .5 inch diam.*

Camels or Camels.—Four, of 40 tons capacity each.

STEAM HOPPER DREDGER. (*Wm. Simons & Co.*)

Iron.

Engine " (English).—Length, 150 feet; breadth, 32 feet.

Dredge from 6 Ins. to 25 Feet. Capacity of Hopper, 500 to 600 Tons.

*Engines.—Two (compound), 375 HP, for dredging and propulsion, and one for
3 bucket-frame and anchor-post.*

*The designed dredger of 1000 tons' capacity has dredged 25000 tons silt per
and transported it 4 miles.*

Dredging 1000 Tons of Mud or Silt per Hour, 5 to 35 Feet in Depth.

Capacity of Hopper, 1000 Tons.

Engines.—Two (compound), HP 1000. Speed.—9 knots per hour.

Steam Dredging Crane. (*English.*)

Lift, 30 Feet per Hour.

| Lifting Power. | Volume of Bucket. | Mud or Silt. | Coal and Sand. | Excava- tion Ground. | Weight of Crane. | Lifting Power. | Volume of Bucket. | Mud or Silt. | Coal and Sand. |
|-------------------|-------------------------|-----------------|-------------------|----------------------------|---------------------|-------------------|-------------------------|-----------------|-------------------|
| Tons. | Lbs. | Tons. | Tons. | C. Yds. | Lbs. | Tons. | Lbs. | Tons. | Tons. |
| 2.5 | 1120 | 25 | 20 | 20 | 18000 | 5 | 2240 | 50 | 40 |
| 3 | 1680 | 37.5 | 32 | 25 | 33480 | 7 | 3360 | 60 | 54 |

HOISTING ENGINES.

For Pile Driving, Hoisting, Mining, etc.

Lidgerwood Manuf'g Co., New York.

| SINGLE CYLINDERS. | | | | DOUBLE CYLINDERS. | | | |
|-------------------|-----------|-----------|---------------------|-------------------|-----------|-----------|---------------------|
| | Cylinder. | Capacity. | Cost, with Boiler.* | HP | Cylinder. | Capacity. | Cost, with Boiler.* |
| | Ins. | Lbs. | \$ | No. | Ins. | Lbs. | \$ |
| 4 | 5 × 5 | 1000 | 600 | 8 | 5 × 8 | 2000 | 950 |
| 5 | 6 × 8 | 1250 | 675 | 12 | 6 × 8 | 2500 | 1050 |
| 6 | 7 × 10 | 1800 | 825 | 20 | 7 × 10 | 3500 | 1350 |
| 7 | 8 × 10 | 2800 | 1050 | 30 | 8 × 10 | 6000 | 1550 |
| 8 | 9 × 12 | 4000 | 1275 | 40 | 9 × 12 | 8000 | 2000 |
| 9 | 10 × 12 | 5000 | 1375 | 50 | 10 × 12 | 9000 | 2350 |

* Complete.

Details and Operation.

| No. | Drum. | Boiler. | | Ram. | Leaders. | Lift. | Blows per Minute. | Piles per 10 Hours. | Fuel per Hour. |
|-----|---------|------------------|-------------|------|----------|---------|-------------------|---------------------|----------------|
| | | Dimen- sions. | Tubes. | | | | | | |
| | Ins. | Ins. | No. | Lbs. | Feet. | Feet. | No. | No. | Lbs. |
| 1 | 12 × 24 | 32 × 75 | 48 of 2 in. | 1953 | 40 | 8 to 12 | 25 | 50 | 70 |
| 2 | 14 × 26 | 40 × 84 | 80 of 2 in. | 2700 | 75 | 8 to 12 | 29 | 100 | 80 |

* Weight complete, 8500 lbs.

Mining Engines and Boilers. (Various Capacities.)

Engine, Boiler, etc., as given for Pile Driving, page 902.

Operation. — 250 to 300 tons of coal in 10 hours. Fuel, 40 lbs. coal per hour.

Weight of Engine and Boiler, 4500 lbs.

Hancock Inspirator. For a Lift of Water of 25 Feet.

| No. | Diameter. | | Discharge at Pressure of 60 Lbs. | No. | Diameter. | | Discharge at Pressure of 60 Lbs. |
|-----|-------------|----------|----------------------------------|-----|-------------|----------|----------------------------------|
| | Steam-pipe. | Suction. | | | Steam-pipe. | Suction. | |
| | Ins. | Ins. | G'ls. per h'r. | | Ins. | Ins. | G'ls. per h'r. |
| 1 | .375 | .5 | 120 | 30 | 1.25 | 1.5 | 1260 |
| 2 | .5 | .75 | 220 | 35 | 1.25 | 1.5 | 1740 |
| 3 | .5 | .75 | 300 | 40 | 1.5 | 2 | 2230 |
| 4 | .75 | 1 | 540 | 45 | 1.5 | 2 | 2820 |
| 5 | 1 | 1.25 | 900 | 50 | 2 | 2.5 | 3480 |

temperature of water not over 145° for a low lift, and 100° for a high lift.

HYDROSTATIC PRESS. (Cotton.)

30 Bales of Cotton per Hour.

Engine (non-condensing).—Cylinder, 10 ins. in diam. by 3 feet stroke of piston. Pressure of Steam, 50 lbs. per sq. inch, full stroke. Revolutions, 45 to 60 per minute.

Presses.—Two, with 12-inch rams; stroke, 4.5 feet.

Pumps.—Two, diam. 2 ins.; stroke, 6 ins.

For 83 Bales per Hour.

Engine (non-condensing).—Cylinder, 14 ins. in diam. by 4 feet

Boilers.—Three (plain cylindrical), 30 ins. in diam. and 26 feet 1 sq. feet. Pressure of Steam, 40 lbs. per sq. inch. Revolutions

Presses.—Four, geared 6 to 1, with two screws, each of 7.5 ins pitch.

Shaft (wrought iron).—Journal, 8.5 ins. Fly Wheel, 16 feet 1 lbs.

Cost.—Average per mile in England, 2.52 pence sterling = 4.48 cents

PILE-DRIVING.

Driving One Pile.

Engine (non-condensing).—*Cylinder*, 6 ins. in diam. by 1 foot stroke. *Boiler* (vertical tubular).—32 ins. in diam., and 6.166 feet in height. *sq. feet.* *Furnace*, 20 ins. in height. *Tubes*, 35, 2 ins. in diam., 4.5 feet in height. *Revolutions*, 150 per minute. *Drum*, 12 ins. in diam., geared 4 to 1. *feet in height.* *Ram.*—2000 lbs., 2 blows per minute. *Fuel*, 30 lbs. cc

Driving Two Piles.

Engine (non-condensing).—*Cylinders*, two, 6 ins. in diam. by 18 in. piston.

Boiler (horizontal tubular).—*Shell*, diam. 3 feet, and 6 feet in length and 3.75 feet in width, 3.5 feet in length, and 6 feet in height.

Pressure of Steam, 60 lbs. per sq. inch. *Revolutions*, 60 to 80 per minute.

Frame, 8.5 feet in width by 26 feet in length. *Leaders*, 3 feet in width in height. *Rams.*—Two, 1000 lbs. each, 5 blows per minute.

PUMPING ENGINES.

CORLISS STEAM-ENGINE Co., Providence, R. I.—**VERTICAL-BEAM ENGINE**.—*Cylinders.*—18 and 36 ins. in diam. by 6 feet stroke of piston.

Pumps.—Four plunger, 19 ins. in diam. by 3 feet stroke of piston. 1 per revolution of engine, 84.96 cube feet.

Boilers.—Three, vertical fire tubular. *Grate.*—93 sq. feet. *Heating* sq. feet. *Pressure of Steam*, 125 lbs. per sq. inch, cut off at .22 feet. 36 per minute. *IHP* 313. *Fly-wheel.*—25 feet in diam., weight 62 000 lbs.

Fuel.—Cumberland coal, 486 lbs. per hour, inclusive of kindling and r. *Ash and Clinkers*, 9.4 per cent. *Duty* for one week, 113 271 000 foot-lbs.

Water delivered, 17 621 gallons per minute, against head of 180 feet.

Duty, average for 1883, per 100 lbs. anthracite coal, 106 048 000 foot-lb.

For Elevating 200 000 Gallons of Water per Hour.

"Gaskill," at Saratoga, N. Y.

Engine (Horizontal Compound).—Cylinders.—High pressure, 2 of 21 ins. diam. pressure, 2 of 42 ins. diam., all 3 feet stroke of piston. **Pumps.**—Two of 20 ins. by 3 feet stroke of piston.

Wheel, 12.33 feet in diam.; weight, 12 000 lbs.

lars (horizontal tubular).—Two of 5.5 feet in diam. by 18 feet in length. **Heat-surface,** 2957 sq. feet. **Grates,** 51 sq. feet of grate; to heating surface, 1 to 58, 0 transverse section of tubes, 1 to 7. **Chimneys,** 75 feet.

ssure of Steam.—Mean of 20 hours, 74.25 lbs. per sq. inch. **Revolutions,** 17.87 minute. **IHP.**—High-pressure cylinders, 109.2; low-pressure, 76 65. Total, 185.8. **el.**—Anthracite, 6.9 lbs. per sq. foot of grate per hour. **Evaporation,** per sq. of heating surface per hour, 1.175 lbs.; per lb. of coal, 9.25 lbs.; per cent. of combustible, 3.2.

ty, 112 899 993 foot-lbs. per 100 lbs. coal. **Heating surface** per IHP, 14.9.

am per sq. foot of surface per hour, 1.19 lbs.; per sq. foot of surface per lb. of per hour from 212°, 11.28 lbs.

Ericsson's Caloric. For an Elevation of 50 Feet.

| Space occupied. | | Volume per Hour. | Pipes, Suction and Discharge. | Fuel per Hour. | | COST. | | | | |
|-----------------|---------|------------------|-------------------------------|----------------|------|----------|-------|-----------------|--------|---------|
| | | | | | | Furnace. | | Deep Well Pump. | | |
| Floor. | Height. | | | Nut Anthr. | Gas. | Gas. | Coal. | Pump. | Plain. | Galvan. |
| 34×18 | 48 | 150 | .75 | — | 15 | 150 | — | — | — | — |
| 39×20 | 51 | 200 | .75 | 2.5 | 18 | 200 | 210 | — | — | — |
| 48×21 | 63 | 350 | 1 | 3.3 | 25 | 235 | 250 | 10 | .64 | .86 |
| 54×27 | 63 | 800 | 1.5 | 6 | — | — | 320 | 15 | .80 | 1.15 |
| 42×52 | 65 | 1600 | 2 | 12 | — | — | 450 | 25† | .92 | 1.25 |

* Over 90 feet, 92 cents.

† Duplex.

cluding engine and pump, oil-can and wrench, complete in all but suction and large-pipe.

SUGAR MILLS.

ressing 40 000 lbs. Cane-juice per day, or for a Crop of 5000 Boxes of 450 lbs. each in four Months' Grinding.

ngine (non-condensing).—Cylinder, 18 ins. in diam. by 4 feet stroke of piston. **iler** (cylindrical flued).—64 ins. in diam. and 36 feet in length; two return flues, 8. in diam. **Heating surface,** 660 sq. feet. **Grates,** 30 sq. feet.

essure of Steam, 60 lbs. per sq. inch, cut off at .5 the stroke of piston. **Revolu-** 40 per minute.

ls.—One set of 3, 28 ins. in diam. by 6 feet in length; geared 1 to 14. **Shafts,** 1d 12 ins. in diam. **Spur Wheel,** 20 feet in diam. by 1 foot in width. **Fly** 1, 18 feet in diam.; weight, 17 400 lbs.

ights.—Engine, 61 460 lbs.; Sugar Mill, 65 730 lbs.; Spur Wheel and Connect Machinery to Mill, 28 680 lbs.; Boiler, 18 520 lbs.; Appendages, 6730 lbs. Total, 20 lbs.

STONE AND ORE BREAKERS. (See p. 957.)

| Re-ceiver. | Pulley. | | V'locity per Minute. | Power re-quired. | Weight. | No. | Re-ceiver. | Pulley. | | V'locity per Minute. | Power re-quired. | Weight. |
|------------|---------|-------|----------------------|------------------|---------|-----|------------|---------|-------|----------------------|------------------|---------|
| | D'm. | Face. | | | | | | D'm. | Face. | | | |
| Ins. | Feet. | Ins. | Feet. | HP. | Lbs. | | Ins. | Feet. | Ins. | Feet. | HP. | Lbs. |
| 4×10 | 1.66 | 6 | 250 | 4 | 4 000 | 5 | 9×15 | 2.5 | 9 | 250 | 9 | 13 360 |
| 5×10 | 2.75 | 6 | 180 | 5 | 6 700 | 6 | 11×15 | 2.33 | 6 | 180 | 9 | 11 600 |
| 7×10 | 2 | 7.5 | 250 | 6 | 8 000 | 7 | 13×15 | 2.33 | 8 | 180 | | |
| 5×15 | 2.33 | 8 | 180 | 9 | 9 100 | 8 | 15×20 | 3.5 | 10 | 150 | | |
| 7×15 | 2.33 | 9 | 180 | 9 | 10 490 | 9 | 18×24 | 6 | 12 | 125 | | |

OTE.—Amount of product depends on distance jaws are set apart. **lute** given in Table is due when jaws are set 1.5 ins. open at **bow** is run at its proper speed and diligently fed. It will also vary ac- **ceter of stone.** Hard stone or ore will crush faster than sandstone. **the yard of stone** is about one and one third tons.

STEAM FIRE-ENGINE.

Amoskeag, N. H. 1st Class.

Steam Cylinder.—Two of 7.625 ins. in diam. by 8 ins. stroke of piston.

Water Cylinder.—Two of 4.5 ins. in diam.

Boiler (vertical tubular).—*Heating surface*, 175 sq. feet. *Grates*, 4.75 sq. ft.

Pressure of Steam.—100 lbs. per sq. inch. *Revolutions*, 200 per minute.

Discharges.—Two gates of 2.5 ins., through hose, one of 1.25 ins. and two of 1.25 ins.

Projection.—Horizontal, 1.25 ins. stream, 311 feet; two 1 inch streams, 200 feet; Vertical, 1.25 ins. stream, 200 feet. *Water Pressure*.—With 1.125 ins. nozzle, 200 feet.

Time of Raising Steam.—From cold water, 25 lbs., 4 min. 45 sec.

Weights.—Engine complete, 6000 lbs.; water, 300 lbs.

SAW-MILL.

Two Vertical Saws, 34 Ins. Stroke, Lathes, etc.

Engine (non-condensing). *Cylinder*.—10 ins. in diam. by 4 feet stroke of piston. *Boilers*.—Three (plain cylindrical), 30 ins. in diam. by 20 feet in length.

Pressure of Steam.—90 lbs. per sq. inch. *Revolutions*, 35 per minute.

NOTE.—This engine has cut, of yellow-pine timber, 30 feet by 18 ins. in 1 min.

STONE SAWING.

Emerson Stone Saw Co. (Diamond Stone Saw, Pittsburgh, Pa.)
20 HP, 150 sq. feet of Berea sandstone, inclusive of both sides of cut, in 1 hour.

CHIMNEYS.

LAWRENCE, Mass. Octagonal, 222 Feet above Ground, and 19 Feet in Diameter. Foundation, 35 Feet square and of Concrete 7 Feet deep. (Hiram F. Millard.)

Shaft.—234 feet in height, 20 feet at base, and 11.5 at top; 28 ins. thick at base and 8 at top. *Core*.—2 feet thick for 27 feet, and 1 foot for 154.

Horizontal Flues.—7.5 feet square, and Vertical flue or cylinder of 8.5 feet in diameter, high, with walls 20 ins. thick for 20 feet, 16 for 17 feet, 12 for 52 feet, and 8 for 154.

Purpose.—For 700 sq. feet grate surface. *Weight*.—2250 tons. *Bricks*, 500,000.

NEW YORK STEAM HEATING Co. Quadrilateral, 220 Feet above Ground, and 1 Foot below. (Chas. E. Emery, Ph.D.)

Shaft.—220 feet in height, and 27 feet 10 ins. by 8 feet 4 ins. in the clear.

Foundation.—1 foot below high water. *Capacity*.—Boilers of 16,000 HP.

Cost of Steam-Engines and Boilers complete, and Operation per Day of 10 Hours, inclusive of Labor, Fuel, and Repairs. (Chas. E. Emery, Ph.D.)

| HP. | Engine. | Water Evaporated per | | Coal per | | Labor. | Supplies and Repairs. | Cost of Coal. |
|------|----------------------|----------------------|--------------|----------|-------|--------|-----------------------|---------------|
| | | HP. per Hour. | Lb. of Coal. | HP. | Day. | | | |
| 6.25 | Portable Vertical | 42 | 7.5 | 56 | 394 | 1.75 | .33 | .73 |
| 12.5 | " | 38 | 7.5 | 51 | 717 | 1.75 | .41 | 1.50 |
| 29 | Horizontal..... | 32 | 8 | 40 | 1308 | 2.25 | .60 | 2.41 |
| 112 | Single Condensing... | 23 | 8.8 | 25.1 | 3300 | 3.75 | 1.17 | 6.11 |
| 276 | " | 22.2 | 8.8 | 25.2 | 7831 | 4.25 | 2.12 | 14.50 |
| 552 | " | 22.2 | 8.8 | 25.2 | 15663 | 6 | 4.25 | 29.00 |

* \$4.42 per ton (2240 lbs.), including cartage.

GRAPHIC OPERATION.

ns of Questions by a Graphic Operation.

man walks 5 miles in 1 hour, how far will he walk in 4 hours?

Operation.—Draw horizontal line, divide it into equal parts, as 1, 2, 3, and 4, representing hours. From each of these points let fall vertical lines A C, 1 x, etc., and divide A C into miles, as 5, 10, 15, and 20, and from these points draw equidistant lines parallel to the horizontal.

Hence, the horizontal lines represent time or hours, and the vertical, distance or miles.

Therefore, as any inclined line in diagram represents both time and distance, course of man walking 5 miles in an hour is represented by diagonal A e; and if he walks for 4 hours, and read off from vertical line A C the distance = 20 miles.

far will a man walk in 2 hours at rate of 10 miles in 1 hour?

is shown by the line A o, representing 20 miles.

Two men start from a point at the same time, one walking at the rate of 3 miles in an hour and the other at 10 miles, how far apart will they be at the end of 2 hours?

rses being shown by the lines A r and A o, the distance r o represents
ce of their distances, $10 \sim 20 = 10$ miles.

long have they been walking?

rses are now shown by the lines A_0 and A_4 , the distance 2_4 represents
ce of their times, or $2 \sim 4 = 2 \text{ hours}$.

1 they are 10 miles apart, how long have they been walking?

rses are again shown by the lines A r and A o , the distance $r o$ represents the difference of their distances of 10 miles, and A 2, 2 hours.

nan walks a given distance at rate of 3.5 miles per hour, and then of distance back at rate of 7 miles, and walks remainder of dis- minutes, occupying 25 minutes of time in all, how far did he run?

Set off Cr equal to 5 minutes, upon same scale as that of A ; let fall vertical rs , and draw diagonal cu at same angle of inclination as cr ; then from point u draw diagonal us , inclined at such a rate as to represent 3.5 miles per hour; thus, if in represents rate of 3.5 miles, sO , being one distance between the two points is thus determined by Cx , and displays us , measured by scale of miles employed.

Ion.—The distances $A e$ and $A i$ are respectively 10 minutes = .166 of an 5833 mile = .166 of 3.5 miles. Hence, $C x = .875$ mile, and $u s = .5833$ mile. Consequently, the man walked $A O = .875$ mile = 15 minutes, ran $O u = .5833$ mile = 5 minutes, and walked $u C = .2916$ mile.

second man were to set out from C at same time the man referred to in the preceding question started from A, and to walk to A and then to C, in the same rate of speed and occupying same time of 25 minutes, in what times will he meet the first man?

2.—As $A C$ represents whole time, and $C x$ distance be-
hind x will represent course of second man walking at
meet the first man, on his outward course, at a distance
"A, represented by $A o$, and at the time $A a$; and on the
1 v , $x m$, and at the time $A c$.

MISCELLANEOUS.

No., Diameter, and Number of Shot. (American Standard)

Compressed Buck Shot.

| No. | Diam. | Shot per Lb. | No. | Diam. | Shot per Lb. | No. | Diam. | Shot per Lb. |
|-----|-------|-----------------|-----|-------|-----------------|-----|-------|-----------------|
| | Inch. | No. | | Inch. | No. | | Inch. | No. |
| 3 | .25 | 284 | 1 | .3 | 173 | 00 | .34 | 115 |
| 2 | .27 | 232 | 0 | .32 | 140 | 000 | .36 | 98 |

Balls, .38 inch, 85 No. per lb.; .44 inch, 50 No. per lb.

Chilled Shot.

| No. | Diam. | Shot per Oz. | No. | Diam. | Shot per Oz. | No. | Diam. | Shot per Oz. | No. | Diam. | Shot per Oz. |
|-----|-------|-----------------|-----|-------|-----------------|-----|-------|-----------------|-----|-------|-----------------|
| | Inch. | No. | | Inch. | No. | | Inch. | No. | | Inch. | No. |
| 12 | .05 | 2385 | 9 | .08 | 585 | 6 | .11 | 223 | 1 | .16 | 73 |
| 11 | .06 | 1380 | 8 | Trap | 495 | 5 | .12 | 172 | B | .17 | 51 |
| 10 | Trap | 1130 | 8 | .09 | 409 | 4 | .13 | 136 | BB | .18 | 38 |
| 9 | .07 | 868 | 7 | Trap | 345 | 3 | .14 | 109 | BBB | .19 | 31 |
| 8 | Trap | 716 | 7 | .1 | 299 | 2 | .15 | 83 | | | |

Drop Shot.

| No. | Diam. | Pellets per Oz. | No. | Diam. | Pellets per Oz. | No. | Diam. | Pellets per Oz. | No. | Diam. | Pellets per Oz. |
|-----------------|-------|--------------------|-----|-------|--------------------|-----|-------|--------------------|-----|-------|--------------------|
| | Inch. | No. | | Inch. | No. | | Inch. | No. | | Inch. | No. |
| Extra Fine Dust | .015 | 84021 | 9 | Trap | 688 | 5 | .12 | 163 | BBB | .19 | 47 |
| Fine Dust | .03 | 10784 | 9 | .08 | 563 | 4 | .13 | 132 | T | .2 | 36 |
| Dust | .04 | 4565 | 8 | Trap | 472 | 3 | .14 | 106 | TT | .21 | 31 |
| 12 | .05 | 2326 | 8 | .09 | 399 | 2 | .15 | 86 | FF | .22 | 27 |
| 11 | .06 | 1346 | 7 | Trap | 338 | 1 | .16 | 71 | F | .23 | 24 |
| 10 | Trap | 1056 | 7 | .1 | 291 | B | .17 | 59 | | | |
| 10 | | 848 | 6 | .11 | 218 | BB | .18 | 50 | | | |

The scale of the Le Roy standard (adopted by the Sportsman's Convention) commences with .21 inch for TT shot, and reduces .01 inch for each size to .05 inch for No. 12. The number of pellets per oz. being the actual number in perfect shot.

The number of pellets by this standard is nearly identical with that of the American Standard.

Tatham's scale is same as Le Roy's, but number of pellets is deduced mathematically, by computing them from the specific gravity of the lead.

Drains, Diameter and Grade of, to Discharge Rainfall.

| Diam. | Grade 1 Inch. | Acres. | Diam. | Grade 1 Inch. | Acres. | Diam. | Grade 1 Inch. | Acres. | Diam. | Grade 1 Inch. | Acres. |
|-------|------------------|--------|-------|------------------|--------|-------|------------------|--------|-------|------------------|--------|
| Ins. | | | Ins. | | | Ins. | | | Ins. | | |
| 4 | 30 | .5 | | 40 | 1.2 | | 60 | 2.1 | | 80 | 5.1 |
| | 20 | .6 | | 20 | 1.5 | 9 | 120 | 2.1 | 15 | 240 | 7.8 |
| 5 | 80 | .5 | 7 | 20 | 1.2 | | 80 | 2.5 | | 120 | 7.8 |
| | 60 | .6 | | 60 | 1.5 | | 60 | 2.75 | | 80 | 9 |
| 6 | 20 | 1 | 8 | 120 | 1.5 | 12 | 120 | 4.5 | 18 | 60 | 10 |
| | 60 | 1 | | 80 | 1.8 | | 80 | 5.3 | | 240 | 10 |

British and Metric Measures, Commercial Equivalents of. (G. Johnstone Stones, F. R. S.)

| Millimeters. | Weight. | Grammas. | Volumes. | Cuba Content. |
|--------------|------------|----------|-------------|---------------|
| 914.4 | Pound..... | 453.6 | Gallon..... | 454 |
| 304.8 | Ounce..... | 28.35 | Quart..... | 118 |
| 25.4 | Grain..... | .0648 | Ounce..... | 1.73 |

MEMORANDA.

and Mechanical Elements, Constructions, and Results.

Double. — $600 \text{ HP} \div \text{velocity of belt in feet per min.} \div \text{number of revolutions per minute} \times \text{diameter of pulley in feet}$
Machine Belts. — $1500 \text{ to } 2000 \text{ HP} \div \text{velocity of belt in feet per min.}$ (Edward Sawyer.)

Boiler of a Locomotive. Best height is from 6 to 8 diameters of boiler effect when expanded to full diam. of pipe at 2 diameters from base.

Riveting. A riveting gang (2 riveters and 1 boy) will drive in shell, about 12.5 rivets per hour.

Compressed Fuel is composed of coal dust agglomerated together, compressed in molds, and subjected to a high temperature in order to expel the moisture or volatile portion of the pitch and any fire-exist in the cells of the coal.

Deepest. At Garabil, France, 413 feet from floor to surface of water, length.

Malleable. P. Dronier, in Paris, makes alloys of copper and adding from .5 per cent. to 2 per cent. quicksilver.

5. Department, Requirements of. (New York.)

Flues of Dwelling Houses hereafter constructed at least 8-inch walls on inner 4 ins. of which, from bottom of flue to a point two feet above top of fire-brick laid with fire-clay mortar; and least dimensions of flue square, or 4 ins. wide and 16 ins. long, inside measure; and when located in the usual stacks, side of flue inside of house to which it is 4 ins. thick. If preferred, furnace flues may be made of fire-clay size, built in the walls, with an air space of 1 inch between them, and a brick wall on outside.

Flues to be lined with fire-brick at least 25 feet in height from bottom, and walls of said flues to be less than 8 ins. thick.

Buildings for furnaces or boilers must be altered to conform to the above before they are used as such.

3. *Protection of Buildings from Lightning.* A wire rope of 3/8 inch held to be the most efficient.

Flues, weighing 8 lbs. per yard and 4 lbs. for duplicated and all others, 50 feet apart, thus bringing every portion of the building to which it is within 25 feet of their protection.

Material for a conductor; it should be continuous, and all joints at right angles are preferable to one, and greater surface should be given with the earth than usually practised. (Sir W. Thompson.)

Information, see *Van Nostrand's Magazine*, N. Y., Aug. 1882, page 154.

Iron to Stone. — Fine iron filings, 20 parts, Part 1; mixed fluid with vinegar, and applied for 60, and 12.

Draught. $\overline{W - w} h = D$. W and w are weights of air at external and internal temperatures, h height of draught. See Weight of Air, page 521.

For India Ink improves with age, should be kept down, the movement should be in a right.

Coal, Effective Value of. Theoretical quantity of heat per lb. 2564 units per hour, and average quantity of heat in a lb. of coal that is utilized in the generation of steam in a boiler is 8500 units; hence, theoretical quantity of coal required per HP per hour = $\frac{2564}{8500} = .3$ lbs., after the water has been heated into atmospheric steam, being theoretically nearly 7.5 per cent. of total heat required to change 30 lbs. water at 60° into steam of 60 lbs. effective pressure.

The total heat developed by the combustion of coal, when utilized evaporatively, ranges from .55 to .8, but in practice it does not exceed 65 per cent.

Coast and Bay Service. A velocity of current of 2.5 feet per second will scour and transport silt, and 3 to 6.5 feet sand. For river scour the velocities are very much less.

Cold, Greatest. —220°, produced by a bath of Carbon, Bisulphide, and liquid Nitrous Acid.

Corrosion of Iron and Steel. The corrosion of steel over iron is, as a mean, fully one third greater.

Cost of Family of Mechanics in France ranges from \$20 to \$600 per annum, of which clothing costs 16 parts, food 61, rent 15, and miscellaneous 8.

Crushing Resistance of Brick. A pressed brick of Philadelphia clay withstood a pressure of 500 000 lbs. for a period of 5 minutes.

Earthwork. Shovelling. —Horizontal, 12 feet. Vertical, 6 feet. When thrown horizontal, 12 to 20 feet, 1 stage is required, and from 20 to 30, 2 stages. When vertical, 6 to 10 feet, 1 stage is required.

Wheelbarrow. —Proper distance up to 200 feet.

Number of Loads and Volume of Earth per Day. One Laborer. (C. Herschell, C. E.)

| Distance. | | | Distance. | | | Distance. | | |
|-----------|-----|-----------|-----------|-----|-----------|-----------|-----|-----------|
| Feet. | No. | Cub. Yds. | Feet. | No. | Cub. Yds. | Feet. | No. | Cub. Yds. |
| 20 | 120 | 23.5 | 150 | 96 | 13.3 | 350 | 88 | 11.6 |
| 50 | 110 | 16.9 | 200 | 94 | 12.8 | 400 | 86 | 11.2 |
| 70 | 100 | 14.4 | 250 | 92 | 12.4 | 450 | 84 | 10.9 |
| 100 | 98 | 13.8 | 300 | 90 | 12 | 500 | 82 | 10.5 |

Volume of a barrow load, 2.5 cube feet.

Portable Railroad and Hand Cars. —For a distance of 550 feet, 60 cube yards can be transported per day.

Horse Cart.—Volume of Earth transported per Day. One Laborer.

| Distance. | | | Distance. | | | Distance. | | |
|-----------|-----|-----------|-----------|-----|-----------|-----------|-----|-----------|
| Feet. | No. | Cub. Yds. | Feet. | No. | Cub. Yds. | Feet. | No. | Cub. Yds. |
| 300 | 86 | 17.1 | 1000 | 43 | 8.6 | 2000 | 25 | 5 |
| 500 | 67 | 13.6 | 1500 | 31 | 6.4 | 2500 | 21 | 4.3 |

Volume of each load, 8 cube feet.

Ox Cart is less in cost at expense of time.

Electric Light, Candle Power of. *Maxim Incandescent Lamp.* —Current with 30 Faure cells, 74 volts, 1.87 Ampères, 16 standard candles. With 9 like cells, 124 volts, and 3.2 Ampères, 333 candles. (Paget Hills, L.L.D.)

The elevated electric lights at Los Angeles, Cal., are distinctly visible at sea for a distance of 80 miles.

Engine and Sugar Mill, Weights of. *ENGINE (non-condensing).* —30 ins. in diam. by 5 feet stroke of piston. *Boilers* (cylindrical) —diam. by 40 feet in length. *Weights.* —Engine, 105 000 lbs.; *Boilers*, 100 000 lbs.; *Sugar-mill*, 40 ins. by 8 feet, 220 050 lbs.; *Connecting Machinery*, etc., 45 787 lbs.

ing Stone. Artificial.—Clay, 15 parts; Levigated Chalk, 1.5; and coarse, 83.5. Mixed in water, molded, and hard burned.

engine, Steam. Relative effect for equal cost compared with a coal engine, as 1 to 113. Each HP requires about 112 weight of engine.

ing Bodies, Velocities of. At low speeds resistance increases less than square of velocity. In a Canal, at a speed of 5 miles per hour, wave is raised, which at a speed of 9 miles disappears, and when speed is that of the wave, resistance of boat is less in proportion to velocity, and is reduced.

f Vessel.—The proper length for a vessel in feet (upon the wave-line) is fifteen sixteenths of square of her speed in knots per hour.

of Air. $67 \sqrt{h} = \text{Velocity per second} \times C$. h representing column in feet, and C a coefficient ranging from 56 to 100.

| | | |
|--|--------|-----|
| or orifices, thin plate..... | .56 to | .79 |
| rical mouth-pieces, short..... | .81 | .84 |
| do. rounded at inner end..... | .92 | .93 |
| l converging mouth-pieces..... | .9 | 1 |
| l mouth-piece, alike to contracted vein..... | .97 | 1 |

, Corrugated. (Wm. Parker.) $\frac{1000 (T - 2)}{D} = \text{Working stress in lbs. per sq. inch}$. T representing thickness in 16ths of an inch, and D diameter in ins. Corrugations 1.5 ins. deep. Experiments upon a furnace 31.875 ins. in diameter, 10 feet in length, and with 13 corrugations.

ation Piles. When piles are driven to a solid foundation, they act as support, and are designated *Columns*, and when they derive their support from the friction of the soil alone, they are termed *Piles*.

Piles differ greatly as to the factor of safety for Piles, varying .1 to .01 of ram. (Weisbach.)

For safe load may be taken at from 750 to 900 lbs. per sq. inch. This gives a higher value (Rankine and Mahon, 1000); but it is to be borne in mind that when piles are driven to a solid resistance, they are frequently split, and consequently their resistance is much decreased.

For the following coefficients for ordinary structures are submitted:

For piles are wholly free from vibration consequent upon external impulse, and when the structures are heavy and exposed to irregular loading, as bridges, etc., .15 to .2.

For the bearing of a properly driven pile not less than 10 ins. in diam. may be taken at 10 tons.

ion of Bottoms of Vessels. At a velocity of 7 knots per hour, a clean bottom requires 2.42 HP over that for a clean bottom.

ion of Planed Brass Surfaces in muddy water is .4 pressure.

Steam, and Hot-air Engines. Relative costs of gas, steam, and hot-air engines per HP: Otto Gas engine, 8.75; Steam engine, 3.5; and Hot-air engine, 1.5.

Available heat per HP per hour = $\frac{16431535}{\text{Total heat of combustion} \times \text{Coef of coal per HP}}$ =

1000×772 units = 10808000. Theoretical evaporative

Efficiency of furnace = .5; then $10808000 \times .5 = 5404000$

per HP per hour.

boats, Speed of. Maj.-Gen. Z. B. Tower, U. S. A., assigns it twice that of the wind, and the angle of sail, to attain 90°.

Coal. Analysis of Bituminous.—Specific Gravity, 1.25; Hydrogen, 5.28. Oxygen, 3.26. Nitrogen, 2.75. Sulphur, 1.38.

Relative effect = 4.16 lbs. water per lb. of coal.

Lee-way. A full modelled vessel, with an immersed section of 1 to longitudinal section, and with an area of 36 sq. feet of sails to 1 of immersion, will drift to leeward 1 mile in 6. A medium modelled vessel, with immersed section of 1 to 8, and with like areas of sail and section, will drift 1

Light, Standard of. *Photometric, English.*—Spermaceti candle lb.; 120 grains per hour. Carcel burner = 9.5 candles.

Locomotive Axles, Friction of. .016 of weight. Hence, if r wheel = .1, axle friction at periphery $\frac{.016}{.60} \div 10 = 3.73$ at periphery.

Mercurial Gauge. To prevent freezing, apply or introduce Gly top of column.

Metal Products of U. S., 1882. Value, \$222,000,000.

Mississippi River, Silt in. Near St. Charles the volume borne per day in 1879 was 475,457 cube yards, and on one day, July 4, 113,600. At times the volume equals 3 ozs. per cube foot of water.

Motive Power. A sailing vessel having a length 6 times the breadth, requires, for a speed of 10 knots per hour, an impelling force of 4 sq. foot of immersed section.

Mowing Machine. Kirby's (Auburn, N. Y.)—670 lbs., 2 horn heavy clover in 46 min.

Ordnance, Energy of. In a competitive test of a 9-inch V gun, and a 5.75-inch Krupp, the energy per inch of circumference of bore respectively 118 and 123 foot-tons; their penetration therefore by the wrought standard being about the same, but their total energies were respective and 5800 foot-tons.

At Meppier a shot of 110 lbs., with a velocity of 1749 feet per second, and ing energy of 2300 foot-tons, passed through a target composed of two plate wrought iron 7 ins. thick, with 10 ins. of wood between them, and passed it beyond.

Petroleum. One lb. crude oil heated 1 lb. water $315.75^\circ = 28.21^\circ$ I at 60° converted to steam at 212° . Relative evaporative effects of Oil and cile coal as 1 to 3.45.

Population, Comparative Density of, and Numl Persons living in a House in different Citie

Chicago, 4; Baltimore and Naples, 4.5; Philadelphia, 6; London, 6; Cairo, 8; Marseilles, 9; Peking, 10; Amsterdam, 11; New York, 13.5; I 17.07; Rome and Munich, 27; Paris, 29; Buda Pesth, 34.2; Madrid, 40; S burg, 43.9; Vienna, 60.5; and in Berlin, 63.

Power of a Volcano. An eruption of that of Cotopaxi has a mass of rock of a volume of 100 cube yards a distance of 9 miles.

Power Required to Draw a Vessel or Load up inclined Hydrostatic Rail or Slip Way. (Wm. Boyd,

$WI = R$; $CdW \div D = F$; and $P'd'c = f$. W representing weight of load and cradle, I inclination of ways, as length \div rise, R resistance of vessel, F friction of cradle and rollers, and f friction of plunger in stuffing-box, a C and c coefficients of friction of cradle and stuffing-box, d diameter of axle, d' product of circumference of plunger and depth of collar or stuffing, all in P pressure per sq. inch on plunger, in lbs.

Hence, $W \frac{\text{rise}}{\text{length}} = I$, and $R + F + f = \text{power in tons}$.

ILLUSTRATION.—Assume weight of a vessel and cradle 2000 tons, pn plunger 2500 lbs. per sq. inch, inclination of ways 1 in 20, diameters of axers and of rollers 3 and 10 ins., depth of collar 2 ins., and circumference 0 50; what would be the power required? $C = .2$, and $c = .6$.

$$\text{an } \frac{2000}{20} = 100 \text{ tons; } \frac{.2 \times 3 \times 2000}{10} = 120 \text{ tons; } \frac{2500 \times 2 \times 2 \times 50 \times 5}{2240} \\ = 1.67 = 287 \text{ tons.}$$

Propeller Steamer, Ordinary Distribution of Power. Power developed by engine, 88 HP; Power expended in its operation,

| | Per cent. | | Per |
|-----------------------|-----------|---|-----|
| Friction of load..... | 7.5 | Power expended by slip of propeller.... | |
| “ of propeller..... | 7.5 | “ “ in propulsion..... | |

Pump, Centrifugal, has lifted water 28 to 29 feet, drawn it horizontally 100 feet, and then lifted it 15 feet. Also drawn it 24 feet, and projected it 50 feet.

Railway Trains. Power and Resistance.—A railway train running at a rate of 60 miles per hour = 88 feet per second, and velocity a body would acquire in falling from 88 feet = $88 \div 8.02 = 120.3$ feet. Consequently, in addition to power expended in frictional and atmospheric resistance to train, as much power must be expended to put it in motion at this speed, as would lift it in mass to a height of 120.3 feet in a second.

If the train weighed 100 tons = 224 000 lbs., then $224\,000 \times 120.3 = 26\,947\,200$ foot-lbs., and if this result was obtained in a period of 5 minutes, it would require $26\,947\,200 \div 5 \times 224\,000 \div 33\,000 = 163.3$ HP in addition to that required for frictional resistances.

To raise the speed of a train from 40 (58.66 feet per second) to 45 (66 feet per second) miles per hour, the power required in addition to that of friction would be

$58.66 \div 8.02 = 53.44$ feet is to $66 \div 8.02 = 67.57$ feet = $67.57 - 53.44 = 14.13$ ft

Assume a train of 100 tons, running at rate of 60 miles per hour, and total retarding power at 1 its weight $100 \div 10 = 10$. Then $224\,000 \times 10 \times 120.3 = 26\,947\,200$ foot-lbs. = 1203 feet, which train would run before stopping. If, however, train ascending a grade of 1 in 100, the retarding force = .11 (11 ÷ 100) of weight = 24 640, distance in which train would come to rest would be $26\,947\,200 \div 24\,640 = 1093.6$ feet.

Relative Non-conductibility of Materials.

| MATERIAL. | Per cent. | MATERIAL. | Per cent. | MATERIAL. | Per |
|---------------------|-----------|---------------------|-----------|-----------------------|-----|
| Hair felt..... | 100 | Mineral wool, No. 1 | 67.5 | Lime, slacked ... | 48 |
| Mineral wool, No. 2 | 83.2 | Charcoal..... | 63.2 | Asbestos..... | 36 |
| “ “ and tar | 71.5 | Pine wood..... | 55.3 | Coal ashes..... | 34 |
| Sawdust..... | 68 | Loam..... | 55 | Air space, 2 ins. ... | 12 |

Resistance to a Steam-vessel in Air and Water. In 20 per cent. of HP, and in water, at a speed of 20 miles per hour, 90 per cent., of HP per sq. foot of immersed amidship section.

Saws, Circular. 30 ins. in diameter, are run at 2000 revolutions per minute = 3.57 miles.

Spur Gear has been driven at a velocity of 1 mile per minute.

Sugar Mill Rollers. 5 feet by 28 ins., at 2.5 revolutions per minute requires 20 HP, and 18 feet per minute is proper speed of such rolls.

Surface Condensation, Experiments on. (B. G. Nichol.)
Tube of Brass, .75 Inch External Diameter. No. 18 B W G. Surface = 1.1 sq. feet. Duration of Experiment, 20 Minutes.

| STEAM. | Vertical. | | Horizontal. | |
|-----------------------------------|------------|------------|-------------|--------|
| Temperature..... | 255° | 256° | 253° | 254° |
| Pressure per sq. inch per gauge.. | 17.75 lbs. | 18.25 lbs. | 16.75 lbs. | 17.25 |
| Condensation by tube surface.... | 18.5835 “ | 29.9585 “ | 24.0835 “ | 43.083 |
| “ per sq. ft. of “ per hour | 52.32 “ | 84.34 “ | 67.8 “ | 121.29 |
| Condensed during experiment.... | 19.0625 “ | 30.4375 “ | 24.5625 “ | 43.562 |

Steamers' Engines, Weights of. Engine, Boiler, Water, and Fittings ready for Service per HP.

| | | | |
|-------------------------|----------|--------------------|--------|
| Mercantile steamer..... | 480 lbs. | Light draught..... | 280 lb |
| English Naval “..... | 360 “ | Torpedoes..... | |

Ordinary Marine Boiler with Water..... 196 lbs.

Wind, Pressure of. Estimate of, upon Structures.—30' Per lineal foot of a locomotive train = 10 feet in height, 300 lbs. per

A Tornado has developed a pressure of 93 lbs. per lineal foot.

The Great Canal. *Proposed by Emerson.*—*See "Shipping Canals"* for details. Estimated to be 25 days in descent and 25 days in ascent, including 2 days in each way in sailing and 2 days in each way in rowing and paddling.

"Gleason." *Steamer to New York, N. Y.*—In 25 days and in descent, including 2 days at each end. From Philadelphia in 12 days.

Zeus Fall in Steam-Boilers. When in an iron steam-bolt or boiler a violent current, which tempests the water, liberating oxygen and gas. The oxygen combines with fatty acids and makes soap, which clogs the tubes, prevents the admission of the water left by evaporation. The only way can then be readily removed.

Prison. To Compute Extreme Load a Formulation File will Solve.

$P + W \times L \times Z$ representing weight of man, P weight of pile, and L and Z food, all in lbs. Z height of fall of man, and a distance of depression of pile with blow, both in feet.

Illustration.—Assume a man 2000 lbs. to fall 20 feet upon a pile of pile which resistance will the earth bear, or what weight will the pile sustain if driven by the last blow, from a height of 20 feet, .5 inch?

$$s = .5 \text{ of 20 ins.} = .0416$$

$$\text{Then } \frac{2000 \times 20}{(400 + 2000) \times .0416} = \frac{20000000}{5124} = 343405 \text{ lbs.}$$

Perimeter. The limits or bounds of a figure, or sum of all its sides. Of a canal it is the length of open and wet sides of its transverse section.

Flood Wave. The flood wave of the Ohio River in March (1881) was 1 foot 1 inch at Cincinnati, being higher than that of any previous record.

Iron. Crushing strength of, as determined by U. S. testing machine, from 327 to 1000 lbs. per sq. inch.

Atmosphere. If pure air is exhausted of 2.5 per cent. of its oxygen, it will not support the combustion of a candle.

Blasting Paper. Unsized paper coated with a hot mixture of potassium persulfate of potash and charcoal, each 17 parts; refined saltpetre, 35; potassium chlorate, 70; wheat starch, 10, and water, 1500.

Dry, cut into strips, and roll into cartridges.

Circular Saws. Speed, 9000 feet per minute. Thus, for an 8 in. saw, 1000 revolutions, and progressively up to a 72 in., 500 revolutions. (*Emerson*)

Foods, Relative Value of, compared with 100 Lbs. of Good Hay.

Additional to page 203.

| Lbs. | | Lbs. | |
|-----------------------|-----|----------------------|-----|
| Acorns..... | 68 | Linseed..... | 59 |
| Barley and Rye, mix'd | 170 | Mangel-wurzel..... | 339 |
| Barley straw..... | 180 | Pease and Beans..... | 45 |
| Buckwheat..... | 64 | Pea-straw..... | 153 |
| Buckwheat straw..... | 200 | Potatoes..... | 175 |
| | | Rye..... | |
| | | Turnips..... | |
| | | Wheat..... | |
| | | Wheat, Feed and | |
| | | chaff..... | |

Depth of the Ocean. Mean depth is estimated by Dr. Dana to be 11,722 fathoms = 1.83 geographical miles.

Gas-engine. A gas engine 1.5 actual HP will cost, with gas at 1 cent per hour, 10 cents per hour for 10 hours. (*Am. Engineer.*)

Locomotive. Average daily run 100 miles at a cost of \$21.50 per mile. Women, fuel and repairs. (*N. Y. Central R. R. Co.*)

Consumption of Fuel per Mile. Passenger, 25 to 30 lbs. per mile. Freight, 30 to 40, or one cord wood per 40 miles.

Mason
course bend

Steel
gal. 32.66
milled iron
Moment

Saw-
Engine (1
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ry. In laying stones in mortar or cement, they should rest upon the ath them, more than upon the material of joint.

Gun (Krupp's). Bore, 15.75 ins.; length of bore, 28.5 feet; of feet. Weight, 72 tons. Charge, 385 lbs. prismatic powder; projectile, 1660 lbs., with an explosive charge of 22 lbs. of powder.

if shot at muzzle, estimated at 31 000 foot-tons, and range 15 miles.

Mill. 7722 feet of 1 inch Poplar boards in One Hour.

(on-condensing). **Cylinder.**—12 by 24 ins. stroke of piston.

Two (cylindrical flued), 38 ins. in diam. by 26 feet in length, two 14 ins. in each. **Heating Surface.**—780 sq. feet. **Grates.**—42.5 sq. feet.

of Steam.—125 lbs. per sq. inch, cut off at 16.5 ins.

ns.—250 to 350 per minute. **Saws.**—Two circular, 60 and 66 ins. in

brates set 28 ins. from under side of boilers, without bridge-wall, and a chamber under boilers, 4 feet in depth. Fuel, sawdust.

Heating. 62 500 cube feet of space requires 6000 sq. feet of heat to attain a temperature of 70° in the vicinity of the city of New York at weather

1 foot of iron pipe will heat 10.5 cube feet of space in an ordinary build- ature of exterior air 70°. (*Felix Campbell.*)

ty of Steam. Steam at a pressure of 60 lbs. + atmosphere has f efflux of 890 feet per second, and as expanded, a velocity of 1445 feet.

ng. In small blasts 1 lb. powder will detach 4.5 tons material, and in 2.75 tons. (See page 443.)

Metal (Iron and Bronze). Specific gravity 8.4. Melting point 1800°. 34).

1 **Wood of Australia.** Impervious to insects and the *Teredo*

al Gas and Bituminous Coal. Relative water evaporat- Gas, 20 to 21 lbs. Coal, 9 lbs.

Board of Vessels. For each foot of depth of hold (from ceiling ie of main deck), .1 inch added to 1.5 ins. for a depth of 8 feet. Thus, lepth $1.5 + .1 \times 8 \div 24 = 3.1$ ins. (*American.*)

for 8 feet depth and .1 for each foot in addition thereto. (*Lloyd's.*)

Colors for Working Drawings.

| | | |
|----------------------------------|-------------|----------------------|
| .. Gamboge. | Steel | Neutral tint, light. |
| .. Carmine. | Water | Cobalt. |
| .. Burnt Umber. | Wood | Burnt Sienna. |
| .. Sepia with dark markings. | | Burnt Umber. |
| .. Lake and Burnt Sienna. | Stones | Yellow Ochre. |
| .. India Ink, light. | and | " " |
| .. Neutral tint. | Earths .. | " " |
| .. Prussian Blue. | | Red and 1 |
| .. Ind. Ink tinged with P. Blue. | | Burnt Si |

ge of Chain Cable. Square of diameter or 35 will give volume of space required to stow 1 fathom

lt Mortar. Bitumen 1 part, powdered asphalt 7. sin oil .28 part.

men, add asphalt broken small, than resin oil and sand.

lt Concreate. Asphalt mortar 11 parts and broken

tos is a fibrous variety of Actinolite or Tremolite, comp agnesia, oxide of iron, and water. It resists heat, moist.

Daily Food of an Esquimau. Flesh of a sea-horse 8; and Broad 1.75 lbs., Soup 1.25, Spirits 1, and Water .9 pint. (Sir W. E. Parry.)

Coignet's Concrete. For walls that resist moisture.—Sand, Gravel, and Pebbles, 7 parts; Argillaceous Earth 3 parts, and Quicklime 1 part.

Hard and quick setting.—Sand, Gravel, and Pebbles, 8 parts; Earth, burned and powdered Cinders, each 1 part, and Unslacked hydraulic Lime 1.5 parts. For a very hard mixture, add cement 1 part.

Transmission or Conductivity of Temperature in the Earth. At Edinburgh thermometers set at a depth of 16 feet in the earth attained their maximum and minimum at about six months after the corresponding maximum and minimum of the surface, being lowest or coldest in July.

The average rate of transmission of heat, as observed at Schenectady, N. Y., was, downwards, 2.9 feet per month, and upwards 3.4 feet. (Olin H. Landreth.)

Shafts. When loaded transversely, the diameters of the journal should first be determined, its dimensions then at any other point can be deduced from those diameters. It being observed that the diameters at any two points should be proportional to the cube roots of the stress at those points.

Journals.—For operation at high speed a greater length is required than for low speed. The less their length, the less may be its diameter for a given stress, and consequently the friction will be less.

When in constant operation, a large surface is required to reduce heating, and as friction increases with diameter, not with length, for like stress, it is best to lengthen.

Wrought Iron.—For 50 revolutions length to diameter as 1.2 to 1, and for every 50 revolutions additional .2 should be added. Thus, for 1000 revolutions the length to diameter should be 5 times. **Cast Iron.**—Length to diameter as .9, and **Steel** as 1.25 of above value. (W. C. Unwin.)

Non-conducting Materials. By the investigations of Prof. J. M. Ordway of New Orleans, he determined the relative non-conducting values of the following materials, compared with a naked pipe, to be:

| | | | |
|--------------------------------------|------|-------------------------------|-----|
| Hair-felt, burlap..... | 1 | Cork in strips..... | 2 |
| Asbestos paper, hair-felt, duck..... | 1.18 | Rice-chaff..... | 2.1 |
| Pine charcoal..... | 1.26 | Clay and vegetable fibre..... | 2.8 |
| Air space..... | 4 | Naked pipe..... | 31 |

(Engineering, vol. 39, page 206.)

Marine Transportation of Troops. Height between decks (deck to under side of beam), men 6 feet, horses 7 feet. **Hatchways.**—Horses at least 10 by 10 feet. **Vessels.**—Horses, beam not less than 30 feet. Men, all ranks, 2 to 2.5 tons capacity; horses, 10 tons. **Rations.**—If biscuit in bags, 10000 require 950 cube feet of volume; if it is in barrels, 1350 cube feet.

Cabins.—Officers, 30 sq. feet and 105 cube feet of volume, two men 42 sq. feet, and 270 cube feet of volume, and for each additional man 10 sq. feet, exclusive of bed space of 6 by 2 feet.

Hammocks.—To compute number that can be swung under a deck.

$$n \times \frac{b}{5} = n. \quad l \text{ representing length under deck in feet, and } b \text{ breadth in inches.}$$

of Boilers.—30 lbs. water evaporated into dry
under a pressure of 70 lbs. per sq. inch mercurial gauge.
34.5 lbs. water as above from feed at 212° into steam
(Engineers.)

Penetration of Light in Water. Mediterranean, clear sunlig
In March, at a depth of 1200 feet; in winter, 600 feet. (*M. M. Fol and Sarasin*.)

Railroad. Horse. First in operation in 1826-7.

Pins. First in use in England about 1450.

Iron Steamers. First build in 1830.

Lucifer Match. First made in 1829.

Watches. First constructed in 1476.

Load on Stone per sq. foot. Church of All-Saints at Angers, 86 000 l
Pantheon at Rome, 60 000 lbs.

Flexible Paint for Canvas. Yellow soap 1.66 parts. Boil
water 1. Grind while hot with .83 parts oil paint.

Fuel. Evaporation of 9 lbs. water from 212°:

| | |
|-----------------------|--------------------------|
| 1 lb. good coal. | .75 lb. petroleum. |
| 2 lbs dry peat. | 2.5 lbs dry wood. |
| 3.25 " cotton stalks. | 3.5 " brush wood. |
| 3.75 " wheat straw. | 4 " megass, or cane refu |

Tramways or Street Railroads.

Resistance on straight and level tracks 15 to 40 lbs. per ton, or an average
30 lbs.

Power required on a good track to start a car, as determined by A. W. Wrig
M.W.S.E., 116.5 lbs., and to maintain it in motion 17.2 lbs. C. E. Emery, Ph.
made it 13 lbs. On a bad track, the power is 134.6 lbs. to start, and 35 to maint
it in motion.

Power required, as determined by Mr. Wright, to start a car is 33.53 HP, with
average load and day's work, and 133.22 to maintain it in motion.

Average work of a car-horse 5.75 hours per day for a term of service of 6 ye
Strong draught-horses will exert a power of 143 lbs. @ 2.75 miles per hour for
miles, and an ordinary one 121 lbs. for 25 miles. (*Gayflier*.)

Cable Railway. Mr. Wright gives the power required per ton * at 1.92 HP.

* All tons here and elsewhere are given at 2240 lbs.

Result of Experiments on Motors for Street Railroad (1885.)

At Antwerp, by Capt. D. Gallon, F.R.S., etc.

1. Locomotive Engine and Car. Ordinary type of steam-engine, surface conden
(*Krauss*).
2. " " Surface condenser, vertical boiler, escape sup
heated (*Black and Hawthorn*).
3. " " Compound engine, compressed air, water-t
boiler (*Beaumont*).
4. " " and car combined. Ordinary type of steam-engine, wa
tube boiler (*Rowan*).
5. " " " combined. Electric Fausse Batteries.

| Weight of Train per Passenger. | Fuel consumed | | Oil, Tallow, etc. | Water per Mile of Course. |
|-----------------------------------|------------------------------------|---------------------------------|----------------------|---------------------------------|
| | Per Mile of Course. | Per Seat per Mile of Course. | | |
| Lbs. | Lbs. | Lbs. | Lbs. | Gallons. |
| 5. Electric....1.78 | 4. Rowan.....5.22 | .1 | .038 | Rowan..... |
| 4. Steam.....2.3 | 5. Electric.....6.16 | .23 | .038 | Comp'd air. 1 |
| 3. Comp'd air, 2.55 | 2. Black and Hawthorn... } 8.82 | .23 | .073 | Black and } 5 |
| | 1. Krauss9.1 | .25 | .101 | Hawthorn } 5 |
| | 3. Comp'd air. 39.48 | .66 | .255 | Krauss.6 |

NOTE.—The economy of the Rowan motor occurred mainly from the extent of its cond
power, by which warm water was supplied to the boiler.

Corrosive Effects of Salt-water on Steel or Iron.*(J. Farquharson.)**Loss of Plates Submerged for Six Months. Area 12 Sq. Feet.*

| | | | | |
|------------|----------|------------|-----------------|-----------|
| Steel..... | .253 lb. | Steel..... | } combined..... | { .07 lb. |
| Iron | .233 " | Iron | | |

Frictional Resistance of a Railway Train. *(C. H. Hudson.)*

Resistance per ton, due to atmosphere at maximum speed, .132 lb.; to start, 17.27 lbs.; and to maintain in motion, 5.1 lbs.

Blasting Gelatine. *(G. McRoberts, F.C.S.)*

Is composed { Nitro-glycerine..... 93 parts }
by weight.... { Nitro-cotton..... 7 " } Effective power.... 1400 foot-lbs.

It freezes hard at a low temperature (35 to 40°). At ordinary temperature above freezing, it does not explode by shock, but when frozen it readily explodes. It is insoluble in water. Specific gravity 1.55 to 1.59.

Effective Power of some other Explosives.

Nitro-glycerine, 1270 foot-lbs.; Dynamite, No. 1, 900; Gun-powder, extra strong, as Curtis and Harvey's, 272; Dynamite, No. 2, of 18 nitro-glycerine, 71 nitrate of potash, 10 of charcoal, and 1 of paraffin, 531, and Fulminate of Mercury, 367.

Bolts of Wrought Iron as Affected by the Thread.*(D. K. Clark.)**Strength per Square Inch of Metal.*

| Diam. of Bolt. | Tool. | Strength when cut. | Loss. | Diam. of Bolt. | Tool. | Strength when cut. | Loss. |
|----------------|-----------|--------------------|-----------|----------------|-----------|--------------------|-----------|
| Inch. | | Lbs. | Per cent. | Inch. | | Lbs. | Per cent. |
| 1.25 | Dies. # | 40 812 | 25 | 1 | Chaser. | 44 845 | 28 |
| 1.25 | Chaser. | 38 528 | 29 | .625 | Old Dies. | 51 005 | 14 |
| 1 | Old Dies. | 55 149 | 11 | .625 | New Dies. | 43 613 | 26 |
| 1 | New Dies. | 42 650 | 30 | .625 | Chaser. | 41 883 | 33 |

* Die not given, evidently new.

Approximate Bottom Velocities of Flow of Water in Channels, at which following Materials begin to Move. *(Haupt.)*

| Feet. | Miles. | | Feet. | Miles. | |
|-------|--------|------------------------------------|-------|--------|-----------------------------|
| Sec. | Hour. | | Sec. | Hour. | |
| .25 | .17 | Microscopic sand and clay. | | | { Small stones, 1.75 inch |
| .5 | .34 | Fine sand. | 3 | 2.04 | in diam. |
| 1 | .68 | Coarse sand and fine gravel. | | | { Flint stones, size of |
| 1.75 | 1.19 | Pea gravel. | 3.33 | 2.3 | hen's eggs. |
| 2 | 1.39 | { Rounded pebbles, 1 inch in diam. | 5 | 3.41 | { 2-inch square brick-bats. |

Scouring force of the current is proportioned to the square of its velocity.

Transporting capacity varies as sixth power of the velocity. Hence the importance of increasing bottom velocities, both to effect a scour and to prevent deposits.

Chimney. *(Metternick Lead Mining Co.)*

Foundation 35 feet square by 11.5 in height; base circular 24.6 feet by 39.37 in height; shaft, 397.5 feet in height, 24.6 feet at base, and 12.48 at top; flue 12.48 and 5.21 feet in diameter. Total height 441.6 feet.

Evaporation of Water. *Mean, as observed at Boston, Mass.*

| | | | |
|------------|-------------|---------------|-------|
| Inch. | Inch. | Inch. | Inch. |
| April..... | 3.1 | July..... | 6.28 |
| May..... | 4.61 | August..... | 5.49 |
| June..... | 5.86 | September.... | 4.09 |
| Total..... | 39.11 inch. | October..... | 2.05 |
| | | November.... | 1.53 |
| | | December.... | 1.12 |

Central Width of a Roadway in a Cut.

| | Feet. | | Feet. |
|---------------------|----------|--------------------|----------|
| 1, single line..... | 18 to 20 | Public road..... | 28 to 30 |
| double line..... | 30 " 33 | Turnpike road..... | 38 " 40 |

Hydraulic Ram.

cy under Heads of Supply from 2 to 24 Feet, and Delivery of Discharge at Elevations from 15 to 100 feet.

Measurements from Valves of Ram.

Compute Per Cent. of Total Volume of Water Expended.

= Per cent. H representing head of supply, and E elevation of discharge, feet, and C = 8.

TRATION. — What is volume of water delivered with a head of 21 feet to an in of 60 feet?

.8 = .28 per cent. Hence, if the volume of discharge is 100 cube feet, vol-
evated is $100 \times .28 = 28$ cube feet.

sely. By formula of E. B. Weston, M. Am. Soc. C. E.

$V = S$ representing number of cube feet expended in ram per minute, h dif-
in elevations of ram and delivery in feet, and V volume raised in cube feet
to 70.

me as preceding, $H = 21$ feet, $E = 60$ feet, and $S = 100$ cube feet.

$$\text{Then, } \frac{100 \times 65 \times 21}{100 \times 60 - 21} = \frac{136500}{3900} = 35 \text{ cube feet}$$

—To conform to the preceding formula C should be 52.

To Compute Elements of a Screw Propeller.

$$\frac{la}{p} = T; \quad \frac{P2laR}{pR} = T; \quad \frac{P2laR}{33000} = \text{HP}; \quad \text{and} \quad \frac{\text{HP}33000}{pR} = T.$$

representing mean pressure on piston per sq. inch in lbs., a area of piston in sq.
pitch of propeller and l length of stroke, both in feet, R number of revolutions
minute, and T thrust of propeller in lbs.

TRATION.—The elements of operation of a steam-engine are: Mean pressure
on, having an area of 1000 sq. ins. is 30 lbs.; length of stroke 2 feet; revolu-
of engine 130 per minute; and pitch of propeller 12 feet. What is the thrust
propeller, and what the power of the engine?

$$\frac{30 \times 1000}{12} = 10000 \text{ lbs., and } \frac{30 \times 2 \times 1000 \times 130}{33000} = \frac{1560000}{33000} = 472.7 \text{ HP.}$$

Centrifugal Pump.

(Southwark Foundry and Machine Co. Non-condensing.)

ups.—Two of 42 ins., with runners 68 ins. in diameter; **dis-**

ins.—Two of 28 ins. in diameter of cylinder, and 24 ins. at

ers.—12 Horizontal tubular. **Heating surface,** 8568 sq ft

Combustion natural.

sure of Steam 70 lbs. per sq. inch, cut off at .625.

utions, 130 to 160 per minute.

ht of Delivery, 0 to 36 feet.

ht.—Pumping plant exclusive of boilers 300 000 lbs.

large.—From Dry-dock from a depth of water of 0 to 36 feet
gas gallons.

Friction of a Non-condensing Engine. (*Prof. E. H. Thurston*)

Friction of a non condensing engine is given at from 2 to 4.75 lbs. per sq. inch of piston, being least at low pressure. The conclusions drawn from a series of experiments are as follows:

1. It is sensibly constant at any given speed of engine at all loads.
2. It is variable with variation of speed of engine, increasing with the speed.
3. It increases with increase of pressure of steam.

NOTE.—This per cent. of friction is somewhat less than that given ante at p. 733.

Visibility of Vessel's Sidelights. The minimum distance of visibility assigned by the International regulations for green and red lights is 5 nautical miles.

Weight of Anvils. The weight of an anvil for forging iron should be 8 times that of the hammer, and for steel 12 times. (*Prof. Friedrich Rich*)

Temperature of Mines. Temperature of copper-mines of Lake Superior increases 1° for every 100.8 feet of depth. The usual gradient is from 9 to 55 feet. (*H. A. Wheeler*.)

Horse. In transportation by sea occupies the space of 10 tons measurement, and requires that of 300 cube feet of air.

Stalls 6 feet in length in the clear of padding and haunch piece, 2 feet in the clear width between padding, 10 per cent. of this width 2 ins. narrower, and 5 per cent. of it 6 ins. longer.

Mule. A pair will draw, including cart, 1500 to 2000 lbs.

Ass. Will carry 100 to 200 lbs. 15 miles per day.

Camel. The Arabian, or Dromedary, has one hump on back, the Bactrian has two. Large animals will carry 1500 lbs. for 3 or 4 days, or 1000 lbs. for several days, and 450 to 600 lbs. for a long march.

One has travelled 115 miles in 11 hours.

Elephant. Weight, 3 to 5 tons; weight one can carry about 1450 lbs.; 2000 lbs. have been carried. Occupies 55 sq. feet; will travel on a good road at a rate of 2.5 miles per hour for 6 hours.

Whales. Greenland Right, length 50 to 60 feet. Finner, 80 feet. Speed, 8 to 12 miles per hour. Extreme weight, 74 tons. HP estimated at 145.

Chimneys. Late experiments as to the draught of chimneys have developed the result that an increase of its area near to the top increases the draught.

Cost of Maintenance of Street Railroads, 1876.

Average of 16 roads, 102 miles in length, with 1297 cars and 10300 horses. (*H. Haupt*.)

Cost per horse, and average number to a car eight.

| | | | |
|-------------------------|---------|---------------------|------------------|
| Repairs of harness..... | \$ 4.06 | Shoeing..... | \$ 2.77 |
| Feed..... | 124.39 | Stall expenses..... | 42.13 |
| Replacing horses..... | 22.1 | | |
| | | Total.... | \$ 215.65 |

Cost per month of each horse, \$ 18. On one of the longest railroads in the City of New York, on the least populous route, the daily cost per passenger, exclusive of other expenses, was 2.88 cents, and inclusive of general expenses 4.1 cents.

Magnesia Covering for Steam-pipes and Boilers.

Experiments made by Bureau of Steam Engineering, U.S.N., developed the following negative results:

100 | Sectional magnesia.... 103.07 | Sandstone.... 94

APPENDIX.

River Steamboat. Wood Side Wheels.

Freight and Passenger.

TONS.—HORIZONTAL LEVER ENGINES (Non-condensing).—Length on deck, 10 ins.; beam, 43 feet 4 ins.; hold, 6 feet. Tons, 993.52.

—Said section of light draught of 26 ins., 83 sq. feet. Capacity for freight, 1200 tons.

Wheels.—Two of 25 ins. in diam. by 8 feet stroke of piston.

Boilers.—Four of steel, 47 ins. in diam. by 30 feet in length, 6 flues in each. Grate surface, 98 sq. feet.

Consumption of Steam, 154 lbs. per sq. inch, cut off at .625.

Revolutions, —per minute. Speed, 10 miles per hour against current of upper river, 5 miles.

To Compute Meta-centre of Hull of a Vessel.

Operation of Formula in Naval Architecture, page 660.

Consider a sharp-modelled yacht, 45 feet in length, 13.5 feet beam, and 9.5 feet depth. The immersed amidship section of 42 sq. feet, and a displacement of 900 tons at a mean draught of water of 6 feet.

$$\frac{2}{3} \int \frac{y^3 dx}{D} = \text{Meta-centre. See pages 650, 659.}$$

Ordinates (dx) taken at intervals of 2.5 feet are as follows:

| | |
|---------------------------|--------------------------|
| $y^8 = 6.5^3 = 287.496$ | $y^{16} = 3.25 = 34.328$ |
| $y^9 = 6.7^3 = 300.763$ | $y^{17} = 2.4 = 13.824$ |
| $y^{10} = 6.75 = 307.547$ | $y^{18} = 1.5 = 3.375$ |
| $y^{11} = 6.5 = 287.496$ | $y^{19} = .8 = .512$ |
| $y^{12} = 6.25 = 244.14$ | $y^{20} = 0 = .0$ |
| $y^{13} = 5.8 = 195.112$ | <hr/> |
| $y^{14} = 5 = 125$ | 2272.814 |
| $y^{15} = 4.2 = 74.088$ | 2.5 |
| | <hr/> |
| | 5682.035 |

Integration of function of cubes of ordinates for value of $\int y^3 dx = 5682.035$.

$$\text{And } \frac{2}{3} \text{ of } \frac{5682.035}{900} = \frac{2}{3} \text{ of } 6.31 = 4.21 \text{ feet.}$$

—The other elements of this vessel are:

of load-line, 401.12 sq. feet; Displacement in weight, 27,974 tons; do. at load-line, 955 tons per inch; Depth of centre of gravity of displacement below load-line, 19 feet; Volume of displacement, to volume of immersed dimensions, 26.8 ft.

To Compute Height of Jet in a Conduit Pipe from a Constant Head. (Weisbach.)

$$\frac{h}{C + C' \frac{l}{d}} \left(\frac{d'}{d} \right)^4 = \frac{v^2}{2g} = h', \text{ and } \frac{h'}{z} = h''. \quad h, h', \text{ and } h'' \text{ representing heights}$$

of load-line, h , loss of head and of ascent, l length of pipe or conduit, and d diameters of pipe and jet, all in feet, v velocity of efflux in feet per second, C coefficients of friction of inlet of pipe and outlet, and z a divisor determined experimentally with diameters of .5 to 1.25 ins., ranging from 1.06 to 1.08.

APPLICATION.—If conduit pipe for a fountain is 350 feet in length, and $z = 1.06$, to what height will a jet of .5 inch ascend under a head of 40 feet? $C = .8$ and $C' = .5$, $h = 25$ feet, $d = 2$ ins. = .166, and $.5 = .5 \div 1$

$$\text{Then } \frac{25}{1 + (.8 + .5 \frac{350}{.166}) \left(\frac{.0416}{.166} \right)^4} = 4.9 \text{ feet.}$$

To Compute Head and Discharge of Water in Pipes of Great Length.

It becomes necessary first to determine the velocity of the flow, which is $\frac{4V}{3.1416 d^2} = v = 1.273 \frac{V}{d^2}$, independent of friction. V representing volume of water in cube feet, and d diameter of pipe in ins.

When head, length, and diameter of pipe are given,
$$\frac{\sqrt{2gh}}{\sqrt{1+C+e\frac{l}{d}}} = v$$

Coefficients of friction C , for velocity of flow, range from .0234 to .0191 for velocities from 3 to 13 feet per second, and e that for the pipe as a mean at .5. See Weisbach's Mechanics, Vol. I., page 431.

ILLUSTRATION.—What head must be given to a pipe 150 feet in length and 5 in diameter, to discharge 25 cube feet of water per minute, and what velocity it attain at that head?

$$C = .024 \text{ and } e = .5$$

Then $1.273 \frac{25 \times 12^2}{60 \times 5^2} = 1.273 \times 2.4 = 3.055$ feet velocity per second, and

$$\left(1 + .5 + .024 \frac{150 \times 12}{5}\right) \frac{3.055^2}{64.33} = 1.5 + 8.64 \times .14 = 1.21 \text{ feet head}$$

Or, $4.72 \frac{\sqrt{d^5}}{\sqrt{l+h}} = V$ in cube feet per minute, and $.538 \sqrt[5]{\frac{lV^2}{h}} = d$ in ins.

ILLUSTRATION.—Assume elements of preceding case.

Then $4.72 \frac{\sqrt{3125}}{\sqrt{150 + 1.42}} = 4.72 \times \frac{55.9}{10.28} = 25.67$ cube feet, and $.538 \sqrt[5]{\frac{150 \times 25^2}{1.42}} = .538 \times 969.607 = .538 \times 9.301 = 5$ ins.

To Compute Fall of a Canal or Open Conduit to Conduct and Discharge a Given Volume of Water per Second.

Coefficient of friction in such case is assumed by Du Buat and others .007565.

$C \frac{l}{A} \times \frac{v^2}{2g} = h$ h representing height of fall, l length of canal, and p perimeter, all in feet; A area of section of canal in sq. feet, and v velocity of flow in feet per second.

ILLUSTRATION 1.—What fall should be given to a canal with a section of 3 feet at bottom, 7 at top, and 3 in depth, and a length of 2600 feet, to conduct 40 cube feet of water per second?

$C = .0076$, $p = 3 + (\sqrt{3^2 + 2^2} \times 2) = 10.21$ feet, $A = \frac{7+3}{2} \times 3 = 15$ sq. feet, and $v = \frac{40}{15} = 2.66$ feet.

Then $.0076 \frac{2600 \times 10.21}{15} \times \frac{2.66^2}{64.33} = 13.45 \times .11 = 1.48$ feet.

—What is volume of water conducted by a canal, with a section of 4 feet at bottom, 12 at top, and 5 in depth, with a fall of 3 feet, and a length of 5800 feet?

$\frac{1}{2} \times 2gh = v$. $A = \frac{12+4}{2} \times 5 = 40$ sq. feet, and $p = 4 + (\sqrt{5^2 + 4^2} \times 2) =$

$$\frac{40}{5800 \times 16.8} \times 64.33 \times 3 = \sqrt{\frac{40}{740.544} \times 193} = 3.23 \text{ feet, and}$$

$$= 129.2 \text{ cube feet.}$$

verse profile of a canal, see Weisbach, page 492.

AGNESIA COVERING FOR STEAM BOILERS, HEATED PIPES, ETC.

Robert A. Keasbey & Co., New York.

s covering is devoid of organic matter, hence it possesses great capacity ist a high temperature, combined with high rank in the order of non-ctors.

s furnished for pipes in the form of hollow cylinders divided longitu-y, and covered with canvas; for boilers, in blocks; and for covering ttings, filling floors, etc., in dry mass in barrels.

ADJUSTABLE-POP SAFETY-VALVES. (Crosby's.)

| ter of Valve. | INCHES. | | | | | | | |
|----------------|---------|------|-----|----|-----|-----|-----|-----|
| | 1 | 1.25 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| ty in HP. | 10 | 20 | 30 | 50 | 80 | 100 | 150 | 200 |
| | | | | | | | | 300 |

STEAM SIPHON. An Independent Lifting Pump.

Capacity for a Discharge Pipe 2 Ins. in Diameter, per Minute.

| er raised. | | Pressura. | Discharge. | Water raised. | | Pressura. | Discharge. |
|------------|------|-----------|------------|---------------|------|-----------|------------|
| Ins. | Lbs. | Gallons. | Feet. | Ins. | Lbs. | Gallons. | |
| 6 | 30 | 63.54 | 13 | 2 | 60 | 119.68 | |
| 2 | 40 | 85.71 | 13 | 2 | 70 | 138.44 | |
| 2 | 50 | 100 | 13 | 2 | 80 | 157.57 | |

DISTANCES, VELOCITIES, AND ACCELERATION.

o Compute Velocities of an Accelerated Body.

+ (2 v' S), Or, $v + \bar{v} = V$. v and v' representing original and accelerated es, and V final velocity, all in feet per second; S distance or space passed over , and t time in seconds. $\frac{v + V}{2} = V'$. V' representing average velocity in r second. $V' t = S$, and $2 V'^2 - V = v$.

STRATION I.—A body moving with a velocity of 10 feet per second, is acceler- t rate of 4 feet per second, per second, for a period of 6 seconds; what are its nt velocities?

10, $v' = 4$, $t = 6$.

1, $10 + 6 \times 4 = 34$ feet final velocity. $\frac{10 + 34}{2} = 22$ feet average velocity.

= 132 feet distance passed over. $\sqrt{10^2 + (2 \times 4 \times 132)} = \sqrt{1156} = 34$ feet,
 $\times 22 - 34 = 10$ feet original velocity.

$\frac{V - v}{t} = v'$, $\frac{V + v}{2} \times t = S$, $\frac{V^2 - v^2}{2t} \times t = v' S$, $v^2 + 2 v' S = V^2$,

= t , and $\sqrt{V^2 - 2 v' S} = v$.

A body is projected vertically with a velocity of 200 feet per second, and is ed at the rate of 30 feet per second, per second; what height will it have through when its velocity is reduced to 80 feet per second, and in what time?

200, $v' = 30$, and $V = 80$.

Then $\frac{200 - 80}{30} = 4$ seconds. $\frac{80 + 200}{2} \times 4 = 560$ feet.

A vehicle being drawn with a velocity of 25 feet per second, is per second, per second; what is its velocity and time of operatio feet?

25, $v' = 5$, and $V = 100$.

Then $\frac{100 - 25}{5} = 15$ seconds. $\frac{100 + 25}{2} \times 15 = 937.5$ feet.

— a stream of water, after flowing a distance of ten feet, is accelerated to a velocity of 10 feet per second, with an accelerating tendency of 1 foot per second per second. What will be the initial velocity and time of flow?

$$v = 10 \text{ ft.} \quad t = 1 \text{ sec.}$$

$$\text{Then } v = 10 \text{ ft.} \quad t = 1 \text{ sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

Delivery and Friction in Hose.

R. F. Hutton, Inc. Sec. 1 E

For delivery and friction. Formulae according to the

$$\text{Delivery of water, and } v = 10 \text{ ft./sec.} \quad t = 1 \text{ sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

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$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

$$\frac{v}{t} = 10 \text{ ft./sec.} \quad \text{and } \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.} \quad \frac{v}{t} = 10 \text{ ft./sec.}$$

For delivery and friction.

Formulae according to the

— a stream of water, after flowing a distance of ten feet, is accelerated to a velocity of 10 feet per second, with an accelerating tendency of 1 foot per second per second. What will be the initial velocity and time of flow?

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— a stream of water, after flowing a distance of ten feet, is accelerated to a velocity of 10 feet per second, with an accelerating tendency of 1 foot per second per second. What will be the initial velocity and time of flow?

Metric Factors. *In addition to pp. 27-37.**By Act of Congress, July, 1866.**By French Metric Computation***Measures.**

Liter per cube meter = .007 48 gallons per cube foot ... | .007 48 gallons.

Weights and Pressures.

| | | |
|--|---|--------------------|
| Centimeter of mercury per sq. inch = .192 91 lb. per | } | .192 91 x 7 lb. |
| sq. inch..... | | |
| Atmosphere (14.7 lbs.) = 6.6679 kilograms..... | | 6.6678 kilograms. |
| Inch of mercury per sq. inch = 2.54 centimeters..... | | 2.54 centimetres. |
| Pound per sq. inch = 453.6029 grams..... | | 453.5926 grammes. |
| Cube foot per ton = .0279 cube meter..... | | .0279 cubic metre. |

Heat.

Caloric per Kilogram = 1.8 heat units per lb..... | 1.8 heat units.

Velocity.

Meter per second = 3.280 833 feet per second..... | 3.280 869 feet.

Power and Work.

| | | |
|--|---|--------------------|
| Kilogrammeter ($k \times m$) = 2.2046 \times 3.280 83..... | } | 7.233 foot-lbs. |
| Foot-pound = .138 26 kilogrammeters..... | | |
| Kilogram per cheval = 2.2352 lbs. per HP..... | | 2.2353 pounds. |
| Sq. foot per HP = .091 63 sq. meter per cheval..... | | .091 63 sq. metre. |

Miscellaneous.

| | | |
|--------------------------------------|--|------------------------------------|
| 1 Avoirs Lb. = .453 6 kilogram. | | 1 Sq. Foot = .092 903 sq. meter. |
| 1 Ton = 1.016 057 tonne. | | 1 Cube Foot = .028 317 cube meter. |
| 1 Sq. Inch = 645.161 29 sq. mill'rs. | | 1 Cube Yard = .764 559 cube meter. |
| 1 Mile per hour | | = 26.8225 meters per minute. |
| 1 Knot " " (6086.44 feet) | | = 30.9192 " " " |
| 1 Cube Meter per minute | | = 7.848 cube yards per hour. |
| 1 " Yard " " | | = 45.8718 " meters " " |

Locomotive Brakes. $\frac{v^2}{64.4 f}$ and $\frac{V^2}{30 f}$ = distance in which a train is
 stopped. v and V representing velocity in feet per second, and miles per hour, and
 proportion of resistance of brakes to weight of train.

Brakes, self-acting, on all wheels, $f = .14$. Ordinary hand, $f = .023$ to $.031$. As-
 cending in .5 resistance is $f + 2$; descending in .5 $f - 2$.

Hydraulic Rams. Efficiency decreases rapidly as height to which water
 to be raised increases above the fall or head.

Number of times the height to which the water is raised exceeds that of the head
 the supply and efficiency per cent. (Waller S. Hutton, C. and M. E.)

| | | | | | | | | | | | | | | | | | |
|---------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Number ... | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 19 | 20 | 25 |
| Efficiency .. | 75 | 72 | 68 | 62 | 57 | 53 | 48 | 43 | 38 | 35 | 32 | 28 | 23 | 17 | 15 | 12 | 0 |

Speed of water in pumps, 200 feet per minute.

To Compute Weight of Water at any Temperature

$$\frac{T + 461.2^{\circ}}{500} + \frac{500}{T + 461.2^{\circ}} = W. \quad W \text{ and } w \text{ represent } \quad \text{per } c$$

foot at temperature T , and at maximum density of 39
 equal absolute temperature.

ILLUSTRATION.—Required weight of a cube foot of wa

$$\frac{62.425 \times 2}{60 + 461.2} + \frac{500}{60 + 461.2} = 62.37$$

Results of Experiments or Performances of Steam-engines and Boilers.

Cylinders, Cut-off, Vacuum, and Diameters in Inches, Revolutions per Minute, Pressure, Water, and Coal in Lbs., and Surfaces and Areas in Sq. In.

| ELEMENTS OF ENGINE. | HARRIS. | | | CORLISS. | | BOILERS. |
|--|-----------------|-------------|-------------|--|-------------|----------|
| | Non-condensing. | Condensing. | Condensing. | Condensing. | Condensing. | |
| Cylinder..... | 18X42 | 18X42 | 24X60* | Number..... | 1 | |
| Revolutions..... | 74.29 | 73.6 | 59.62 | Diameter..... | 60 | |
| Pressure in Pipe..... | 58.5 | 76.37 | 92.88 | Length..... | 12 | |
| Cut off..... | 4.74 | 7.94 | 18.02 | Tubes 50..... | 4 | |
| Mean effective Pressure | 26.93 | 29.47 | 89.38 | Heating Surface..... | 1536.7 | |
| IHP..... | 105.47 | 115.43 | 270.58 | Grate..... | 51.75 | |
| Friction HP..... | 12.64 | 13.07 | 12.55 | Calorimeter..... | 1256.4 | |
| Net HP..... | 92.83 | 102.36 | — | Heating to Grate..... | 29.7 | |
| Water per net HP } per hour..... | 18.59 | 25.39 | — | Grate to Calorimeter.. | 5.5 | |
| Coal per do..... | 2.34 | 3.18 | — | Temperature of Feed.. | 114.5 | |
| Coal per IHP per } hour..... | 2.07 | 2.82 | 1.98 | Steam per Lb. of Combustible.. | 8.5 | |
| Vacuum..... | 21.83 | — | 26.4 | Steam per Lb. of Coal..... | 8.5 | |
| Combustible per } IHP per hour..... | — | 1.83 | 1.83 | Coal per Sq. Foot of Grate per hour | 10.3 | |
| Relative efficiency.... | — | .753 | — | Steam per Temp. 212°C | 9.4 | |

* Weight of engine, 40,000 lbs.

† Steam per lb. of coal 8.21 lbs., and evaporation 9 to 1.

WINDMILLS. (Andrew J. Corcoran, New York.)
(Improved. Patented June and August, 1888; March and June, 1889.)
Volume of Water Pumped per Minute.
From 10 to 200 Feet.

| Diameter of Wheel. | VERTICAL DISTANCE FROM WATER TO POINT OF DELIVERY IN FEET. | | | | | | | | |
|--------------------------|--|----------|----------|----------|----------|----------|----------|----------|--|
| | 10 | 15 | 25 | 50 | 75 | 100 | 150 | 200 | |
| Feet. | Gallons. | Gallons. | Gallons. | Gallons. | Gallons. | Gallons. | Gallons. | Gallons. | |
| 8.5 | 15.242 | 10.162 | 6.162 | 3.016 | | | | | |
| 10 | 48.262 | 32.175 | 19.179 | 9.563 | 6.638 | 4.25 | | | |
| 12 | 86.708 | 57.805 | 33.941 | 17.952 | 11.851 | 8.485 | 5.68 | | |
| 14 | 111.665 | 74.443 | 45.139 | 22.569 | 15.304 | 11.246 | 7.807 | 4.991 | |
| 16 | 155.982 | 103.988 | 64.8 | 31.654 | 19.542 | 16.15 | 9.771 | 8.095 | |
| 18 | 249.93 | 159.954 | 97.682 | 52.165 | 32.513 | 24.421 | 17.485 | 12.211 | |
| 20 | 309.604 | 206.403 | 124.95 | 63.75 | 40.8 | 31.248 | 19.284 | 15.991 | |
| 25 | 532.517 | 355.012 | 212.381 | 106.964 | 71.604 | 49.725 | 37.349 | 26.741 | |
| 30 | 1080.112 | 728.828 | 430.848 | 216.172 | 146.608 | 107.712 | 74.8 | 54.011 | |

Velocity of Wind.

over the United States, as determined by the Signal Service of the
over 60 miles per month, or about 8 miles per hour.
determined that, to operate a windmill, there is required an
wind of six miles per hour.

pressure of wind per sq. foot of surface in lbs.

venting velocity of air in feet per second, and

Compute Head in Lbs. per Sq. Inch to Resist Friction of Air in Long and Rectilineal Pipes, etc.

$$\frac{1728}{60''} = v; \quad \frac{V^2 L}{(3.7 d)^5 83.1} = H; \quad \frac{H (3.7 d)^5 83.1}{V^2} = L; \quad \sqrt{\frac{H (3.7 d)^5 83.1}{L}} = V;$$

$\frac{V^2 L}{3.7 d} = H$, and $\frac{a 60'' v \overline{P+H}}{12'' \times 33000} = \text{HP}$. V representing volume discharged in cube feet per minute, L length of pipe in feet, d diameter of pipe in ins., H head in lbs. per sq. inch, v velocity of discharge in feet per second, P pressure, both in lbs. and per sq. inch, v velocity of discharge in feet per second, a area of discharge in sq. ins., and HP horse-power of friction of air alone.

ILLUSTRATION.—Assume volume of air discharged 44000 cube feet per minute, diameter of discharge pipe 40.5 ins. (say 1280 sq. ins. net), length of pipe 1000, and pressure at discharge 3.5 lbs. per sq. inch.

$$\text{then } \frac{44000 \times 1728}{1280 \times 60} = 990 \text{ feet, and } (3.7 \times 40.5)^5 \times 83.3 = 6310406250000.$$

$$\frac{14000^2 \times 1000}{310406250000} = .3068 \text{ lbs.}; \quad \sqrt{\frac{.3068 \times 6310406250000}{1000}} = 44000 \text{ cube feet};$$

$$\frac{1936000000 \times 1000}{83.1 \times .3068} \div 3.7 = 40.5 \text{ ins., and } \frac{1280 \times 60 \times 990 \times 3.5 + .3068}{12 \times 33000} = 5 \text{ HP.}$$

Volume of Enclosed Air at 0° that may be Heated by One Square Foot of Iron Heating Surface.

| ENCLOSURE. | Heater | | ENCLOSURE. | Heater | |
|--------------------|---------------|-------------|--------------------------|---------------|-------------|
| | in Cellar. | in Room. | | in Cellar. | in Room. |
| Wellings | Cub. ft. | Cub. ft. | Large stores, average... | Cub. ft. | Cub. ft. |
| Offices | 40 | 50 | Hotels | 90 | 110 |
| House stores | 60 | 70 | Churches | 100 | 125 |
| | 70 | 80 | | 150 | 200 |

Commercial HP of Chimney for a Given Diam. of Flue.
Height of Chimney in Feet.

| 1. of fl. | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 125 | 135 | 150 | 175 | 200 | 225 | 250 | 300 |
|--------------|----|----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 18. | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP | HP |
| 5 | 16 | 18 | 19 | — | — | — | — | — | — | — | — | — | — | — | — |
| 10 | 84 | 92 | 100 | 107 | 113 | 119 | 124 | — | — | — | — | — | — | — | — |
| 5 | — | — | 250 | 270 | 288 | 305 | 321 | 345 | 353 | 385 | — | — | — | — | — |
| 5 | — | — | — | 330 | 370 | 405 | 438 | 465 | 490 | 530 | 560 | — | — | — | — |
| 5 | — | — | — | — | 860 | 900 | 935 | 1000 | 1036 | 1090 | 1185 | 1270 | 1345 | 1415 | 1480 |
| 10 | — | — | — | — | — | 1350 | 1410 | 1500 | 1556 | 1640 | 1770 | 1890 | 2000 | 2100 | 2190 |
| 10 | — | — | — | — | — | 1665 | 1725 | 1820 | 1880 | 1970 | 2115 | 2255 | 2390 | 2520 | 2645 |

For Intermediate Diameters and Powers, take proportionate Diameters and Powers.
For Square Chimney deduct one-ninth to one-twelfth of Diameter of Round, for Side

Friction of Water in Pipes. (Weisbach.)

$$\frac{865 l v^2}{d} = C = h. \quad l \text{ representing length of pipes in feet, } v = \frac{183.34}{d^2} V, \text{ or velocity}$$

feet per second, V volume of water in cube feet elevated per second, d diameter in ins., and C a coefficient, ranging from .069 when velocity = .1 foot, .037 for .1 foot, .0265 for 2 feet, .023 for 4 feet, .0214 for 6 feet, .0205 for 12 feet, and .0182 for 20 feet.

ILLUSTRATION.—Assume volume 125 cube feet, raised 25 feet per hour, 12 ins. in diameter and 500 feet in length; how many feet of water friction in the pipe be equal to?

$$\text{Then } \frac{183.34 \times 125}{3600 \times 2} = 3.18 \text{ velocity, and } C = .028.$$

$$\text{now, } \frac{.865 \times 500 \times 3.18^2}{2} \times .028 = 14.6 \text{ feet, and } 25 \div 14.6 = 3$$

Water Tube Boiler.

Proportions of Grate and Heating Surfaces of Water Tube Boilers, as Determined by an Extended Series of Experiments and Observations, with Different Fuels and at Different Localities. (1876 to 1884.)

(Committee of U. S. Centennial Exhibition and Individuals.)

Babcock and Wilcox Co.

Water evaporated from and at 212°.

| Duration of Test. | Surfaces. | | | Combustible consumed. | | Coal per Grate per Hour. | Evaporation | | Ash per cent. |
|-------------------|-----------|-----------|--------|-----------------------|----------------------|--------------------------|-----------------|----------|---------------|
| | Grate. | Heating. | Ratio. | Per Grate. | Per Heating Surface. | | by Combustible. | by Coal. | |
| Hours. | Sq. Ft. | Sq. Feet. | | Sq. Feet. | Sq. Feet. | Sq. Feet. | Lbs. | Lbs. | |
| 8 | 44.5 | 1676 | 37.7 | 8.88 | .256 | — | 12.131 | — | — |
| 120 | 50.7 | 1980 | 39.1 | 11.21 | .26 | 12.99 | 11.62 | 9.71 | 13.7 |
| 216 | 54.7 | 2148 | 39.1 | 12.22 | .292 | — | 11.982 | — | — |
| 24 | 61.9 | 2760 | 44.6 | 8.22 | .198 | — | 11.626 | 10.09 | 13.2 |
| 22 | 59.5 | 2757 | 46.3 | 14.25 | .307 | 9.93 | 11.43 | 9.96 | 12.9 |
| 13.5 | 39.7 | 1680 | 42.3 | 5.8 | .137 | 6.26 | 12.495 | 11.53 | 7.5 |
| 4 | 25 | 1403 | 56.1 | 12.41 | .276 | 13.44 | 12.38 | 11.52 | 7 |
| 10.25 | 70 | 3126 | 44.7 | 18.15 | .406 | 20 | 12.42 | 11.32 | 8.8 |

Coals: * Anthracite, American. † Bituminous, Welsh. ‡ Bituminous, Scotch. § Bituminous, Powelton. ¶ Test in London.

A Galloway boiler of standard efficiency, at this exposition, having a ratio of heating surface to grate of 25 to 1, and feed water at a temperature of 56°, gave the following results:

Consumption of coal, 8.87 lbs. per hour per sq. foot of grate. Pressure of steam, 70 lbs. per sq. inch. Water evaporated per hour per sq. foot of heating surface, 3 lbs.; water evaporated per hour per sq. foot of heating surface in lbs. per lb. of combustible, 9.68 lbs., and per lb. of coal, 8.63 lbs.

Average Results of Thirty Tests at Different Locations and with Different Experts.

Coal consumed per hour per grate surface, 15.028 lbs. Water evaporated from and at 212° per hour per sq. foot of heating surface, 3.71 lbs.; water evaporated per hour per sq. foot of heating surface per lb. of combustible from and at a like temperature, 11.442 lbs.

Elements of a Comparative Test with a Return Fire Tubular Boiler. (W. Barnet Le Van and J. C. Hoadley.)

Water evaporated from and at 212° per lb. of combustible.—Water tubular, 11.255 lbs. Fire tubular, 10.571 lbs. Comparative economy by evaporation test, 6.47 per cent.

Combustible consumed per HP per hour.—Water tubular, 4.321 lbs. Fire tubular, 4.648 lbs. Comparative economy by engine operation, 7.57 per cent.

Heat in chimney.—Water tubular, 20.54 per cent. Fire tubular, 25.47 per cent. Comparative economy, 7 per cent.

Wing Arc Lights.—Combustible consumed: Water tubular, 4.6567 lbs. Fire tubular, 5.038 lbs. Comparative economy, 6.81 per cent.

1 result was 7 per cent. in favor of the Water Tubular.

—Average evaporation from and at 212°: Combustible, 11.292 lbs. From 100° and at 212°: Combustible, 10.96 lbs.; Coal, 10.67 lbs. of grate, 15.396 lbs. per hour.

ned by 20 tests and different coals, in excess of nominal range moisture in steam 1.045 per cent.

coal with 12 per cent. of ash, 11.121 lbs. water evaporated 9.252 of coal.

demanded in some manufacturing operations, most economical operation of triple surfaces of these boilers are made

APPENDIX.

To Compute Area of Cylinder of a Steam-engine Grate and Heating Surfaces of a Boiler.

When Required Power is Given.—It is assumed that HP of a steam-engine obtained by evaporation of 33.6 lbs. water per hour, at a temperature of 212° from water at 100°.

NOTE.—This is a deduction from the elements of the estimate as given by the Am. Soc. of Engineers, in order to put temperature of the feed at 100° instead of 212°.

Non-condensing (Single Cylinder).
$$\frac{V \times 33.6 \times \text{HP}}{60 \times 2 \times R \times 2 \times S \times 12} \times 1728 = \text{area of cyl in sq. ins.}$$
 V representing volume of 1 lb. water at terminal pressure of steam in feet, *P* pressure of steam at termination of stroke of piston in lbs., *R* number of revolutions per minute, and *S* stroke of piston in feet.

ILLUSTRATION.—Required HP of an engine is 300, initial pressure of steam is exceed 70 lbs. mercurial gauge, cut off at 5 stroke of piston of 4 feet, and number of revolutions 60 per minute. What should be areas of cylinder of engine and heating surfaces of boiler?

Steam at 70 lbs. pressure, clearance in cylinder and steam passages = 1.8 ft 15 foot, point of cutting off = $4 \div .5 = 2$ feet.

Then (formula p. 711), $70 \times (2 + 15 \div 4 + .15) = 36.26$ lbs. terminal pressure steam at this pressure has a density or volume, which is its reciprocal (for p. 706) of 11.26 cube feet for each lb. of water contained.

Hence, $\frac{11.26 \times 33.6 \times 300}{60 \times 2 \times 60 \times 2 \times 4 \times 12} \times 1728 = 283.75$ sq. ins., to which is to be added

for friction of engine and of load 10 per cent. = 312 sq. ins.

Inversely.—By ordinary formula area = 336 sq. ins.

Grate Surface.—Evaporation of fresh water in an efficient marine boiler, at temperature of feed of 100°, is assumed, with a proportion of heating surface of 30 to 1, to be, with a combustion of 20 lbs. coal per sq. foot of grate per hour, 213 lbs. per sq. foot of grate, and 10.3 lbs. per lb. of coal.

Hence, $\frac{E \text{ HP}}{L} = \text{area.}$ *L* representing evaporation per sq. foot of grate per hour.

ILLUSTRATION.—Assume elements of preceding, with evaporation as above.

$$\frac{33.6 \times 300}{213} = 47.32 \text{ sq. feet.}$$

Heating Surface.—Then $47.32 \times 30 = 1419$ sq. feet area.

For the several types of boilers the following units should be used:

| T Y P E. | Ratio of Heating Surface to Grate 30 to 1 50 to 1 | | | | |
|-----------------|---|-----|-----|-----|-----|
| | Coal consumed per Sq. Foot of Grate per hr in Lbs. | | | | |
| | 15 | 20 | 30 | 15 | 20 |
| Marine..... | 164 | 214 | 314 | 183 | 242 |
| Stationary..... | 159 | 207 | 299 | 182 | 241 |
| Portable..... | 132 | 174 | 257 | 145 | 187 |
| Locomotive..... | 150 | 197 | 290 | 164 | 211 |
| “ Coke..... | 131 | 170 | 247 | 159 | 197 |

Units of Heat in Fuels.

| | | |
|-----------------------|--------|-------------------------|
| Anthracite..... | 14 500 | Petroleum, refined..... |
| Bituminous..... | 14 200 | “ crude..... |
| Petroleum, light..... | 22 600 | Coal Gas..... |
| “ heavy..... | 19 440 | Water Gas..... |

To Resist Oxidation in Cast-iron!

A coating of hot lime, which is much preferable to tar.

To Compute Relative Velocities of Steam Yachts, from Elements of their Construction, Capacity, and Operation.

RULE.—Multiply area of their grate surfaces by *Constant* due to the character of the combustion of their furnaces, divide product by cube root of square of their gross tonnage (U. S.), and cube root of quotient will give their relative velocities.

Or, $\sqrt[3]{\frac{G \cdot C}{T^2}} = V$. *G* representing area of grate surface in sq. feet, *T* gross tonnage,

and *C* a constant, viz. natural draught 1. Jet or exhaust 1.25, and blast 1.6.

In the application of this rule, as alike to all others when there is material difference in the elements, as with large and small vessels, those that approach each other in general dimensions or capacities, as determined by certain ranges or limits of tonnage, should be classed together.

ILLUSTRATION.—The grate surface of a yacht is 27.5 sq. feet, her tonnage 71.24, and the combustion in her furnaces, jet.

$$\text{Hence, } \sqrt[3]{\frac{27.5 \times 1.25}{71.24^2}} = \sqrt[3]{\frac{34.75}{17.185}} = 1.26.$$

This result is an index of the capacity of the vessel, when compared with another in like manner.

Thus, assume one to be a fair exponent of her class, as from 40 to 60, 60 to 80, or 80 to 100, etc., tons, and her speed to be 12 knots per hour, or 60 minutes.

If then a competitor possessed the elements that by the above formula would give a result of 1.3, their relative capacities over a like course would be as 1.26 : 1.3 :: 60 : 61.9, and 61.9 - 60 = 1.9 minute = 1 minute 54 seconds, which is the time the yacht of greatest capacity would have to allow the other.

If the course was for a greater distance, as for 80 knots, then $\frac{80}{12} \times 1.9 = 12 \frac{2}{3}$ 4 sec. the allowance.

For Large Steamers.

$$3.95 \sqrt[3]{\frac{HP}{S^2}} = V. \quad S \text{ representing area of immersed amidship section in sq. meters.}$$

NOTE.—A sq. meter is 10.764 sq. feet.

This formula is used in Europe, and is applicable only for vessels of great capacity and with a blast combustion.

Simple Water Tests.

For Hard or Soft Water.—Dissolve a small quantity of soap in alcohol. Put a drop of it in a vessel of water. If it becomes milky, it is hard, if not, it is soft.

For Earthy Matters or Alkali.—Dip litmus paper in vinegar, and if on immersion the paper returns to its true shade, the water is free from earthy matter. Syrup added to a water containing earthy matter will turn it green.

For Carbonic Acid.—Take equal parts of water and clear lime-water. If carbonic acid is present, a precipitate is produced, to which, if a few drops of acid be added, an effervescence occurs.

Put the water to a twentieth part of its weight, drop a few grains of ammonia and a few drops of phosphate of soda into it, and it will precipitate to the bottom.

Put a little nutgall and mix it with the water; if it turns gray or blue. —(2.) Dissolve a little prussiate of potash, and mix it with the water, it will turn blue.

Put water put two drops of oxalic acid and blow upon it with the mouth.

Put litmus paper in it. If it turns red, it is acid. If it turns blue, it is carbonic acid. If a blue paper is turned

TOBIN BRONZE. (Hot rolled.)

sonia Brass and Copper Co., New York, N. Y.,
Sole Manufacturer.

Specific gravity 8.379. Weight of a cube inch .3021 of a lb. Tensile strength 1-inch round rod 79 600 lbs. per sq. inch. Elastic limit 54 257 lbs. per sq. inch.

Reduction in a rod 1 inch in diameter and 8 ins. in length 15.4 per cent. Reduction 37.26 per cent.—*Fairbanks.*

Easily forged into bolts and nuts at a dark-red heat, Torsional strength elastic limit equal to machinery steel.

Torsional Strength.

1.5 inch in diameter and 1 inch in length, load at end of lever 1 foot. Torsion 2.67°. Elastic limit 328 lbs. Rupture 633 lbs. Torsion point temperature 92.2°.—*J. E. Denton.*

Tensile Strength, maximum, 181 000 lbs. per sq. inch.

Plates.*Weight per Square Foot.*

| Size. | Weight. | Thickness. | Weight. | Thickness. | Weight. | Thickness. | Weight. |
|-------|---------|------------|---------|------------|---------|------------|---------|
| | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| 5 | 2.72 | .4375 | 19.03 | .8125 | 35.35 | 1.1875 | 51.66 |
| | 5.44 | .5 | 21.75 | .875 | 38.06 | 1.25 | 54.38 |
| 5 | 8.16 | .5625 | 24.47 | .9375 | 40.78 | 1.3125 | 57.1 |
| | 10.88 | .625 | 27.19 | 1 | 43.5 | 1.375 | 59.82 |
| 5 | 13.59 | .6875 | 29.91 | 1.0625 | 46.22 | 1.4375 | 62.53 |
| | 16.31 | .75 | 32.63 | 1.125 | 48.94 | 1.5 | 65.25 |

Bolts and Rods.*Weight per Lineal Foot.*

| Size. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diameter. | Weight. | Diam. | Weight. |
|-------|---------|-----------|---------|-----------|---------|-----------|---------|-------|---------|
| | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| | .177 | .75 | 1.6 | 1.5 | 6.42 | 2.5 | 17.8 | 4 | 45.57 |
| | .279 | .8125 | 1.88 | 1.625 | 7.5 | 2.625 | 19.6 | 4.25 | 51.44 |
| | .399 | .875 | 2.18 | 1.75 | 8.7 | 2.75 | 21.53 | 4.5 | 57.64 |
| | .544 | .9375 | 2.5 | 1.875 | 10 | 2.875 | 23.52 | 4.75 | 64.24 |
| | .711 | 1 | 2.84 | 2 | 11.38 | 3 | 25.53 | 5 | 71.16 |
| | .899 | 1.125 | 3.6 | 2.125 | 12.87 | 3.25 | 30.05 | 5.25 | 78.46 |
| | 1.11 | 1.25 | 4.46 | 2.25 | 14.43 | 3.5 | 34.86 | 5.5 | 86.11 |
| | 1.34 | 1.375 | 5.36 | 2.375 | 16.06 | 3.75 | 40.01 | 6 | 102.45 |

Is supplied in the form of Round, Square, and Hexagon bars; Plate, Wire, and Finished Piston Rods.

Due to its great strength and non-corrosive properties it is extensively used in bolts, etc., for Marine and Naval machinery, Sugar-houses, Presses, Pump rods and Pump linings, Condenser tube sheets, Yacht shafting, etc.

Is used for Forgings of every description.

Weights of Steam-engines and Boilers with*Per Indicated HP in Lbs.*

| | | | |
|---------------------|-----|---------------------|--|
| Marine Steamer..... | 480 | Torpedo Boats..... | |
| Naval "..... | 360 | Marine Boilers..... | |
| Locomotive..... | 280 | Locomotive "..... | |

Weight and Strength of Ordinary Stud-Link Chain Cable.

| Dimensions of Link. | | | Weight per Fathom. | Admiralty Proof- stress.* | Dimensions of Link. | | | Weight per Fathom. | Admiralty Proof- stress.* |
|---------------------|---------|--------|--------------------------|---------------------------------|---------------------|---------|--------|--------------------------|---------------------------------|
| Diam. | Length. | Width. | | | Diam. | Length. | Width. | | |
| Ina. | Ina. | Ina. | Lbs. | Lbs. | Ina. | Ina. | Ina. | Lbs. | Lbs. |
| .375 | 2.25 | 1.35 | 7.55 | — | 1.375 | 8.25 | 4.95 | 107.6 | 516 |
| .4375 | 2.625 | 1.575 | 11.3 | 7 840 | 1.5 | 9 | 5.4 | 121 | 579 |
| .5 | 3 | 1.8 | 13.4 | 10 080 | 1.625 | 9.75 | 5.85 | 142 | 640 |
| .5625 | 3.375 | 2.025 | 17.2 | 12 320 | 1.75 | 10.5 | 6.3 | 164.6 | 747 |
| .625 | 3.75 | 2.25 | 21 | 15 680 | 1.875 | 11.25 | 6.75 | 189 | 840 |
| .6875 | 4.125 | 2.475 | 25.4 | 19 040 | 2 | 12 | 7.2 | 215 | 960 |
| .75 | 4.5 | 2.7 | 30.2 | 22 680 | 2.125 | 12.75 | 7.65 | 242.8 | 1080 |
| .875 | 5.25 | 3.15 | 41.2 | 30 800 | 2.25 | 13.5 | 8.1 | 276.2 | 1224 |
| 1 | 6 | 3.6 | 53.8 | 40 320 | 2.375 | 14.25 | 8.55 | 302.2 | 1377 |
| 1.125 | 6.75 | 4.05 | 69 | 50 960 | 2.5 | 15 | 9 | 330 | 1500 |
| 25 | 7.5 | 4.5 | 84 | 63 000 | 2.75 | 16.5 | 9.9 | 405.6 | 1800 |

* Adopted by Lloyd's.

NOTE 1.—*Safe Working-stress* is taken at half the *Proof-stress*.2.—*Proof-stress* and *Safe Working-stress* for close-link chains are respectively two-thirds of those of stud-link chains.3.—*Average Proof-stress* is 72 per cent. of ultimate strength, or 17 000 lbs. per sq. inch of section of both sides. *Safe working-stress* is half the *proof-stress*, or 8 500 lbs. per sq. inch of section.

Weight of close-link chain is about three times the weight of the bar from which it is made, for equal lengths.

4.—*Ultimate Strength* per sq. inch of section of metal is 35 000 lbs.

Comparing the weight, cost, and strength of the three materials, hemp, iron wire, and chain iron, the proportion between the cost of hemp rope, wire rope, and chain is as 2 : 1 : 3; and, therefore, for equal resistances, wire rope is only half the cost of hemp rope, and a third of the cost of chains. (Karl von Ott.)

Height and Retrocession of Niagara Falls. (J. Hall)

| | | |
|-------------------------------|-----------|----------------|
| 1842. Height.—American..... | 167 feet. | } Jas. Hall |
| Horseshoe..... | 158 " | |
| Width.—American..... | 600 " | |
| Horseshoe..... | 1800 " | } Lake Survey. |
| 1875. Recessed.—American..... | 60 " | |
| Horseshoe..... | 160 " | |

1886.—Average retrocession 2.5 feet per annum (Woodward).

200 feet in 11 years, and 9 feet per year in 42 years.

Descent of the river below 15 feet per mile.

Bridge.—Over Orus on Caspian sea, 6230 feet in length

LENGTHS OF ENGLISH RACE-COURSES.

| Course. | Miles. | Course. | Miles. | Course. | Miles. |
|------------------------|--------|---------------------|------------|--------------------|--------|
| NEWMARKET. | | | DONCASTER. | | |
| Across the Flat..... | 1.292 | Circular..... | 1.545 | GOODWOOD. | |
| Beacon..... | 4.206 | Fitzwilliam..... | 1 | Cup Course..... | |
| Cambridgeshire..... | 1.136 | Red House..... | .711 | LIVERPOOL. | |
| Cesarewitch..... | 2.266 | St. Leger..... | 1.825 | New Course..... | |
| Round..... | 3.579 | Cup Course..... | 2.634 | New Castle..... | |
| Rowley Mile..... | 1.009 | SPRUE. | | | |
| Summer Course..... | 2 | Craven..... | 1.25 | THUR. | |
| Two-year old, new..... | .702 | Derby and Oaks..... | 2.5 | Stakes Course..... | |
| Yearling..... | .277 | Metropolitan..... | 2.25 | Two-mile..... | |

Railway Speed in England.

1884.—North Western Railway. To Crewe, 128.5 miles in 178 minutes with a stop.

Calcuttium. Carlisle to Edinburgh, 200.75 miles, including 100 miles of elevation of 1 in 50, in 104 minutes.

Dimensions of Wrought-iron Floor-beams and their Distances from Centres. The New Jersey Steel and Iron Co., Trenton, N. J.

Loads per Sq. Foot from 100 Lbs. to 300 Lbs.

| SPAN. | 100 Lbs. | | | 150 Lbs. | | | 200 Lbs. | | | 250 Lbs. | | | 300 Lbs. | | |
|-------|----------|----------------------------|-----------|----------|----------------------------|-----------|----------|----------------------------|-----------|----------|----------------------------|-----------|----------|----------------------------|-----------|
| | Feet. | Depth and Weight per Yard. | Distance. | Feet. | Depth and Weight per Yard. | Distance. | Feet. | Depth and Weight per Yard. | Distance. | Feet. | Depth and Weight per Yard. | Distance. | Feet. | Depth and Weight per Yard. | Distance. |
| 8 | { | 4 30 | 4.0 | 3.1 | 5 30 | 3 | 3 | 6 40 | 3.9 | 6 40 | 3.2 | 3.2 | 6 40 | 3.2 | 3.2 |
| 10 | { | 5 30 | 5.9 | 4 | 6 40 | 4.1 | 4 | 7 55 | 4.7 | 7 55 | 3.9 | 3.9 | 8 65 | 3.9 | 3.9 |
| 12 | { | 5 40 | 3.8 | 5 | 6 50 | 4.1 | 5 | 7 55 | 3 | 7 55 | 3.3 | 3.3 | 8 65 | 3.3 | 3.3 |
| 14 | { | 6 40 | 4.2 | 6 | 7 55 | 3.4 | 6 | 8 65 | 3.7 | 8 65 | 3.6 | 3.6 | 9 70 | 3.6 | 3.6 |
| 16 | { | 6 50 | 5.2 | 7 | 8 65 | 4.6 | 7 | 9 70 | 4.5 | 9 70 | 4.5 | 4.5 | 10 85 | 4.5 | 4.5 |
| 18 | { | 7 55 | 5 | 8 | 9 70 | 3.3 | 8 | 10 5 | 3.3 | 10 5 | 3.3 | 3.3 | 11 105 | 3.3 | 3.3 |
| 20 | { | 8 65 | 6.7 | 9 | 10 5 | 4.5 | 9 | 11 105 | 4.1 | 11 105 | 3.8 | 3.8 | 12 125 | 3.8 | 3.8 |
| 22 | { | 9 70 | 6.3 | 10 | 11 105 | 3.3 | 10 | 12 125 | 3.7 | 12 125 | 3.6 | 3.6 | 13 150 | 3.6 | 3.6 |
| 24 | { | 10 85 | 4.9 | 11 | 12 125 | 3.9 | 11 | 13 150 | 4.2 | 13 150 | 4.3 | 4.3 | 14 170 | 4.3 | 4.3 |
| 26 | { | 10 90 | 5.9 | 12 | 13 150 | 4.9 | 12 | 14 170 | 4.6 | 14 170 | 4.5 | 4.5 | 15 200 | 4.5 | 4.5 |
| 28 | { | 10 90 | 6 | 13 | 14 170 | 6 | 13 | 15 200 | 5.2 | 15 200 | 5.2 | 5.2 | 16 220 | 5.2 | 5.2 |
| 30 | { | 10 90 | — | 14 | 15 200 | — | 14 | 16 220 | 5.1 | 16 220 | 5.1 | 5.1 | 17 240 | 5.1 | 5.1 |
| | | 10 90 | 4.9 | 15 | 16 220 | 4.9 | 15 | 17 240 | 4.9 | 17 240 | 4.9 | 4.9 | 18 260 | 4.9 | 4.9 |
| | | 10 90 | 5.6 | 16 | 17 240 | 5.6 | 16 | 18 260 | 5.6 | 18 260 | 5.6 | 5.6 | 19 280 | 5.6 | 5.6 |
| | | 12 125 | 5 | 17 | 18 260 | 5 | 17 | 19 280 | 5.1 | 19 280 | 5.1 | 5.1 | 20 300 | 5.1 | 5.1 |
| | | 12 125 | 6.1 | 18 | 19 280 | 6.1 | 18 | 20 300 | 5.1 | 20 300 | 5.1 | 5.1 | 21 320 | 5.1 | 5.1 |
| | | 12 125 | — | 19 | 20 300 | — | 19 | 21 320 | 5.1 | 21 320 | 5.1 | 5.1 | 22 340 | 5.1 | 5.1 |
| | | 12 125 | 5.1 | 20 | 21 320 | 5.1 | 20 | 22 340 | 5.1 | 22 340 | 5.1 | 5.1 | 23 360 | 5.1 | 5.1 |
| | | 15 150 | 5.5 | 21 | 22 340 | 5.5 | 21 | 23 360 | 5.5 | 23 360 | 5.5 | 5.5 | 24 380 | 5.5 | 5.5 |
| | | 15 150 | — | 22 | 23 360 | — | 22 | 24 380 | 5.5 | 24 380 | 5.5 | 5.5 | 25 400 | 5.5 | 5.5 |
| | | 15 150 | 5.6 | 23 | 24 380 | 5.6 | 23 | 25 400 | 5.6 | 25 400 | 5.6 | 5.6 | 26 420 | 5.6 | 5.6 |
| | | 15 150 | — | 24 | 25 400 | — | 24 | 26 420 | 5.6 | 26 420 | 5.6 | 5.6 | 27 440 | 5.6 | 5.6 |
| | | 15 150 | — | 25 | 26 420 | — | 25 | 27 440 | 5.6 | 27 440 | 5.6 | 5.6 | 28 460 | 5.6 | 5.6 |
| | | 15 150 | — | 26 | 27 440 | — | 26 | 28 460 | 5.6 | 28 460 | 5.6 | 5.6 | 29 480 | 5.6 | 5.6 |
| | | 15 150 | — | 27 | 28 460 | — | 27 | 29 480 | 5.6 | 29 480 | 5.6 | 5.6 | 30 500 | 5.6 | 5.6 |
| | | 15 150 | — | 28 | 29 480 | — | 28 | 30 500 | 5.6 | 30 500 | 5.6 | 5.6 | 31 520 | 5.6 | 5.6 |
| | | 15 150 | — | 29 | 30 500 | — | 29 | 31 520 | 5.6 | 31 520 | 5.6 | 5.6 | 32 540 | 5.6 | 5.6 |
| | | 15 150 | — | 30 | 31 520 | — | 30 | 32 540 | 5.6 | 32 540 | 5.6 | 5.6 | 33 560 | 5.6 | 5.6 |

APPENDIX.

Niagara River Power.

al through which the water is to be drawn commences at a dist above the Falls. The water-storage of the river is computed at les, viz., 87 620 of lake and 241 235 of shed. The annual rainfall b

ng the rainfall to be but 30 inches, the flow over the Falls would be per second. The Lake survey computes it at 265 000 cube feet.

' designed to be used by the company constructing the canal is 1200 and corrected determination of levels gives Lake Ontario 246 and Lal above mean tide at city of New York.

Height of Towers, Spires, etc.

(Additional to page 180.)

| | |
|------------------------------------|-------------------------------------|
| Tower, Paris 984.3 feet. | Cathedral, Strasburg. 46 |
| dral, Rouen 492 " | City Hall, Philadelphia. 53 |

11th and Meridian Distance and Altitude of
at New York. O. H. (Lat. 40° 42' 44".)

| | |
|--|-------------------------------------|
| 8 21st, Zenith distance. . . 17° 15' 44" | Dec. 21st, Zenith distance. . . 64° |
| Meridian altitude. 72° 44' 16" | Meridian altitude. 25° 5 |

Water-pump.—First in use 283 years B.C. *Rotating* introduced in ntury. *Plunger pistons* invented by Morland (England), 1674. *Double acti* la Hire (France).

Symbolic Hatching and Designations.

As adopted by Engineer Department, U. S. Navy.



CAST IRON.



WROUGHT IRON.



CAST STEEL.



WROUGHT STEEL.



BR.



COPPER.



LEAD.



BRICK.



GLASS.



STONE.



EARTH.



WOOD.



LEATHER.



following are designed and added by th



NICKEL.



GUM.



ZINC.

Results of Experiments on Operation of Steam-Engine. (C. E. Emery, M. E.)

| Condensing. | Non-Condensing. | Condensing. | Non-Condensing. |
|---------------------------------|-----------------|-------------------|-----------------|
| 1er..... 16 X 42 ins. | 18 X 42 ins. | Cut off.... .189 |189 |
| 2d..... 8.69 lbs. | 73.37 lbs. | HP.....78.79 | 115.43 |
| mean effective, 31.06 " | 29.47 | Friction HP 10.09 | 13.07 |
| utions per min... 60.3 | 73.6 | Net HP...68.7 | 102.36 |
| ter per net HP per hour in lbs. | |29.23 | 25.39 |
| l..... | |3.25 | 2.82 |
| per HP per hour in lbs. | |3.97 | 3.44 |
| ative efficiency per steam..... | |988 | I |
| " " coal..... | |805 | I |

Safety Valves of Steam-Boilers.

lers operated at a low pressure of steam require proportionately larger safety valves than when operated at a high pressure. Thus: If steam at 20 lbs. pressure is raised to 30 lbs., a valve nearly one half more capacity is required; raised from 100 lbs. to 110 lbs., a valve of nearly one tenth more capacity is required.

Belting. Double belts will transmit one and one half times the power of

le belts are less effective per unit of area than narrow. Long belts are more ve than short. Driving belts may be driven at a velocity of 3500 feet per min-
Lathe belts from 1500 to 2000 feet. Economy of wear requires less velocities.

Non-Conductors of Temperature.

Their Comparative Efficiency.

ials in Italics are wholly free from Carbonization or Ignition from slow contact with Boilers or Steam-pipes.

following Materials were used as Covering to a
Steam-pipe 2 Ins. in Diameter.

is of Water Heated 10° per Hour through One Square Foot of the Material.
(J. M. Ordway.)

| Cal x Inch in Thickness. | Lbs. | Relative Solidity. | Material x Inch in Thickness. | Lbs. | Relative Solidity. |
|---|------|--------------------|--|------|--------------------|
| ool, loose..... | 8.1 | .56 | 13. Anthracite coal pow- der..... | 35.7 | 5.06 |
| athers of live geese.. | 9.6 | .5 | 14. Magnesia, calcined and compressed.. | 42.6 | 2.85 |
| mp black, loose..... | 9.8 | .56 | 15. Air alone..... | 48 | — |
| lf of hair..... | 10.3 | 1.85 | 16. Asbestos, fine..... | 49 | .8x |
| ton wool, carded..... | 10.4 | .2 | 17. Sand..... | 62.1 | 5.27 |
| mp black, compressed | 10.6 | 3.44 | 18. Stag wool ¹ threads of | | — |
| arcoal of cork..... | 11.9 | .53 | 19. Paper.. | | |
| agnesia, calcined and loose..... | 12.4 | .23 | 20. Rice chi | | |
| agnesia, carbonate d, and light..... | 13.7 | .6 | 21. Bit. coal | | |
| arcoal of white pine. | 13.9 | 1.19 | 22. Asbesta | | |
| agnesia, carbonate f, and compressed | 15.4 | 1.5 | 23. Anth. co | | |
| water of Paris..... | 30.9 | 3.68 | 24. Clay and | | |

ow of Several Rivers. Minimum

In Cube Feet per Minute.

| | | | |
|-------------------------------------|-------------|----------------|--|
| France, at Brockville, Ont., | 18 000 000. | Seine, at Park | |
| Mississippi, at St. Pauls, Minn.... | 2 000 000. | Mohawk, at Gt | |
| Colonel, at Holyoke, Mass.... | 300 000. | Thames, at La | |
| at Pittsburgh, Penn. | 100 000. | Chicago, at C. | |
| Illinois, at La Salle, Ill. | 35 000. | | |

Standard U. S. Weights and Measures.

(U. S. Coast Survey.)

Lineal.

| Inch to Millimetres. | Foot to Metre. | Yard to Metre. | Mile to Kilometre. | Metre to Inches. | Metre to Feet. | Metre to Yards. | Kilometre to Miles. |
|-------------------------|-------------------|-------------------|-----------------------|---------------------|-------------------|--------------------|------------------------|
| 25.4 | .304 801 | .9144 | 1.609 35 | 39.37 | 3.280 83 | 1.093 611 | .621 37 |

Chain = 20.1169 metres. Fathom = 1.829 metres. Sq. mile = 259 hectares.

Knot = 1853.27 metres.

Square.

| Inches to Centimetre. | Feet to Decimetre. | Yard to Metre. | Acre to Hectare. | Centimetre to Inch. | Metre to Feet. | Metre to Yards. | Hectare to Acre. |
|--------------------------|-----------------------|-------------------|---------------------|------------------------|-------------------|--------------------|---------------------|
| 6.452 | 9.29 | .836 | .4047 | .155 | 10.764 | 1.196 | 2.471 |

Volume. (Fluid.)

| Dram to Milli- litres.* | Ounce to Milli- litres. | Quart to Litre. | Gallon to Litres. | Milli- litre † to Dram. | Millilitre to Ounce | Litres to Quarts. | Deca- litre to Gallons. | Hecto- litre to Bushels. |
|-------------------------------|-------------------------------|--------------------|----------------------|-------------------------------|------------------------|----------------------|-------------------------------|--------------------------------|
| 3.7 | 29.57 | .946 36 | 3.785 44 | .27 | .338 | 1.0567 | 2.6417 | 2.8375 |

Cube.

| Inch to Centimetres. | Foot to Metre. | Yard to Metre. | Bushel to Hectolitre. | Centimetre to Inch. | Decimetre to Inches. | Metre to Feet. | Metre to Yards. |
|-------------------------|-------------------|-------------------|--------------------------|------------------------|-------------------------|-------------------|--------------------|
| 16.387 | .028 32 | .765 | .352 42 | .061 | 61.023 | 35.314 | 1.36 |

Weight.

| Grain to Milligrams. | Av. Ounce to Grains. | Av. Pound to Kilogram. | Tr. Ounce to Grains. | Milligram to Grain. | Kilogram to Grains. | Hectogram to Av. Ounces. | Kilogram to Pounds. |
|-------------------------|-------------------------|---------------------------|-------------------------|------------------------|------------------------|-----------------------------|------------------------|
| 64.7989 | 28.3495 | .453 59 | 31.103 48 | .015 43 | 15 432.36 | 3.5274 | 2.204 62 |

Quintal to Av. Pounds, 220.46. Tonnes ‡ to Av. Pounds, 2204.6. Grams to Oz.
Ounce, .032 15. Av. Pound = 453.592 427 7 grams. Kilogram = 15 432.356 39 grains.

NOTE.—The U. S. yard is equal to the British yard. British gallon = 4.546 09
litres. Bushel = 36.3477 litres.

Value of the Metre in terms of the British Imperial Yard, and of the
Committee Metre (C.M.) of the U. S. Coast and Geodetic Survey. (O. H. Tittmann)

| Authority. | Value. |
|--------------|------------|
| Hassler..... | 39.380 917 |
| Kater..... | 39.370 79 |
| Gay..... | 39.369 678 |
| | 39.370 432 |
| | 39.369 85 |
| Mean.... | 39.369 8 |

and Valley of the Jordan. Portions of these are
level — the sea. (R. E. Peary.)

1501 to 1889 the ratio of gold and silver varied

Wood columns or posts, set in earth opposite
durable than when set with it.

‡ 100 Grams. § Millions.

Miscellaneous Operations.

move Paint. Apply chloroform.

store Color of a Fabric. When destroyed by an acid, apply to neutralize it, and then chloroform.

ware. Warm, and cover with a mild solution of collodion in alcohol, with a soft brush.

rames. To restore, rub with a sponge moistened with spirits of

rain. On silver, rub with salt.

rust. To remove from white fabrics, saturate the spots with lemon-juice, and expose to the sun.

stains. Wash with pure fresh water, and apply oxalic acid. If this stain to a red color, apply ammonia.

res on Brick. Apply oyster-shells on the top of a clear fire.

Antidotes for Poisons.

Additional to page 185.

al Wine or Tartar Emetic.—Warm water to induce vomiting.

r Fowler's Solution.—Emetic of mustard and salt, a tablespoonful, sweet-oil, or milk.

—Oil of vitriol, corrosive sublimate, sugar of lead.

ada or Potash, and Volatile Alkali.—Drink freely of lemon-juice or water.

acid.—Flour and water, and glutinous drinks.

of Soda, Copperas, or Cobalt.—Administer emetic; soap or mucilagi-

a.—Apply cold water to head and face, artificial respiration, and gal-

i. Morphine, or Opium.—Administer strong coffee, mustard flour, butter and warm water, and exercise.

or Oxalic Acid.—Give magnesia mixed, and soap dissolved with fresh

Silver.—Salt in water.

f Zinc or Red Precipitate.—Give milk or white of eggs copiously.

Acid.—Aqua fortis.

z.—Emetic of mustard or sulphate of zinc, aided by warm water.

Motive Power of the World.

Steam-engines. In Horse-Power.

8.... 7 500 000 | Germany..... 4 500 000 | Austria 1 500 000
..... 7 000 000 | France..... 3 000 000 | Other countries. 19 000 000

Steam-boilers in Foreign Countries.

uding Locomotive.. 51 390 | Germany 60 700 | Austria 120

Locomotives in Foreign Countries.

7000 | Germany... 10 000 | Austria.... 2800 | Other countries...

—engines of the world represent the power or work

(Bureau of Stat)

Destructive Stress of Belting. (H)

In Lbs. per Sq. Inch.

| | Maxi- mum. | Minimum. | Exten- sion.* | Material. | Max mun. |
|----|---------------|--------------|------------------|---------------|-------------|
| r. | Lbs. 8000 | Lbs. 2850 | Inch. .018 | Rubber | Lbs. 388 |
| . | 6750 | 3000 | .18 | Cotton belt'g | 291. |

* At 400 lbs. per sq. inch.

Largest Constructions and Natural Formations.

New Opera-House, Paris.—Covers 3 acres, and has a volume of 4 257 000 feet.

Popocatepetl, Highest active Volcano, Mexico.—Has a crater one mile in diameter and 1000 feet in depth. (See p. 182.)

Telegraph Wire over river Kistnah, India.—6000 feet in length and 1000 feet in elevation. (See p. 179.)

Chinese Wall, Built 220 B.C. (See p. 179.)

Lambert Coal Mine, Belgium.—3490 feet in depth.

Mammoth Cave, Kentucky.—Some of its chambers are traversed by marginal branches of the subterranean river Echo.

St. Gotthard Tunnel.—Its summit is 900 feet below the surface at Andermat, and 6600 feet below the peak of Kastlehorn. (See p. 179.)

Bibliothèque Nationale, Paris.—Founded by Louis XIV., contains 140 000 volumes, 300 000 pamphlets, 175 000 MSS., 300 000 maps and charts, and 150 000 medals and medals. Engravings 1 300 000, contained in 1000 volumes, and 200 000 portraits.

Desert of Sahara, Africa.—Length 3000 miles, average breadth 900 miles, and area 2 000 000 sq. miles.

Pyramid of Cheops, Egypt.—Volume of masonry 89 000 000 cube feet; weight of stone computed at 6 300 000 tons. (See p. 174.)

Bell, Moscow.—Circumference at base 68 feet, height 21 feet. (See p. 21.)

Bridges. Rialto, Venice.*—A single arch of marble, 98.5 feet in length.

Clifton Suspension, Bristol, Eng.—Span 702 feet, elevation 245 feet.

Niagara Suspension, U.S.—Cantilevers, of steel, length 512 feet. Elevation the rigids 245 feet.

Britannia, England.—1472 feet in length, and elevation 205 feet.

Forth, Firth of Forth, Scotland.—Length 808.5 feet, exclusive of approach of 300.3 feet. Two Cantilever spans of 1710 feet each. Piers 350 feet apart and 150 feet in the clear above water. Iron and steel 24 000 tons. Built 1850-1859.

Fyfe, Scotland.—Length 2 miles, 83 piers, and elevation 77 feet.

Columns or Pillars.

When a column or pillar is without its vertical line, one with slight lateral ends becomes capable of greater resistance than one with square ends.

Experiments at the U. S. Arsenal at Watertown, Mass., developed that the lateral resistance of timber, to transverse compression or crushing, was about one third of its resistance to longitudinal compression, and hence, that the area of wood at the head of a timber-column, should proportionately exceed that of the shaft.

Steam-engine Notes.

True power's Equivalent.—Is usually computed from the volume of gas discharged from the cylinder. Its measure for an ordinary non-condensing engine is about $\frac{1}{4}$ of its actual power. It refers more to the dimensions of an engine than its capacity.

Indicated.—Is the measure of the force exerted by an engine, and from which is deduced the losses, friction of its parts, and of its connecting parts, and so on.

Real Power.—Ordinarily $\frac{1}{2}$ to $\frac{3}{4}$ gallons of water at 70° F. at water in engine per inch HP.

Real.—The ordinary consumption of fuel may be taken at 12 lbs. per HP. for non-condensing engine, and 1 lb. for a condensing.

Indicated.— $\frac{1}{2}$ to $\frac{3}{4}$ lbs. of heating surface, or $\frac{1}{4}$ to $\frac{1}{2}$ of grate surface, at normal draught, will give one HP.

Speed of Steam.—The velocity of the piston per second may be determined by the formula $V = \frac{1}{2} \pi D N$, where V is the square root of the area of the piston, D is the diameter, and N is the number of strokes per second. The velocity of the piston may be obtained. (See Appendix, Table.)

antic and Pacific Oceans. There is not any difference in the levels of these Oceans at Aspinwall and Panama, as determined by Geo. M. who constructed the Panama Railroad.

Origin and Period of Great Inventions.

See also *Chronology*, pp. 71, 72, 915.

ngine.—Amonton, 1699. Stirling, 1827. Ericsson, 1855.
ump.—Otto Guericke, 1650. *Anemometer*.—Wallius, 1709.
on.—First, Lyons, France, 1783. *Barometer*.—Torricella, 1643.
ry.—Electric, 1745; claimed by Kleist, Cunnæus, and Muschenbroch.
es (Suspension).—Of chains, China, 100 B. C.
nets.—At Bayonne, 1670. Socket bayonet, 1699.
—In Christian church, 400; in France, 550. *Bellows*.—Egypt, 1490 B. C.
mer Steel.—Sir Henry Bessemer, 1856. *Blankets*.—England, 1340.
ing.—Germany, 1620. *Bullets*.—Of stone, 1418; of iron, 1550.
Printing.—Egypt; introduced in England 1696.
ra Obscura.—Roger Bacon, 1214; Newton, 1700; Daguerre, 1839.
les.—Of tallow, 1290. *Cannon*.—1118; England, 1521.
ages.—Vienna, 1515; England, 1580.
z.—To strike, by Arabians, 800; by Italians, 1200.
**—1184* B. C.; China, 1200 B. C.; Rome, 576; England, 1101.
ass.—China, 2634 B. C. *Cotton Gin*.—Whitney, 1793.
ng.—1490 B. C. Prussian Blue, Berlin, 1710.
mite.—Sobrero, 1846; Nobel, 1867.
ric Discoveries.—Leyden Jar, Cunnæus, 1746; Electric Light, Davy, 1800;
tent of it, Greene & Staite, 1846.
ro-Magnetism.—Oersted, Copenhagen, 1819.
rotyping.—Jacobi of Russia and Spencer of England, 1837.
aving.—China, 1000 B. C.; on metal, 1423; line or steel, 1450; etching, 1512.
—Murdock Cornwall, 1792; Meter, Clegg, 1807; Dry-meter, Malam, 1820.
**—Egypt*, 1740 B. C. Windows, France, 12th century.
Leaf.—Egypt, 1700 B. C. *Gunpowder*.—Unknown; rediscovered 1324.
shoes.—300; of iron, 480.
aulic Press.—Bramah, 1796. *Hydraulic Ram*.—Whitehurst, 1772.
ogen.—Isolated by Cavendish, 1766. *Iron Vessels*.—J. Wilkinson, England,
ship, 1821; Steam-boat, 1830; Ship building, 1833.
Idoscope.—Sir Daniel Brewster, 1814-17. *Knives*.—Table, England, 1550.
boat.—1817. *Lithography*.—Senefelder, about 1796.
notive.—Watt, 1769 and 1784. Cugnot, 1769.
ies.—Friction, 1829. *Medicine*.—From Greece, in Rome 200 B. C.
we.—Glass, Venice, 13th century. *Newspaper*.—First authentic, 1494.
ibus.—Paris, 1827.
ze.—755. England, 951. *Oxygen*.—Priestley, 1774.
er.—From silk, China, 120 B. C.; from rags, Egypt, 1085.
z.—Of steel, 1803; gold, 1825. *Pencils*.—Of lead, 50. *Eno*
tephone.—Italy, 1710. *Phonograph*.—Edison, 1877.
tegraph.—England, 1802; perfected, 1841. *Pottery*.—Old
toffes.—Vienna and Brussels, 1516. *Stamps*.—England
king.—Types, I. Coster, 1423.
and.—Passenger, England, Sept. 27, 1825.
the machine.—Patented, England, 1755.
er.—1858; Pullman, 1864. *Soap*.—England, 16th c.
Italy, 15th century.
—A. S. Bell and C. J. Blake, Boston, 1874.
to D. Bushnell, 1777.

Notes that the subject is also given at pp. 71, 72.

Values of some Precious Metals.

Per Pound Troy.

| | | | | | |
|--------------|-------|----------------|--------|----------------|--------|
| Cobalt..... | \$ 10 | Osmium..... | \$ 590 | Rhodium..... | \$ 475 |
| Gold..... | 250 | Platinum..... | 102 | Ruthenium..... | 575 |
| Iridium..... | 295 | Potassium..... | 25 | Silver*..... | 11 |

* Variable.

Expenditure in England for Various Purposes and of Articles Compared with that of Spirituous Liquors.

In Millions of Pound Sterling.

| | | | | | |
|--------------------------|----|------------------------|----|--------------------|----|
| Missions..... | 1 | Tea, Coffee, etc..... | 20 | Woollen Goods..... | 4 |
| Education..... | 11 | Sugar..... | 25 | Bread..... | 7 |
| Fuel for Households..... | 15 | Milk..... | 30 | Rents..... | 19 |
| Linen and Cotton..... | 20 | Butter and Cheese..... | 35 | Liquors..... | 17 |

Aluminum.

Elastic limit of bars in tension 14 000 lbs. per sq. inch. Specific heat .2185. Melting point at 1400°. Malleable at from 200° to 300°.

Tensile strength, ultimate, 26 000 lbs. Modulus of elasticity, 12 000 000.

Shrinkage .022 per linear foot. It is comparatively unaffected by exposure to air or water. Specific Gravity .0926. A cube foot weighs 160.013 lbs.

Linear expansion 32° to 212° .0000206.

(Continued on page 976.)

Bushels of Seed Required Per Acre.

In Bushels per Acre.

| | | | | | |
|-----------------|------------|-----------------|-------------|------------|----------|
| Barley..... | 1.5 to 2.5 | Flax..... | .5 to 2 | Oats..... | 2 to 4 |
| Beans..... | 1 " 2 | Grass, blue.... | .625 " .875 | Parsnips.. | .5 " 1 |
| Buckwheat.... | .75 " 1.5 | " orchard..... | 1.5 " 2.25 | Pease..... | 2.5 " 4 |
| Carrots..... | .75 " 1.5 | " Herds'..... | .375 " .5 | Potatoes.. | 5 " 10 |
| Clover, red.... | .16 " .33 | " Timothy.. | .5 " 1 | Rice..... | 2 " 4 |
| " white..... | .16 " .33 | Hemp..... | 1 " 1.5 | Rye..... | 1 " 2 |
| Corn, brown.. | 1 " 1.5 | Millet..... | 1 " 1.5 | Turnips.. | .06 " .2 |
| " Indian.. | .25 " 1 | Mustard..... | .25 " .625 | Wheat.... | 1.5 " 2 |

* See also page 193.

Domestic Remedials.

2.—Discharged by an acid, can be restored by Ammonia.

-Carbolic Acid (20 drops), evaporated on a hot surface, as a shovel, or from a room.

remove stains from a white fabric, wet with Milk and cover with Salt.

3.—May be discharged by Buttermilk.

Amphor Gum, vaporized over the chimney of a gas-burner or lamp, or from a room.

To remove them off, apply Chloride of Lime to their locality.

To remove the noxious effects removed by Chloride of Lime.

Remove patient to a cool place, administer water freely, or Soda.

Values of Food for Sheep.

Wool and Tallow Produced.

| Wool. | Tallow. | Food. | Wool. | Tallow. |
|-------|---------|-------------------------|-------|---------|
| Lbs. | Lbs. | | Lbs. | Lbs. |
| .97 | .99 | Corn-meal, wet..... | .83 | .93 |
| .7 | .7 | Buckwheat..... | .79 | .7 |
| .78 | 1 | Rye, without salt.... | .58 | .97 |
| 1 | .7 | Potatoes, with salt.... | .3 | .45 |
| .97 | .58 | " without salt..... | .28 | .45 |

Croton Aqueduct. New York, 1890.**Dimensions, Length, and Capacity.**

Tunnel proper..... 29.63 miles }
 Aqueduct in open trench..... 1.12 " } 30.75 miles in length.

Pipes to Central Park reservoir, 2.37 miles in length.

Tunnel under Harlem river, 307 feet below tide-water level.

Course.—From Croton Lake, 350 feet above the Dam, and runs generally Southerly, through Westchester Co. and the 24th Ward of New York, to a point 7000 feet N. of Rome Park, with a uniform inclination of .7 feet per mile; its general form, that of a horse-shoe with curved invert; being 13.33 feet in height and 13.6 feet in width; having a computed capacity of 318 millions of gallons per day. From hence, where it is contemplated to construct a large reservoir, for the supply of the annexed districts of the city, to its termination at 135th Street and 10th Avenue, its capacity is reduced to 250 millions of gallons per day, and the Aqueduct which from there is to be operated under pressure, is circular in its section, 12.3 feet in diameter, with varying inclinations, the portion under the Harlem river being 15 feet.

From 135th Street it is connected to 12 cast-iron pipes, 48 ins. in diameter, 4 of which connect with the old Aqueduct, 4 with the present City distribution, and 4 leading through Convent, (6th) and 8th Avenues to the Reservoir in Central Park. The operating capacity of all being equal to that of the Aqueduct, 250 millions of gallons.

The Aqueduct is for the greater portion of its length a tunnel, it raising to the surface but at four points, from which it can be emptied through gates into the adjacent rivers.

Capacity.—The water-shed of the Croton, in extreme dry weather, with storage, 50 million gallons per day.

The present storage system includes Croton Lake, Reservoir at Boyd's Corners, the middle branch Reservoir of the Croton valley, and several lakes, with a total capacity of 10,000 million gallons: three dams being in progress of construction and others contemplated, viz., one at Carmel and one at Quaker Bridge.

The Capacity of the Reservoir in Central Park is computed at 1000 million gallons.

Ice.*Additional to p. 195.*

2.5 ins. thick will support a man; 5 ins., an 84-lbs. cannon; 10 ins., a body of men; 18 ins., a railroad train.

Yield of Oil in Seeds.*Per Cent.*

| | | |
|-----------------------|------------------------|--------------------|
| Rape..... 55 | Mustard, white..... 37 | Oats..... 6.5 |
| Almond, sweet..... 47 | Hemp..... 19 | Clover-hay..... 5 |
| " bitter..... 37 | Linseed..... 17 | Flour-wheat..... 3 |
| Corn/np..... 45 | Corn, Indian..... 7 | Barley..... 2.5 |

Additional to page 189.

Historical Events and Notes.**Facts.**

Australia.—Discovered 1622.

Banana.—Produce per acre 44 times greater than wheat.

*times greater

Camels.—Some can travel 800 miles in 8 days.

Catacombs.—Of Rome, remains of 6,000,000 bones.

1.—Authentic history of it, 3000 B.C. C

iry of Alexandria—47 B.C. contained 400

—Steel, consumption 4,000,000 per day.

ry.—Abolished in Eng. West Indies, 1834; -

***de reached by Explorers**

The distance from this to the

Influence of the Rotation of the Earth on Moving Bodies.

The Rotation of the Earth on its axis effects an appreciable displacement of the rails in a line of railroad.

In the case of an express train weighing 400 tons, running N. at the rate of 5 miles per hour, the pressure on the right hand or Eastern rail is computed at 34 lbs., and with a steamer, alike to the Inman Line "City of New York," the pressure is computed at 936 lbs. This lateral force increases to the Poles. (*T. Von Baer*.)

Bacteria in Earth-soil.

In Virgin soil; soil from beneath Roadways; from Gardens; adjacent to Factories; from Courtyards and Cemeteries.

| Depth below Surface. | Germs per Cube Centimeter.* | Depth below Surface. | Germs per Cube Centimeter.* | Depth below Surface. | Germs per Cube Centimeter.* | Depth below Surface. | Germs per Cube Centimeter.* |
|----------------------|-----------------------------|----------------------|-----------------------------|----------------------|-----------------------------|----------------------|-----------------------------|
| Meters. | No. | Meters. | No. | Meters. | No. | Meters. | No. |
| 1 | 124 800 | 2 | 750 | 1 | 64 200 | 2 | 590 |

* .061 022 cube inches.

The number very rapidly decreases in the deeper layers of the earth, both in virgin soil and in that which has been polluted. (*John Reimera*.)

Water-meters.

Worthington's. New York.

| Diam. of Receiving Pipe. | Volume delivered per Minute. | Diam. of Receiving Pipe. | Volume delivered per Minute. | Diam. of Receiving Pipe. | Volume delivered per Minute. | Diam. of Receiving Pipe. | Volume delivered per Minute. |
|--------------------------|------------------------------|--------------------------|------------------------------|--------------------------|------------------------------|--------------------------|------------------------------|
| Inch. | Cube ft. Galls. | Inch. | Cube ft. Galls. | Inch. | Cube ft. Galls. | Inch. | Cube ft. Galls. |
| .625 | 1.5 11.25 | 1 | 5 37.5 | 2 | 8 60 | 4 | 60 45 |
| .75 | 3 22.5 | 1.5 | 6 45 | 3 | 18 130 | 6 | 120 90 |

NOTE 1.—The volume of delivery here given, for each meter, can be exceeded.

2.—Extreme velocity of a meter produces incessant and improper resistance hence, in order that the instrument may operate only within a perceptible reduction of the head of the supply, it should be of a capacity to effect its duty at a moderate velocity of operation.

Telescopes.

Galileo's first telescope magnified but three times; but by the addition of a concave eye and convex object glass he attained a magnifying power of 30 times.

The construction of large lenses is at present limited by the chromatic aberration or separation of light in a telescope.

Kepler was the first to discover the principle governing this aberration and the method of abolishing it.

Diameters of the Principal Objective Glasses.

United States.

| Location. | Diameter. | Focal Length. | Location. | Diameter. | Focal Length. |
|-----------|-----------|---------------|-----------|-----------|---------------|
| Inch. | Feet. | | Inch. | Feet. | |
| 2 | — | | 16 | 22 | |
| 3 | — | | 26 | 32.472 | |
| 2.5 | 15 | | 26 | 25 | |
| 2.56 | 20.2 | | 36 | 56.2 | |

California contemplates the construction of one of 40 in

the U. S. are, Gates Head, England, 24 in; Vienna 16 in; Pulkowa, Russia, 30 in.

Four lenses of 24 inches.

Manufacture of Ice. Machinery and Apparatus.

| c. of 24 h. | Steam-Engine. | | Com- pressors. | Blocks of Ice. | | Water required per Minute. | Operators. | | | | Weight of Engine and Plant. |
|----------------------|---------------|------|-------------------|--------------------------|----------|-------------------------------------|------------|-----------------|---------------|----------------|--------------------------------------|
| | Ina. | Rev. | Ina. | Ina. | Gallons. | T's.* | Coal. | Engi- neers. | Fire- men. | Lab- orers. | Lbs. |
| | 7×9 | 90 | 5×10† | 8×8×28 | 5 | .5 | 2 | 2 | — | — | 20 000 |
| | 8×16 | 80 | 5×15 | 8×15×28 | 15 | 1 | 2 | 2 | 2 | — | 58 000 |
| | 10×20 | 75 | 6×18 | 8×15×28 | 20 | 1.5 | 2 | 2 | 2 | — | 69 000 |
| | 12×30 | 70 | 8×20 | 11×22×28 } 11×11×28 } | 30 | 2 | 2 | 2 | 3 | — | 101 000 |
| | 14×30 | 65 | 8×25 | 11×22×28 } 11×11×28 } | 35 | 2.5 | 2 | 2 | 3 | — | 129 000 |
| | 14×30 | 65 | 10×20 | 11×22×28 } 11×11×28 } | 40 | 3 | 2 | 2 | 4 | — | 167 000 |
| | 16×30 | 55 | 10×30 | 11×22×28 } 11×11×28 } | 50 | 4 | 2 | 2 | 5 | — | 190 000 |
| | — | — | — | 11×22×28 } 11×11×28 } | 60 | 5 | 2 | 2 | 6 | — | 225 000 |
| | 18×36 | 50 | 12×30 | 11×11×28 | 90 | 6.5 | 2 | 2 | 7 | — | 260 000 |
| | 20×36 | 50 | 15×30 | 11×11×28 | — | — | 2 | 2 | 8 | — | — |
| | 24×36 | 45 | 12×30‡ | 11×11×28 | — | — | 2 | 2 | 9 | — | — |
| | 26×48 | 45 | 20×36 | 11×22×28 | 100 | 13 | 2 | 2 | 10 | — | 360 000 |

* 2000 lbs.

† One compressor.

‡ And one 16×36 ins. additional.

others two compressors, and all single acting.

ssure of Steam.—For all 75 lbs. per square inch.

t-off.—For the three first, which are slide valves, three eighths. For the s, as Corliss engines, one fifth.

a volumes of ice above given cover that lost in thawing the molds to release it, coal given as that required is inclusive of that required to distil water from h to make the ice.

TE.—In order that the proper dimensions of engine and plant may be arrived a required volume of ice, it is necessary that the quantity and temperature e water supply should be furnished.

—The ice is produced from water of distillation; hence, it is clear and trans- sil.

When a Machine is operated by Water-power. As the et from which the ice is made is not distilled from steam, as in the case where m is the motive power, the ice produced is less clear or transparent, and is n as "white ice."

Refrigerating.

achines for Refrigerating are in all respects alike to eaking, with
thirds more capacity. As distilled water is not r
ing of fuel in consequence is fully thirty per cent. F
lves a much less expenditure of water than when i

ements of a Test of Operation a Refrigerating Machir

Ice Liquefied in 24 Consecutive Hours, 78.41

Steam-engine.—Non-condensing, 18×36 ins. Compress.

Pressure of Steam, 86 lbs. Revolutions per minute, 56.5

HP.—Steam-cylinder, 84.3. Of compressors, 67.78.

Temperature of condensing water, 76.2°. Of condenser ro.

Volume of condensing water per minute, 21.19 gallons. Of
be fed = 2 496 900 lbs.

Evaporating pressure, 25.22 lbs. Condensing pressure, 157.1

Anthracite coal, consumed, 6108 lbs. Combustible, 83.63 per

Coal per HP per hour, 3.02 lbs. Consumption equivalent to
of ice, 77.85 lbs.

Asphalt and Asphalt Pavement.

Barber Asphalt Paving Co., New York.

Asphalt rock is an amorphous limestone, impregnated with about 10 per cent bitumen. Trinidad Asphalt is a mixture of sand, pulverized limestone, and bitumen forming an artificial impregnated sandstone.

In the cities of Europe, where asphalt pavement has been laid, the practice spread from 1.5 to 2.5 inches of it on a bed of concrete 6 ins. in depth.

The process of preparing the material for use is to crush the rock to powder it to about 280°, spread it on the concrete, and then compress it by rammer.

The use of natural bitumen, found in the United States, as the Albert Grahamite, was resorted to, but without success, when the pitch or asphalt Trinidad, W. I., was discovered; by combining this material with highly refined petroleum, a satisfactory cement was produced, which being mixed with a silicious sand and powdered limestone, a desired sandstone was formed; a compound possessing the necessary firmness and resistance to the changes of temperature, under the wear of loaded vehicles, combined with smoothness, noise, and comparative freedom from noise; without the facility for the slip horses' feet, usual with pavements with smooth surface. So evident was the full application of this construction, that in 1870 an essay of its merits was made at Newark and New York, and in 1876 it was further essayed on an extended Washington, its merits being evidenced by a Board of U. S. Engineers. Since that time it has been laid in over 60 other cities in the U. S. to an extent of 12,000,000 square yards.

The advantages of such a pavement are the reduction of the resistance to economy of transportation, and freedom from jolting in travel, added to cleanliness and public health, as it is without seams or joints wherein filth may be collected.

Its durability in wear is less than granite trap, and greater than sandstone or macadam.

As regards the cost of its maintenance, it is less than that of any other pavement maintained in like condition of repair.

Origin and Development.

The utility of asphalt for covering of a road was not discovered until 1805. Asphalt rock, broken up, was laid in the manner of a macadamized road, and asphalt was such, that in 1854 a street in Paris was laid with compressed asphalt on a foundation bed of concrete.

In 1869 it was first laid in London, and is now extensively laid in the cities of Europe to an extent in excess of 1,500,000 square yards.

Substitutes. — *Tar.* As a substitute for it it was essayed to use the heavy tar, obtained from gas-works; but as it is deficient in the required qualities, susceptible of being rendered viscid by the heat of summer, and brittle by the cold of winter, the use of it was abandoned.

Wood. — Wood-pavement is laid in London and Paris on a foundation of concrete and it lasts from 4 to 6 years.

Stone-blocks filled in with bitumen water-proof filling has been practised with success. In some of the principal cities of Europe, the uniformity in the dimensions and shape of the blocks contribute to their durability. The cost of such a pavement is less than that of all others.

Macadam. — Macadam pavement is unsuited for cities from the wear and tear and the great cost of maintenance.

Brick. hard burnt brick laid in two courses on 6 inches of sand, tamped down, its longitudinal edge, has been used in England. The success of such a pavement depends wholly on the quality of the material and the manner of laying. In general practice it was found to be unsuited for cities.

In his report (1879) submits the following:

— A good pavement must be smooth, and to give a safe foothold for animals, and not polished or as nearly as possible, noiseless and free from material, laid upon a firm foundation, and be durable in all seasons of the year.

Suitable Foundations for Pavements.

and unyielding foundation is quite as necessary for stability and endurance as for any other structure.

ring are suitable foundations for street-pavements, in order of value, proper thickness is adapted to character of subsoil and nature of traffic, viz.: 1. concrete 5 to 8 ins. in thickness; 2. rubble-stone set on edge side by side, in close contact, with interstices filled in with hydraulic concrete; 3. an old pavement properly brought to slope and grade; 4. rubble-stone set on edge closely in contact like sub-pavement of a Telford road; 5. an old pavement-blocks, cobble, or rubble stone; and 6. an old Macadamized or gravel a compost layer of broken stone or gravel, 8 or 10 inches thick.

est pavements now prominently before the public, classified with respect to materials of which they are made, are Asphalt, Stone block, Wooden block, and pavements. The wooden-block pavement is not entitled to a place in the

Pavements.—The best is formed with rectangular blocks from 3.5 to 4.5 ins. in length on wearing surface, and 8 to 9 inches deep, set upon their edge across the street, upon a foundation of hydraulic concrete.

Asphalt.—Best asphalt is one having for a foundation a bed of hydraulic cement, or something equivalent thereto in firmness and durability, and for wearing surface either the natural bituminous limestone known as asphalt rock, from the Jurassic region on the confines of Switzerland, or, preferable thereto, a specially compounded mixture of refined asphaltum and silico-calcareous sand, in which the calcareous ingredient is finely pulverized limestone. As the material for the best pavement comes principally from vicinity of Neufchatel, the pavement is known as the Neufchatel. Asphaltum for the other pavements referred to is from the Island of Trinidad, and the pavement is sometimes called the Trinidad

Asphalt pavement.—Has been extensively laid in London, Paris, and other European cities.

ough these two pavements represent the best type of street surface, there is a characteristic and somewhat important difference between them, due to the fact that the Trinidad contains nearly 75 per cent. of sharp silicious sand, and does not become polished and slippery by wear; while the Neufchatel, being made entirely of bituminous limestone (a species of amorphous pulverulent limestone without grit, impregnated with bitumen), is by no means free from this fault. Any asphalt pavement adapted to streets of exceptionally steep grade, is one made with rectangular blocks of compressed asphalt concrete.

Comparative Merits of the Several Pavements.

First Cost.—In cost of construction, wood is the cheapest; Coal-tar comes second; Sheet asphalt like the Trinidad third; Stone-blocks fourth, and Wooden-blocks fifth.

Durability.—Assuming each of the four pavements named to be the best, stone and asphalt will possess the longest life, and wood and coal-tar the shortest. Between the first two and the last two there is no comparison. If the stone be of good quality, asphalt will take first place and wood last.

Order of Maintenance.—Order of merit under this head would be stone, asphalt, and wood and coal-tar last. If the asphalt is of good quality, stone must be both tough and hard in order to maintain

Relative Resistance of Roadways to Traction

At Low Speed. (Rudolph Herring, C. E.)

| slippery. | Resistance in Terms of the Load. | Authority. | Roadway. | Resistance in Terms of the Load. |
|-----------|----------------------------------|------------|------------------|----------------------------------|
| dry... | .005 | Gordon. | Plank | .025 |
| | .007 | do. | Stone-block.... | .04 |
| | | Morin. | Gravel..... | .04 |
| id. | | Kopack. | Cob. stone, good | .033 |
| ... | | MacNiel. | Earth | .125 |

Sub-Marine Torpedoes.

Formula for Determination of Pressure per Square Inch of Various Explosives at Different Distances.

$$\sqrt[3]{\frac{6636 (\Delta + E) C^2}{(D + .01)^{2.1}}} = P. \quad \Delta \text{ representing angle with the vertical passing through}$$

the centre of the charge, made by a line drawn from it to the surface exposed to the shock, determined from the nadir,* in degrees; E a constant for the explosive, as determined by experiment; C weight of the explosive in lbs.; D distance from centre of the explosive to the surface exposed, in feet; and P the mean pressure, corresponding to that which would be transmitted to a disc of copper, by a Rodman indenting ball, per square inch of surface exposed to the shock, in lbs. (Brev. Brig. General H. L. Abbott, U. S. A., 1831.)

Value of E, or Relative Strength of Explosives Fired under Water.

| Explosive. | Nitro-glycerine. | W | Downward Δ = 0°. | Horizontally Δ = 90°. | Upward Δ = 180°. | Explosive. | Nitro-glycerine. | W | Downward Δ = 0°. | Horizontally Δ = 90°. | Upward Δ = 180°. |
|---------------------|------------------|-----|---------------------|--------------------------|---------------------|--------------------|------------------|-----|---------------------|--------------------------|---------------------|
| Dualin. | — | 232 | 116 | 111 | 108 | Forcite No. 1. . . | — | 333 | — | — | — |
| Dynamite No. 1† | 75 | 186 | 100 | 100 | 100 | Tonite. | — | 118 | — | — | — |
| No. 2. . . | 36 | 120 | 75 | 83 | 88 | Rackarock. . . . | — | 220 | — | — | — |
| Explosive Gelat. . | 89 | 259 | 125 | 117 | 113 | Nitro-glyc'ne. . . | 100 | 111 | 71 | 81 | 81 |
| Gun-cotton. . . . | — | 135 | 81 | 87 | 91 | Rendrock. | 20 | 101 | 67 | 78 | 84 |
| Electric No. 1. . . | 33 | 67 | 51 | 69 | 77 | " | 40 | 160 | 91 | 94 | 94 |
| No. 2. . . . | 28 | 43 | 38 | 62 | 72 | " | 60 | 166 | 91 | 95 | 95 |
| Hercules No. 1. . . | 77 | 211 | 109 | 106 | 105 | Vulcan No. 1. . . | 30 | 99 | 66 | 66 | 66 |
| No. 2. . . . | 42 | 118 | 74 | 83 | 87 | No. 2. . . . | 35 | 114 | 72 | 82 | 82 |

ILLUSTRATION.—Assume the distance between the line of the centre of a charge of dynamite No. 1 and the bottom of a vessel to be 5 feet, the angle between the line of centre of the distance and the bottom, measured from the nadir, to be 180°, the constant for the charge 186, and its weight 100 lbs. What would be the mean pressure on the object in lbs. per sq. inch?

$$\Delta = 180^\circ, E = 186, C = 100, \text{ and } D = 5.$$

$$\sqrt[3]{\frac{6636 (180 + 186) 100^2}{(5 + .01)^{2.1}}} = \sqrt[3]{\frac{2428736 \times 100}{29.489}} = \sqrt[3]{8236210^2} = 40724 \text{ lbs.}$$

* A point of the globe directly under our feet, or that opposite the zenith.

† Standard of comparison.

‡ For $(5 + .01)^{2.1}$, see p. 310. Thus, $\frac{5.01 \times 21}{10} = \frac{21}{10} \times \log 5.01 = 2.1 \times .699837 = \text{Number } 27.41$

When the Object is not in a Vertical line with the Explosion.

ILLUSTRATION.—Assume a charge of gun-cotton weighing 882 lbs., set in water, at a horizontal distance of 24, and a vertical of 86 feet from the object; what would be the effect?

To obtain Δ, or angle of divergence, $180^\circ - \tan^{-1} \frac{24}{86} = 15^\circ 25'$, and $180^\circ -$

$$5' = 164.58^\circ. D = \sqrt{24^2 + 86^2} = 89, \text{ and } E = 135. \text{ Hence,}$$

$$\sqrt[3]{\frac{6636 (164.58 + 135) 882^2}{(89 + .01)^{2.1}}} = P$$

$$= 3.821906$$

$$89 + .01 = 89.01$$

$$2.945469$$

$$243888$$

$$23822$$

$$5066$$

$$2$$

$$3813$$

$$\text{Log. of } 89 + .01 = 1.949439$$

$$2.1$$

$$1.949439$$

$$3898878$$

$$\text{Log. of } 89.01^{2.1} = 4.0938219$$

$$2$$

$$3813$$

$$2712.5 \text{ lbs.} = P.$$

Efficiency of Water-Tube Steam-Boilers.

a late test by J. J. Thorneycroft of his patented boiler, the following elements results are reported to the Institute of Civil Engineers. See Vol. XCIX., 1889. *Engine.*—Triple expansion, Cylinders 14, 20, and 31.5, by 16 ins. stroke of piston, jacketed. Independent engines for circulating pump, Blower, Donkey, and ring. All exhausting into engine condenser.

Results of Trials.

| Elements and Dimensions. | Furnace. | | | | | |
|--|------------------|-------|-------|----------------|-------|--|
| | Natural Draught. | | | Blast Draught. | | |
| Surface in sq. feet..... | 30 | 26.2 | 30 | 30 | 26.2 | |
| ing " " | 1837 | 1837 | 1837 | 1837 | 1837 | |
| Surface to grate..... | 61.2 | 70.1 | 61.2 | 61.2 | 70.1 | |
| Pressure of steam in boiler, p'r sq. in. | 200.8 | 196.3 | 186 | 164.2 | 194.9 | |
| " " blast in fire-room in ins | — | — | 27 | 49 | 2 | |
| Revolutions of engine per minute.. | 192.8 | 165.2 | 234.2 | 268.7 | 318.4 | |
| per sq. foot of grate per hour.. | 11.1 | 7.74 | 18.6 | 29.8 | 66.8 | |
| Evaporated from and at 212° | — | — | — | — | — | |
| 1 lb. of coal, ash utilized } | — | 11.22 | 10.48 | 10.2 | 8.89 | |
| 10. do. per lb. of carbon.. | — | 13.08 | 12.18 | 11.7 | 10.04 | |
| 10. do. per sq. foot of } | — | 1.24 | 3.2 | 4.7 | 8.05 | |
| Grating surface per hour | — | — | — | — | — | |
| Temperature of gases in chimney.. | 474° | 421° | 540° | 610° | 777° | |
| " " of air in fire-room.... | — | 69.3° | 71.4° | 60.3° | 62.1° | |
| per IHP per hour..... | 2.22 | 2.28 | 1.981 | 1.99 | 2.26 | |
| on " " | 2.28 | 2.334 | 2.03 | 2.04 | 2.32 | |
| Efficiency of boiler per cent..... | 150.3 | 89.1 | 282.1 | 449.2 | 774.7 | |
| Water used for jacket per IHP per } | — | 86.8 | 81.4 | 78.2 | 66.6 | |
| hr in lbs. } | 1 | 1.43 | .84 | .42 | .38 | |

el. Calorific value of 14 900 thermal units per lb., equal to 1.025 of a lb. of carbon. Each lb. of coal, if completely consumed, is capable of evaporating 15.41 lbs. of water from and at 212°.

Barbed Steel-wire Fencing. (*Galvanized or painted.*)

J. A. Roebling's Sons Co., New York.

Four points, barbs 6 inches apart, 15 feet = 1 lb.

" " " 3 " " 12 " = 1 "

1 Spools.—15 feet in length of the regular measures and 12 feet of the thickest, each one lb.

Coil, about 18 × 18 × 17 ins., measuring 3.5 cube feet, weighing from 60 to 100 and length of wire ordinarily 1500 feet. Thickest or Hog weighs 12 more.

Compute Volume of Boards that can be Sawed out of a Round Log. (*M. J. Butler, C. E.*)

RE.—From diameter of log in inches subtract 4, multiply remainder by one of it, multiply proceed by length of log in feet, and divide product by 8; result give number in feet.

$$-4 \times \frac{d}{2} \times l \div 8 = V. \quad d \text{ representing least diameter in inches, } l \text{ length of log ft., and } V \text{ volume in feet of board measure.}$$

ILLUSTRATION.—Assume a log 30 ins. in diameter and 15 feet in length.

$$30 - 4 \times 26 \div 2 \times 15 \div 8 = 633.75 \text{ feet B. M.}$$

Foot-Pound—When for Unit of Work—Is 1 lb. lifted, thrust, or projected through 1 foot, against gravity or inertia, and is expressed in pounds or tons, with regard to the period of its action.

Ten for Unit of Rate of Work—Is 1 lb. lifted, etc., as above, 1 foot in a given $\frac{1}{10}$ as in 1 second or minute.

ency of Water-Tube Steam-Boilers.

by J. J. Thornycroft of his patented boiler, the following elements reported to the Institute of Civil Engineers. See Vol. XCIX., 1889.
the expansion, Cylinders 14, 20, and 31.5, by 16 ins. stroke of piston.
 Independent engines for Circulating pump, Blower, Donkey, and exhausting into engine condenser.

Results of Trials.

| nd Dimensions. | Furnace. | | | | | |
|------------------------|------------------|-------|-------|----------------|-------|--|
| | Natural Draught. | | | Blast Draught. | | |
| sq. feet..... | 30 | 26.2 | 30 | 30 | 26.2 | |
| " | 1837 | 1837 | 1837 | 1837 | 1837 | |
| to grate..... | 61.2 | 70.1 | 61.2 | 61.2 | 70.1 | |
| n in boiler, p'sq. in. | 200.8 | 196.3 | 186 | 164.2 | 194.9 | |
| in fire-room in ins. | — | — | 27 | 49 | 2 | |
| engine per minute.. | 192.8 | 165.2 | 234.2 | 268.7 | 318.4 | |
| of grate per hour.. | 11.1 | 7.74 | 18.6 | 29.8 | 66.8 | |
| d from and at 212° } | — | 11.22 | 10.48 | 10.2 | 8.89 | |
| ash utilized } | — | 13.08 | 12.18 | 11.7 | 10.04 | |
| per lb. of carbon.. } | — | 1.24 | 3.2 | 4.7 | 8.05 | |
| per sq. foot of } | — | — | — | — | — | |
| e per hour | 474° | 421° | 540° | 610° | 777° | |
| gases in chimney.. | — | 69.3° | 71.4° | 60.3° | 62.1° | |
| air in fire-room.... | 2.22 | 2.28 | 1.98 | 1.99 | 2.26 | |
| r hour..... | 2.28 | 2.334 | 2.03 | 2.04 | 2.32 | |
| " | 150.3 | 89.1 | 282.1 | 449.2 | 774.7 | |
| ler per cent..... | — | 86.8 | 81.4 | 78.2 | 66.6 | |
| acket per HP per } | 1 | 1.43 | .84 | .42 | .38 | |

3 value of 14 900 thermal units per lb., equal to 1.025 of a lb. of car-
 of coal, if completely consumed, is capable of evaporating 15.41 lbs.
 at 212°.

1 Steel-wire Fencing. (*Galvanized or painted.*)

J. A. Roebling's Sons Co., New York.

arbs 6 inches apart, 15 feet = 1 lb.

" 3 " " 12 " = 1 "

5 feet in length of the regular measures and 12 feet of the thickset,
 lb.

8 × 18 × 17 ins., measuring 3.5 cube feet, weighing from 60 to 100
 of wire ordinarily 1500 feet. Thickset or Hog weighs .2 more.

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diameter of log in inches subtract 4, multiply remainder by one
 ly proceed by length of log in feet, and divide product by 8; result
 r in feet.

$l \div 8 = V$. *d* representing least diameter in inches, *l* length of log
 lume in feet of board measure.

—Assume a log 30 ins. in diameter and 15 feet in length.

$$30 - 4 \times 26 \div 2 \times 15 \div 8 = 633.75 \text{ feet B.M.}$$

nd—*When for Unit of Work*—Is 1 lb. lifted thrust, or
 against gravity or inertia, and is expressed in pounds or
 e period of its action.

of Rate of Work—Is 1 lb. lifted, etc
 ond or minute.

Wire Rope.*

Galvanizing decreases strength of unannealed wire 5 per cent., and its ductility 15 per cent.

Breaking Weight of No. 20, B W G (.035 in.) crucible steel rope of 6 strands, 17 ins. in circumference: Wires, 78 to 102 tons per square inch, and Ropes 575 to 1047 tons.

Annealed Wire is not affected by galvanizing, but its ductility is reduced in 179 twists to 58, = 68 per cent.

Annealing Wire reduces its strength 45 per cent., but increases its ductility 77 per cent.

Tensile Strength of crucible steel wire averages 85 tons (80 to 90) per sq. in.

Permanent set, Bessemer iron wire 12 tons per sq. in., or .25 of ultimate tenacity.

Variation of tensile strength of like pieces of steel wire, galvanized or plain, is but 3 per cent. for the former and 8 for the latter.

Modulus of Elasticity (ME). Iron wire 22 400 000, Steel 35 000 000, and crucible Steel 33 000 000.

Bending. Stress due to it, in a wire of the material and dimensions given.

$$ME = 32\,000\,000.$$

Diam. of pulley..... 10.5 13.125 16.875 18.75 24 ins.

Stress per sq. inch..... 50 40 31.4 28.2 22 tons.

Durability. Life of steel wire ropes over iron pulleys, of material and dimensions above.

Number of times rope passed over the pulleys without Breaking. Load 1568 lbs.

| | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. |
|---------------------|------|--------|---------|---------|--------|----------|
| Diam. of pulley ... | 5.25 | 7.875 | 10.5 | 13.125 | 16.875 | 18.75 |
| Number of times.. | 6075 | 10 300 | 16 000 | 23 400 | 46 800 | 72 700 |
| | — | — | 53 100† | 85 200† | — | 392 500† |

Over Pulleys 24 Inches in Diameter. Load 1568 Lbs.

| Manufacture of T. & W. Smith. | No. | Ins. | Diameter. | Number of Bands before Breaking. | | |
|-------------------------------|-----|------|-----------|--|------------|--|
| | | | | Wire and strands laid in opposite direction. | | Wire and strands laid in same direction. |
| | | | | 1-24 Inch. | 3-24 Inch. | 3-24 Inch. |
| Ordinary crucible steel..... | 20 | .035 | | No. 74 100 | No. 51 000 | No. 126 000 |
| Patent improved steel..... | 20 | .035 | | No. 96 000 | No. 57 000 | No. 142 800 |
| Plough steel..... | 20 | .035 | | No. 100 000 | No. 54 000 | No. 134 400 |
| Iron wire..... | 19 | .042 | | No. 66 000 | No. 32 000 | No. 79 000 |
| Crucible steel..... | 18 | .049 | | No. 87 000 | No. 47 400 | No. 117 100 |
| Crucible steel..... | 22 | .028 | | No. 111 000 | No. 48 700 | No. 120 300 |

NOTE.—By author: diameter of pulleys should be = 10 circumferences of rope.

Tenacity of Dovetails.

White Pine, 6 inches square. Notch in Length equal to Depth of Timber.

S and D each representing proportion or depth of cuts to width of

| Destruction. | | Destruction. | |
|--------------|-----------|--------------|---------|
| S .25 | 3.9 tons. | D .125 | 6 tons. |
| .33 | 5.75 " | .167 | 6 " |
| .41 | 5.1 " | .208 | 6 " |

Strength in a double dovetail is attained when $D = .167$, and in a single, (Gen'l O. M. Poe, U. S. E.)

Shafting for Lathes and Mills.

ould be given in inches or quarters only. *Length.*—Not to exceed *ty.*—Machinery, 125 to 150 revolutions per minute; Woods, 200 to 1 Applied at middle of length of shaft whenever practicable. *Hanger* ble boxes, in order the easier to maintain a shaft in line.

Cost of Sawing and Dressing Stone.**Sawing.***Per Cube Foot.*

Marble Stone.—20 cents. At Chicago, *Soft*, medium, 8 to 10 cents; *Best*, 10 to 12 cents; *Hard*, 12 to 14 cents. **Magnesian, and Oolites.**—*Medium*, 13 to 17 cents; *Hard*, 17 to 21 cents. **Granite.**—*Soft*, 15 to 20 cents; *Hard*, 20 to 25 cents.

Rate in 10 Hours.

| | Ins. | | Ins. | | Ins. |
|--------|----------|-----------------|----------|----------------|----------|
| 1..... | 12 | Marble, Tenn... | 9 | Limestone..... | 10 to 15 |
| 2..... | 8 | " Vermont. | 20 | " magnesia. | 36 |
| 3..... | 36 to 40 | Brownstone.... | 20 to 25 | " oolite.... | 40 to 70 |

—Depth of cut without reference to its length or number of saws. (R. J.)

Dressing.*Per Square Foot. Labor \$3 per Day.*

Marble Limestone.—Bush hammered, rough, 25 cents; Medium work, 30 cents; Fine work, 35 cents.

Cost of Raising Water.

1000 Imperial or 1200 000 U. S. Gallons 1 Foot.

Average of 15 Years.

| <i>Low Service.</i> | <i>High Service.</i> |
|-----------------------|------------------------|
| By steam, 1.23 cents. | By steam.... 25 cents. |

Adhesion of Drifted Bolts.*Steel, One Inch in Diameter.—Hole Six Inches in Depth.*

| Wood. | Mean Holding Resistance per Lineal Inch. | | | | | | | |
|-----------|--|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| | Hole 15-16ths. | Hole 14-16ths. | Hole 13-16ths. | Hole 12-16ths. | Hole 15-16ths. | Hole 14-16ths. | Hole 13-16ths. | Hole 12-16ths. |
| | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. |
| Pine..... | 361 | 616 | 761 | 400 | .47 | .8 | 1 | .53 |
| Oak..... | 1300 | 1778 | 2499 | 1133 | .52 | .71 | 1 | .45 |

Block in 15-16ths hole 415 lbs. per lineal foot to withdraw it, and White or Pine 12-16ths hole 830 lbs.

Obtain maximum holding resistance of timber, diam. of hole to bolt as 13 to 16. Give holding resistance between driving parallel or perpendicular to the fibre to 2. (J. B. Tschamer.)

Resistance of the Air to Falling Bodies.

| Falling Body In Vacuo. | | Lead Ball, 2 ins. in Diameter, Weight 1 lb. | | | Body Falling Horizontally. Weight 1 lb. | | | | | |
|------------------------|-------|---|-------|----------------------|---|-------|----------------------|------------------|-------|----------------------|
| | | In Air. | | Retardation per Sec. | One Foot Square. | | | Two Feet Square. | | |
| Velocity. | Fall. | Final Velocity. | Fall. | | Final Velocity. | Fall. | Retardation per Sec. | Final Velocity. | Fall. | Retardation per Sec. |
| Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. |
| 16 | 32 | 30 | 15.5 | .5 | 28 | 14.33 | 1.66 | 13.33 | 13.33 | 2.66 |
| 48 | 64 | 55 | 43.5 | 4.5 | 35 | 33 | 15 | 24 | 37.3 | 24 |
| 80 | 96 | 77 | 67.5 | 12.5 | 38 | 38.5 | 41.5 | — | 61 | 66.66 |

(P. H. Van Der Weyde.)

Retardation is Inversely as Density of Body. Velocity after fall of one second is measurably uniform; the increased velocity being balanced by the increased resistance.

Resistance of the air at moderate velocities, to the velocity of a fall is in the square of its velocity.

When the velocity is doubled, the resistance is quadrupled; or 4 times greater. Applicable alike to a cannon ball in air or

Cost of a Horse-Power by Steam.*

Joule's equivalent (p. 504) = 772 units = heat required to raise 1 lb. water 1° = elevation of 1 lb., 1 foot high.

Unit of evaporation, to evaporate 1 lb. water to steam at the pressure of the atmosphere = 966.1 British thermal units.

Horse-power 33 000 lbs.

$\frac{33\,000}{772} = 42.75$ heat units = 1 HP 1°, and 42.75×60 min. = 2565 per HP per hour.

$\frac{2565}{966.1} = 2.655$ lbs. water required to generate 1 HP.

Anthracite coal has. . . 14 500 heat units. } For other fuels see p. 486.
Bituminous " British. 14 320 " " }

Then $\frac{14\,500}{966.1} = 15.01$ lbs. water evaporated per lb. of coal.

Hence, $\frac{2.655}{15.01} = .1769$ lbs. coal per HP per hour, or $\frac{1}{.1769} = 5.65$ HP per hour per lb. of coal.

Assuming in all of the above, the normal condition, that there is neither expenditure of water or temperature in the operation.

Operatively.—From elements furnished, in part by Thos. Pray, C.E., the cost of 1 HP at the pressures and expansions given is as follows:

Coal at \$ 3 per 2000 lbs.

| Engine. | Condensing. 20" X 45" | Non-condensing. 18" X 42" |
|----------------------------------|--------------------------|------------------------------|
| Initial pressure of steam..... | 83.6 lbs. | 58.5 lbs. |
| Cut off..... | — ins. | 4.74 ins. |
| Terminal Pressure..... | 8.71 lbs. | 8.12 lbs. |
| Evaporation per lb. of coal..... | 10.31 " | 9.99 " |
| " " " IP..... | 16.84 " | 18.59 " |
| Coal per IP per hour..... | 2.28 " | 2.07 " |
| Cost per IP per 10 hours..... | 2.6 cents. | 6.21 cents. |

A Condensing Pumping Engine has been operated at a cost of 2.28 cents.

* For Horse-power see pp. 441, 733, 758, and 914.

Cost of Water Power on Driving Shaft.

Per IP.

Power is variable, depending upon variation in head of water, as when it is delivered in a river subject to rise by freshets, cost of water, and of plant. In order to attain an average daily power, the power must be increased to meet the loss of head by back water in freshets.

| LOCATION. | IP | Distance From Supply to Wheel. | Average Head. | Cost per IP | LOCATION. | IP | Distance From Supply to Wheel. | Average Head. | Cost per IP |
|-----------------------|------|--------------------------------|---------------|-------------|-------------------|------|--------------------------------|---------------|-------------|
| Manchester, N. Y. . . | 890 | 30 | 30 | 44 | Lowell, Mass. . . | 1000 | 575 | 13 | 100 |
| Lawrence, Mass. . . | 1000 | 490 | 28 | 42 | " " " " . . . | 1000 | 290 | 18 | 57 |

Cost of a 1000 IP Plant independent of cost of water about \$ 45 per IP.

Cost of a like Plant under different Heads.

| From Supply to Discharge in Feet. | | | | | | | From Supply to Discharge in Feet. | | | | | | |
|-----------------------------------|-----|-----|-----|-----|-----|-----|-----------------------------------|-----|-----|-----|-----|-----|-----|
| Head. | 100 | 200 | 300 | 400 | 500 | 600 | Head. | 100 | 200 | 300 | 400 | 500 | 600 |
| Feet. | \$ | \$ | \$ | \$ | \$ | \$ | Feet. | \$ | \$ | \$ | \$ | \$ | \$ |
| 10 | 95 | 110 | 125 | 140 | 155 | 170 | 30 | 26 | 32 | 39 | 45 | 51 | 58 |
| 20 | 38 | 46 | 54 | 62 | 70 | 77 | 40 | 20 | 25 | 31 | 37 | 42 | 48 |

At Lawrence and Lowell, a Mill Power = 30 cube feet of water per second, with a head of 25 feet. At Manchester it is 38 cube feet, with a head of 20 feet.

(Chas. T. Main, M.E.)

a Mill Wheel see pp. 565, 566.

Steam Plant.

Daily and Yearly Cost of Coal and Labor in Operating a Plant of 1000 HP.

Year of 308 days of 10.25 hours, coal at \$3 per ton of 2000 lbs.

Deduced from Reports of Chas. T. Main, M.E.

| ENGINE. | *Exhaust steam used. | Coal per HP per hour. | Attendance and Stores per HP per Day. | | | Coal per HP per day. | †Daily per HP | †Daily for 1000 HP | Yearly. |
|-------------------|----------------------|-----------------------|---------------------------------------|---------|---------|----------------------|---------------|--------------------|----------|
| | | | Boller. | Engine. | Stores. | | | | |
| | Per cent. | Lbs. | c | c | c | \$ | \$ | \$ | \$ |
| Compound.. | { 0 | 1.75 | .53 | .60 | .25 | 2.14 | 3.52 | 3520 | 10841.6c |
| | { 25 | 1.5 | .45 | .60 | .25 | 1.84 | 3.16 | 3160 | 9732.8c |
| | { 50 | 1.25 | .38 | .60 | .25 | 1.53 | 2.76 | 2760 | 8500.8c |
| Condensing. | { 0 | 2.5 | .75 | .40 | .22 | 3.06 | 4.43 | 4430 | 13644.4c |
| | { 25 | 2.06 | .62 | .40 | .22 | 2.52 | 3.76 | 3760 | 11580.8c |
| | { 50 | 1.63 | .49 | .40 | .22 | 2 | 3.11 | 3110 | 9578.8c |
| Non-Condensing... | { 0 | 3 | .90 | .35 | .20 | 3.67 | 5.12 | 5120 | 15769.6c |
| | { 25 | 2.44 | .73 | .35 | .20 | 2.99 | 4.27 | 4270 | 13151.6c |
| | { 50 | 1.88 | .56 | .35 | .20 | 2.3 | 3.47 | 3470 | 10687.6c |

* For heating.

† Including coal.

Yearly Cost of 1000 HP and of a HP.

Year of 308 days of 10.25 hours. Coal at \$3 per ton of 2000 Lbs.

Deduced from Reports of Chas. T. Main, M.E.

| ENGINE. | *Exhaust Steam used. | Engine and House. | †Operating Expenses. | Boller-house and Shed. | †Operator's Expense. | ‡Coal and Labor and Stores. | \$Total per HP |
|---------------------|----------------------|-------------------|----------------------|------------------------|----------------------|-----------------------------|----------------|
| | | | | | | | |
| | Per cent. | \$ | \$ | \$ | \$ | \$ | \$ |
| Compound.... | { 0 | 40 | 5.02 | 18.36 | 2.50 | 10841.60 | 18361.60 |
| | { 25 | 40 | 5.02 | 16.16 | 2.20 | 9732.80 | 16952.8c |
| | { 50 | 40 | 5.02 | 13.90 | 1.89 | 8500.80 | 15410.8c |
| Condensing... | { 0 | 33 | 4.14 | 24.80 | 3.38 | 13644.40 | 21164.4c |
| | { 25 | 33 | 4.14 | 21.12 | 2.88 | 11588.80 | 18608.8c |
| | { 50 | 31 | 3.95 | 17.33 | 2.36 | 9578.80 | 16888.8c |
| Non-Condensing..... | { 0 | 29.50 | 3.70 | 24 | 3.95 | 15769.60 | 23419.6c |
| | { 25 | 29.50 | 3.70 | 24.28 | 3.31 | 13151.60 | 20161.6c |
| | { 50 | 29.50 | 3.70 | 19.46 | 2.65 | 10687.60 | 17037.6c |

* For Heating.

† Injector, Depreciation, Taxes, Interest, and Insurance.

‡ As per previous Table.

§ Not including Cost of Plant in column 3 and 5.

Sugar in Mortar.

It has been demonstrated that the addition of saccharine matter to lime-mortar is very beneficial, as it enables it to be laid in frosty weather.

It is claimed also that it causes the mortar to set very soon and strengthens it, and that it can be laid with dry bricks.

As sugared water dissolves lime, it is necessary to dissolve the sugar first, and then add the water to the lime slowly and cautiously. The mortar should be very stiff.

Proportions.—For mortar, coarse brown sugar, 2 lbs.; lime, 1 bushel; sand, 2 bushels.

If sugar is added to mixed mortar, it renders it too thin. (*Manufacturer and Builder.*)

Belting. Speed of belts, single and double, 1 inch in width, speed for the first, 800 feet per minute, and for the second 500 feet, each.

Railroad Speed.

London, North Western, and Caledonian.

London to Edinburgh, 400 miles. Speed, 55.4 miles per hour; w. miles. Engine, Tender, and Cars, 348000 lbs.

Cost of Irrigation per Acre.

California.—From \$7.18 to \$53.33. Colorado.—\$3.75 to \$10.80. Utah, \$4
 France.—Average of several, \$58. India.—Average of several, \$1.75 to \$10.

Alloy

That expands in cooling: Lead 9 parts, Antimony 2, Bismuth 1.

Extremes of Temperature.

Artificial, 135° (Faraday). Atmosphere, 77° (Back).

Extension of Woods by Water. (de Volson Wood.)

| | | | | | |
|-------------|------------|------|----------|------------|------|
| Elongation. | Pine..... | .065 | Lateral. | Pine..... | 2.6 |
| | Oak..... | .085 | | Oak..... | 3.5 |
| | Chestnut.. | .165 | | Chestnut.. | 3.65 |

Smokeless Powder. Gun 6 ins. in diam. Charge 17.64 lbs. Energy
 at muzzle, 4609 foot-tons. Per lb. of powder 139.7, and per weight of gun 720.

Volume of Water Flowing over Niagara Falls.

270000 cube feet per second. Since 1842, Horseshoe Fall has receded 140.5 feet,
 and American 36.5 feet. (J. Bogart, S. E.)

ROOFS.

To Compute Stress on Roofs.

Velocity and Pressure of Wind.

RULE.—Multiply square of velocity of wind in feet per second by .0023, or square
 of its velocity in miles per hour by .005, and product will give pressure in pounds
 per sq. foot.

$$\text{Or, } v^2 \times .0023 = P, \text{ and } V^2 \times .005.$$

Also, $.0023 v^2 \sin. z = P$. P representing pressure per sq. foot in lbs., z angle
 of incidence of wind with plane of surface in degrees, V velocity of wind in miles per
 hour, and v velocity in feet per second.

Direction of wind usually makes an angle of 10° with the horizon, hence 10° is
 to be added to horizontal plane of direction of the wind.

ILLUSTRATION 1.—Assume wind with a velocity of 100 feet per second to impinge
 upon a plane roof set at an angle of 45°; what would be the pressure per sq. foot?

$$\text{Sin. } 45^\circ + 10^\circ = .819. \quad .0023 \times 100^2 \times .819 = 18.837 \text{ lbs.}$$

2.—Assume the wind to have a velocity of 150 feet per second, and angle of roof
 60°; what would be the pressure per sq. foot?

$$\text{Sin. } 60^\circ + 10^\circ = .94. \quad .0023 \times 150^2 \times .94 = 48.75 \text{ lbs.}$$

Pressure of Snow.

This pressure decreases per square foot in Ratio of half space, to length of rafters,
 or height divided by space.

Pressures for Various Angles or Ratios.

At 15 Pounds Weight per Square Foot.

| h + s | Degrees. | Lbs. | h + s | Degrees. | Lbs. | h + s | Degrees. | Lbs. |
|-------|----------|------|-------|----------|------|-------|----------|------|
| .5 | 45° | 10.6 | .2 | 21° 48' | 13.9 | .125 | 14° 2' | 14.5 |
| .33 | 33° 40' | 12.6 | .16 | 17° 45' | 14.3 | .11 | 12° 31' | 14.6 |
| .25 | 26° 34' | 13.4 | .14 | 15° 39' | 14.4 | .10 | 11° 19' | 14.7 |

Weights on Roofs.

Per Square Foot in Lbs.

| | | | |
|----------------------|----|---------------------------|----|
| 1" Iron, sheet..... | 8 | Iron, corrugated, on iron | 43 |
| 7" inc. sheet..... | 8 | Zinc " " | 41 |
| lates on iron..... | 10 | Snow..... | 10 |
| ron, sheet on iron.. | 5 | Wind..... | 10 |

Comparative Operations of a Simple and a Compound Locomotive.

Brooklyn and Union Elevated Railway of Brooklyn, N. Y. Forney Type.

| UNITS. | Simple. | Compound. | ELEMENTS. | Simple. | Compound. |
|---------------------------------|-----------|--------------|---------------------------|-------------|-------------|
| Boiler, ins. | 11 X 16 | 11.5 18 X 16 | Coal per car mile. | 11.05 lbs. | 6.88 lbs. |
| Cylinders, diam. | 42 ins. | 42 ins. | Water | 26 070 lbs. | 19 862 lbs. |
| Revolutions per mile } | 480 | 480 | Gain in fuel. | — | 37.7% |
| | 42 ins. | 42 ins. | Evaporation from 212° | 8.09 lbs. | 9.97 lbs. |
| | 1.5 ins. | 1.5 | Gain in water. | — | 23.8% |
| | 124 | 124 | Water per car mile | 73.85 lbs. | 56.27 lbs. |
| | 3.25 ins. | 3 ins. | Pressure of steam, ave' } | 136 lbs. | 136 lbs. |
| | 15.6 | 15.6 | Revolutions per min. | — | 222 |
| | 289.46 | 289.46 | Miles per hour. | — | 27.73 |
| | 18.5 | 18.5 | HP | — | 223.6 |
| | 3899 lbs. | 2430 lbs. | Weight, loaded. | 45 350 | 45 850 |
| | | | Miles run. | 122 | 122 |

High Explosives.

Firing Point and Relative Strength.

| IGNITION. | Firing Point. | Order of Strength. | DESIGNATION. | Firing Point. | Order of Strength. |
|-----------|---------------|--------------------|-------------------------|---------------|--------------------|
| | Degree. | | | Degree. | |
| | 365 | 106.17 | Tonite. | — | 68.24 |
| | — | 106.17 | Bellite. | — | 65.7 |
| | 365 | 100 | Rack-a-rock. | — | 61.71 |
| | — | 92.37 | Atlas powder. | — | 60.43 |
| | — | 81.85 | Ammonia dynamite. | — | 60.25 |
| | — | 92.38 | Volney's powder No. 1 | — | 58.44 |
| | 346 | 83.12 | " No. 2 | — | 53.18 |
| | — | 81.31 | Melinite. | — | 50.82 |
| | — | 81.31 | Fulminate, silver. | — | 50.27 |
| | 301 | 77.86 | " mercury. | 315 | 49.91 |
| | — | 69.51 | Mortar powd., Dupont | 500 | 23.13 |
| | — | 69.87 | Forcite No. 1. | 330 | — |

(Lieut. W. Walke, U. S. Army.)

Electrical. (Additional to p. 34.)

Units in Electrical Engineering.

Resistance.—The *Ohm*, symbol ω , equal to the resistance of a column of mercury 106.3 cms. long, and 1 \square mm. in uniform cross-section at 0° C. Symbol Ω , 1 000 000 ohms. *Microhm*, one millionth of an ohm.

Current.—The *Ampere*, that strength of current that will transfer 1.118181818 grams of silver per second between electrodes of pure silver immersed in a 20 to 30% solution of pure silver nitrate.

Units: *Kilo-ampere*, *Hecto-ampere*, *Deca-ampere*, *Deci-ampere*, *Centi-ampere*, *Milli-ampere*, *Micro-ampere*.

Electromotive Force.—The *Volt*, the EMF. which will force one ampere through a circuit of one ohm resistance. Derivative, *Microvolt*.

Capacity (Electrostatic).—The *Farad*, Φ , the capacity of a condenser that will store one Coulomb, under a difference of potential of one volt. Deriv., *Microfarad*.

Work.—The *Joule*, the work done by transferring one Coulomb through one volt. Deriv., *Microjoule*.

Units: *Kilojoule*, *Millijoule*, *Microjoule*.

Power of Working.—The *Watt*, the rate of 1 Joule per second. Derivative, *Kilowatt*.

Inductance.—The *Henry*, H , 1 000 000 000 centihenries. Deriv., *Millihenry*, *Microhenry*.

STEAM-ENGINES.

Compound.

Duration of Operation 2 Hours.

Cylinders.—5.5, 9, and 15.5 ins. in diameter. Stroke of piston 14 ins.

Revolutions.—150 per minute. IHP 40.

Boilers.—Fire tubular. Tubes, 38 of 2 ins.; 6.25 feet in length.

Heating Surface.—158 sq. feet. *Grates.*—5.7 sq. feet.

Pressure of Steam.—175 lbs. per sq. inch.

Water.—Weight consumed, 1140 lbs. *Evaporation* per lb. of coal, 9.8 lbs. *Drawn* from jackets, 84 lbs. *Consumption* per HP per hour, 1.425 lbs. *Temperature of feed,* 55°.

Consumed 116 lbs.—per IHP per hour 1.45 lbs.; per sq. foot of grate 10.2 lbs.

Indicator Diagrams.—Mean HP 54 = 13.31 IHP; Intermediate 18 = 12 IHP; Condensing 7.5 = 14.7 IHP = 40.

Fly-wheel.—5.5 feet in diameter and 10.5 ins. in width.

Weight of Engine and Boilers, without water, 14 560 lbs.

Builders.—Marshall & Co., Kreigly, Eng.

PUMPING ENGINE.

Vertical Compound.

Cylinders.—34 and 66 ins. in diameter by 60 ins. stroke of piston.

Pressure of Steam.—74.81 lbs. per sq. inch. *Vacuum,* 26.25 ins.

Revolutions.—25.51 per minute. *Grate Surface.*—70 sq. feet.

Pressure of Water by Gauge.—62.02 lbs. *Head, including lift,* 155.17 feet = 67.62 lbs. *Fuel.*—675 lbs. per hour.

Duty.—104 820 431. *Stack, in height,* 125 feet.

Constructors.—The Edward P. Allis Co., Milwaukee, Wis.

ELECTRIC DYNAMO ENGINE.

Triple Expansion.

Arc Lights.—500. *Water entrained in steam* 7.39%.

Cylinders.—14, 25, and 33 ins. in diam. by 48 ins. stroke of piston.

Condenser.—Separate. *Circulating Pump,* 16 × 16 ins.; *Air-pump,* single-acting, 24 × 16 ins. *Cylinders,* 12 × 16 ins., operating both pumps. *Revolutions,* 61.29 IHP 16.4.

Pressure of Steam.—125 lbs. per sq. inch; *Revolutions,* engine, 99.12; *Steam per IHP per hour,* 12.94 lbs. IHP 516. *Injection Water.*—72°. *Reservoir,* 90°.

Constructors.—The Edward P. Allis Co., Milwaukee, Wis.

Railroad Signals and Significations.

up," one pull of bell-cord.

ahead," Two pulls.

ack up," three pulls.

own breaks," one whistle.

ahead," a sweeping parting of the hands, on level with the eyes.

slowly," a slowly sweeping meeting of the hands, over the head.

downward motion of the hands with extended arms.

beckoning motion of a hand.

ger," a red flag or light waved up the track.

red flag raised at a station.

light raised and lowered vertically.

light angels across the track.

hung in a circle.

"Off breaks," two whistles.

"Back up," three whistles.

"Danger," continued whistles.

"A cattle alarm," rapid short whistles.

Opera House, New York.

ending 400. If the saloons attached to the prima
apacity would be 5000.

Distillation of Fresh Water.

Process of G. W. Baird, U. S. Navy, New York.

Marine Steamers for long voyages, operated under a high pressure of steam, are necessarily provided with Evaporators, to replace the water expended in leaks and vents, and to provide for the ordinary requirements for fresh water.

This process is an improvement upon existing methods, inasmuch as it furnishes the water potable, and it is as follows:

The *Evaporator* contains a series of tinned metallic coils and a volume of sea-water; which is designed to be evaporated by the passage of steam from the engine boilers through the coils. The water condensed in them is returned to the boilers; the water vaporized from the sea-water, external to the coils, is either led to the Engine condenser, to replenish that lost by leaks and vents, as from gauge cocks, &c.; or if required for potable purposes, is led to a *Distiller*, where it is aerated, condensed, and filtered, from which it is drawn for use.

As the sea-water is evaporated in vacuo, vaporization occurs at a temperature below that at which much scale is precipitated. Hence the shell and coils are both measurably free from it.

Results of an Experiment.

Pressure in coils, 20 lbs. above atmosphere; temperature of steam in coils, 253.3°; temperature of feed water, 131.66°; temperature of the water vaporized, 212°; water vaporized per hour, 103.33 lbs.; water condensed in the coils per hour, 112.12 lbs.; total heat in the steam, 1193.7°, and in the water vaporized, 1178.6°.

Capacities of Evaporators and Distillers.

Gallons per day of 24 hours.

| No. | Evapo- rator. | Dis- tiller. | No. | Evapo- rator. | Dis- tiller. | No. | Evapo- rator. | Dis- tiller. | No. | Evapo- rator. | Dis- tiller. |
|-----|------------------|-----------------|-----|------------------|-----------------|-----|------------------|-----------------|-----|------------------|-----------------|
| | Gallons. | Gallons. | | Gallons. | Gallons. | | Gallons. | Gallons. | | Gallons. | Gallons. |
| 1 | 600 | 600 | 3 | 2000 | 1600 | 4 | 3000 | 2000 | 5 | 4000 | 2500 |
| 2 | 1200 | 1200 | 3.5 | 2000 | 1600 | 4.5 | 3000 | 2500 | 6 | 6000 | 3000 |

Coal Production and Consumption

Of the World Per Diem.

Production.—Estimated at 3 360 000 000 to 3 696 000 000 lbs.

Consumption.—Generation of steam, Land and Marine, 624 000 000 lbs.; Smelting Iron Ore, 28 800 000 lbs.; other metals, 23 000 000 lbs.; Forges, 20 000 000 lbs.; Domestic use, 57 600 000 lbs. Total, 2 700 000 000 lbs.

Corrosion of Wrought Iron.

The purer the water, the more active it is in corroding and pitting Wrought-iron plates. This arises from the greater presence of air in pure water, and hence a greater proportion of Oxygen. (*Locomotive.*)

Earth Boring and Heat of Mines.

Sperenberg, near Berlin. Bore, 4172 feet in depth, about 1000 feet in excess of Artesian well at St. Louis.

In lower levels of some of the shafts in the Comstock mines, prior to the draining into the Sutro Tunnel, the water was at a temperature of 120°.

Preservatives of Iron.

Pitch, Black Varnish, Asphalt and Mineral waxes are among the best, provided the acid and ammonia salts, which frequently occur in tar and tar products, are removed.

If in addition these substances are applied hot to warm asphaltic substances form on the surface of the iron and other coatings, is not microscopically porous, and consequently water

injurious and
exposed to
rust

Spirits and Naptha varnishes are injurious. (Prof. Lea.

Felting, Covering, Lagging, etc.

Steam Pipe and Boiler Coverings, etc.

In the protection of heated surfaces, or from freezing, Hair-Felt, in consequence of its destructibility at high temperatures and the loss in removing it for repairs, the arrest of leaks in boilers or pipes which it covers, has very generally been superseded by the adoption of Asbestos fibre with its composition with other materials and conformations, whereby it is rendered available for all requirements as well as conformable to, the several and various conditions in which it is required.

Asbesto-Sponge.—Composed of the fibres of Asbestos and canvas sponge, by which combination porosity is attained and the non-conducting power increased.

Adapted for the lining of Cars, Chests, Floors, Walls, etc., and furnished in barrels and bags containing 25 lbs.

Asbestos, Cement Felting.—Composed of Asbestos fibre, inferior earth, and a cementing compound, applied to pipes, boilers, etc., while heated.

Furnished in barrels and bags. One bag will supply material to cover a surface of 40 square feet, 1 inch in thickness, and weighs about 150 lbs.

Asbestos Roll and Sheet Fire-Felt.—Composed wholly of Asbestos, similar in texture to hair-felt, can be readily cut and applied.

In Rolls of 300 square feet, 3 feet in width, and from .09378 to .25 ins. in thickness.

The *Sheet felt* is furnished in sections, in order that they may be removed and replaced, and are especially adapted for Marine Boilers.

In Sheets, 2 by 3 feet, and from .5 to 1.5 ins. in thickness.

Asbesto-Sponge, Molded, Fire-Felt, and Standard Sectional Coverings.—The *Sponge and Molded* are adapted both for low and high pressure of steam, felted and formed in cylinders, cut lengthwise in one piece, that they may be laid over pipes, and are finished with a canvas jacket, secured by metal bands.

The *Fire-Felt* is adapted for high pressures and Superheated steam. The *Standard* is composed of alternate layers of Asbestos sheathing and soft-wool felt applied together, and is adapted for medium pressures of steam.

In Sections, 3 feet in length, with bands, staples, and straps for securing Elbows, Tees, etc., &c.

"National," Removable, Air-Chamber, and Protective Sectional Coverings.—"National" is composed of soft-wool felt, lined on the inside and alternated for three layers with Asbestos hair-felt, and it is an economical covering for low pressures of steam and steam boiler pipes. *Removable* is composed of hair-felt and Asbestos of equal thickness, composed in a case of wool sheathing and a lining of Asbestos.

In Sections of 3 feet in length, with straps and staples to secure.

Asbesto-Sponge Lined and Asbestos Lining Felt. Suitable for Furnaces, Furnace Pipes, and Fire-heated surfaces.

In Rolls, from 12 to 36 inches in width, and 50 feet in length.

Asbesto-Sponge Hair-Felt.—Is very elastic, and, in consequence of the large proportion of Asbestos in it, it is not liable to injury from steam heat.

In Rolls of 250 square feet, and .25 ins. in thickness.

Superator.—An Asbestos fire and water proof sheet has a foundation of wire mesh through which Asbestos is pressed and the sheet rendered waterproof by a special process. It is suited for ceilings, walls, partitions, and all surfaces exposed to fire and water.

In Rolls of 300 square feet, 39 ins. in width.

Corded Sheathing.—For fire-heated surfaces, of pure Asbestos, in the form of which are attached loosely twisted rolls of it, forming a material 1/2 inch in thickness, suitable for Furnaces, Furnace Pipes, etc.

Rolls of 100 and 150 square feet, and 3 feet in width.

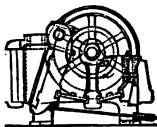
Locomotive Lagging.—Of pure Asbestos, woven with a heavy wire with a pure Asbestos sheathing. It is fire and acid proof.

Rolls of 36 inches in width and .5 inch in thickness.

Capair-Felt.—Of various thicknesses, in bales of 300 square feet, and in boxes.

Stone Breaker and Ore Crusher.

Farrel Foundry and Machine Co., Ansonia, Conn.



Stone Breakers and Ore Crushers are used in making Macadam for construction of roads; material for concrete; ballasting railroads, crushing ores, quartz, corundum, and all brittle substances; they can be adjusted to pass a mass from the size of a pea to larger diameters, depending upon the capacity of the machine.

Crushed to Cubes of 2 Inches. Per Hour.

| No. | Receiver. | Volume. | Extreme Weight of Stone. | Weight Produced. | Dimensions. | | | Pulley. | Speed. | HP |
|-----|-----------|-----------|--------------------------|------------------|-------------|----------|----------|---------|--------|-----|
| | Ins. | Cub. yds. | | | Length. | Breadth. | Height. | | | |
| | Ins. | Cub. yds. | Lbs. | Lbs. | Ft. ins. | Ft. ins. | Ft. ins. | Ins. | | No. |
| 1 | 3X1.5 | — | 40 | 100 | 1.1 | .6 | .10 | 5X1 | 250 | .5 |
| 2 | 6X2 | 1 | 560 | 1200 | 2.10 | 2.1 | 2.3 | 11X5 | 250 | 4 |
| 3 | 10X4 | 3 | 1800 | 4900 | 4 | 3.3 | 3.9 | 20X6 | 250 | 6 |
| 4 | 10X7 | 5 | 3800 | 7800 | 5.1 | 3.9 | 4.5 | 24X7.5 | 250 | 8 |
| 5 | 15X9 | 8 | 7400 | 15500 | 6.6 | 5 | 5.11 | 30X9 | 250 | 15 |
| 6 | 15X10 | 9 | 7800 | 16000 | 6.6 | 5.5 | 5.11 | 30X10 | 250 | 15 |
| 7 | 20X6 | 10 | 5300 | 11200 | 5.3 | 2.11 | 4.6 | 30X10 | 250 | 15 |
| 8 | 20X10 | 10 | 8100 | 18300 | 6.10 | 5.9 | 5.11 | 36X12 | 250 | 20 |
| 9 | 12X30 | 16 | 14200 | 33000 | 7.10 | 8.4 | 6.4 | 36X12 | 250 | 30 |
| 10 | 15X30 | 20 | 14200 | 35000 | 7.10 | 8.4 | 6.4 | 36X12 | 250 | 30 |

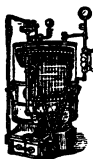
NOTE.—The 30X15 and the 36X24 are preparatory Crushers, the former breaking 30 cube yards in 10 hours to 4 ins., and the latter 800 cube yards to 8 ins.

Crusher with Revolving Screen.

| Dimensions. | Volume. | Extreme Weight of Stone. | Weight Produced. | Dimensions. | | | Pulley. | Speed per Min. | HP |
|-------------|-----------|--------------------------|------------------|-------------|----------|----------|---------|----------------|-----|
| | | | | Length. | Breadth. | Depth. | | | |
| Ins. | Cub. yds. | Lbs. | Lbs. | Ft. ins. | Ft. ins. | Ft. ins. | Ins. | Rev. | No. |
| 10X7 | 5 | 3800 | 10200 | 5.1 | 3.9 | 4.5 | 2X7.5 | 250 | 8 |
| 15X9 | 8 | 6800 | 17700 | 6.6 | 5 | 5.11 | 2.6X9 | 250 | 15 |
| 15X10 | 9 | 7300 | 18100 | 6.6 | 5.5 | 5.11 | 2.6X10 | 250 | 12 |
| 20X10 | 10 | 7700 | 21500 | 6.10 | 5.9 | 5.11 | 3X11 | 250 | 14 |

Steam Heating and Boilers.

Gorton & Lidgerwood Co., New York.



Steam Heating.—Is effected *Directly* or *Indirectly*. In the first case, the steam is conveyed through a pipe, or to a cluster of them, at whatever point they are required, termed a Radiator; air being heated by contact with the exterior surface of the pipes, and the water of the condensed steam flows back (by gravity) through the return pipes discharging into the boiler.

In the second case, steam is conveyed in like manner to a cluster of pipes enclosed in a chamber, in the lowest part of the building, usually the cellar, the air within the chamber, upon being heated, ascends by its rarefaction, and is led to the space or apartment required to be heated.

Hot-water Heating.—This system consists of circulating hot water in the radiators instead of steam. The boiler, pipes, and radiators are fully filled with water—the flow or circulation pipes attached to the top of the boiler and the return pipes to the bottom. The water in the boiler, when heated, rises and circulates through the pipes and radiators, and parting with a portion of its heat it becomes denser, and gravitates through the return pipe to the boiler, where it is again heated.

This system requires a much greater proportion of radiating surface than of steam.

Steam-boilers.

| ELEMENTS. | No. | Wrought-iron Water Legs. | | | | | | Cast-Iron Legs. | | |
|--|-----|--------------------------|-----|-----|------|------|------|-----------------|-----|-----|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Shell, diam.....Ins. | 32 | 35 | 41 | 43 | 51 | 54 | 54 | 24 | 28 | 32 |
| " over jacket... " | 35 | 38 | 45 | 46 | 55 | 57 | 57 | 30 | 31 | 35 |
| " height..... " | 33 | 37 | 37 | 45 | 45 | 45 | 45 | 30 | 33 | 33 |
| " extreme..... " | 69 | 72 | 80 | 87 | 90 | 92 | 92 | 64 | 67 | 69 |
| Furnace, diam..... " | 21 | 24 | 30 | 32 | 38 | 40 | 40 | 18 | 19 | 21 |
| Tubes, No., do. 2..... " | 44 | 56 | 84 | 91 | 124 | 160 | 30 | 36 | 44 | |
| " length..... " | 30 | 34 | 34 | 42 | 42 | 42 | 27 | 30 | 30 | |
| Steam-out'lets, diam. " | 2 | 2 | 2.5 | 2.5 | 3 | 3 | 1.5 | 1.5 | 2 | |
| Chimney flue, diam. " | 8 | 8 | 10 | 10 | 12 | 12 | 7 | 7 | 8 | |
| Water-line from base..... " | 55 | 59 | 63 | 70 | 73 | 74 | 51 | 54 | 55 | |
| Heating surface... □ feet | 75 | 105 | 140 | 185 | 260 | 320 | 45 | 60 | 75 | |
| Direct radiating } surface supplied } | " | 450 | 630 | 830 | 1050 | 1500 | 1900 | 260 | 350 | 450 |

For Direct radiation, each □ foot of radiating surface will heat from 50 to 100 feet of air space, and for Indirect, from 25 to 50 cube feet; the range depending on the conditions of construction of building and its exposure to external air.

HYDRAULIC CEMENT.

In addition to pp. 589, 590.

New York and Rosendale Cement Co. (Hiram Snyder, Sec'y), New York.

Cements.—Are classed as natural and artificial; first being used in construction of Fortifications, Breakwaters, Aqueducts, Sewers, Bridges, Concrete work of all description, as in Canals, Cellars, Cisterns, and Walls.

The stone from which hydraulic cement is made in the U. S. is found stratified beds of aqueous deposits, which in extent cover about one third the area of the State of New York, the western part of Vermont, and in New Jersey, Pennsylvania, Maryland, Virginia, and East Tennessee.

The largest deposits of aqueous limestone, producing hydraulic quality are located in the Town of Rosendale, Ulster Co., New York.

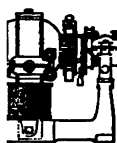
The color and quality of most of the hydraulic cements depend principally upon the relative volumes of Oxide of Iron, Lime, Alumina, and Magnesia which may be present; and as these elements are very perceptible in the highest grades of Rosendale cement, it resembles the celebrated Portland Cement of England and France.

Tensile Strength. Per Square Inch.

| Tests. | Area. | In Air. | In Water. | Period. | Tests. | Area. | In Air. | In Water. | Tests. |
|--------|----------|---------|-----------|---------|--------|----------|---------|-----------|--------|
| No. | Sq. Ins. | Lbs. | Lbs. | Hours. | No. | Sq. Ins. | Lbs. | Lbs. | No. |
| 200 | 2 | 101.32 | 86 | 25 | 5 | 3 | 82 | 65 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 86 | 78 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 87 | 69 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 88 | 63 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 94 | 74 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 94 | 77 | 1 |
| 100 | 2 | 101.32 | 86 | 25 | 5 | 3 | 99 | 92 | 1 |

7.5 lbs. per sq. in.
lay, then tested
1.100 lbs. per

The Crocker-Wheeler, New York.



This Motor has been designed to remove difficulties which experience has developed to be attendant upon other instruments of like purpose.

Care has been taken in its design and construction. The bearings are oiled automatically, and magnetic circuit is made as perfect as practicable. Its centre of gravity is low, machine strongly built, weight of it comparatively low, and its efficiency high.

ned to run at low speed, in order to reduce wear, heating journals, etc.

| Light | Velocity. | Pulley. | | Dimensions. | | | Between Bolt-holes. | | Shafts. | |
|-------|-----------|----------------|-------|-------------|----------|---------|---------------------|----------|---------|---------------------|
| | | Diam. | Face. | Length. | Breadth. | Height. | Length. | Breadth. | Diam. | From Base of Motor. |
| 18 | 2100 | G 1.5 F 1.5 | .375 | 7.375 | 5.5 | 7.875 | 4.625 | 2.75 | .25 | 3.375 |
| 26 | 2000 | G 1.5 F 1.5 | .375 | 9.75 | 7.5 | 8.5 | 6.375 | 3.375 | .375 | 3.6875 |
| 26 | 1800 | 2.5 | 1.25 | 9.75 | 7.5 | 8.5 | 6.375 | 3.375 | .375 | 3.6875 |
| 55 | 1500 | 3 | 2 | 14.75 | 9.5 | 10.75 | 9.75 | 4.3125 | .5625 | 4.75 |
| 50 | 1350 | 3.5 | 3 | 18.25 | 11 | 13 | 11.5 | 5 | .6875 | 5.75 |
| 57 | 1050 | 4 | 3.5 | 19 | 13.25 | 15.5 | 12.25 | 7 | .875 | 7.0625 |
| 90 | 1050 | 6 | 3 | 25 | 12.625 | 18.375 | 18.25 | 9.25 | 1 | 8.25 |
| 90 | 1000 | 8 | 4 | 26.25 | 15.625 | 18.375 | 18.25 | 9.25 | 1 | 8.25 |
| 85 | 1000 | 7.5 | 4.5 | 18 | 8.75 | 21 | 18.75 | 9.75 | 1.125 | 9.25 |

G Grooved. F Flat.

ation of the Motor —For Printing-rooms and mechanical Shops of medium and Elevators, one of 5 HP is sufficient.

To Compute Power required for Elevators.

—Multiply twice * product of weight to be raised in lbs., and height of ascent per minute; divide by 33 000, and the quotient will give the number of red.

all Motors, of .166, .125, and .0833 HP are adapted for operating Fans, Sewing-machines, Small Lathes, Presses, Tools, Models in operation, Road-ventisements, Organ blowing, Baffling wheels, Knife sharpeners, Cloth and sitting, Experimental models, etc., etc.

Electric Fans.

entilation of Offices, Restaurants, Kitchens, Sick-rooms, etc., etc.
ructed in various styles, Plain and Nickel-plated.

Fans 12 Inches in Diameter

er, .0833 HP motor. — Fast, .125 HP motor — Double (a Fan at each side), F motor, or one 16-inch Fan.

Construction. —Variable speed, Fan 24 ins. in diam. —20 Inch, 125 HP motor.

Electric Pumps.

IP, will elevate 500 gall. water per day of 10 hours 100 feet in height.
are arranged to operate automatically, so that when a receiving tank
ump is arrested.

Capacity of Pump per Hour.

| Feet. | HP | Gallons. | HP | Gallons. |
|-------|----|----------|----|----------|
| 5 | .5 | 370 | 3 | 1670 |
| 1 | 1 | 750 | 5 | 2600 |

Are Circuit Motors.

all others. The manner of connecting the circuit as
n that of other motors.

.125 to 5 HP.
nnected to the are circuit

to cover loss of power

Dynamo Leather Belts.

Belting.—For Dynamos and Electric Light Machinery should be double and endless, and not over .33 inch thick, when run at a velocity of 4000 feet or more per minute; should be perforated to prevent air-cushioning, the perforations may be .09375 inch in width, .28125 inch in length, and placed 1.5 inches apart; furnishing about 50 openings per sq. foot of belt, without material injury to the tensile or operating strength of it.

In order to protect the surface of a Dynamo Belt it should be rendered impervious to the mineral oil used on it, which is destructive to the fibres of the leather.

LEATHER LINK BELTS.

Where Belts are run at right angles and at short distances apart, Leather Link Belts are recommended, as they are very pliable and have uniform oscillation.

Link Belts made .6875 inch thick, forming when combined two full circles, assure the required uniformity of oscillation.

WIRES AND CABLES.

Telegraph, Telephone, and Electric Light Wires and Cables.

For Aerial, Sub-marine, and Underground.

The International Okonite Co., Ltd., New York.

Insulation.—In consequence of the decomposing influence of the elements upon insulated wires exposed to them, it is necessary that their insulation should be as perfect in construction and enduring in material as it is practicable to attain. In order to effect an enduring insulation this Company use a compound, termed Okonite, a material possessing both tenacity and resistance to abrasion, while it is equally unaffected by extremes of temperature with insulation of a high order.

Telegraph and Telephone Wire.

Size of Insulation and Diameter per B W G; External Diameter of Insulation in 32ds; Weights per Mile in Lbs., and Insulation Resistance per Mile in Megohms.

| No. | Diameter. | Insulation. | | | | No. | Diameter | Insulation. | | | |
|-----|-----------|-------------|--------------------|---------|-------------|-----|----------|-------------|--------------------|---------|-------------|
| | | Gauge. | External Diameter. | Weight. | Resistance. | | | Gauge. | External Diameter. | Weight. | Resistance. |
| 40 | 24 | 14 | — | 29 | 1000 | 43 | 1000 | 55 | 16 | — | 6 |
| 41 | 24 | 13 | 3 | 33 | 1200 | 48 | 1200 | 56 | 16 | 6 | 6.5 |
| 42 | 22 | 13 | 3 | 45 | 1000 | 60 | 1000 | 57 | 16 | 5 | 7 |
| 43 | 20 | 12 | 3.5 | 56 | 1000 | 71 | 1000 | 58 | 16 | 3 | 8 |
| 44 | 20 | 11 | 4 | 65 | 1200 | 80 | 1200 | 59 | 16 | 2 | 9 |
| 45 | 18 | 11 | 4 | 81 | 1000 | 96 | 1000 | 60 | 14 | 8 | 5 |
| 46 | 18 | 10 | — | 91 | 1200 | 107 | 1200 | 61 | 14 | 7 | 5.5 |
| 47 | 18 | 8 | 5 | 119 | 1600 | 137 | 1600 | 62 | 14 | — | 6 |
| 48 | 18 | — | 6 | 155 | 2000 | 175 | 2000 | 63 | 14 | 5 | 7 |
| 49 | 18 | 5 | 7 | 197 | 2400 | 220 | 2400 | 64 | 14 | 3 | 8 |
| 50 | 18 | 3 | 8 | 244 | 2600 | 271 | 2600 | 65 | 14 | 2 | 9 |
| 51 | 18 | 2 | 9 | 299 | 3000 | 329 | 3000 | 66 | 12 | 6 | 6.5 |
| 52 | 16 | 2 | 4.5 | 120 | 1000 | 143 | 1000 | 67 | 12 | 5 | 7 |
| 53 | 16 | 2 | 5 | 140 | 1400 | 158 | 1400 | 68 | 10 | 5 | 7 |
| 54 | 16 | 7 | — | 157 | 1500 | 176 | 1500 | 69 | 10 | 4 | 7.5 |

Voltmeters and Ammeters.

Esteston Electrical Instrument Co., Newark, N. J.

Readings are Direct Reading.—A multiplying constant being unnecessary, with voltmeters of high range, as a simple inspection of position of pointer indicates value in amperes or volts, and as pointer immediately becomes a "dead beat," it reduces period for reading, added to which there does not exist a "magnetic lag" which induces different deflection with like current or voltage, as in instruments in which vibrating parts are of iron.

Readings of the scale commence at 0, and the uniformity of the divisions insures the visual subdivision of them.

Adjustments for Temperature are unnecessary, as it is for the long coil of the voltmeter, being not in excess of .25 per cent. for a range of 35° above or below 70°, and the ammeter less than 1 per cent. for a like range of temperature.

Use.—These instruments can be retained in circuit without injury, as the effect, except in the high ranges, is inappreciable.

Rating Coil.—In Voltmeters provided with one, changes in the scale value without accidental injury are readily checked.

The respective parts are made to a uniform gauge, and are interchangeable.

| able in olts, up to | Read- able to | Standard Voltmeters. DESCRIPTION. | Telegraphic Code. |
|---------------------------|------------------|---|----------------------|
| 150 | .1 | By single volt divisions..... | Reprint. |
| 150 | .1 | " " " " " " " " " " " " | Repraisal. |
| 150 | .1 | Scale of double values, with ratio of 30..... | — |
| 15 | .1 { | Each division on upper scale-values = 1 volt; and on lower scale-values .033 volt, readable to .0033.. | Reproach. |
| 150 | .1 { | Scale of double values with ratio of 10..... | — |
| 15 | .1 { | Each division on upper scale-values = 1 volt; and on lower scale-values .1 volt, readable to .0r..... | Reprune. |
| 300 | .2 | By 2-volt divisions..... | Reptile. |
| 450 | .33 | By 3- " " " " " " " " " " " " | Republic. |
| 600 | .5 | By 5- " " " " " " " " " " " " | Repulse. |
| 600 | .5 | " " " " " " " " " " " " " " " " " " | Requitall. |
| 150 | .1 { | Scale of double values, with ratio of 4, reading by 4- volt divisions, to .5 on upper scale-values; and by single-volt divisions to .1 on lower scale-values .. | Resemble. |
| 750 | .5 { | Scale of double values with ratio of 5, reading by 5-volt divisions, to .5 volt on upper scale-values, and by single-volt divisions to .1 volt on lower scale-values. | Reservoir. |
| 150 | .1 { | " " " " " " " " " " " " " " " " " " | |
| 600 | .5 { | Scale of double values, with ratio of 2, readings by 4- volt divisions, to .5 volt or upper scale-values; and by 2-volt divisions to .2 on lower scale-values.... | Reside. |
| 300 | .2 { | " " " " " " " " " " " " " " " " " " | Residue. |
| 150 | .5 { | By 5-volt divisions..... | Resin. |
| 750 | .5 { | By 10- " " " " " " " " " " " " | |

1.—Calibrating coils and contact keys are attached to instruments only as stated.—2. Instruments provided with a contact key can be retained in per cent circuit by depressing it and giving it a quarter turn.—3. All instruments above 150 volts are provided with India-rubber-covered binding-posts.—4. Meters of other ranges and scale-values are furnished as required.

High Range Voltmeters No. 1.

These instruments part of the resistance is contained in a separate box, designed to obtain high insulation, and without calibrating coil or contact key. The scale is graduated from 0 to 150 volts and a resistance box, multiplying valve of scale divisions.

| Volts to 150. | Scale. | No. | Volts to 150. | Scale. | No. | Volts to 150. | Scale. |
|---------------|--------|-----|---------------|--------|-----|---------------|--------|
| 2250 | By 15 | 15 | 3750 | By 25 | 17 | | |
| 3000 | " 20 | 16 | 4500 | " 30 | 18 | | |

Word Codes:—13. Resolute; 14. Resolve; 15. Resolvent; 16. Resonant

—Above are graduated to read in lamps in addition to simple inspection number of arc lamps open circuit giving intermediate ranges, applicable to these lamps.

—Reversing Key. When many and rapid tests of unknown polarity are required, it is possible by turning the milled head through 180° to reverse the current in the instrument by an additional

962 VOLTMETERS AND AMMETERS.—RAILROAD CRANE.

| No. | Scale in Volts. 0, up to | Description. | Telegraphic Code. |
|-----|-----------------------------|--|-------------------|
| | | | |
| 1 | .02 | By 100 divisions, each readable to .1..... | Restless |
| 2 | .01 | Right and left by 100 divisions, each representing .0001 volt. | Restless |
| 3 | .01 | Right and left by 100 divisions, supplied with contact key, reducing sensibility 10 times..... | Restless |

Volt-Ammeters.

No. 1. Scales 0 to 150 volts and 0 to 1500 mil-amperes. Reading to 150 volts by single-volt divisions, or to 1.5 amperes by .01-ampere divisions.—Retiary.

Mil-Ammeters and Ammeters.

| No. | Scale in mil-amperes. | Single-scale divisions, readable | No. | Scale in mil-amperes. | Single-scale divisions, readable |
|-----|-----------------------|----------------------------------|-----|-----------------------|----------------------------------|
| 0 | 150 | 1 mil-ampere, to .5 do. | 3 | 1000 | 10 mil-ampere, to 1 do. |
| 1 | 300 | 2 do. to .2 do. | 4 | 1500 | 10 " to 1 do. |
| 2 | 600 | 5 do. to .5 do. | | | |

Telegraphic Codes.—0. Retiary; 1. Retiary; 2. Retouch; 3. Reuben; 4. Reunite.

No. In volts up to

For Physicians.

| No. | Scale | Description | Telegraphic Code. |
|-----|---------------|---|-------------------|
| 5 | { 500 50 } | Scale of double values, with ratio of 10, each division on upper scale-values is 5 mil-amperes, and each on lower scale-values .5 mil-ampere readable to .05..... | Revel |
| 6 | { 500 10 } | Do. do. with ratio of 50, each division on upper scale-values is 5 mil-amperes; each division on lower scale-values is .1 mil-ampere readable to .01..... | Revel |
| 7 | { 500 10 } | Do. No. 6, with a resistance box having 2 values which, when used in connection with lower scale, reads to 10 volts by .1-volt divisions, and to 100 volts by 1-volt divisions..... | Revel |

Ammeters.

| No. | Scale in amperes. | Single-scale divisions of an am- pere, readable | No. | Scale in amperes. | Single-scale divisions of an am- pere, readable |
|-----|-------------------|--|-----|-------------------|--|
| 1 | 5 | .05 ampere, to .005 do. | 4 | 50 | .5 ampere, to .05 do. |
| 2 | 15 | .1 do. to .01 do. | 5 | 100 | 1 do. to .1 do. |
| 3 | 25 | .25 do. to .025 do. | 6 | 150 | 2 do. to .1 do. |

Telegraphic Codes.—1. Reverse; 2. Reversed; 3. Reversal; 4. Review; 5. Revile; 6. Revial.

Railroad Crane.

The Farrell Foundry and Machine Co., Ansonia, Conn.



Post.—Of cast iron, in one piece, fitted to deck-plate, with faced joints and secured by bolts running through a stone foundation, set up on anchor plates on its under side.

Jib.—Of two wrought-iron beams, bolted at head and foot to a bonnet and shoe, with tie bolts between them, and secured to the post by bolts which lead from its head to a yoke, which turns on a pin in the hub.

With a pin is fitted into head of post, on which the jib turns.

Is secured by two bolts, which lead down through and are secured to the foundation.

Double and set for both fast and slow motions, and detachable, moving load by a brake.

One, and all sheaves have roller bushes.

Capacity.

| Radius. | Capacity. | Weight. | Radius. | Capacity. | Weight. |
|---------|-----------|---------|---------|-----------|---------|
| Feet. | Tons. | Lbs. | Feet. | Tons. | Lbs. |
| 15 | 6 | 10 400 | 15 | 15 | 11 400 |
| 15 | 10 | 14 600 | 20 | 20 | 21 400 |

or operation on Wrecking and Constructing cars.

Vacuum Pumps.

Guild & Garrison, Brooklyn, N. Y.

Vacuum Pumps.—Air pumps are so termed when they are used in connection with vacuum pans, multiple effects, or filters.

It is impracticable to define a general rule for their capacity, as the circumstances of their operation vary in different cases.

Vacuum Evaporators.—Their dimensions depend upon the temperature to which they are submitted, the evaporation, character of the liquid concentrated, vacuum desired, and type and efficiency of the condenser.

Dry Exhaustion.—When air alone is withdrawn.

$\left(\frac{V'}{V+V'}\right)^n M = Q$ V and V' representing volumes of cylinder and receiver, M volume of air in receiver at commencement of operation, both in cube feet, n number of strokes of piston, and Q volume of air remaining after n strokes of piston.

Condensation.—There are two systems in operation for vacuum pans and multiple effects, viz.—

Dry System.—Where the condenser is fitted with a leg pipe or barometric tube, through which the injected water passes off by gravitation.

Wet System.—When the pump receives and discharges the condensing water, in addition to its maintaining a vacuum.

In either system the pump is required to discharge: 1st. The air contained in the injection water, in the liquid, and in the pan, pipes, and condenser.—2d. The incondensable gases evolved from the liquid in operation.

Notes.—The Pan and its immediate connections are made of iron, copper, bronze, or alloys.

In designating the design and construction of pump required, the liquor, the volume, the degree of concentration required, and the time in which the operation must be completed, should be furnished.

To facilitate transportation, the bed plates of the large sizes are cast in two parts and bolted together.

An order for a pump should state: 1st. What liquor, and volume of it, is to be evaporated in a given period, as an hour? 2d. What the diameter of pan or evaporating vessel, and what that of vapor pipe when it enters condenser? 3d. What the heating surface of pan, and has it a steam-jacket and coils, and if coils, what is their diameter and length? 4th. If heating surface is of iron, brass, or copper? 5th. What the average temperature of condensing water, and what the volume of it?

Duplex Vacuum Pumps.

Fly-wheel Type for "Dry" or "Wet" System.

| DIMENSIONS. | | | Stroke. | Volume per Revolution. | Displacement, at 75 Feet Piston Speed per Minute. | | Diameter of Pipes. | | |
|-------------------------------|------------------------------|------|------------|------------------------|---|-----------|-------------------------------|--------|----------|
| Diameter of Vacuum Cylinders. | Diameter of Steam Cylinders. | | | | Per Min. | Per Hour. | Suction and Discharge. | Steam. | Exhaust. |
| Ins. | Ins. | Ins. | Cub. feet. | Cub. feet. | Cub. feet. | | *a circumference of the case. | Ins. | Ins. |
| 6 | 5 | 6 | .589 | 29.45 | 1 767 | | | 1 | 1.25 |
| 8 | 6 | 6 | 1.047 | 52.35 | 3 141 | | | 1.25 | 1.5 |
| 10 | 7 | 6 | 1.635 | 81.82 | 4 909 | | | 1.25 | 2 |
| 10 | 8 | 6 | 1.635 | 81.82 | 4 909 | | | 1.5 | 2 |
| 12 | 8 | 9 | 2.356 | 117.81 | 7 068 | | | 1.5 | 2 |
| 12 | 9 | 9 | 2.356 | 117.81 | 7 068 | | | 1.5 | 2 |
| 14 | 9 | 9 | 3.207 | 160.35 | 9 621 | | | 1.5 | 2 |
| 14 | 10 | 9 | 3.207 | 160.35 | 9 621 | | | 1.5 | 2 |
| 16 | 10 | 9 | 4.188 | 209.4 | 12 564 | | | 1.5 | 2 |
| 16 | 12 | 9 | 4.188 | 209.4 | 12 564 | | | 2 | |
| 18 | 12 | 9 | 5.301 | 265 | 15 898 | | | | |
| 18 | 14 | 9 | 5.301 | 265 | 15 898 | | | | |
| 20 | 12 | 9 | 6.545 | 327.25 | 19 635 | | | | |
| 20 | 14 | 9 | 6.545 | 327.25 | 19 635 | | | | |
| 22 | 14 | 9 | 7.919 | 395.95 | 23 757 | | | | |
| 22 | 16 | 9 | 7.919 | 395.95 | 23 757 | | | | |
| 24 | 16 | 9 | 9.424 | 471.25 | 28 275 | | | | |
| 24 | 18 | 9 | 9.424 | 471.25 | 28 275 | | | | |

Drawing, Tracing, Profile, Cross-Section, Photo-printing Papers and Drawing Cloths.

For Designations and Dimensions see p. 29.

In Sheets. Whatman's Hand-made.—Three styles of **HP.** signifies "Hot Pressed," has a smooth surface, suited for pencil and fine drawings.—**N.** signifies "Not Hot Pressed," has a finely grained surface, for general purposes and water-colors.—**R.** signifies "Rough," has a coarsely grained surface, used for very bold drawing and sketching (Torchon Paper).—"Selected Best" and "Retree" are of like quality. "Selected Best" is free from imperfections.

In eight sizes, from 13×17 ins. to 31×53 ins. Royal, Imperial, and Double Elephant, also with rough surface and with all surfaces extra thick and extra heavy.

Universal, for general drawing and water-colors, six sizes, 14×17 ins. to 31×53 ins.—**Normal**, Not hand-made, but very similar to the Not Hot Pressed, in Royal, Imperial, and Double Elephant.—**Duplex**, Cream color, for fine detail and general drawings, in Royal, Imperial, and Double Elephant.—**Duplex**, Drab Color, in Double Elephant only.—**Bristol-Board** (Reynolds's), Five sizes, 12.5×15.5 ins. to 21.5×28.75 ins., 2, 3, or 4 sheets in thickness. Patent, 10×15 ins. and 15×20 ins.

Tracing Papers.—**Vegetable** (French), Five sizes, 13×17 to 29×42 ins.—**Acanthus**, Very thin, 30×40 ins.—**Hermes** (common), 20×30 and 30×40 ins.—**Paragon**, Tough, 20×27 and 27×40 ins.—**Corona**, Thick, 27×40 ins. Of these the Vegetable, Ceres, and Corona are natural tracing paper (not oiled, etc.).

In Rolls.—**Duplex**, Medium, cream color, 36, 42, and 56 ins. in width.—**Thick**, drab color, 36 and 56 ins. in width.—**Universal**, For general drawing and tracing, etc., 36, 42, 56, and 62 ins. in width.—**Anvil**, Medium and thick, surface and appearance similar to Whatman's Not Hot Pressed, medium, 36, 42, and 56 ins. in width; thick, 62 ins. in width.—**Paragon**, Pebbled surface (similar to egg-shell), thin, medium, thick, and extra thick. All 58 ins. in width, except medium, which is also 36 and 42 ins. in width.—With smooth surface (similar to Whatman's), on one side, smooth on the other), medium and thick, both 58 ins. in width. can be had by the yard, in 10-yard lengths or in rolls of about 35 lbs.

Detail.—**Economy**, Nearly white, thin, 50-yard rolls, 60 ins. in width.—**Duplex**, Medium and heavy (Manilla), 36, 42, and 54 ins. in width, in 100-yard and 200-yard rolls.

Mounted.—**Universal**, **Duplex**, **Anvil**, and all the **Paragon Paper**, Extra thick, are mounted on muslin, in all the widths, by the yard, or a 20 yard rolls. All the sheet papers are also to be obtained mounted.

Photo-printing Papers.—**Helios Blue Print**, medium and thick compared (sensitized), 24 to 42 ins. in width; unprepared, 24 to 54 ins. in width. in sheets, five sizes. **E. T. Paper**, very thin, for mailing, prepared and unprepared, 24, 30, and 36 ins. in width. The prepared paper is in 10-yard rolls; unprepared, 50-yard rolls. **Blue Process Cloth**, prepared and unprepared, in 10-yard rolls and 42 ins. in width. **Nigrosine** (Positive Black Process), 10-yard rolls, 36, 42, and 42 ins. in width, prepared only; Cloth, 36 and 42 ins. in width, prepared only.

Profile Paper.—In orange or green, rulings 4×20, 4×30, and 5×30 sheets about 15×42 ins.; also continuous in 50-yard rolls or by the yard.

Cross Section Papers.—10×10, 16×16, per inch, 5×5 to the half inch and 8×8, millimeter, all in sheets about 16×20 ins. 10×10, 16×16 and millimeter also continuous in rolls and in the usual variety of colors.

Ruled Cross Section, common, 5×5, 10×10, 8×8, and 4×4 per inch.

Parchment.—Medium and thick, for Blue Prints direct from drawing table out tracing.

Tracing Papers.—**Parchment**, 20 yards, 37 ins. in width.—**Albion**, thin, 40 yards, 42 ins. in width.—**Patera** and **Gothic**, very tough, 30 yards, 42 ins. in width.—**Doric**, medium, 20 yards, 42 ins. in width.—**Alba**, for transferring, 30 yards, 54 ins. in width.—**Lotus**, thin, 20 yards, 42 ins. in width.—**Libra**, thin, 30 yards, 42 ins. in width. The **Parchment**, **Alba**, **Lotus**, and **Libra** are natural **Tracing Papers** (not oiled, etc.). All these papers can also be obtained by the yard.

Tracing Cloths.—**Excelsior**, Extra fine, very thin, 30, 36, and 42 ins. in width.—**Imperial**, Both sides glazed or one side dull, 30, 36, and 42 ins. in width.—**Union**, Thick, for coarse tracings, 30, 37, 40, and 43 ins. in width.

Mechanical Refrigeration.

The De La Vergne Refrigerating Machine Co., New York.

Mechanical Refrigeration is effected by *Compression, Condensation, and Expansion* of a liquefiable gas.

The Refrigerating or Heat-absorbing agents are Ammonia, Ether, Sulphurous Oxide, Carbonic Acid, etc., which undergo the operations at given. The De La Vergne Machine is operated with Ammonia.

Compression.—The gaseous agent is compressed if Ammonia is it to from 125 to 175 lbs. per sq. inch; during which operation heat is developed in proportion to the pressure exerted upon the gas, or the relative time to which it has been reduced.

Condensation.—The heat developed in the operation of compression is withdrawn from the compressed gas, which is forced through coils of metal pipe, surrounded with cold water. As soon as the condition of saturation is reached, the gas assumes a liquid state.

Expansion.—The liquefied gas is also passed through coils of metal pipe, suspended or seated in a space where the substance to be cooled, as water, brine, beer, etc., is introduced; the pressure in the interior of the coils being at a lower point than that required for the maintenance of the gas in the liquid state.

The liquefied gas, upon entering these coils, again expands, and extracts from them and the substance around them the same quantity of heat that was previously given up by the gas to the water of condensation.

The gas, having passed through this routine of operation of refrigeration is now in a condition to be used in a repetition of it.

The gas is forced through these coils by the pressure in the condenser, which the use of Ammonia, is generally from 125 to 175 lbs. per sq. inch. Under this pressure and the cooling action of the water, liquefaction occurs, and the resulting liquefied gas flows to a stop-cock, having a minute opening, by which the pressure in the condenser is reduced to a pressure of 10 to 30 lbs. per sq. inch in the expansion coils, and where the liquid through reduction in pressure is again transformed into a gas. By the exhausting operation of a gas-pump, this pressure is maintained and then the gas is forced by compression into the condenser again.

Thus the expansion coils, although similar to those for condensation, are operated for the reverse, which is the absorption of heat by the liquefied gas, instead of extraction of heat from it.

In Operation, heat is transmitted from the outside through the walls of the expansion or cooling coils, and is absorbed by the expanding liquefied gas within the coils. This heat is borne by the gas through the pump into the condenser, where it is in turn transferred to the cooling water through the walls of the condensation coils, and ultimately carried away by this water.

NOTE.—Liquefied ammonia in a gaseous condition at atmospheric pressure and temperature of expands about 1000 times, and upon its expansion re-absorbs a quantity of heat equal in amount to originally held and evolved from it during liquefaction.

The liquefied gas, entering the coils through the minute opening in stop-cock immediately relieved of a pressure of 125 to 175 lbs., that requisite to maintain it in a liquid state, when it boils and expands into gas. To obtain this, heat is required, and which alone can be supplied from the substance surrounding the coils, such as air, brine, water, etc.

As a result, the surrounding substance is reduced in temperature, the quantity of heat withdrawn by the gas being the same as that which was withdrawn during its liquefaction in the condenser.

Consequently, if the expansion coils are set in an insulated space, refrigerated; and if brine or any liquid surrounds the coils, it will, by its expansion, and brine, in this condition led into a space through which it will refrigerate it.

Results of Operation of Refrigerating Machines of 200* Tons.

At Lion Brewery, New York. Duration of Test 11 h 20 min.

Steam Cylinders.—Diameter, 36 ins.; *Stroke of Piston*, 36 ins.——*Pressure of steam* (mean), 48.4 lbs.

Gas Compressors.—Two double acting; diam. 18 ins.; *Stroke of Piston*, 36 ins.; back-pressure, 28.22 lbs.; condenser, 180.78 lbs. per sq. inch; *Revolutions*, 39.55 per minute.

Test for cooling made by running water of a mean temperature of 100.95° over wort, Baudet Cooler, and cooling same to a mean temperature of 50.77°.

Refrigeration, equal to melting of 210 tons Ice per day of 24 hours.

Horse Power.—1HP = 313, and assuming consumption of coal at 3 lbs. per hour per 1HP, ratio of refrigeration = 20.84 lbs. ice per lb. of coal.

If operated under ordinary condensing pressure of 156 lbs., the 1HP would be 274 and ratio 23.47 lbs. ice per lb. of coal; 1HP per ton of ice per day = 1.183.

Of a 20-Ton Machine. At Bohlen-Huse Machine and Lake Ice Co., Memphis, Tenn. Duration of Operation 20 Days.

Steam Cylinder: Diameter, 22 ins.; *Stroke of Piston*, 28 ins.; *Steam*, 93.49 lbs. per sq. inch.—*Gas Compressors*, Two single-acting: diam., 14 ins.; *Stroke of Piston*, 14 ins.—*Revolutions*, 40 13 per min.—*Temperatures:* Cooling water 63°, brine 18.6°, coal consumed, 180 597 lbs.; Ice produced. 1 221 172 lbs.—*Ice-making*, 26.83 tons per day of 24 hours.—*Steam-boiler* evaporated 5.5 lbs. water per lb. coal.

* All tons are given at 2240 lbs. See foot-note, p. xxvi.

FORCITE POWDER.

American Forcite Powder M'g Co., New York.

Forcite.—Is an improvement in Nitro-glycerine compounds, and it presents the following elements:

It is less sensitive to shock than other explosives.

Assuming Dynamite No. 1 as the Standard = 100.

Forcite No. X, 95 per cent. Nitro-glycerine, 133 per cent. intensity.

| | | | | | | | |
|----|----|---|---|---|-----|---|---|
| 1* | 75 | " | " | " | 125 | " | " |
| 3† | 40 | " | " | " | 95 | " | " |

* 25 per cent. stronger than Dynamite No. 1. † Within 5 per cent. the strength of No. 1, 75 per cent.

It is more powerful than any other known explosive in our market.

See Report of Henry L. Abbott, Lieut.-Col. E. U. S. A.

It is safe in handling and transportation, quintuple force-caps being applied to explode it, and free from noxious fumes. Water-proof, free from the absorption of moisture, and is not injured by submersion in water.

Directions in Use.

In Blasting, fill the hole, and thoroughly tamp the charge.

Thaw it, if frozen, as frozen powder will not explode with its proper effect.

— or caps should be maintained dry, and are not to be stored in sand the powder.

— ignited by weak caps, instead of being exploded, emits noxious vapors.

Int. o' glycerine in Brands of Forcite.

Int. o'

No. 2..... 50 | No. 3..... 40 | No. 3 B..... 3

" 3 X..... 45 | " 3 A..... 35 | " 3 C..... 3

der, one and a half times stronger. Forcite

SURFACE CONDENSATION.

Wheeler Condenser & Engineering Works, New York.

Construction.—The Wheeler Condenser, alike to others for the same use, is an elongated vessel, cylindrical or cubical, with the necessary attachments for Steam and water connections.

distinguishing features are: The exhausted steam, upon entering the condenser, impinges upon a perforated scattering plate, which distributes it all over the tubes and thus diverts the deteriorating effect of the direct impingement of it upon one portion of the tubes; the steam, expanding and flowing above the tubes, is reduced in pressure, and consequent temperature, and it flows into contact with the surfaces of the tubes.

Each pair of tubes is composed of an external and internal tube, set horizontally, the inner tube having an open end, the other end being screwed on a removable head or vertical diaphragm, which is set at a space of a few inches from a like head, into which one end of this large tube is screwed, the other end being closed by a screw cap.

This design permits the tubes to expand or contract, without the use of packings or ferrules of any kind, as only one end of each tube is fixed. The tubes are tinned both externally and internally, and can be readily drawn for cleaning, etc.

Operation.—The tubes are divided into two distinct tiers; the condenser water flowing through the small tubes in the lower division passes out at its open ends and through the annular space between their external surfaces and the internal surfaces of the larger tubes, and from thence into the upper division, and through its tubes in like manner to the space between the heads referred to, and finally out through the discharge pipe.

The circulation of the condensing water is by this manner of flowing very active, and consequently a less volume of it is required, and therefore a less tube surface needed for a required volume of condensation.

Results of an Operation to Determine the Efficiency of this Condenser, with and without a Vacuum.

Steam Condensed per Hour per Sq. Foot of Condensing Surface.

| Instr. | Vacuum. | Temperatures. | | | | Condenser. | Temperatures. | | | | Steam Condensed. |
|-------------|---------|------------------|------------------|------------|------------------|-----------------|------------------|------------------|------------|------------------|------------------|
| | | Injection Water. | Discharge Water. | Reservoir. | Steam Condensed. | | Injection Water. | Discharge Water. | Reservoir. | Steam Condensed. | |
| | Inch. | Deg's. | Deg's. | Deg's. | Lbs. | | Deg's. | Deg's. | Deg's. | Lbs. | |
| High Vacuum | 24.5 | 56.5 | 98 | 138 | 101.8 | Without Vacuum* | 78.5 | 139 | 201 | 204.2 | |

* As a simple surface condenser without air pump attached.

REFRIGERATING AND ICE-MAKING.

A Refrigerating Machine is one that produces as low a temperature as a volume of ice, at the temperature attained, would in melting from the nature of the air, or void to be refrigerated = 142° (142.6°) of temperature are required to transfer one lb. ice at 32° to one lb. water at 32° , which difference represents the Latent heat.

In order to operate such a machine for the formation of ice, instead of 142° , about 236° .

Assume the water from which the ice is to be formed is at a temperature of 72° ; then to reduce it to 32° , before ice can be formed, 40 units are to be abstracted from each lb. of water; then from the lb. of water of 32° to reduce it to one lb. ice

If the ice is produced at the general temperature of 18° , and the Specific Heat is taken at $.5^{\circ}$; then, $32 - 18 \times .5 = 7^{\circ}$. To reduce this water from 72° to 18° there is a reduction of 40° or thermal units from each lb. of water.

If ice is produced at 18° , Then 7° additional, as deduced above, are required.

In practice it is observed that the average loss of temperature by radiation from the freezing tank, melting the external surface of the ice, to withdraw it from the molds, etc., is fully 20 per cent. of the total capacity of the machine. Hence the 236° which are to be abstracted from the water per lb. of ice, in order to freeze it to ice, 47.2° are lost by radiation. And $40 + 142 + 7 + 47 = 236^{\circ}$ are to be abstracted from each lb. of water of 72° , in order to produce 1 lb. ice at 18° .

Consequently, If 142° are required in Refrigerating machine and 236° in ice-making, the relative requirements are as 1 to 1.66 or as 6 to 10.

Refrigerating Capacity.—Of a machine is designated by number of lbs., or tons of Ice, which it is capable of producing.

One lb. of ice at 32° absorbs 142° or thermal units in melting. Hence, one of ice absorbs $142^{\circ} \times 240 = 34080^{\circ}$, and a machine of 50 tons' capacity absorbs $3408000 \times 50 = 15900000^{\circ}$ every 24 hours of its operation.

Ice-making Capacity.—Of a machine is also designated by number of lbs., or tons of Ice, which it is capable of producing.

To freeze one lb. of water at 72° to ice at 18° , it requires the absorption of 236° viz., To reduce one lb. of water at 72° to 32° , it requires the absorption of 40° ; to freeze it requires 142° ; to reduce ice from 32° to 18° requires $14 \times .5 = 7^{\circ}$ (Specific heat of ice = .5). Reduction of temperature from surface of freezing tank to withdrawing the ice from its molds by the application of heat, about 25% of capacity of machine = 20% of $236 = 47^{\circ}$. Hence, Total heat to be absorbed per lb. of ice = $40 + 142 + 7 + 47 = 236^{\circ}$.

Ratio of Capacity of Refrigerating to Ice-making.—As $142 : 236 :: 6 : 10$, a machine of 10 tons capacity, or a Refrigerating machine of 9.97 tons capacity will produce about 10 tons of ice in the same period.

Highest Elevation of a Lake.

Colorado.—"Green Lake" is 10252 feet above level of the sea and 300 feet depth.

Magnifying.

Bavaria, Munich, possesses a microscope that magnifies 16000 diameters.

Power of Screw Bolts.

Results of an Experiment.

Wrought-iron.—Diameter, 2 ins. Thread, V. Pitch, .22 ins.

Mean Power applied at a circumference of 78.85 ins., 213 lbs.

Loss by friction, 10.19 per cent.

(Jas. McBride, M. Am. Soc. E. E.)

Duration of Railroad Cross-ties.

Duration of Following Woods.

| Wood. | Years. | Wood. | Years. | Wood. |
|--------------------|--------|-----------------|--------|------------------|
| White Cedar..... | 8.75 | Chestnut..... | 7.5 | Yellow Pine..... |
| White Oak..... | 8 | Red Spruce..... | 6 | Hemlock..... |
| Black Cypress..... | 8 | Red Oak..... | 5.5 | Tamarack..... |

The elements of durability are Resistance to decay and to wear. White Oak and Yellow Pine are the best, and are the most durable. Yellow Pine resists decay, but not wear. Red Cedar and Black Cypress resist decay, but not wear.

Ties should not be cut when the tree is in leaf, and should be well reserved by some antiseptic process before being laid.

Proper draining of a road-bed will add to the duration of ties, and all on their surface by tools, etc., should be avoided, and all spike-holes should be filled with tar or oil. (H. W. Reel.)

GAS AND ELECTRIC LIGHTING.

(In Addition to pp. 583-587). Gas.

The Power and Consumption of Different Burners.

| | Candle Power. | | Consumption per Hour per Lamp. | Burner. | Candle Power. | | Consumption per Hour per Lamp. |
|---------|---------------|--------------------|--------------------------------|---------|---------------|--------------------|--------------------------------|
| | No. | Per Foot per Hour. | | | No. | Per Foot per Hour. | |
| | No. | Feet. | | | No. | Feet. | |
| | 20 | 2.33 | 4.3 | Flat | 60 | 4 | 15 |
| from .. | 11.5 | 2.5 | 4.6 | Clus- | 90 | 5.5 | 20 |
| to..... | 13.8 | 3 | 4.8 | Flame | 150 | 5 | 30 |

Electric.

Arc Lamps.

| Candle Power. | | | | | Watts Required. | Units per Hour. | Relative Costs* of Gas, Electric=1. |
|---------------|-----------|------------|------------|------------|-----------------|-----------------|-------------------------------------|
| Horizontal. | Angle 7°. | Angle 20°. | Angle 30°. | Angle 40°. | | | |
| No. | No. | No. | No. | No. | No. | Hour. | |
| 98 | 175 | 307 | 322 | 460 | 300 | .3 | 2.67 |
| 156 | 300 | 350 | 546 | 780 | 400 | .4 | 3.77 |
| 220 | 420 | 495 | 770 | 1100 | 500 | .5 | 4.83 |

* Per Candle Power for Batawing Burner.

lights should be set high and for the following causes:
 1. **High candle power** and distance apart being in excess of gaslights.
 2. **Light radiating at a depressed angle** is greater than when cast horizontally.
 3. **Horizontal rays** are not as steady as angular.
 4. **Light of greatest intensity** with continuous currents is at an angle of 40° below horizontal line.

Determine the Coefficient of Minimum Lighting Power in Streets.

Formula. L representing candle power of lamps, D maximum distance and H height of lamp, both in feet, and C , coefficient.
 For Gaslighting is assumed for a unit of pavement 50 feet distance of 12 candle power 9 feet in height. Hence,

$$12 \times 9 \div 50^2 = .000864.$$

is coefficient, the following capacities of arc lights will give the same light at the following height and distance.

standard would increase the coefficient to .001728.

arc light can replace from 3 to 6 gas-lamps, according to locality and light adopted.

lighting, based on the substitution of one light for 3.5 to 4 gas-lamps, the minimum standard of light; while the average standard would be from 10 to 12 times.

(Eliminated, etc., from Papers of Henry Robinson, M.I.C.E.)

Railroad Speed.

N. Y. Central and Hudson River R.R.—
 Buffalo, N. Y. 436 miles in 426 minutes, at
 hour. Weight of Train 230 tons.

Fairport, 361 miles in 360 minutes, there del
 Philadelphia and Reading R.R.—One mile in 39.7
 minutes.

London to Edinburgh, 400 miles;
 = 47.05 miles per hour.
 including locomotive, 80 tons.

Tenacity of Round and Square Wrought-Iron Bolt Holes of Different Diameters.

Round.—.75-inch, driven into a hole of .625 inch, in White Pine, for 12 ins., required 6875 lbs. to withdraw it.

1-inch, driven into a hole of .75 inch, in White Pine, for 12 ins., required 10 830 lbs. to withdraw it; and in Norway Yellow Pine, 10 830 lbs.

1-inch, screwed, 8-threads per inch into a hole .8125 inch, in White Pine, for 12 ins., required 15 125 lbs. to withdraw it, and one of 12 threads required 15 250 lbs.

1.125-ins., driven into a hole of .875 inch, in Hemlock, for 12 ins., required 15 125 lbs. to withdraw it.

Square.—The difference between that and Round, under like conditions, is essentially different, and when a hole was bored 10 ins. in depth, the difference was not essential.

Railway Spikes.

| Length in Tie. | To Withdraw | | | | | Remarks. |
|----------------------|-------------|----------|-----------|---------|----------|-----------------------------|
| | Chestnut. | Y. Pine. | W. Cedar. | W. Oak. | Hemlock. | |
| Ins. | Lbs. | Lbs. | Lbs. | Lbs. | Lbs. | |
| 4.6 | 3264 | 3198 | 2305 | 4330 | 3485 | In solid wood, sharp point. |

Ship Spikes.—.375 inch square and 7 ins. in depth, driven 3 ins. in White Pine and drawn back, required 1617 lbs., their edge with the grain of the wood 1317 lbs. with it across.

Note.—The above are deduced from Experiments of Gen. Weitzel, U. S. A. 1874-77.

Resistance of Bolts, after being 7 months driven = 10 per cent. greater than immediately after, and when driven through in direction of fibre it is but 60 per cent. of that of being withdrawn.

Smooth bolts have greater retention than ragged, either driven or withdrawn.

Moderate "ragging" reduces their power 25 per cent., and extreme 50 per cent. Relation between diameters of bolt and hole showed that the resistance of a bolt of 1 inch in a .6875-inch hole was greater than in one of .75 or .8125 inch.

With a .75-inch bolt the resistance was greater in a hole of .625 inch, and one quarter greater than in one of a sixteenth greater or less.

One-inch square bolt in a .875-inch hole was the same as a round bolt in a 1.5 inch hole.

Screw-bolts are about 50 per cent. more effective than plain round.

Long pointed blunt bolts are more effective than short pointed.

Experiments of Mr. F. Collingwood and Wm. H. Paine, made in connection with the construction of the New York and Brooklyn Bridge, gave for a 1-inch round bolt driven in a .9375-inch hole, in best Georgia Pine, a resistance of 15 000 lbs. per inch bolt, and in a .875-inch hole 12 000 lbs. In lighter woods the tenacity was less.

J. B. Tscharnier, in the laboratory of the University of Illinois, determined the resistance of a bolt (1-inch round), under like conditions in White Pine, was 6000 lbs. at a bolt driven parallel to the grain of the wood has but half of the resistance that driven perpendicular to it. Further, that assuming a bolt of 1 inch diameter as 1, that if driven in a .75-inch hole it would be 1.69, and in a .625 inch hole 1.3.

Driving Resistance of Round and Square Steel Bolts.

Diameter. Drive into Pine Wood. Six Inches in Depth.

| | Square. | | | Round. | | |
|---|---------|------|-------|--------|------|-------|
| 1 | .9375 | .875 | .8125 | .9375 | .875 | .8125 |
| 2 | 4260 | 4060 | 4050 | 2250 | 3150 | 3150 |
| 3 | 710 | 777 | 675 | 315 | 315 | 315 |

(J. H. Powell and A. F.

of metal in the Round bolts is the least expensive.

Mortar.

els.—Clean and smooth the surface of the mortar, and make it large to a thickness of 1/2 inch. The mortar should be made by taking the lime as specified, and mixing it with the sand thoroughly, in the proportion of 1 part of lime to 3 parts of sand; and when this mixture is made, it should be allowed to settle to the bottom of the mortar, and the surface should be smoothed over the mass.

The condition of the mortar should be such that it is as preferable.

is required for use, and it should be kept in a dry place.

air Mortar.—If the mortar is to be used for the purpose of setting and setting of the mortar, it should be made in the proportion of 1 part of lime to 3 parts of sand, and it should be allowed to settle to the bottom of the mortar, and the surface should be smoothed over the mass.

rest on a layer of sand, and it should be kept in a dry place.

Large Trees in Australia.
Victoria. Eucalyptus — 100 ft. high, 1 ft. in diameter.

Speed of Vessels.

Determine the True Speed of a Vessel, by the Log, and Alternate Runs over a Measured Distance.

Log 100 fms. 100 fms. 100 fms.

| Miles or Knots. | Result. | Log 100 fms. | Log 100 fms. | Log 100 fms. |
|-----------------|---------|--------------|--------------|--------------|
| 15.6 | 12.3 | 12.3 | 12.3 | 12.3 |
| 10.2 | 12.3 | 12.3 | 12.3 | 12.3 |
| 14.4 | 12.3 | 12.3 | 12.3 | 12.3 |
| 11 | 12.3 | 12.3 | 12.3 | 12.3 |
| 13.2 | 12.3 | 12.3 | 12.3 | 12.3 |
| 11.5 | 12.3 | 12.3 | 12.3 | 12.3 |

11.5 $62.5 \div 5 = 12.5$ *Observed speed.*

11.5 — The mean of second result is 12.5.

Velocity of the Current.

Determine the Velocity of the Current, by the Log, and the Vessel's Course.

On the observed speed of the vessel, find the true velocity of the current.

ILLUSTRATION.—Assume preceding rules.

| Obs. | Speed. | True |
|------|--------|------|
| 1 | 15.6 | 12.3 |
| 2 | 15.2 | 12.3 |
| 3 | 14.4 | 12.3 |
| 4 | 11 | 12.3 |
| 5 | 13.2 | 12.3 |
| 6 | 11.5 | 12.3 |



Native Corrosion of Wrought Iron

| | |
|-----------------------------|------|
| In Air..... | 1. |
| In contact with brass..... | 3.4 |
| In contact with copper..... | 4.0 |
| In contact with tin..... | 8.7. |

PILE-DRIVING.

(Continued from page 672.)

To Compute Weight of Ram. (Molesworth.)

$P \left(\frac{hP}{5AL} - 1 \right) = R$. *P* representing weight of pile in lbs., *h* height of fall of ram, and *L* length of pile, both in feet, and *A* area of section of pile in sq. ins.

Piles are distinguished according to their position and purpose: thus *Gauge Piles* are driven to define limit of area to be enclosed, or as guides to the permanent piling.

Sheet or Close Piles are driven between gauge piles to form a compact and continuous enclosure of the work, and are driven as close and uniform to each other as practicable of attainment, and the intervening space or joint, however close, is made water-tight by the introduction of a "feather" driven in a groove on the sides of the piles.

Crushing.—Crushing resistance of a pile, unless of very hard wood, should not be estimated to exceed a range of from 500 to 1000 lbs. per sq. inch.

Refusal of a pile intended to support a weight of 13.5 tons can be safely taken with a ram of 1350 lbs., falling 12 feet, and depressing the pile .8 of an inch at final stroke.

Pneumatic Piles.—A hollow pile of cast iron, 2.5 feet in diameter, was depressed into the Goodwin Sands 33 feet 7 ins. in 5.5 hours.

Water Jets.—A stream of water is ejected under pressure at the point of a pile, and, rising around it, removes the end and surface resistance, so that it will be much easily driven. Suited for sand or fine soil.

Nasmyth's Steam Pile-hammer has driven a pile 14 ins. square, and 18 feet in length, 15 feet into a coarse ground, imbedded in a strong clay, in 17 seconds, with 20 blows of ram, making 70 strokes per minute.

Shaw's Gunpowder Pile-driver is operated by cartridges of powder on head of pile, which are ignited by fall of the ram. 30 to 40 blows per minute have been made under a fall of 5 and 10 feet.

Sheet Piling.

Bevelling..... 120° | Shoeing..... 25°

To Compute Coefficient of Resistance of the Earth.

$\frac{R h}{d} = C$. *R* representing resistance of the earth, *h* height of fall of ram, and *d* final depression, both in feet.

Ringing Engine

Requires 1 man to each 40 lbs. of ram, which varies from 450 to 900 lbs.

To Color Brass (Copper and Zinc) Blue.

Mix in a close vessel 100 grains = 6.5 oz. Troy, of Carbonate of Copper and 750 grains = 4.06 lbs. Troy, of Ammonia; shake until solution is effected and then add distilled water; shake, and the solution is ready for use.

Keep it cool and effectively stopped. If deteriorated, add a little Ammonia.

Articles to be colored, to be perfectly clean, suspended in motion in the solution; remove therefrom in from 2 to 3 minutes, wash in pure water, and dry in sawdust or like effluvia.

Exposure to operation as little to the air as practicable.

Other articles of iron, copper and tin and argentine, not available.

(Chemical Jov

STEEL SPRINGS. (Additional to page 779.)

To Compute Safe Elements of Springs.*

$$\frac{1}{n} = D; \quad \sqrt[3]{\frac{D b t^3 n}{.8}} = l; \quad \frac{.8 l^3}{D t^3 n} = b; \quad \sqrt[3]{\frac{.8 l^3}{D b n}} = t; \quad \frac{.8 l^3}{D b t^3} = n; \quad \frac{b t^2 n}{5 l} = L$$

representing deflection and t thickness of plates, both in 16ths of an inch; l length of rod or bearing when weighted, and b breadth of plates of springs, both in ins.; n number of plates, and L load or stress in 1000 lbs.

RE.—The plates are assumed to be similar and regularly formed.

ILLUSTRATION.—Assume a spring of the following elements:

$l = 20$ and $b = 3$ ins., $t = 4$ 16ths, $n = 5$, and L 2400 lbs.

$$\frac{.8 \times 20^3}{\times 4^3 \times 5} = \frac{6400}{960} = 6.66 \text{ 16ths}; \quad \sqrt[3]{\frac{6.66 \times 3 \times 4^3 \times 5}{.8}} = \sqrt[3]{\frac{6400}{.8}} = 20 \text{ ins.};$$

$$\sqrt[3]{\frac{.8 \times 20^3}{6.66 \times 3 \times 5}} = \sqrt[3]{\frac{6400}{100}} = 4 \text{ 16ths}; \quad \frac{.8 \times 20^3}{6.66 \times 3 \times 4^3} = \frac{6400}{1280} = 5;$$

$$\frac{.8 \times 20^3}{6.66 \times 4^3 \times 5} = \frac{6400}{2133} = 3 + \text{ins.}; \quad \frac{3 \times 4^2 \times 5}{5 \times 20} = \frac{240}{100} = 2.4 \text{ 1000 lbs.}$$

NOTE.—When back or short plates are added, they are to be added to the number of plates if of the ruling breadth and thickness.

When extra thick back or short plates are added, they are to be represented by the use of ruling thickness having an equivalent resistance, prior to computation by the rules for D and L , and are thus ascertained: multiply number of additional plates by cube of their thickness, and divide product by cube of ruling thickness.

ILLUSTRATION.—Assume as preceding, thickness of plates = 4 16ths, number of plates = 5, and 3 extra plates of 5 16ths to be added.

$$\text{Then, } \frac{3 \times 5^3}{4^3} = \frac{375}{64} = 5.86 = \text{no. of plates, and } 5 + 5.86 = 10.86, \text{ the no. of plates 16ths in thickness. } 10.86 \times 4^3 = 695, \text{ and } \frac{5 \times 4^3 = 320}{3 \times 5^3 = 375} \left. \vphantom{\frac{5 \times 4^3 = 320}{3 \times 5^3 = 375}} \right\} 695.$$

Hence, 3 plates of 5 16ths added to the 5 of 4 16ths = 10.86 plates of 4 16ths.

Conversely, $695 \div 5^3 = 5.56$ plates of 5 16ths are equal to the 10.86 of 4 16ths.

Helical Steel Springs.

$$\frac{d^3 L}{C t^4} = D; \quad \sqrt[4]{\frac{d^3 L D}{C}} = t; \quad \sqrt[3]{\frac{C t^4 D}{L}} = d; \quad \frac{C t^4 D}{d^3} = L$$

addition of .125 to .25 should be added to the diameter or square to compensate for a set of the wire.

$$\text{Safe Load. } \sqrt[3]{\frac{L d}{3}} = t \text{ for round, and } \sqrt[3]{\frac{L d}{3.8}} = t \text{ for square.}$$

representing diameter or distance between the centres of the rod or bar of the spring, and D compression of the spring, both in ins.; L load or stress applied in lbs.; d diameter of rod or side of square of bar in 16ths of an inch, and C a coefficient = 22 for round rods and 30 for square bars.

ILLUSTRATION.—Assume as follows: $d = 7$ ins. square; $L = 3363$ lbs.; $t = 16$ sixteenths, and $C = 22$.

$$\frac{\times 3363}{\times 16^4} = \frac{153434}{1441792} = .8 \text{ inch}; \quad \sqrt[4]{\frac{7^3 \times 3363 \times .8}{22}} = 16 \text{ 16ths};$$

$$\frac{22 \times 16^4 \times .8}{3363} = \sqrt[3]{\frac{153434}{3363}} = 7 \text{ ins.}; \quad \frac{22 \times 16^4}{7^3} = 3363 \text{ lbs.};$$

$$\sqrt[3]{\frac{3363 \times 7}{3}} = \sqrt[3]{\frac{23541}{3.8}} = 18.$$

load and deflection obtained for one coil of wire of coils for the respective total load and deflection. The spring is approximately equal to a round

* Essentially from D. K. Clark's

Blast Draught in Ashpit of a Marine Boiler

S. S. "Resolute."

| Of Boiler Engine. | IHP | Of Engine. | Per IHP per hour. | Coal Consumed per hour. | Water Evaporated per lb. of Coal. | Re ES |
|-------------------|-----|------------|-------------------|-------------------------|-----------------------------------|-------|
| No. | | No. | Lbs. | Lbs. | Lbs. | Per |
| Natural Draught | .. | 57.5 | 3.72 | 214 | 10.77 | 1 |
| .96 | | 88.8 | 3.26 | 290 | 8.82 | 1 |
| 2 | | 100.5 | 3.12 | 314 | 8 | 1 |
| 3 | | 106.1 | 2.04 | 323 | 7.82 | 1 |
| 4.2 | | 118.8 | 2.93 | 348 | 7.82 | 1 |
| 5 | | 119.8 | 3.12 | 374 | 7.53 | 1 |
| 6 | | 127.9 | 3.12 | 399 | 7 | 1 |
| 7.4 | | 135.7 | 3.1 | 421 | 7.03 | 1 |

When the Power was Doubled.—The fuel consumed was as 1.5 to 1, the evaporated as .73 to 1, and the saving of coal was 19 per cent.

An average of the above results gave a saving of 15.8 per cent.

By trials in the R. N., it was ascertained that a blast draught increased the efficiency of the engines 52.5 per cent., and the boilers 65 per cent. per ton of their weight.

First Steam-Launch.

"SWEETHEART."—Was built at the Navy Yard, New York, in 1837.

Length, 35 feet; beam, 4.25; depth, 1.83.

Engine, vertical cylinder beam, 4 ins. in diam. by 12 ins. stroke of piston

Water-wheels, 4 feet by 10 ins. Boiler, horizontal fire tubular.

On her trial trip she was saluted by steamboats and assemblages of pleasure boats and on the pier. Designed by and constructed under the direction of the Author.

Bearings without Lubricants.

Graphite or Plumbago—Is the essential element in dry bearings.

"Fibre graphite"—Consisting of finely-powdered plumbago mixed with wood fibre, is pressed in a mold of the required form, then saturated with oil and oxidized in a hot dry air.

NOTE.—This bearing * has been favorably reported on by a committee of the Franklin Institute.

"Carboid"—Is carbon mixed with finely-powdered steatite; its specific gravity = 1.66, that of carbon being 1.48. It can be molded, turned, bored, and shaped in any form.

NOTE.—The coefficient of friction with dry bearings is lower than that of oil bearings in good condition.

Tests for Water.

(Additional to page 852.)

To Ascertain

If Hard or Soft.—Into a clean glass tube put a solution of soap, add a small amount of the water, when, if hard, the mixture will become milky.

If Alkaline.—It will turn red litmus-paper blue.

If Acid.—It will turn blue litmus-paper red.

If Carbonic Acid is present.—Equal volumes of it and lime-water will become milky. Add a little hydrochloric acid to the mixture and it will become clear.

If Sulphate of Lime (Gypsum) is present.—Add to it a little chloride of calcium; a white precipitate is formed, which will not dissolve when a small volume of acid is added, it contains the sulphate.

Anchoring Bolts in Stone.

Test of the relative value of Lead, Sulphur, and Portland Cement, for anchoring bolts in limestone rock, give similar results.

* Philip H. Holmes patent.

Gate Valves.

Eddy Valve Co., Waterford, N. Y.

Gate Valves, Double Seated, have faces set at a slight angle to line of stem, and as the gates, in consequence of their angular faces, cannot fill the space between the valve-seats until they are fully down to their position, the adhesion of them to the valves in their progress down, from the interposition of sediment or other obstructions, is not only not arrested, but they are impracticable of arrest before being fully seated, and left partially open, under the impression on the part of the operator that they are in position and the flow of the fluid arrested.

The valves are attached to the stem by an articulated ball joint, hence they are rendered free to revolve, and their faces varying with that of their valve-seats, cutting or grooving is measurably avoided.

The valves are two independent pieces, whereby a single defect involves the repair or removal of but one of them.

The stem rotates in a screw-collar connected to the ball joint, and hence it is not elongated outside its glands upon the raising of the valves.

| Classes 1 and 6. | | | | Class 2. | | | | Class 2. | | | | Class 3 & 4. | | | |
|-----------------------------|-------------------------|------------------------------|-------|-----------------------------|-------------------------|------------------------------|-------|-----------------------------|-------------------------|------------------------------|-------|-----------------------------|-------------------------|------------------------------|-------|
| Measurement. | | | | Measurement. | | | | Measurement. | | | | Measurement. | | | |
| End to End of screw socket. | Face to Face of Flange. | Diameter of Standard Flange. | Size. | End to End of Screw Socket. | Face to Face of Flange. | Diameter of Standard Flange. | Size. | End to End of Screw Socket. | Face to Face of Flange. | Diameter of Standard Flange. | Size. | End to End of Screw Socket. | Face to Face of Flange. | Diameter of Standard Flange. | Size. |
| Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. | Ins. |
| .375 | 2.5 | 3 | 2 | 5.5 | 5.75 | 6 | 14 | 15 | 21 | 4 | 12 | | | | |
| .75 | 3 | 4.5 | 2.5 | 6 | 6.5 | 7 | 15 | 15 | 22 | 5 | 12.5 | | | | |
| .875 | 3.25 | 4.75 | 3 | 7.25 | 7.75 | 8 | 16 | 16.25 | 23 | 6 | 13.5 | | | | |
| .375 | 3.5 | 5 | 3.5 | 7.5 | 7.75 | 8.5 | 18 | 16.25 | 25 | 8 | 14.5 | | | | |
| .5 | 4 | 5.25 | 4 | 8 | 7.75 | 9 | 20 | 17.75 | 27 | 10 | 15.5 | | | | |
| .375 | 4.625 | 5.75 | 4.5 | 8.5 | 8.5 | 9.5 | 24 | 20 | 31 | 12 | 16 | | | | |
| .125 | 5.5 | 6 | 5 | 9 | 9.25 | 10 | 30 | 22.5 | 38 | 14 | 17.25 | | | | |
| .375 | 5.875 | 6.5 | 6 | 10.25 | 9.75 | 11 | 36 | 25.75 | 45 | 16 | 17.5 | | | | |
| .625 | 6.25 | 7.25 | 7 | 11 | 11 | 12 | | | | 18 | 18 | | | | |
| .625 | 7 | 8 | 8 | 11 | 11 | 13 | | | | 20 | 18 | | | | |
| .25 | 7.5 | 9.5 | 9 | 13.25 | 12 | 16 | | | | 24 | 23 | | | | |
| .75 | 7.75 | 11 | 10 | 14.75 | 13.5 | 18 | | | | 30 | 25 | | | | |
| | | | | | | | | | | 36 | 30 | | | | |

II Brass, Screw and Flange Ends. 2. Iron Body, Brass Mounted. [Mains. ub Valves, all Iron, for Gas Mains. 4. Hub Valves, Brass Mounted, for Water uick Opening Valves, with Rack and Pinion Stem, Iron Body, Brass Mounted. uick Opening Valves, Rack and Pinion Stem, all Brass, Screw Ends.

Eddy Hydrants.

Eddy Valve Co., Waterford, N. Y.

| Diameters of | | | Nozzles. | | | | | | Steamer and | | |
|--------------|------------|--|----------|----------|----------|----------|----------|----------|-------------|----------|--|
| Stand. Pipe. | Seat Ring. | | 2.5 Ins. | 2.5 Ins. | 2.5 Ins. | 2.5 Ins. | 2.5 Ins. | Steamer. | 2.5 Ins. | 2.5 Ins. | |
| Ins. | Ins. | | No. | No. | No. | No. | No. | No. | No. | No. | |
| 4.5 | 3 | | 1 | — | — | — | — | — | — | — | |
| 5.5 | 4 | | 1 | 2 | 3 | — | — | 1 | 1 | 2 | |
| 5.5 | 4 | | 1 | 2 | 3 | — | — | 1 | 1 | 2 | |
| 6 | 4.5 | | — | 2 | 3 | — | — | 1 | — | — | |
| 6.625 | 5 | | — | 2 | 3 | 4 | — | — | — | — | |
| 7.625 | 6 | | — | 2 | 3 | 4 | — | — | — | — | |
| 7.625 | 6 | | — | 2 | 3 | 4 | — | — | — | — | |
| 9.75 | 8 | | — | — | — | — | 6 | — | — | — | |

ard Length from Pavement to bottom of Main or branch

Aluminum.*(Continued from page 938.)*

The available properties of Aluminum are its relative lightness, freed from tarnish, not being affected by sulphurous fumes and being slowly oxidized in moist atmosphere, its extreme malleability, its facility of being cast, its high heat and electrical and heat conductivity, and its extreme ductility.

Its transverse and torsional resistances are very low, its maximum resistance for castings 12 000 lbs., and forgings 16 000 lbs. per square inch.

It is adapted for structures under water, can be welded by electricity, annealed if heated and gradually cooled just below a red heat. The tensile strength of its wire is greater than that of its rolled metal.

Its properties are materially changed and impaired by alloying it with percentages of other metals, and its tensile resistance, relative to its weight, plates as strong as steel at 80 000 lbs. per square inch, and in cold draw strong as it is at 180 000 lbs. (*Alfred E. Hunt.*)

Magnesium.

Specific gravity 1.74, is .33 lighter than Aluminium; is harder, less dense; less affected by alkalis, and takes a higher polish.

Staff.

Staff is composed of Plaster of Paris, water, and hemp fibre, the latter binds the mass.

For ornamental pieces, matrices of hardened gelatine are used.

It resists the weather and even frost after being saturated.

Boiler Setting.

The fire-brick should be laid with very thin joints, and set in Kaolin mixed fire-clay, so thin that it is necessary to lay it with a spoon and trowel.

Every fifth course should be a header course. (*"The Locomotive."*)

Glue.—Its tenacity varies from 500 to 700 lbs. per square inch.

Friction of Engines and Gearing.*(In addition to pages 469-478, etc.)*

Deduced from Experiments of Alfred Saxton, Manchester Assn. of Eng.

| | | | |
|-------------------|----------------------|--------------------|------|
| Spur Gearing..... | 25.9 per cent. | Belt Driving..... | 28.6 |
| Rope Driving..... | 29.6 | Direct Acting..... | 23.8 |
| Engines..... | 6 and 10.3 per cent. | | |

Spur gearing gave the best result when not complicated with rope driving.

Rope driving gave best results at high speeds.

Belt driving for developing large power is only equal to an average rope engine.

* A variety of clay, one of the two ingredients in Oriental porcelain; the other is termed *petunee*.

Spirally Riveted Iron or Steel Pipe.

Abendroth & Root Mfg. Co., New York.

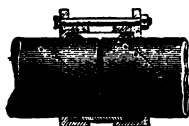
Spirally Riveted Metal Pipe,

Compared with Wrought or Cast iron Pipe, has the advantage of low original cost and expense of transportation, maintaining a nearly equal bursting pressure with that made of heavier material. It is made of Sheet Iron or Sheet Steel, varying in thickness from No. 20 to No. 12 B.W.G., according to diameter and pressure. The rivets in the seam are set by compression, while the laps are thoroughly coated with hydraulic cement to make it water tight.



Connections.—When a moderate pressure is maintained, these pipes, their ends being crimped, are usually connected by a cement joint, as shown in the annexed cut.

When the pressure is excessive, a bolted joint is resorted to, as also shown, and which is in effect a stuffing box or sleeve joint, dispensing with lead calking, and admitting of a slight flexure of the pipe.



For service connections the collar may be tapped. When lead calking is required, the inner ends of the pipe are reinforced by an iron collar.

Bursting Pressure.

| Diameter Internal. | Per Sq. Inch. | Diameter Internal. | Per Sq. Inch. | Diameter Internal. | Per Sq. Inch. | Diameter Internal. | Per Sq. Inch. | Diameter Internal. | Per Sq. Inch. |
|--------------------|---------------|--------------------|---------------|--------------------|---------------|--------------------|---------------|--------------------|---------------|
| Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. | Ins. | Lbs. |
| 3 | 900 to 1300 | 6 | 350 to 800 | 10 | 275 to 650 | 16 | 190 to 400 | 22 | 125 to 300 |
| 4 | 700 " 1000 | 8 | 350 " 825 | 12 | 225 " 550 | 18 | 150 " 375 | 24 | 110 " 275 |
| 5 | 550 " 800 | 9 | 300 " 750 | 14 | 200 " 470 | 20 | 140 " 325 | — | — |

In order to enable an estimate of the relative cost of these pipes, compared with cast and ordinary wrought-iron pipes, the weights of each are submitted.

Weights.

| Diam. | Heavy Spiral. | Wrought Iron.* | Cast Iron.† | Diam. | Heavy Spiral. | Wrought Iron.* | Cast Iron.† | Diam. | Heavy Spiral. | Wrought Iron.* | Cast Iron.† |
|-------|---------------|----------------|-------------|-------|---------------|----------------|-------------|-------|---------------|----------------|-------------|
| Ins. | Lbs. | Lbs. | Lbs. | Ins. | Lbs. | Lbs. | Lbs. | Ins. | Lbs. | Lbs. | Lbs. |
| 3 | 2 | 7.5 | 13 | 8 | 8 | 28 | 40 | 14 | 20 | 28 | 180 |
| 4 | 2.5 | 10.75 | 20 | 10 | 10 | 40 | 55 | 16 | 31 | — | 200 |
| 6 | 5 | 18.75 | 30 | 12 | 13 | 49 | 70 | 18 | 33 | — | 250 |

* Standard.

* 4 N

In Mechanical and Engineering Operations.



Slip-Knot.



Square or Reef Knot.



Flemish Loop.

Sheet Bend or
Weaver's Knot.

Bowline.

Marline-spike
Hitch.

Carrick Bend.



Reef Knot.



Bowline on a Bight.



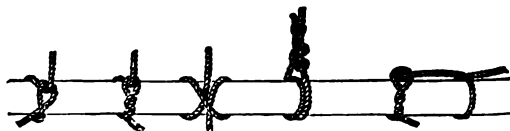
Water Knot.



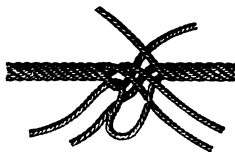
Sheepshank.



Chain Knot with Four.

Halt
Hitch.Timber
Hitch.Clove
Hitch.Rolling
Hitch.Timber Hitch
and Round Turn.

Cat's-paw.



Short Splice.



Fishermans Knot.

Round
Seizing.

Cask Sling.



For

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 Secto...
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 Square...
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 Baromet...
 Breadth...
 Centre...
 Centre...
 Circumf...
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 Compos...
 Cube...
 Cylinder...
 Dead...
 Depth...
 Depart...
 Diamet...
 Dist...
 F...

Knot...
 and...
 Knot...

SYMBOLS

Elements and Formulæ, proposed by the Author.

For the purpose of inducing a uniformity in their expression (1891).

| | | |
|-----------------------------|----------------------------------|--------------------------------------|
| Δ | Henry, -s..... H | Sum, -s..... Sm |
| idence..... z | Joule, -s..... J | Tangent..... Tan. |
| A. a. a' | Kilojoule, -s..... KJ | Cotangent..... Cotan. |
| meter..... Cal. | Watt, -s..... wt | Thrust..... Tt |
| Gt | Kilowatt, -s..... kwt | Time, -s..... T. t. t' |
| g surface..... Hs | Millihenry, -s..... mh | Second, -s. Sec. sec. or " |
| 1.. Scn, H, L or T | Milliampere, -s..... ma | Minute, -s Min. min. or ' |
| cial..... Sup. | Megohm (Greek c. omega) Ω | Degree, -s.... Deg. or ° |
| Sq. or \square | Microvolt, -s..... Mv | Hour, -s..... Ho. ho |
| foot, feet... \square ft. | Ohm, -s. (Greek omega) ω | Day, -s..... Ds |
| ere, -s..... At. | Volt, -s..... Vo | Month, -s..... Mo |
| ric..... Bc | Evaporation, -ive.... Evp | Year, -s..... Ys |
| -s..... b. b' | Foot-pound, -s, tons Fp. Ft | Triangle, -s..... Δ Δ' |
| gal force..... Cf | Force..... F | Triple..... Tpl |
| f gravity..... Cg | Friction..... Fn | Unit, -s. Heat..... Hu |
| rence, -s. C. c. c' | Gravity..... g | Calorific or French.. Cc |
| nt or Factor.. Co. | Height, -s..... H. h. h' | Vacuum..... Vm |
| id..... Cpd | Horse-power..... HP | Velocity..... V. v. v' |
| Cub. or \boxtimes | Effective..... EHP | Versed sine..... v-sin |
| cal..... Cyl. | Indicated..... IHP | Vertical..... Vt |
| \boxtimes | Nominal..... NIP | Volume, -s..... Vol. vol. |
| dp. dp' | Inclination..... In | Chaldron, -s..... Ch |
| e..... Dpt | Joule's Equivalent.... JE | Chord, -s..... Co |
| , -s.... D. d. d' | Latitude..... Lat. | Bushel, -s..... Bl |
| 3. Inch, -es. ins. | Length, -s..... L. l. l' | Cube foot, feet.... Cf |
| ft. | Logarithm..... Log. | Barrel, -s..... bbl |
| s..... Yds | Hyperbolic... Hyp. log. | Gallon, -s..... gl |
| -s..... Chn | Longitude..... Lon. | Microliter (Greek lambda) λ |
| s..... Rd | Mercurial gauge..... Mg | Milliliter, -s..... ml |
| s..... K | Meridian..... M | Centiliter, -s..... cl |
| Ms | Modulus of Elasticity. ME | Deciliter, -s..... dl |
| ter, -s..... mm | Moment, -s..... Mt | Liter, -s..... lr |
| oter, -s..... cm | Number, -s..... No | Dekaliter, -s..... dr |
| oter, -s..... dm | Ordinate, -s..... O. o. o' | Hektoliter, -s..... hr |
| -s..... m | Perpendicular..... Pr | Kiloliter or Stero, -s. Kr |
| oter, -s..... dk | Pitch, -s..... Ph. Ph' | Water-line..... Wl |
| eter, -s..... hk | Pressure, -s..... P. p. p' | Weight, -s..... W. w. w' |
| ter, -s..... kl | Quadruple..... Ql | Ounce, -s..... oz |
| oter, -s..... mr | Radius, -il..... R. r. r' | Pound, -s..... lb. lbs. |
| orsq. meter, -s Ce | Revolution, -s.. Rev. rev. | Ton, -s (2240) Tons |
| Ae | Secant..... Sec. | " (2000) —Tons |
| -s..... He | Cosecant..... Cosec. | Milligram, -s..... mg |
| F water..... Dw | Sine..... Sin. | Centigram, -s..... cg |
| -s..... El | Cosine..... Cosin. | Decigram..... d. |
| E | Slip..... Sp | Grar |
| U. S. or } Eq. | Solid..... Sd | Dek |
| Eq. | Specific gravity..... Sg | Hek |
| Ampere, -s. Am | Span..... Sn | Kik |
| -s. (Greek esp. phi) Φ | Stability..... St | Myr |
| rad, -s (Greek phi) ϕ | Steam..... Stm | Qul |
| | Stroke..... S. s | Mil. |

ORTHOGRAPHY OF TECHNICAL WORDS AND TERMS

Orthography in ordinary use of following words and terms is so varied, that they are here given for the purpose of aiding in the establishment of a uniformity of expression.

Abut. To meet, to adjoin to at the end, to border upon. *Abut end* of a log, etc., is that having the greatest diameter or side.

But and *Butt end*, when applied in this manner, are corruptions.

Adit. In *Mining*, the opening into a mine.

Amidships. The middle or centre of a vessel, either fore and aft or athwartships. The amidship frame of a vessel is at \boxtimes , and is termed *dead flat*.

Arabesque. Applied to painted and carved or sculptured ornaments of imaginary foliage and animals, in which there are no perfect figures of either. Synonymous with *Moresque*.

Arbor. The principal axis or spindle of a machine of revolution.

Arris. A term in *Mechanics*, the line in which the two straight or curved surfaces of a body, forming an exterior angle, meet each other. The edges of a body, as a brick, are arrises.

Ashlar. In *Masonry*, stones roughly squared, or when faced.

Athwart. Across, from side to side, transverse, across the line of a vessel's course.

Athwartships, reaching across a vessel, from side to side.

Bagasse. Sugar-cane in its crushed state, as delivered from the rollers of a mill.

Balk. In *Carpentry*, a piece of timber from 4 to 10 ins. square.

Baluster. A small column or pilaster; a collection of them, joined by a rail, forms a *balustrade*.

Banister is a corruption of *balustrade*.

Bark. A ship without a mizzen-topsail, and formerly a small ship.

Bateau. A light boat, with great length proportionate to its beam, and wider at its centre than at its ends.

Batten. In *Carpentry*, a piece of wood from 1 to 2.5 ins. thick, and from 1 to 7 ins. in breadth. When less than 6 feet in length, it is termed a *deal-end*.

Berne. In *Fortifications* and *Engineering*, a space of ground between a rampart and a moat or fosse, to arrest the ruins of a rampart. The level top of the embankment of a canal, opposite to and alike to the towpath.

Bevel. A term for a plane having any other angle than 45° or 90° .

Binnacle. The case in which the compass, or compasses (when two are used), is set on board of a vessel.

Bit. The part of a bridle which is put into an animal's mouth. In *Carpentry*, a boring instrument.

Bitter End. The inboard end of a vessel's cable abaft the bitts.

Bitts. A vertical frame upon a deck of a vessel, around or upon which is secured cables, hawsers, sheets, etc.

Bogie. Pivoted truck, to ease the running of an engine or car around a curve.

Boomkin. A short spar projecting from the bow or quarter of a vessel, to extend the tack of a sail to windward.

Boulder. A stone rounded by natural attrition; a rounded mass of rock transported from its original bed.

Breast-summer. A lintel beam in the exterior wall of a building.

Buhr-stone. A stone which is nearly pure siliceous, full of pores and cavities, and used for Mills.

Woolen texture of which colors and flags are made.

A load. The quantity that a ship will carry. Hence *burdensome*.

Small cask, differing from a barrel only in size. Commonly written *in*

ber. An instrument with semi-circular legs, to measure diameters of spheres, prior and interior diameters of cylinders, bores, etc.
of Calibers is superfluous and improper.

l. To stop seams and pay them with pitch, etc. To point an iron shoe so as to prevent its slipping.

l. An irregular curved instrument, having its axis eccentric to the shaft which it is fixed.

ber. To camber is to cut a beam or mold a structure archwise, as deck of a vessel.

boose. The stove or range in which the cooking in a vessel is effected. The stowage-room of a vessel; this term is usually confined to merchant vessels; in ships of war it is termed *Galley*.

vel. In *Engineering*, a decked vessel, having great stability, designed for use in lifting of sunken vessels or structures. Also to transport loads of great weight or bulk.

ve is open decked.

tle. A fragment; a piece; the raised portion of the hind part of a saddle.

ttine. The space between the sides of two casks stowed aside of each other. When a cask is laid in the cantline of two others, it is said to be stowed *bilge and ve*.

stan. A vertical windlass.

avel. A small vessel (of 25 or 30 tons' burden) used upon the coast of France for fishing fisheries.

lings. Pieces of timber set fore and aft from the deck beams of a vessel, to strengthen the ends of the ledges in framing a deck.

vel built.—A term applied to the manner of construction of small boats, so that the edges of their bottom planks are laid to each other like to the manner of planking vessels. Opposed to the term *Clincher*.

ler. A small phial or bottle for the table. *Casters.* Small wheels placed under the legs of tables, etc., to allow them to be moved with facility.

zmaran. A small raft of logs, usually consisting of three, the centre one being larger and wider than the others, and designed for use in an open roadstead upon a sea-coast.

mfer. A slope, groove, or small gutter cut in wood, metal, or stone.

ypelling. Wearing a ship around without bracing her fore yards.

mney. The flue of a fireplace or furnace, constructed of masonry in houses, chimneys, and of metal, as in a steam boiler. See *Pipe*.

nse. To *chinse* is to calk slightly with a knife or chisel.

ck. In *Naval Architecture*, small pieces of wood used to make good any decay in a piece of timber, frame, etc. See *Furrings*.

ke. To stop, to obstruct, to block up, to hinder, etc.

zls. Pieces of wood or metal of various shapes, according to their uses, either strung upon ropes, to resist or support weights or strains, as *sheet*, *shoar*, *beam*, etc.

acher built. A term applied to the construction of vessels' bottoms, when the lower edges of the planks overlay the next under them.

z. A cylinder, cube, or triangle of hard wood let into the ends or faces of two pieces of timber to be secured together. The metallic eyes in a sheave through which the pin runs. In *Naval Architecture*, the oblong ridges banded on the masts and spars.

mings. Raised borders around the edges of hatches.

le. A small fishing-boat.

mon. The ease which certain insects make for a covering in their metamorphosis to the pupa state.

~~the word~~

Cog. In *Mechanics*, a short piece of wood or other material let into the bore of a body to impart motion to another. A term applied to a tooth in a wheel which is made of a different material than that of the wheel. In *Mining*, an instrument used to matter into fissures of rocks, as when a mass of unstratified rocks appears to be projected into a rent in the stratified rocks.

Cogging. In *Carpentry*, the cutting of a piece of timber so as to leave a piece alike to a cog, and the notching of the upper piece so as to conform to and mesh with it. Alike to *indenting* or *tabling*.

Colter. The fore iron of a plough that cuts earth or sod.

Compass. In *Geometry*, an instrument for describing circles, measuring figures, &c. A pair of *Compasses* is superfluous and improper.

Connecting Rod. In *Mechanics*, the connection between a prime and secondary mover, as between the piston-rod of a steam-engine and the crank of a water wheel, or fly-wheel shaft.

The term *Pistons* is local, and altogether inapplicable.

Contrariwise. Conversely, opposite. *Crossways* is a corruption.

Corridor. A gallery or passage in or around a building, connected with various departments, sometimes running within a quadrangle; it may be opened or enclosed. In *Fortifications*, a covert way.

Cyma. A molding in a cornice.

Damasquinerie. Inlaying in metal.

David. A short boom fitted to hoist an anchor or boat.

Deals. In *Carpentry*, the pieces of timber into which a log is cut or sawed. Their usual thickness is 3 by 9 ins. and exceeding 6 feet in length. Improperly restricted to the wood of fir-trees.

Dike. In *Engineering*, an embankment of greater length than breadth, impervious to water, and designed as a wall to a reservoir, a drain, or to resist the action of a river or sea.

Dingy (Nautical). A ship or vessel's small boat.

Dock. In *Marine Architecture*, an enclosure in a harbor or shore of a river, for the reception, repair, or security of vessels or timber. It may be wholly or partially enclosed. See *Pier*.

When applied to a single pier or jetty, it is a misapplication.

Dowel. A pin of wood or metal inserted in the edge or face of two board-like pieces, so as to secure them together.

This is very similar to coaking, but is used in a diminutive sense. An illustration of it is best in the manner a cooper secures two or more pieces in the head of a cask.

Draught. A representation by delineation. The depth which a vessel or floating body sinks into water. The act of drawing. A detachment of men from the main body, etc.

Ordinarily written *draft*.

Dutchman. In *Mechanics*, a piece of like material with the structure, set into a slack place, to cover slack or bad work. See *Shim*.

Edgewise. An edge put into a particular direction. Hence *endwise* and *sidewise* have similar significations with reference to an end and a side.

Edgeway is a corruption.

Expendee. A piece of wood by which the crowfoot of an awning is extended.

Fault. In *Mining*, a break of strata, with displacement, which interrupts operations. Also, fissures traversing the strata.

Felloe, Felloes. The pieces of wood which form the rim of a wheel.

Fetch. Length of a reservoir, pond, etc., along which the wind may blow towards the embankment or dam.

Flange. A projection from an end or from the body of an instrument, or a part composing it, for the purpose of receiving, confining, or of securing it to a support or to a second piece.

Flier. In *Carpentry*, a straight line of steps in a stairway.

Frap. To bind together with a rope, as to *frap* a fall, etc.

Pier. I

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2. In *Architecture*, the part of the entablature of a column which is between the architrave and the cornice.

um. The part of a solid next the base, left by the removal of the top or the bottom.

um, although used by some lexicographers, is erroneous.

ings. Strips of timber or boards fastened to frames, joists, etc., in order to give them the required shape or level.

ing. Putting gallets into pointing-mortar or cement.

3. Pieces of stone chipped off by the stroke of a chisel. See *Spall*.

t. A small galley built for speed, having one mast, and from 16 to 20 thwarts or benches. A Dutch-constructed brigantine.

In *Mechanics*, the hole through which molten metal is poured into a mold. ing. *Geat* and *Gett* are corruptions.

ing. A series of teeth or cogged wheels for transmitting motion. To *gear* a machine is to prepare to connect its parts as by an articulation.

ie. To shake so as to produce a sharp, clattering noise, commonly *Jingle*.

The circumference of a tree or piece of timber. *Girth*. The band or strap which a saddle or burden is secured upon the back of an animal, by passing over his belly. In *Printing*, the bands of a press.

led. Knotty.

2. To clean a vessel's bottom by burning.

ing. Burning off grass, shells, etc., from a ship's bottom. *Synonymous* *reaming*.

met. A wreath or ring of rope.

al Ring. A circular *rynd* for the connection of the upper mill-stone to the lower, by which the stone is suspended, so that it may vibrate upon all sides.

ings. The fore part of the wales of a vessel which encompass her bows, and are fastened to the stem. *Cat harpings*, ropes which brace in the shrouds of a vessel.

ing. A term applied to the hull of a vessel when her ends drop below her beam. See *Sagging*.

ing. In *Naval Architecture*, calking with a large maul or beetle.

To press, to crowd, to wedge in. In *Nautical language*, to squeeze tight.

4. A pier; the sides of an opening in a wall.

The projecting beam of a crane from which the pulleys and weight are suspended. A sail in a vessel.

To shift a boom-sail from one tack to another; hence *Jibing*, the shifting of sails.

ng. Washing minerals in a sieve.

on. The timber within a vessel laid upon the middle of the floor timbers, and projecting over the keel. When located on the floors or at the sides, it is termed a *side keelson*.

Slit made by cut of a saw.

5. Large wooden cleats to belay hawsers and ropes to, commonly *Cavil*.

uer. A spirituous solution of *lac*. To varnish with lacquer.

n. Articles sunk in the water with a buoy attached.

ince. A pulpy, gelatinous fluid washed from the cement of concrete deposits.

sided. A term expressive of the condition of a vessel or any body when it is not to float or sit upright.

to. To arrest headway of a vessel, without anchoring or securing her to a wharf, as by counterbracing her yards, or stopping her engine.

A trench to conduct water to or from a mill-wheel.

In *Nautical language*, the perpendicular or the distance from the spar to stay.

Luf. The fullest part of the bow of a vessel.

Mall. A large double-headed wooden hammer.

Mantle. To expand, to spread. **Mantelpiece.** The shelf over a fireplace in front of a chimney.

Marquetry. Checkered or inlaid work in wood.

Matrass. A chemical vessel with a body alike to an egg, and a tapering neck.

Mattress. A quilted bed; a bed stuffed with hair, moss, etc., and quilted.

Mitred. In *Mechanics*, cut to an angle of 45° , or two pieces joined so as to make a right angle.

Missen-mast. The aftermost mast in a three-masted vessel.

Mold. In *Mechanics*, a matrix in which a casting is formed. A number of pieces of vellum or like substance, between which gold and silver are laid for the purpose of being beaten. Thin pieces of materials cut to curves or any required figure. In *Naval Architecture*, pieces of thin board cut to the lines of a vessel's timbers, etc. Fine earth, such as constitutes soil. A substance which forms upon bodies in warm and confined damp air.

This orthography is by analogy, as *gold, sold, old, bold, cold, fold, etc.*

Molding. In *Architecture*, a projection beyond a wall, from a column, wainscot, &c.

Moresque. See *Arabesque*.

Mortise. A hole cut in any material to receive the end or tenon of another piece.

Muck. A mass of dung in a moist state, or of dung and putrefied vegetable matter.

Mullion. A vertical bar dividing the lights in a window; the horizontal bars termed *transoms*.

Net. Clear of deductions, as *net weight*.

Newel. An upright post, around which winding stairs turn.

Nigged. Stone hewed with a pick or pointed hammer instead of a chisel.

Ogee. A molding with a concave and convex outline, like to an S. See *Cyma* and *Talon*.

Paillasse. Masonry raised upon a floor. A bed.

Pargeting. In *Architecture*, rough plastering, alike to that upon chimneys.

Parquetry. Inlaying of wood in figures. See *Marquetry*.

Parrel. The rope by which a yard is secured to a mast at its centre.

Pawl. The catch which stops, or holds, or falls on to a ratchet wheel.

Peek. The upper or pointed corner of a sail extended by a gaff, or a yard set obliquely to a mast. To *peek* a yard is to point it perpendicularly to a mast.

Pendant. A short rope over the head of a mast for the attachment of tackle thereto; a tackle, etc.

Pennant. A small pointed flag.

Pier. In *Marine Architecture*, a mole or jetty, projecting into a river or sea, to protect vessels from the sea, or for convenience of their lading. See *Dock*.

Erroneously termed a *Dock*.

Pile. In *Engineering*, spars pointed at one end and driven into soil to support superstructure or holdfast. *Spile* is a corruption.

Pipe. In *Mechanics*, a metallic tube. The flue of a fireplace or furnace when constructed of metal; usually of a cylindrical form.

The term or application of *Stack* (which refers solely to masonry) to a metallic pipe is a misapprehension.

Piragua. A small vessel with two masts and two boom-sails.

Commonly termed *Perry-augur*.

Pirogue. A canoe formed from a single log, propelled by paddles or by a sail with the aid of an outrigger.

Plag. In *Architecture*, covering with plaster cement or mortar upon wall or ceiling, termed *laying*, if in one or two coat work; and *pricking* if in three.

Plumb. To receive and support the journal of a shaft or axle, without tops.

Poppets. In *Naval Architecture*, pieces of timber set perpendicular to a vessel's bilge-ways, and extending to her bottom, to support her in launching.

Porch. An arched *vestibule* at the entrance of a building. A vestibule supported by columns. A *portico*.

Portico. A gallery near to the ground, the sides being open. A *piazza* encompassed with arches supported by columns, where persons may walk; the roof may be flat or vaulted.

Pozzuolana. A loose, porous, volcanic substance, composed of silicious, argillaceous, and calcareous earths and iron.

Prize. In *Mechanics*, to raise with a lever. To *pry* and a *pry* are corruptions.

Proa, Flying. A narrow canoe, the outer or lee side being nearly flat. A framework, projecting several feet to the windward side, supports a solid bearing, in form of a canoe. Used in the Ladrone Islands.

Purlin. In *Carpentry*, a piece of timber laid horizontal upon the rafters of a roof, to support the covering.

Ramp. In *Architecture*, a flight of steps on a line tangential to the steps. A concave sweep connecting a higher and lower portion of a railing, wall, etc. sloping line of a surface, as an inclined platform.

Rarefaction. The act or process of distending bodies, by separating their particles, rendering them more rare or porous. It is opposed to *Condensation*.

Rebate. In *Mechanics*, to pare down an edge of a board or a plate for the purpose of receiving another board or plate by lapping. To lap and unite edges of boards and plates. In *Naval Architecture*, the grooves in the side of the keel for receiving the garboard strake of plank.

Commonly written *Rabbit*.

Remous. Eddy water without progressive action, in bed of a river; a return of water against direction of flow of a river.

Rendering. In *Architecture*, laying plaster or mortar upon masonry or wood. *Rendered* and *Set* refers to two coats or layers, and *Rendered*, *Flushed*, and *Set* three coats or layers.

Reniform. Kidney-shaped.

Resin. The residuum of the distillation of turpentine. *Rosin* is a corruption.

Riband. In *Naval Architecture*, a long, narrow, flexible piece of timber.

Rimer. A bit or boring tool for making a tapering hole. In *Mechanics*, to *R* is to bevel out a hole. *Rimming.* The opening of the seams between the planks of a vessel for the purpose of calking them.

Rotary. Turning upon an axis, as a wheel.

Rynd. The metallic collar in the upper mill-stone by which it is connected to the spindle.

Sagging. A term applied to the hull of a vessel when her centre drops below the ends. The converse of *Hogging*.

Scallop. To mark or cut an edge into segments of circles.

Scarcement. A set back in the face of a wall or in a bank of earth. A footing.

Scarf. To join; to piece; to unite two pieces of timber at their ends by running the end of one over and upon the other, and bolting or securing them together.

Scend. The settling of a vessel below the level of her keel.

Selvahee. A strap made of rope-yarns, without being twisted or laid up, and fastened in form by knotting it at intervals.

Sennit. Braided cordage.

Sewage. The matter borne off by a sewer.

Sewed. In nautical language, the condition of a vessel aground; she is said to be *sewed* by as much as the difference in depth of water around her and her floating depth.

Sewerage. The system of sewers.

Shaky. Cracked or split, or as timber loosely put together.

Shammy. Leather prepared from the skin of a chamois goat.

Sheer. In *Naval Architecture*, the curve or bend of a ship's deck or sides, to slip or move aside.

Sheers. Elevated spars connected at the upper ends, and used to elevate bodies, as masts, etc.

Shim. In *Naval Architecture*, a piece of wood or iron let into a slack place in frame, plank, or plate to fill out to a fair surface or line.

Shoal. A great multitude; a crowd; a multitude of fish.

Shoel is a corruption.

Shoar. An oblique brace, the upper end resting against the substance to be supported.

Sholes. Pieces of plank under the heels of shoars, etc.

Shoot. A passage-way on the side of a steep hill, down which wood, coal, etc. is thrown or slid. The artificial or natural contraction of a river. A young pig.

Sidewise. See *Edgewise*.

Signalled. Communicated by signals.

Signalized, when applied to signals, is a misapplication of words.

Sill. A piece of timber upon which a building rests; the horizontal piece of timber or stone at the bottom of a framed case.

Siphon. A curved tube or pipe designed to draw fluids out of vessels.

Skeg. The extreme after-part of the keel of a vessel; the portion that supports the rudder-post.

Slantwise. Oblique; not perpendicular.

Sleck. To make smooth. Refuse; small coal.

Sleeker. A spherical-shaped, curved, or plane-surfaced instrument with which smooth surfaces.

Slue. The turning of a substance upon an axis within its figure.

Snying. A term applied to planks when their edges at their ends are curved rounded upward, as a strake at the ends of a full-modelled vessel.

Spall. A piece of stone, etc., chipped off by the stroke of a hammer or the force of a blow. *Spalling*, breaking up of ore into small pieces.

Spandrel. In *Architecture*, the irregular triangular space between the outer curve or extrados of an arch, a horizontal line drawn from its apex, and a vertical line from its springing.

Sponson. An addition to the outer side of the hull of a steam vessel, connected near the light water-line and running up to the wheel guards; applied for the purpose of shielding the deck-beams from the shock of a sea.

Sponson-sided. The hull of a vessel is so termed when her frames have the line of a sponson, and the space afforded by the curvature is included in the bottom.

Sponding, *Sponaing*, etc., are corruptions.

Squilgee. A wooden instrument, alike to a hoe, its edge faced with leather or vulcanized rubber, used to facilitate the drying of wet floors, or decks of a vessel.

Stack. In *Masonry*, a number of chimneys or pipes standing together. The chimney of a blast furnace.

The application of this word to the smoke-pipe of a steam-boiler is wholly erroneous.

Stage. In *Engineering*, the interval or distance between two elevations, in shafting, throwing, or lifting.

Sleeving. The elevation of a vessel's bowsprit, cathead, etc.

Strake. A breadth of plank.

Strut. An oblique brace to support a rafter.

Style. The gnomon of a sun-dial.

Sump. In *Mining*, a pit or well into which water may be led from a mine or mine-work.

Surcingle. A belt, band, or girth, which passes over a saddle or blanket on a horse's back.

Swoye. To bear or force down. An instrument having a groove on one side for the purpose of giving shape to any piece subjected to it when struck with a blow from a hammer.

Sphered. Overlapping the chamfered edge of one plank upon the chamfered edge of another in such a manner that the joint shall be a plane surface.

Talus. In *Architecture*, the slope or batter of a wall, parapet, etc. In *Geology*, a sloping heap of rubble at foot of a cliff.

Template. In *Architecture*, a wooden bearing to receive the end of a girder to distribute its weight.

Templet. A mold cut to an exact section of any piece or structure.

Tenon. The end of a piece of wood, cut into the form of a rectangular prism, designed to be set into a cavity of a like form in another piece, which is termed the *mortise*.

Terring. The earth overlying a quarry.

Tester. The top covering of a bedstead.

Tholes. The pins in the gunwale of a boat which are used as rowlocks.

Thwarts. The athwartship seats in a boat.

Tide-rode. The situation of a vessel at anchor, when she rides in direction of the current instead of the wind.

Tire. The metal hoop that binds the felloes of a wheel.

Tompson. The stopper of a piece of ordnance. The iron bottom to which grape-shot are secured.

Treenails. Wooden pins employed to secure the planking of a vessel to the frames.

Trepan. In *Mining*, the instrument used in the comminution of rock in earth-boring at great depths.

Trestle. The frame of a table; a movable form of support. In *Mast-making*, two pieces of timber set horizontally upon opposite sides of a mast-head.

Trice. In *Seamanship*, to haul or tie up by means of a rope or tricing-line.

Tue-iron or Tuyere. The nozzle of a bellows or blast-pipe in a forge or smelting-furnace.

Vice. In *Mechanics*, a press to hold fast anything to be worked upon.

Voyal. In *Seamanship*, a purchase applied to the weighing of an anchor, leading to a capstan.

Wagon. An open or partially enclosed four-wheeled vehicle, adapted for the transportation of persons, goods, etc.

Wear. In *nautical language*, to put a vessel upon a contrary tack by turning her around stem to the wind.

Weir. A dam across a river or stream to arrest the water; a fence of twigs or stakes in a stream to divert the run of fish.

Whipple-tree. The bar to which the traces of harness are fastened.

Wind-rode. The situation of a vessel at anchor, when she rides in direction of the wind instead of the current.

Windrow. A row or line of hay, etc., raked together.

Withe. An instrument fitted to the end of a boom or mast, with a ring, through which a boom is rigged out or mast set up.

Woold. To wind; particularly to bind a rope around a spar, etc.

Addenda.

Astragal. In *Architecture*, a round molding, surrounding the head or base of a column. In *Gunnery*, a like molding on cannon near the mouth.

Cresote. An oily colorless liquid, procured from coal-tar.

Flume. a channel for conducting water, as that by which the surplus water of a canal is led to a lower level.

Forebay. The part of a Mill-race or Penstock, from which water flows upon a water-wheel.

Grillage. A frame, constructed of beams laid in parallel right angles, with others notched over them.

Designed to uniformly distribute or extend the area of a foot

Hypotenuse. Commonly, but incorrectly, *hypothénuse*.

Jetty. In *Naval Architecture*, a pier that juts out or projects into a river or sea, a landing-place.

Kibble. In *Mining*, a metallic bucket in which ore is drawn up to the surface.

Lewis. One or two frustums of a right-angled metallic wedge, set inverted or dove-tailed and keyed in a wedge-formed slot, in stone or like solid substance, whereby it may be lifted and laid without the use of slings.

Newel. In *Engineering*, a cylindrical pillar terminating a wing wall of a bridge or viaduct.

Parcelled. *Nautical.* Wrapped with canvas or tarred rope, to resist wear from friction.

Payed. *Nautical.* Painted, tarred, or greased, to resist moisture and wear.

Penstock. An artificial conduit for water to a water-wheel, and furnished with a flood-gate.

Ravel. To disentangle, untwist, or unweave.

The usual prefix of *Un* is wholly superfluous.

Roil. To render turbid, to stir or mix.

Scabble. The dressing of the faces of rough stones, as with a broad chisel.

Served, Service. *Nautical.* The layer of wrapping, as spun yarn, lines, etc., around a stay or rope, to resist friction and wear.

Shackle, or Clevis. An open link set in a chain, secured by a pin running through eyes in its ends, which, when withdrawn, admits the chain to be parted at that point.

Soffit. In *Architecture*, the under side of an opening; the lower surface of a vault or arch; also the under surface of an arch between columns.

Splay. In *Architecture*, a sloped surface, or one making an oblique angle with another. A large chamber.

Strike, in *Geology*, is the compass direction of the intersection of the plane of stratified rock with the plane of the horizon.

Altars. In *Naval Architecture*, the steps on the sides and end of a marine dock.

Gin. An instrument operated by men or animals for the raising or drawing of heavy bodies; usually a vertical revolving windlass and lever.

Sump. In *Salt-works*, a pond in which the sea or saline water is retained for use in the future.

Skeel. *Nautical.* A scoop with a long handle, for use in wetting the sails or the sides of a vessel.

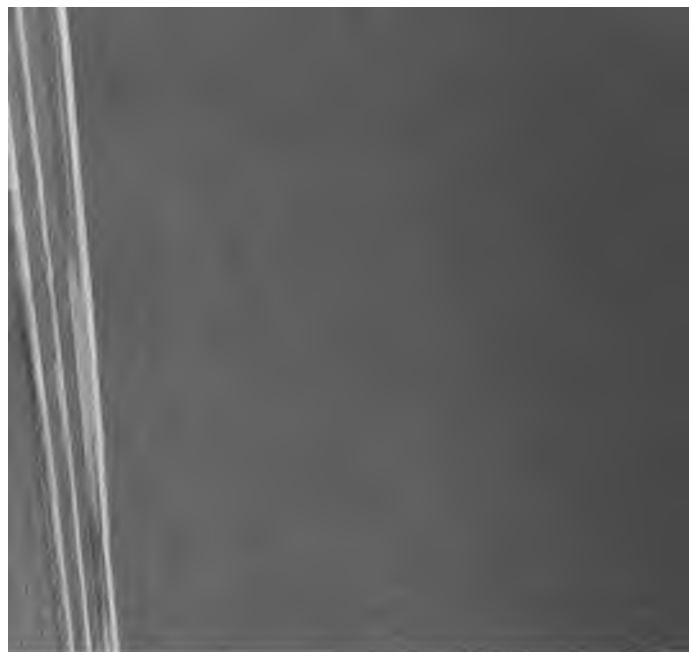
Wyes. The vertical standards on which the telescope of a Theodolite or Level is supported, and which admits of their being reversed by a reversal of its ends. When the telescope is reversed by rotation on its trunnions, the instrument is termed a Transit.

THE END.

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